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PUMPING-INDUCED LEAKAGE IN A BOUNDED AQUIFER: AN EXAMPLE OF A SCALE-INVARIANT PHENOMENON

by

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ABSTRACT

A new approach is presented for calculation of the volume of pumping-induced leakage entering an aquifer as a function of time. This approach simplifies the total leakage calculation by extending analytical-based methods developed for infinite systems to bounded aquifers of any size and shape. The simplification is possible because of the relationship between drawdown and leakage in aquifers laterally bounded by impermeable units. This relationship produces a scale-invariant total leakage; i.e. the volume of leakage as a function of time does not change with the size or shape of the aquifer, or with the location of the pumping well. Two examples and image-well theory are used to demonstrate and prove, respectively, the generality of this interesting hydrologic phenomenon.

INTRODUCTION

Pumping-induced flow through units of relatively low permeability (leakage) is a common hydrologic mechanism of importance for a wide range of practical applications (Figure 1). Leakage is often a significant component of the hydrologic budget of an aquifer undergoing development. If a stream overlies the confining bed, leakage may also be a critical factor in water-rights and minimum-streamflow considerations. In many situations, pumping-induced leakage can be the mechanism by which contaminants move into an aquifer used for drinking-water supplies. In addition, leakage may often be the primary determinant of well yields.

The quantification of the volume of pumping-induced leakage entering an aquifer (henceforth, total leakage) as a function of time is necessary for many such applications. In aquifers that resemble the hypothetical infinite systems of the well hydraulics literature, analytical expressions (e.g., Hantush and Jacob, 1955; Hantush, 1960; Neuman and Witherspoon, 1969; Butler et al., 2001) can be integrated through space to estimate the total leakage produced by pumping at individual wells. In less-ideal systems, numerical models are used to determine total leakage through a summation over the active cells (e.g., Anderson and Woessner, 1992). Obviously, in either case, the computation becomes trivial once steady-state conditions are achieved.

A new approach for calculation of total leakage in aquifers that are laterally bounded by impermeable units is introduced here. The approach simplifies the calculation by extending analytical-based methods developed for infinite systems to bounded aquifers of any size or shape. The simplification is possible because of the relationship between drawdown and leakage in aquifers bounded by impermeable units. This relationship produces a scale-invariant total leakage, an interesting hydrologic phenomenon that has not previously been reported.

The purpose of this note is to present this new analytical-based approach for the calculation of total leakage, and to demonstrate that total leakage does not change with the size of the aquifer or the distance from, or geometric form of, a no-flow boundary. The note begins with a brief derivation of analytical expressions for total leakage in laterally infinite aquifers. The derivation will be presented in Laplace-space to more clearly emphasize the points of interest here. Two examples are then used to demonstrate the scale invariance of total leakage in bounded systems. In both cases, image-well theory (e.g., Ferris et al., 1962) and the previously derived analytical expressions are utilized to prove the universality of the scale invariance. The note concludes with a brief summary of the major points.

ANALYTICAL CALCULATION OF TOTAL LEAKAGE

A number of analytical solutions have been proposed for transient flow to a pumping well in a semiconfined aquifer. The earliest reported solution is that of Hantush and Jacob (1955) in which the storage properties of the confining unit are neglected and the head in the adjacent aquifer is assumed to be constant (Figure 1). In that case, the drawdown in a laterally infinite semiconfined aquifer can be written in dimensionless form in Laplace space as

$$\bar{s}_d = \frac{K_0(\lambda\, r_d)}{p} \qquad (1)$$

where $\bar{s}_d$ = dimensionless drawdown = $(2\pi T\bar{s})/Q$,

$\bar{s}$ = Laplace-space drawdown in pumped aquifer, equal to $\int_0^\infty (e^{-pt}s)\,dt$,

s = drawdown in pumped aquifer, T = transmissivity of aquifer = Kb,

K = hydraulic conductivity of aquifer, b = aquifer thickness, Q = pumping rate,

K_0 = modified Bessel function of the second kind of order zero,

p = Laplace-transform variable, $r_d = r/b$, r = radial distance from the pumping well,

$\lambda = (\frac{1}{B_d^2} + p)^{1/2}$; B_d = dimensionless leakance parameter = $(Kb'/K'b)^{1/2}$,

b' = thickness of confining unit, and K' = hydraulic conductivity of confining unit.

The leakage through a concentric ring of infinitely small width centered on the pumping well can be written in dimensionless form in Laplace space as

$$\overline{q}_{zd} = (r_d dr_d \overline{s}_d)/B_d^2 \qquad (2)$$

where q_{zd} = dimensionless leakage (q_z/Q) and q_z = pumping-induced leakage.

An expression for total leakage can be obtained by integrating from r_d=0 to ∞ and substituting equation (1) for $\overline{s}_d$:

$$\overline{Q}_{zd} = \frac{1}{B_d^2}\int_0^\infty r_d \overline{s}_d dr_d = \frac{1}{pB_d^2}\int_0^\infty r_d K_0(\lambda r_d)\, dr_d \qquad (3)$$

where Q_{zd} = dimensionless total leakage (Q_z/Q) and Q_z = pumping-induced total leakage. The Stehfest (1970) algorithm can be used to numerically invert the Laplace-space expression for Q_{zd}. By definition, Q_{zd} must vary between zero (negligible amount of water moving across the confining bed) and one (the rate of water movement across the confining bed equals the pumping rate (Q)).

Figure 2a is a plot of Q_{zd} versus dimensionless time ($t_d = Tt/Sb^2$) for a range of values of the dimensionless leakance parameter (B_d). Although Q_{zd} must eventually reach one (steady-state leakage) with continued pumping, the dependence on B_d can produce large differences in total leakage prior to the attainment of steady-state conditions.

Analytical expressions for total leakage can also be obtained by relaxing some of the more restrictive assumptions of the preceding development. For example, if a non-negligible storage (S') is assumed for the confining unit, an expression for total leakage can be obtained using the solution of Hantush (1960):

$$\overline{Q}_{zd} = \frac{1}{p\Gamma}\int_0^\infty r_d K_0(\gamma r_d)\,dr_d \qquad (4)$$

where

$$\Gamma = \frac{B_d^2}{\sqrt{pS_d B_d^2}\,\coth(\sqrt{pS_d B_d^2})} \text{ if confining bed abuts a constant-head aquifer,}$$

$$= \frac{B_d^2}{\sqrt{pS_d B_d^2}\,\tanh(\sqrt{pS_d B_d^2})} \text{ if confining bed abuts an impermeable formation,}$$

$\gamma = (p + 1/\Gamma)^{1/2}$, $S_d = S'/S$, and S = storage coefficient of semiconfined aquifer.

Figure 2b is a plot of Q_{zd} versus dimensionless time generated using the Hantush (1960) solution for the case of a constant head in the unpumped aquifer. The inclusion of storage in the aquitard significantly alters the form of the Q_{zd} plots. If the head in the non-pumped aquifer is affected by pumping, the solution of Neuman and Witherspoon (1969) can be used to obtain an expression for total leakage. Although not given here, that expression has the same general form as equations (3) and (4), albeit with more parameters.

Analogous expressions can also be obtained for the case of a stream and aquifer separated by a thin unit (streambed) of low permeability. Figure 3 is an areal and cross-sectional view of a stream-aquifer system in which pumping induces flow from the stream through the low-permeability streambed. In this case, the total pumping-induced leakage from the stream is termed stream depletion. For a streambed of negligible storage and a stream of shallow penetration and constant head, the dimensionless Laplace-space expression for stream depletion can be written as (Butler et al., 2001):

$$\overline{Q}_{zd} = \frac{1}{B_{dsa}^2} \int_{-\infty}^{+\infty} \int_{-1}^{0} \overline{s}_d \, dx_d \, dy_d \qquad (5)$$

where

$\overline{s}_d$ = dimensionless drawdown underneath the stream = $T\overline{s}/Q$,

B_{dsa} = dimensionless leakance parameter for stream-aquifer system = $(b'T/K'w^2)^{1/2}$,

w = stream width, $x_d = x/w$, and $y_d = y/w$.

Note that the inner integral of equation (5) is over the width of the stream (dimensionless width of one), and the notation of Butler et al. (2001) has been modified to be consistent with the notation of this paper.

SCALE INVARIANCE OF TOTAL LEAKAGE

The expressions presented in the preceding section were derived for a semiconfined aquifer of infinite lateral extent (equations (3)-(4)) and a stream-aquifer system of infinite length (equation (5)). In many cases, however, the confining bed and underlying aquifer are truncated by impermeable units that are close enough to the well to affect drawdown. The issue of how total leakage changes in such situations is one of considerable practical importance. Fortunately, the scale invariance of total leakage greatly simplifies the required calculations. This scale invariance is defined and demonstrated in the following paragraphs.

The term scale-invariant leakage is used here to designate the condition when total leakage does not change with the size of the aquifer or the distance to, or geometric form of, a lateral no-flow boundary. Two examples are given to demonstrate this phenomenon.

A simple example of scale-invariant leakage can be observed in the truncated stream-aquifer system of Figure 4a. In this case, pumping-induced drawdown and stream depletion can be calculated with the solution of Butler et al. (2001) using a single image well to model the no-flow boundary in the y direction (Figure 4b). The drawdown and stream-depletion plots for a stream-aquifer system of infinite length and the truncated system of Figure 4a are presented in Figures 5a-b. Although drawdown varies with the distance from the pumping well to the no-flow boundary in the y direction (Figure 5a), stream depletion does not (Figure 5b). In all cases, the stream-depletion curves for the infinite and truncated systems coincide. Additional calculations have shown that this coincidence of stream-depletion curves occurs regardless of the distance from the pumping well to the boundary in the y direction.

The universality of this scale invariance for systems similar to that in Figure 4a can be proven using image-well theory. In the case of a single impermeable boundary perpendicular to the stream (Figure 4b), stream depletion is the summation of the leakage produced by the pumping well and that produced by the single image well. The component of stream depletion produced by each well is calculated by integrating the analytical expression for total leakage (equation (5)) over the truncated aquifer. The component produced by the pumping well is

$$\overline{Q}_{zd_1} = \frac{1}{B_{dsa}^2} \int_{-\infty}^{Y_b} \int_{-1}^{0} \overline{s}_d \, dx_d dy_d \qquad (6a)$$

where Y_b equals y_{bnd}/w, and the origin of the coordinate system is at the pumping well. The component produced by the image well is

$$\overline{Q}_{zd_2} = \frac{1}{B_{dsa}^2} \int_{Y_b}^{+\infty} \int_{-1}^{0} \overline{s}_d \, dx_d dy_d \qquad (6b)$$

where the origin of the coordinate system is at the image well. The summation of these two terms is

$$\overline{Q}_{zd_1} + \overline{Q}_{zd_2} = \frac{1}{B^2_{dsa}} \int_{-\infty}^{+\infty} \int_{-1}^{0} \overline{s}_d \, dx_d dy_d = \overline{Q}_{zd} \qquad (6c)$$

which is exactly the expression for total leakage in an aquifer of infinite length (equation (5)). This proof can be readily extended to show that the same agreement is obtained for the case of boundaries in both the positive and negative y directions. Thus, stream-depletion plots for an aquifer of infinite length can be used to calculate stream depletion in a bounded aquifer of any length. This scale invariance should be of considerable interest to water managers and water-right adjudicators working in interconnected stream-aquifer systems.

The scale invariance of total leakage is not restricted to the case of pumping-induced stream depletion. Consider the case of a rectangular semiconfined aquifer bounded on all sides by impermeable units. In this situation, total leakage is most easily calculated using a numerical model. Figures 6a-b are plots of drawdown and total leakage, respectively, versus dimensionless time for a series of rectangular bounded aquifers that vary in size by six orders of magnitude. Although drawdown varies with the size of the aquifer (Figure 6a), total leakage does not (Figure 6b). In all cases, the total leakage is the same as that for an aquifer of infinite lateral extent (Hantush and Jacob, 1955).

Image-well theory can again be used to prove the universality of this scale invariance. Figure 7 illustrates a portion of the infinite network of image wells that would arise for a rectangular aquifer bounded on all sides by impermeable units. The total leakage is the summation of the leakage produced by each well, which is again calculated by integrating the analytical expression for leakage derived for an infinite aquifer over the actual finite aquifer. In this case, the contribution of each well is written using the Cartesian coordinate form of equation (3). The component of leakage produced by the actual pumping well on Figure 7 is

$$\overline{Q}_{zd_{A1}} = \frac{1}{B_d^2}\int_{-y_{d1}}^{y_{d2}}\int_{-x_{d1}}^{x_{d2}} \overline{s}_d \, dx_d dy_d \quad (7a)$$

where x_{di} $(=x_i/b)$ and y_{di} $(=y_i/b)$ are normalized distances in the x and y directions, respectively. The component of leakage produced by the two closest image wells in the center row on Figure 7 is

$$\overline{Q}_{zd_{A2}} = \frac{1}{B_d^2}\int_{-y_{d1}}^{y_{d2}}\int_{-x_{d1}-\Delta x_d}^{-x_{d1}} \overline{s}_d dx_d dy_d + \frac{1}{B_d^2}\int_{-y_{d1}}^{y_{d2}}\int_{x_{d2}}^{x_{d2}+\Delta x_d} \overline{s}_d dx_d dy_d \quad (7b)$$

where Δx_d is the dimensionless width of the aquifer $(x_{d2} + x_{d1})$ and the origin of the coordinate system for each double integral is at the respective image well. The first double integral is the contribution of the closest image well from the group denoted Series 1 on Figure 7, while the second double integral is the contribution of the closest image well from the group denoted Series 2 on Figure 7. The component of leakage produced by all wells in the center row of Figure 7 is

$$\overline{Q}_{zd_A} = \frac{1}{B_d^2}\int_{-y_{d1}}^{y_{d2}}\int_{-x_{d1}}^{x_{d2}} \overline{s}_d dx_d dy_d + \frac{1}{B_d^2}\sum_{j=1}^{\infty}\left(\int_{-y_{d1}}^{y_{d2}}\int_{-x_{d1}-j\Delta x_d}^{-x_{d1}-(j-1)\Delta x_d} \overline{s}_d dx_d dy_d + \int_{-y_{d1}}^{y_{d2}}\int_{x_{d2}+(j-1)\Delta x_d}^{x_{d2}+j\Delta x_d} \overline{s}_d dx_d dy_d\right) \quad (7c)$$

$$= \frac{1}{B_d^2}\int_{-y_{d1}}^{y_{d2}}\int_{-\infty}^{+\infty} \overline{s}_d dx_d dy_d \quad (7d)$$

where the first term in the summation of equation (7c) is the contribution of the Series 1 image wells and the second term is the contribution of the Series 2 image wells.

Summation over all rows of image wells yields

$$\overline{Q}_{zd} = \frac{1}{B_d^2}\int_{x_{d1}}^{x_{d2}}\int_{-y_{d1}}^{y_{d2}} \overline{s}_d dy_d dx_d + \frac{1}{B_d^2}\sum_{j=1}^{\infty}\sum_{i=1}^{\infty}\left(\int_{-y_{d1}-j\Delta y_d}^{-y_{d1}-(j-1)\Delta y_d}\int_{-x_{d1}-i\Delta x_d}^{-x_{d1}-(i-1)\Delta x_d} \overline{s}_d \, dx_d dy_d + \int_{-y_{d1}-j\Delta y_d}^{-y_{d1}-(j-1)\Delta y_d}\int_{x_{d2}+(i-1)\Delta x_d}^{x_{d2}+i\Delta x_d} \overline{s}_d \, dx_d dy_d \right) +$$

$$\frac{1}{B_d^2}\sum_{j=1}^{\infty}\sum_{i=1}^{\infty}\left(\int_{y_{d2}+(j-1)\Delta y_d}^{y_{d2}+j\Delta y_d}\int_{-x_{d1}-i\Delta x_d}^{-x_{d1}-(i-1)\Delta x_d} \overline{s}_d dx_d dy_d + \int_{y_{d2}+(j-1)\Delta y_d}^{y_{d2}+j\Delta y_d}\int_{x_{d2}+(i-1)\Delta x_d}^{x_{d2}+i\Delta x_d} \overline{s}_d dx_d dy_d \right) +$$

$$\frac{1}{B_d^2}\sum_{j=1}^{\infty}\left(\int_{x_{d1}}^{x_{d2}}\int_{-y_{d1}-j\Delta y_d}^{-y_{d1}-(j-1)\Delta y_d} \overline{s}_d dy_d dx_d + \int_{x_{d1}}^{x_{d2}}\int_{y_{d2}+(j-1)\Delta y_d}^{y_{d2}+j\Delta y_d} \overline{s}_d dy_d dx_d \right) \qquad (8)$$

where Δy_d is the dimensionless length of the aquifer ($y_{d2} + y_{d1}$), the first term is the component of leakage produced by the actual pumping well, the first double summation is the contribution of Series 1 and 2 image wells in the negative y direction, the second double summation is the contribution of Series 1 and 2 image wells in the positive y direction, and the final summation is the contribution of Series 3 and 4 image wells. In all cases, the contribution of each image well is computed assuming that the origin of the coordinate system is at the location of that image well.

Consideration of the contribution of all the wells in equation (8) allows the equation to be rewritten in a concise form as

$$\overline{Q}_{zd} = \frac{1}{B_d^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \overline{s}_d dy_d dx_d \qquad (9)$$

which is equivalent to the Cartesian coordinate form of equation (3). Equation (9) therefore demonstrates that the total leakage for a bounded system is the same as that for an aquifer of infinite lateral extent. Thus, regardless of the size of the aquifer or the distance to a no-flow boundary, the total leakage is the same as in the case of a laterally infinite system. Although the image-well proof was based on a simple rectangular aquifer, a similar proof could be developed for an aquifer of any geometric form. This is possible because an impermeable boundary of any shape can be approximated by linear segments if small enough segments are used, and the effect of a linear boundary segment can always be simulated with image wells. Scale invariance of total leakage is therefore a universal phenomenon in aquifers bounded laterally by impermeable units.

The scale invariance of total leakage is not an intuitively obvious phenomenon. As the areal extent of an aquifer that is laterally bounded by impermeable units decreases, drawdown and the leakage per unit area clearly must increase. What is not as clear, however, is that the relationship between drawdown and leakage is such that the integral of leakage over the aquifer (total leakage) remains the same, regardless of the size of the aquifer. Note that this scale invariance of total leakage is accompanied by a scale invariance in the time to steady state (Figures 6a and 7a) and in the integral of drawdown over the aquifer (equations (3)-(5)). These additional scale-invariant phenomena also have considerable practical significance.

CONCLUSIONS

The quantification of the volume of pumping-induced leakage entering an aquifer as a function of time is necessary for a wide range of practical applications. In this paper, a new approach for calculation of this total leakage was developed for aquifers that are laterally bounded by impermeable units. This approach is based on a simplification that allows estimates of total leakage calculated for aquifers of infinite extent to be used for aquifers of any size and shape. The practical ramifications are significant, as leakage estimates for infinite aquifers can be readily calculated and presented in a type-curve format.

The simplification upon which the new approach is based is possible because of the relationship between drawdown and leakage in aquifers bounded by impermeable units. This relationship produces a condition in which the total leakage does not change as a function of the size or shape of the aquifer, or the proximity of the pumping well to an impermeable boundary. Image-well theory was used here to prove the universality of this scale invariance. The scale-invariant total leakage demonstrated and proven here is an interesting hydrologic phenomenon that undoubtedly has parallels in other branches of the physical sciences. Some of those parallels may also be of practical significance.

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FIGURE CAPTIONS

Figure 1 – Cross-sectional view of a hypothetical semiconfined aquifer and adjacent units. Pumping in the semiconfined aquifer induces vertical flow across the confining layer; head in the unconfined aquifer is unaffected by pumping.

Figure 2 – a) Dimensionless total leakage (Q_z/Q) versus dimensionless time (Tt/Sb^2) plot calculated using the Hantush and Jacob (1955) solution for drawdown in a leaky aquifer of infinite lateral extent; b) Dimensionless total leakage versus dimensionless time plot calculated using the Hantush (1960) solution for drawdown in a leaky aquifer of infinite lateral extent ($B_d = (Kb'/K'b)^{1/2}$, $S_d=S'/S$).

Figure 3 – Cross-sectional (a) and areal (b) views of a hypothetical stream-aquifer system (stream depletion in this configuration consists of vertical leakage across the low-permeability streambed; after Butler et al. (2001)).

Figure 4 – a) Areal view of hypothetical stream-aquifer system truncated by a dam; b) Image well representation of the impermeable boundary perpendicular to the stream created by the dam.

Figure 5 – a) Dimensionless drawdown (sT/Q) versus dimensionless time (Tt/Sw^2) plot for truncated (Figure 4a) and infinite (Figure 3) stream-aquifer systems; b) Dimensionless stream depletion (Q_z/Q) versus dimensionless time plot for these same systems ($Y_B = y_{bnd}/w$; pumping and observation wells located at (5,0) and (3.5,0), respectively).

Figure 6 – a) Dimensionless drawdown (sT/Q) at the pumping well versus dimensionless time (Tt/Sb^2) plot for rectangular aquifers bounded by impermeable units; b) Dimensionless total leakage (Q_z/Q) versus dimensionless time plot for the same aquifers (dimensionless aquifer widths and lengths defined in legend; curves for rectangular aquifers generated with Modflow (Harbaugh and McDonald, 1996) using the same number of grid cells in all simulations with the pumping well at the center of the aquifer).

Figure 7 – Image well representation of rectangular aquifer bounded by impermeable units on all sides (near-aquifer image wells are shown from a network that extends to infinity in all directions; Series 1 image wells – image wells related to the image well at a distance of x_1 from the left boundary of the aquifer, Series 2 image wells – image wells related to the image well at a distance of x_2 from the right boundary of the aquifer, Series 3 image wells – image wells related to the image well at a distance of y_1 from the lower boundary of the aquifer, Series 4 image wells – image wells related to the image well at a distance of y_2 from the upper boundary of the aquifer, image wells not on the axes centered on the actual pumping well are shown as Series 1 or Series 2 to allow a more straightforward derivation in text).

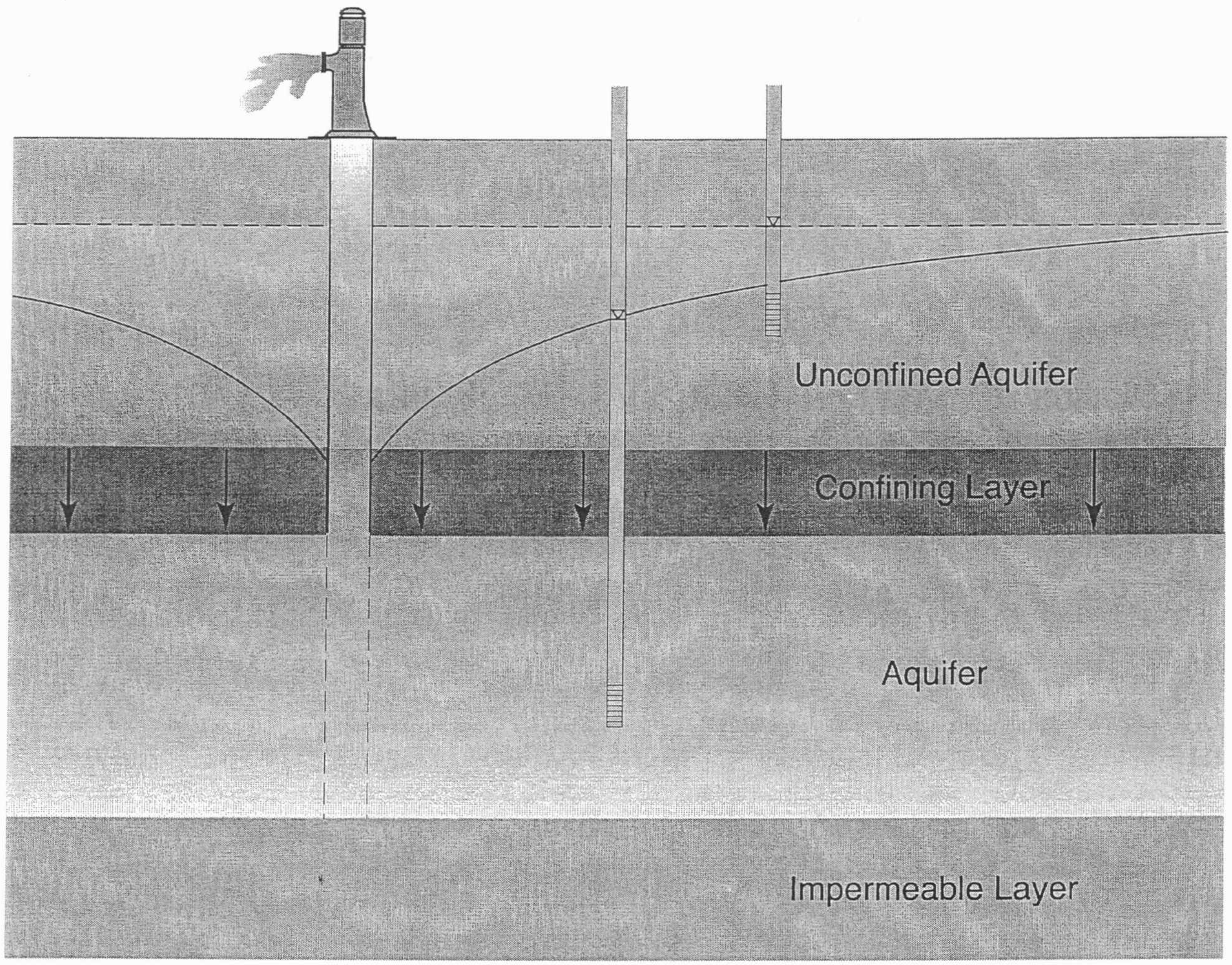


Figure 1 – Cross-sectional view of a hypothetical semiconfined aquifer and adjacent units. Pumping in the semiconfined aquifer induces vertical flow across the confining layer; head in the unconfined aquifer is unaffected by pumping.

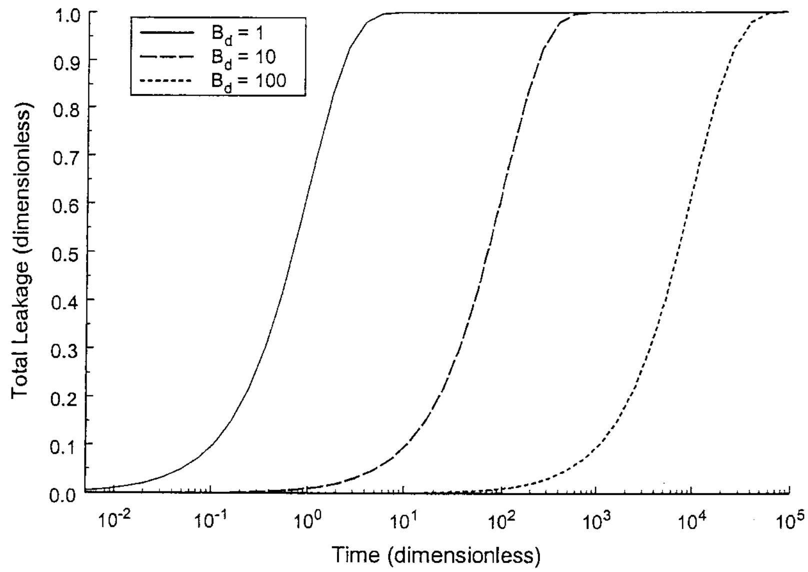
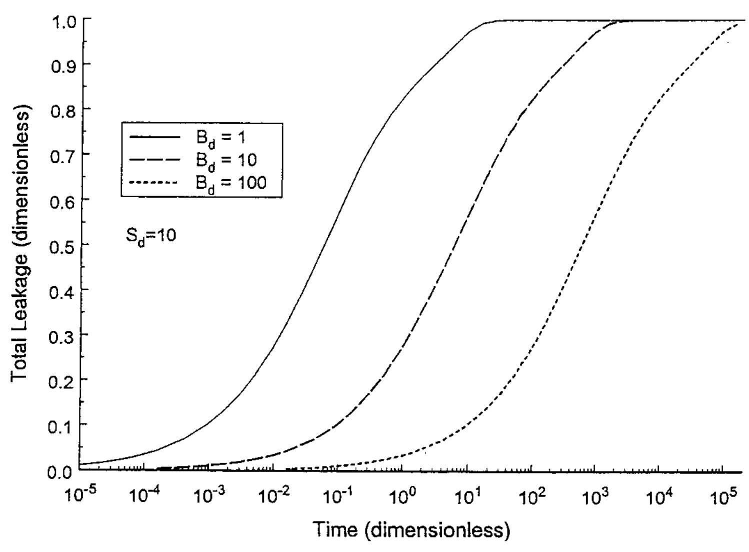


Figure 2 – a) Dimensionless total leakage (Q_z/Q) versus dimensionless time (Tt/Sb^2) plot calculated using the Hantush and Jacob (1955) solution for drawdown in a leaky aquifer of infinite lateral extent;



b) Dimensionless total leakage versus dimensionless time plot calculated using the Hantush (1960) solution for drawdown in a leaky aquifer of infinite lateral extent ($B_d = (Kb'/K'b)^{1/2}$, S_d=S'/S).

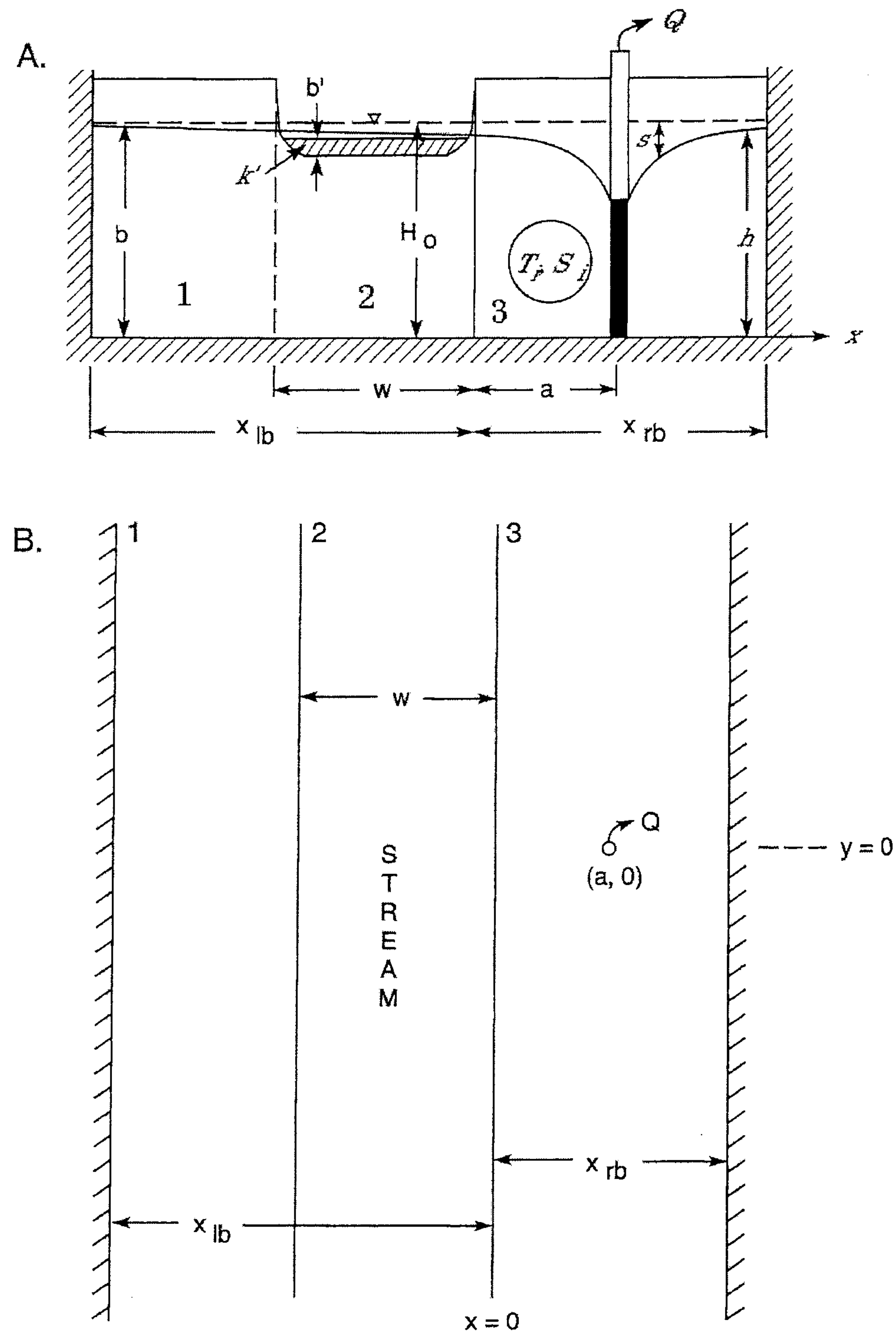


Figure 3 – Cross-sectional (a) and areal (b) views of a hypothetical stream-aquifer system (stream depletion in this configuration consists of vertical leakage across the low-permeability streambed; after Butler et al. (2001)).

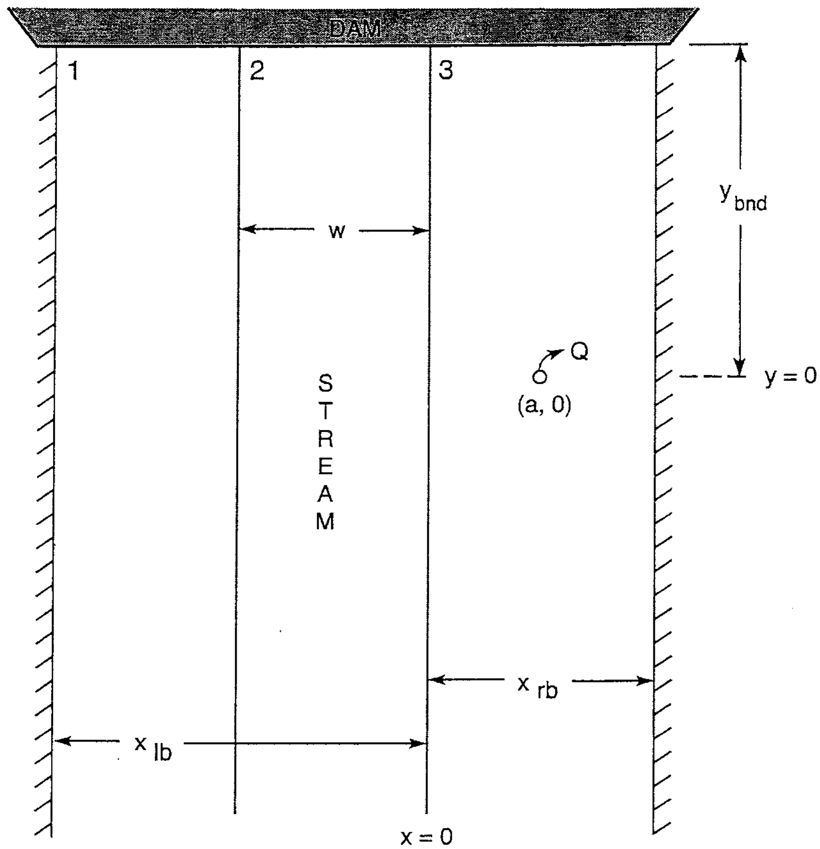
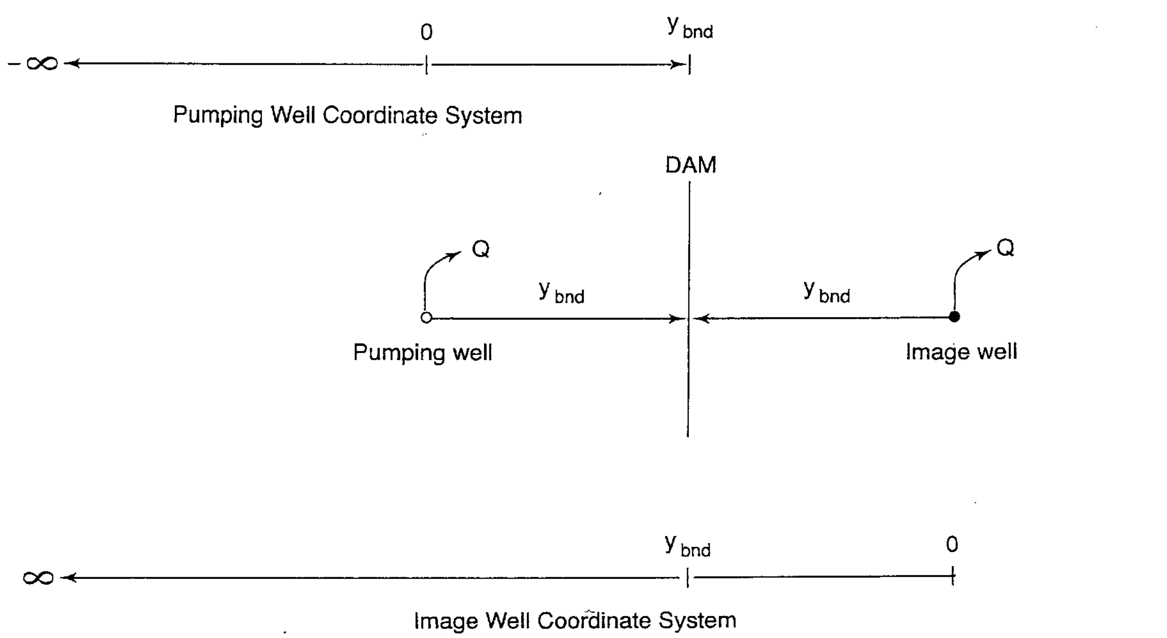


Figure 4 – a) Areal view of hypothetical stream-aquifer system truncated by a dam;



b) Image well representation of the impermeable boundary created by the dam (rotated 90 degrees with respect to Figure 4a).

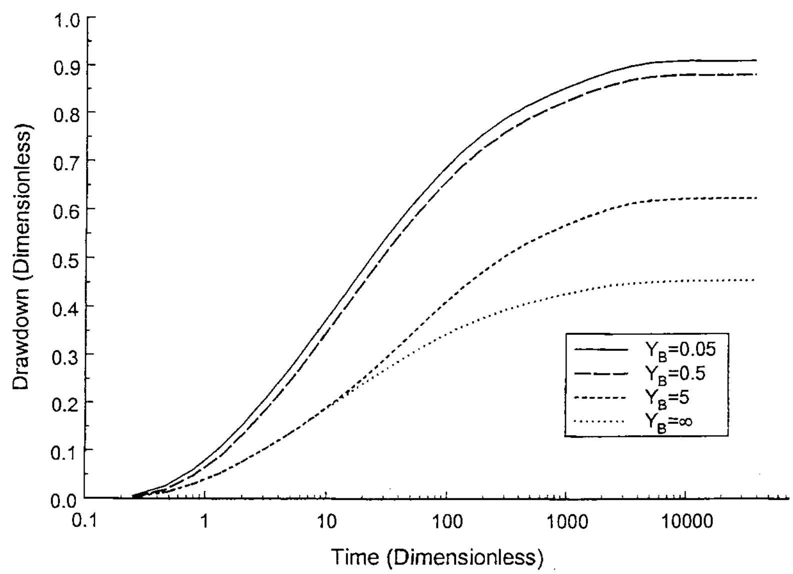
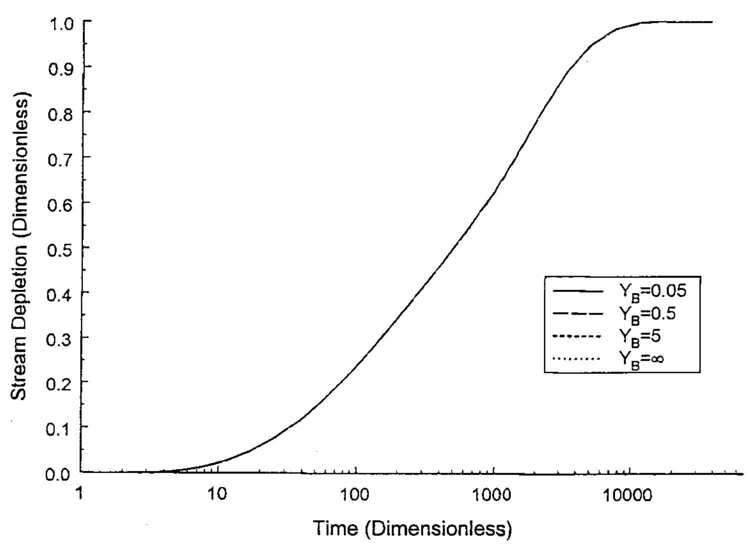


Figure 5 – a) Dimensionless drawdown (sT/Q) versus dimensionless time (Tt/Sw^2) plot for truncated (Figure 4a) and infinite (Figure 3) stream-aquifer systems;



b) Dimensionless stream depletion (Q_z/Q) versus dimensionless time plot for these same systems ($Y_B = y_{bnd}/w$; pumping and observation wells located at (5,0) and (3.5,0), respectively).

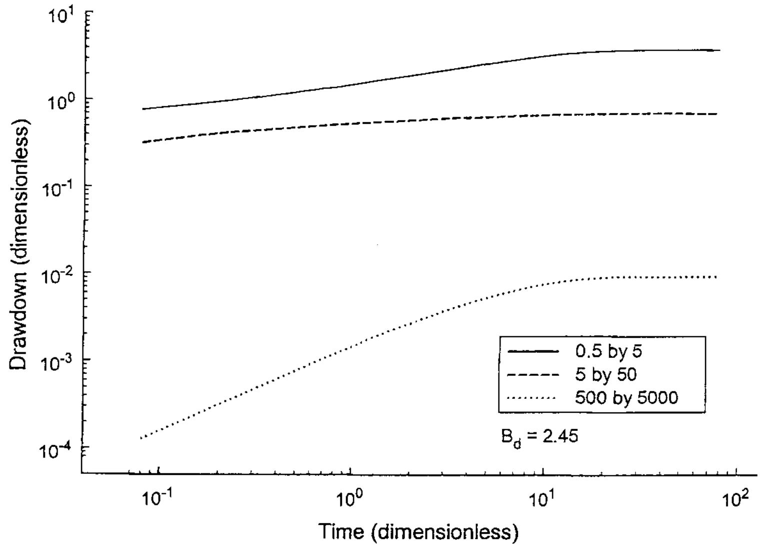
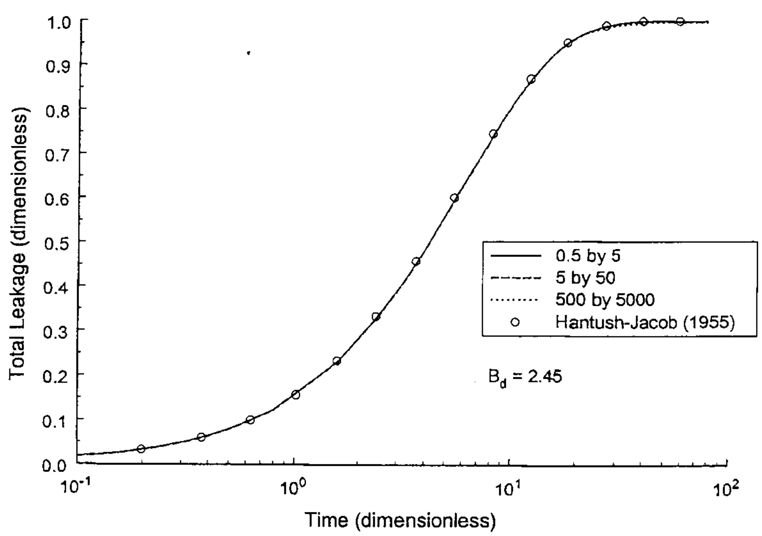


Figure 6 – a) Dimensionless drawdown (sT/Q) at the pumping well versus dimensionless time (Tt/Sb^2) plot for rectangular aquifers bounded by impermeable units;



b) Dimensionless total leakage (Q_z/Q) versus dimensionless time plot for the same aquifers (dimensionless aquifer widths and lengths defined in legend; curves for rectangular aquifers generated with Modflow using the same number of grid cells in all simulations with the pumping well at the center of the aquifer).

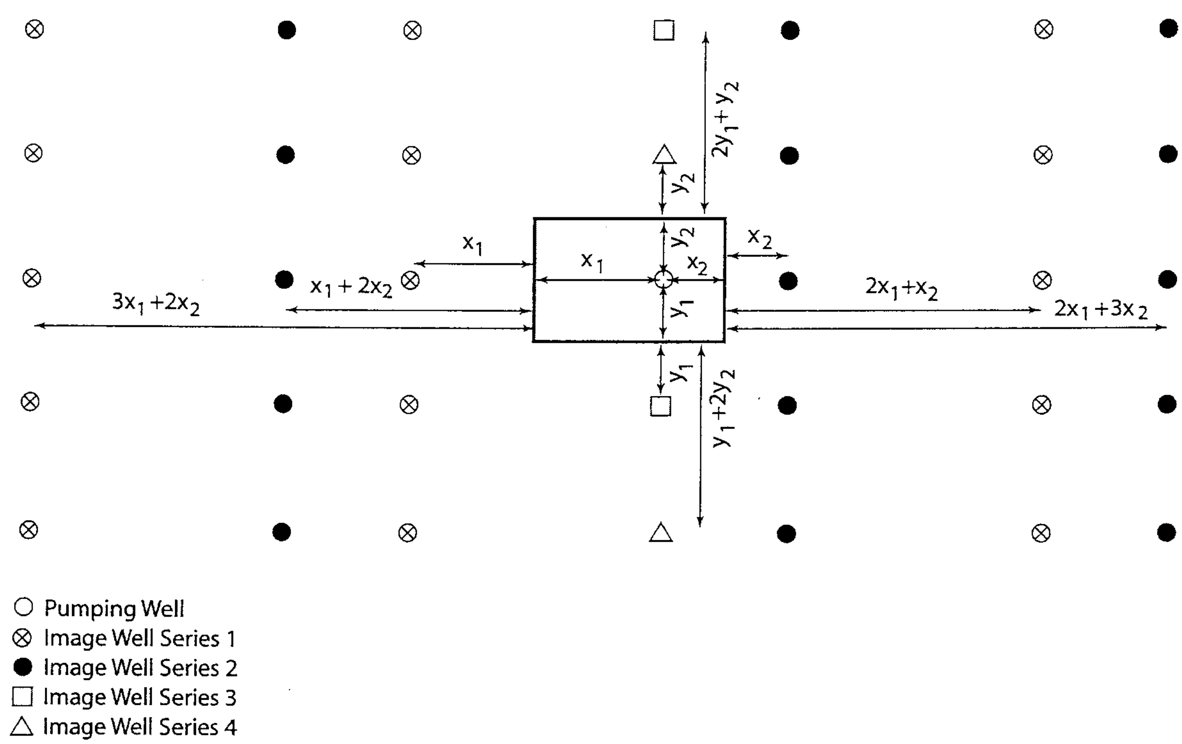


Figure 7 – Image well representation of rectangular aquifer bounded by impermeable units on all sides (near-aquifer image wells are shown from a network that extends to infinity in all directions; Series 1 image wells – image wells related to the image well at a distance of x_1 from the left boundary of the aquifer, Series 2 image wells – image wells related to the image well at a distance of x_2 from the right boundary of the aquifer, Series 3 image wells – image wells related to the image well at a distance of y_1 from the lower boundary of the aquifer, Series 4 image wells – image wells related to the image well at a distance of y_2 from the upper boundary of the aquifer, image wells not on the axes centered on the actual pumping well are shown as Series 1 or Series 2 to allow a more straightforward derivation in text).