Multichannel Analysis of Surface Waves (MASW)

Theory and Applications

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Outline

- Theory
- Verifications
- 2-D S-wave Velocity Sections
- Future Studies

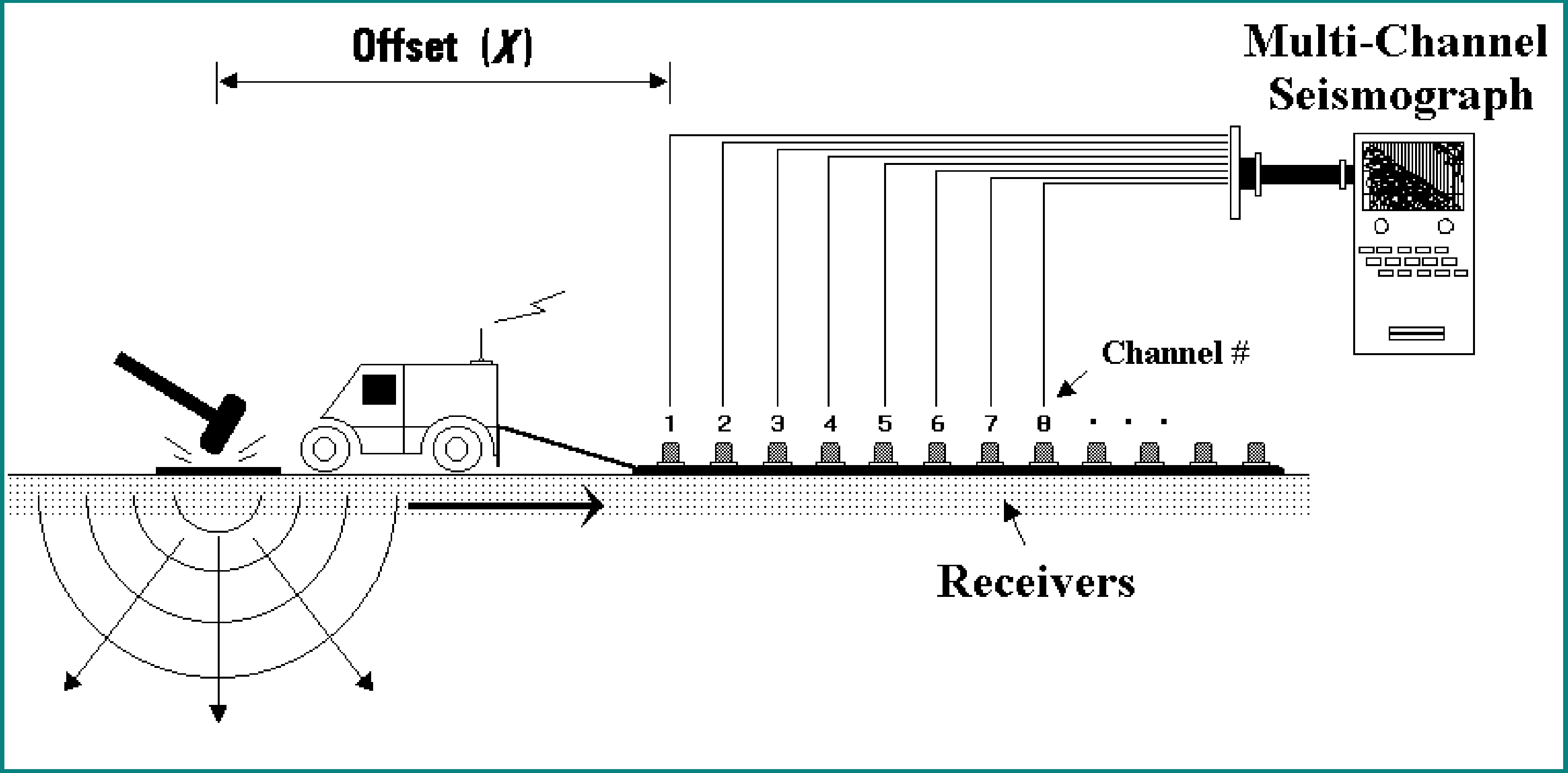
Part 1

Theory

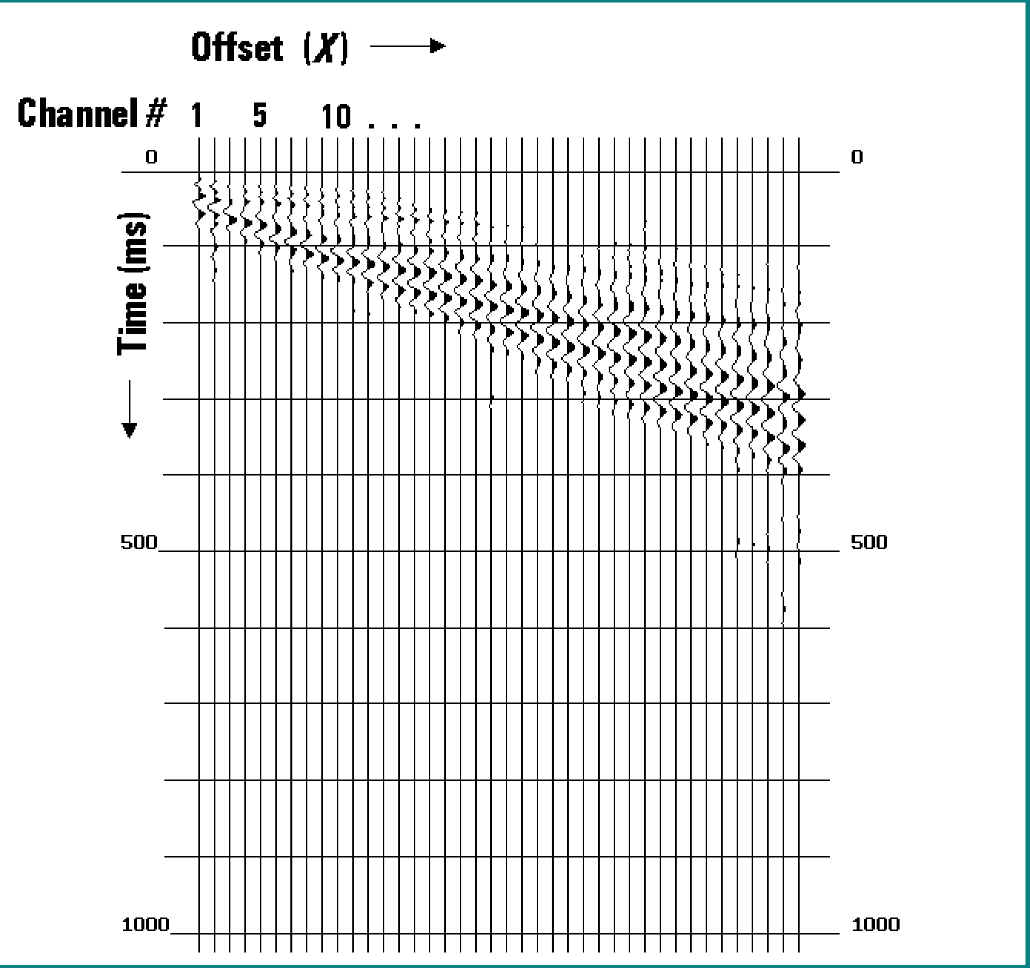
From field shot gather to

S-wave velocity profile

Multichannel recording system



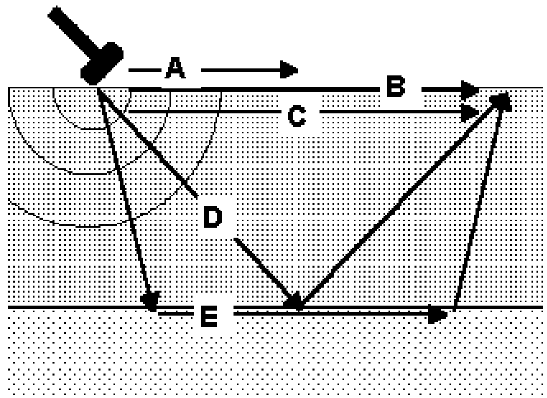
Raw Field Data



Theory—Outline

- Surface wave background
- Calculation of dispersion curve
- Inversion of dispersion curve
- Parameters of a layered earth model
- Equipment and data acquisition parameters

Theory—Surface wave

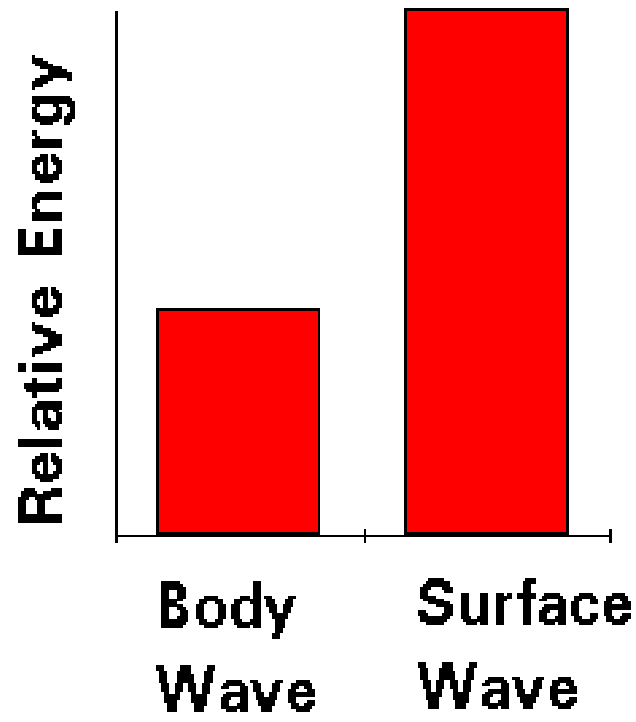


BODY WAVE

A: Air Wave
B: Direct Wave
D: Reflected Wave
E: Refracted Wave

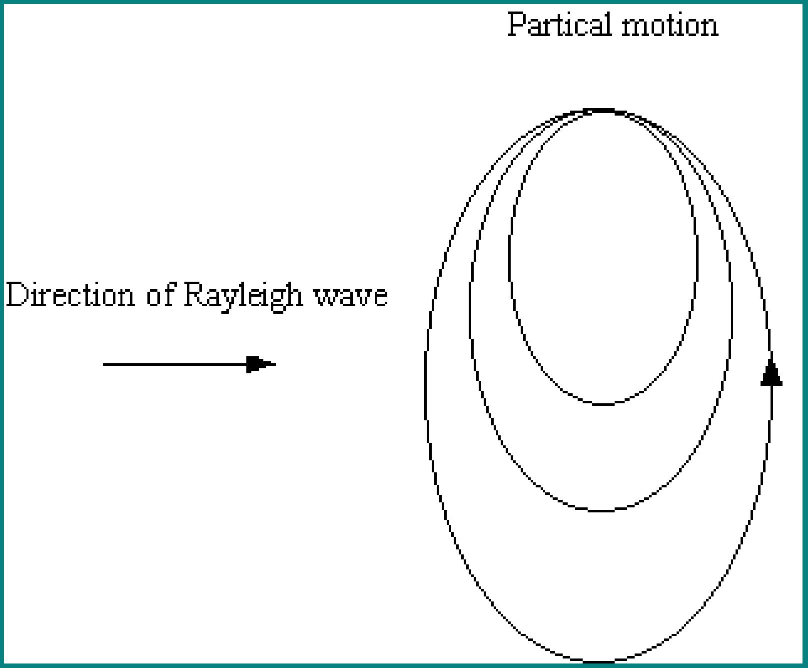
SURFACE WAVE

C: Ground Roll



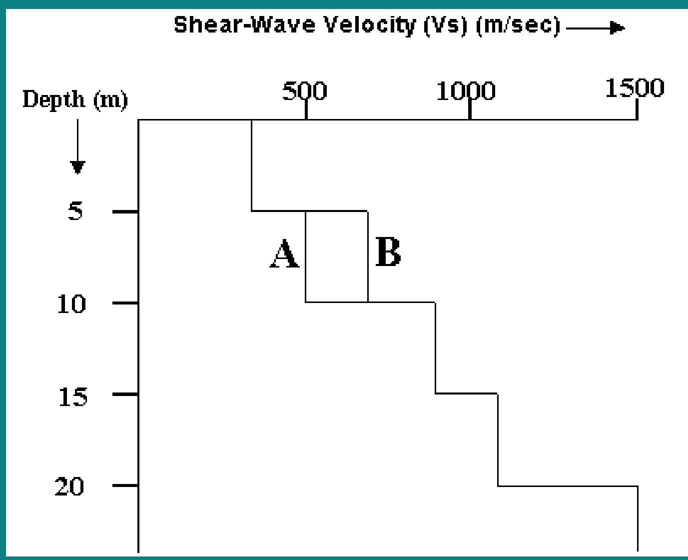
Theory—Penetrating depth

- Penetrating depth is about one wavelength.
- Longer wavelengths can "see" deeper than shorter wavelengths.
- In a homogeneous half-space, Rayleigh wave velocity is about 0.92Vs if Poisson's ratio = 0.25.

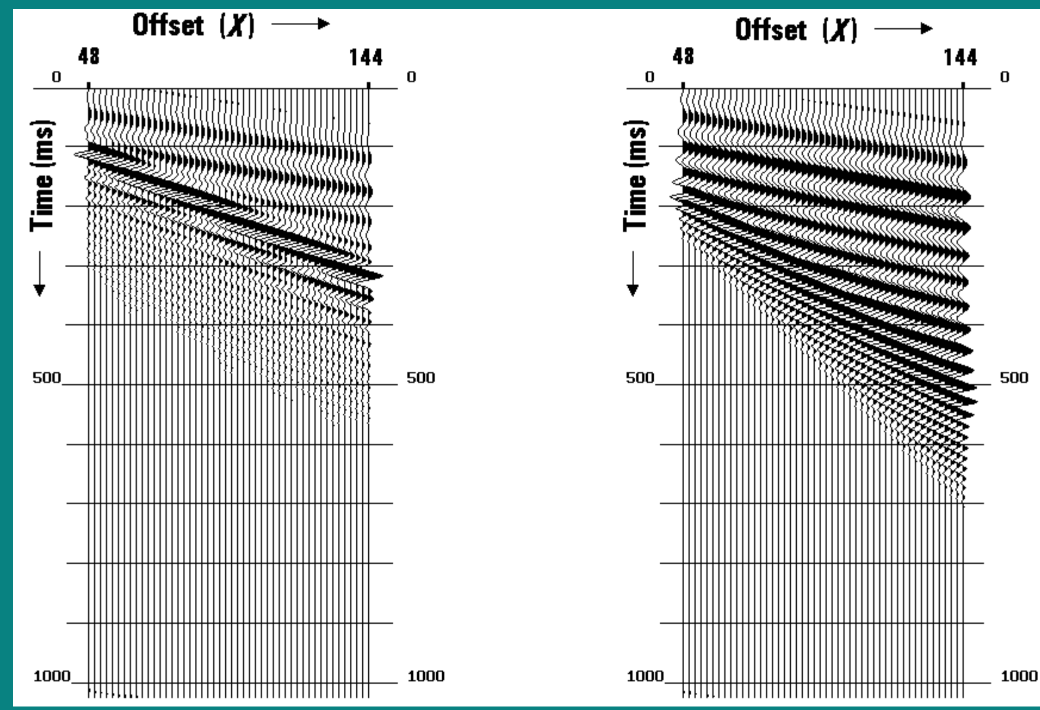


Theory—Model response

S-wave velocity



A B



Theory—Calculation of dispersion curve

1. $$U(x,w)=\int u(x,t)e^{iwt}dt$$

U(x,t) is a shot gather in the offset-time domain

U(x,w) is a shot gather in the offset-frequency domain after applied the Fourier transform to *U(x,t)*.

U(x,w) can be expressed as the multiplication of phase and amplitude spectrum

$$U(x,w)=e^{-i\Phi x}A(x,w) \qquad \Phi = w/c_w$$

w is frequency in radian and c_w is phase velocity for frequency w.

Theory—Calculation of dispersion curve

2. **Applying integral transformation to *U(x,t)***

$$V(\phi,w)=\int e^{i\phi x}[U(x,w)/|U(x,w)|]dx$$

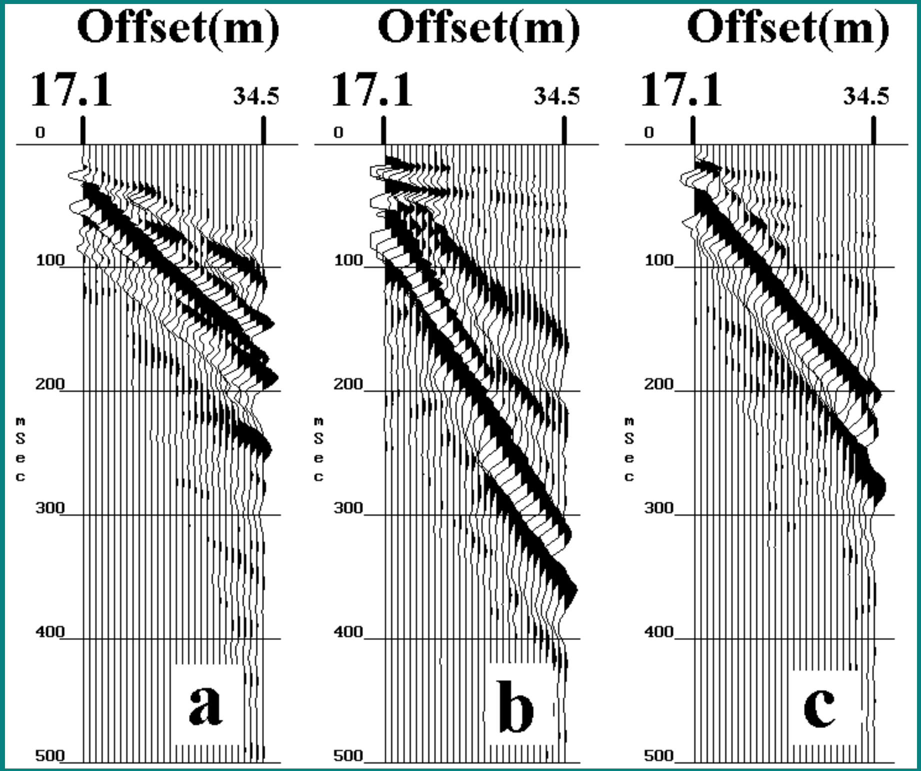
$$=\int e^{-i(\Phi-\phi)x}[A(x,w)/|A(x,w)|]dx$$

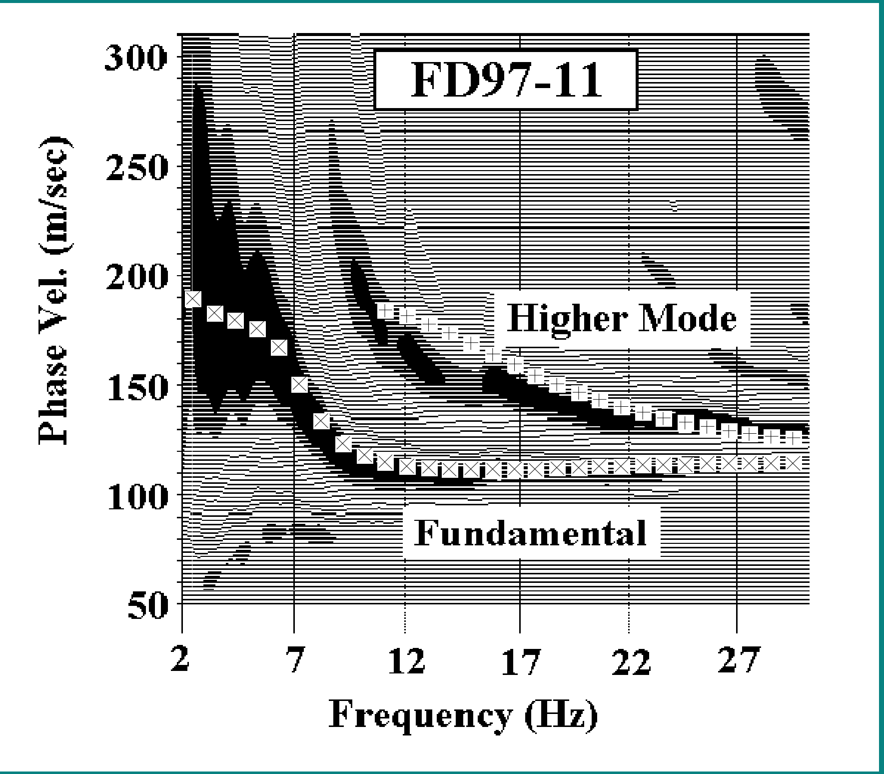
Because *A(x,w)* is both real and positive, $V(\phi,w)$ **will have a maximum if**

$$\Phi=\phi$$

Example of dispersion curve image

FD97-1





Theory—Inversion of dispersion curve

Outline

- Forward calculation
- Partial derivatives of phase velocity function
- Sensitivity of earth model parameters
- Inversion algorithms

Theory—Inversion of dispersion curve

Layered earth model, four parameters

Free surface

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v_{s1} v_{p1} ρ_1 h_1

---

v_{s2} v_{p2} ρ_2 h_2

.
.
.

---

v_{si} v_{pi} ρ_i h_i

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.
.

---

v_{sn} v_{pn} ρ_n infinite

Forward calculation

$F_j(f_j, c_{Rj}, \mathrm{v_s}, \mathrm{v_p}, \mathrm{d}, \mathrm{h}) = 0, \qquad (j = 1, 2, \ldots, m)$

m: the number of data points,

f_j: the frequency,

c_{Rj}: the Rayleigh wave phase velocity,

$\mathrm{v_s} = (v_{s1}, v_{s2}, \ldots, v_{sn})^T$: the S-wave velocity vector,

$\mathrm{v_p} = (v_{p1}, v_{p2}, \ldots, v_{pn})^T$: the P-wave velocity vector,

$\mathrm{d} = (\mathrm{d}_1, \mathrm{d}_2, \ldots, \mathrm{d}_n)^T$: the density vector, and

$\mathrm{h} = (h_1, h_2, \ldots, h_{n-1})^T$: the thickness vector

Partial derivatives of the phase velocity function

- The Jacobian matrix calculated by Ridder's method—one numerical method.

Sensitivity of earth model parameters

The six-layer model is used to analyze the sensitivity of higher modes of surface waves.

A Layered Earth Model

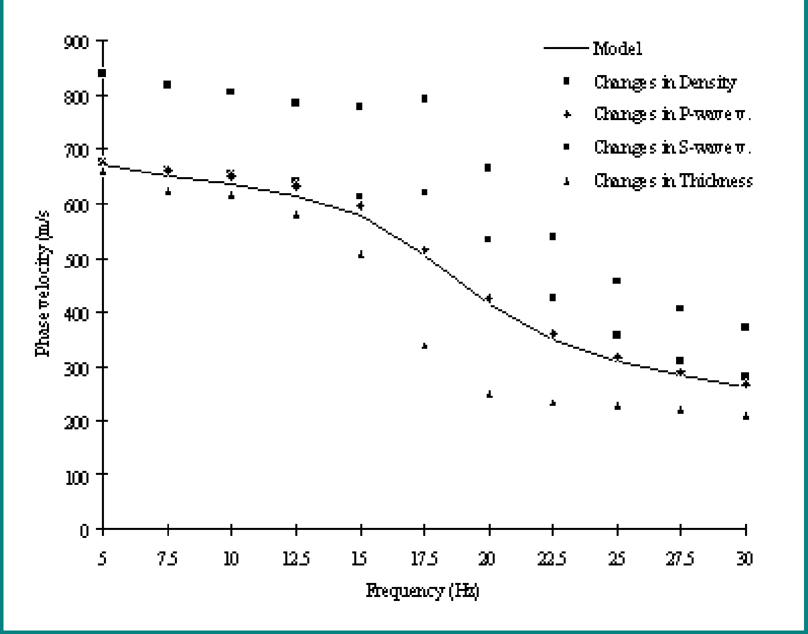
S (m/s)	P (m/s)	d (g/cm3)
194	650	1.82
270	750	1.86
367	1400	1.91
485	1800	1.96
603	2150	2.02
740	2800	2.09

0
4
8
12
Meters

Sensitivity of earth model parameters

Why only S-wave velocity?

Model Parameters	model (%)	data (%)
P-wave Velocity	25	3
Density	25	10
S-wave Velocity	25	39
Thickness	25	16



S-wave velocity is the dominant property for the fundamental mode of high-frequency Rayleigh wave dispersion data.

Why only S-wave velocity?

Based on the sensitivity analysis of four groups of earth model parameters: S-wave velocity, P-wave velocity, density, and thickness of layers, S-wave velocity is dominant. If we can get good estimates of P-wave velocity and density, we can only invert S-wave velocity from phase velocities of surface waves.

The following discussion assumes P-wave velocity and density are known. Only S-wave velocities are updated during the inversion procedure based on the layered earth model.

Inversion algorithms

Objective function:

$$\Phi = \left\| Jx - b \right\|_2 W \left\| Jx - b \right\|_2 + \lambda \left\| x \right\|_2^2$$

Solution

$$x = V\left(\Lambda^2 + \lambda I\right)^{-1}\Lambda U^T d$$

Where *d* is the vector of difference between modeled and measured data, V, Λ, and U are the SVD matrixes of the weighted Jacobian matrix A.

Theory—
Parameters of a layered earth model

1. Initial values of S-wave velocities:

$v_{s1} = c_R(\text{high})/A$,   (for the first layer)
$v_{sn} = c_R(\text{low})/A$,   (for the half space)
$v_{si} = c_R(i)/A$,   ($i = 2, 3, ..., n-1$)
$A = 0.88$

Initial values of S-wave velocities are determined based on dispersion curve data.

Theory—
Parameters of a layered earth model

Based on analysis of sensitivity of earth model parameters, the other three groups of parameters—P-wave velocities, densities, and thickness of layers—are not changed during inversion procedure.

2. P-wave velocities can be determined from the first arrivals of surface wave data. The first arrivals are refraction information on P-wave velocities.

Theory—
Parameters of a layered earth model

3. Densities can be chosen from 1.6–2.2 g/cc for shallow sedimentary geology. Based on our experience, this range of density gives enough accuracy for inverted S-wave velocities up to 100 ft depth.

Theory—
Parameters of a layered earth model

4. The depth to the top of the half-space is determined by your investigation depth. Ten to fifteen layers is a good place to start with testing. After determining the number of layers, the thickness of each layer can easily be defined.

Make sure the maximum wavelength is greater than the investigation depth.

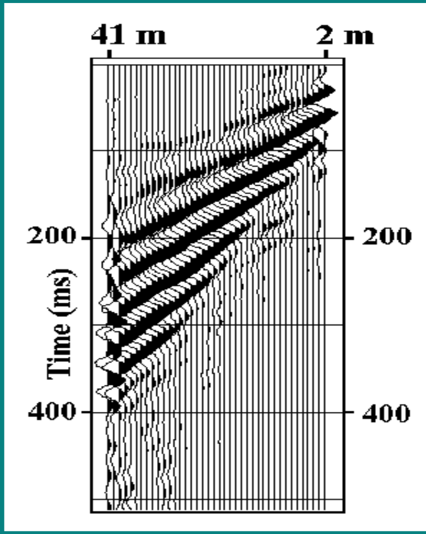
Theory—
Parameters of a layered earth model

Trade-off between resolution and accuracy

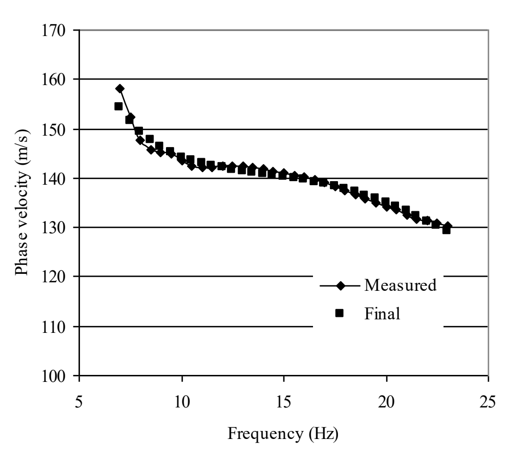
The thickness of layers basically is a measurement of the vertical resolution. The vertical resolution is limited by accuracy of the dispersion curve. In the case of low accuracy of dispersion curve data, you should reduce the number of layers (increase thickness of each layer) to reduce uncertainty of the inverted S-wave velocities (stabilize inversion).

Summary—
From shot gather to S-wave velocity profile

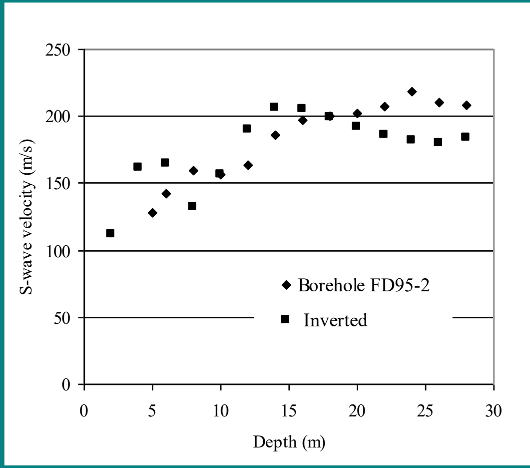
f-k transformation   Inversion



Multichannel raw data



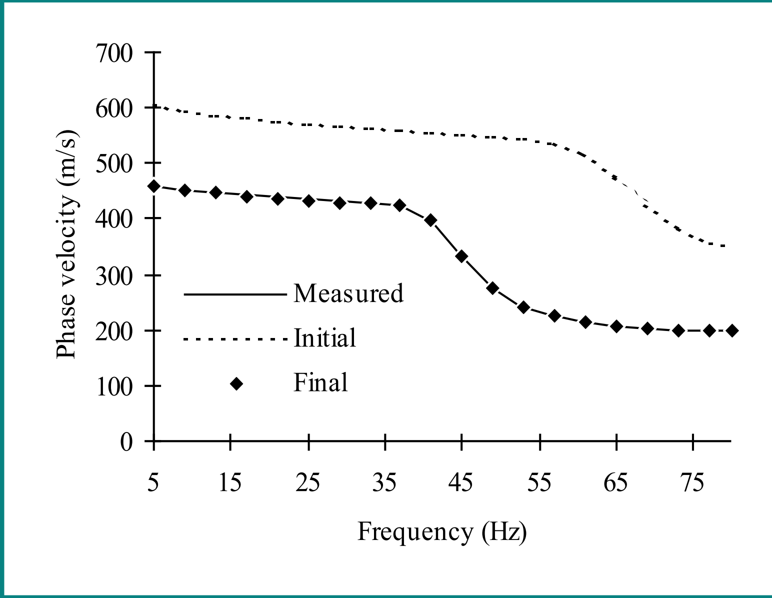
Dispersion curve

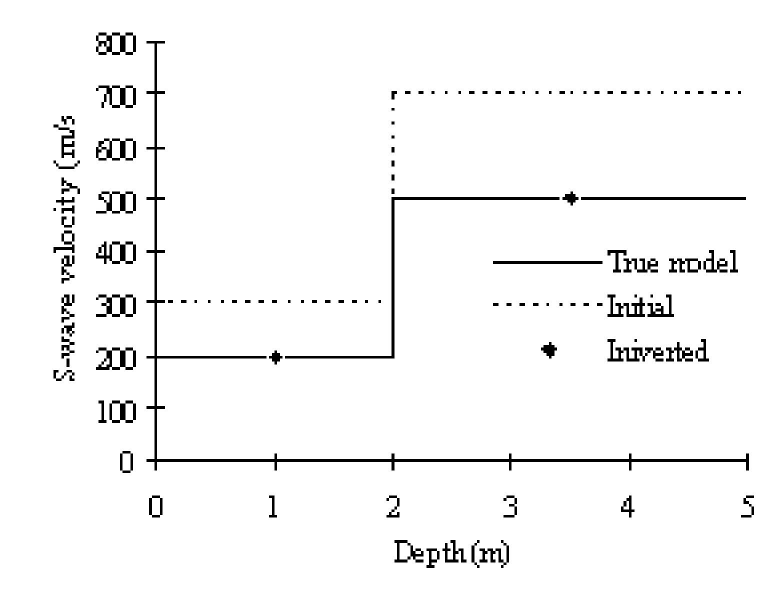


S-wave velocity

Synthetic Examples

Two-layer models

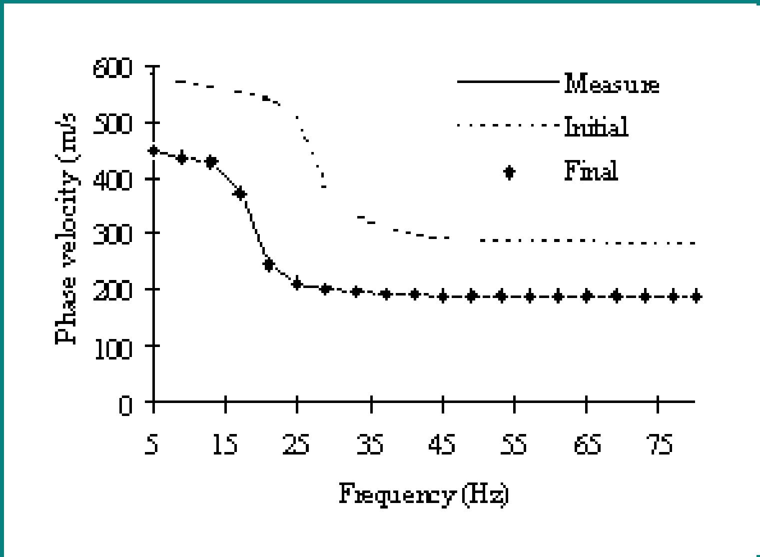


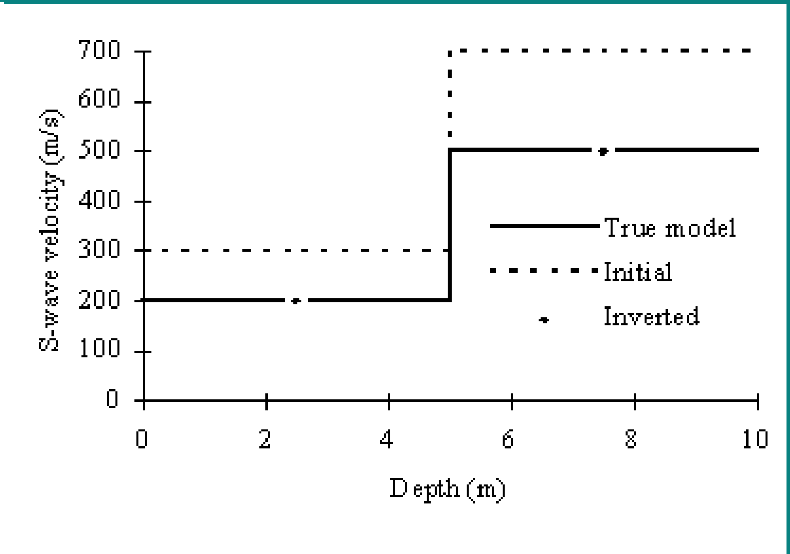


Thickness of top layer: 2 m

Synthetic Examples

Two-layer models

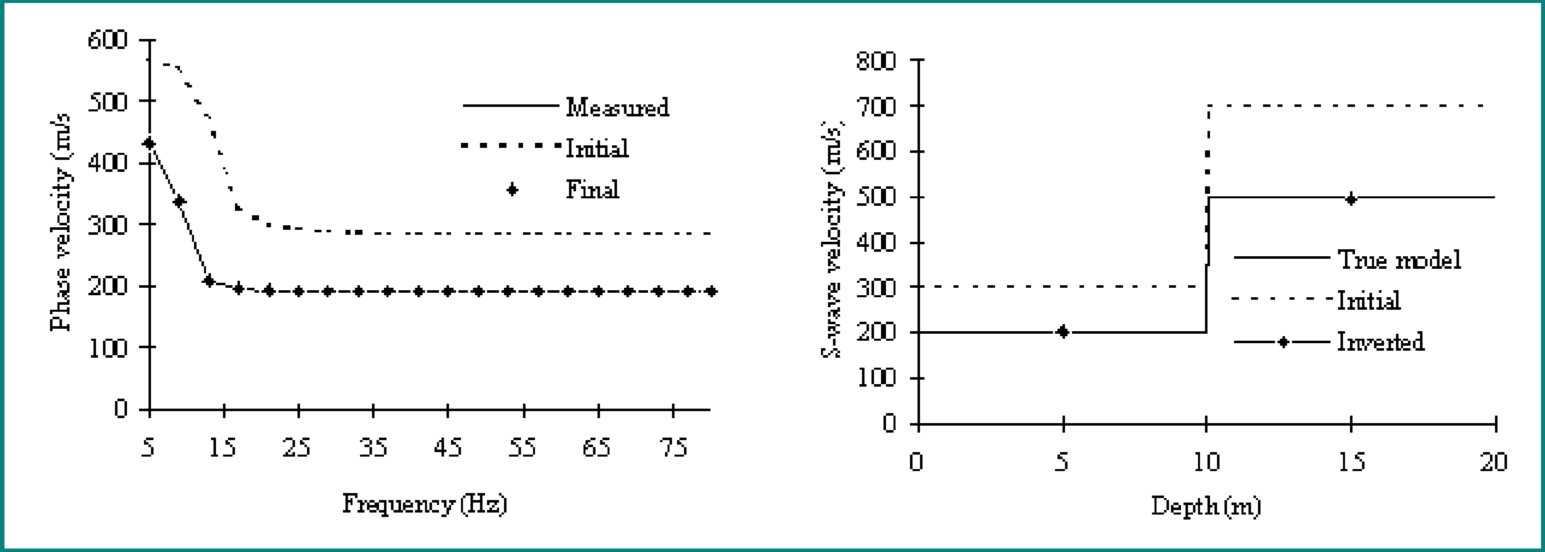




Thickness of top layer: 5 m

Synthetic Examples

Two-layer models



Thickness of top layer: 10 m

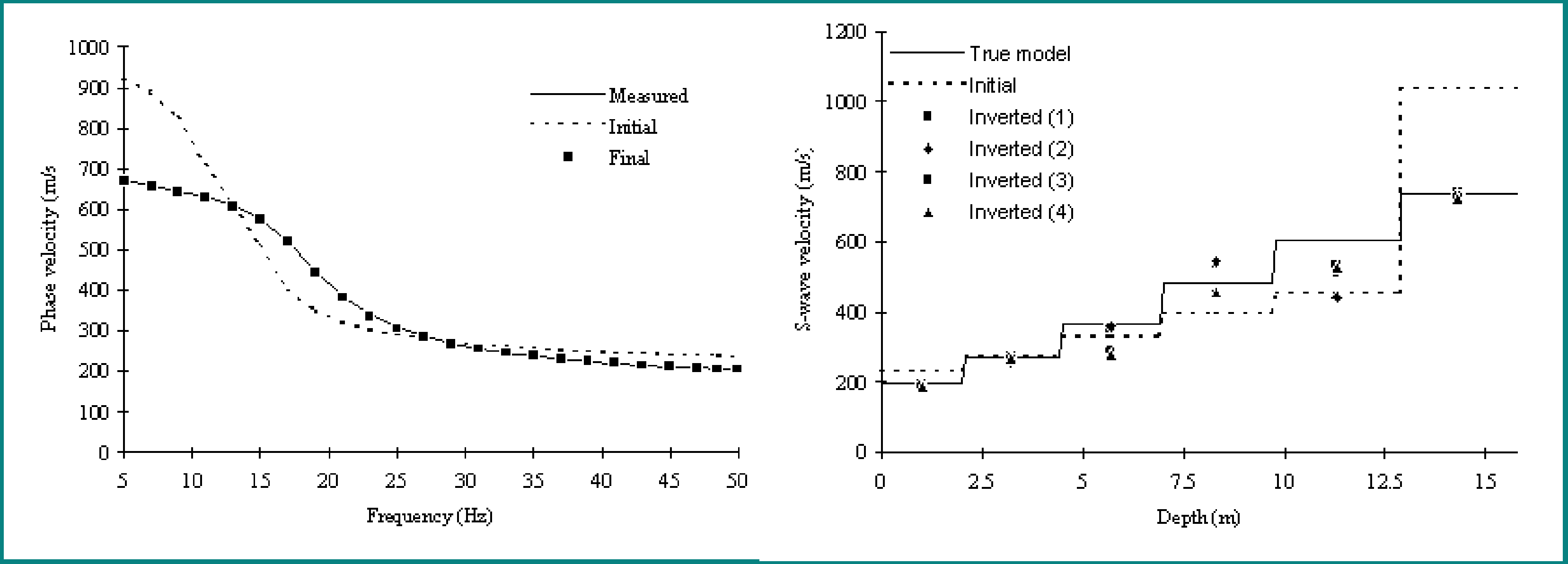
Synthetic Examples

Why use two-layer models?

One direct application of a two-layer model is static correction in S-wave reflection and refraction survey in oil industry.

Synthetic Examples

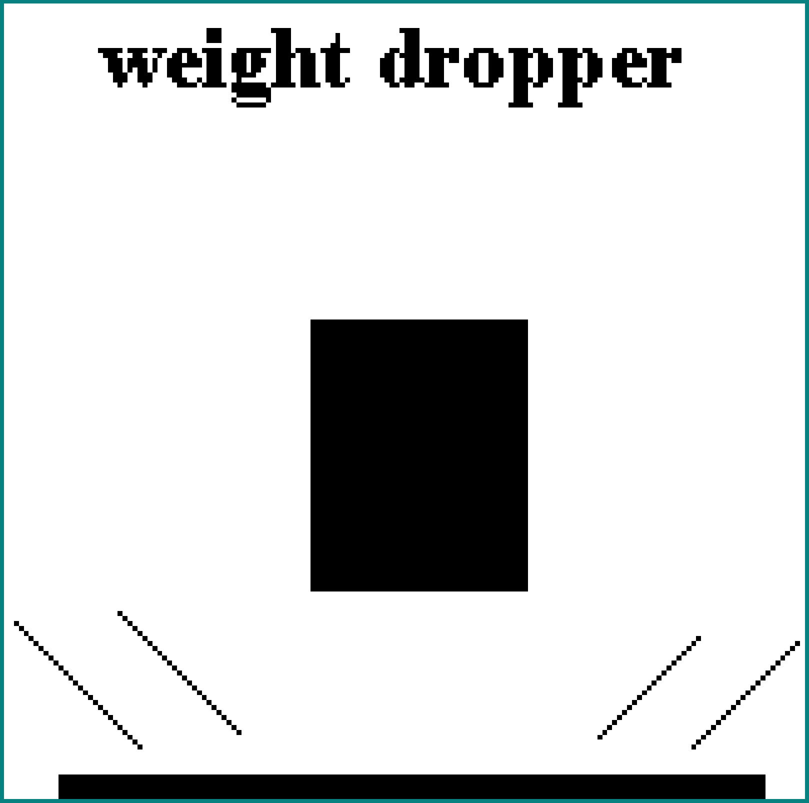
A multilayer model—Effects of P-wave velocity and density



25% change (1) in S; (2) in S and P; (3) in S and density; and (4) in S, P, and density.

Data Acquisition Equipment: Seismic Sources

A surface impact source can generate surface wave "enriched" records. "Enriched" means wavelengths of surface waves evenly cover the range of investigation depth.



Seismic Sources

1. Industrial Vehicle International (IVI) Minivib

Downward weight 6,000 lb.

Investigation depth: 1 to 30 meters

Seismic Sources

2. KGS-built weight dropper

Investigation depth: 2 to 30 meters

Seismic Sources

3. Sledgehammer and plate

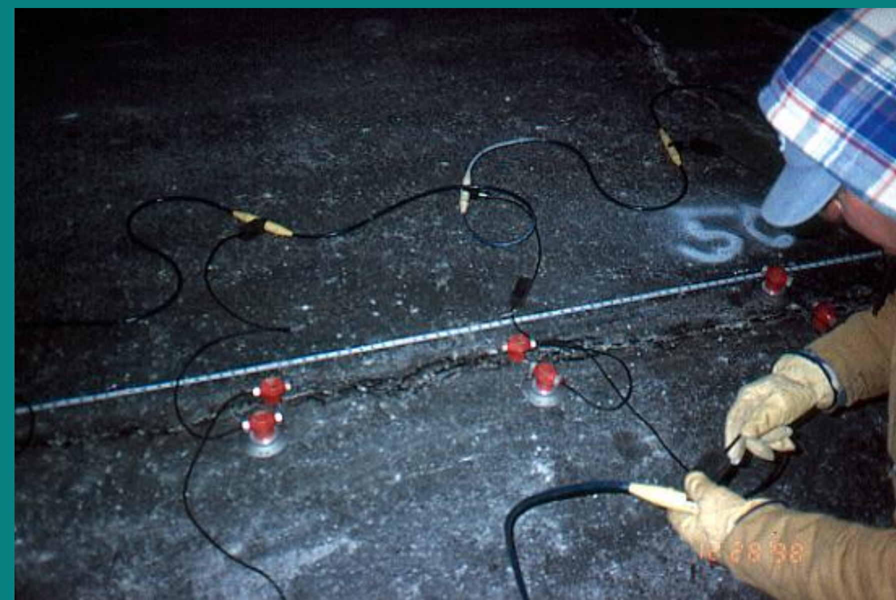
Investigation depth: 0.5 to 15 meters

Seismograph—48 to 60 channels

60-channel Geometrics StrataView

Geophones—4.5 to 10 Hz vertical component geophone

Geophone with spike



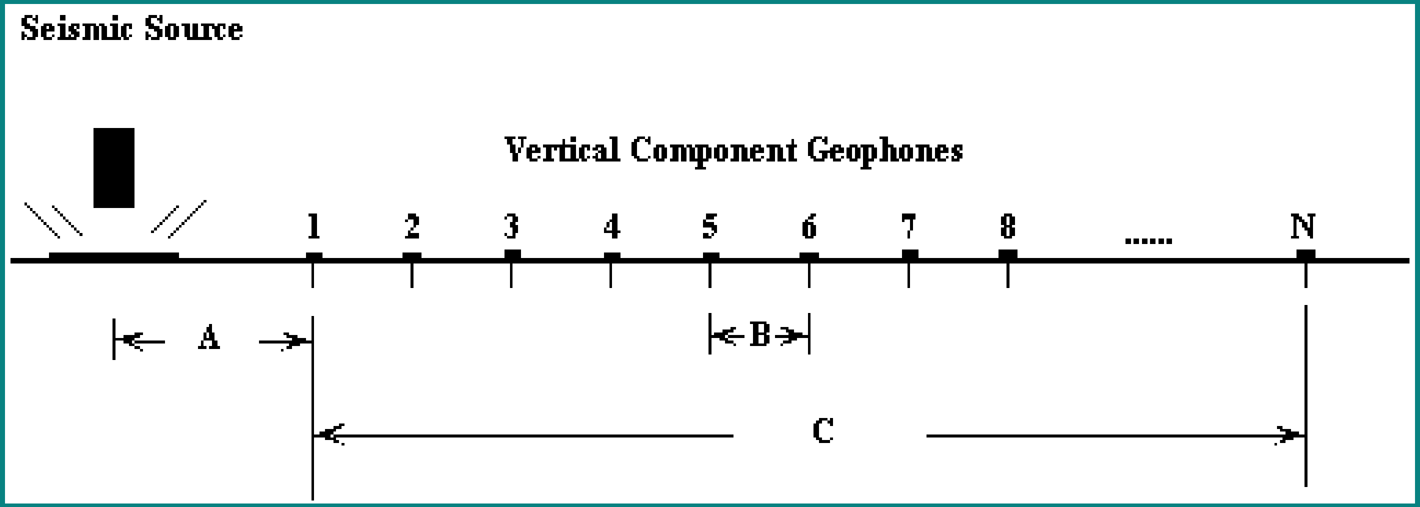
Geophone with baseplate

Geophones—4.5 to 10 Hz vertical component geophone

Geophone on tiles

Geophone on carpet

Data Acquisition Parameters



A. Nearest source-receiver offset

B. Receiver spacing

C. Receiver spread: distance between the first receiver and the last receiver

Data Acquisition Parameters

Nearest source-receiver offset

Near-offset effect: Lower frequency components are not fully developed as plane waves.

Plane-wave propagation of surface waves occurs when the nearest source-receiver offset is greater than half the maximum desired wavelength.

The maximum desired wavelength is about equal to the maximum investigation depth so that the nearest source-receiver offset is about equal to the maximum investigation depth.

Data Acquisition Parameters

Receiver spacing

Receiver spacing should follow the Nyquist sampling theorem. Receiver spacing determines the shortest wavelength in recorded data, which is a guideline for determining thickness of a layer model and is also a limit in the inverted S-wave velocity model.

Data Acquisition Parameters

Receiver spread

Receiver spread should also follow the Nyquist sampling theorem. Receiver spread determines the longest wavelength in recorded data, which is a guideline for determining total thickness of layers on the top of the half-space.

The receiver spread is limited by ***far-offset effect***.

Far-offset effect: Higher frequency components of surface waves are contaminated by body waves due to high-frequency attenuation.

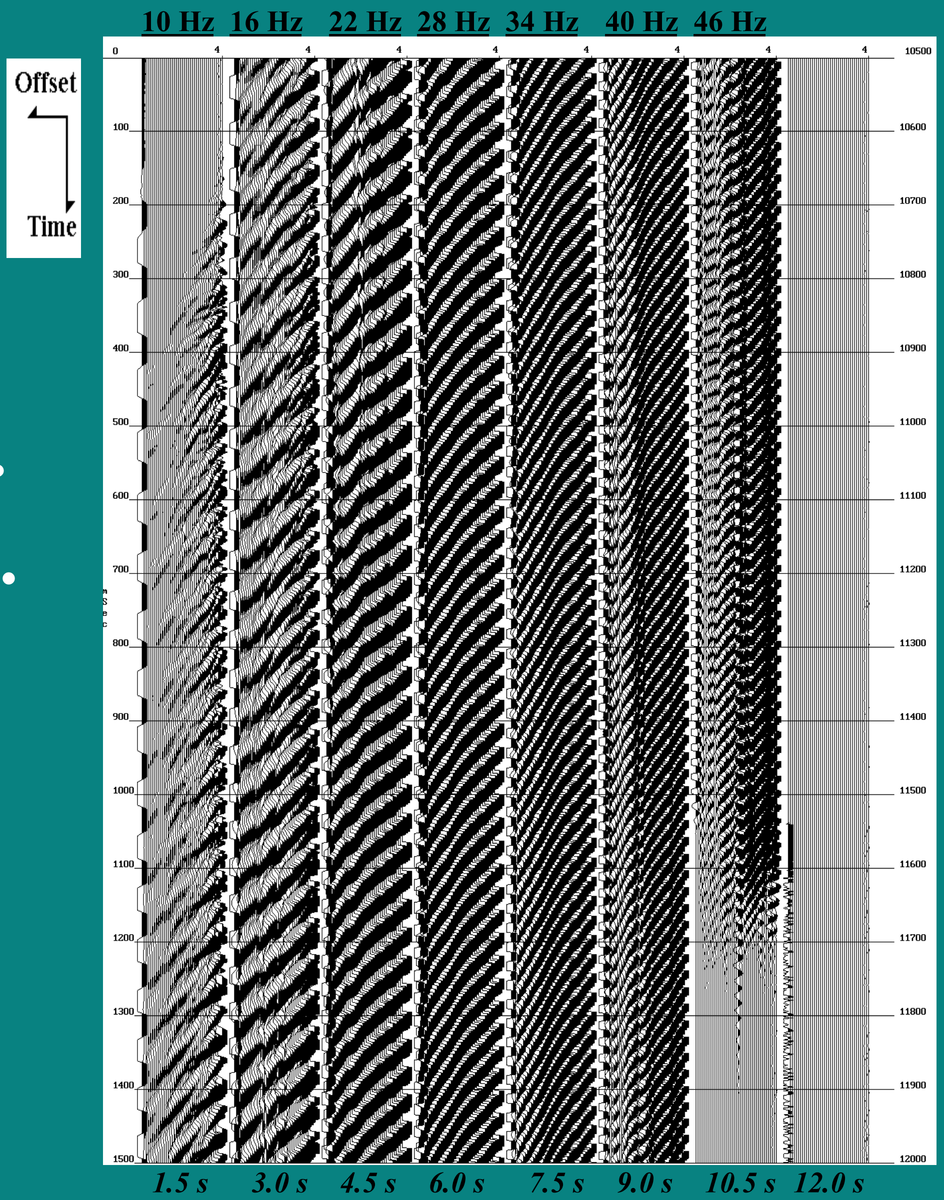
Near-offset effects

Nearest offset: 1.8 m.

Receiver spacing: 1 m.

Receiver spread: 40 m.

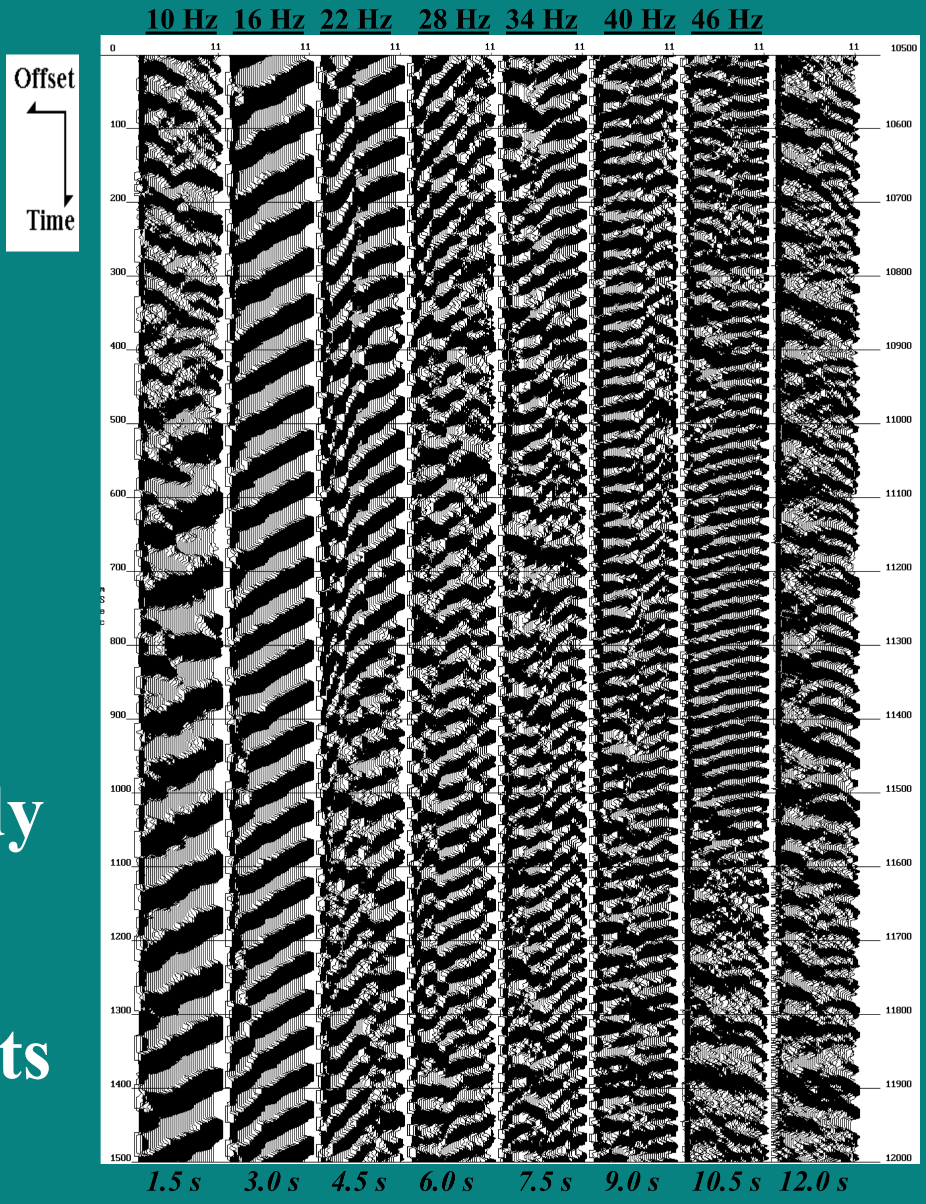
Lower frequency components are not fully developed as plane waves.



Far-offset effects

Nearest offset: 89 m.
Receiver spacing: 1 m.
Receiver spread: 40 m.

Higher frequency components are contaminated by body waves due to attenuation of high frequency components of surface waves.



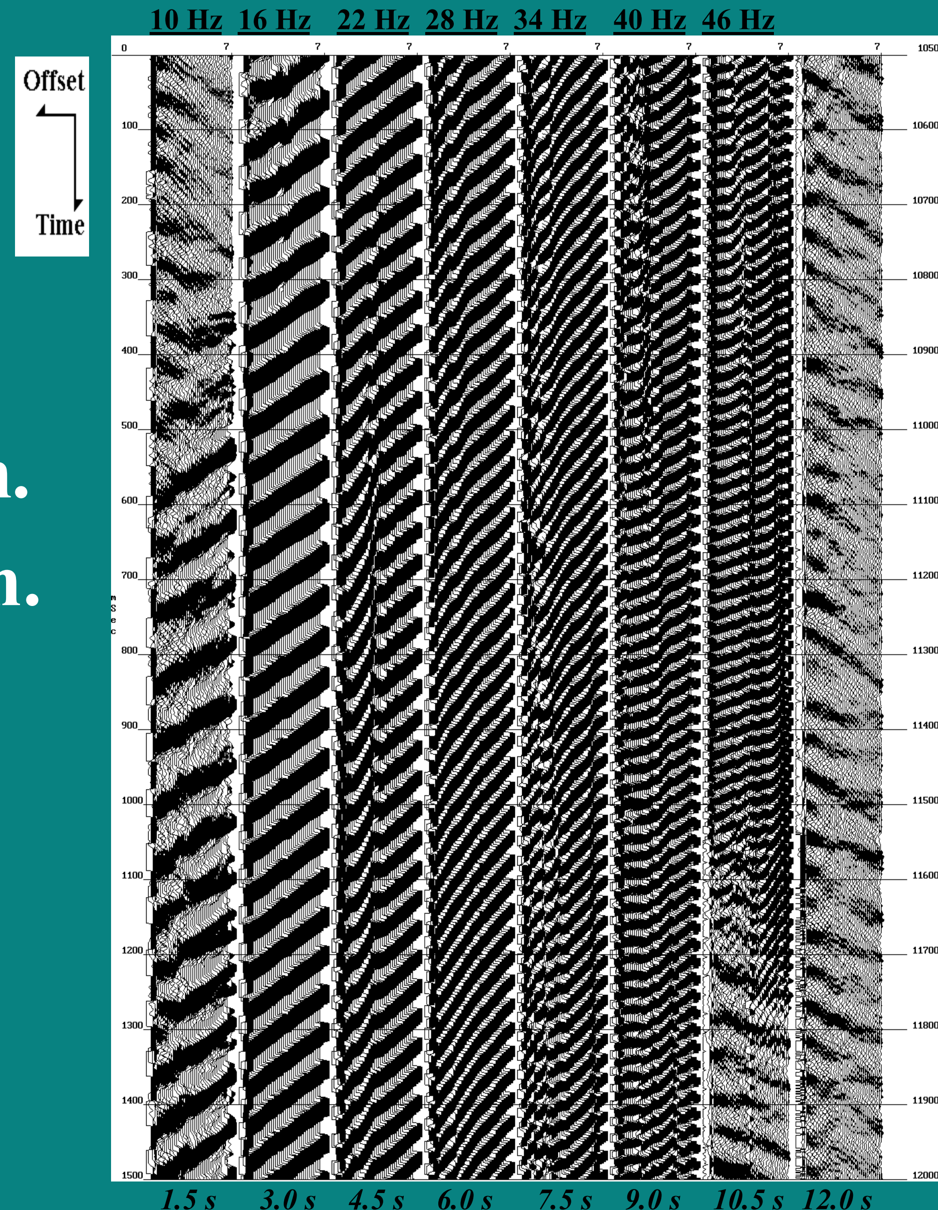
Optimum offset

Nearest offset: 27 m.

Receiver spacing: 1 m.

Receiver spread: 40 m.

Linearity of surface wave is clearly improved from 4 Hz to 35 Hz.



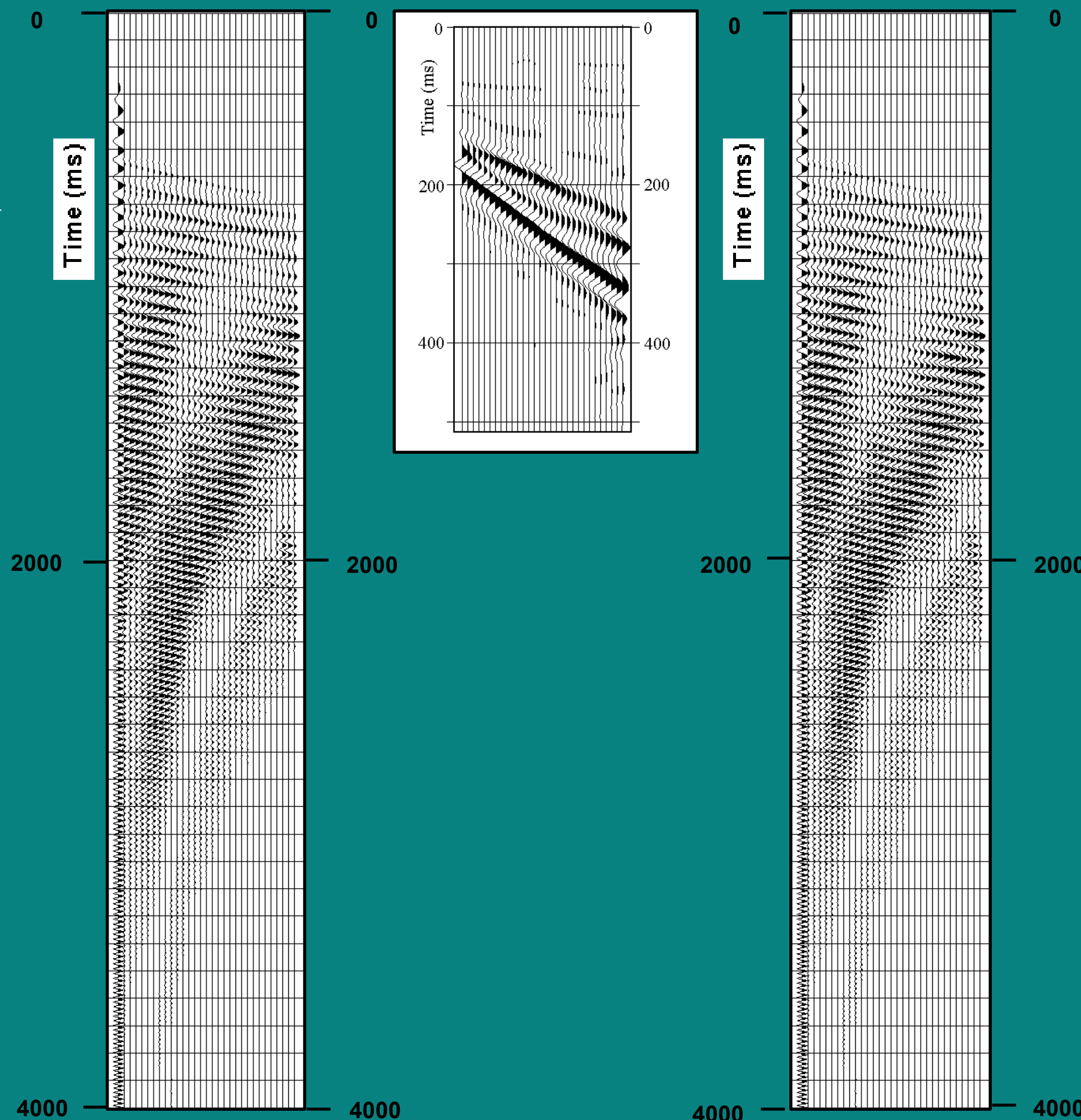
How to check near-offset effects or far-offset effects on impulsive data?

Impulsive data to swept data:

Convolution

Swept data to impulsive data (frequency decomposition):

Correlation



Summary—Rule of thumb

- The nearest source-receiver offset = 1/3 to 1/2 of the maximum investigation depth.
- Receiver spacing = the thinnest layer of the layer model.
- Receiver spread = 1 to 2 times of the maximum investigation depth.