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The Behavior of Weather in Terms of Dynamical Systems Theory and Chemical Records in Greenlandic Ice

by

Sergey Kotov

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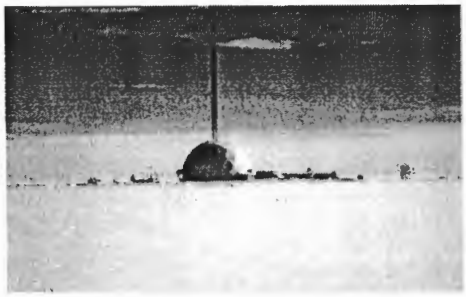
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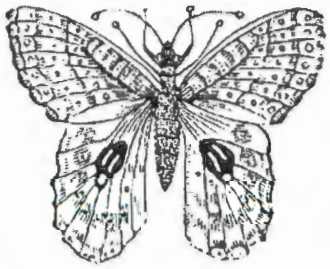
The Behavior of Weather in Terms of Dynamical Systems Theory and Chemical Records in Greenlandic ice.

Sergey Kotov

Mathematical Geology Section
Kansas Geological Survey
1930 Constant Avenue
Lawrence, Kansas 66047-3726

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"One meteorologist remarked that if the theory were correct,
one flap of a seagull's wings would be enough
to alter the course of the weather forever"
E. Lorenz

Abstract

The problem of weather prediction is the one of the most difficult problems of science. E. Lorenz has shown that changes in weather could be determined by a dynamical system with chaotic behavior. In the framework of this model the so-called "butterfly effect" is operable: one flap of a butterfly's wings would be enough to alter the course of the weather forever.

The classical examples of nonlinear dynamical systems (Henon map, Lorenz system) are shown. The delay time technique as a method for extracting new information from measured data sequences is discussed. The records of weather in Greenland ice for the last 100000 years indicate that the temperature behavior for this period of time was probably defined by a dynamical system like the Lorenz model. Thus, for prediction of weather one should use the modern technique from the theory of nonlinear prediction.

Keywords: dynamical system, attractor, fractal, dimension, ice core, wavelett

I. Introduction

Deterministic chaos is a fundamental concept well established and described in a rich literature for many fields of science [1-7]. The deterministic unpredictable behavior has became in some case a contra verse for randomness in classical understanding of unpredictability. In this case the current state of a system depends on the previous state in a rigidly determined way. In the same time, measurements made on the system do not allow the prediction of the state of the system even moderately far into the future.

The theory or deterministic chaos stems from the Theory of Dynamical Systems. So, we have to recall some necessary basic definitions.

a) Dynamical System – the system defining the behavior of researched phenomenon in time entirely (as a means of describing how one state develops into another state over the course of time);
b) Phase Space – the space of variables describing the system in which the each state of the system is described as point;
c) Trajectory – the sequence of states of a system for some period of time;

d) Attractor – the subset of phase space to which the arbitrary trajectory can be attracted with time. More strictly, an attractor is a set of states (points in the Phase Space), invariant under the dynamics, towards which neighboring states in a given basin of attraction asymptotically approach in the course of dynamic evolution.

II. Deterministic chaos

Whenever dynamical chaos is found, it is accompanied by nonlinearity. It means, that the dynamical system is supposed to be described entirely by some system of differential (difference) equations of the form 1a, 1b correspondingly.

$$\text{a)}\quad \frac{\partial X}{\partial t} = \mathbf{F}(X,\lambda)\ , \qquad \text{b)}\quad X(n+1) = \mathbf{G}(X(n))\,. \tag{1}$$

Here $X(t)=(x_1(t),\ldots,x_k(t))$ is a trajectory of dynamical system in k-dimensional phase space, **F** (**G**) – some nonlinear vector-function, determining the dynamical system. It is known from the theory of dynamical systems, that in common case there exist only three types of trajectory – equilibrium points, periodical trajectories, "others". "Others" will be of our higher interest, because they can be attracted with time to some subsets of embedded space (attractors) with exotic features. Particularly, such a set can have no such habitual characteristics as length, volume, etc. For describing such sets Hausdorff and Bezikovich [8,9] have proposed special measures, generalized the notion of dimension of measure from integer numbers (length ~ 1, area ~ 2 …) on real ones. Afterwards, sets characterized by noninteger dimension measures were called by Mandelbrot [10] fractals.

It is now common for these sets to be described by a spectrum of dimensions, D_q [11]. These are defined by

$$D_q = \frac{1}{q-1}\lim_{r\to 0}\frac{\log\sum_i p_i^q}{\log r}, \tag{2}$$

where p_i is the share of points in the i-th box of a partition of phase space consisting of boxes of size r covering the attractor.

On practice the D_2 is used most frequently and called "correlation dimension " because it can be easily estimated through the correlation integral [12]:

$$C(r) = \frac{1}{N^2}\sum_{i\neq j}\Theta(r-|X_i - X_j|)\,, \tag{3}$$

where Θ is the Heaviside function, N- quantity of observed points on trajectory. On sufficiently small and appropriate scales r (the so-called "scaling intervals"), $C(r)\propto r^{D_2}$, so we can estimate the correlation dimension D_2 as the slope of the log-log plot of $C(r)$ versus r.

Let us consider two classical examples of dynamical systems.

1. **Henon system** [13].

This is two-dimensional discrete dynamical system:

$$x_{n+1} = 1 - ax_n^2 + y_n;$$
$$y_{n+1} = bx_n. \qquad (4)$$

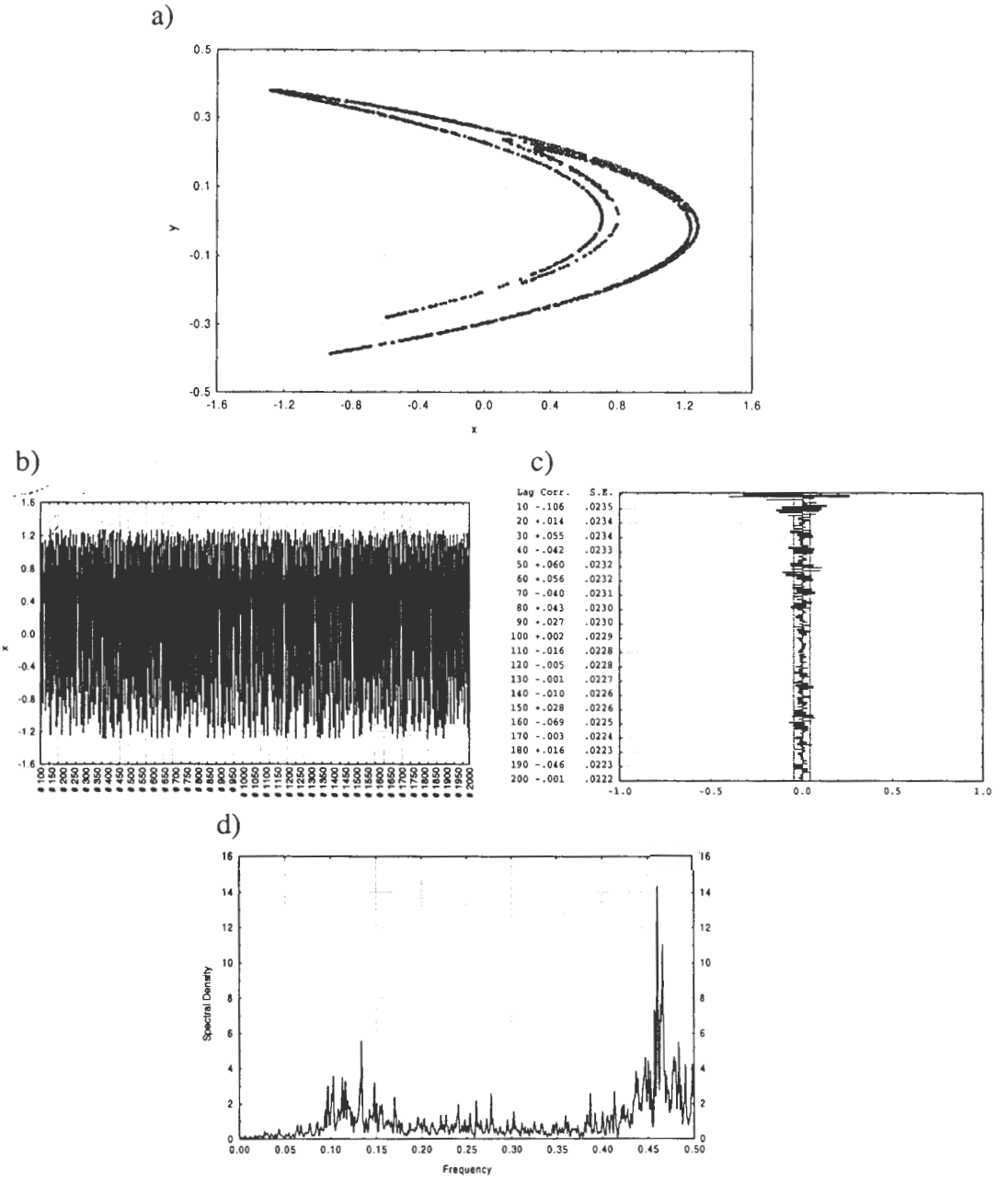


Fig. 1. Characteristics of the Henon map: a) The image of the attractor (2000 points); b) x coordinate of the map, on horizontal axis – discrete time; c) autocorrelation function for x coordinate; d) spectral density for x coordinate.

At b=0.3 and a=1.4 the system demonstrates chaotic behavior on the attractor with fractal structure. The correlation dimension of the attractor D_2 =1.21. The embedding dimension of a system (i.e. the minimal dimension of space entirely including the trajectory) is 2.

Figures 1a-1d demonstrate stochastic properties of Henon map. Like in a random process, autocorrelation function quickly tends to zero, spectral density is represented by a sum of many different frequencies.

2. Lorenz system:

$$\begin{aligned} \frac{\partial X}{\partial t} &= -\sigma X + \sigma Y; \\ \frac{\partial Y}{\partial t} &= -XY + rX - Y; \\ \frac{\partial Z}{\partial t} &= XY - bZ. \end{aligned} \qquad (5)$$

a) b)

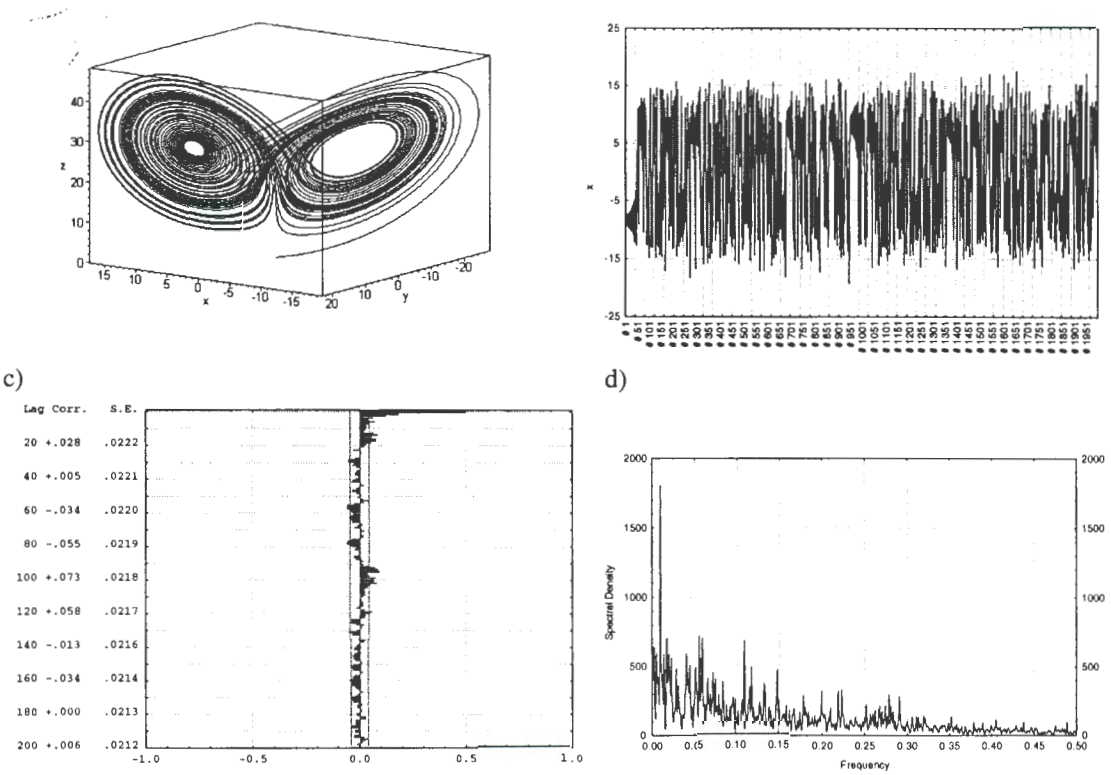


Fig. 2. Characteristics of the Lorenz system (σ=10, b=8/3, r=28): a) the trajectory; b) x coordinate of the map, on horizontal axis – discrete time (δt =0.25); c) autocorrelation function for x coordinate; d) spectral density for x coordinate.

This system was obtained by Edward Lorenz [14] by simplification and linearization of Navier-Stokes equations during his research into weather pattern. The parameter σ is a Prandtl number, the parameter r is the ratio of the Rayleigh number and critical Rayleigh number. The third parameter b is related to the horizontal wave number of the system.

The system demonstrates chaotic behavior when $\sigma=10$, $b=8/3$ and $r>24.74$. At $r=28$ any trajectory is attracted to a subset of phase space with fractal dimension. The moving of this system is wandering, i.e. the trajectory makes several right coils, then several left ones and so on... Moreover, the trajectory is very sensitive to small variations of initial parameters. It is impossible to predict how many coils will make the concrete trajectory during some period of time (see fig. 2a). This effect is well known now as the "butterfly effect" - a butterfly flapping its wings in Moscow can affect the weather in Lawrence (probably, meteorologists have supposed that seagull's flaps are less convincing).

The stochastic features of Lorenz system are similar to ones of Henon map (see fig.2a-2d). The correlation dimension of the attractor D_2 =2.05. The embedding dimension of a system is 3.

III. Delay time technique

In the foregoing paragraph it is supposed that one knows the system with exactness within the system of equations. What can we do if we have no ideas about the whole system but have only some measured characteristics from experiment?

There exists a number of algorithms for extracting new information from measured data. They are generally based on a phase space reconstruction using "time delay" procedure [12]. This procedure is based on the Takens's theorem according to which it is possible to reconstruct some properties of the attractor of the dynamical system from time sequence of one component of the entire vector [15]. It is now a common technique to estimate such characteristics of hypothetical attractor as correlation dimension and embedding dimension (in other words – the quantity of degrees of freedom of system) from single measured sequence.

Let $X(i)$ be the sequence of vectors defined as follows:

$$X(i)=(x(i),x(i+L),\ldots,x(i+L(M-1))), \qquad (6)$$

Here, L is the "lag", or the number of sampling intervals between successive components of embedding vectors, M – dimension of delay vector. The choice of L is somewhat arbitrary and there may not even be a unique good selection for this parameter [16]. According to Grassberger and Procaccia's algorithm the correlation dimension defined in formulae (3) will not change from some value M corresponding to the correlation dimension of attractor D_2 [12]. Moreover, the first integer number greater than D_2 will define the embedding dimension, i.e. the quantity of degrees of freedom excited in the system.

In our case we used the correlation integral in the form

$$C(M,r,w)=\frac{1}{N_{pairs}}\sum_{j=1}^{N}\sum_{k<j-w}\Theta(r-|X(j)-X(k)|), \qquad (7)$$

where N is the number of observations, $X(j)$ is a M –dimensional vector defined by (6), $N_{pairs} = ((N-w)^2 - N + w)/2$ is the number of pairs of points covered by the sums, Θ is Heaviside step function, $|\cdot|$ is the suitable M-norm (Euclidian norm in our case) and w is the so-called Theiler window [17]. The w allows us to exclude false correlation due to samples close it time in highly sampled flow data. In such data subsequent delay vectors can be highly correlated. Normally the choice of w is defined by the first zero of the autocorrelation function.

On the next figure one can see the results of applying the delay time technique to Henon map and Lorenz system.

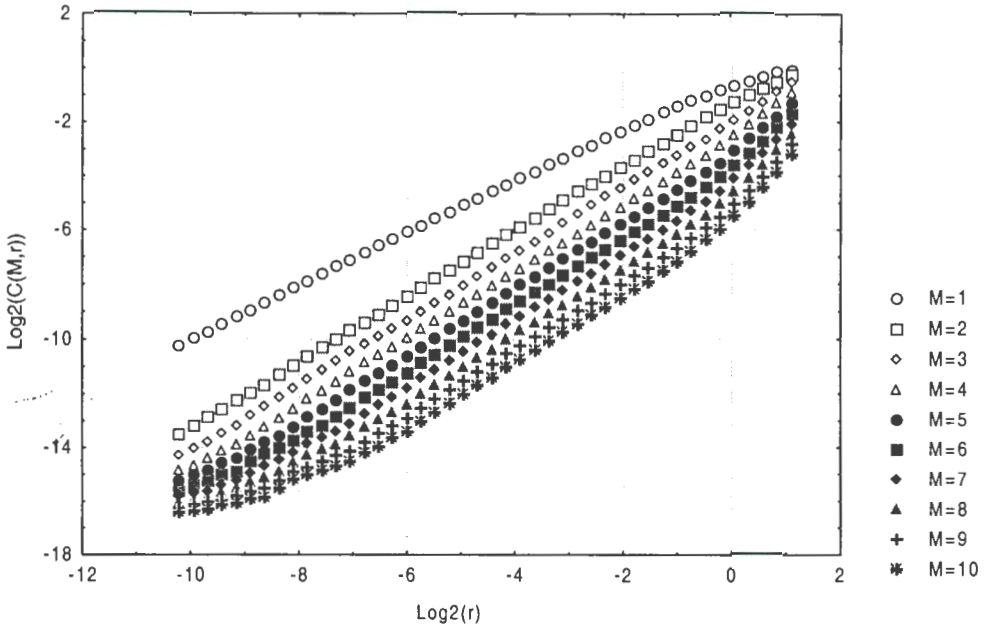


Fig. 3. Log_2 (C) vs. Log_2 (r) for the Henon map; M – dimension of delay vectors.

One can see the transition period at high scales r and scaling period (region of linear dependence) where Log_2 (r) is between –6 and –1.

The estimated correlation dimension of the Henon attractor is 1.208 (see fig. 4), the embedding dimension M=2 (N=2000). The slopes of lines within scaling intervals were estimated by the method of least squares.

For comparison the estimations for "white noise" are represented also. Note that they are not very close to integer numbers as it possible to expect from theory (M-dimensional vectors of white noise must fill up the M-dimensional space entirely). It is due to edge effects and scarcity of data [18].

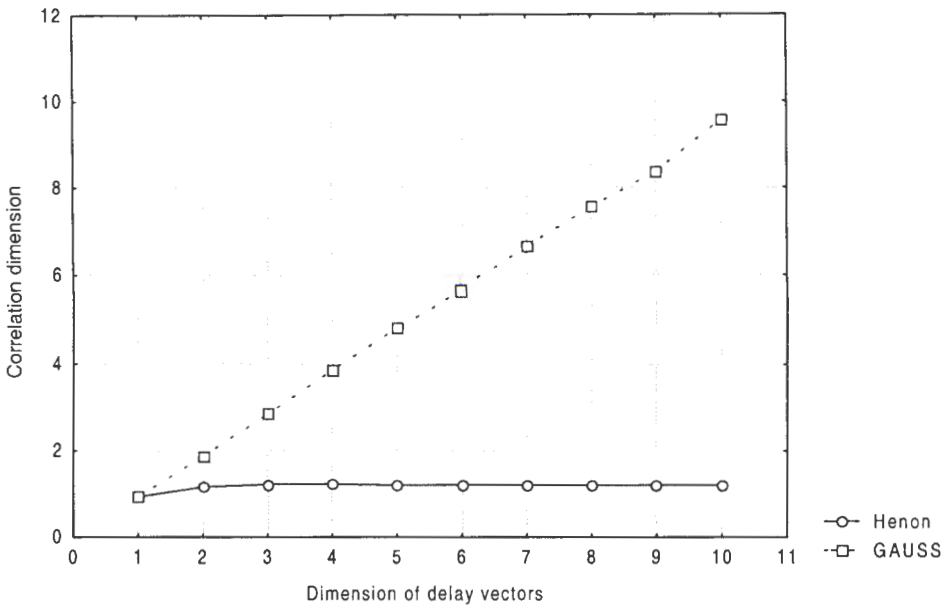


Fig. 4. Correlation dimensions for the Henon map and "white" (Gaussian) noise at different dimensions of delay time vectors.

So, using information only about one coordinate of trajectory we have reconstructed the embedding and correlation dimension of the attractor with high precision.

The next figures (fig. 5-6) are the same but for the Lorenz system.
The estimated correlation dimension of the Lorenz attractor is 2.05 and the embedding dimension M=3 (N=2000, δt =0.25, L=1). The estimations are practically coincident with literature values for entire system.

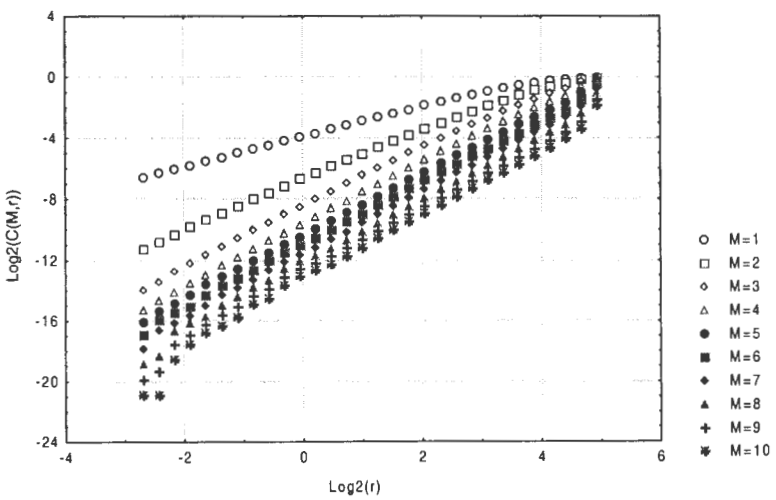


Fig. 5. Log_2 (C) vs. Log_2 (r) for the Lorenz map; *M* is the dimension of delay vectors.

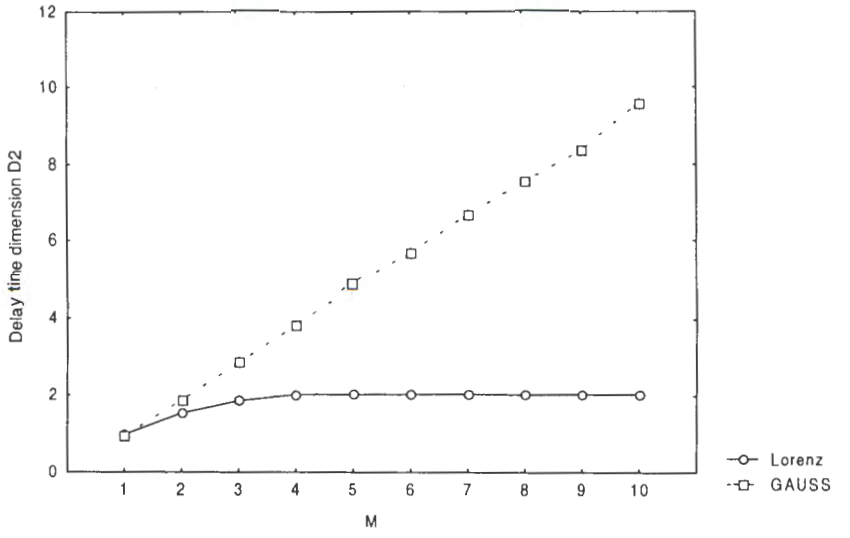


Fig. 6. Correlation dimensions for the Lorenz map and "white" (Gaussian) noise at different dimensions of delay time vectors.

Another application of the correlation integral technique is estimation of the level of noise imposed on a dynamical system [19]. Let us demonstrate it on the figure 7.

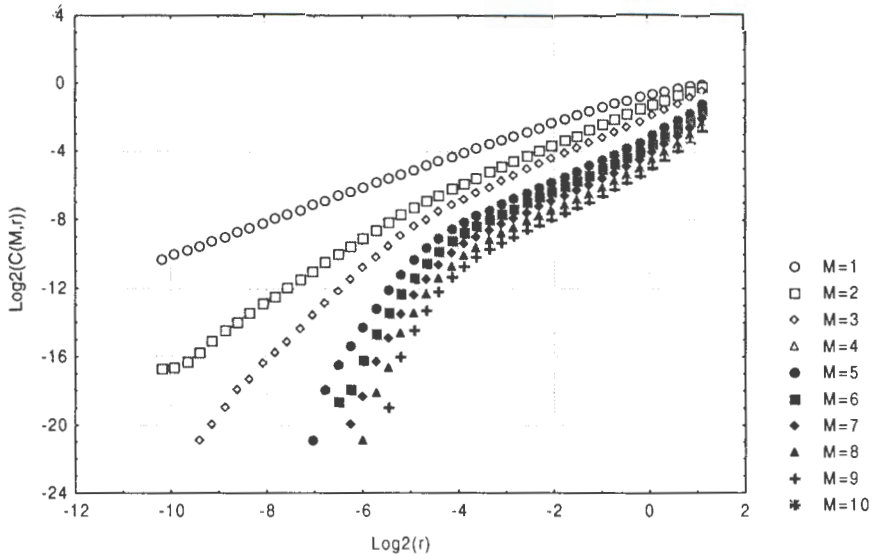


Fig. 7. Log_2 (C) vs. Log_2 (r) for Henon map, complicated by Gaussian noise; M – dimension of delay vectors.

On this figure one can see two scaling regions. The first region is observed at logarithm of r between –4 and 0 and corresponds to attractor of Henon map. The smaller scales are in the region of influence of white noise (left of the fold of curves). So, the estimation of level of noise is $2^{-4}\approx 0.063$ which is in good agreement with the reality (in this example we have imposed on the Henon map trajectory a Gaussian noise with mean 0 and dispersion 0.05).

IV. Determinate dynamics in Greenland's ice?

In this paragraph we will try to apply the delay time technique to the data from Greenlandic ice. The Greenland Ice Sheet Project Two (GISP2) as well as the Greenland Ice Core Project (GRIP) were devoted to investigation of the climatic and environmental changes of the past 250,000 years by drilling and analysing an ice core in the central part of the Greenland ice sheet [20] (recall that investigations of Lorenz was devoted to the problem of weather prediction also).

One of the characteristics used by us is the $\delta^{18}O$ ($\delta^{18}O=(^{18}O/^{16}O)/(^{18}O/^{16}O)_{smow}-1$, where the SMOW is standard mean of ocean water). In polar glacier ice, δ is mainly determined by its temperature of formation [21].

The distribution of $\delta^{18}O$ for the last 1kyr is shown on the fig. 8a (1 year intervals, back to 1133 years B.P.).

In fig.8b-8c one can see the weak modulations of auto correlation function and multiple peeks of spectral density. In the case of stationary processes it means the presence of multi-periodicity in the flow of data. In a common case the wavelet transform seems to be more appropriate [22]. The literature about the wavelet transform is huge. In this article it is important for us that this transform allows us to get the information not only about periodicity in data but also about variation of the periodicity in time.

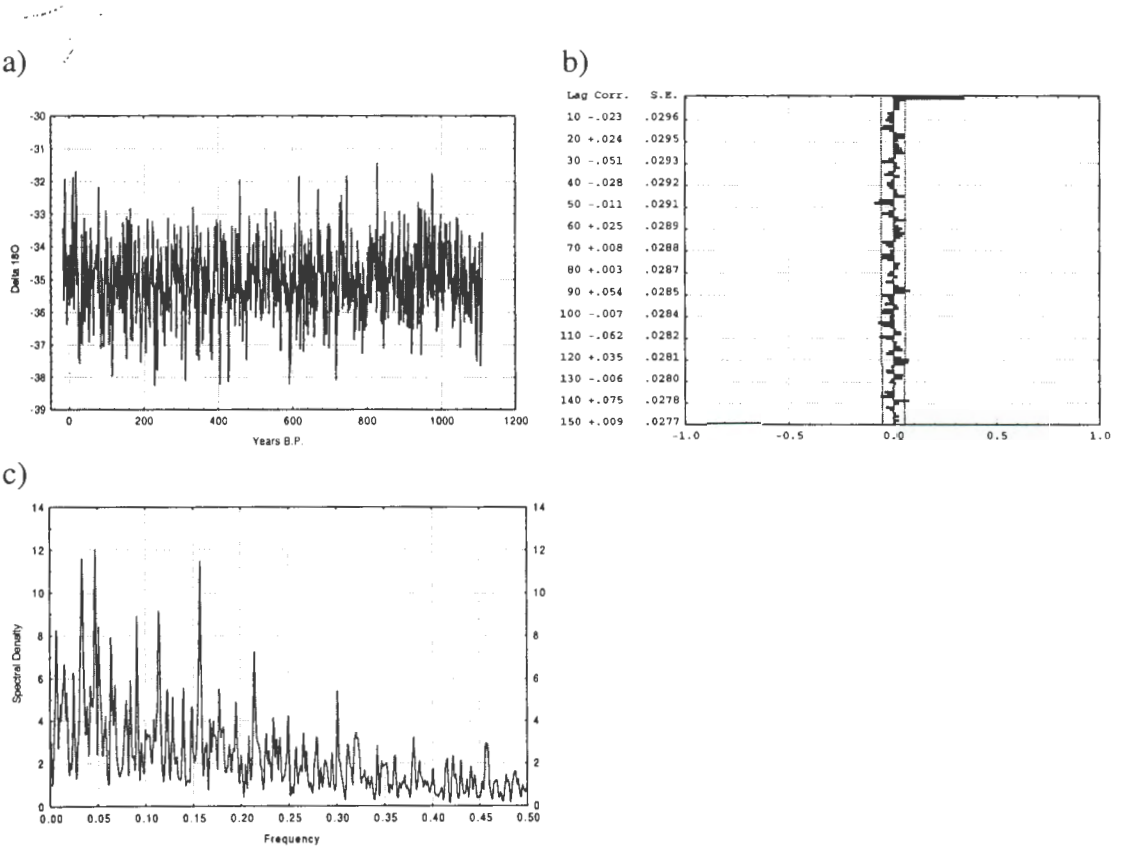


Fig. 8. a) The 1-year average record of $\delta^{18}O$ (GISP2); c) autocorrelation function; d) spectral density.

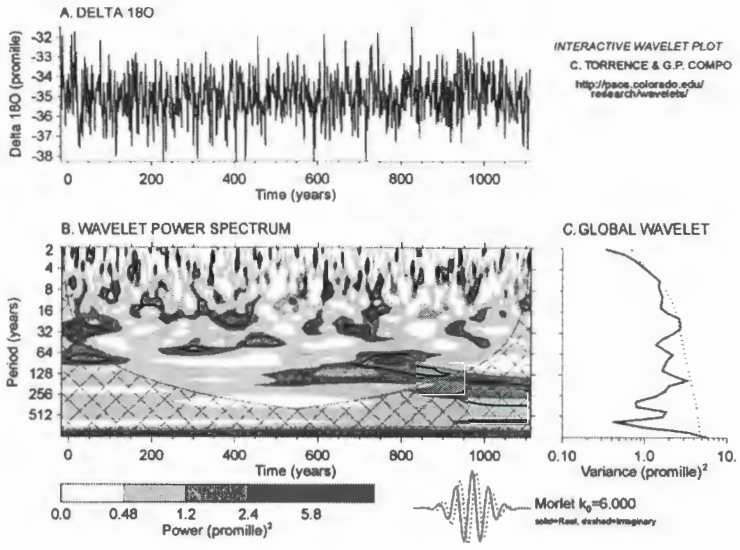


Fig. 9: (a) The 1-year average record of $\delta^{18}O$ (GISP2); (b) The wavelet power spectrum. The contour levels are chosen so that 75%, 50%, 25%, and 5% of the wavelet power is above each level, respectively. Cross-hatched region is the cone of influence, where zero padding has reduced the variance. Black contour is the 10% significance level, using the global wavelet spectrum as the background. (c) The global wavelet power spectrum (black line). The dashed line is the 10% significance level for the global wavelet spectrum, using a red-noise (autoregressive lag1=0.72) background spectrum.

Figure 9 demonstrates the sporadic distribution of periodicity in time. In some way it looks like the Lorenz system, in which the periodical motions with different frequencies are intermitted randomly. To compare see the wavelet transform for the Lorenz system on fig. 10.

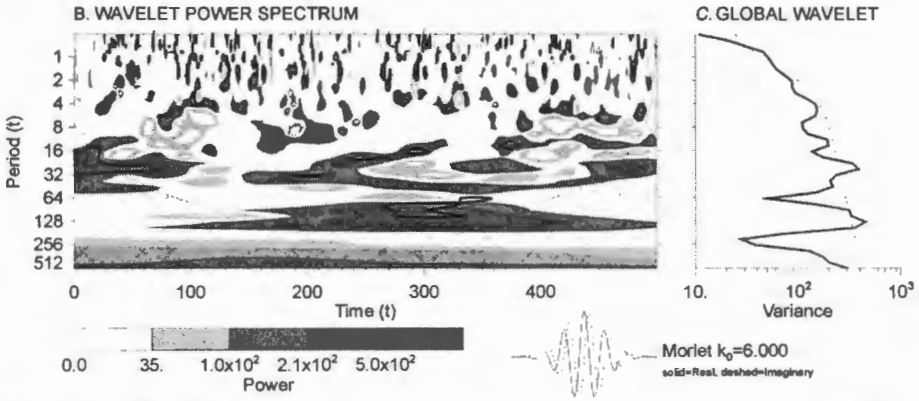


Fig. 10: (b) The wavelet power spectrum for Lorenz system. (c) The global wavelet power spectrum (black line).

On the figure 11 the results of delay time method are shown (N=1130, w=7).

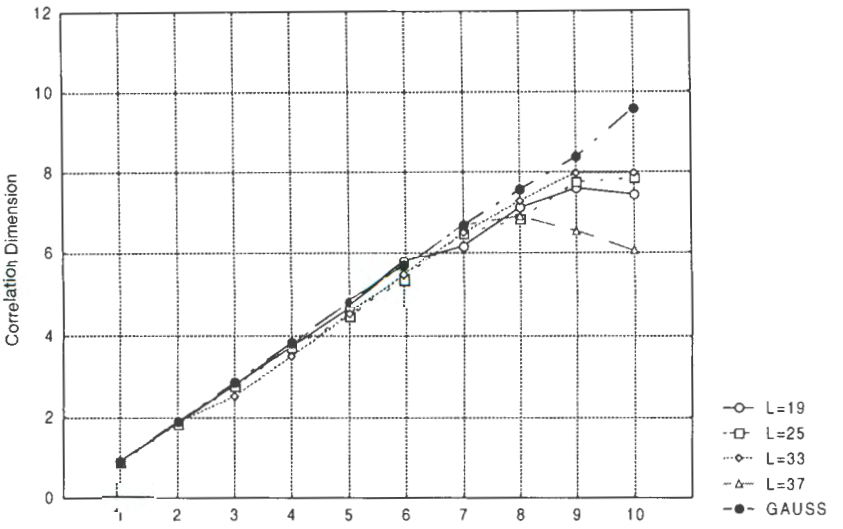


Fig. 11. Correlation dimensions for the 1-year average record of $\delta^{18}O$ and "white" (Gaussian) noise at different dimensions of delay time vectors.

From this graphic one can conclude that dynamics of behavior of temperature over the last 1kyr belongs to the class of relatively low-dimensional chaos, as in the Lorenz system, but probably the number of degrees of freedom of the system is not more then 8.

The next example – the 20-year average record of $\delta^{18}O$ values from GISP2 for last 10kyr (N=500, w=11).

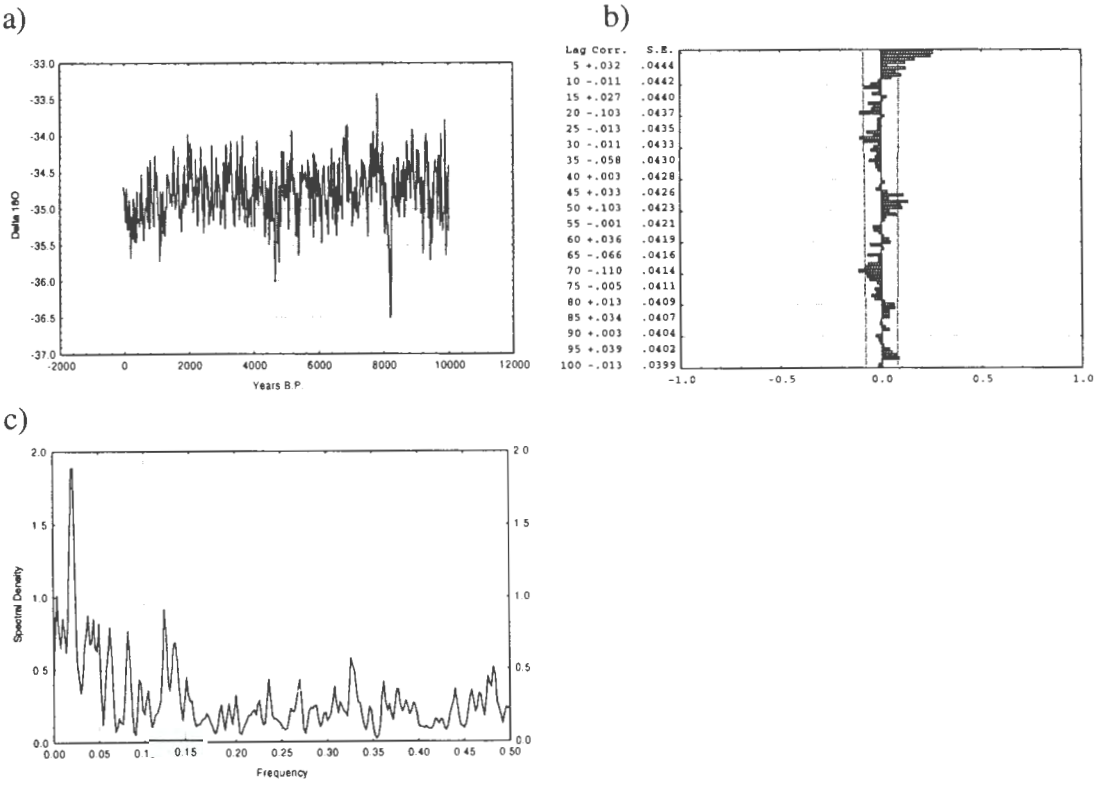


Fig. 12. a) The 20-year average record of $\delta^{18}O$ (GISP2); c) autocorrelation function; d) spectral density.

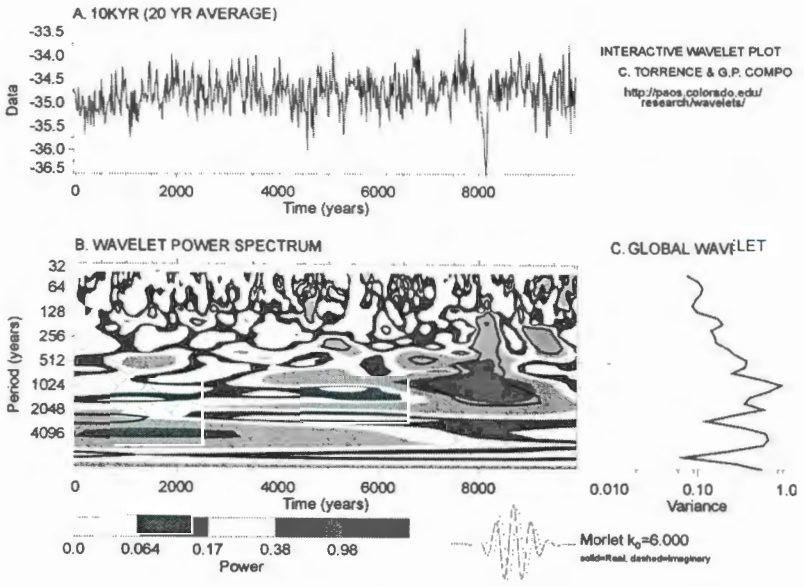


Fig. 13: (a) The 20-year average record of $\delta^{18}O$ (GISP2); (b) The wavelet power spectrum. (c) The global wavelet power spectrum (black line).

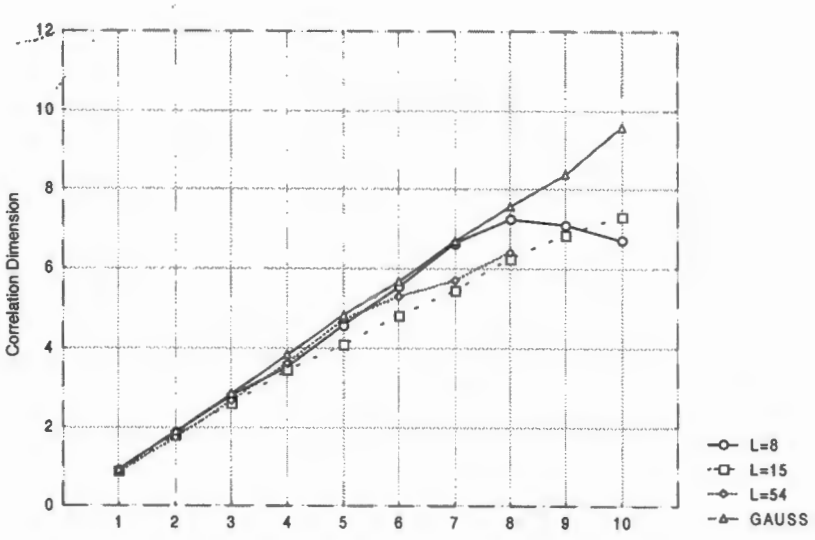


Fig. 14. Correlation dimensions for the 20-year average record of $\delta^{18}O$ (GISP2).

The graphics from figures 12-14 allow us to suppose the dynamics of the last 10 kyr to be the same as the dynamics of the last 1 kyr. But one must understand that this conclusion is tentative because of the scarcity of data (only 500 values). However it has been shown that it is possible to get an acceptable estimate of the attractor's dimension with as few as 500 data points [23].

One more example is data from the period 10-100 kyr B.P. The dynamics of this period strongly differ from dynamics in Holocene (0-10 kyr B.P., see fig. 15).

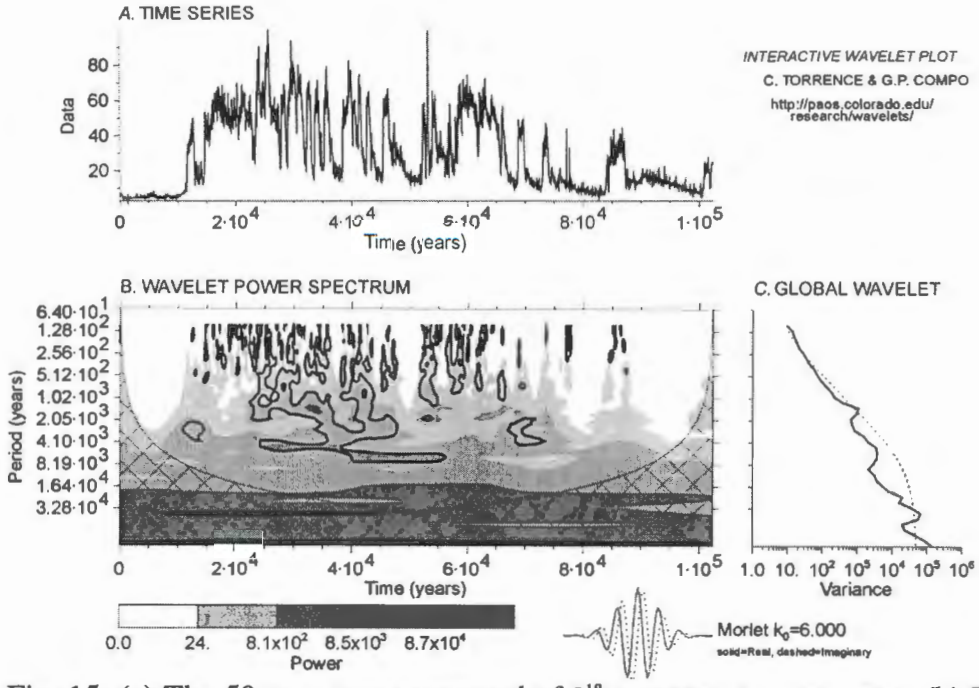


Fig. 15: (a) The 50-year average record of $\delta^{18}O$ (GISP2), 0 –110kyr B.P.; (b) The wavelet power spectrum. (c) The global wavelet power spectrum (black line).

The next figure is the results of delay time approach applied to the 50-year average record of Na values from GISP2 for period 10-100 kyr B.P. (N=2000, w=360).

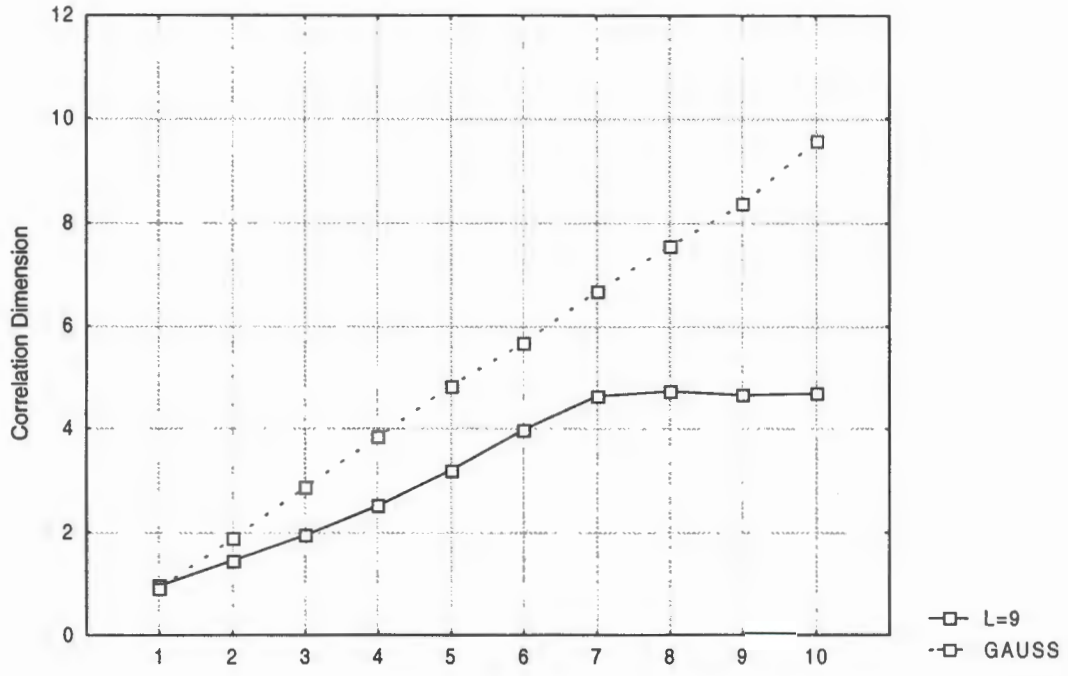


Fig. 16. Correlation dimensions for the 50-year average record of Na and "white" (Gaussian) noise at different dimensions of delay time vectors.

It follows from the graphic that the embedding dimension of the dynamical system is M=5 and the correlation dimension D approximately equals 4.7.

V. Conclusions

1. The dynamics of weather conditions for the last 10 kyr seem to exhibit relatively low dimensional behavior like the Lorenz model. The quantity of **degrees of freedom** for this period of time is **8**. The estimation of **correlation dimension** of the attractor (**$7<D_2\leq 8$**) does not contradict the idea of the presence of determinate chaos in the behavior of temperature.

   This conclusion must be excepted with strong precaution due to the scarcity of data.

2. Characteristics of weather before Holocene (10-110 kyr B.P.) strongly differ from the ones of Holocene. There is a high probability that the weather for this period was determined by a **dynamical system with a strange attractor and chaotic behavior**. The **embedding dimension** of system is **5**, the **correlation dimension of the attractor is 4.7 ± 0.05.**

3. In terms of dynamical systems the Holocene weather was **more complex** in comparison with the foregoing period (**8** degrees of freedom against **5**). But this does not mean that the **dynamics** of weather before Holocene was more predictable. In the framework of deterministic chaos in both cases the **"butterfly effect"** might have been equitable: **whether a butterfly in America flapped its wings three times or only twice could alter totally the weather in Greenland some days hence.**

Note that these conclusions do not deny the possibility of prediction of weather. But one has to take into account the possible presence of instability in nonlinear dynamical systems ("butterfly effect") and use an adequate technique from the theory of nonlinear prediction. The discussion of nonlinear forecasting is beyond the scope of this report and is planned further.

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