

**KANSAS GEOLOGICAL SURVEY
OPEN-FILE REPORT 97-59**

**THEORETICAL ANALYSIS OF IMPACT OF INCOMPLETE
RECOVERY OF SLUG TESTS**

by

J. J. Butler, Jr.

Disclaimer

The Kansas Geological Survey does not guarantee this document to be free from errors or inaccuracies and disclaims any responsibility or liability for interpretations based on data used in the production of this document or decisions based thereon. This report is intended to make results of research available at the earliest possible date, but is not intended to constitute final or formal publications.

Kansas Geological Survey
1930 Constant Avenue
University of Kansas
Lawrence, KS 66047-3726

**THEORETICAL ANALYSIS OF IMPACT OF INCOMPLETE
RECOVERY ON SLUG TESTS**

James J. Butler, Jr.
Kansas Geological Survey
1930 Constant Avenue
Lawrence, Ks. 66047

December, 1997

KGS OPEN FILE REPORT #97-59

ABSTRACT

Recently, a series of practical guidelines for the design, performance, and analysis of slug tests were proposed. One of the most important of these guidelines is that three or more tests should be performed at each well. The recommendation to perform multiple slug tests at a well leads to the question of what degree of head recovery is required prior to initiation of the next test. This question is examined in a simulation study performed with mathematical models of slug tests in fully and partially penetrating wells. Regardless of whether quasi-steady-state or fully transient approaches are used for the analysis of slug-test responses, incomplete recovery will have virtually no impact on estimates of hydraulic conductivity as long as the residual deviation is a small fraction of the initial displacement. The largest errors in conductivity estimates will occur when quasi-steady-state approaches are used to analyze tests performed in well-formation configurations with large dimensionless storage parameters. However, even when the residual deviation is relatively large (20% of the initial displacement), the error introduced by incomplete recovery will be acceptable for most practical applications.

INTRODUCTION

Butler et al. (1996) and Butler (1997) propose a series of practical guidelines for the design, performance, and analysis of slug tests. These guidelines were the product of an extensive program of field and theoretical research carried out by the Kansas Geological Survey. One of the most important of these guidelines is the recommendation that three or more slug tests, preferably closely spaced in time, be performed at a given well. The comparison of normalized responses from slug tests performed at the same well using two or more different initial displacements (H_0) is an important step in verifying the validity of the theory upon which conventional analysis methods are based. As Butler (1997) emphasizes, there is no point in analyzing response data with a particular theoretical model if there is a significant difference between the underlying assumptions of that model and actual conditions at a site.

The need to perform multiple slug tests at a well naturally leads to the question of what degree of head recovery is required prior to repeating a test at that well. Several of the mathematical models on which commonly used methods for the analysis of slug-test response data are based assume that formation heads are at equilibrium conditions prior to test initiation, so a conservative approach would be to wait until static conditions have been reestablished before initiating the next test. However, since the process of head recovery during a slug test can be approximately represented as an exponential decay, the time

required for recovery of the last few percent of the original deviation from static can be quite long. Although this does not present much of a problem in units of high hydraulic conductivity, it can greatly lengthen the duration of a test program in less permeable formations. Thus, an issue of considerable practical importance is that of how much error is introduced into the hydraulic-conductivity estimate when a slug test is initiated prior to complete recovery from a previous test.

Apparently, this issue of the impact of incomplete recovery has not been fully assessed in either field or theoretical studies. Shapiro and Greene (1995) describe a method for the analysis of pneumatic slug tests initiated before complete recovery from the pressurization of the air column. Their focus, however, is on the development of a new set of type curves, so they do not explicitly address the impact of incomplete recovery on parameter estimates obtained with conventional analysis methods. Clearly, the ramifications of initiating a slug test prior to complete recovery from the previous test need further investigation. That is the purpose of the study described in this report.

This report will present the results of a simulation investigation of the ramifications of incomplete recovery for slug tests performed in fully and partially penetrating wells in confined formations (see Figure 1). Semianalytical solutions to models of slug tests in fully and partially penetrating wells will be used to simulate a series of hypothetical slug tests. Effects of incomplete recovery will be incorporated using the principle of

superposition. Parameter estimates will be obtained from the simulated responses using conventional methods for the analysis of response data. The error introduced by incomplete recovery will be quantified by comparing estimates obtained from tests initiated prior to complete recovery with those from tests initiated after static conditions had been reestablished in the formation.

METHODOLOGY

Slug-test simulations were performed for this work using semianalytical solutions to mathematical models of slug tests in fully and partially penetrating wells in confined formations. Tests in fully penetrating wells were simulated using the model of Cooper et al. (1967), while slug tests in partially penetrating wells were simulated with the KGS model (Hyder et al., 1994). In both cases, the simulations were performed using the computer code of Liu and Butler (1995).

The Cooper et al. and KGS models assume that the formation is at static conditions prior to test initiation. Since both are linear mathematical models, conditions of incomplete recovery can be simulated using the principle of superposition, which is frequently invoked in well hydraulics applications for modeling the recovery period following pumping, the effects of pumping at nearby wells, etc. (Kruseman and de Ridder, 1990). Superposition is applied in the following manner to simulate slug tests initiated prior to complete recovery:

- 1) A slug test initiated at static conditions is simulated

with the Cooper et al. or KGS model;

2) The degree of incomplete recovery, i.e. the normalized deviation from static immediately prior to initiation of the next test (henceforth designated as the residual deviation), is selected;

3) A record of the slug-test responses simulated in 1) is truncated by deleting all normalized heads larger than the residual deviation selected in 2);

4) The times in the truncated data record are reinitialized to a zero time equal to the time of the residual deviation;

5) The truncated and reinitialized data record is added to the responses of the slug test simulated in 1);

6) The new record is normalized by the summation of the original deviation from static and the residual deviation.

A short example can help clarify this procedure. The solid line in Figure 2A is a simulated record of normalized head versus logarithm of time for a slug test initiated at static conditions. Assume that the next test is going to be initiated at a residual deviation of 0.20 (Point A on Figure 2A). The dashed line in Figure 2A is the record produced by deleting all normalized heads for times less than that of point A and reinitializing the data record to a zero time equal to that of point A. Figure 2B displays the two curves of Figure 2A and their superposed and normalized superposed counterparts. In this case, the superposed record (top plot in Figure 2B) is normalized by 1.20 to produce the solid line with

circles, which is a response record that incorporates the effects of incomplete recovery. This superposed and normalized record is utilized for the parameter estimation phase of this work. Note that for the particular example shown in Figure 2B, the impact of incomplete recovery appears quite small.

Parameter estimates were obtained from normalized superposed records of slug-test responses using both quasi-steady-state and fully transient analysis approaches. The Hvorslev (1951) method was the quasi-steady-state approach used for analysis of tests in both fully and partially penetrating wells. In this work, the hydraulic conductivity estimate was obtained with the Hvorslev method using T_0 , which is designated as the time lag by Hvorslev (1951) and is equal to a normalized head of 0.37. Although Butler (1997) recommends that conductivity estimates be obtained with the Hvorslev method utilizing normalized heads between 0.15 and 0.25, use of T_0 was considered a reasonable approach for this work because of the emphasis on relative changes in conductivity estimates between tests rather than on the estimates themselves. Note that results obtained for the Hvorslev method are also considered valid for the Bouwer and Rice (1976) and Dagan (1978) methods, both of which are based on a quasi-steady-state representation of the slug-induced flow system.

The Cooper et al. (1967) method was the fully transient approach used for the analysis of tests in fully penetrating wells, while the KGS model (Hyder et al., 1994) was used for tests in partially penetrating wells. In both cases, automated analogues of

standard type curve matching methods were utilized to obtain estimates of both hydraulic conductivity and specific storage.

The impact of incomplete recovery is quantified using the ratio of the parameter estimate obtained from a test initiated prior to complete recovery over the estimate obtained from a test initiated at static conditions. The significance of the magnitude of this ratio will be application dependent, so the figures presented in the following section can be utilized to assess if the error is acceptable for a particular application.

RESULTS

A series of slug-test simulations were performed to assess the error introduced by incomplete recovery for the range of conditions expected in field applications. Tests in both fully and partially penetrating wells were considered. The effects of incomplete recovery on tests in partially penetrating wells were assessed for wells of both small (15) and large (150) aspect ratios (b/r_w , where b is screen length and r_w is screen radius). For each well-formation configuration, tests were simulated using four different residual deviations, ranging from 0.20 to 0.01. Three different dimensionless storage coefficients ($\alpha = (r_w^2 S_s b) / (r_c^2)$, where b is formation thickness for fully penetrating wells and screen interval for partially penetrating wells, and r_c is casing radius) were examined for each well type.

Figure 3 present the results obtained using the Hvorslev

method for the analysis of tests in fully penetrating wells. These results indicate that the impact of incomplete recovery is a function of the dimensionless storage parameter (α). Figure 3 shows that, for moderate to small values of α , the error introduced by incomplete recovery will be under 10% when the residual deviation is less than 20% of H_0 . For large α values, the residual deviation should be less than 5% of H_0 to keep the error introduced by incomplete recovery under 10%. Figures 4 and 5 show that the impact of incomplete recovery is diminished in partially penetrating wells. These results indicate that the effect of incomplete recovery is quite small for wells of small to moderate aspect ratios, and thus can be ignored as long as the residual deviation is not much larger than 0.20. In all cases, the error introduced by incomplete recovery leads to an underestimation of hydraulic conductivity.

Figures 3-5 present results found for analysis methods based on a quasi-steady-state representation of the slug-induced flow system. However, many slug tests are analyzed using methods based on fully transient models of the flow system. Butler (1997) points out that methods based on fully transient models are often preferred because they can be used as screening tools for identifying the influence of vertical flow and well skins. Figures 6A and 6B presents results obtained using the method of Cooper et al. (1967) for the analysis of tests in fully penetrating wells. Figure 6A shows that the impact of incomplete recovery on hydraulic conductivity estimates is again dependent on α and leads to a

relatively small (in terms of practical applications) underestimation of hydraulic conductivity. Although Figure 6B shows that test initiation prior to complete recovery can lead to a much larger error in the estimate of specific storage, this error must still be considered relatively small when viewed in the context of practical applications. As Cooper et al. (1967) and others have pointed out, the similarity in the shape of the α type curves makes it difficult to obtain reliable estimates of specific storage from single-well slug tests. Note that this similarity in the shape of the α type curves is primarily responsible for the nonmonotonic relationships displayed in Figures 6A and 6B.

Use of the KGS model for the analysis of tests in partially penetrating wells produced results analogous to those found with the Hvorslev method, i.e. the underestimation of hydraulic conductivity was less than that for a test in a fully penetrating well. The circles on Figure 6A correspond to estimates obtained with the KGS model for tests in partially penetrating wells of small aspect ratios. Points are not plotted for the specific storage estimates because of the insensitivity of tests in partially penetrating wells to specific storage, which is particularly pronounced in well-formation configurations with low to moderate values of α (Butler, 1997).

Since the Cooper et al. and KGS models can be utilized as screening tools to assess the viability of certain assumptions regarding the slug-induced flow system, it is of interest to examine if these methods can provide any clues that incomplete

recovery is affecting test data. Figure 7A presents simulated normalized head versus logarithm of time plots for two slug tests performed in a well-formation configuration with a large dimensionless storage parameter. As in all the cases examined here, the primary effect of incomplete recovery is to translate the response plot to larger times, the shape of the plot is changed relatively little. Figure 7B shows that an excellent match is obtained between a theoretical type curve and the simulated responses when the method of Cooper et al. is used to analyze the test initiated at a residual deviation of 0.20. The small deviation seen between the best-fit type curve and the simulated responses at large times, which was found in all the cases examined here, will be very difficult to discern in practical applications. Thus, the effect of incomplete recovery will be virtually impossible to recognize from the fit of Cooper et al. or KGS model type curves to the test data. Note that this particular example was designed to accentuate the impact of incomplete recovery (i.e. a large α was used). Smaller deviations will be found for tests performed in partially penetrating wells and in well-formation configurations with small to moderate values of α .

Since the impact of incomplete recovery will be virtually impossible to recognize from the type curve fit, a program of pre- and post-test measurements must be used to identify conditions when incomplete recovery may be affecting response data. However, as long as the residual deviation is relatively small (0.20 or less), the effect of incomplete recovery can be neglected for most

practical applications. Although these results were obtained for tests performed in confined formations, they are equally applicable for tests in unconfined formations when a linear mathematical model is a reasonable representation of the governing physics. Note also that the results reported here were obtained assuming that repeat slug tests were performed using the same H_0 . If H_0 changes between tests, the residual deviation should be normalized relative to the H_0 for the second test.

CONCLUSIONS

The primary conclusion of this investigation is that incomplete recovery will have a very limited impact on hydraulic-conductivity estimates obtained from slug tests. As long as the residual deviation is less than 20% of the initial displacement (H_0), the small underestimation of hydraulic conductivity produced by incomplete recovery can be ignored for virtually all practical applications. Although the error in the specific storage estimate can be much larger, it is still relatively small when considered in the context of the reliability of specific storage estimates obtained from single-well slug tests.

REFERENCES

- Bouwer, H., and R.C. Rice. 1976. A slug test for determining hydraulic conductivity of unconfined aquifers with completely or partially penetrating wells. *Water Resour. Res.*, v. 12, no. 3, pp. 423-428.
- Butler, J.J., Jr. 1997. *The Design, Performance, and Analysis of Slug Tests*. Lewis Press, Boca Raton, 252 pp.
- Butler, J.J., Jr., C.D. McElwee, and W.Z. Liu. 1996. Improving the quality of parameter estimates obtained from slug tests. *Ground Water*. v. 34, no. 3, pp. 480-490.
- Cooper, H.H., Bredehoeft, J.D., and I.S. Papadopoulos. 1967. Response of a finite-diameter well to an instantaneous charge of water, *Water Resour. Res.* v. 3, no. 1, pp. 263-269.
- Dagan, G. 1978. A note on packer, slug, and recovery tests in unconfined aquifers., *Water Resour. Res.*, v. 14, no. 5, pp. 929-934.
- Hvorslev, M.J. 1951. Time lag and soil permeability in ground-water observations. Bull no. 36. *Waterways Exper. Sta., Corps of Engrs., U.S. Army*, 50 pp.
- Hyder, Z., J.J. Butler, Jr., C.D. McElwee, and W.Z. Liu. 1994. Slug tests in partially penetrating wells. *Water Resour. Res.* v. 30, no. 11, pp. 2945-2957.
- Kruseman, G.P. and N.A. de Ridder. 1990. *Analysis and Evaluation of Pumping Test Data - ILRI publication 47*. ILRI, The Netherlands. 377 pp.

Liu, W.Z., and J.J. Butler, Jr. 1995. The KGS model for slug tests in partially penetrating wells (version 3.0). Kansas Geological Survey Computer Series Rept. 95-1.

Shapiro, A.M., and E.A. Greene. 1995. Interpretation of prematurely terminated air-pressurized slug tests. Ground Water. v. 33, no. 4, pp. 539-546.

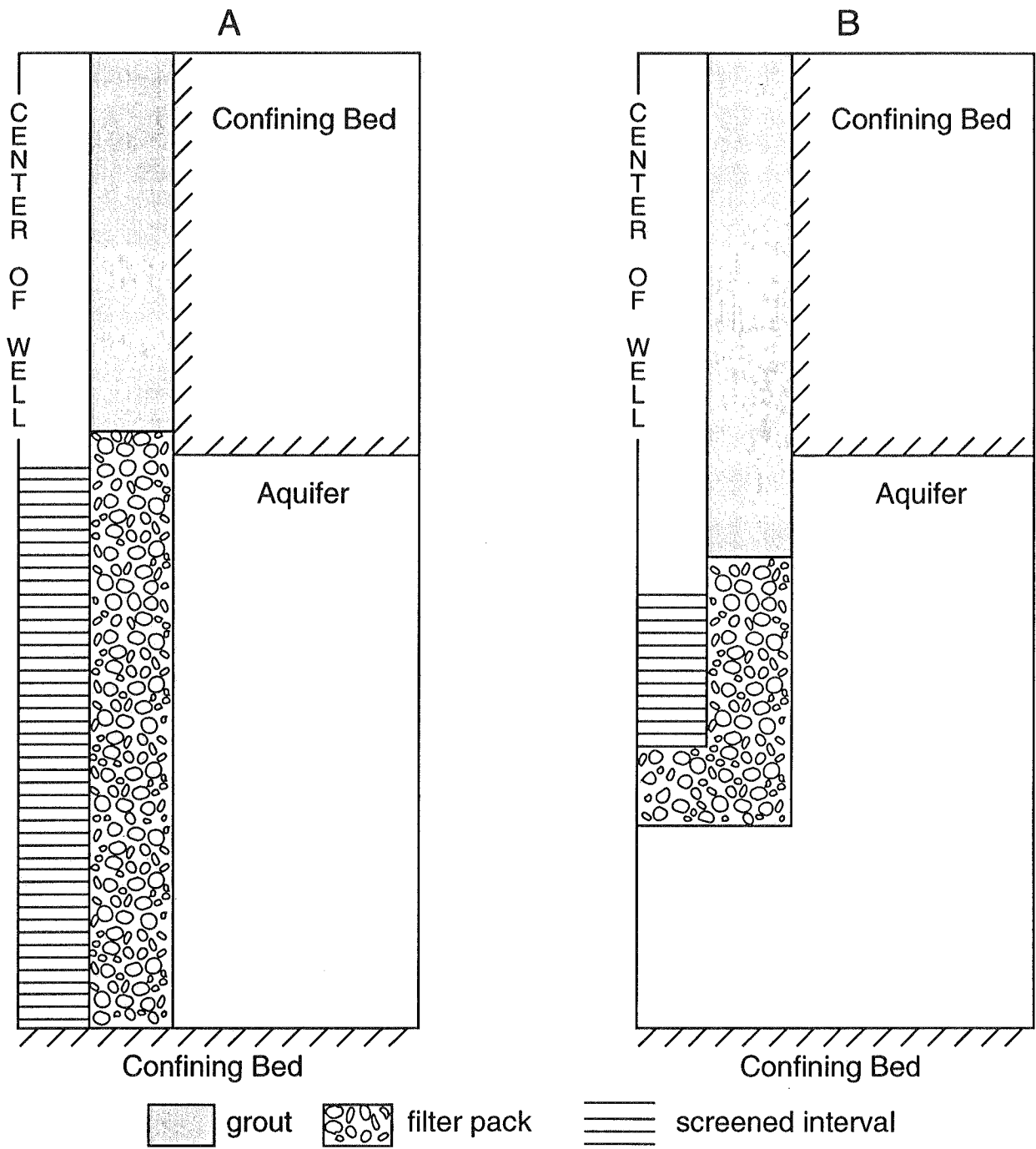


Figure 1 - Hypothetical cross sections depicting a fully penetrating (A) and partially penetrating (B) well in a confined aquifer (figures not to scale).

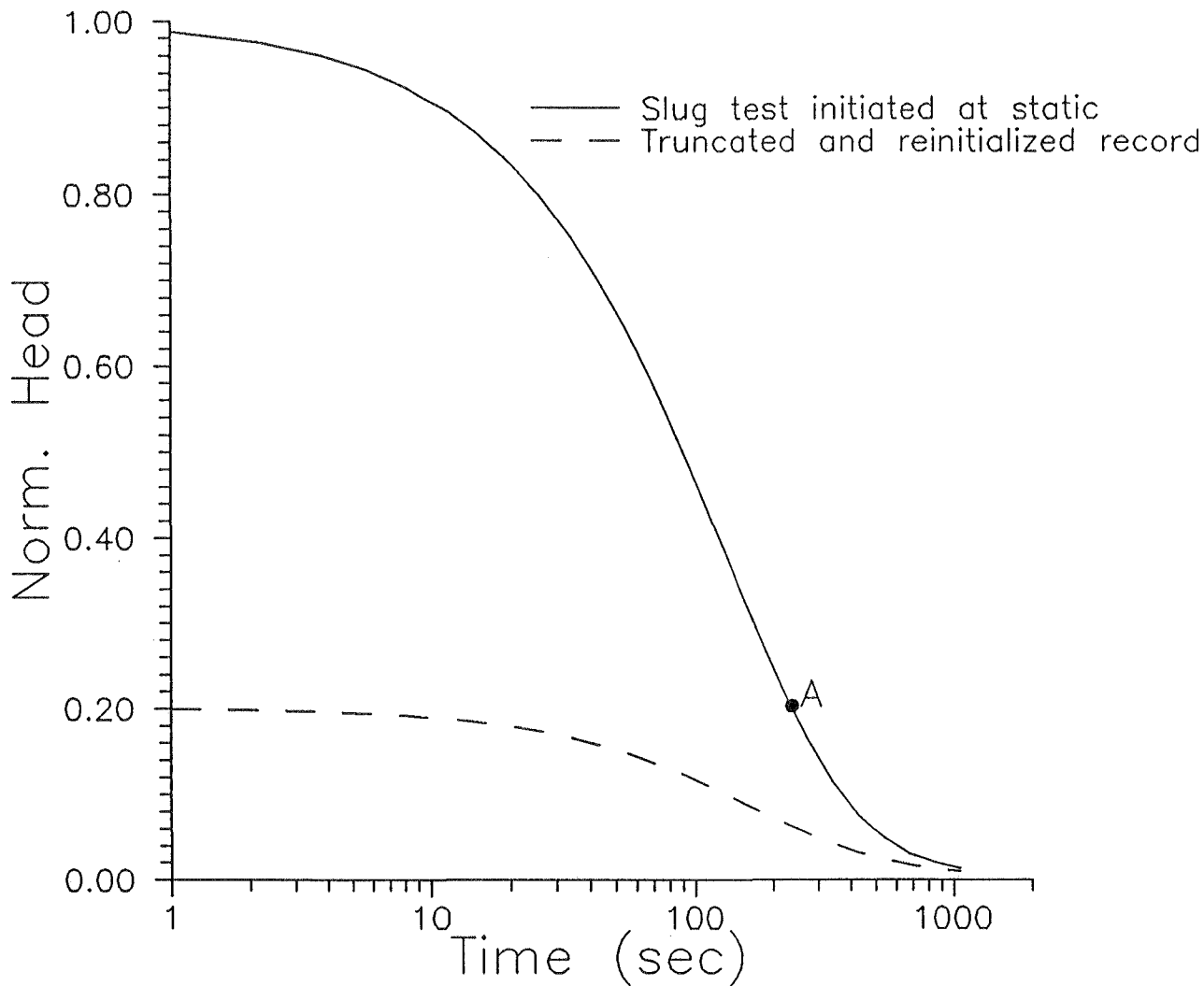


Figure 2A - Simulated normalized head ($H(t)/H_0$, where $H(t)$ is deviation from static and H_0 is magnitude of the initial displacement) versus log time plot of a slug test initiated at static conditions and the truncated record corresponding to a residual deviation of 0.20 (residual deviation designated by A).

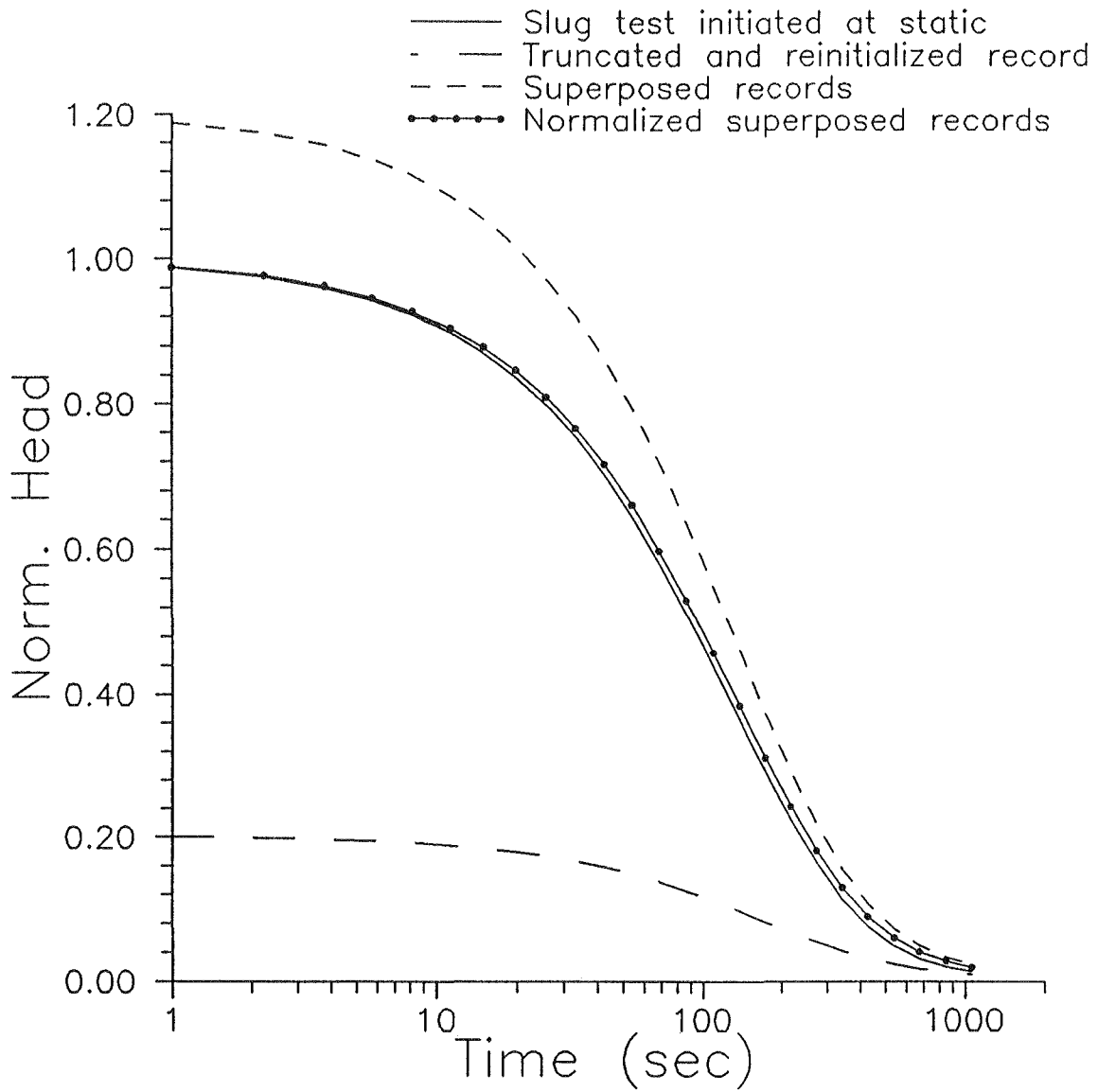


Figure 2B - Simulated normalized head versus log time plot of a slug test initiated at static conditions and a slug test initiated at a residual deviation of 0.20 (truncated and superposed records included for comparative purposes).

Impact of Incomplete Recovery Fully Penetrating Well Hvorslev Analysis

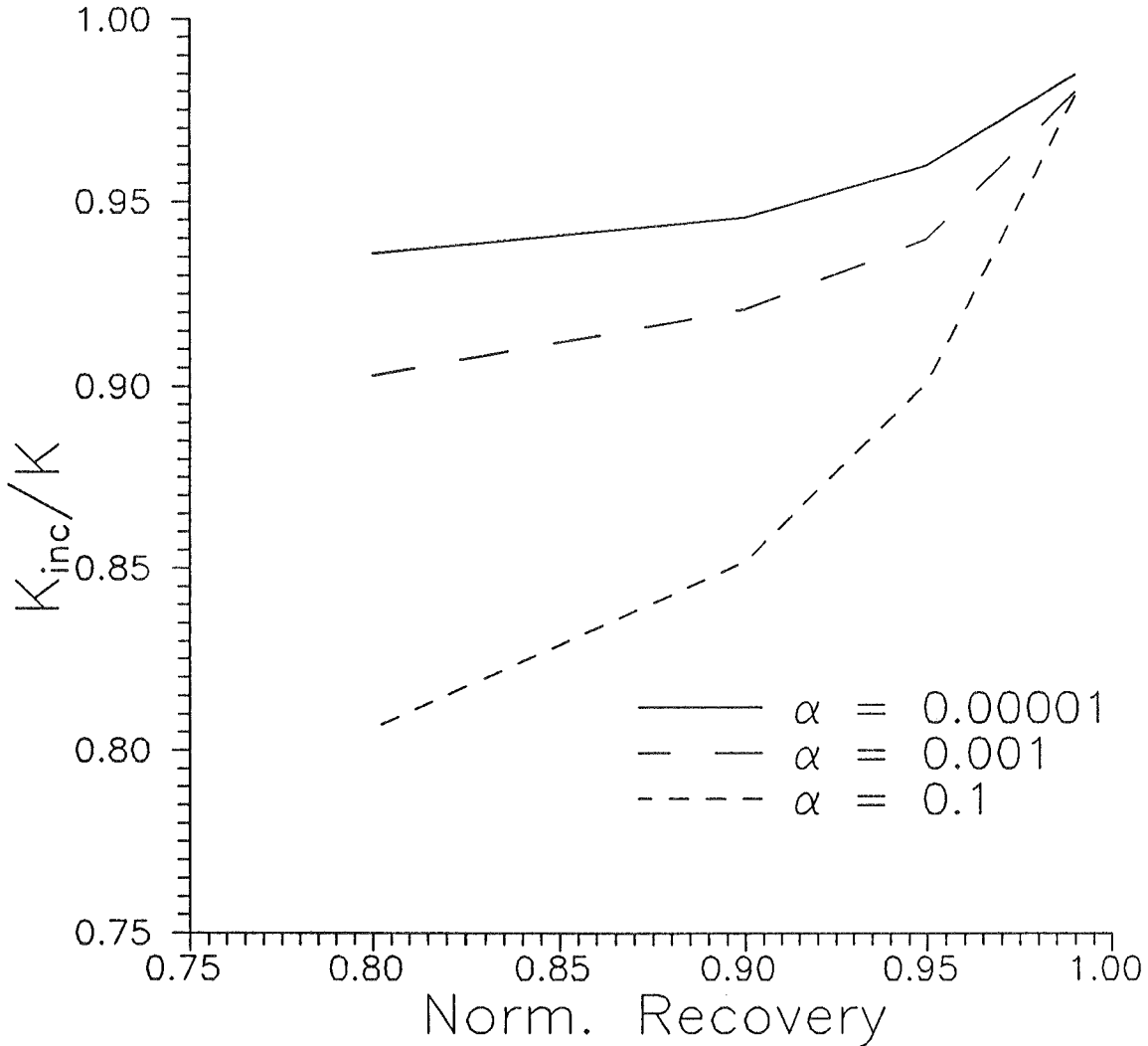


Figure 3 - Plot of hydraulic conductivity ratio (Hvorslev conductivity estimate for test initiated in fully penetrating well prior to complete recovery (K_{inc}) over actual conductivity (K)) versus normalized recovery (H_{inc}/H_0 , where H_{inc} is the residual deviation) as a function of α (α defined in text).

Impact of Incomplete Recovery
 Partially Penetrating Well
 Hvorslev Analysis ($b/r_w=160$)

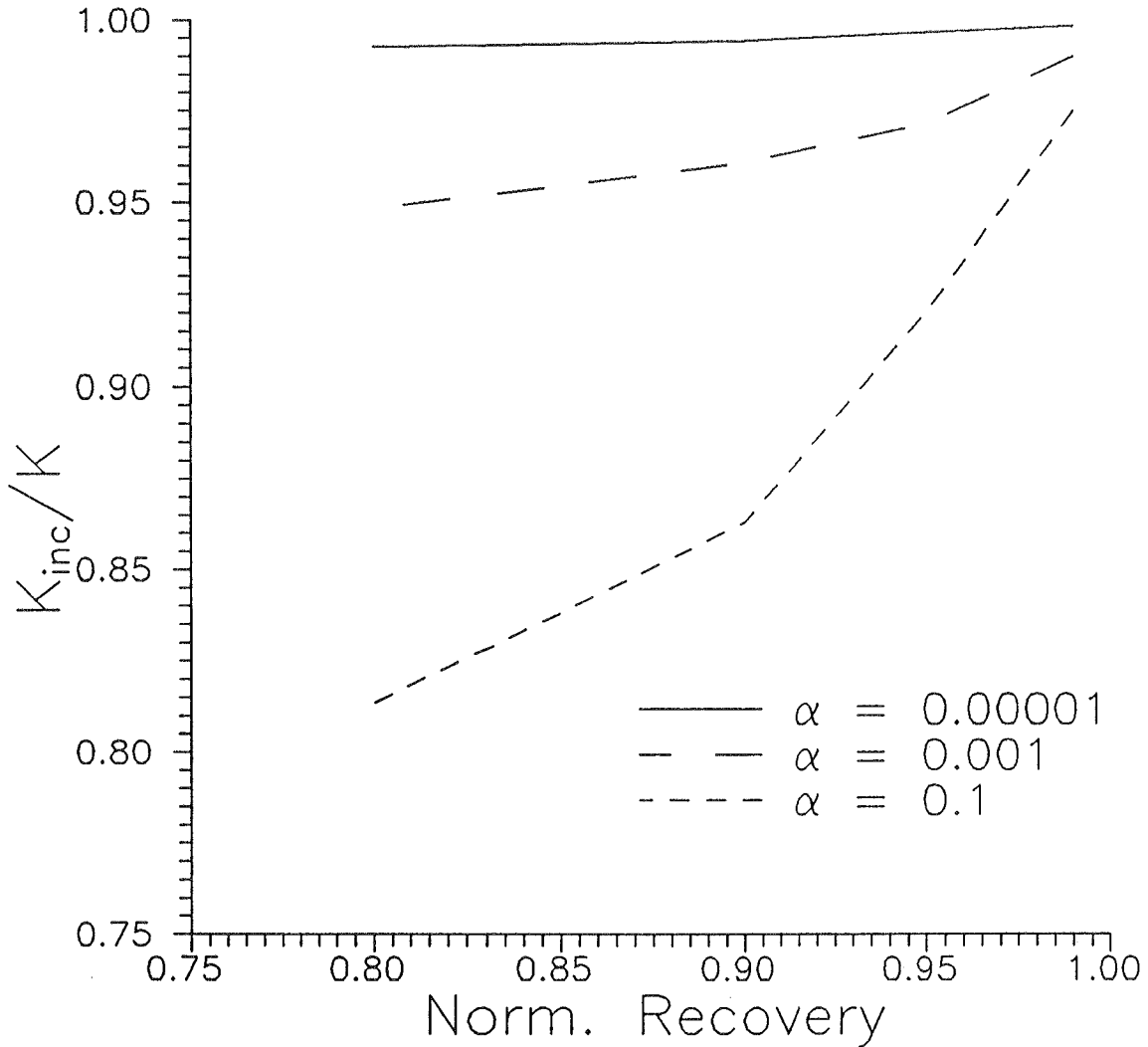


Figure 4 - Plot of hydraulic conductivity ratio (Hvorslev conductivity estimate for test initiated in partially penetrating well prior to complete recovery (K_{inc}) over actual conductivity (K)) versus normalized recovery as a function of α ($b/r_w=160.$).

Impact of Incomplete Recovery
 Partially Penetrating Well
 Hvorslev Analysis ($b/r_w=15.7$)

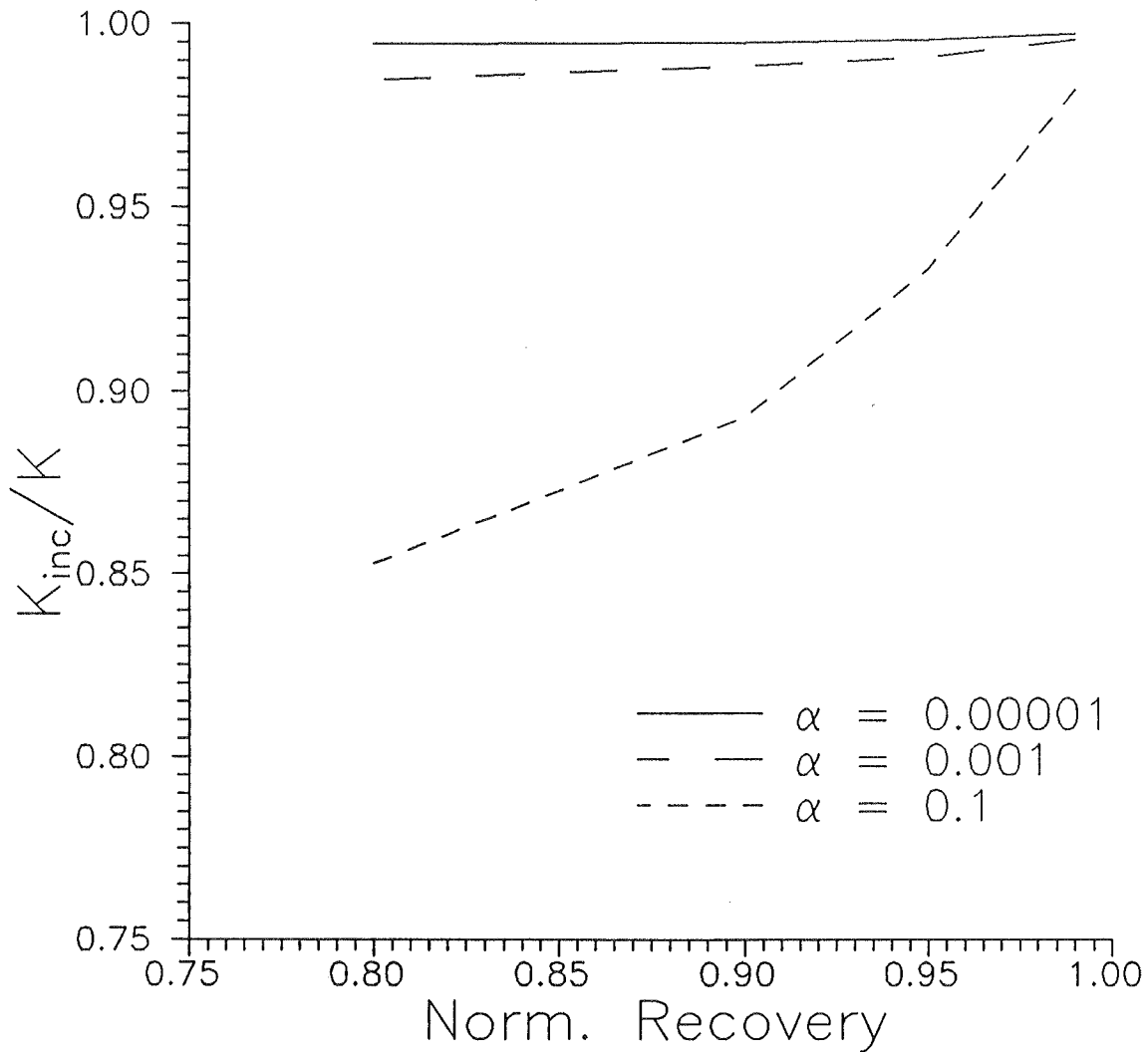


Figure 5 - Plot of hydraulic conductivity ratio (Hvorslev conductivity estimate for test initiated in partially penetrating well prior to complete recovery (K_{inc}) over actual conductivity (K)) versus normalized recovery as a function of α ($b/r_w=15.7$).

Impact of Incomplete Recovery Fully Penetrating Well Cooper et al. Analysis

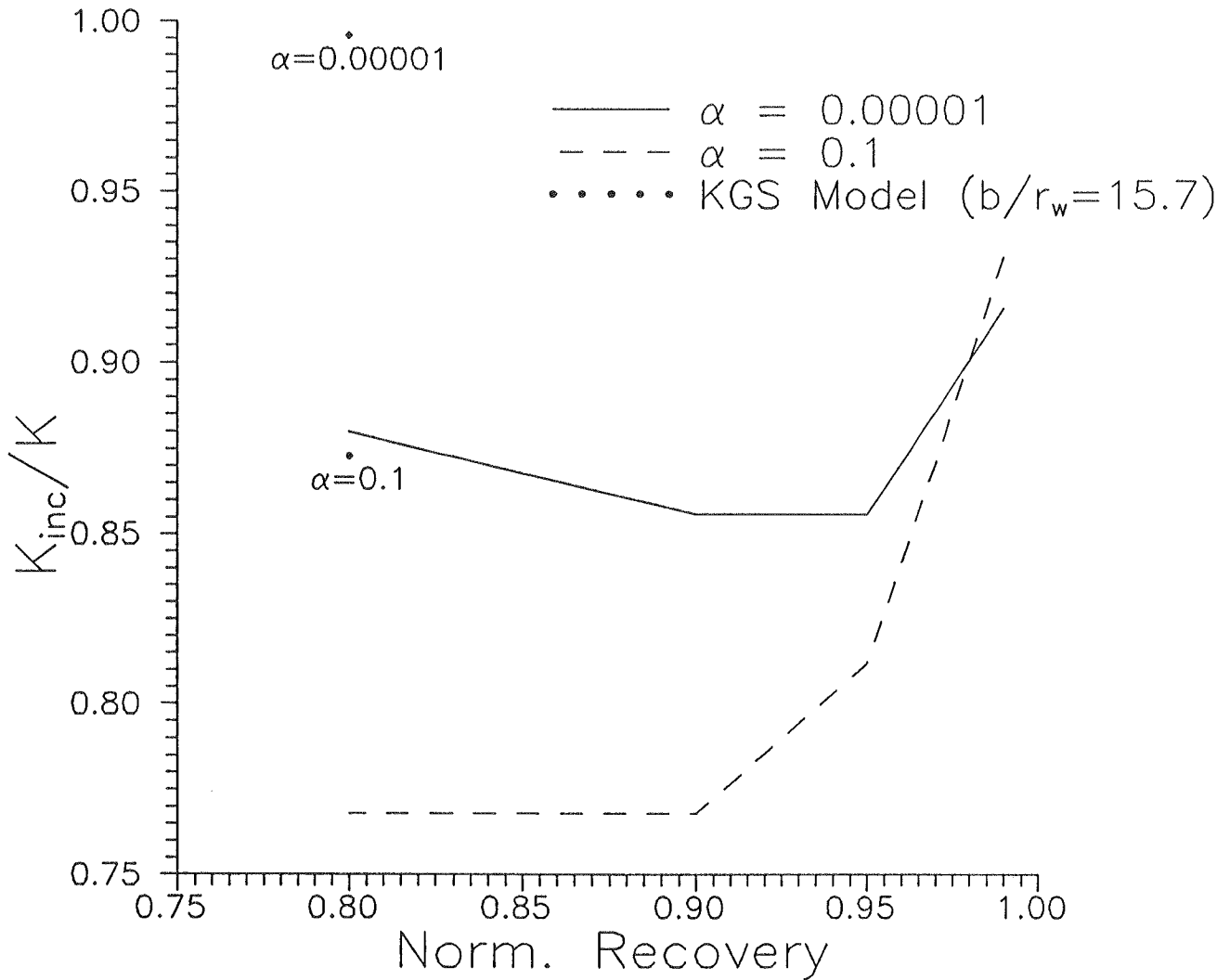


Figure 6A - Plot of hydraulic conductivity ratio (Cooper et al. conductivity estimate for test initiated in fully penetrating well prior to complete recovery (K_{inc}) over actual conductivity (K)) versus normalized recovery as a function of α (circles designate estimates obtained with the KGS model for tests performed in partially penetrating wells ($b/r_w = 15.7$)).

Impact of Incomplete Recovery
 Fully Penetrating Well
 Cooper et al. Analysis

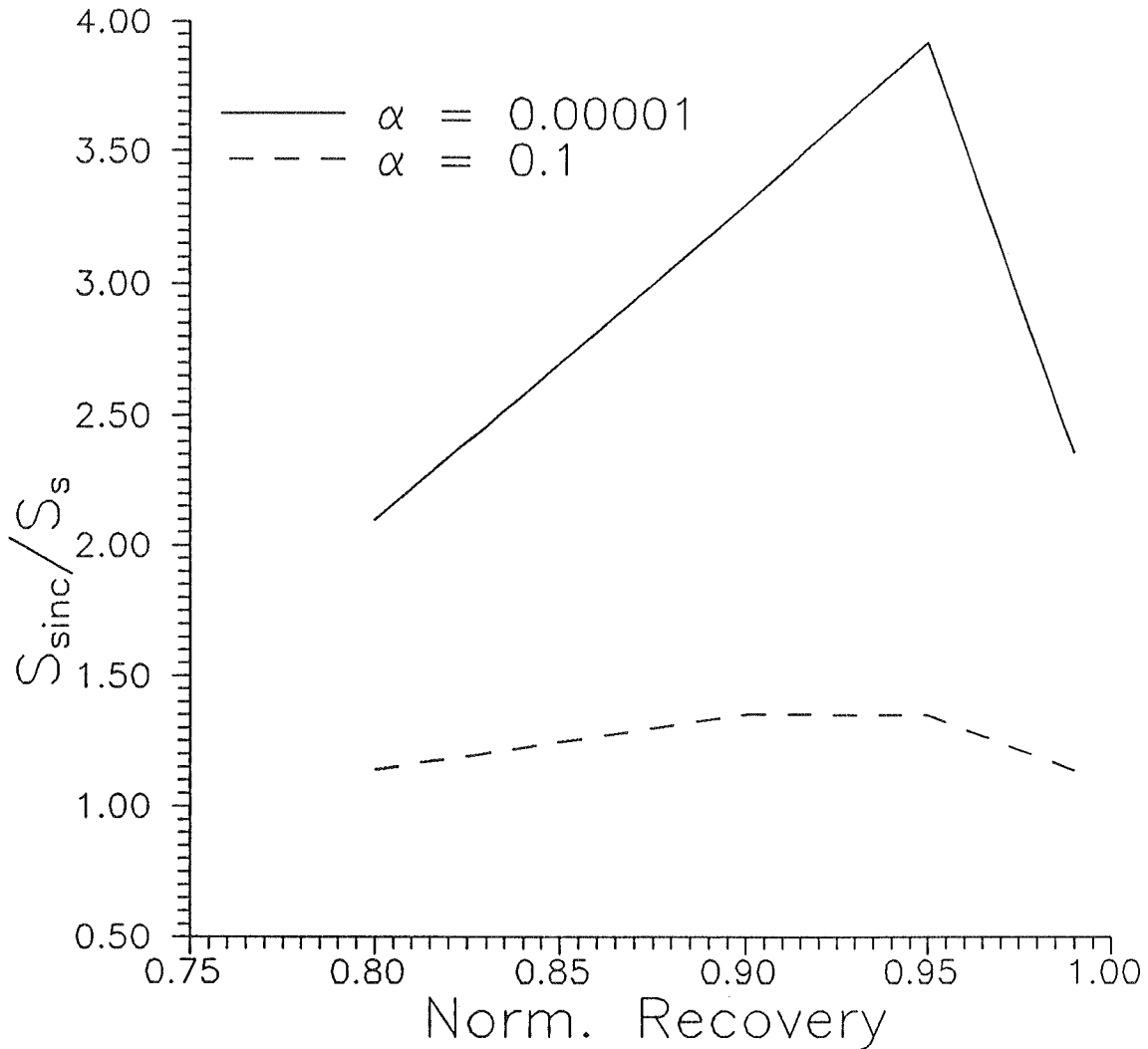


Figure 6B - Plot of specific storage ratio (Cooper et al. specific storage estimate for test initiated in fully penetrating well prior to complete recovery (S_{sinc}) over actual specific storage (S_s)) versus normalized recovery as a function of α .

Incomplete Recovery Analysis
Fully Penetrating Well - $\alpha=0.1$
Residual Deviation = 0.20

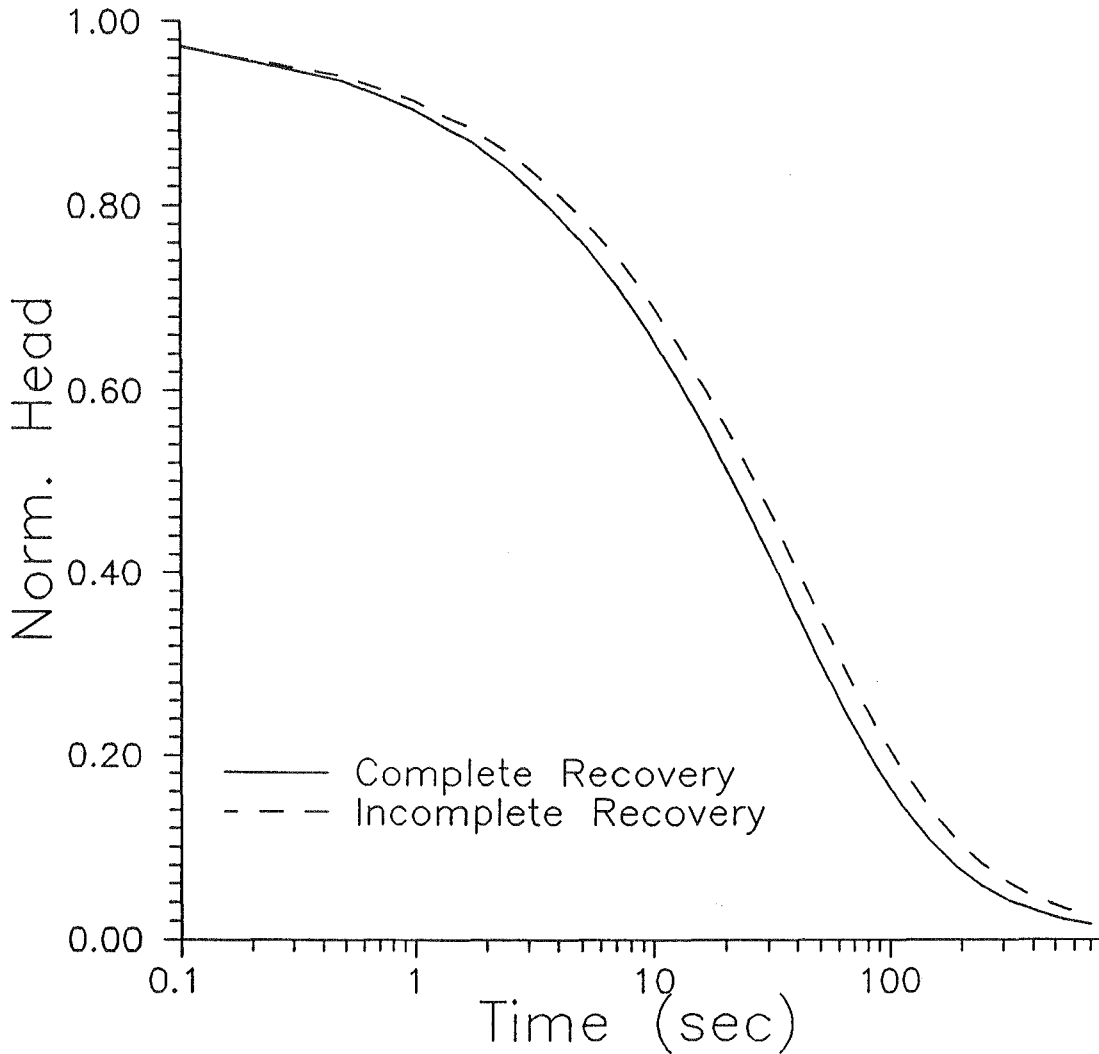


Figure 7A - Simulated normalized head versus log time plots of a slug test initiated at static conditions and a slug test initiated at a residual deviation of 0.20.

Incomplete Recovery Analysis
Fully Penetrating Well - $\alpha=0.1$
Residual Deviation = 0.20

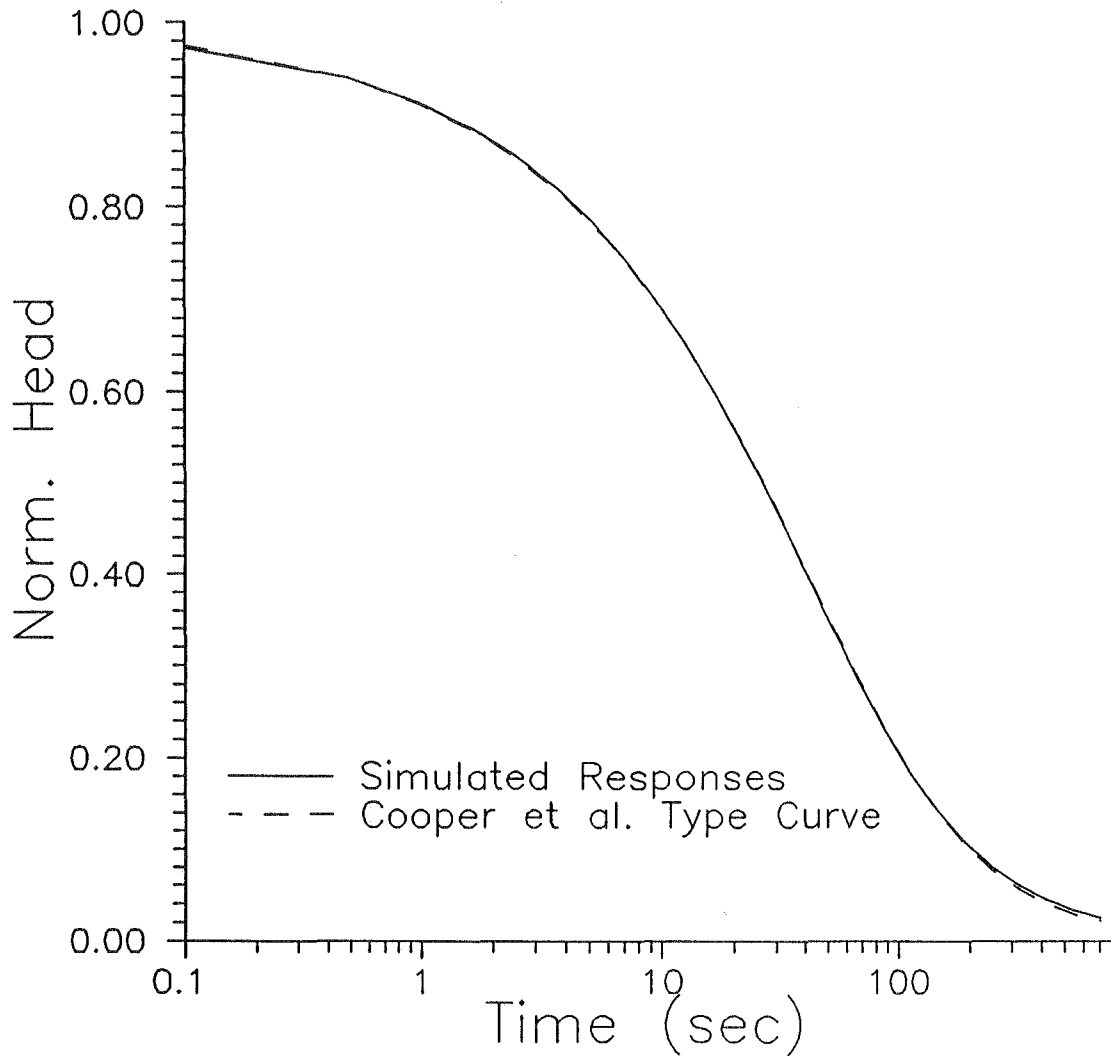


Figure 7B - Simulated normalized head versus log time plot of a slug test initiated at a residual deviation of 0.20 and the best-fit Cooper et al. model type curve.