# SLUG TESTS IN UNCONFINED AQUIFERS

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# ABSTRACT

A fully transient analytical solution has been developed for the case of slug tests in unconfined aquifers. The solution, which is developed using Laplace and Fourier integral transforms, represents the water table as a constant head upper boundary. This approximation should be reasonable for all cases except those when the well is screened very close to or across the water table. The solution has no constraints with respect to aspect ratio (screen length/screen radius) or range of formation parameters. Well skins, anisotropy, and the effects of a lower impermeable boundary can all be incorporated into the model. This solution is most useful in identifying conditions when conventional approaches (i.e. the Bouwer and Rice model) introduce unacceptably large errors into parameter estimates. In general, the Bouwer and Rice model provides acceptable estimates (within 25% of actual field values) for slug tests performed in homogeneous, isotropic aquifers. As would be expected, the Bouwer and Rice model introduces the largest error (can easily exceed an order of magnitude or more) in the presence of a low-permeability well skin. Uncertainty about anisotropy at small aspect ratios can also be the source of considerable error. The solution introduced here can be employed for parameter estimation under conditions when the Bouwer and Rice model introduces unacceptably large errors into parameter estimates. This solution can be rapidly evaluated, allowing easy incorporation into an automated well-test analysis package.

# INTRODUCTION

The slug test is probably the most commonly used technique by hydrogeologists for estimating hydraulic conductivity in the field (Kruseman and deRidder, 1989). This technique has become a preferred field method because of its several logistical and economic advantages. These include the small amount of equipment and manpower required to perform a test, the relatively short duration of the test, the perceived ease of the analysis of the recovery data, and the need for only a small amount of water (if any) to be added/removed from the well during the course of the test.

Approaches for the analysis of the recovery data collected during a slug test are based on analytical solutions to mathematical models describing the flow of groundwater to/from the test well. Over the last thirty years, solutions have been developed for a number of test configurations commonly found in the field (see Chirlin (1990) for a summary of much of the past work). For slug tests in unconfined aquifers, solutions for the mathematical model describing flow in response to the induced disturbance are difficult to obtain because of the nonlinear nature of the model in its most general form. Currently, most field practitioners use the technique of Bouwer and Rice (Bouwer and Rice, 1976; Bouwer, 1989), which is based on steady-state simulations using an electrical analog model, for the analysis of slug tests in unconfined flow systems. Dagan (1978) presents an analytical solution based on assumptions similar to those of Bouwer and Rice (1976). Amoozegar and Warrick (1986) summarize related methods employed by agricultural engineers. All of these techniques result from the application of several simplifying assumptions to the mathematical description of flow to a well in an unconfined aquifer. The most important of these assumptions are: 1) the specific storage of the aquifer is negligible, 2) changes in the position of the water table due to a slug test are so small that the water table can be represented as a constant-head boundary, 3) flow above the water table can be ignored, 4) there is no zone of disturbance created by drilling or development, and 5) the formation is isotropic with respect to hydraulic conductivity. The error that is introduced into hydraulic conductivity estimates by employing these techniques in cases where their assumptions are inappropriate has not yet been fully evaluated.

In this paper, a semianalytical solution to a general mathematical model describing flow of groundwater in response to an instantaneous change in water level at a central well is presented. The model incorporates the effects of partial penetration, anisotropy, finite-radius well skins of either higher or lower permeability than the formation as a whole, and an upper constant-head boundary. This model can be employed for the analysis of data from slug tests in a wide variety of commonly met field configurations. The major purpose of this paper is to delineate the conditions when such a model is needed by assessing the ramifications of assumptions 1), 4), and 5) described in the previous paragraph. The error that is introduced into parameter estimates by using the Bouwer and Rice model for the analysis of data from slug tests in configurations not strictly addressed in the derivation of that method will be quantified. The ultimate objective of this work is to use the estimate of the error arising in different configurations as the basis for recommendations for approaches to the analysis of slug-test data that can be utilized by field practitioners.

# PROBLEM STATEMENT

The problem of interest here is that of the head response produced by the introduction of an instantaneous slug of water in the screened or open section of a well. For the purposes of this development, the well will be assumed to be located in the unconfined aquifer shown in Figure 1. Note that there is a well skin of radius $r_{sk}$ that extends through the full width of the aquifer. The skin has transmissive and storage properties that may differ from the formation as a whole. Flow properties are assumed uniform within both the skin and formation, although the vertical ($K_z$) and radial ($K_r$) components of hydraulic conductivity may be different.

The partial differential equation representing the flow of groundwater in response to an instantaneous pressure disturbance introduced at a central well is the same for both the skin and the aquifer and can be written as

$$\frac{\partial^2 h_i}{\partial r^2}+\frac{1}{r}\frac{\partial h_i}{\partial r}+\left(\frac{K_{z_i}}{K_{r_i}}\right)\frac{\partial^2 h_i}{\partial z^2}=\left(\frac{S_{s_i}}{K_{r_i}}\right)\frac{\partial h_i}{\partial t} \qquad (1)$$

where

$h_i$ = head in zone i, [L];

$S_{s_i}$ = specific storage of zone i, [1/L];

t = time, [T];

r = radial direction, [L];

z = vertical direction, $z=0$ at the top of the aquifer and increases downward, [L];

i = zone designator, for $r_w \le r \le r_{sk}$, $i = 1$, and for $r_{sk} \le r$, $i = 2$.

The initial conditions can be written as

$$h_1(r,z,0) = h_2(r,z,0) = 0, \ r_w < r < \infty, \ 0 \leq z \leq B \tag{2}$$

$$h_1(r_w,z,0) = \begin{cases} H_0, \ d \leq z \leq d + b \\ 0, \ \text{elsewhere} \end{cases} \tag{3}$$

where

B = aquifer thickness, [L];
$H_0$ = height of initial slug, [L];
d = distance from the top of the aquifer to the top of the screen, [L];
b = screen length, [L].

The boundary conditions are the following:

$$h_2(\infty,z,t) = 0, \ t \geq 0, \ 0 \leq z \leq B \tag{4}$$

$$h_i(r,0,t) = 0, \ r_w < r < \infty, \ t \geq 0 \tag{5}$$

$$\frac{\partial h_i(r,B,t)}{\partial z} = 0, \ r_w < r < \infty, \ t \geq 0 \tag{6}$$

$$h_1(r_w,z,t) = H(t),\ t > 0,\ d \leq z \leq d + b \qquad (7)$$

$$2\pi r_w K_{r_1} \frac{\partial h_1(r_w,z,t)}{\partial r} = \frac{\pi r_c^2}{b} \frac{dH(t)}{dt} \square(z),\ t>0 \qquad (8)$$

where

$H(t)$ = level of water in well, [L];

$r_c$ = radius of well casing, casing and screen do not have to be of equal radius, [L];

$\square(z)$ = boxcar function = 0, $z < d$, $z > b+d$,
= 1, elsewhere.

Note that (8) indicates that the horizontal hydraulic gradient is assumed equal along the well screen. This assumption could be relaxed by using a function other than a boxcar function on the right side of (8).

In order to ensure continuity of flow between the skin and the formation, auxiliary conditions at the skin-formation boundary ($r=r_{sk}$) must also be met:

$$h_1(r_{sk},z,t) = h_2(r_{sk},z,t),\ 0 \leq z \leq B,\ t>0 \qquad (9)$$

$$K_{r_1} \frac{\partial h_1(r_{sk},z,t)}{\partial r} = K_{r_2} \frac{\partial h_2(r_{sk},z,t)}{\partial r},\ 0 \leq z \leq B,\ t>0 \qquad (10)$$

Equations (1)-(10) describe the flow conditions of interest here. Hyder et al. (in preparation - available from authors on request) provide the details of the solution derivation. In summary, the approach employs a series of integral transforms (Laplace transform in time and a modified finite Fourier sine transform in the z direction) to obtain functions in transform space that satisfy the transform-space analogues of (1)-(10). Given the complexity of the transform-space solutions, the analytical back transformation from Fourier-Laplace space to real space is only readily performed under quite limited conditions. In the general case, the transformation is best performed numerically. Numerical evaluation of the Fourier transforms and their inversions are done here using Discrete Fourier Transforms (Brigham, 1974), thereby allowing computationally efficient Fast Fourier Transform techniques (Cooley and Tukey, 1965) to be utilized. The Stehfest algorithm (Stehfest, 1970) was used to perform the numerical Laplace inversion.

# RAMIFICATIONS FOR DATA ANALYSIS

As discussed in the Introduction, the primary purpose of this paper is to evaluate the error that is introduced into parameter estimates when employing the Bouwer and Rice method to analyze the response data from slug tests performed in unconfined aquifers. This evaluation is carried out by using the derived solution to simulate slug tests in a variety of configurations. The simulated response data are analyzed using an automated version of the Bouwer and Rice method. The parameter estimates are then compared to the parameters employed in the original simulations to assess the magnitude of the error introduced into the parameter estimates through use of the Bouwer and Rice method. Note that, in this work, the simulation and analysis of slug tests were performed using SUPRPUMP, an automated well-test analysis package developed at the Kansas Geological Survey (Bohling and McElwee, 1992). In all of the simulations, the following set of parameters for a slug test in a homogeneous, isotropic aquifer is used as the base case:

$K_r = K_z = .001$ m/s;
$S_s = .00001$ /m;
$r_w = r_c = .05$ m.

## DEPENDENCE ON ASPECT RATIO

Figure 2 displays the dependence of the error in the Bouwer and Rice conductivity estimates on aspect ratio (screen length (b)/ screen radius ($r_w$)) for the case of a well screened at the center of a very thick isotropic aquifer. Note that the Bouwer and Rice estimates are poorest at an aspect ratio of 5, an expected result since that aspect ratio is at one end of the range for which the empirical coefficients were developed. This plot indicates that in geologic systems that can be treated as isotropic the Bouwer and Rice estimates will be within 25% of the actual field values for the aspect ratios commonly employed in practice, when the test response is not strongly controlled by a hydrologic boundary.

# DEPENDENCE ON ANISOTROPY

Many natural systems will be characterized by an anisotropy in hydraulic conductivity ($K_r > K_z$). The Bouwer and Rice method, however, is based on the assumption of isotropy. Figure 3 displays the dependence of the error in the Bouwer and Rice conductivity estimates on the degree of anisotropy for the case of a well screened at the center of a very thick aquifer. Note that the Bouwer and Rice estimates considerably underpredict hydraulic conductivity as the degree of anisotropy increases. This effect is exacerbated at smaller aspect ratios since a greater proportion of vertical flow is expected at such ratios. Clearly, the Bouwer and Rice method must be used with caution when analyzing results from tests performed in formations that are expected to exhibit a considerable degree of anisotropy.

## DEPENDENCE ON SPECIFIC STORAGE

The results displayed in Figures 2-3 were obtained assuming a specific storage of $1.0e^{-5}$. As stated in the Introduction, the Bouwer and Rice method is based on the assumption that specific storage can be neglected. Figure 4 displays the dependence of the error in the Bouwer and Rice conductivity estimates on the magnitude of the specific storage for the case of a well screened at the center of a very thick, isotropic aquifer. These results indicate that this error is not large for the range of specific storages commonly expected in the field. Only in the case of a very large specific storage and wells with moderate to large aspect ratios will the error introduced by this assumption become worrisome.

## DEPENDENCE ON DISTANCE FROM BOUNDARY

Figures 2-4 display results for the case of a well screened at such a large distance from a boundary that boundary effects are negligible. In many field situations, however, the effect of the water table or a lower impermeable boundary can be considerable. Figure 5 displays the dependence of the error in the Bouwer and Rice conductivity estimates on the distance below the water table. The quality of the estimates clearly depends on the distance from the boundaries. Note that there is a break in the plot near the water table because the original relationships developed by Bouwer and Rice did not extend completely to the water table due to a limitation in the electrical analog model. Note also that there is a sharp increase at the lower impermeable boundary as a result of using the "fully penetrating well" formula of Bouwer and Rice for the case of the well screened up to the impermeable boundary. These results demonstrate that wells screened at different levels in the same homogeneous formation can be expected to have Bouwer and Rice conductivity estimates that differ by as much as 50% simply due to differences in their positions with respect to the boundaries. Figure 6 indicates how anisotropy and a higher specific storage affect these relationships. The results shown in Figure 6 again demonstrate that the Bouwer and Rice estimates are suspect in the presence of anisotropy.

## DEPENDENCE ON A LOW PERMEABILITY SKIN

Figures 2-6 display results for the case of slug tests performed in ideal, homogeneous aquifers. Often, however,as illustrated in Figure 1, well drilling and development creates a disturbed, near-well zone (well skin) of properties differing from those of the formation as a whole. Depending on the drilling method, the method of well emplacement, the type of development activities, and the nature of the formation, this well skin can be either of lower or higher permeability than the formation in which the well is screened. Clearly, the effect of well skins on parameter estimates obtained from slug tests must be understood in order to avoid using parameter estimates representative of skin properties to characterize the formation as a whole.

Figure 7 is a plot of the dependence of the error in the Bouwer and Rice conductivity estimates on the aspect ratio for the case of wells with skins one and two orders of magnitude less conductive than the formation. The Bouwer and Rice estimates are clearly heavily weighted towards the conductivity of the skin. Note that the estimates improve with increases in aspect ratio due to the lessening importance of vertical flow at higher aspect ratios. Figure 8 displays a plot of a simulated slug test and the best-fit Bouwer and Rice model that is representative of all the low-conductivity skin cases shown in Figure 7. As can be seen from Figure 8, the Bouwer and Rice model provides an excellent fit to the simulated data. In fact, a large number of additional simulations have shown that the Bouwer and Rice fit for the low-conductivity skin case is almost always better than that for the homogeneous-aquifer case. This is especially true at moderate to large aspect ratios, where the response data for the homogeneous case generally will display a distinct concave upward curvature (e.g., Chirlin, 1989). Thus, the fact that the response data plot as a straight line on a log head versus time plot should not be necessarily taken as definitive proof that the assumptions of the Bouwer and Rice are being upheld. The possibility that the behavior is reflective of a low permeability skin must be first discounted prior to acceptance of the validity of the parameter estimates.

## DEPENDENCE ON A HIGH PERMEABILITY SKIN

Figure 9 is a plot of the dependence of the error in the Bouwer and Rice conductivity estimates on the aspect ratio for the case of wells with skins one and two orders of magnitude more conductive than the formation. In this case, the analysis is performed assuming that the screen radius is equal to the radius of the skin as recommended by Bouwer and Rice. Note that the Bouwer and Rice estimates appear reasonable except in the case of a well of a small aspect ratio with a very conductive skin.

# CONCLUSIONS

The results of the investigation described in this presentation can be summarized as follows:

1) In a homogeneous, isotropic aquifer, the Bouwer and Rice model should provide acceptable estimates (within 25%) of the hydraulic conductivity of the formation for aspect ratios commonly employed in the field. There will, however, be a considerable variation in the estimates as a function of the distance of the screened interval from the boundaries, with the error being largest near the boundaries of the aquifer;

2) In a homogeneous, anisotropic aquifer, the Bouwer and Rice estimates may differ by more than 50% from the actual field values for aspect ratios commonly employed in the field;

3) In the case of a low conductivity skin, the Bouwer and Rice estimates will be heavily weighted towards the conductivity of the skin. Estimates that differ from the formation conductivity by over an order of magnitude and more can easily be obtained. In addition, data plots exhibit behavior that Bouwer and Rice hypothesized would be seen in the homogeneous-aquifer case. Unless healthy skepticism in conjunction with geological insight is employed, parameters reflective of the well skin can be inadvertently used to represent formation properties;

4) In the case of a high conductivity skin, the Bouwer and Rice model will provide acceptable estimates for moderate to large aspect ratios if the screen radius is set equal to the radius of the skin. At small aspect ratios, however, estimates that are 50% greater than the actual field values can be obtained.

Despite the concerns expressed by Dagan (1978) and others, the Bouwer and Rice model does appear to provide reasonable estimates of field values in many situations. However, in cases of anisotropy or a low conductivity skin, the Bouwer and Rice model will not provide estimates that will be acceptable for most applications. In those cases, models such as that introduced here must be employed for the analysis of the response data.

## ACKNOWLEDGEMENTS

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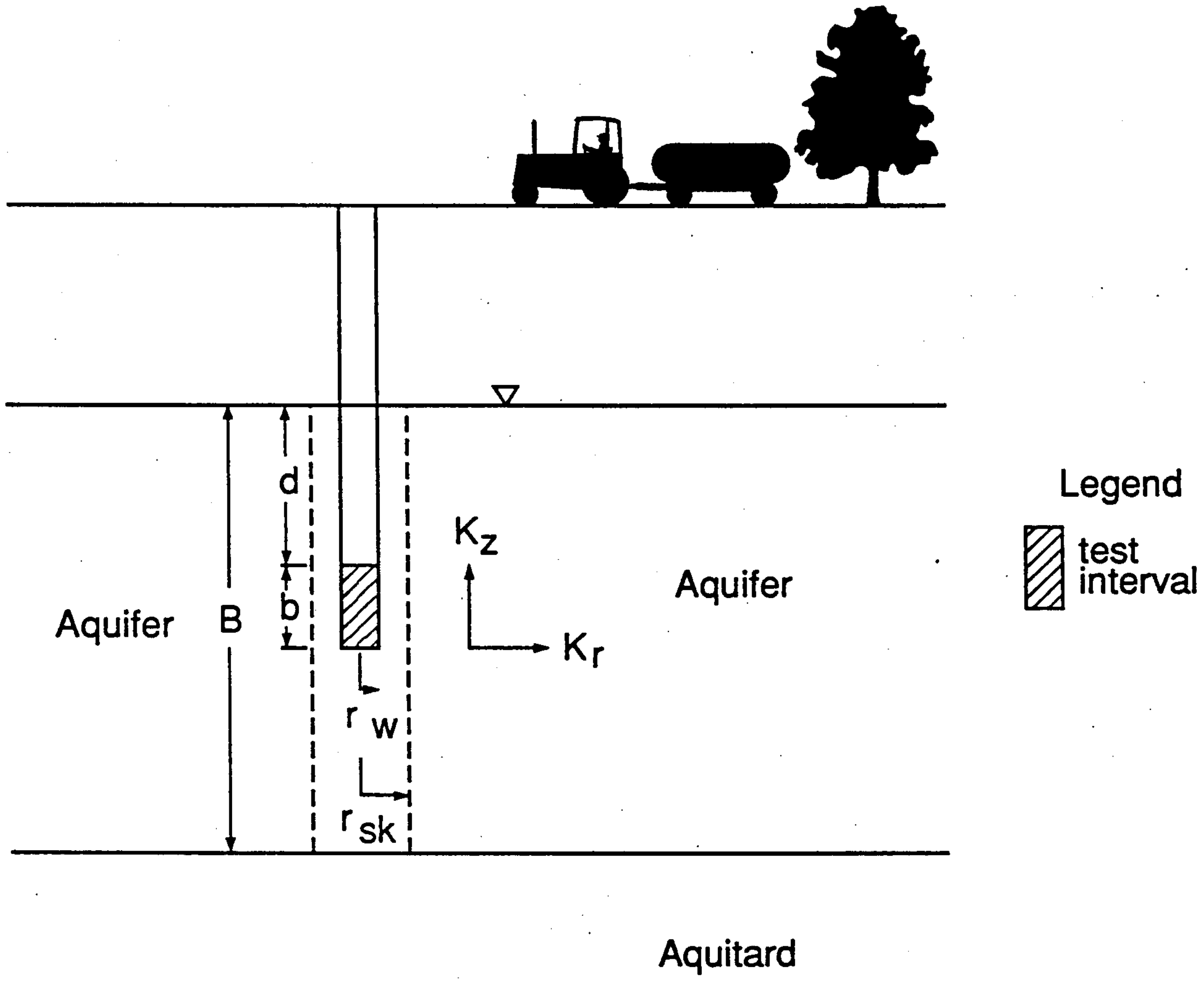

FIGURE 1 - Cross-sectional view of hypothetical unconfined aquifer

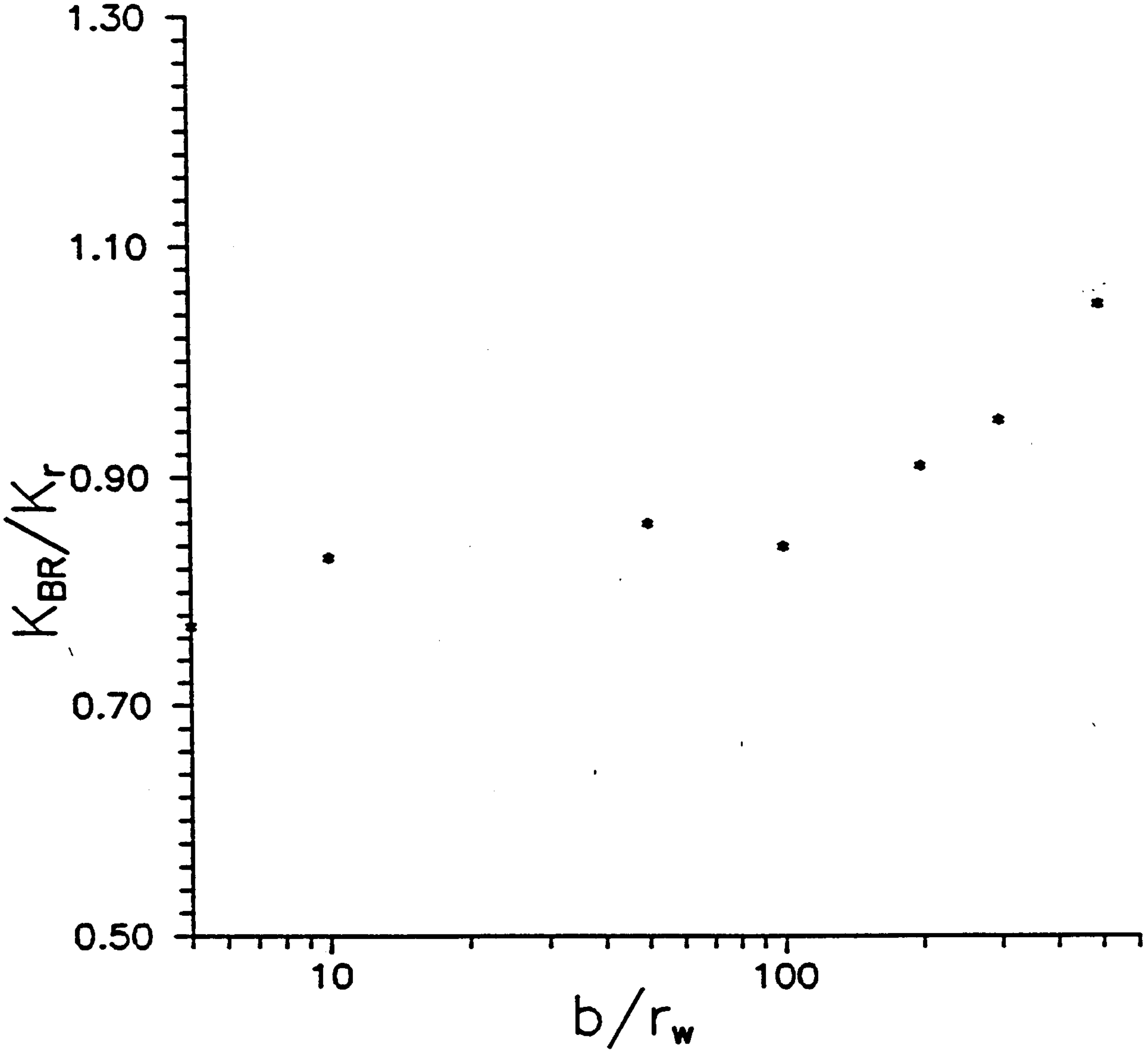

FIGURE 2 - Plot of conductivity ratio (Bouwer and Rice estimate ($K_{BR}$) over actual conductivity ($K_r$)) versus aspect ratio ($b/r_w$) for the case of a well screened at the center of a very thick aquifer.

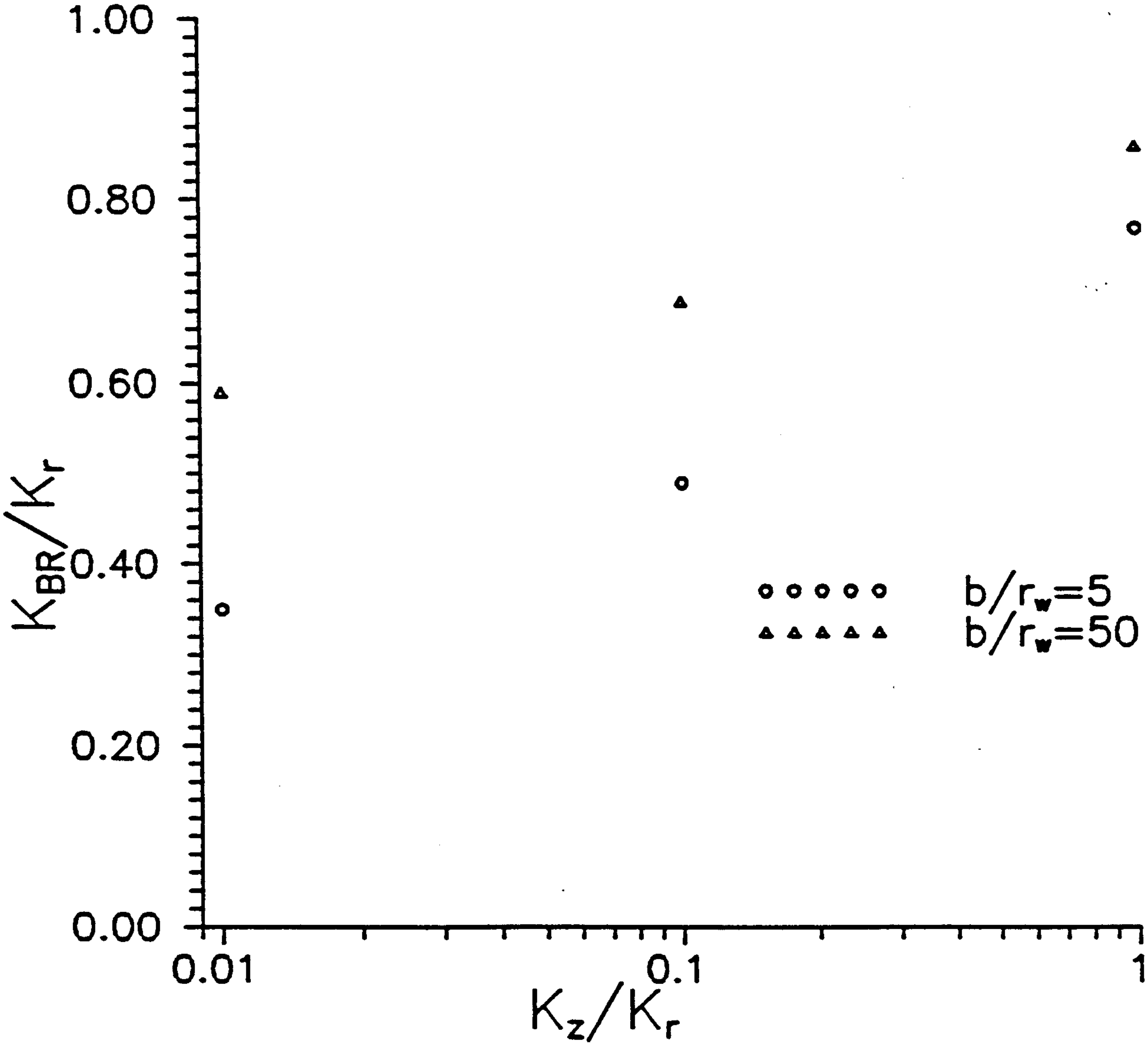

FIGURE 3 - Plot of conductivity ratio (Bouwer and Rice estimate ($K_{BR}$) over actual conductivity ($K_r$)) versus anisotropy ratio for the case of a well screened at the center of a very thick aquifer.

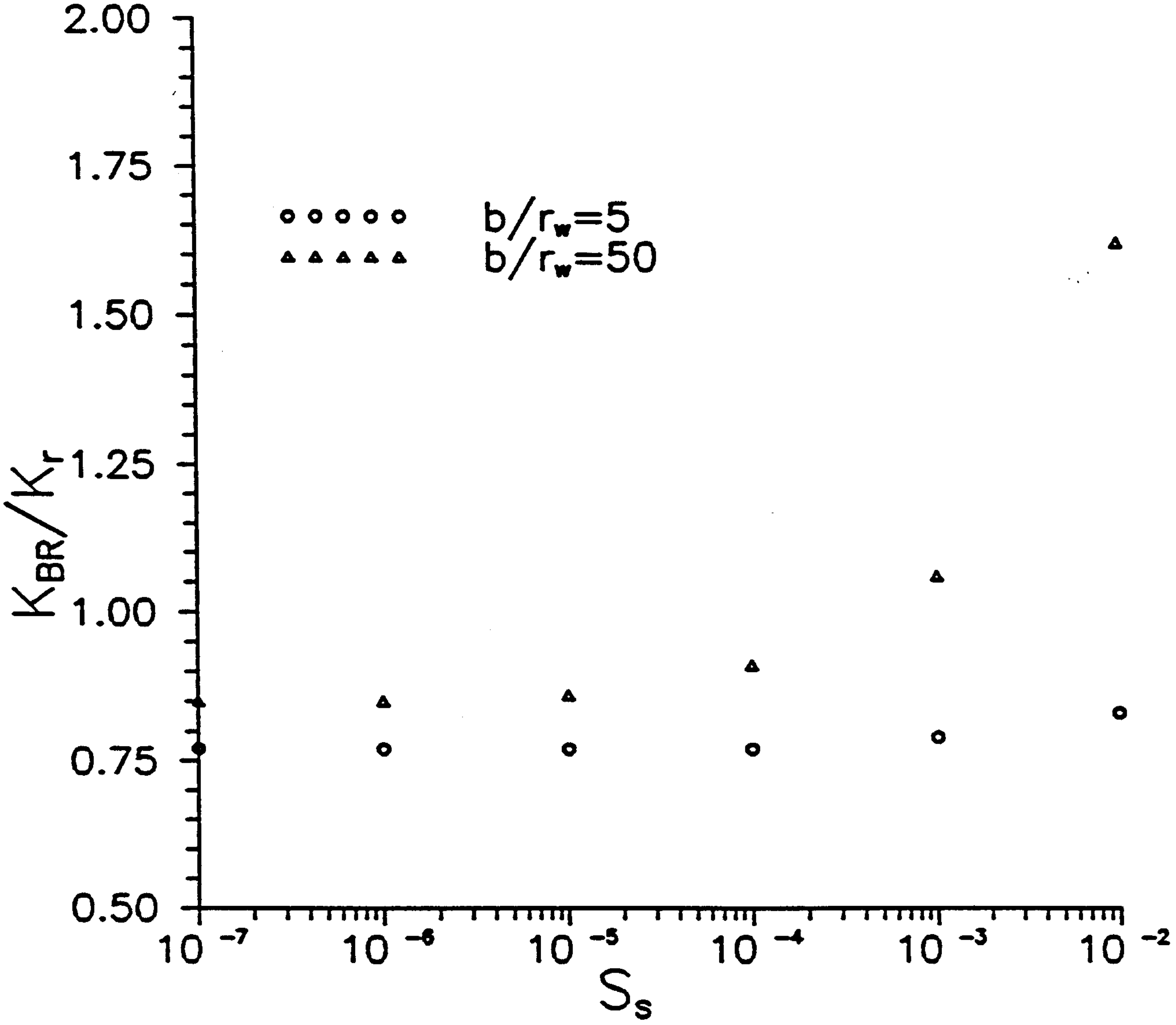

FIGURE 4 - Plot of conductivity ratio (Bouwer and Rice estimate ($K_{BR}$) over actual conductivity ($K_r$)) versus specific storage for the case of a well screened at the center of a very thick aquifer.

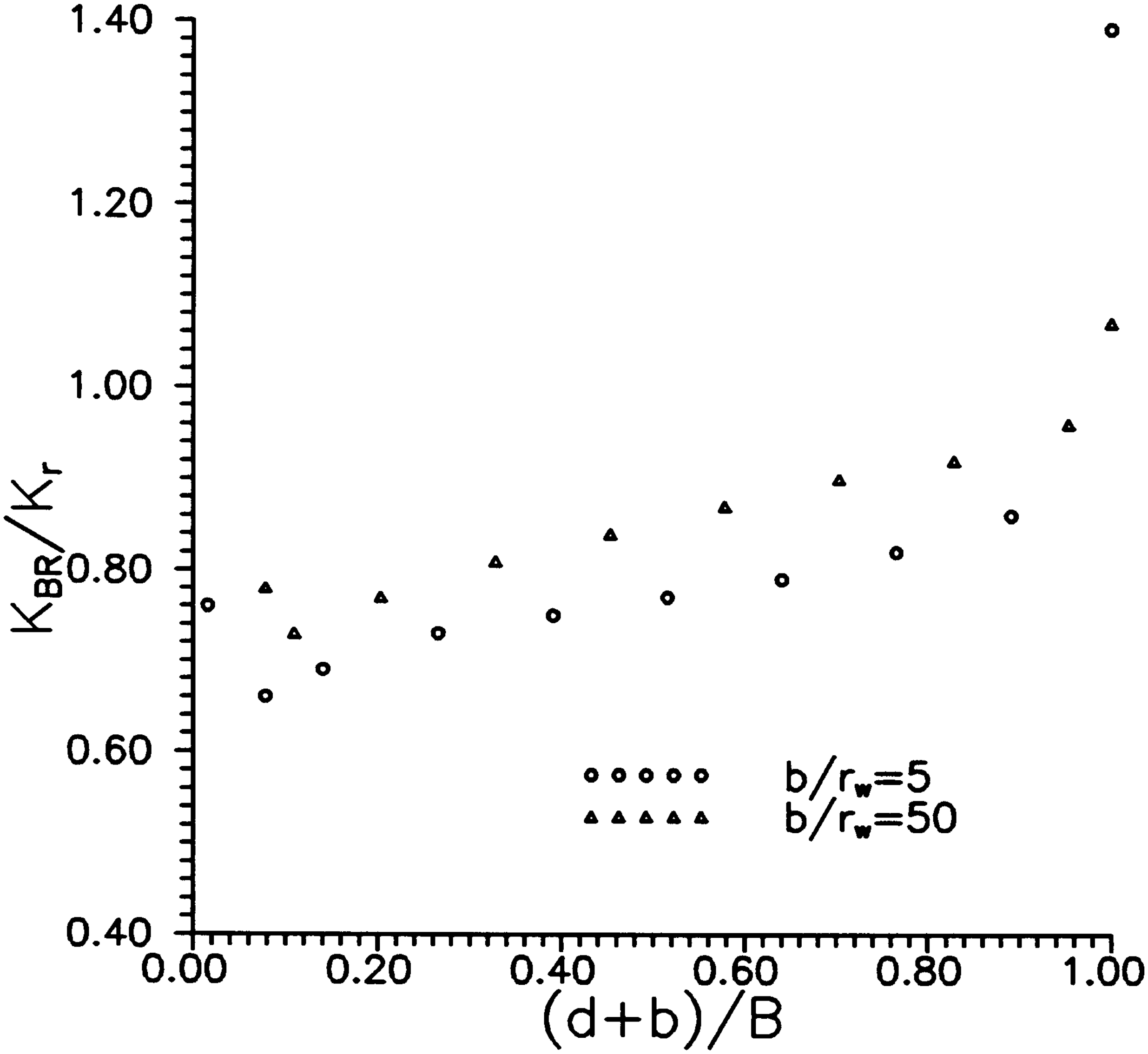

FIGURE 5 - Plot of conductivity ratio (Bouwer and Rice estimate ($K_{BR}$) over actual conductivity ($K_r$)) versus normalized depth below water table ((d+b)/B).

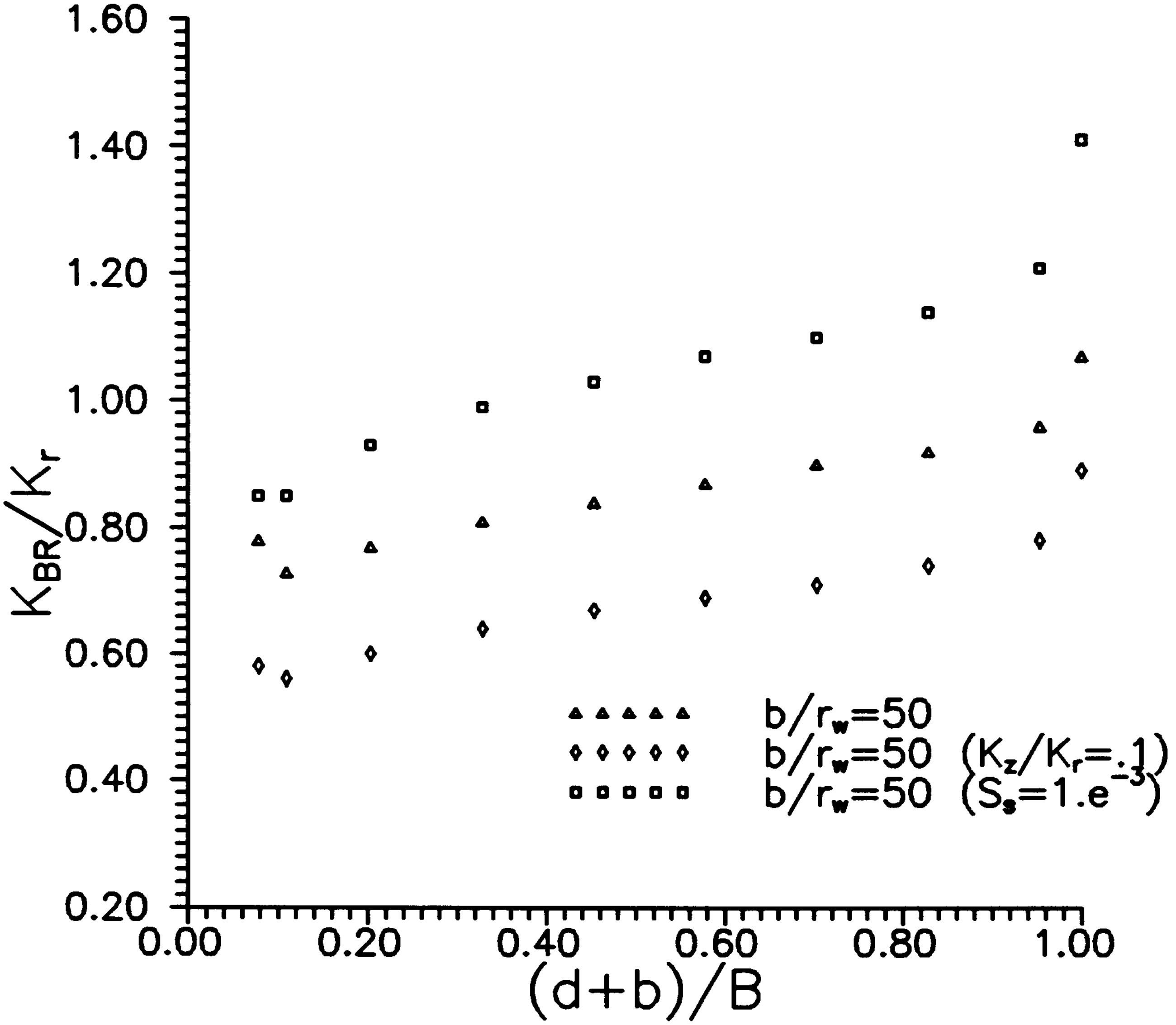

FIGURE 6 - Plot of conductivity ratio (Bouwer and Rice estimate ($K_{BR}$) over actual conductivity ($K_r$)) versus normalized depth below water table ((d+b)/B) as a function of specific storage and anisotropy.

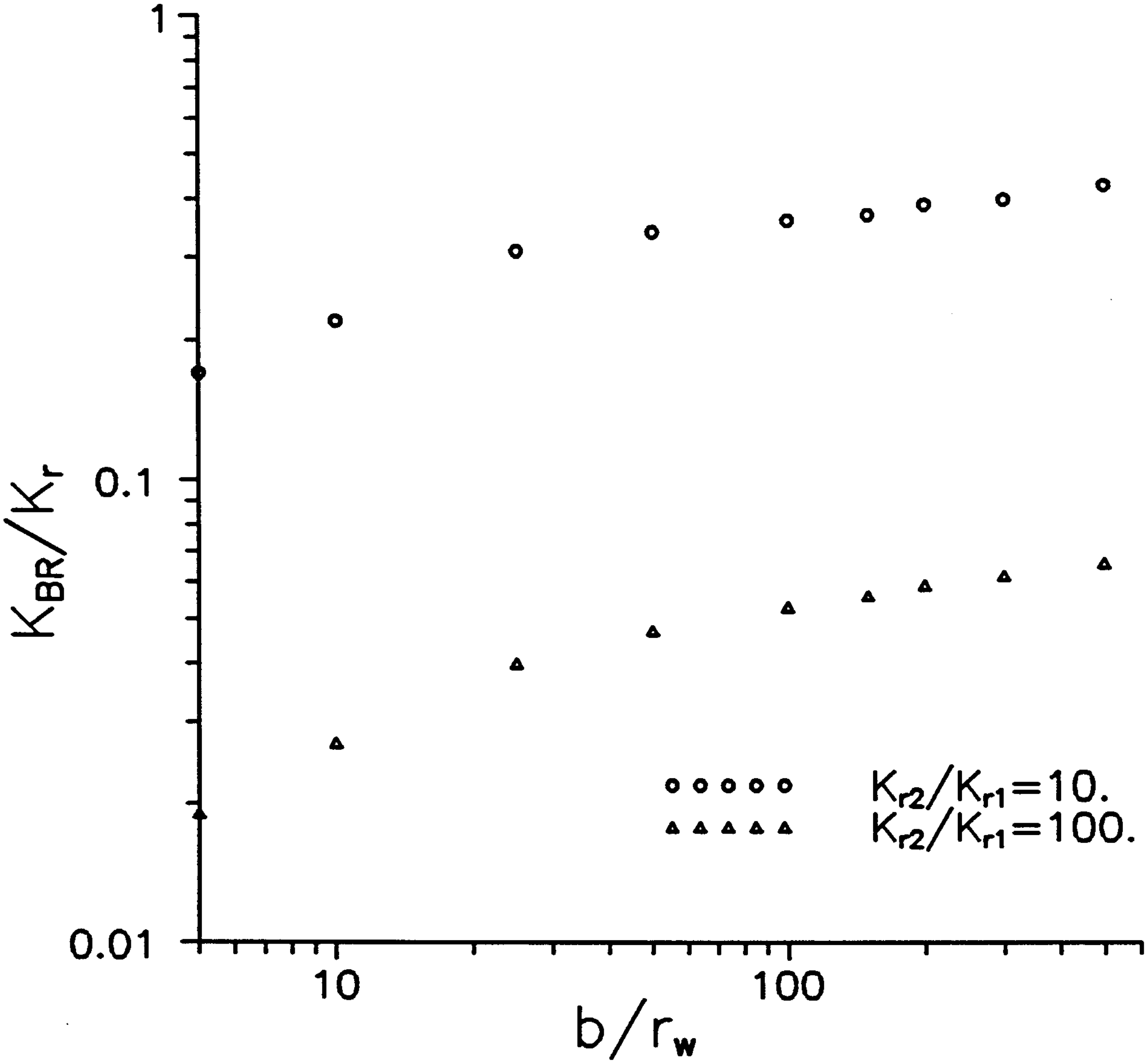

FIGURE 7 - Plot of conductivity ratio (Bouwer and Rice estimate ($K_{BR}$) over actual formation conductivity ($K_r$)) versus aspect ratio for the low-conductivity skin case ($r_{sk}=.10$ m).

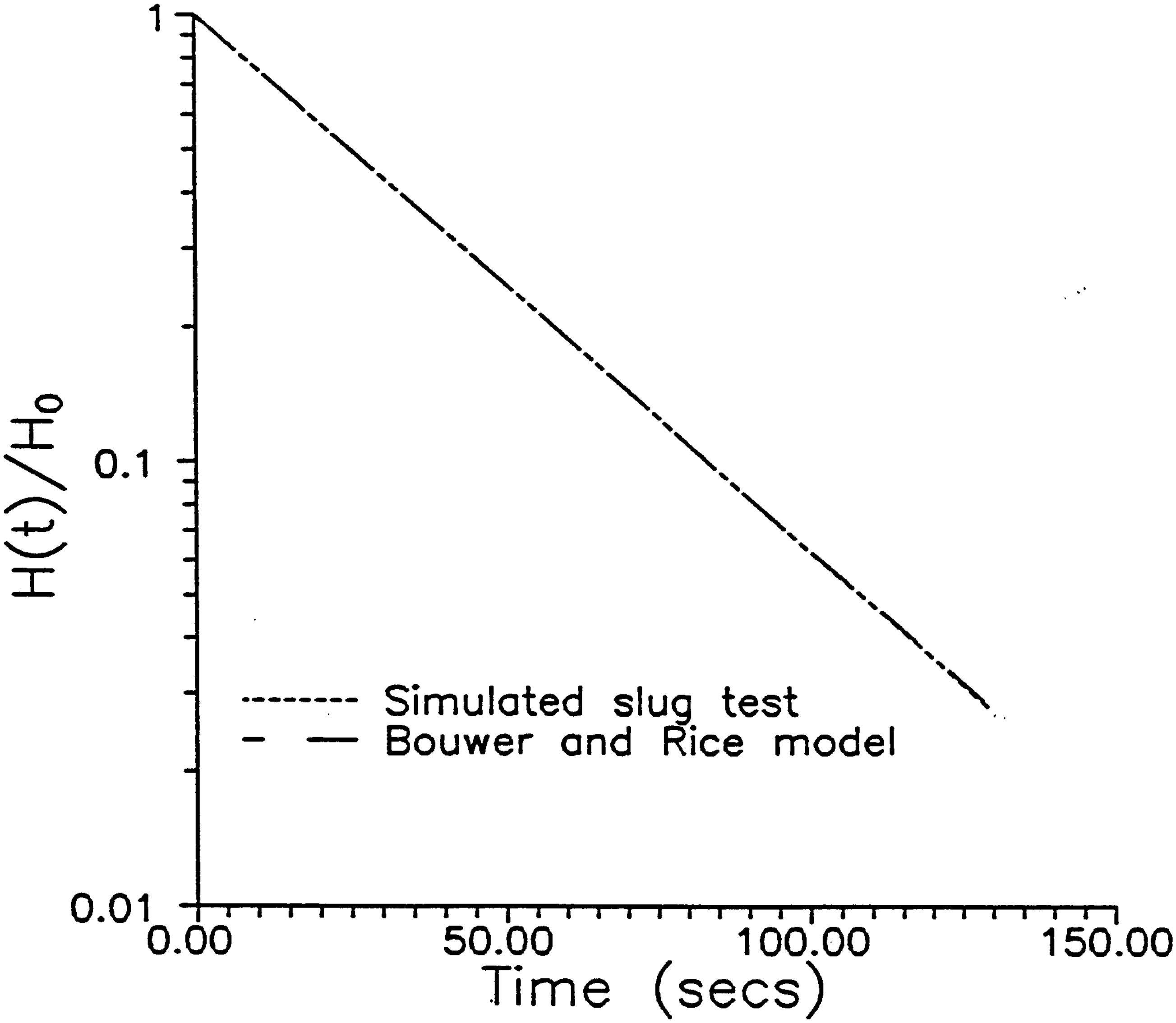

FIGURE 8 - Plot of simulated slug-test data and the best-fit Bouwer and Rice model for the case of a skin two orders in magnitude less conductive than the formation ($b/r_w = 50$, $r_{sk} = .10$ m).

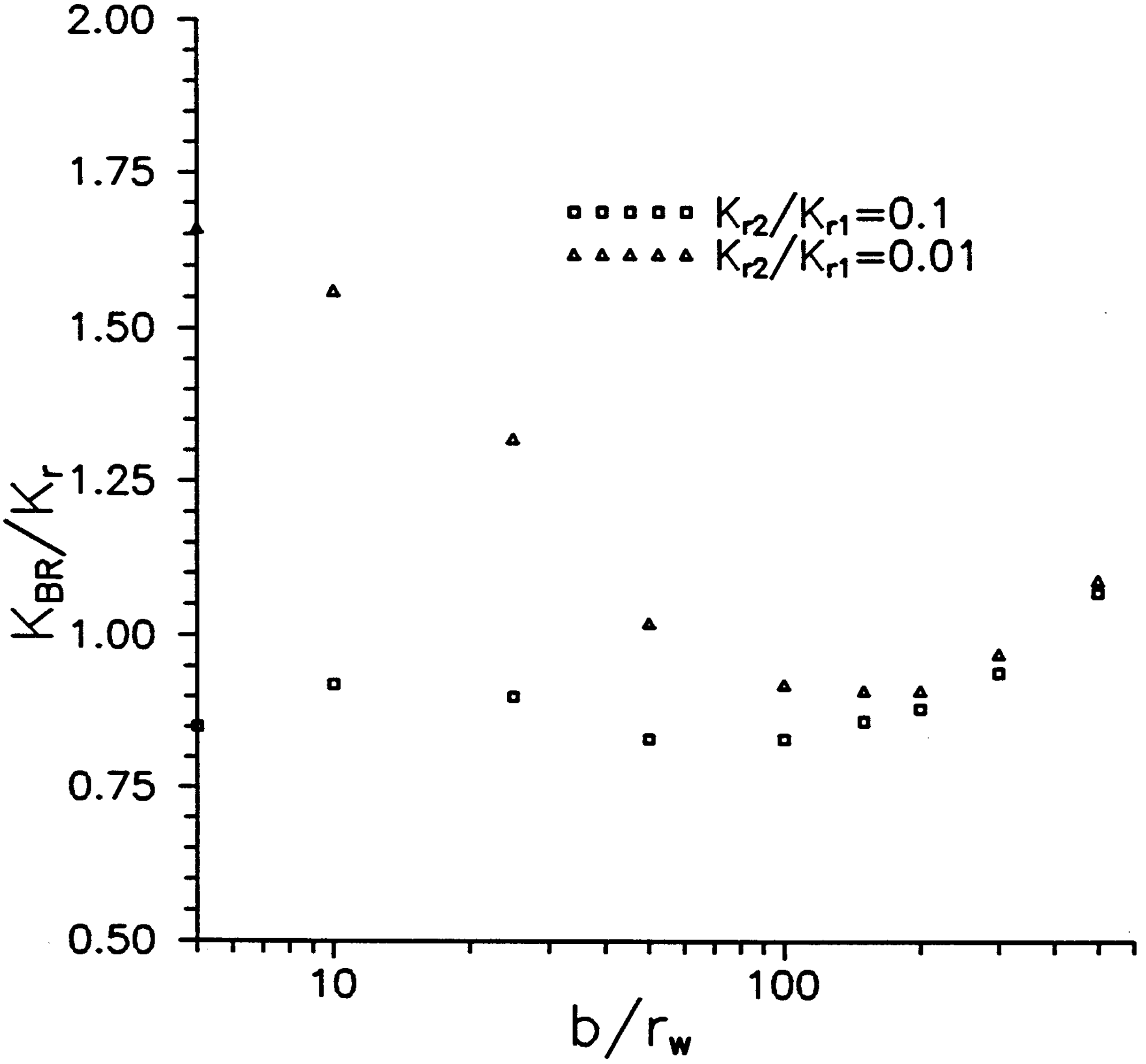

FIGURE 9 - Plot of conductivity ratio (Bouwer and Rice estimate ($K_{BR}$) over actual formation conductivity ($K_r$)) versus aspect ratio for the high-conductivity skin case ($r_{sk}=.10$ m).