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**NONLINEAR ANALYSIS OF SLUG TESTS IN HIGHLY-PERMEABLE
AQUIFERS USING A HVORSLEV-TYPE APPROACH**

by

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**Nonlinear Analysis of Slug Tests in Highly-Permeable Aquifers Using a
Hvorslev-Type Approach**

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Abstract

Slug tests are frequently used to characterize the transmissivity of an aquifer. However, in highly permeable aquifers problems arise when the conventional analytical techniques are applied. In an aquifer consisting of coarse sand and gravel overlain by silt and clay, we have consistently seen deviations from the expected response of linear theoretical models. For example, in the Hvorslev method the log of the slugged head in the well is plotted against time on a linear scale. Ideally this plot is a straight line, the slope of which is proportional to the hydraulic conductivity of the aquifer. Typically, we do not see a straight line on this plot, but rather a concave downward curve. Also, we see a dramatic dependence of the duration of the slug test on the initial height. If the slug-test data are normalized to the initial height, conventional models predict that all curves for different initial heights should be the same. We find that the curves are dramatically shifted to larger times for our field data. The sand and gravel at our field site is so coarse that the slug tests generally only last a few tens of seconds. This means that the water velocity is much faster than in finer sediments. We have investigated two non-linear variants of the Hvorslev model. One considers frictional effects caused by the flow of water in the casing and through the screen. We assume this frictional effect is proportional to a power of the velocity. The other model assumes non-Darcian flow, with the hydraulic gradient proportional to the first and second powers of velocity. Both models can be shown to have the same mathematical form, even though they represent different physical processes. There are two parameters in the models: one is the hydraulic conductivity, and the other is related to the non linearity. We have developed a numerical solution for these models and have found that the solutions do exhibit downward curvature and do show the correct form of dependence on initial head. We have applied these models to field data and have found that the downward curvature and the dependence on head can be fit very well. Initial findings are that the hydraulic conductivity can be determined with some consistency, at a given well, for tests with different initial heads. However, a traditional Hvorslev analysis gives very different results for the hydraulic conductivity, with these same data for varying initial heads.

Introduction (Figure 1.)

Slug tests are frequently used to characterize the hydraulic conductivity of an aquifer. However, in highly permeable aquifers problems arise when the conventional analytical techniques are applied. We have developed a research and teaching site in the Kansas River alluvium, which we call GEMS (Geohydrologic Experimental and Monitoring Site). In this aquifer, consisting of coarse sand and gravel overlain by silt and clay, we have consistently seen deviations from the expected response of linear theoretical models.

For example (Figure 2), in the Hvorslev (1951) method the log of the slugged head in the well is plotted against time on a linear scale. Ideally this plot is a straight line, the slope of which is proportional to the hydraulic conductivity of the aquifer. Typically, we do not see a straight line on this kind of plot.

Field Data

When we plot field data (Figure 3.) on a Hvorslev-type plot, concave downward curves are seen. Chirlin (1989) has shown that for the transient C-B-P model (Cooper et al, 1967) concave upward curves should result. Also, we see a dramatic dependence of the duration of the slug test on the initial height. If the slug-test data are normalized to the initial height, conventional models predict that all curves for different initial heights should be the same. We find that the curves are dramatically shifted to larger times for our field data.

Nonlinear Models

The sand and gravel at our field site is so coarse that the slug tests generally only last a few tens of seconds. This means that the water velocity is much faster than in finer sediments. We have investigated two nonlinear variants of the Hvorslev model. One considers frictional effects (Figure 4.) caused by the flow of water in the casing and through the screen. Equation (1) in this figure is the usual equation used to start the Hvorslev derivation, except that the right hand side contains $h(t)$ which may be different from the head in the casing due to frictional effects. The flow through the

screen $[Q(t)$, equation (2)] is assumed to be proportional to the head loss in the casing and screen and inversely proportional to a resistance factor (R). Combining equations (1) and (2) we obtain equation (3), which involves only the head in the casing $H(t)$. $Q(t)$ can be replaced by using equation (1).

R is an empirical resistance factor that we have introduced. We assume this factor is proportional to a power of the velocity (Figure 5). Equation (1) assumes that R is proportional to the first power of the velocity. A is the assumed constant of proportionality and is a measure of the non linearity (nonlinear factor), if $A = 0$ then we have the usual Hvorslev solution. Replacing R in the last equation of the previous figure, yields equation (2) here. It is clear that equation (2) is a nonlinear equation which must be solved for the head in the casing, $H(t)$. There is other work in the literature that suggests the frictional effects may be proportional to higher powers of the velocity (Barker and Herbert, 1992; Singh and Shakya, 1989). Consequently, equations (3) and (4) of this figure give the generalization to the case when R is proportional to the Nth power of the velocity.

The other model we shall employ assumes non-Darcian flow (Figure 6), with the hydraulic gradient proportional to the first and second powers of velocity (Bear, 1972; Guppy et al., 1982). b is the nonlinear factor here in equation (1), again if $b = 0$ we have the usual Hvorslev solution. After some algebra, both models can be shown to have the same mathematical form for $N = 1$ [equation (2)], even though they represent different physical processes. The relationship between the two empirical constants A and b is shown in equation (3). From here on, we shall use only A for the nonlinear factor, realizing that both models can be combined for $N = 1$. There are two parameters in the models that may be fitted: one is the hydraulic conductivity (K), and the other is the nonlinear factor (A). We have also done some analysis with the $N = 2$ case but preliminary results indicates that the $N = 1$ model is more consistent with the data.

Numerical Solution of Nonlinear Models

The nonlinear equations of the two models can not be solved in closed form. We have developed an iterative numerical solution (Figure 7.) for these models based

on equation (1). At any given time step equation (1) must be iterated until convergence occurs. n is the time step index and m is the iteration index. t_0 is the usual Hvorslev time lag factor. Equation (1) can be used for sequential time steps to generate the entire nonlinear type curve.

Figure 8 shows some of these type curves for typical parameters that might occur at the GEMS site. The solutions do exhibit downward curvature and do show the correct form of dependence on initial head. As the initial head increases the length of the test increases also. This implies that, when all data is given equal weight and a normal Hvorslev analysis is applied, one will estimate a lower hydraulic conductivity for tests with larger initial heads. However, looking carefully at the type curves in Figure 8., one can see that the long time behavior of all the curves for differing initial heads become parallel with a slope that is proportional to the hydraulic conductivity. Therefore, the late time data is more sensitive to the hydraulic conductivity, while the early time behavior may be heavily influenced by the nonlinear behavior.

Fitting and Editing Considerations

Figures 9. and 10. show some data from a 2 inch PVC well at GEMS with approximately 2.5 feet of screen. The initial slug height is about 23 feet. The field data are shown by asterisks and the solid curves are fitted by using an equal weight regression program (Bohling and McElwee, 1992). There are problems in fitting both the traditional Hvorslev model (Figure 9.) and the nonlinear model (Figure 10.) using an equal weight regression program. First of all, the late-time data is small in magnitude so its effect on the least squares fit is minimized. This shows up as substantial deviation of the data from the curve at late time. Of course the deviation in Figures 9 and 10 is emphasized by taking the log of the head. The second problem in fitting the field data centers about the fact that field data will be dominated by noise at late time when the head has decayed to nearly the static level. Unfortunately, the late-time data is most sensitive to the hydraulic conductivity, so these two problems must be dealt with carefully in order to achieve the best estimate of hydraulic conductivity. First of all the late-time data can be emphasized by fitting the log of the slugged head; this tactic will give greater weight to the late-time data. Secondly, careful editing of the log of the slugged head must be done before fitting, since taking the log of the slugged

head emphasizes the importance of the late-time data. We must be certain that the signal level of all late-time data used in the fit is substantially above the ambient noise level. The last data point in Figures 9. and 10. is an example of data that is not substantially above the ambient noise level and should be edited out.

Figure 11. shows field data from four slug tests with varying heights from well GEMS 02, which is a 2 inch PVC well that is approximately 45 feet deep with a screen length of approximately 2.5 feet. Clearly, the late-time data must be edited. The edit line we used is shown in the figure. It is uncertain what causes all the ambient noise (some sites will be noisier than others due to cultural effects), however, part of the late time noise level comes from elastic effects which are probably greater for larger initial heads.

Results

We have applied the nonlinear models to carefully edited field data and have found that the downward curvature and the dependence on head can be fit very well to the log of the slugged head (Figure 12.). The data in Figure 12 are also from GEMS 02; the initial head is 8.62 feet. The fit is good; the early time curvature and late time behavior are well represented by the nonlinear models. Fits for other values of initial head are similar. However, the systematic deviations from the model fit seem more pronounced at early time for the smaller values of initial head (not actually shown here).

Analysis of eight separate slug tests at GEMS 02 are summarized in a table in Figure 13. The initial heads range from 2.75 feet to 23.1 feet. This table shows that the hydraulic conductivity can be determined with some consistency at this well, for tests with different initial heads. The fitted hydraulic conductivity (K) varies from .00485 ft./sec. to .00296 ft./sec., with an average value of .00388 ft./sec. The nonlinear factor (A) varies from 177 to 63.9, with an average of 108. Ideally, K and A should be constant within the limitations of the data noise level, if the physical model is correctly describing the test. It is clear that there are some systematic trends in the table of Figure 13; K and A both decrease as the initial head is increased. This systematic trend indicates that our physical model is still not quite correct. However, these

nonlinear models have certainly described the major features of the data: that of downward curvature and shifting to longer times for higher heads. A traditional Hvorslev analysis, giving equal weight to all data, would yield very different results for the hydraulic conductivity, for varying initial heads.

Summary (Figure 14.)

In summary, slug tests in highly permeable aquifers seem to exhibit non-linear behavior. Field data show a downward curvature when plotted in the normal Hvorslev fashion. The duration of the slug test in field data is dependent on the value of the initial head. A model based on frictional loss or non-Darcian flow seems to explain the gross features of this noted field behavior. Through careful editing and fitting the log of the slugged head, reasonable consistency in hydraulic conductivity values for various initial heads can be obtained. Further refinement of the non-linear models is needed because systematic trends in the fitted parameters with initial head are observed. One possible refinement of the model would be to fit the power of the velocity dependence, rather than assume $N = 1$. The treatment given here is based on a Hvorslev type analysis, and therefore essentially quasi steady state. A fully transient model incorporating nonlinear effects would be an additional possible improvement.

Acknowledgment

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Introduction

1. Slug tests are frequently used to characterize the hydraulic conductivity of an aquifer.
2. However, in highly permeable aquifers problems arise when the conventional analytical techniques are applied.

Figure 1.

Typical Hvorslev Analysis

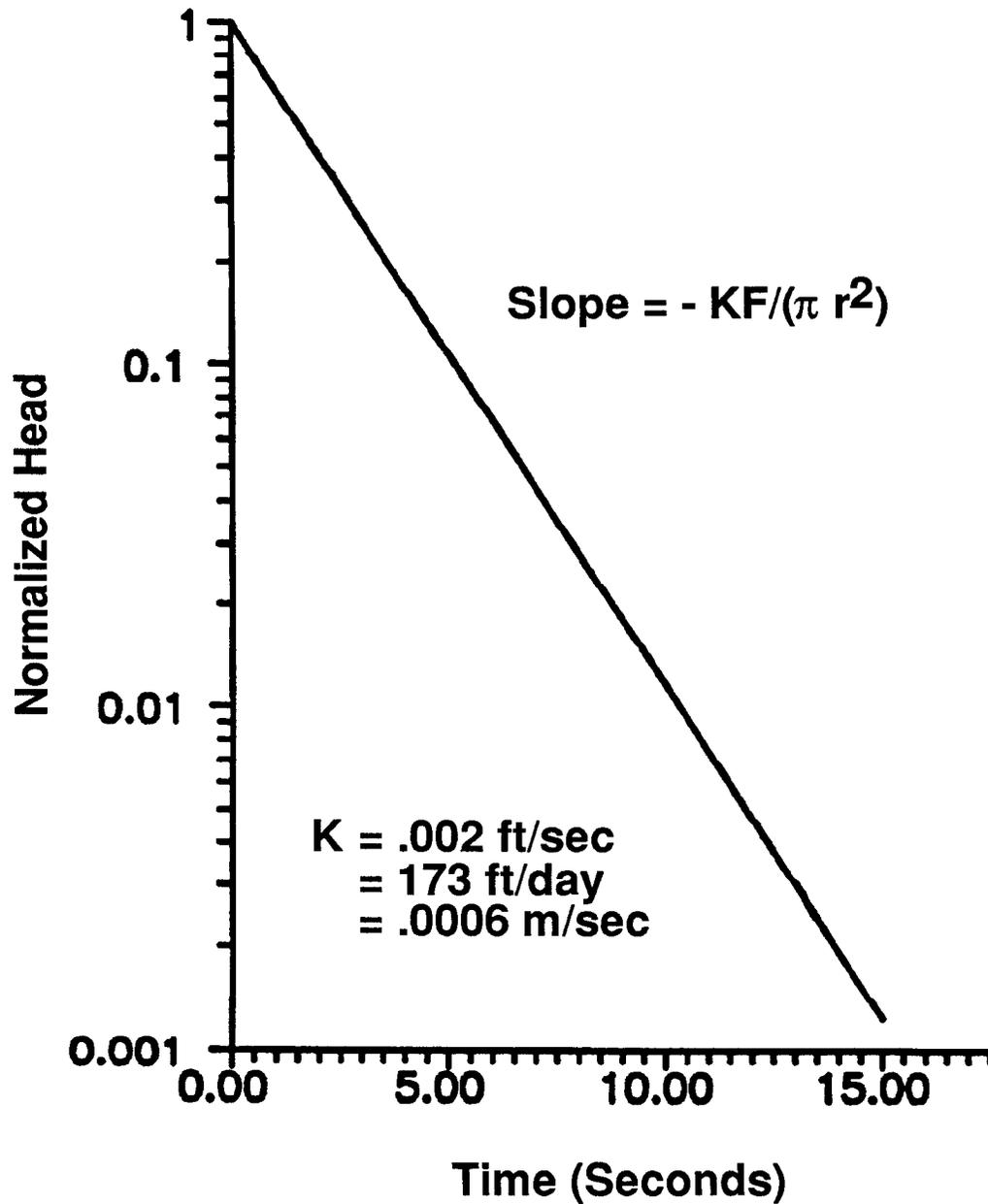


Figure 2.

Typical GEMS Field Data

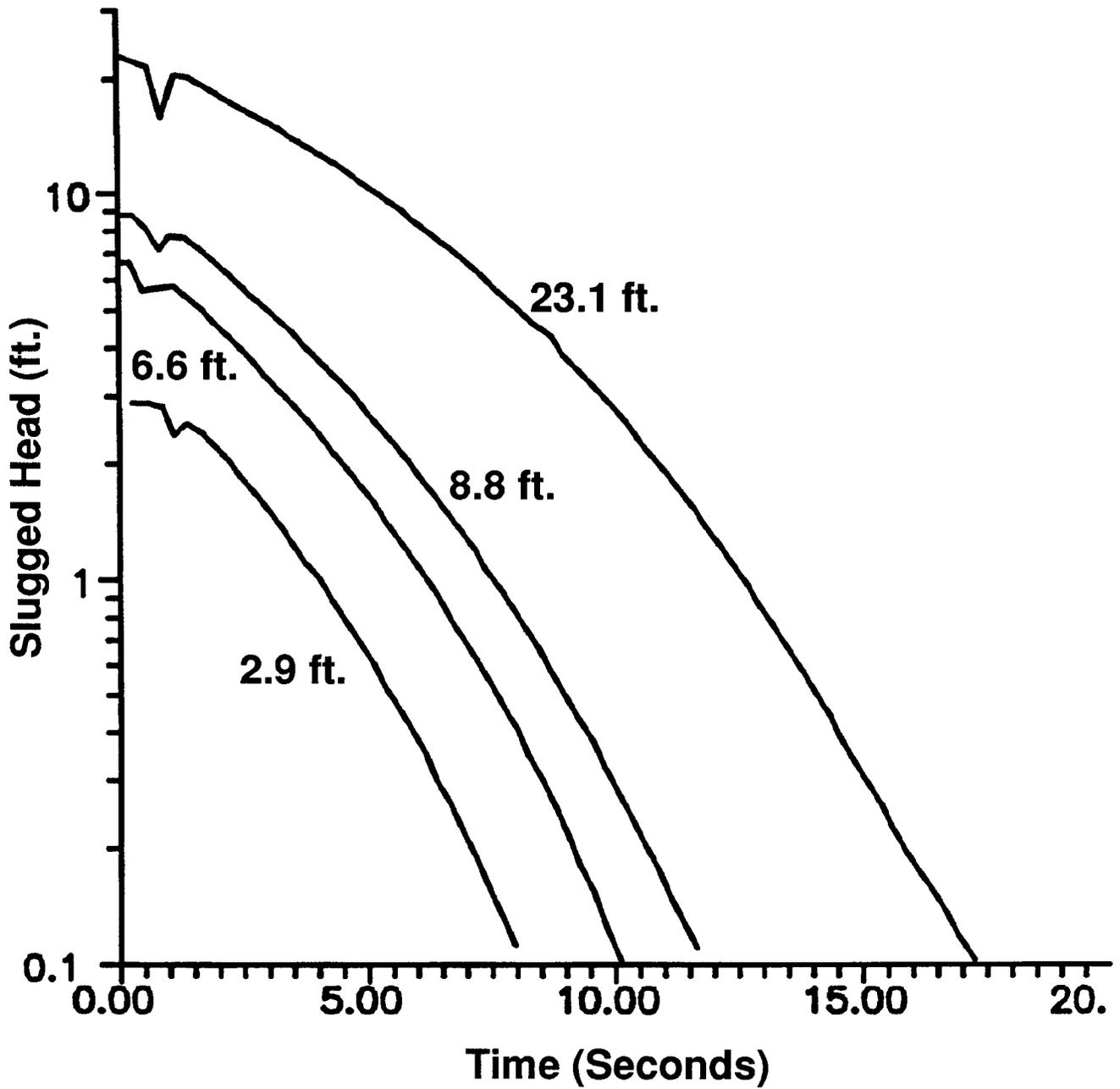


Figure 3.

Hvorslev Analysis With Frictional Head Loss

$$Q(t) = \pi r_c^2 \frac{\partial H(t)}{\partial t} = -FK \cdot h(t) \quad (1).$$

where

$Q(t)$ = flow of water into aquifer

$H(t)$ = height of water in well at any time,

$h(t)$ = head of water just outside the screen in the aquifer,

K = hydraulic conductivity,

F = Hvorslev geometric factor, and

r_c = casing radius.

Assume a loss in head across the screen due to friction

$$Q(t) = \left(\frac{H(t) - h(t)}{R} \right) \quad (2).$$

R is the resistance factor.

Replacing $h(t)$ in equation (1) with equation (2) gives

$$\pi r_c^2 \frac{\partial H(t)}{\partial t} = -FK \cdot (Q(t)R + H(t)) \quad (3).$$

Figure 4.

Resistance Proportional to a Power of the Velocity

Proportional to first power of velocity

$$R = A \cdot |V| = A \left| \frac{dH(t)}{dt} \right| \quad (1).$$

where A is an assumed constant of proportionality.
The result is:

$$\frac{dH(t)}{dt} \left[1 + FKA \left| \frac{dH(t)}{dt} \right| \right] = -\frac{FK}{\pi r_c^2} H(t) \quad (2).$$

The generalization for any power (N) of the velocity is:

$$R = A \cdot |V| = A \left| \frac{dH(t)}{dt} \right|^N. \quad (3).$$

Similarly, the generalization of equation (2) is:

$$\frac{dH(t)}{dt} \left[1 + FKA \left| \frac{dH(t)}{dt} \right|^N \right] = -\frac{FK}{\pi r_c^2} H(t). \quad (4).$$

Figure 5.

Non-Darcian Flow in the Hvorslev Model

Assume the hydraulic gradient is proportional to the first and second power of velocity

$$\frac{\partial h}{\partial r} = -\frac{1}{K}V + bV^2 \quad (1).$$

where b is a constant. After some algebra the result is the same as for the resistive loss case with $N = 1$.

$$\frac{dH(t)}{dt} \left[1 + FKA \left| \frac{dH(t)}{dt} \right| \right] = -\frac{FK}{\pi r_c^2} H(t) \quad (2).$$

where the relationship between A and b is

$$A = \frac{br_c^2}{4\pi L^2 r_w}. \quad (3).$$

L is the length of the well screen and r_w is the screen radius.

Figure 6.

Numerical Solution of Nonlinear Equations

Both the resistive and non-Darcian flow models can be represented by the same nonlinear equation in the variable $H(t)$. In general, it can not be solved in closed form. Using an iterative numerical solution technique, one obtains the following form:

$$H^{n+1(m+1)} = H^n - \frac{\frac{\Delta t}{2t_0} (H^{n+1(m)} + H^n)}{\left[1 + FKA \left| \frac{H^{n+1(m)} - H^n}{\Delta t} \right|^N \right]} \quad (1).$$

where we have also used the usual definition of the Hvorslev time lag

$$t_0 = \frac{\pi r_c^2}{FK} \quad (2).$$

and $H^{n(m)}$ is the m th iteration value at the n th time level. Equation (1) must be iterated for each time step until there is relatively little difference in the $H(t)$ for consecutive iterations.

Figure 7.

Theoretical Nonlinear Solutions

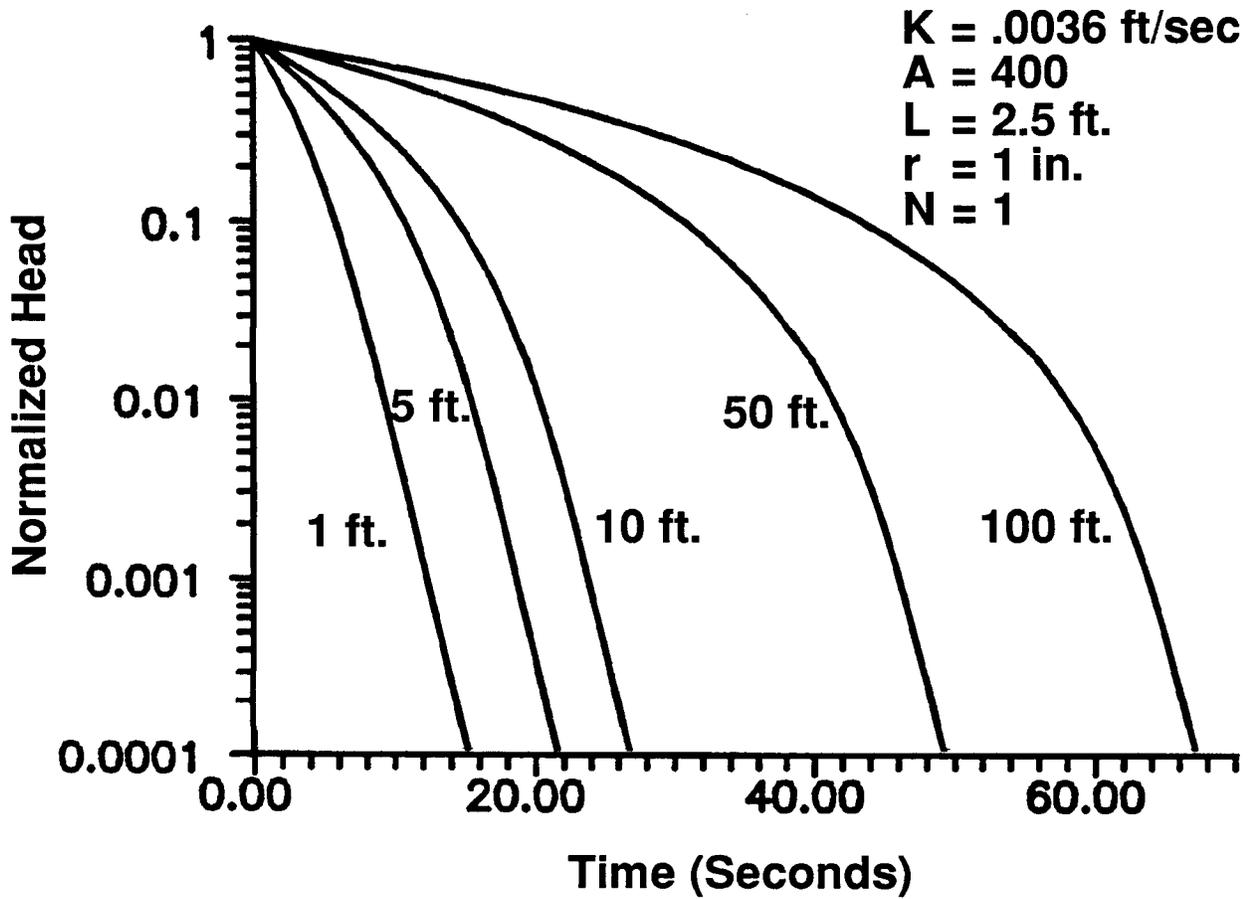


Figure 8.

Regression Fit of Head to Hvorslev Model

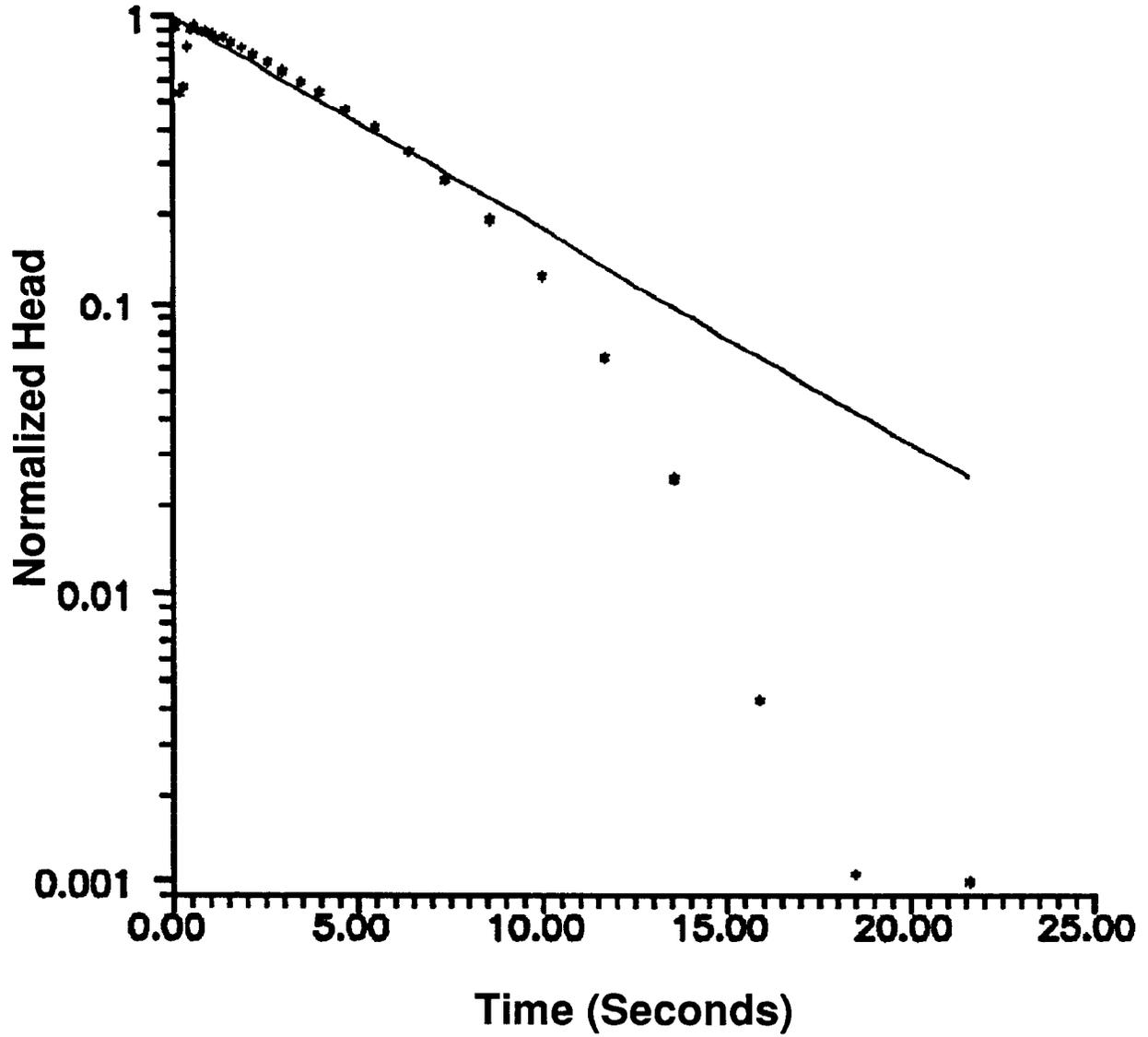


Figure 9.

Regression Fit of Head to Nonlinear Model

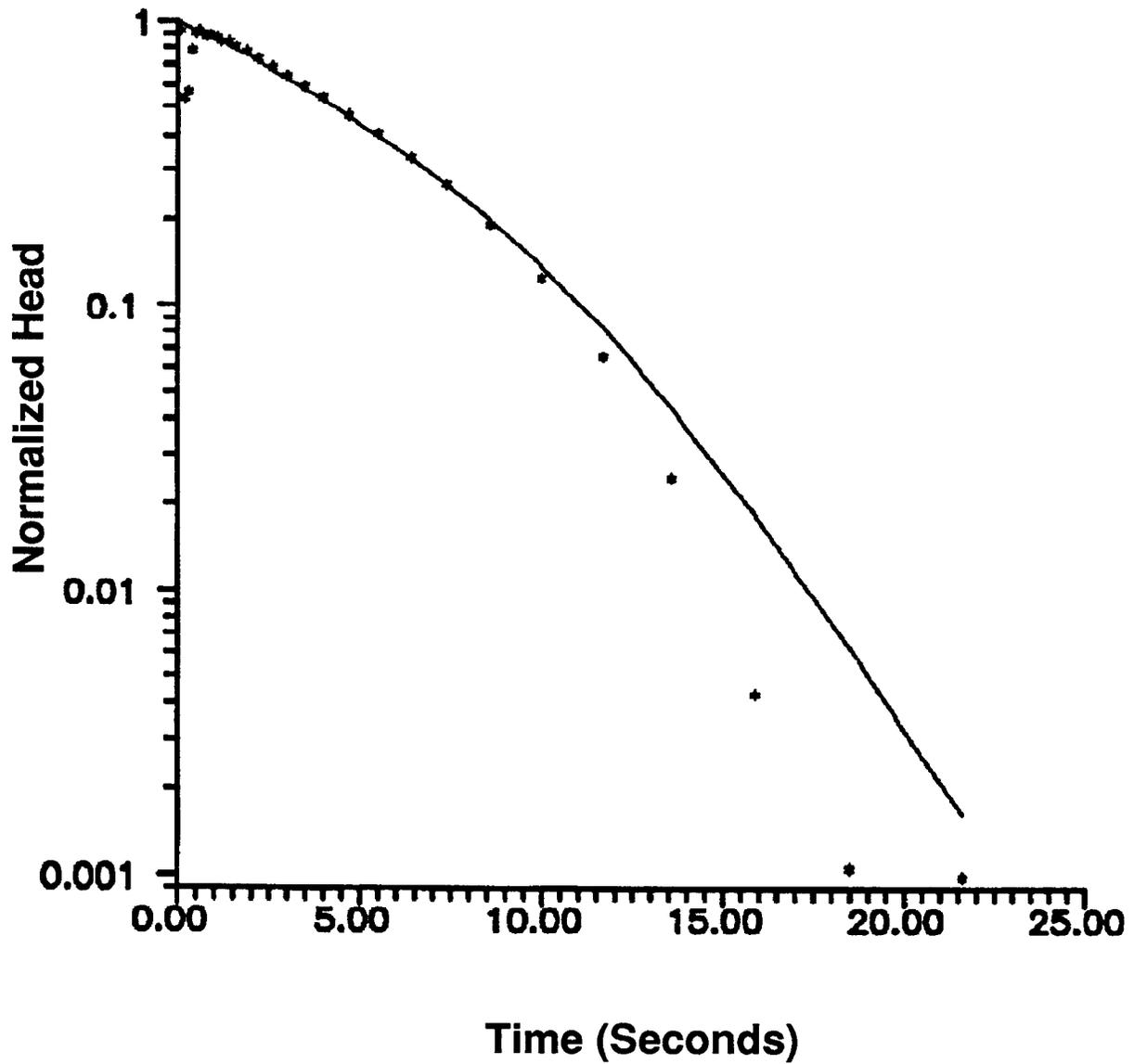


Figure 10.

Multiple Slug Tests at GEMS 02

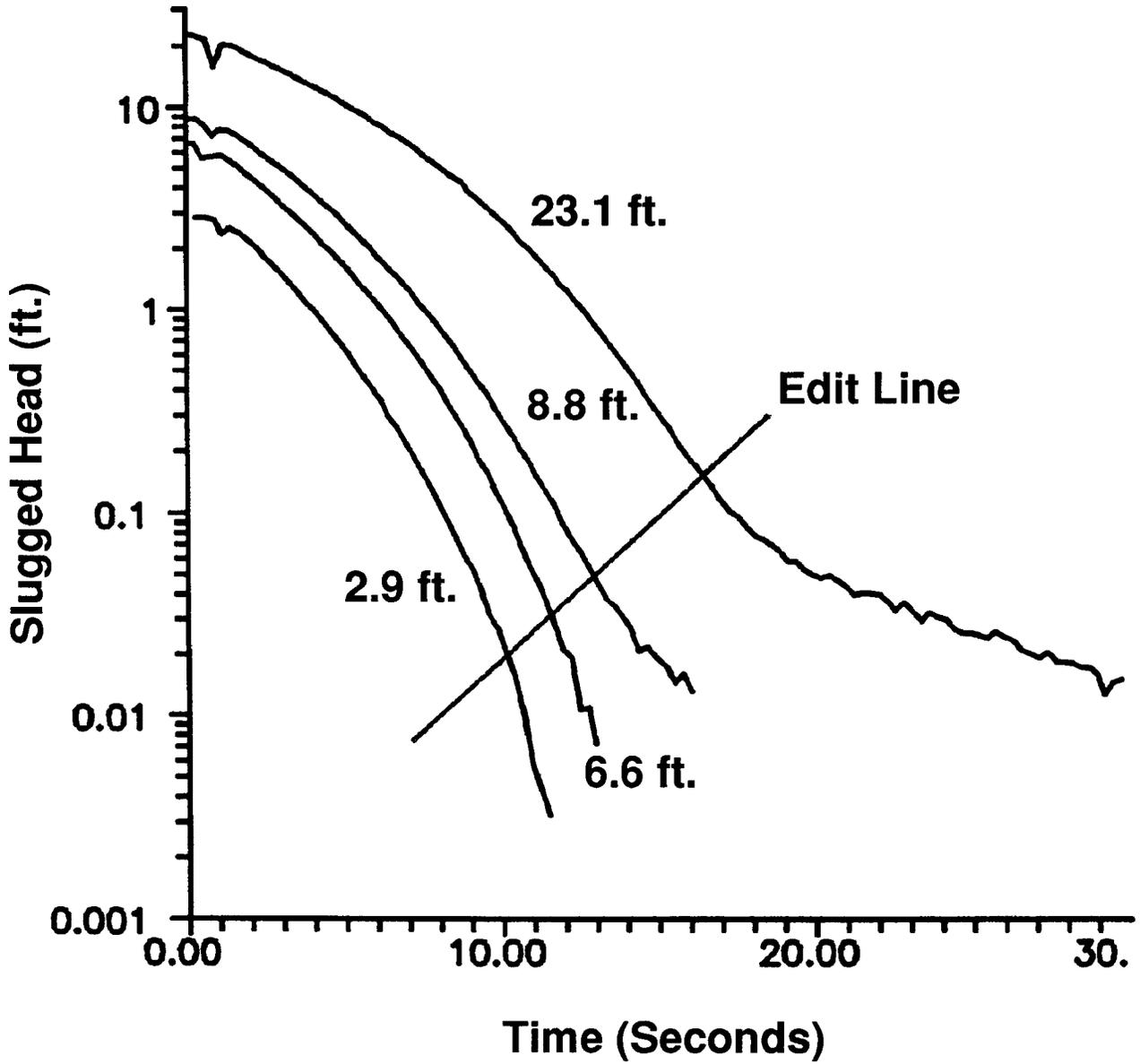


Figure 11.

Typical Fit of Nonlinear Model at Well GEMS 02

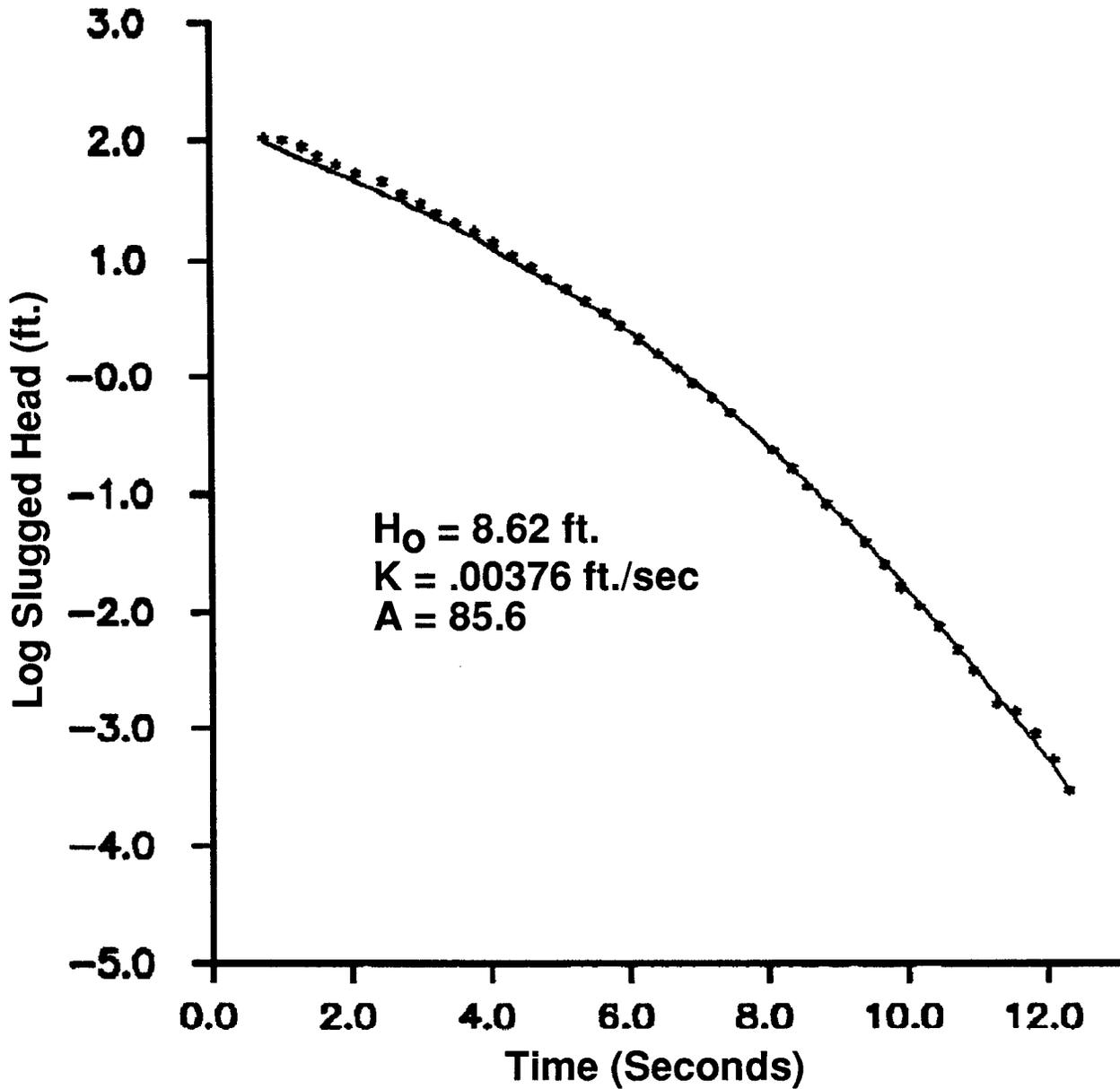


Figure 12.

Summary of Eight Slug Tests at Well GEMS 02

INITIAL HEAD	HYDRAULIC COND.	NON-LINEAR FACTOR
2.88 ft.	.00450 ft./sec	177.
6.63	.00414	116.
8.81	.00349	103.
23.1	.00308	77.0
2.75	.00485	131.
7.14	.00430	108.
8.62	.00376	85.6
23.1	.00296	63.9
Average	.00388	108.

Figure 13.

Summary

1. Slug tests in highly permeable aquifers seem to exhibit non-linear behavior.
2. Field data show a downward curvature.
3. Field data are dependent on the initial head.
4. A frictional loss or non-Darcian flow model seems to reproduce the noted behavior.
5. Reasonable consistency of hydraulic conductivity for various initial heads can be obtained.
6. Further refinement of non-linear models is needed.

Figure 14.