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EFFECTIVE TRANSMISSIVITIES FROM SLUG TEST IN WELLS WITH A SKIN

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Abstract

Slug tests are frequently used to characterize the transmissivity of an aquifer. However, in the presence of heterogeneity it is uncertain how the slug test is averaging the aquifer properties. In this paper, we look at the effect of one kind of heterogeneity: a well skin. A well skin can be created by the process of establishing the well, and may have a transmissivity value (T1) greater or less than that of the aquifer transmissivity (T2). We have investigated this problem using sensitivity analysis and an automated well-test analysis program we have developed. The sensitivity coefficients for T1 and T2 are similar is shape but shifted slightly in time. Thus, it is difficult to obtain good estimates for both T1 and T2 due to correlation. The maximum amplitude of the sensitivities is inversely proportional to transmissivity. Varying the skin radius shifts the head response curve along the dimensionless time axis. When fitted to the Cooper-Bredehoeft-Papadopulas (C-B-P) model, the data show a systematic lack of fit, however, it is not large. The effective transmissivity obtained from the C-B-P fit is a weighted average of T1 and T2 and can be predicted with a simple empirical formula. The effective transmissivity is highly weighted by the smallest transmissivity and is a weak function of the skin radius. Addition of observation wells makes little difference in the C-B-P fit results. The value of the effective transmissivity does not depend on the initial slug height. If one attempts a three parameter fit using T1, T2, and the skin radius (Rs), it is found that T1 and Rs are usually very highly correlated leading to a nonunique situation. If Rs is assumed known and one tries to solve for T1 and T2, the situation is better, but the fit is still difficult and depends on the initial estimates and the quality of the data. Having copious amounts of accurate data and multiple observation wells is the best situation for obtaining an accurate fit.

Introduction

Slug tests are frequently used to characterize the transmissivity of an aquifer. However, in the presence of heterogeneity it is uncertain how the slug test is averaging the aquifer properties. In this paper, we look at the effect of one kind of heterogeneity: a well skin. A well skin can be created by the process of establishing the well, and may have a transmissivity value (T1) greater or less than that of the aquifer transmissivity (T2).

MODEL FOR SLUG TESTS WITH SKIN

The physical situation is shown schematically in Figure 1. Mathematically the model is given by the following equations (Moench and Hsieh, 1985):

$$\frac{\partial^2 H_1}{\partial r^2} + \frac{1}{r}\frac{\partial H_1}{\partial r} = \frac{S_1}{T_1}\frac{\partial H_1}{\partial t} \qquad r_w \le r \le R_s$$

$$\frac{\partial^2 H_2}{\partial r^2} + \frac{1}{r}\frac{\partial H_2}{\partial r} = \frac{S_2}{T_2}\frac{\partial H_2}{\partial t} \qquad r \ge R_s$$

At the well bore

$$\pi r_c^2 \left(\frac{\partial H_1}{\partial t}\right)_{r_w} = \left(2\pi r T_1 \frac{\partial H_1}{\partial r}\right)_{r_w} \qquad r_c \equiv \text{casing radius}$$

Initial conditions:

$$H_1 = H_2 = 0 \quad at\ t = 0\ for\ r > r_w$$
$$H_1 = H_0 \ \ at\ \ t = 0\ for\ r = r_w$$

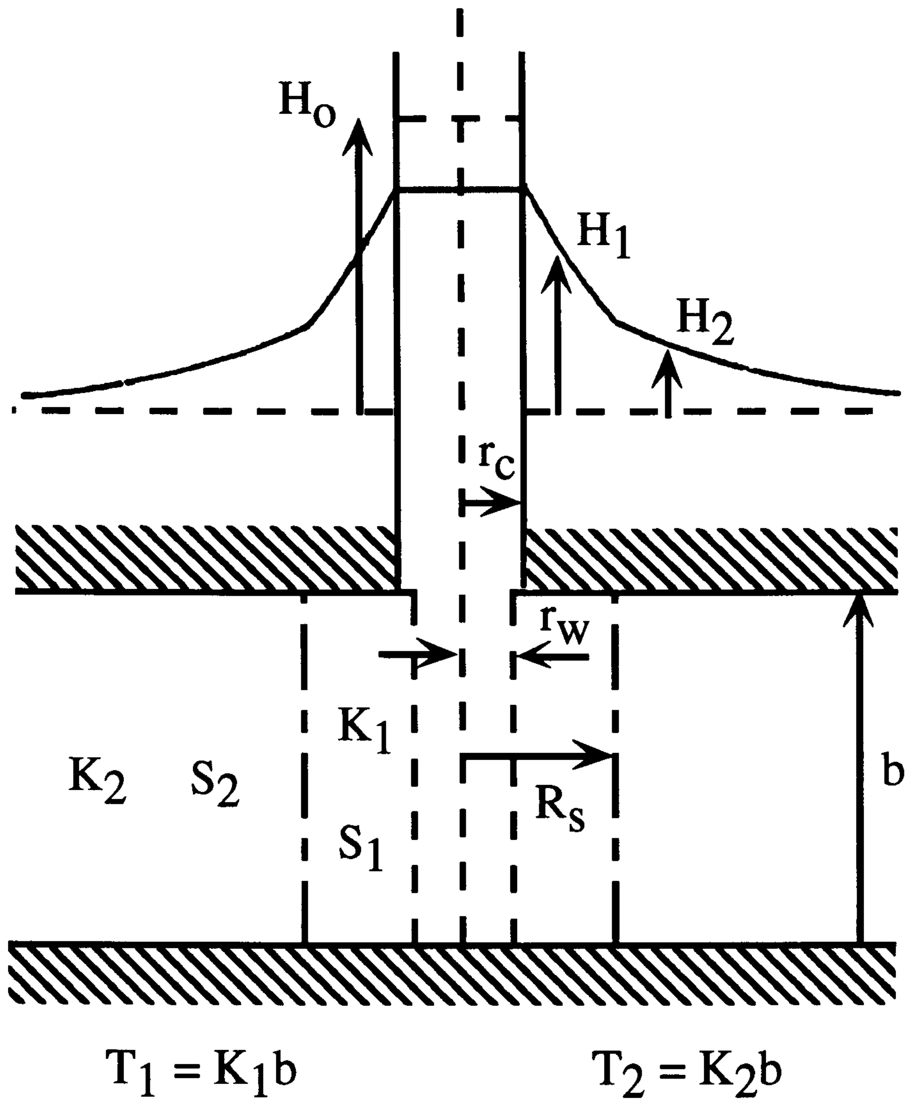
The boundary conditions between the skin and aquifer are

$$H_1 = H_2\ at\ r = R_s$$
$$T_1 \frac{\partial H_1}{\partial r} = T_2 \frac{\partial H_2}{\partial r}\ \ at\ r = R_s$$

Far away from the slugged well

$$H_2 = 0\ \ at\ \ r = \infty$$

Figure 1. Schematic of a slug test in a well with a skin.



NORMALIZED VARIABLES

The solutions can be expressed more conveniently for a range of parameters by using normalized variables.

$$h = \begin{cases} H_1 / H_0 & r_w \le r \le R_s \\ H_2 / H_0 & r \ge R_s \end{cases}$$

$$R = r / r_w$$

$$\beta_1 = \frac{T_1}{r_w^2} t$$

$$\beta_2 = \frac{T_2 t}{r_w^2}$$

In this paper we only look at the effects of T_1, T_2 and R_s. The storage coefficient is assumed constant at 10^{-3}.

SENSITIVITY COEFFICIENTS

We apply first order sensitivity analysis (McElwee, 1987) to the problem. The first order Taylor expansion for the head is

$$H^* \cong H^m + U_{T_1}^m \Delta T_1^m + U_{T_2}^m \Delta T_2^m + U_{R_s}^m \Delta R_s^m$$

H^*: vector of heads based on true parameters T_1^*, T_2^*, R_s^*

H^m: vector of heads based on current estimates T_1^m, T_2^m, R_s^m

$$U_{T_1}^m = \frac{\partial H^m}{\partial T_1^m}, U_{T_2}^m = \frac{\partial H^m}{\partial T_2^m}, U_{R_s}^m = \frac{\partial H^m}{\partial R_s^m}\text{: sensitivities to } T_1^m, T_2^m, R_s^m$$

$\Delta T_1^m, \Delta T_2^m, \Delta R_s^m$: unknown perturbations in skin transmissivity, aquifer transmissivity and skin radius.

NORMALIZED SENSITIVITIES TO RELATIVE HEAD

$$U'_{T_1} = T_1 \frac{\partial H}{\partial T} = H_0 T_1 \frac{\partial h}{\partial T_1} = H_0 u'_{T_1}$$

$$u'_{T_1} = T_1 \frac{\partial h}{\partial T_1}$$

$$U'_{T_2} = T_2 \frac{\partial H}{\partial T_2} = H_0 \, T_2 \frac{\partial h}{\partial T_2} = H_0 \, u'_{T_2}$$

$$u'_{T_2} = T_2 \frac{\partial h}{\partial T_2}$$

$$U'_{R_s} = R_S \frac{\partial H}{\partial R_s} = H_0 \, R_s \frac{\partial h}{\partial R_s} = H_0 \, u'_{R_s}$$

$$u'_{R_s} = R_s \frac{\partial h}{\partial R_s}$$

u'_{T_1}, u'_{T_2}, and u'_{R_s} are functions of $T_1 \, T_2$, and R_s. We shall look at these functions in greater detail later.

PARAMETER ESTIMATION

OBJECTIVE: Minimize $E = \sum_i \left[H_i^e - H_i\right]^2$

H_i^e = observed head at index point i

H_i = calculated head at index point i

ASSUME: $H^e = H^* + \varepsilon$

ε = error vector

THUS: $H^e - H^m = U_{T_1}^m \Delta T_1^m + U_{T_2}^m \Delta T_2^m + U_{R_s}^m \Delta R_s^m + \varepsilon$

SOLVE: New parameter estimates

$T_1^{m+1} = T_1^m + \Delta T_1^m \quad T_2^{m+1} = T_2^m + \Delta T_2^m \quad R_s^{m+1} = R_s^m + \Delta R_s^m$

SENSITIVITY DESIGN MATRIX

$$a_{ij} = [A]_{ij} = \sum_{r,t} U_i(r,t)U_j(r,t)$$

$$i,\ j = T_1,\ T_2,\ R_s$$

The sensitivity design matrix [A] defined here is a sum over time and space of products for any two sensitivity coefficients. If we are fitting all three parameters T_1, T_2, and R_s then the sensitivity design matrix is 3 x 3. The least squares solution for the delta parameter changes can be expressed in terms of the inverse of [A]. In general, the solution is well behaved if the diagonal elements are large and nearly equal and the off-diagonal elements are small. This will be the case if the sensitivity coefficients are large and do not have similar shapes over the measurement times and locations.

SENSITIVITY CORRELATION MATRIX

$$c_{ij} = [C]_{ij} = \frac{a_{ij}}{\sqrt{a_{ii}a_{jj}}}$$

One way to measure the similarity of the sensitivity coefficients is to define the sensitivity correlation matrix as shown here; it will have ones on the diagonal and the off-diagonal terms will vary between ± 1. If any of the off-diagonal terms are exactly one, the inverse of [A] does not exist and the inverse problem can not be solved for aquifer parameters. From a practical standpoint, anytime the off-diagonal elements of the correlation matrix get above .9 the [A] matrix becomes ill-conditioned rather rapidly and the inverse solution becomes more unreliable.

PARAMETER COVARIANCE MATRIX

$$COV(P) = [B] = [A]^{-1}\sigma^2$$

σ^2 = head variance

Estimated Standard Error of Parameter i is given by

$$\sqrt{b_{ii}}$$

As long as an inverse of [A] can be found, the reliability of the parameter estimates can be assessed by looking at the parameter covariance matrix defined here. The form shown here results from some simplifying assumptions about the errors in head such as additive, zero mean, noncorrelated and constant variance. With these assumptions the estimated standard errors of the parameters are given by the square roots of the diagonal elements of the parameter covariance matrix.

Sensitivities for T_1 and T_2

The sensitivities for T_1 and T_2 are very similar in shape as shown in Figure 2. However, the sensitivity for T_2 is shifted slightly to larger time values. Figure 2 shows the sensitivity to the skin region when it has the same value for transmissivity as the aquifer. Figure 3 shows the sensitivities when the skin transmissivity is an order of magnitude smaller. Note that the maximum amplitude of the sensitivity is inversely proportional to the transmissivity. The lower sensitivity for T_2 coupled with the similarity in shape (meaning correlation is high) indicates that it is going to be very difficult to get accurate estimates for T_2. Away from the slugged well there is some difference in shape between sensitivities for T_1 and T_2, however, the amplitude decays rapidly making it difficult to utilize these differences.

Figure 2.
Variation of u'_T With Time at the Slugged Well

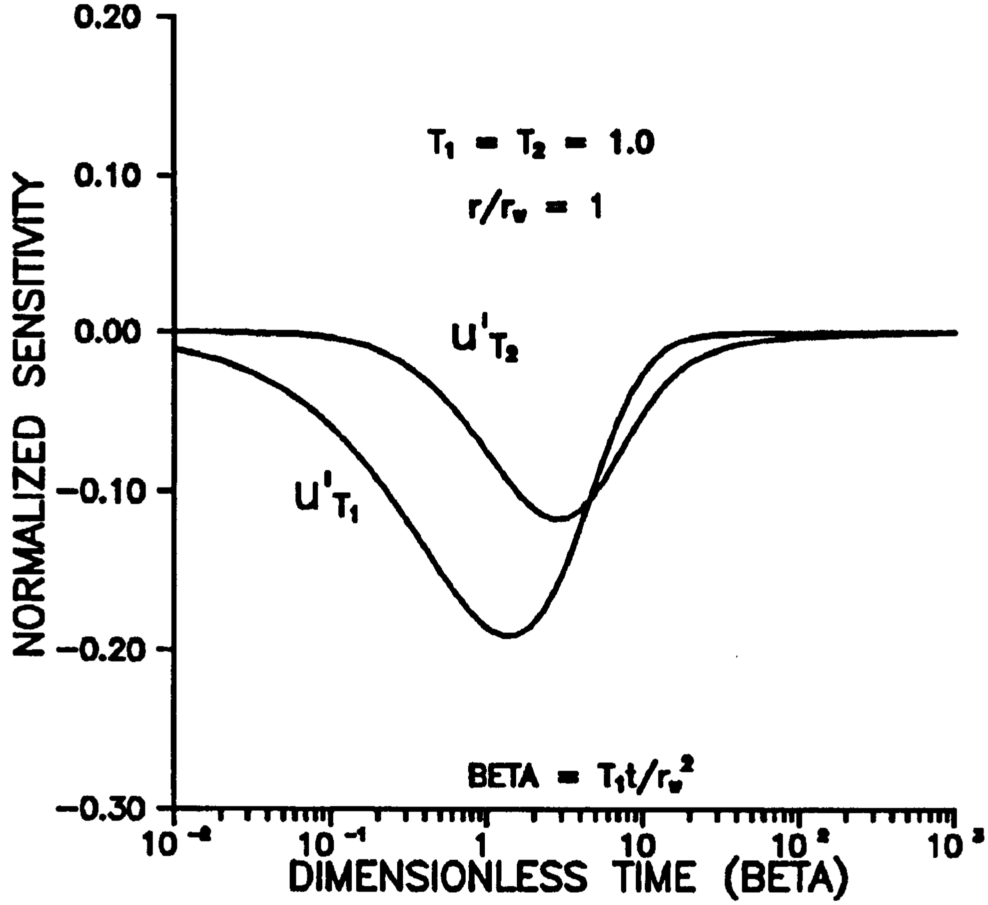
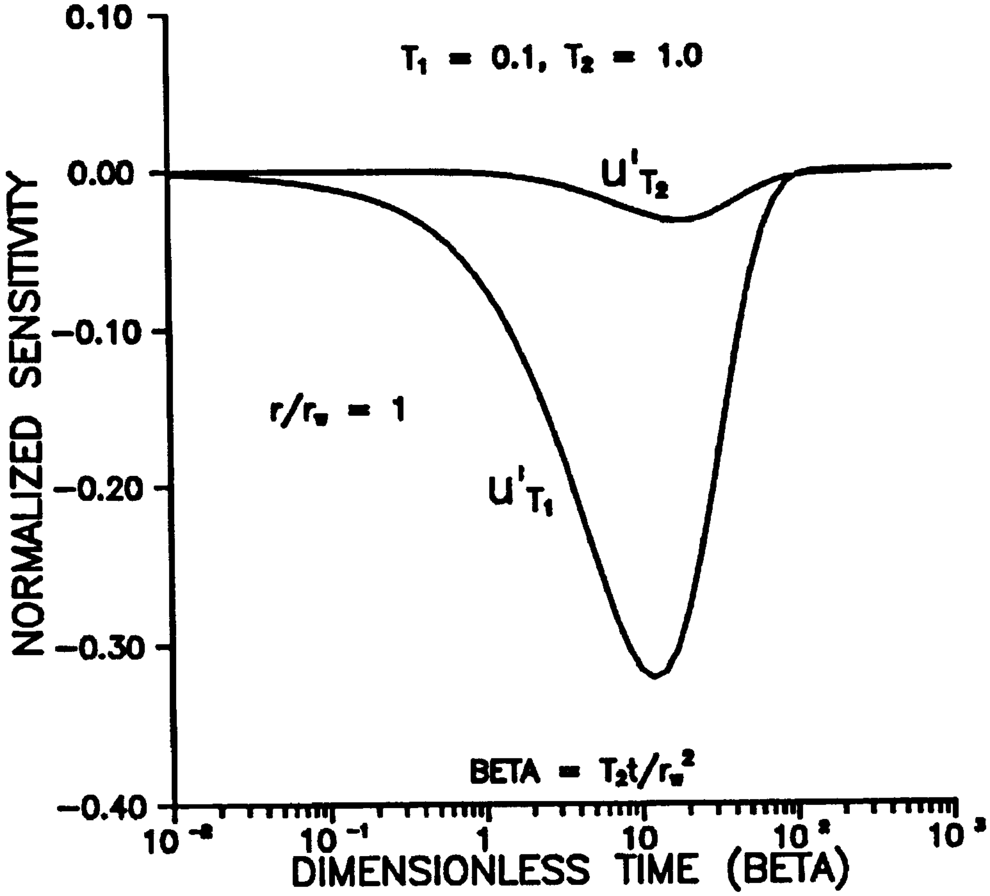


Figure 3.
Variation of u'_T With Time at the Slugged Well



Effect of Varying the Skin Radius

Varying the skin radius shifts the head response curve along the dimensionless time axis for both the slugged well and observation wells. Figure 4 shows the normalized head in the slugged well for various skin radii. Figure 5 is a similar plot for an observation well located at 100 well radii away from the slugged well. Increasing the skin radius shifts the head response curve to larger dimensionless time (beta) when $T_1 << T_2$. This is true for both the slugged well and the observation well. However, the response in the observation well declines with increasing skin radius for $T_1 << T_2$.

Figure 4.
Variation of Head in Slugged Well With Skin Radius

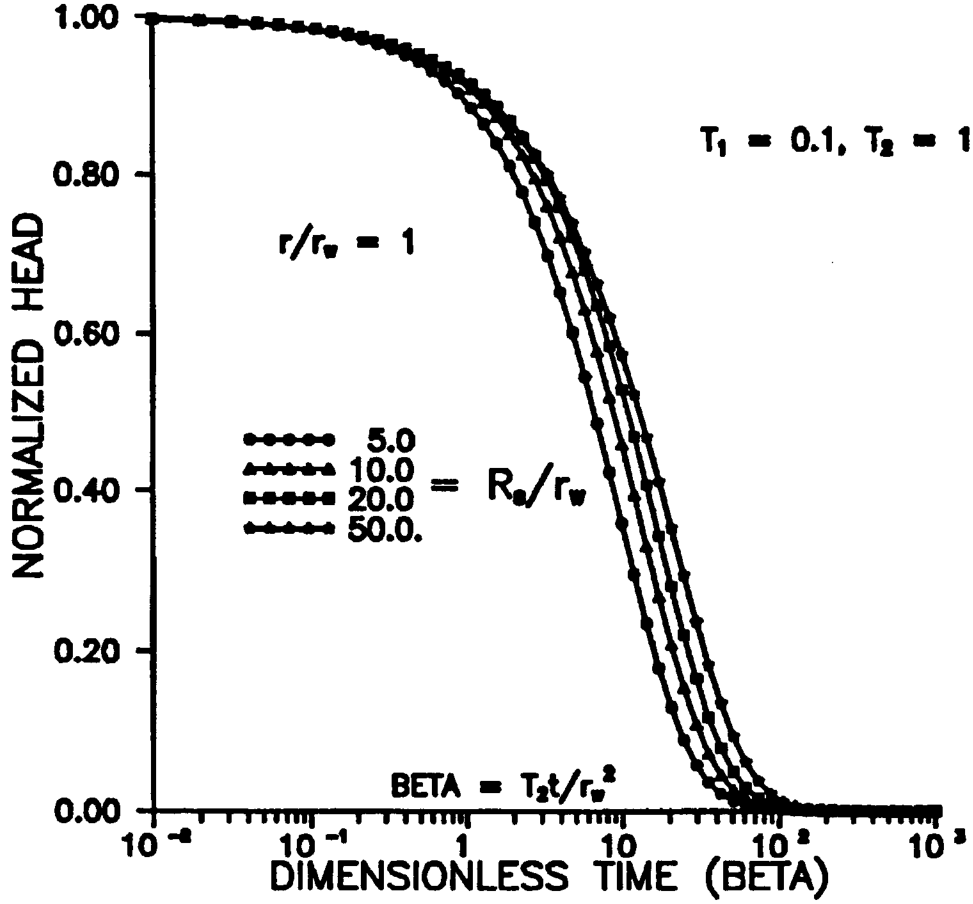
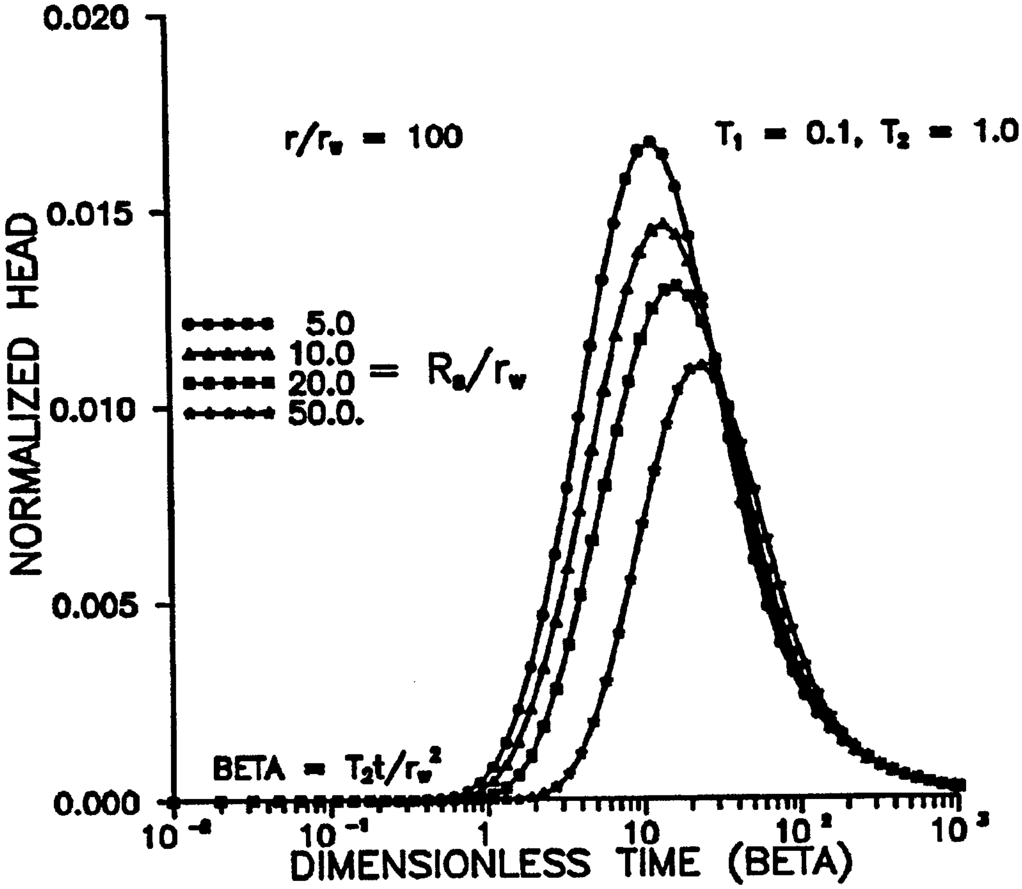


Figure 5.
Variation of Head in Obs. Well With Skin Radius



Fitting Well Skin Data to the C-B-P Model

Many times the C-B-P model (Cooper et al., 1967) is used to fit slug test data. The obvious question is: What is the effective transmissivity when a well skin is present? Figure 6 shows a resulting fit with SUPRPUMP (Bohling and McElwee, 1992). The fitted data show a systematic deviation which may be diagnostic. The skin data is greater than the C-B-P model for early time and less than the C-B-P model for larger times, when $T_1 << T_2$. In general, the effective transmissivity resulting from the application of the C-B-P model to well skin data is an average of the skin and aquifer transmissivities. We have found that a good empirical equation for the effective transmissivity is

$$\frac{1}{T_{eff}} = \left[\frac{\ln(R_s / r_w)}{T_1} + \frac{\ln(r_{eff} / r_w) - \ln(R_s / r_w)}{T_2} \right]$$

r_{eff} = effective radius influenced by the slug test.

$$r_{eff} / r_w = [C / S]^{1/2} \quad 1 \leq C \leq 2$$

S = storage coefficient

The result given by this equation is highly weighted by T_1 and has a weak dependence on the skin radius when $T_1 << T_2$. In general, the lowest value of transmissivity, (whether it is the aquifer or the skin) will be the dominant factor in determining the effective transmissivity. Table 1 gives some typical results for effective transmissivity for varying skin radii and transmissivity distributions. As one might expect the effective transmissivity is independent of initial slug height.

Figure 6.
Fit of Well Skin Data to the C–B–P Model

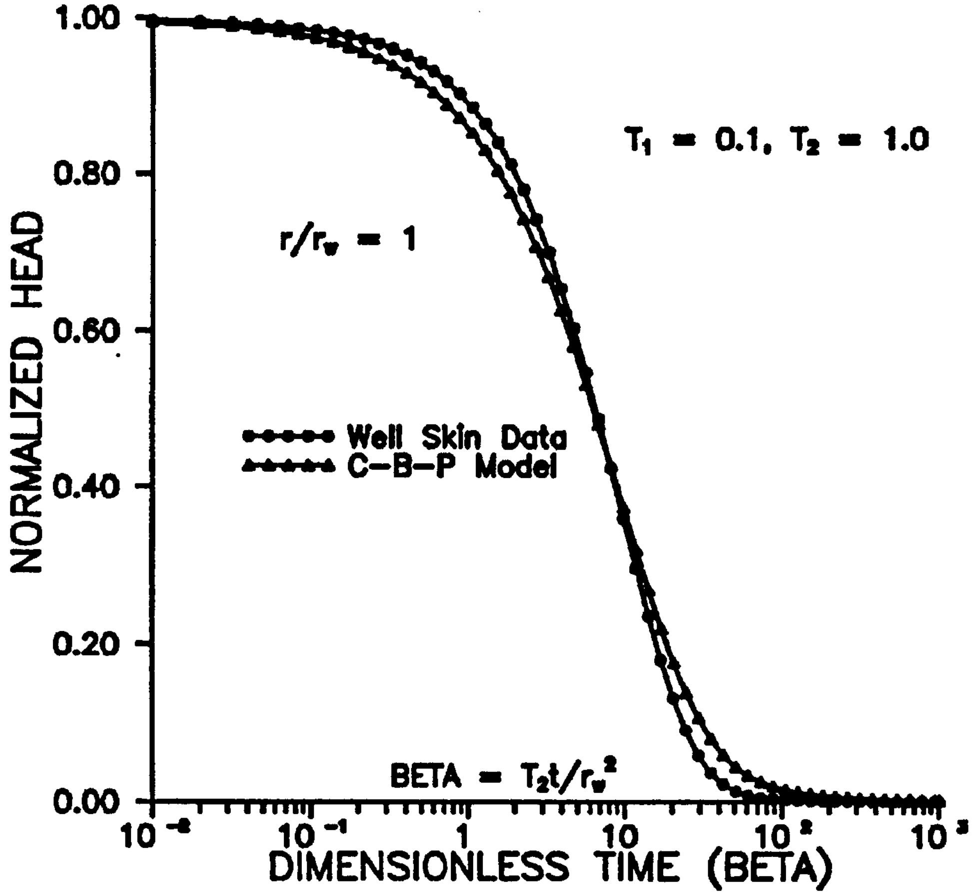


Table 1
Effective Transmissivities in the Presence of a Skin

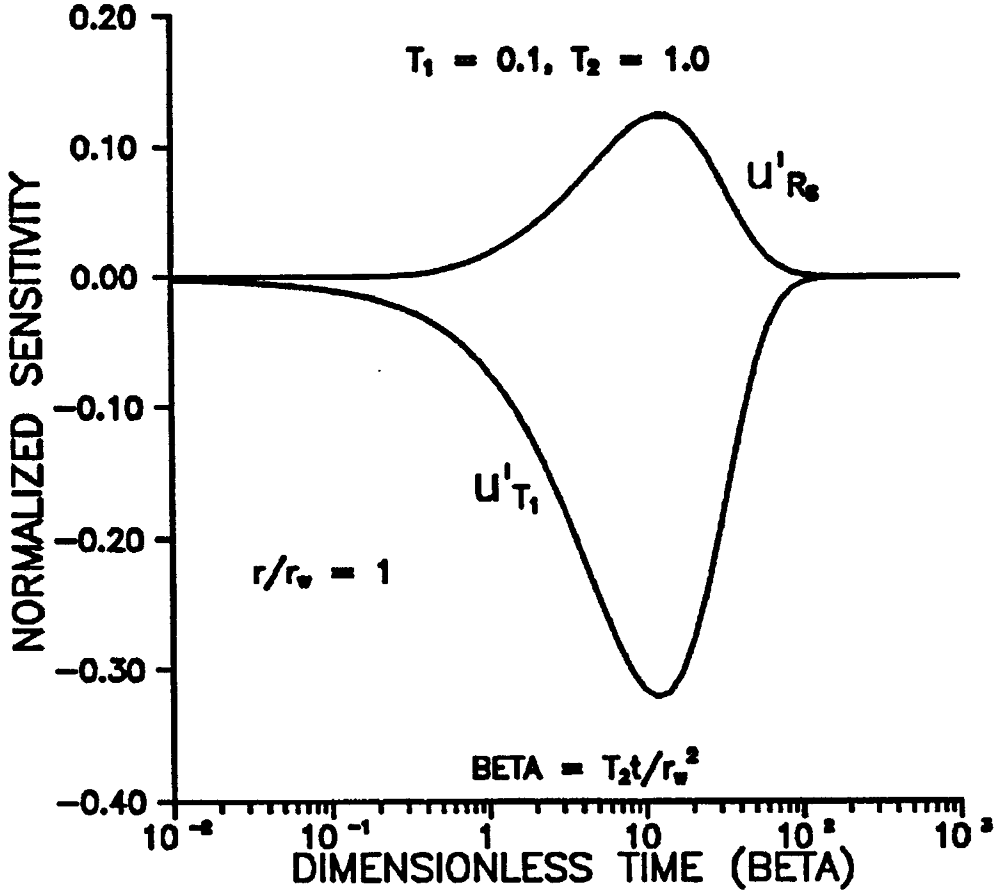
T_1	T_2	R_s/r_w	T_{eff} C-B-P	C (emp.)	T_{eff} (emp.)
0.1	1.0	5	.205	2	.208
0.1	1.0	10	.155	2	.155
0.1	1.0	20	.126	2	.124
1.0	0.1	5	.175	1	.172
1.0	0.1	10	.260	1	.250
1.0	0.1	20	.456	1	.456

The storage coefficient was 10^{-3} for all the above simulations and an observation well was used at 100 r_w.

Parameter Estimation with the Well Skin Model

The well skin model of Moench and Hsieh has been implemented in SUPRPUMP and can be used to look at sensitivities and to fit the data. Figure 7 is a plot of the sensitivities to T_1 and R_s. The sensitivity to R_s is smaller than to T_1 and is of opposite sign. However, the shape of the two sensitivities in Figure 7 is almost identical and very high correlation is the general rule. Away from the slugged well there is some difference in shape between sensitivities for T_1 and R_s, however, the amplitude decays rapidly making it difficult to utilize these differences. Therefore, it is extremely difficult to obtain good estimates for both T_1 and R_s. Usually there will be many pairs of values of T_1 and R_s that will give equally good results. In tests we have run, the high correlation persists even if observation wells are available and used. In some cases the approximate value for the skin radius may be known from the diameter of the hole made by the drilling equipment; in this case one can try holding R_s constant and just fitting T_1 and T_2.

Figure 7.
Comparison of u'_{T_1} and u'_{R_s} at the Slugged Well



Three Parameter Fit

Table 2 shows the output from the SUPRPUMP package used in design mode for the three parameter fit (T_1, T_2, and R_S) of the Moench and Hsieh well skin solution. Four observation wells were used at r/r_W of 1, 25, 50 and 100 for 56 time measurements between .01 and 1000. units of dimensionless time (BETA). The 95% confidence limits show that T_1 and T_2 could be determined within about ± 40%, however, the skin radius (R_S) is very poorly determined. The root causes of these results can be seen by looking at the matrix of normalized sensitivities, the sensitivity correlation matrix, and the parameter correlation matrix in Table 2. The diagonal elements of the matrix of normalized sensitivities shows that T_1 has the highest sensitivity by far. The sensitivity correlation matrix and the parameter correlation matrix shows that the correlation between T_1 and R_s is greater than .99 . These results are for perfect model data with an assumed accuracy of ± .025 Ho. In the real world things are likely to be much worse.

TABLE 2
SUPRPUMP OUTPUT
THREE PARAMETER FIT

The estimated root-mean-squared residual is .2500E-01

The parameter values with approximate 95% confidence intervals are:

Parameter	Value	Lower Bound	Upper Bound
TRANSMISS. OF AQUIFER	1.000	.6227	1.377
TRANSMISS. OF SKIN	.1000	.5668E-01	.1433
SKIN RADIUS	10.00	-1.459	21.46

For the following arrays:
Col-Row 1 represents TRANSMISSIVITY OF AQUIFER (T2)
Col-Row 2 represents TRANSMISSIVITY OF SKIN (T1)
Col-Row 3 represents SKIN RADIUS (Rs)

Raw crossproducts matrix of normalized sensitivities:

	1	2	3	
1	.2357E-01	.8306E-01	-.3155E-01	This matrix shows that
2	.8306E-01	1.093	-.4113	T1 has the highest
3	-.3155E-01	-.4113	.1566	sensitivity by far.

The reciprocal condition number of the
sensitivity crossproducts matrix is .1079E-02

Sensitivity correlation matrix:

	1	2	3	
1	1.000	.5174	-.5192	
2	.5174	1.000	-.9940	--------Very high correlation
3	-.5192	-.9940	1.000	between T1 and Rs.

Covariance matrix of normalized parameter variations:

	1	2	3
1	.3630E-01	-.6035E-03	.5727E-02
2	-.6035E-03	.4787E-01	.1256
3	.5727E-02	.1256	.3349

Parameter correlation matrix:

	1	2	3	
1	1.000	-.1448E-01	.5194E-01	
2	-.1448E-01	1.000	.9918	--------Very high correlation
3	.5194E-01	.9918	1.000	between T1 and Rs.

Alluvial Aquifer Example

Figures 8 and 9 show a real example of a slug test performed in the Kansas River valley near Lawrence, Ks. Figure 8 shows the fit of the data to the C-B-P model. There is a systematic deviation present and we thought it might be explained by a skin effect. Figure 9 shows the same data fit to the Moench and Hsieh model. The fit is much better, but it was difficult to obtain and is very sensitive to the number of parameters fit and the initial estimates for those parameters. Many of the analyses did not converge. Consequently, even though the fit is much better, we do not believe very much accuracy can be ascribed to the estimated aquifer conductivity. On the other hand, the effective conductivity from the C-B-P model and the skin conductivity from the Moench and Hsieh model agree fairly well and can probably be used with some confidence. Unfortunately, it is the aquifer conductivity that is of greatest interest.

Figure 8.
Analysis of an Alluvial Aquifer Slug Test
With the C-B-P Model

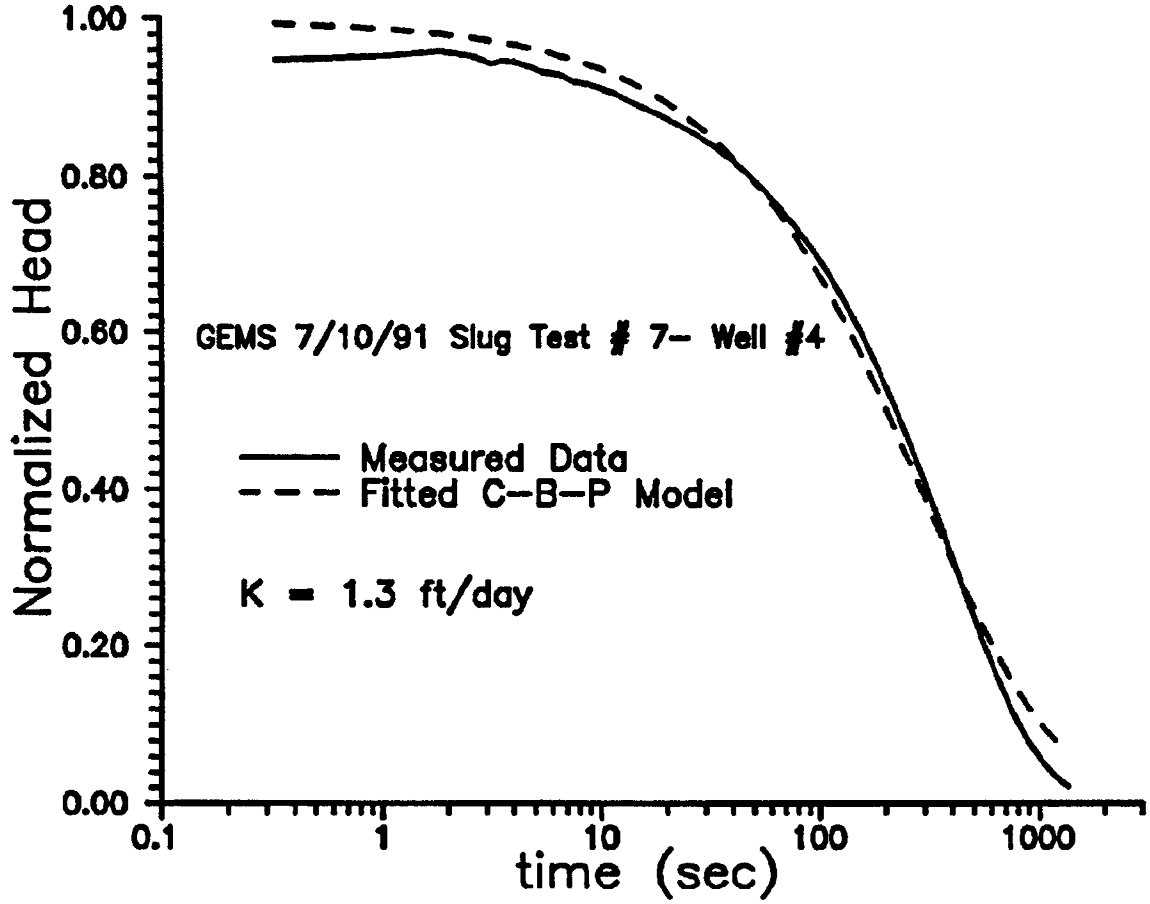
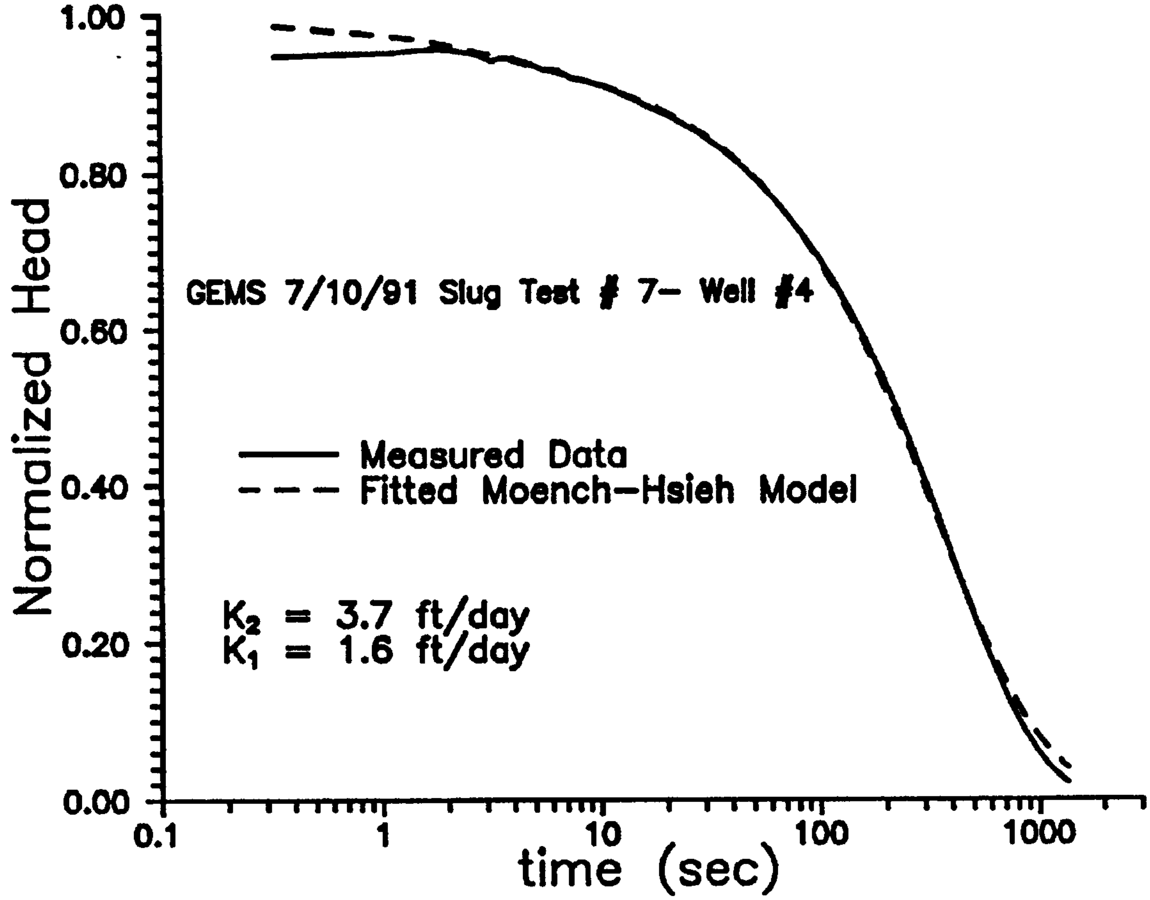


Figure 9.
Analysis of an Alluvial Aquifer Slug Test
With the Skin Model



Summary

We have investigated the problem of a slugged well having a well skin using sensitivity analysis and an automated well-test analysis program we have developed. The sensitivity coefficients for T_1 and T_2 are similar is shape but shifted slightly in time. Thus, it is difficult to obtain good estimates for both T_1 and T_2 due to correlation. The maximum amplitude of the sensitivities is inversely proportional to transmissivity. Varying the skin radius shifts the head response curve along the dimensionless time axis. When fitted to the Cooper-Bredehoeft-Papadopulas (C-B-P) model, the data show a systematic lack of fit, however, it is not large. The effective transmissivity obtained from the C-B-P fit is a weighted average of T_1 and T_2 and can be predicted with a simple empirical formula. The effective transmissivity is highly weighted by the smallest transmissivity and is a weak function of the skin radius. If one attempts a three parameter fit using T_1, T_2, and the skin radius (R_s), it is found that T_1 and R_s are usually very highly correlated leading to a nonunique situation. If R_s is assumed known and one tries to solve for T_1 and T_2, the situation is better, but the fit is still difficult and depends on the initial estimates and the quality of the data. Having copious amounts of accurate data and multiple observation wells is the best situation for obtaining an accurate fit.

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