Kansas Geological Survey

Stream-Aquifer Numerical Modeling of the Kinsley to Great Bend Reach of the Arkansas River in Central Kansas

Report of First-year Results

by

Marios Sophocleous and Seid Pourtakdoust

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Introduction

The objectives of this study, as outlined in the Kansas Water Office–Kansas Geological Survey cooperative agreement (Kansas Water Office Contract No.91-9) are as follows:

1. To define the geologic and hydrologic relationship between groundwater and surface water in the reach of the Arkansas River from Kinsley to Great Bend;
2. To evaluate the impacts of groundwater management alternatives on streamflows in the river reach; and
3. To evaluate recovery of regional groundwater in response to increased streamflow in the river reach.

In this report we outline the construction and application of a stream-aquifer numerical model for the Kinsley to Great Bend reach of the Arkansas River and compile the necessary data for this purpose. We also report on the calibration of such a model using parameter optimization techniques.

The report is organized into four parts: (1) methodology employed and basic data analysis; (2) model implementation and calibration; (3) numerical modeling results and related analyses; and (4) management alternatives to be tested and other work ahead.

Methodology and Data Analysis Results

The methodology we employed in this study consists of three approaches: (1) compilation and analysis of existing information; (2) limited field data collection, and (3) numerical modeling based on items 1 and 2. A brief summary of the main components of each approach together with some basic data analysis results follows.

Compilation and analysis of existing information

1. A comprehensive bedrock and predevelopment water-level map for the Great Bend Prairie region (in which the study area belongs) based on all available data accessible and/or known to us

has been prepared and documented separately (Sophocleous et al., 1990). A predevelopment water-level map and a bedrock map of the study area are shown in Figs. 1 and 2.

2. A soils map of the Arkansas River valley from Kinsley to Great Bend has been constructed (Fig. 3) based on Soil Conservation Service county reports. Also, the available water capacities of the different soils have been compiled (Table 1).

Table 1. Available Water Capacities (AWC, in inches) for the upper 60 Inches of the Study Area Soils

Soil Association	Corresponding Percent Membership in Soil Association	AWC
Platte–Waldeck	50–55	4.7
Pratt–Tivoli	59–41	5.1
Pratt–Carwile	81–19	7.0
Attica–Pratt–Carwile	57–31–12	7.9
Naron–Farnum	75–25	9.3
Naron–Carwile	75–25	9.8
Farnum–Lubbock	65–35	10.8

3. All current (as of 1990) water rights for the Great Bend Prairie region were obtained on tape from the Division of Water Resources, and the groundwater rights have been processed and displayed on a 1:250,000 map (Sophocleous, 1990). Figure 4 displays the groundwater rights in the study area, and Fig. 5 depicts the number of groundwater rights issued in the study area versus time.

4. Current and historic streamflow data for the area streams have been compiled and analyzed. Average annual streamflows of the Arkansas River at Kinsley and Great Bend and of the Pawnee River at Larned are shown in Figs. 6, 7, and 8, respectively. Streamflow gains or losses between Kinsley and Great Bend are shown in Figs. 9 and 10. As can be seen in these last figures, the frequency of streamflow losses from Kinsley to Great Bend increases after the early 1960's. Other stream-related data, such as stream widths and stream slopes were obtained from topographic and other maps.

5. Climatic data were obtained from existing NOAA climatic stations in Kansas, and from an ongoing Groundwater Management District 5–Kansas Geological Survey (GMD5, KGS) cooperative study on recharge assessment in GMD5, which encompasses most of the Great Bend Prairie region, including the study area. The 1980–1990 average annual precipitation at Kinsley is 25.19 in., at Larned 22.76 in., and at Great Bend 24.68 in.

6. Water level data from the Great Bend aquifer for various years since the 1970's have been examined and analyzed to delineate groundwater flow lines for possible selection of one boundary flow line separating the Arkansas River valley system from the Great Bend aquifer proper.

7. Most, if not all, available reports related to the study area have been examined for useful information transferable to this study.

8. A number of driller's logs from the study area, characterized by their detailed and careful descriptions, were selected and used in a computer program (Ed Gutentag, U.S. Geological Survey, personal communication, 1991) to estimate the hydraulic conductivity and storativity of the aquifer based on the lithology and thickness of the water-saturated strata. The results of this effort are shown in Figs. 11 and 12.

Based on items 7 and 8, a compilation of hydrogeologic properties of the Arkansas River alluvial aquifer has been constructed (Table 2).

Hydrogeology and Pleistocene history of the Great Bend Prairie with emphasis on the study area

Knowing the geologic history and geologic composition and structure of the study area is a prerequisite to understanding the water-bearing and water-yielding properties of the modeled stream-aquifer system.

The Great Bend Prairie is covered with a veneer of loess deposits and sand dunes, with underlying Pleistocene alluvium forming the major aquifer of the area (Latta, 1950; Fader and Stullken, 1978). This alluvium was deposited by the ancestral Arkansas River and a small number

Table 2 Hydrogeologic properties of the Great Bend Prairie aquifer including the Arkansas River alluvium

Methodology	Transmissivity $\underline{T}$(ft^2/d)	Hydraulic Conductivity $\underline{K}$(ft/d)	Storativity $\underline{S}$	Average Saturated Thickness (ft)	Source
5 aquifer tests	7,000–16,000	56–128	0.004–0.17	125	Fader & Stullken, 1978
Specific capacities of 235 irrigation wells	2,500–35,000 (ave. = 11,000)			125	Fader & Stullken, 1978
6-hr aquifer test near St. John	10,026	72	0.025	139	Cobb, 1979; 1980
8-day stream-aquifer test near Great Bend	19,404 (geom. mean)[a] 19,768 (arith. mean)[a] 4,979 (std. dev.)[a]	223 230 57	0.00056 0.000742 0.000664	87	Sophocleous et al., 1987; 1988
68 drillers' logs in the model area and vicinity (Figs. 11 and 12)	6,132 (mean) 3,171 (std. dev.)	85 37	0.15 0.05	76 30	This study

a. Average of drawdown- and recovery-derived values of 12 observation wells.

of local streams. The Pleistocene alluvium overlies Cretaceous and Permian bedrock. A generalized columnar action of the geologic units and their water-bearing properties is given in Table 3 from Fader and Stullken (1978).

The Permian bedrock crops out in an approximately north-south trend in the vicinity of US–281 and constitutes a source of poor-quality (saline) water east of US–281 in northeast Stafford County. The Permian formations in the area, known as redbeds, consist of reddish-brown sandstone, siltstone, shale, salt, gypsum, anhydrite, and limestone. Rocks of Cretaceous age form the bedrock surface in the western part of the Great Bend Prairie, including the model area. These rocks consist of interbedded shales, sandy shales, and fine- to coarse-grained sandstones (Fader and Stullken, 1978). Of the three Cretaceous units given in Table 3, only the lower unit (Cheyenne Sandstone) is a potential source of water to large-capacity wells, but the water is believed to be highly mineralized (Fader and Stullken, 1978).

Table 3. Generalized columnar section of geologic units and their water-bearing properties, (from Fader and Stullken, 1978)

System	Geologic unit	Maximum thickness, in feet	Physical character	Remarks
Quaternary	Undifferentiated Pleistocene deposits	360	Unconsolidated deposits of sand and gravel with interbedded lenses of clay, silt, and caliche. Windblown silt (loess) and dune sand occur at the surface over most of the area. Stream-laid deposits (alluvium) of late Quaternary age ranging from clay to gravel occur along the principal stream valleys	Comprises principal aquifer. Water generally is of good chemical quality,* but may be of poor chemical quality in the northeastern part of the area and in deep buried valleys in the south-eastern part. Yields as much as 2,000 gal/min to wells.
Tertiary	Ogallala Formation (Pliocene deposits)	65	Unconsolidated deposits of silt and fine sand with interbedded caliche. Some interbedded sand and gravel.	
Cretaceous	Undifferentiated Lower Cretaceous rocks	380	Upper unit (Dakota Formation) brown to gray fine- to medium-grained sandstone interbedded with gray sandy shale and varicolored shale. Middle unit (Kiowa Formation) dark-gray to black shale interbedded with tan and gray sandstone. Lower unit (Cheyenne Sandstone) gray and brown fine to medium grained sandstone interbedded with dark-gray shale.	Water probably of poor chemical quality. Yields 10 to 100 gal/min to wells locally in the western part of the area.
Permian	Undifferentiated Permian rocks	350	Interbedded reddish shale, siltstone, and sandstone with some beds of dolomite and anhydrite. Includes in descending order, Whitehorse Formation, Dog Creek Formation, Blaine Formation, and Flower-pot Shale.	Water generally of poor chemical quality. May yield as much as 10 gal/min to wells
	Cedar Hills Sandstone	200	Reddish shale, siltstone, silty shale and sandstone.	Sandstone may contribute highly mineralized water to the principal aquifer where the two units are in contact.
	Salt Plain Formation	300	Reddish-brown sandy siltstone and fine grained sandstone.	May contribute highly mineralized water to the principal aquifer where the two units are in contact.
	Harper Sandstone	250	Brownish-red siltstone and silty shale with a few thin beds of silty sandstone. Kingman Sandstone Member is near the top of the formation.	Water may be of poor chemical quality. May yield no water or as much as 100 gal/min to wells in the eastern part of area
	Stone Corral Formation	20	White and light-gray anhydrite and dolomite	Not known to yield significant amounts of water to wells in the area.
	Ninnescah Shale	400	Red and grayish-green shale, siltstone and very fine grained silty sandstone.	May yield water of fair to poor chemical quality to wells in the outcrop areas.
	Wellington Formation	550	Calcerous gray and blue shale containing several thin beds of limestone, gypsum, and anhydrite. The Hutchinson Salt Member, when present, is near the middle of the formation.	Not known to yield significant amounts of water to wells in the area.

* Chemical quality of water is classed as good if the concentrations of dissolved solids is less that 500 mg/l (milligrams per liter) or the concentrations of chloride and sulfate are less than 250 mg/L, fair if dissolved solids are 500 to 1,000 mg/L or chloride and sulfate are 250 to 500 mg/L, and poor if dissolved solids are greater than 1,000 mg/L or chloride and sulfate are greater than 500mg/L.

Most of the Tertiary deposits making up the Ogallala Formation were removed by erosion before deposition of Pleistocene material. The stratigraphy of the Quaternary alluvium in descending order is generally (1) sand dunes; (2) a relatively continuous near-surface silt-clay bed, probably a loess deposit; (3) alternating sequences of fining-upward sandy silt-clay, and sand and gravel lenses (not always present); (4) a basal sand and gravel bed of fluvial origin; (5) bedrock (Rosner, 1988).

The Arkansas valley alluvium consists of stream-laid deposits that range in texture from clay and silt to sand and very coarse grained gravel. The upper 2–20 ft (0.6–6 m) of the alluvium on the Arkansas valley consists of silt and fine- to coarse-grained sand (Latta, 1950). Beneath these finer surficial deposits are thick beds of coarse-grained granitic sand and gravel. Most of the area lying south of the Arkansas River in Pawnee and Edwards counties is overlain by dune sand composed of uniform-grained, moderately well-rounded fragments of quartz, and lesser amounts of silt and clay (McLaughlin, 1949). The dune sand lies above the water table and hence yields no water to wells.

In general, the present drainage system of central Kansas is the result of events that took place during the Pleistocene Epoch. The Pleistocene history of the area is complex and is marked by the cutting and filling of deep valleys and by major changes in drainage (Fent, 1950; Frye and Leonard, 1952). During early Pleistocene time, the ancestral Arkansas River, instead of following the present course around the great bend, is thought to have flowed eastward or southeastward across south-central Kansas. This can be seen on the bedrock map of the area (Sophocleous et al., 1990), where a number of west-east paleodrainage channels progress from south to north throughout the basin.

The Pleistocene drainage patterns of central Kansas record the history of the northeastward migration of through-flowing streams from the Rocky Mountain area. According to Fent (1950), this migration was caused by successive captures of the southern trunk of the ancestral stream by its own northern tributaries. The captures seem to have resulted from the difference in the debris load available in the headwater areas of the streams. Through-flowing streams originating from the

Rocky Mountains, such as the Arkansas River, filled their channels throughout the Pleistocene with coarse gravel and sandy alluvium derived from igneous rocks. This material built up the surface over which they flowed, causing stream avulsions and the consequent spreading of alluvial material over wide areas. In contrast, the northern tributaries to the southern trunk stream carried only the finer-grained, less-permeable sediment load obtained by downcutting in their immediate headwater areas. The silt and fine-grained sand of local origin in the northern Great Bend Prairie, with its low permeability, favored runoff and consequently more erosion and downcutting below the level of the through-flowing streams; this downcutting led to the eventual capture of the through-flowing streams. This is evident in the relative abundance of northern tributaries to the Arkansas River in central Kansas (Fent, 1950).

The Quaternary deposits constitute an excellent source of good-quality water across most of the area. Some water-quality problems relating to mineral intrusion from the underlying Permian bedrock units render the groundwater in certain areas unusable, such as the area around the Big and Little Salt marshes in northeastern Stafford County.

Field data collection

An extensive water-level survey in the Kinsley to Great Bend area was conducted in 1985 by the KGS and the GMD5. The resulting water-level map from that survey is shown in Fig. 13. Also a stream-gaging survey of the Arkansas River (at Kinsley, Nettleton, Garfield, Larned, Dundee, Great Bend, and other stations) was conducted during March, April, and May 1986 by a cooperative group from the Kansas Water Office, the Stafford Field Office of the Division of Water Resources, the GMD5 and the Kansas Department of Wildlife and Parks under the direction of the KGS. The results of that stream-gaging survey are presented in Table 4.

During January and February 1991 a limited water level survey in the study area was conducted by the GMD5 at the request of the KGS. These data combined with the annual water level measurements taken by the Division of Water Resources in cooperation with the US Geological Survey were used to produce the January 1991 water-level map of the study area (Fig. 14).

Table 4. Stream-gaging Surveys along the Kinsley to Great Bend Reach of the Arkansas River

Date	Width (ft)	Discharge (cfs)	Station	Comments
03/04/86	34.5	1.143	Ark River at	
03/18/86		1 (est.)	Kinsley	Impossible to get velocity with pygmy method
04/01/86		1 (est.)		Flow unchanged from last measurement
04/15/86				Noticeable flow over beaver dams
05/06/86				Same as 4/15/86
03/06/91	10.0	0.86		
03/04/86	1.0	0.029	Ark River at	
03/18/86	(channel in	0.097	Nettleton	
04/01/86	center of	0.097		Flow unchanged from last measurement
04/15/86	road)			Flow reduced slightly but still a noticeable trickle
05/06/86				Same as 04/15/86
03/06/91				Unmeasurable
03/04/86	33.5	0.771	Ark. River at	
03/18/86		>1 (est)	Garfield	Impossible to get pygmy meter velocity
04/01/86		>1 (est)		Flow unchanged from last measurement
04/15/86				Noticeable flow over beaver dams
05/06/86				Same as 04/15/86
03/06/91	14.0	2.093		
03/03/86		<0.5 (est.)	Ark River at Larned	No flow under bridge To the east, an estimated 0.5 cfs flow before confluence with Pawnee River; there is ponded water east and west of bridge.
03/31/86		0		No flow under bridge
04/15/86		0		No flow under bridge Ponded areas are reduced
05/06/86		0		No flow under bridge Ponded areas are slightly larger than 2 weeks ago
03/06/91		0		No flow below confluence of Pawnee either
03/03/86		<0.5 (est.)	Pawnee River	
03/18/86		0.5–1 (est.)	at Larned	
03/31086		<1 (est.)		
04/15/86		0		No flow over beaver dams
05/06/86		0		Water is ponded under the bridge but there is no distinct flow
03/06/91		0.26		
03/06/91		0.61	Ark River at Pawnee Rock	
03/03/86			Ark River at Dundee	No flow under bridge 0.5 cfs (est.) on stretch of ponded water about 100 yds west of bridge
03/18/86		0		No flow under bridge
04/01/86		0		No flow under bridge

Table 4 (continued)

Date	Width (ft)	Discharge (cfs)	Station	Comments
04/15/86		0	Ark River at Dundee (cont.)	Ponded areas have diminished since 04/01/86
05/06/86		0		No flow under bridge Ponded areas are slightly larger
03/04/86	14.5	1.81	Ark River at Great Bend	
03/18/86	17.0	2.695		
04/02/86	15.0	3.436		
04/16/86	15.0	2.286		
05/01/86	16.0	3.079		
03/06/91		4.84		
03/04/86	28.0	1.16	Wet Walnut Creek at Great Bend	
03/18/86	17.9	0.199		
04/02/86	15.0	2.379		
04/16/86	15.0	1.436		
05/01/86	14.0	2.69		
03/04/86	29.0	9.861	Ark River at Dartmouth	
03/18/86	26.0	10.715		
04/02/86	26.0	12.935		
04/16/86	25.0	8.049		
05/01/86	25.0	13.688		

A stream-gaging survey of the same stations used in 1986 was also conducted during March 1991 by a cooperative group from the Kansas Water Office, the Stafford Field Office Division of Water Resources, the GMD5, and the KGS. The results of that survey are also presented in Table 4.

Numerical Modeling

The major thrust of this study is to implement and analyze an appropriate stream-aquifer numerical model for the study area. The chosen simulation model to evaluate the Kinsley to Great Bend stream-aquifer system is a modified two-dimensional version of the popular modular three-dimensional finite-difference groundwater model (MODFLOW) of MacDonald and Harbaugh (1984) with streamflow routing capabilities as documented by Prudic (1989). MODFLOW solves the three-dimensional groundwater flow equation using finite-difference approximations and includes the effects of many processes, such as areal recharge, rivers, drains, evapotranspiration, and pumpage. The finite-difference procedure requires that the aquifer be divided into cells. The aquifer properties in each cell are assumed uniform. The unknown head in each cell is calculated at a point or node at the center of the cell. The head is calculated by iterating through the finite-difference equations for all nodes until the maximum head change in any cell between the previous iteration and the current iteration is less than a specified small value. Once this criterion is met, the program advances to a new time step and the process of computing heads at each node is repeated.

Streams superimposed on the aquifer are divided into *reaches* and *segments*. A segment consists of one or more *reaches*. Each *reach* corresponds to individual cells in the finite-difference equation used to simulate groundwater flow. Streamflow is accounted for by specifying flow for the first *reach* in each *segment* that enters the model area and then computing streamflow to adjacent downstream *reaches* in each *segment* as equal to inflow in the upstream *reach* plus or minus leakage from or to the aquifer in the *reach*. Leakage is calculated for each *reach* on the basis of the head difference between the stream and aquifer, and a conductance term:

$$Q_\ell = C_{str}\,(H - h) \qquad (1)$$

where Q_ℓ is the leakage to or from the aquifer through the streambed, H is the head in the stream, h is the head in the aquifer side of the streambed, and C_{str} is the conductance of the streambed, which is the hydraulic conductivity of the streambed times the product of the width of the stream reach and its length divided by the thickness of the streambed.

The stage in each reach can be computed by using the Manning formula under the assumption of a rectangular stream channel:

$$Q = \frac{c}{n}\left(AR^{\frac{2}{3}}S_0^{\frac{1}{2}}\right), \qquad (2)$$

where Q is the stream discharge, n is Manning's roughness coefficient, A is the cross-sectional area of the stream R is the hydraulic radius, S_0 is the slope of the stream channel, and c is a constant, which is 1.486 for units of cubic feet per second (cfs). The cross-sectional area and the hydraulic radius for a rectangular channel are

$$A = wd, \text{ and} \quad (3)$$

$$R = wd/(w + 2d) \quad (4)$$

where d is the depth of the water in the stream and w is the width of the channel.

The amount of leakage in each reach either into or out of the aquifer is incorporated into the groundwater flow model by adding appropriate terms to the finite-difference equation. Recharge to the aquifer in a reach ceases when all the streamflow in upstream reaches has leaked into the aquifer and the stream is dry. A stream is permitted to flow again in downstream reaches if the head in the aquifer is above the elevation of the streambed.

The groundwater flow model with the streamflow-routing package has an advantage over the analytical solution in simulating the interaction between aquifer and stream because it can be used to simulate complex systems that cannot be readily solved analytically.

Required input data for the stream-aquifer model include (1) the areal distribution of aquifer-related parameters, such as transmissivity or hydraulic conductivity, storativity, and natural recharge; (2) water levels in the aquifer and the stream(s); (3) bedrock and land surface elevations; (4) the input stream and tributary hydrograph; (5) stream width, slope, streambed elevations, and Manning's roughness coefficients; (6) the streambed conductance (i.e., hydraulic conductivity of streambed or canal and ditch sediments divided by their thickness); (7) the location and pumping rate of wells; and (8) the initial and boundary conditions.

Calibration

One of the most important steps in setting up a groundwater model is calibration. Development of the computer model as a predictive tool is based on the premise that, if historic hydrologic phenomena can be satisfactorily approximated by the model, then so should future conditions. Calibration involves adjusting model input parameters, based on field data, to accurately predict real-world cause-and-effect relationships. The task of manually adjusting parameter and past recharge values over different parts of the aquifer until the model nearly replicates previously measured water-level measurements in a set of observation wells is an arduous one requiring many model runs. Adjustments are often made in a hit-or-miss fashion until the fit between the model and the observed water levels is acceptable. This process is often time consuming and expensive, and sometimes can result in no answer. Also, questions about whether or not the derived solution is the optimum and how many other solutions are equally good are difficult to answer when trial-and-error methods are used. To avoid problems related to manual calibration, one can use a parameter estimation computer program that uses the MODFLOW program as its forward processor to obtain an optimum set of parameter or input values. The process by which one is attempting to solve for one or a number of the model parameters or inputs is known as *inverse modeling* (or *inverse problem*). Once we know the parameters or inputs to the model (e.g., hydraulic conductivity, storativity, recharge), it is a relatively simple matter to obtain model outputs such as heads or water levels in the aquifer. This modeling process is known as the *forward problem* or *forward modeling*. In this study we employ a parameter optimization software for MODFLOW known as MODINV (for *mod*flow *inv*erse). Using MODINV, we can optimize the specific values taken by any parameter type that MODFLOW can read as a two-dimensional data array such that model-generated heads are as well matched as possible to those observed in the field. Steady-state and transient, single-layer and multilayer, and confined and unconfined models can all be calibrated in this manner. MODINV adjusts parameter and/or recharge values pertaining to a set of constant-value zones chosen by the modeler (based on field data) for each parameter type until the optimum fit between observed and model heads is obtained. MODINV then provides the covariance matrix, which indicates the reliability or uncertainty levels of these parameter estimates. Model and observed heads are matched according to the weighted least-squares criterion, and optimization is achieved using the Gauss–Newton–Marquardt method (Draper and Smith, 1981).

Regression Problem

The calibration or parameter estimation problem can be viewed as a classical nonlinear regression problem with a solution of the appropriate flow equation forming the regression equation and all unknown quantities, such as hydrogeologic parameters, sources, sinks, and boundary fluxes, as parameters (Sophocleous, 1984). The set of measured hydraulic heads are observations of the dependent variable for which a set of least-squares estimates is to be obtained. This viewpoint not only has the advantage of finding the parameters of a given model that produce the best fit of the calculated hydraulic head (dependent) variable to the observed dependent variable but also allows implementation of many methods and tests that have been developed to analyze on a probabilistic basis the propagation of data errors in the estimates of parameters and the predictive capability of the model (Draper and Smith, 1981).

The basic equation that is to be fitted to the observed head data is the general form of the two-dimensional groundwater flow equation (which the MODFLOW program is designed to solve):

$$\frac{\partial}{\partial x}\left(T_{xx}\cdot\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(T_{yy}\cdot\frac{\partial h}{\partial y}\right)+R(H-h)+W+\sum_{\ell=1}^{N}\delta(x-a_\ell)\delta(y-b_\ell)Q_\ell=S\frac{\partial h}{\partial t} \quad , \qquad (5)$$

where T_{xx} (= K_{xx} b) and T_{yy} (= K_{yy} b) are the transmissivities in the x and y directions, respectively; K_{xx} and K_{yy} are the hydraulic conductivities of the aquifer in the x and y directions, respectively; b (x, y) is the saturated thickness of the aquifer, R (x, y) is the hydraulic conductance of the streambed, which is the hydraulic conductivity of the streambed times the product of the width of the stream and its length divided by the thickness of the streambed; H (x, y, t) is the head in the stream; h (x, y, t) is the hydraulic head in the aquifer; and W (x, y, t) is a source-sink term (positive for a source, such as recharge) distributed areally. The expression $\sum_{\ell=1}^{N}\delta(x-a_\ell)(y-b_\ell)Q_\ell$ is the Dirac delta designation for N wells, each one pumping at rate Q^ℓ(t) (positive for injection) and located at coordinates (a_ℓ , b_ℓ). S is the storativity (storage coefficient or specific yield); x, y are the Cartesian coordinates; and, finally, t is time.

To approximate the variability of a given parameter, the region of interest is subdivided into a number of zones in which the parameter is assumed to be constant within each zone. Zones of one type of parameter, such as hydraulic conductivity, do not necessarily correspond to zones for another type, such as recharge.

Boundary conditions, such as lateral model inflow rate or constant head levels, are often considered to be part of the model itself, being neither an input nor a parameter. Along internal discontinuities in hydraulic conductivity the hydraulic head and flux normal to the boundary remain unchanged as the boundary is crossed.

The classical problem of groundwater hydrology is to directly solve Eq. (5) together with its associated boundary conditions (including the initial head at $t = 0$) for the hydraulic head $h = h(x, y, t)$ in the aquifer, whereas an inverse solution involves solving Eq. (5) and boundary conditions for one or more of the parameters, such as T, K, S, or W.

Sources of error in groundwater data

Numerous problems involving groundwater flow modeling of real field systems exist because the data necessary for the direct or inverse solutions are usually lacking. Head distribution is never known exactly because measurements do not exist at all points and because, where the measurements do exist, they are not exact. Estimates of the parameters either are completely unknown or have been obtained by spot measurements, few of which are directly useful for construction of appropriate effective values for use in Eq. (5). It should be clear that modeling problems in groundwater hydrology involve an incomplete combination of several types of data in which error and error propagation are important considerations.

Some major potential sources of random error in head data with respect to the model [Eq. (5)] are enumerated by Cooley (1979):

1. Areal groundwater models assume that the head used is the average over the vertical, but wells may not be opened over the entire interval modeled, and, if they are, they may not measure the average;
2. Hydraulic conductivity varies from point to point, which causes water levels to vary from values they would have if hydraulic conductivity were uniform. However, models usually do not take this detailed variation into account;

3. Water levels measured in wells in use may contain unknown amounts of residual drawdown. In addition, unused wells may be near wells that are in use, with resulting unknown drawdown in the unused well;
4. Measurement of well-head elevation may be in error;
5. Measurement of water levels may be in error [although usually of the order of 0.1–0.2 ft (0.03–0.06 m)].

Actual total error from the above sources is highly problem dependent, but it is easy to imagine errors of several feet. In addition, interpolation errors are also of the order of several feet (Sophocleous, 1983). Major model errors in Eq. (5) and associated boundary conditions can be detected relatively easily and can be eliminated by analysis of model results.

Because there are several different parameters to be considered and because each can be estimated or measured in several different ways, numerous sources of error exist in parameter data. Some examples of errors in parameter data illustrating the nature of the problem are given by Cooley and Naff (1985):

1. Too few estimates of parameters are available to compute stable estimates of statistics such as mean and variance,
2. Results of point sampling are often biased because a large amount of data does not necessarily allow computation of nearly true or effective values of a parameter and its variance. For example, permeability values from core analyses often are not representative of regional values because flow through large fractures is not reproduced by core analyses,
3. Transmissivities estimated from specific capacity data collected by drillers are subject to numerous sources of error. Common sources include mismeasured water levels or pumping rates, recovery of water level after bailing, clogging the slots or screen, and inaccurate reporting. A persistent source of bias results because drillers drill wells in favorable locations and screen only the most productive zones,

4. Transmissivities and storativities estimated from pumping-test analyses are subject to many of the same errors in item 3, but the more carefully controlled tests should reduce their frequency and magnitude. In addition, a single test may not be representative of an entire hydrostratigraphic unit,
5. Transmissivities and storativities estimated from lithologic data are usually biased to an unknown extent.

Numerical regression solution procedure

To form the regression problem, Eq. (5) must be solved subject to the appropriate boundary conditions. For the present study, the regression solution is based on a numerical solution of Eq. (5) described in detail by MacDonald and Harbaugh (1984). In matrix form the solution can be written

$$\underline{\underline{D}}\,\underline{h} = \underline{q} \qquad (6)$$

where $\underline{\underline{D}}$ is a square coefficient matrix involving parameters T_{ij} and R of order m, the number of nodes used to discretize the model region; $\underline{h}$ is the hydraulic head vector of order m; and $\underline{q}$ is the known vector involving parameters W, Q, specified head, and boundary fluxes.

The set of optimal parameters is defined as the set that minimizes the objective function

$$SS = \underline{e}^T\,\underline{w}\,\underline{e} = (\underline{h}^{obs} - \underline{h})^T\,\underline{w}\,(\underline{h}^{obs} - \underline{h}) \qquad (7)$$

where $\underline{h}^{obs}$ is the vector of observed heads, $\underline{h}$ is a vector of predicted heads, $\underline{e} = (\underline{h}^{obs} - \underline{h})$ is the residual vector consisting of the deviations of calculated heads from observed heads, superscript T indicates transpose, and $\underline{w}$ is a diagonal weight matrix that describes the reliability of h^{obs} at each node. If for observation l, $w_l = 0$, then there is no observed head at that node. SS is the weighted sum of squared deviations of calculated heads from observed heads, which is to be minimized. The use of the objective function [Eq. (7)] is equivalent to minimizing the error variance.

If the parameters to be computed (such as all the different values of K_{xx}, K_{yy}, S, and W are designated vector $\underline{b}$, then the normal equations (Draper and Smith, 1981) derived by minimizing Eq. (7) with respect to each parameter can be written

$$\underline{e}^T \; \underline{w} \frac{\partial \underline{e}}{\partial \underline{b}} = 0 \tag{8}$$

The necessary elements of e and their derivatives are obtained through use of a Guass–Marquardt linearization scheme applied to Eq. (6). The technique yields a regression equation, which can be written

$$\Delta \underline{b}_i = -\underline{N}^{-1} \underline{f}_i \tag{9}$$

where $\Delta \underline{b}_i = \underline{b}_{i+1} - \underline{b}_i$; i is the iteration number; $\underline{N}$ is the normal matrix ($\underline{J}^T \underline{wJ}$) consisting of the derivatives of the elements of h with respect to each of the elements of b, and $\underline{f}_i$ is the gradient of the objective function (i.e., the weighted sum of squared head differences between the model and the observed heads).

The sensitivity coefficients J_{ij}, or simply sensitivities, indicate the change in the value of head h_i for a unit change in parameter b_j. The regression algorithm uses only observed values of head in the criterion SS for the best fitting solutions.

Assumptions for the regression analysis

The nonlinear model—assumed to be the true model—represented by the solution of Eq. (6) for h, which is the subset of h_m applying at nodes that are observation nodes, can be written for observation ℓ as

$$h_\ell^{obs} = f(\xi_\ell, \beta) + \varepsilon_\ell \tag{10}$$

where f indicates a function that is the solution of Eq. (6); ξ_ℓ is a vector of independent variables that is an undetermined but observable function of coordinates x, y, the problem geometry and boundary conditions; β is the vector of true parameters; and ε_ℓ is an error in observation.

To analyze statistically the results of and the predictions made by the regression model, we assume (Draper and Smith, 1981) that

$$E(\varepsilon_\ell) = 0 \tag{11}$$

$$Var(\varepsilon_\ell) = \sigma^2 \tag{12}$$

$$Cov\ (\varepsilon_\ell, \varepsilon_m) = 0 \qquad \ell \neq m \tag{13}$$

where E, Var, and Cov are the expected value, variance, and covariance operators, respectively. These assumptions indicate that ε_ℓ is considered a random variable with zero mean and constant variance σ^2 and that ε_ℓ and ε_m ($\ell \neq m$) are uncorrelated. In addition, it is often assumed that ε_ℓ is normally distributed with mean 0 and variance σ^2 such that

$$\varepsilon \sim N\ (\underline{0}, \underline{I}\sigma^2) \tag{14}$$

This means that the elements of ε are independent and uncorrelated and allows the use of statistical tests and measures involving the *F* and *t* distributions (Draper and Smith, 1981).

Because β is unknown, ε is not observable, and the assumptions cannot be checked directly. However, they can often be checked indirectly, after the regression and model analysis have been performed, as demonstrated later.

Model Implementation and Calibration

Grid Selection

The study area consists of a 48 x 15 mile (77 x 24 km) rectangle in a southwest to northeast direction incorporating the Arkansas River from Kinsley to Great Bend (Fig. 15). The Pawnee River joins the Arkansas River at approximately the middle of the northwest side of the study rectangle. This rectangle is divided into 720 squares or cells of 1 square mile area each, thus forming a rectangular cell-centered finite-difference grid used by MODFLOW.

Boundary and Initial Conditions

The model boundaries for the study area, as shown in Fig. 15, were arrived at by superimposing the water-level, soil, and bedrock maps mentioned previously. The southeast boundary of the Arkansas River Valley system was thus separated from the rest of the Great Bend aquifer by following a bounding southwest to northeast flow line, which is equivalent to a no-flow boundary condition (because no flowlines can cross each other). The northwest boundary (no-flow) separation presents no significant problems because the valley is mostly bounded by relatively impervious Lower Cretaceous (Dakota Formation) strata and clayey soils in that direction. The northeast and southwest boundaries were cut along appropriate iso-water level contours near Kinsley and Great Bend, respectively, thus forming assumed constant head end boundaries.

Model Stresses

The period of simulation is divided into a series of "stress periods" within which specified stress parameters are constant. Each stress period, in turn, is divided into a series of yearly time steps. The system of finite-difference equations of the form of Eq. (6) is formulated and solved to yield the head at each node at the end of each time step.

Groundwater Pumpage

A computer program was written to read and reformat the water rights tape obtained from the Division of Water Resources, and to sort water rights according to year, application number, or legal location. Figure 5 is a plot of all groundwater rights versus year of issue in the model area. To simplify matters and to avoid excessive input files to the model, we decided to approximate this curve by dividing it into segments of uniform number and distribution of wells starting in 1955, which is considered an indicator year of predevelopment conditions. Thus the time period from 1955 to 1962 is represented by the 1955 groundwater rights distribution, the 1963–1969 period by the 1967 groundwater rights distribution, the 1970–1975 period by the 1972 distribution, the 1976–1982 period by the 1979 distribution, and the 1983–1990 period by the 1990 distribution, as shown by the filled squares in Fig. 16. Therefore pumping-well matrices for the different pumping stress periods, as indicated by the chosen index years 1955, 1967, 1973, 1979, and 1990, were prepared as input to the model. The distribution and increase with time of the pumping irrigation wells are shown in Figs. 17–21. In those figures each model cell with an asterisk represents one or more wells in that cell.

Incoming Streamflows

Another stress to the model system is represented by the fluctuating amount of incoming streamflow in the model area from the Arkansas River, as monitored at the Kinsley station, and from the Pawnee River, as monitored west of Larned. To simplify matters in a way similar to that done for the pumping stresses, we divided the average annual incoming streamflows at Kinsley (and similarly at Larned) into periods of relatively uniform incoming streamflows (Fig. 16). These periods, and the average Arkansas River streamflow (in parentheses) are (1) 1955–1962 (153.5 cfs); (2) 1963–1964 (63.1 cfs); (3) 1965 (711.1 cfs; flood); (4) 1966–1969 (158.7 cfs); (5) 1970–1973 (143.3 cfs); (6) 1974–1975 (65.3 cfs); (7) 1976–1978 (35.2 cfs); (8) 1979–1980 (19.3 cfs); (9) 1981–1982 (4.6 cfs); (10) 1983–1986 (4.4 cfs); (11) 1987 (334.0 cfs; flood); and (12) 1988–1990 (18.7 cfs). The progressive decline in incoming streamflow is clearly evident from these data and in Fig. 16.

Therefore, by combining the pumping and incoming streamflow stress periods, we obtained 12 pumping and stream stress periods and therefore 12 corresponding input data matrices for the model in simulating stream-aquifer conditions from 1955 to 1990. Table 5 details these 12 stress periods.

Table 5. Stress Periods

Pumping Averaging Period	Incoming Streamflow Averaging Period	Stress Period
1955–1962	1955–1962	1
1963–1969	1963–1964	2
1963–1969	1965	3
1963–1969	1966–1969	4
1970–1975	1970–1973	5
1970–1975	1974–1975	6
1976–1982	1976–1978	7
1976–1982	1979–1980	8
1976–1982	1981–1982	9
1983–1990	1983–1986	10
1983–1990	1987	11
1983–1990	1988–1990	12

Aquifer-Related Data

Aquifer Base

The aquifer base was extracted from the compiled bedrock map (Fig. 2) by superimposing the model grid on that map and reading (or interpolating) a bedrock elevation value at the center of each cell block.

Predevelopment and Other Water Levels

The same procedure as that used to define the aquifer base was followed for the predevelopment water-level map (Fig. 1). Water levels from the 1985 (Fig. 13) and 1991 (Fig. 14) surveys were used to create additional observed water-level matrices for comparisons with corresponding simulated results.

Hydrogeologic Properties

Several existing values of hydrogeologic properties from previous reports (Table 2), including the stream-aquifer pumping test by the Arkansas River near Great Bend (Sophocleous et al., 1987, 1988) were considered. Initial parameter values were used by averaging such data. In addition, hydrogeologic property distributions resulting from analyzing drillers logs, as indicated in the Methodology and Data Analysis section (Figs. 11 and 12), were used to divide the model region into zones of higher and lower values of a particular hydrogeologic property (Figs. 22 and 23).

Recharge Data

Recharge data from an ongoing study on groundwater recharge assessment of the GMD5 have been used as initial estimates. Also, the study area was divided into higher and lower recharge zones (Fig. 24) based on the available water capacity of soil (Table 1; Fig. 3).

Stream-Related Data

Stream widths and slopes for the model area were estimated, as mentioned previously, from topographic and other maps. Streambed hydraulic conductance was approximated based on the knowledge of the area geology. Manning's coefficients were obtained from tables (Chow, 1959; White 1979) based on our knowledge of the area streams.

Calibration

Calibration involves adjustment of model input using alternative combinations of parameter values and/or zones to obtain reasonable agreement with measured data. The model was calibrated for both steady-state and transient conditions. A principle adhered to in this regard for this study is the *principle of parsimony*, according to which in a choice among competing hypotheses, other things being equal, the simplest (i.e., the smallest possible number of parameters for adequate representation) is preferable.

Steady-State Calibration

Because of the relatively large number of data points used in the construction of the predevelopment water-level map (Sophocleous et al., 1990) and the minimal amount of external stresses (e.g., pumpage) imposed on the aquifer, the interpolated predevelopment water levels for each active cell of the model grid were considered of similar accuracy to the actually measured ones. Thus all active model grid cells are considered as cells with observed (measured) values, which were identical to the initial (starting) water-level cell entries in the model.

Initially, the model area was divided into three zones of uniform but different hydraulic conductivity [high (170), medium (100), and low (60) hydraulic conductivity, *K* (ft/d)] based on an analysis of drillers' logs (Figs. 11 and 12). The high *K* values are based on the stream-aquifer pump test in Great Bend (Sophocleous et al., 1987). Using this *K* zonation and a uniform recharge estimate of 2 in./yr based on ongoing GMD5 recharge assessment measurements (Sophocleous, 1991), we ran the MODINV model to optimize the hydraulic conductivity values using the predevelopment (steady-state) conditions, as exemplified by the predevelopment water levels and the 1955 irrigation well distribution. However, when we reran the same computer run using only two hydraulic conductivity zones [high and (medium + low)], the results and the sum of squares of the deviations between simulated and observed heads were nearly identical. Therefore the two *K* zonations indicated in Fig. 22 were adopted. Using a single *K* zone increased the sum of squares of the deviations significantly, and thus it was not adopted. Also, addition of another zone consisting of the inferred buried channels in the area (Sophocleous et al., 1990; Fig. 22) did not result in any marked improvement.

Following this *K* optimization and assuming that the hydraulic conductivity is known, we then optimized the amount of recharge. The final result of these parameter optimization iterations is shown in the Simulation Results and Model Analysis section. Reducing the amount of pumpage by 20% did not result in a significant difference, most probably because of the relatively small amount of total pumpage at that period. Simultaneous optimization of both recharge and hydraulic conductivity in a steady-state model results in a singular normal matrix [Eq. (5)] because the values taken by one parameter type (hydraulic conductivity) for a particular head distribution depend on the values taken by the other parameter type (recharge).

Transient-State Calibration

To ensure that the transient model will simulate future conditions in the real system, we found it necessary to first simulate with reasonable accuracy as much hydrologic history as practical. Therefore the transient calibration was run in two stages, using yearly time steps: first, from 1955 to 1985 using the detailed 1985 water-level survey (Fig. 13) for comparison of predictions versus observation; and, second, from 1985 to 1990, using the January 1991 water-level measurements (Fig. 14) for comparison. This second step was undertaken mainly so that we could demonstrate a reasonable ability to simulate observed historic responses to historic conditions different from those for which the model was calibrated. When the model is thus validated (Sophocleous, 1988), it can be used to project stream-aquifer responses to hydrologic conditions.

Starting with the optimized parameter estimates from the steady-state calibration and employing the stress periods indicated in the Model Stresses section, we ran the MODINV parameter estimation program to optimize, in sequence, storativity and recharge, keeping the already optimized hydraulic conductivity values constant (recharge was reoptimized for the new simulation periods to allow for fluctuating climatic condition influences). Employing two zones of storativity, based on the results of drillers' log analysis (Figs. 12 and 23), versus one zone did not result in any significant difference in the sum of squares of deviations between simulated and observed values of hydraulic head, and thus a single storativity zone was employed. In the transient runs the groundwater pumpage was held at 100% and 80% of the appropriated amounts in separate runs, with the latter pumpage resulting in somewhat improved model fit. We judged that 80% of the appropriated water rights was closer to the one actually used, and therefore all transient runs were run under this assumption.

Simulation Results and Model Analysis

As mentioned previously, the calibration or parameter estimation problem is in essence a regression problem, and the various methods and tests that have been developed to analyze regression problems are also applicable to studies using groundwater flow models. Many of these procedures are used in the following steady and transient model analyses.

Predevelopment (Steady-State) Conditions

The results of the steady-state analysis are shown in Table 6, in which the fit of simulated and observed values of head is very good, as indicated by the high value of the correlation coefficient ($R = 0.9994$). The standard error of the estimate for the ith parameter [given by the square root of the ith diagonal component of the parameter variance-covariance (or simply covariance) matrix] is a measure of the range over which the parameter can be varied to produce a similar solution for the dependent variable (i.e., hydraulic head) as that obtained using the estimated parameter. Standard errors are indications of the precision of the determined parameters. Examination of Table 6 indicates that the standard errors for the parameters are generally less than 5% of the magnitude of the parameters. Converting such standard errors into a confidence interval, requires the assumption of some probability distribution for these errors. For example, if the central limit theorem holds, a 95% confidence interval for the parameter values will be given by $\hat{b} \pm 1.96\ \sigma_E$, where $\hat{b}$ is the estimated value of the parameter and σ_E is the standard error.

The parameter covariance matrix is one of the most useful pieces of information to come out of the inversion process. The diagonal elements of this matrix are the variances of the individual parameter values, whereas the off-diagonal elements are the covariances between parameter pairs; these covariances are indicative of how highly correlated two different parameter values are. The principal role of the parameter covariance matrix is as an indicator of how well the

Table 6. Steady-state analysis results

Zone	K(ft/d)	Std Error	r_{12}	Recharge (in)	Std. Error
1	164	3.9	0.28	1.0	0.02
2	242	12.9	0.28	1.0	0.02

s = square root of the error variance = 3.0 ft
R = correlation between simulated and observed water levels = 0.9994.
N = no. of observations = 407.
SS = weighted sum of squares of the deviations between simulated and observed values of head = 3,638.
$s/\Delta h$ = 0.0091 (Δh = 330 ft).
$r12$ = correlation between hydraulic conductivity (K) estimates for zones 1 and 2.
Std. error = $\sigma/\sqrt{N}$

observation-well head measurements are able to define aquifer properties (including recharge); for although the model heads may be well matched to the measured heads (the reference or error variance may be satisfactory), some parameter value standard deviations may still be large. This indicates that, as mentioned earlier, these parameter values can be made to vary by large amounts with little effect on the model heads at the observation wells. If this applies to a single parameter value, the value will have a high variance and will be uncorrelated with other parameter values. If, however, two or more parameter values can be simultaneously varied in a certain relationship to each other while causing minimal change to the model heads at the observation wells over time, then these parameter values will each have a high standard deviation, and the covariance between pairs of such parameters, as indicated by the pertinent off-diagonal elements of the covariance matrix, will also be large. This indicates high parameter value correlation. Thus the parameter covariance matrix tells us something about our model that the goodness of fit between model and observed heads cannot tell us. For example, if the density of observation wells is low or zero over a certain part of the aquifer, parameter values estimated in that area may not be well defined, and this will be indicated in the covariance matrix. Although the model may appear to be well calibrated because the model replicates observed heads at the existing observation wells with a good degree of accuracy, its capacity to predict water levels in other parts of the aquifer may be highly suspect if the calculation of these latter heads relies on parameter values that are locally ill-defined.

The correlation r_{12} between the hydraulic conductivity estimates of zones 1 and 2 (Table 6) gives an estimate of the degree of linear dependence of one parameter (K in this case) in one zone with the corresponding parameter in another zone throughout the course of repeated experiments if such experiments were to be carried out. A high degree of correlation between parameter value pairs (1 and 2) is indicated by a correlation coefficient r_{12} close to 1 or −1. The higher the degree of parameter value correlation, the closer the normal matrix resulting from the weighted least-squares minimization of the residuals will approach singularity, and the greater the possibility of numerical instability. Examination of Table 6 indicates that the correlation of the K parameters in zones 1 and 2 is negligible, implying a good K parameter estimation.

The error variance s^2 of the hydraulic head values is another measure of overall goodness of fit of the model. [It is calculated as the ratio of the weighted sum of squares of the deviations between simulated and observed values of head (SS) over the number of observation points minus the number of estimated parameters]. A good overall fit between modeled and measured heads indicates that the head measurement standard deviations (i.e., the square root of the error variance) are small. The value of the ratio of the square root of the error variance over the difference between the highest and the lowest value of head in the model region ($s/\Delta h$) is 0.009 (Table 6), a relatively small value, so that errors in the model are considerably less than the model response, as indicated by the maximum head loss (Δh) between Kinsley and Great Bend of 300 ft (91 m).

The main reasons for analyzing residuals are to examine the validity of the various assumptions concerning their distribution given earlier [Eqs. (11)–(14)], and to investigate the correctness of the model. Aspects that could be investigated include evidence for spatial nonrandomness and evidence that the residuals are not approximately normally distributed (Sophocleous, 1984). Draper and Smith (1981) give a number of methods for examining residuals, and they emphasize that graphical procedures involving visual analysis are valuable tools for detecting nonrandomness, because violations of assumptions serious enough to require corrective action generally are apparent on the various plots. In Fig. 25 residuals are plotted against values of estimated head. Under the given assumptions the plot should display a roughly horizontal band of residuals having no apparent trend, and this is exactly how the plotted residuals in Fig. 25 behave. The residuals were also plotted in Cartesian coordinates and contoured (Fig. 26). The residuals show no obvious systematic variation of significant degree over the map area, indicating that this model is probably adequate for these data.

To check whether the residuals are normally distributed, we plotted them on normal probability paper (Fig. 27). A relatively good-fitting straight line can be drawn through the bulk of the points plotted, indicating that the calculated residuals are approximately normally distributed. For the parameters derived by the least-squares analysis to be maximum-likelihood estimates of the true parameters (i.e., parameter estimates that give the greatest probability of obtaining the observed data), residuals must be normally distributed.

Water budget

A summary of all inflows and outflows to a region is generally called a water budget. Because in the model program the water budget is calculated independently of the equation solution process, it provides independent evidence of a valid solution. The difference between total inflow and outflow is printed as a percent error, calculated using the formula

$$D = \frac{100(\text{In} - \text{Out})}{(\text{In} + \text{Out})/2}, \qquad (15)$$

where In is the total inflow to the system, Out is the total outflow, and D is the percent error term. If the model equations are correctly solved, the percent error should be small. The overall model water budget is presented to check the acceptability of the solution and to provide summarized information on the flow system. The volumetric water budget for the model area under predevelopment conditions is presented in Table 7. It is evident from the table that the bulk input to the stream-aquifer system is groundwater recharge and that the largest outflows from the system are outflows from the constant head boundary by Great Bend and streamflow gains from groundwater (stream baseflows). Note that the irrigation pumpage is a minor element of total system outflow for the considered period.

Table 7. Volumetric Budget for Entire Model at End of Stress Period 1 (Predevelopment period 1955–1962)

	Cumulative Volumes (acre-ft)
Inflows	
constant head	4,350.3
recharge	23,028.0
stream leakage	8,514.0
Outflows	
constant head	16,522.0
pumping wells	4,627.2
stream leakage	14,738.1
Discrepancy = 0.01%	

Transient-State Simulations

The results of the transient state simulations from 1955 to 1985 are shown in Table 8. The value of the ratio $s/\Delta h$ (0.015) is still relatively small, indicating that errors in the model are considerably less than the model response, as indicated by the maximum head loss. Comparisons of predicted versus observed values of hydraulic head depicted in Figs. 28 (for 1985) and 29 (for 1990) are satisfactory. However, because of the smaller number of cells with observed hydraulic head values (compare the N values of Tables 6 and 8), the standard errors of the parameter estimates are relatively larger. However, a plot of the residuals versus predicted values of hydraulic head for both the 1955–1985 and 1985–1990 periods (Fig. 30) reveals that no relationship of significant concern is obvious, indicating that this model is probably adequate for these data. A normal probability plot of the residuals for both the 1955–1985 and the 1985–1990 periods (Fig. 31) reveals a well-fitting straight line through the bulk of the points plotted, indicating that the residuals are approximately normally distributed.

Table 8. Transient 1955–1985 Analysis Results

Zone	K (ft/d)	Std. Error	r_{12}	Recharge (in.)	Std. Error	Storativity	Std. Error
1	164	19.0	0.43	1.1	0.06	0.10	0.01
2	242	79.6	0.43	1.1	0.06	0.10	0.01

s = square root of the error variance = 5.1 ft
R = correlation between simulated and observed water levels = 0.9985.
N = no. of observations = 90.
SS = weighted sum of squares of the deviations between simulated and observed values of head = 2,282.
$s/\Delta h$ = 0.0155.
r_{12} = correlation between hydraulic conductivity (K) estimates for zones 1 and 2.
Std. error = $\sigma/\sqrt{N}$.

Comparison of predicted groundwater discharge and observed average annual streamflow for the 1955–1990 simulation period at Great Bend shows a satisfactory match, as indicated in Fig. 32. As can be seen from that figure, the model underpredicts groundwater discharge during periods of high streamflow because the model does not simulate overland runoff. During periods of low flow, however, most streamflow is derived from groundwater discharge.

Under natural conditions the water table gradient slopes toward the river, and groundwater discharges from the aquifer into the river. This can be seen by the curvature of the iso-water-level contours pointing upstream (Fig. 1). However, under pumping conditions the water-table gradient decreases, and groundwater discharge to the stream is reduced. If pumping is of sufficient volume and duration, the gradient may be reversed, and water from the stream will move by induced infiltration through the streambed into the alluvial aquifer. Figures 33 to 35 depict the model simulated reaches of the Arkansas River and Pawnee River that are gaining or losing water based on average yearly streamflows. Note the progressively increasing number of river reaches that are losing water with time because of progressively decreasing groundwater contributions to streamflow. These model results are satisfactorily consistent with the observed streamflow measurements shown in Table 4.

The overall volumetric water budgets for the model area during the 1955–1985 and 1985–1990 transient-state simulations are presented in Table 9. The convention followed in MODFLOW is that flow into or out of storage is considered part of the overall budget in as much as accumulation in storage effectively removes water from the flow system, and storage release effectively adds water to the flow—even though neither process, in itself, involves the transfer of water into or out of the groundwater regime (McDonald and Harbaugh, 1988).

The major inflow and outflow for the 1955–1985 period is groundwater recharge and pumping, respectively. Also note the significant increase in stream losses and decrease in stream gains compared to the 1955 period; this trend continues into the 1985–1990 period to such an extent that the current major source of aquifer inflow comes from streamflow losses (instead of natural recharge), and the bulk of aquifer outflows goes to pumping.

Sensitivity Analysis and Predictive Runs

Sensitivity analysis, which quantifies the model's response to input parameter changes, gives insight into mechanisms and dependencies. Therefore an analysis was made to determine the sensitivity of the model to variations in the values of selected parameters on both the aquifer and the stream. The input and aquifer parameters considered were pumpage, recharge, hydraulic conductivity, and storativity. The stream parameters considered were conductance of the streambed, Manning's roughness coefficient, stream slope, and stream width. Sensitivity to each of these parameters was determined by running the model with the optimized parameters for 1990

Table 9. Volumetric Water Budgets for 1955–85 and 1985–90

	Cumulative Volumes (acre-ft)
1955–1985 Inflows	
constant head	156,352.2
recharge	784,458.2
stream leakage	326,010.1
net water released from storage	202,394.4
1955–1985 Outflows	
constant head	486,111.1
pumping	573,783.3
stream leakage	409,366.4
percent discrepancy = 0.00	
1985–1990 Inflows	
constant head	77,933.9
recharge	108,450.4
stream losses	135,555.6
net water released from storage	55,676.3
1985–1990 Outflows	
constant head	58,395.3
pumping	252,089.1
stream gains	23,758.0
percent discrepancy = 0.00	

in a predictive mode from 1990 to 2010 and by varying (increasing and decreasing) each parameter by 50%. Corresponding changes in groundwater hydraulic heads at typical nodes near Kinsley [node at row 10, column 6, (10, 6)] Larned [node (8, 24)] and Great Bend [node (7, 42)] were observed, tabulated, and graphed (Figs. 36, 37, and 38), and the corresponding changes in streamflow (groundwater runoff or baseflow) were displayed for the Arkansas River near Great Bend (Figs. 39 and 40) and Garfield [node (5, 16); Figs. 41 and 42].

Sensitivity of Hydraulic Head to Changing Aquifer and Input Parameters

Examination of Figs. 36, 37, and 38 indicates that water levels (hydraulic head values) at different parts of the aquifer respond to changing input and parameter values in both similar and different ways. The similarity of response is based on the fact that pumpage and recharge have, in general, the greatest effect on water levels, followed, to a lesser degree, by hydraulic conductivity and storativity. However, different parts of the aquifer respond differently in absolute amount to changing parameters, with the relative significance of some parameters altered in some instances. For example, near Kinsley (Fig. 36) the hydraulic conductivity parameter seems to be more sensitive (and thus relatively more important) than groundwater recharge, whereas near Great Bend the opposite seems true. As we move from Kinsley to Great Bend (Fig. 38), groundwater pumpage, already the most sensitive parameter of all, becomes progressively even more important (Figs. 36, 37, and 38). In the area around Kinsley increasing or decreasing groundwater pumpage by 50% has approximately the same but opposite effect on increasing or decreasing the groundwater levels [i.e., the water levels increase or decrease by 4–8 ft (1.2–2.4 m) in a nearly symmetrical fashion around the base water-level conditions]. In the area close to Great Bend, however, an increase of 50% in groundwater pumpage has 2 to >4 times the effect on water levels than decreasing pumpage by 50%. Figures 36, 37, and 38 clearly show which parameters are the more important ones in different parts of the model area with regard to their impact on groundwater levels.

Sensitivity of Streamflows to Changing Aquifer, Input, and Stream Parameters

Examination of Figs. 39–42 indicates that, similar to what was observed with regard to water levels, streamflows respond differently to various parameters in different parts of the modeled area. The aquifer and aquifer input-related parameters in this case have a much more pronounced effect on streamflows than stream-related parameters. For example, groundwater pumpage, recharge, and hydraulic conductivity are much more sensitive parameters than streambed conductance, Manning's roughness coefficient, stream slope, or stream width (compare Figs. 39 and 40 to Figs. 41 and 42).

Management Alternatives to Be Tested

The predictive capabilities of the calibrated model permit hypothetical conditions to be explored by simply changing the data input to emulate the situations desired. After further checks on model sensitivity and reliability analyses, the following initial set of scenarios will be tested (suggestions for other options to be tested are solicited):

1. How would groundwater levels and streamflows respond to increased incoming streamflows in the Arkansas River?
2. What effect do climatic fluctuations (i.e. sequence of flooding and drought years) have on the stream-aquifer system?
3. What effect do changing pumping patterns, including water conservation and improved irrigation efficiency, have on the stream-aquifer system?
4. How can specified minimum desirable streamflows be maintained throughout the Kinsley to Great Bend reach of the Arkansas River?
5. What effect would protective stream corridors of different sizes have on streamflows?
6. In case of drought what is the most vulnerable subregion of the study area, and what ameliorating options are available?

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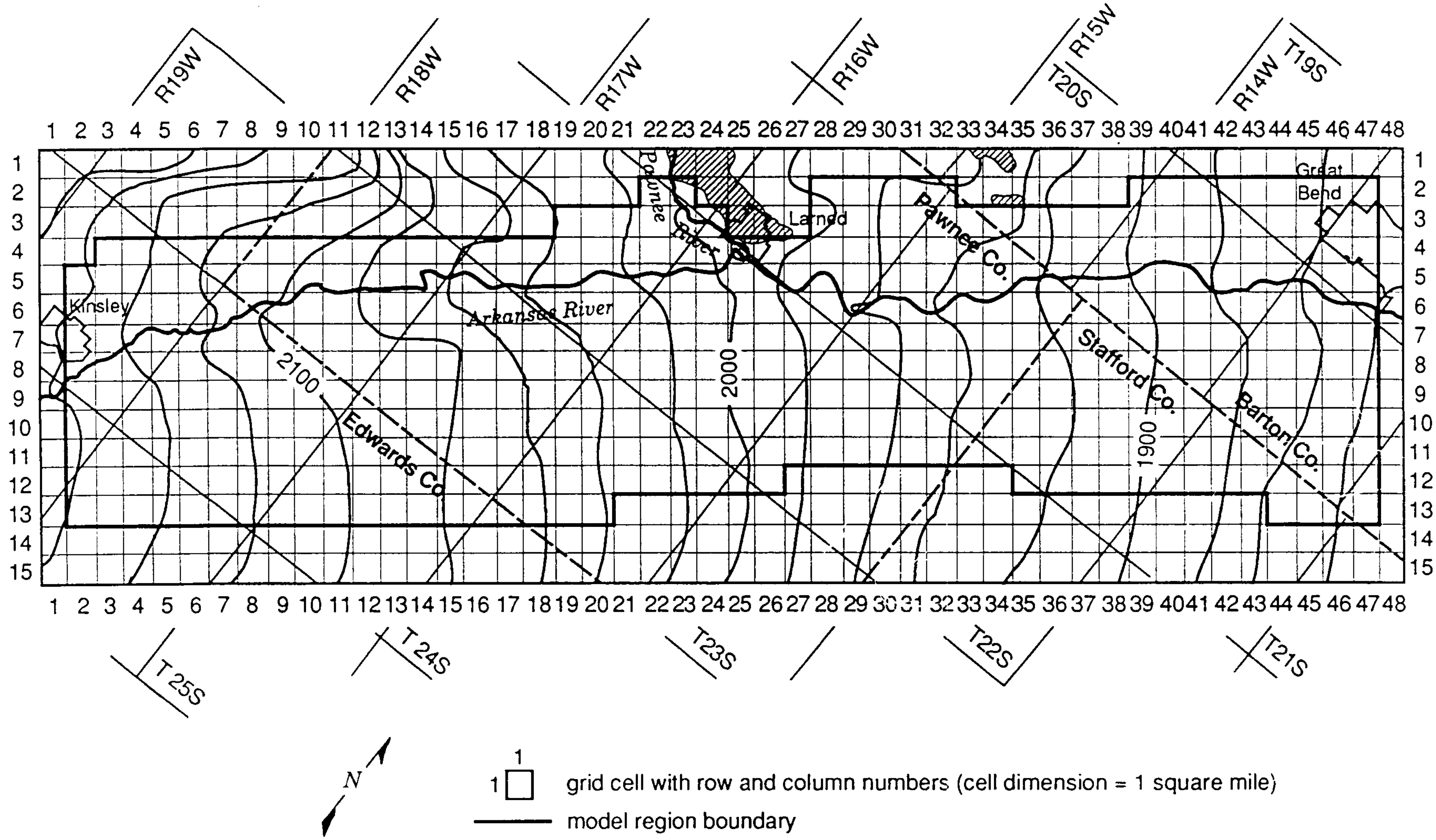
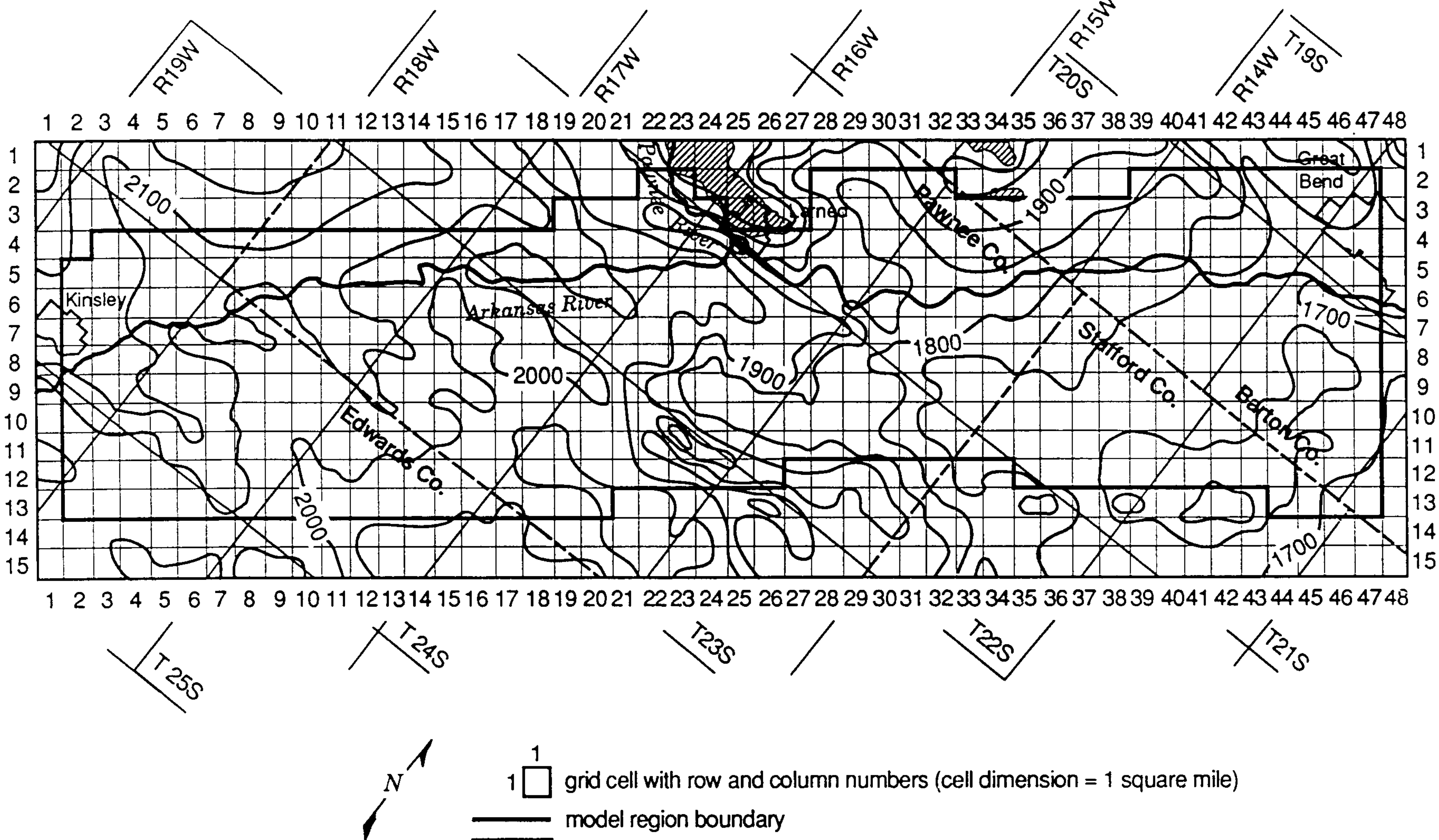
Predevelopment water level contours (ft)



Figure

Bedrock contours (ft)

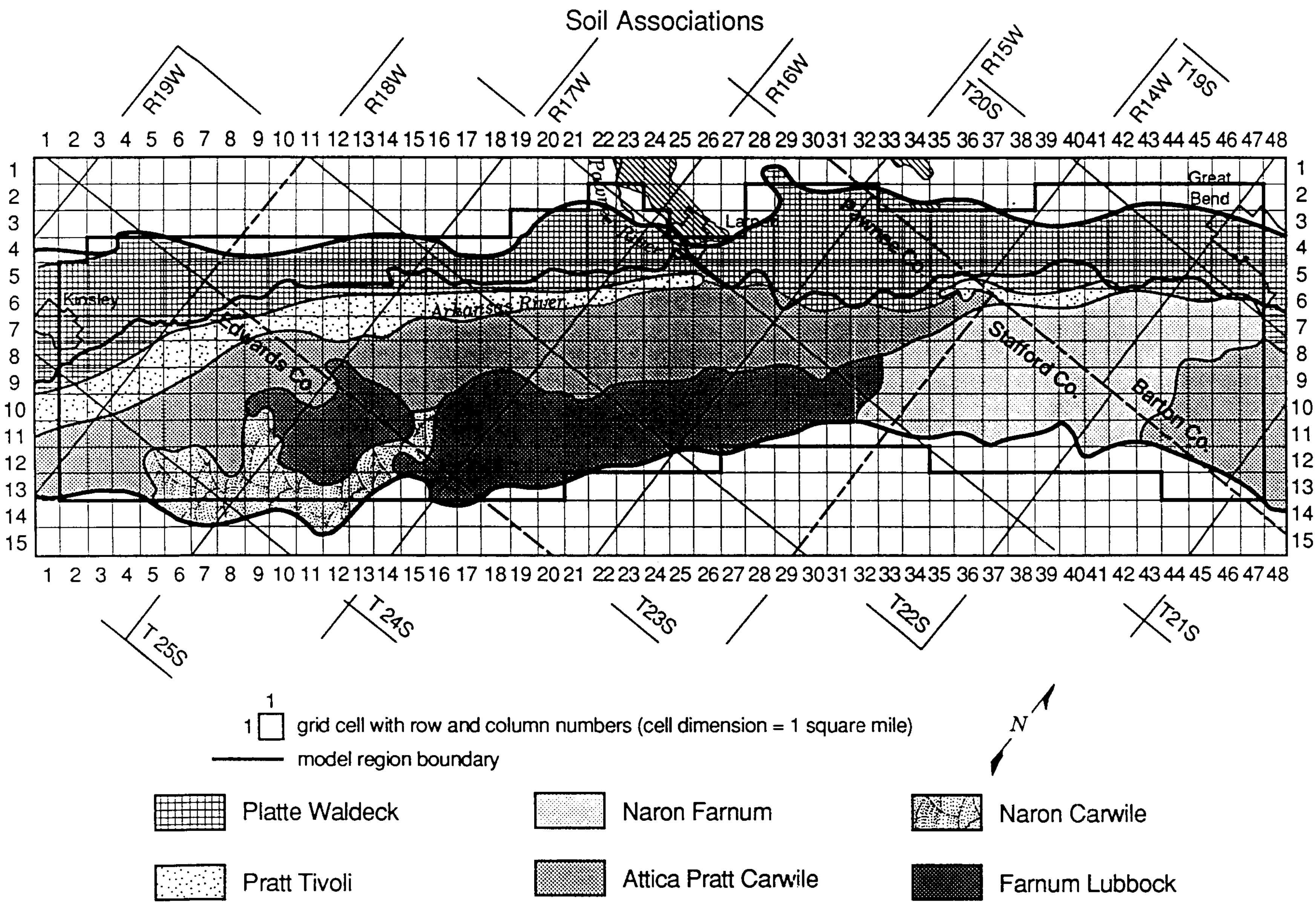


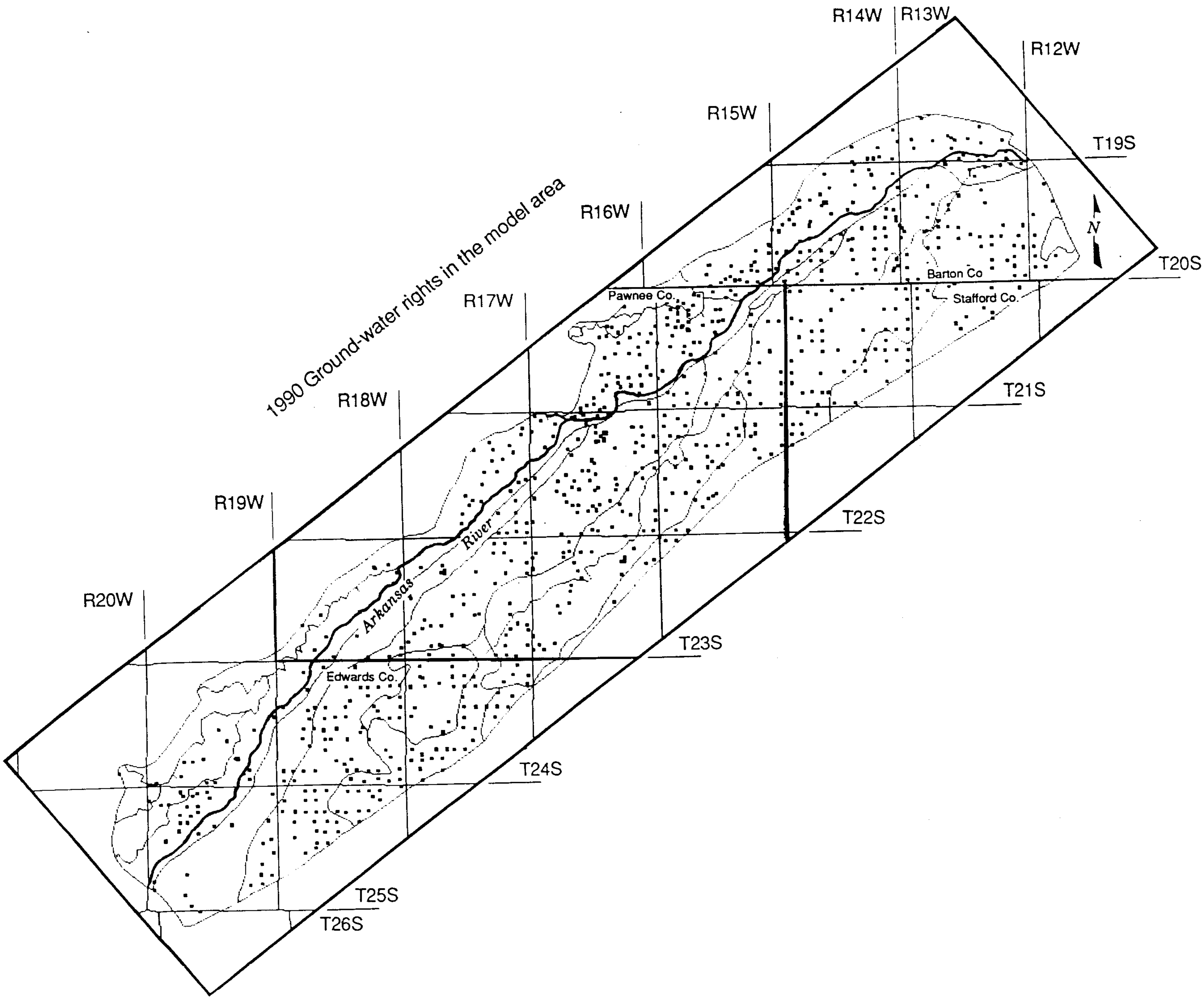
1 grid cell with row and column numbers (cell dimension = 1 square mile)

model region boundary

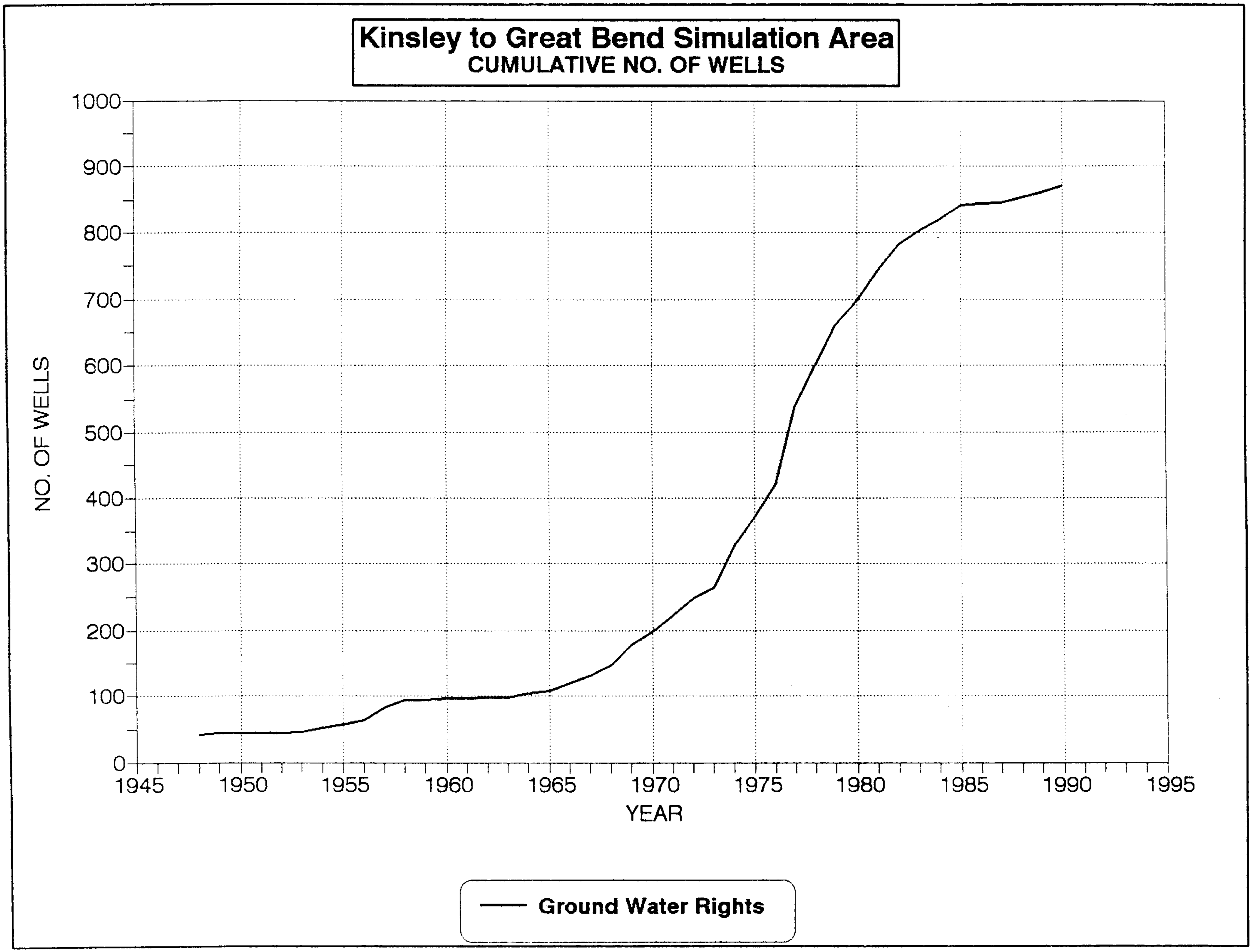
bedrock (Dakota Formation) outcrop

Figure

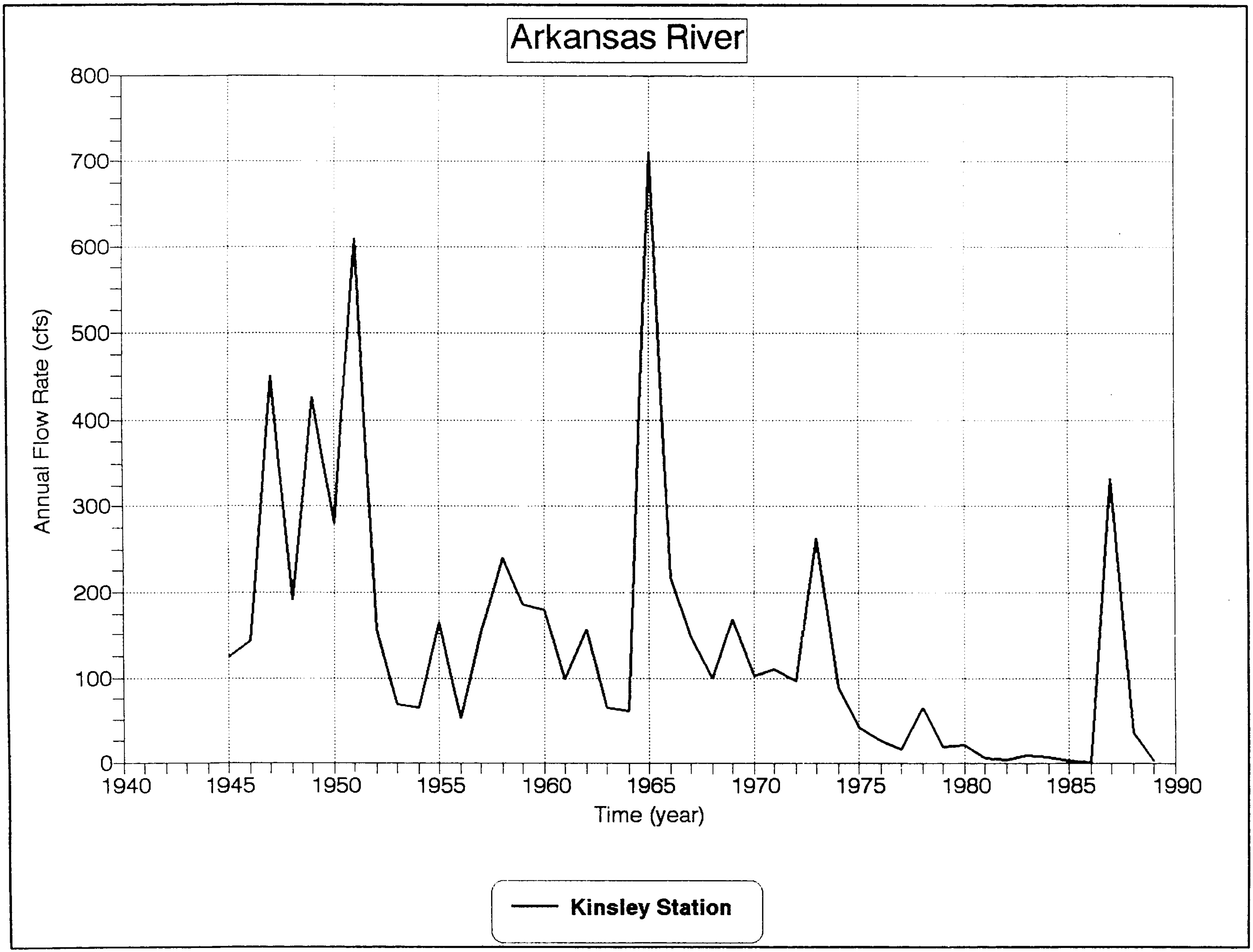

Soil Associations
R19W
R18W
R17W
R16W
R15W
R14W
T19S
T20S
T21S
T22S
T23S
T 24S
T 25S
Kinsley
Edwards Co.
Arkansas River
Pawnee River
Larned
Pawnee Co.
Stafford Co.
Barton Co.
Great Bend
N
1
1 grid cell with row and column numbers (cell dimension = 1 square mile)
model region boundary
Platte Waldeck
Pratt Tivoli
Naron Farnum
Attica Pratt Carwile
Naron Carwile
Farnum Lubbock


1990 Ground-water rights in the model area
R20W
R19W
R18W
R17W
R16W
R15W
R14W
R13W
R12W
T19S
T20S
T21S
T22S
T23S
T24S
T25S
T26S
Pawnee Co.
Barton Co
Stafford Co.
Edwards Co.
Arkansas River
N

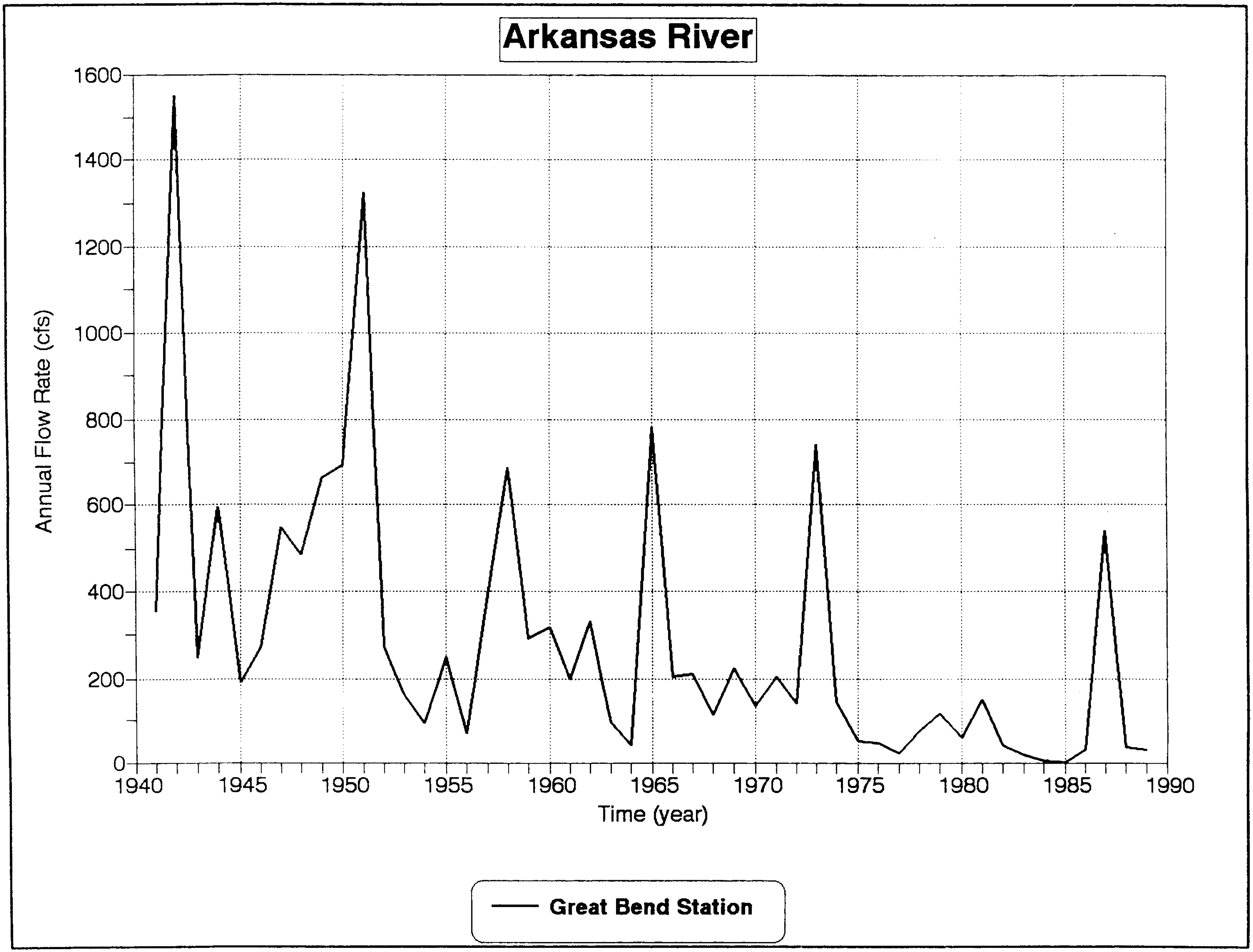
Figure

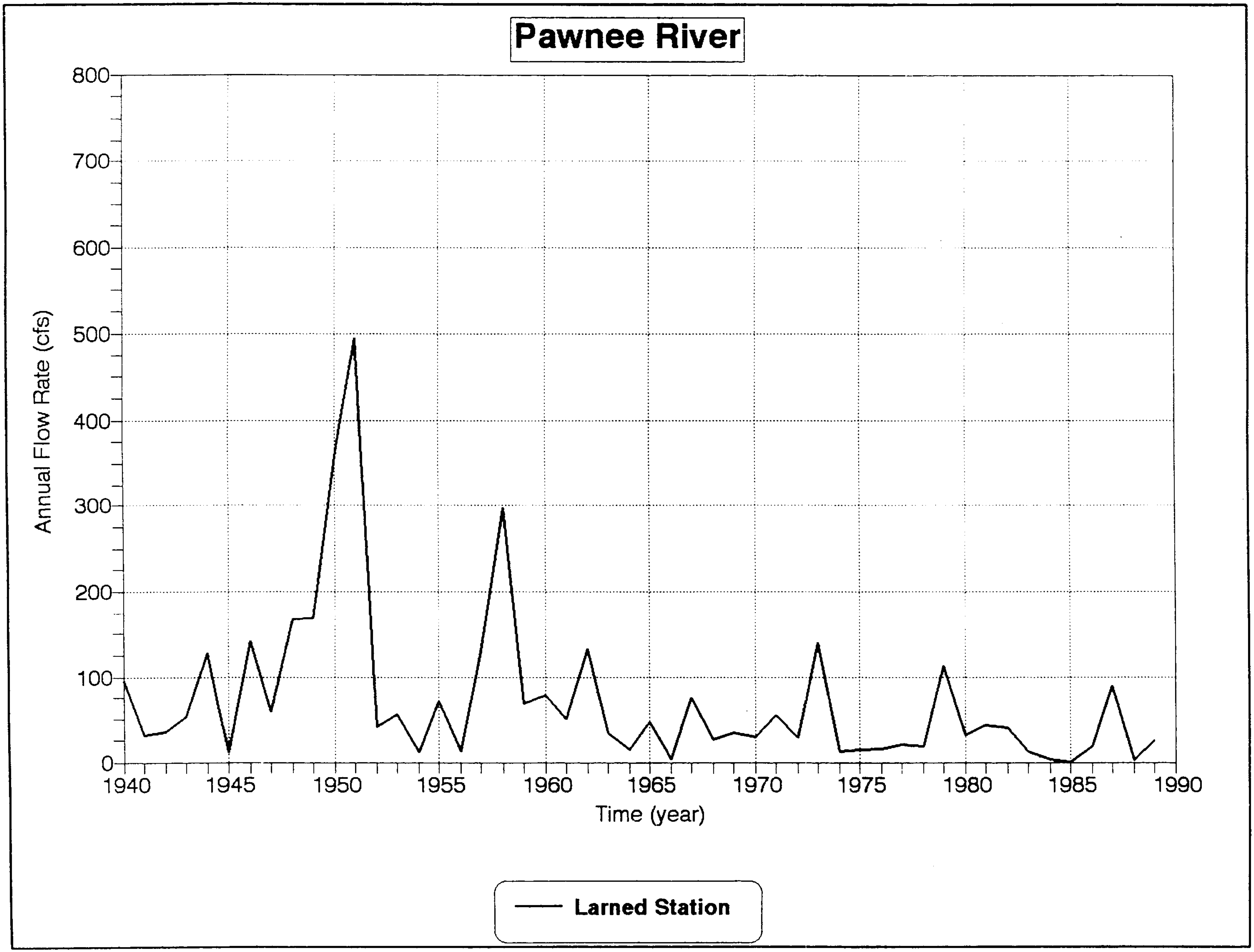


Kinsley to Great Bend Simulation Area
CUMULATIVE NO. OF WELLS
NO. OF WELLS
0
100
200
300
400
500
600
700
800
900
1000
YEAR
1945
1950
1955
1960
1965
1970
1975
1980
1985
1990
1995
Ground Water Rights

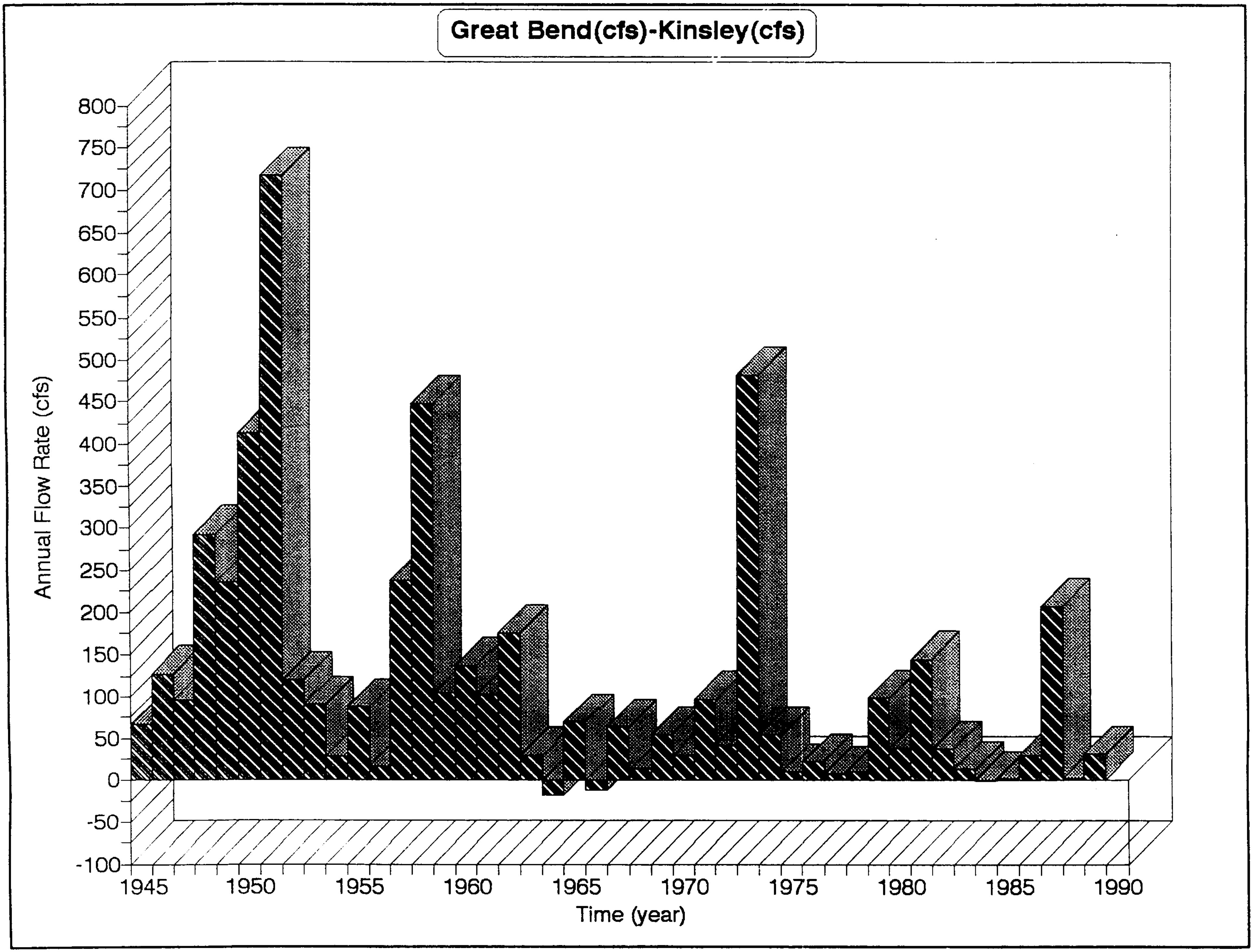


Arkansas River
Annual Flow Rate (cfs)
800
700
600
500
400
300
200
100
0
1940
1945
1950
1955
1960
1965
1970
1975
1980
1985
1990
Time (year)
Kinsley Station


Arkansas River
Annual Flow Rate (cfs)
0
200
400
600
800
1000
1200
1400
1600
1940
1945
1950
1955
1960
1965
1970
1975
1980
1985
1990
Time (year)
Great Bend Station



Pawnee River
800
700
600
500
400
300
200
100
0
Annual Flow Rate (cfs)
1940
1945
1950
1955
1960
1965
1970
1975
1980
1985
1990
Time (year)
Larned Station



Great Bend(cfs)-Kinsley(cfs)
Annual Flow Rate (cfs)
800
750
700
650
600
550
500
450
400
350
300
250
200
150
100
50
0
-50
-100
1945
1950
1955
1960
1965
1970
1975
1980
1985
1990
Time (year)

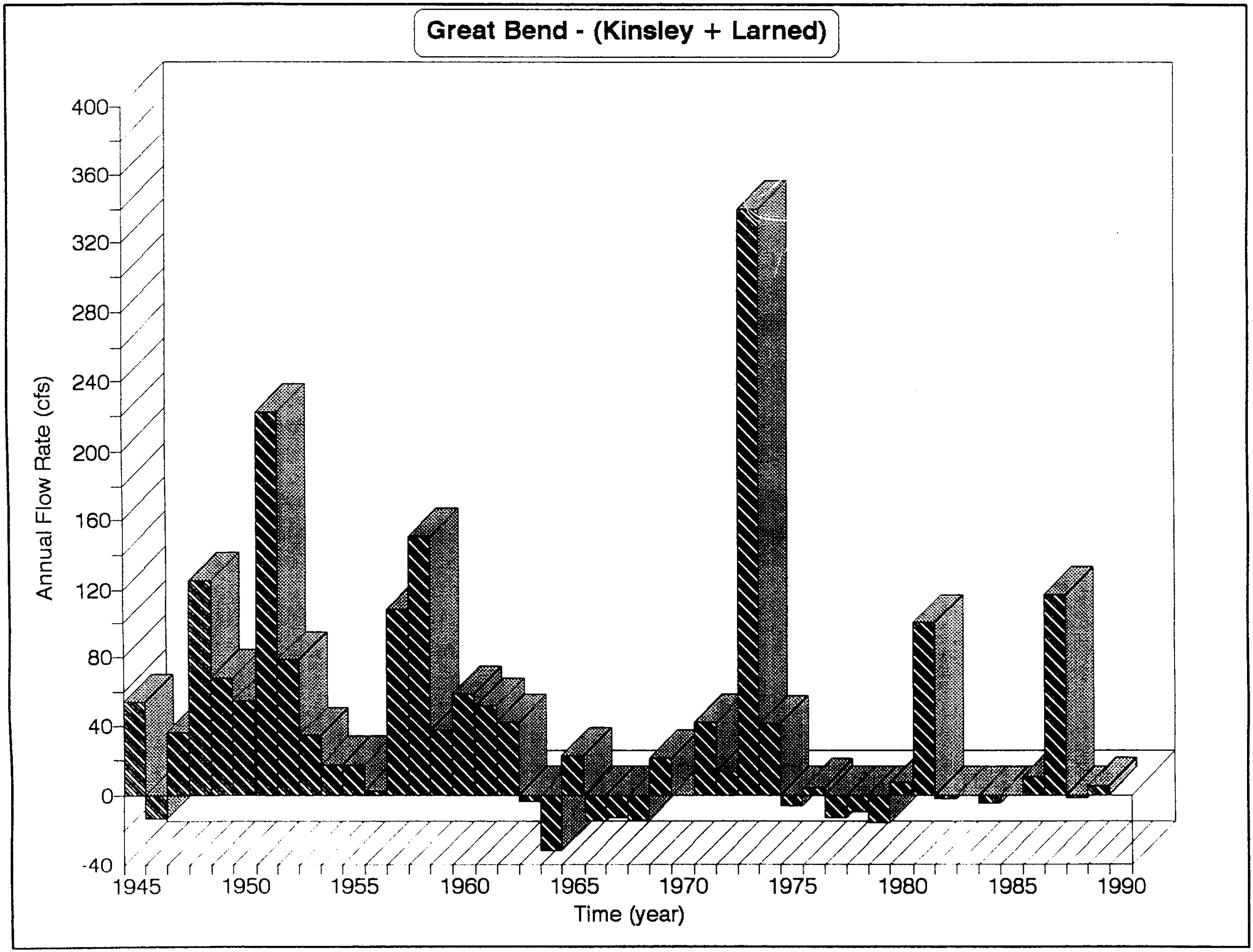
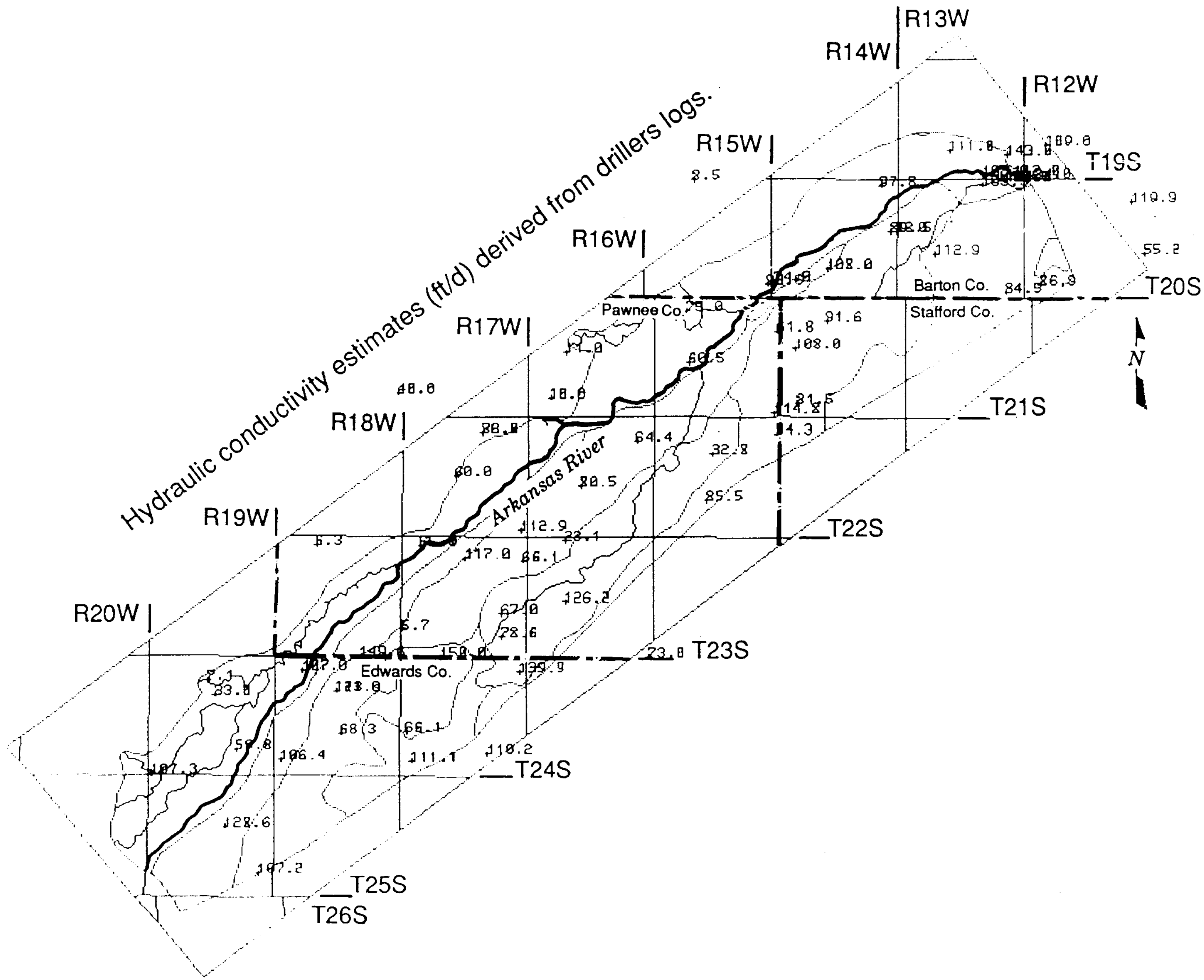

Great Bend - (Kinsley + Larned)
Annual Flow Rate (cfs)
400
360
320
280
240
200
160
120
80
40
0
-40
1945
1950
1955
1960
1965
1970
1975
1980
1985
1990
Time (year)

Figure 1



Hydraulic conductivity estimates (ft/d) derived from drillers logs.
R12W
R13W
R14W
R15W
R16W
R17W
R18W
R19W
R20W
T19S
T20S
T21S
T22S
T23S
T24S
T25S
T26S
Arkansas River
Pawnee Co.
Barton Co.
Stafford Co.
Edwards Co.
N

Figure

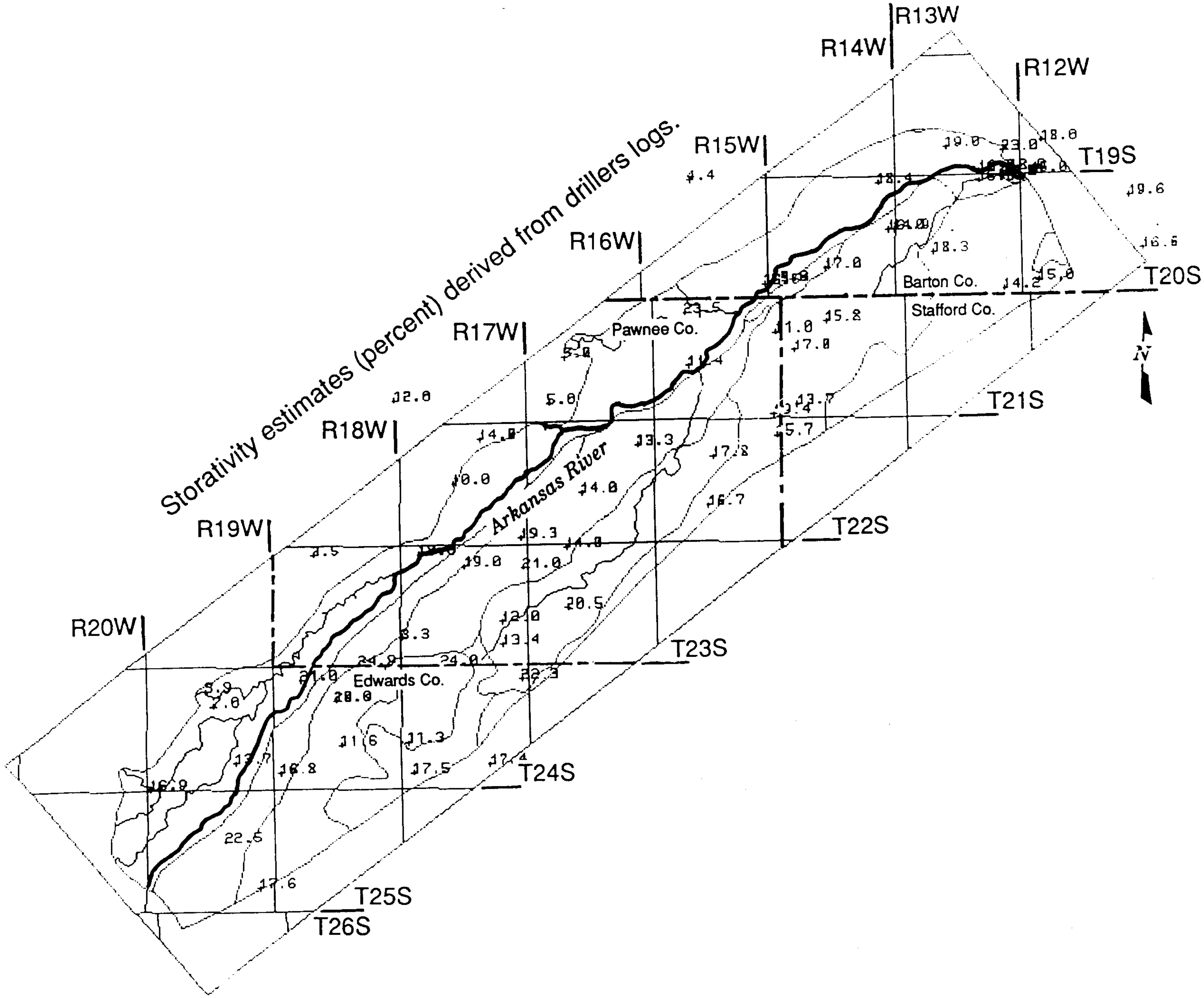

Storativity estimates (percent) derived from drillers logs.
R12W
R13W
R14W
R15W
R16W
R17W
R18W
R19W
R20W
T19S
T20S
T21S
T22S
T23S
T24S
T25S
T26S
Pawnee Co.
Barton Co.
Stafford Co.
Edwards Co.
Arkansas River
N

Figure 1

January 1985 water-table elevations (ft) in the study area (boxed) and vicinity.
(+ = measurement point)

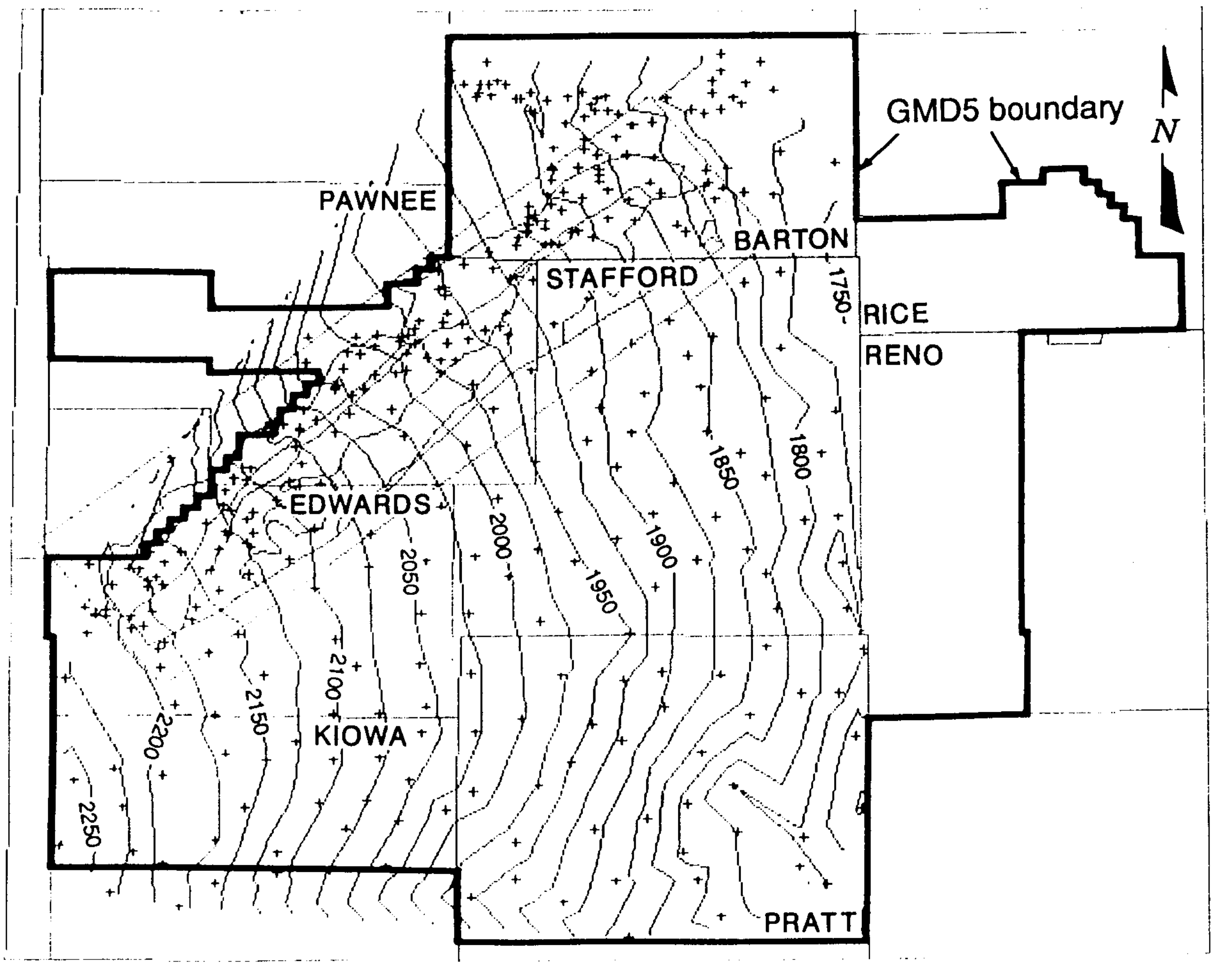


Figure1

January 1991 water-table elevations (ft) in the study area (boxed) and vicinity.
(+ = measurement point)

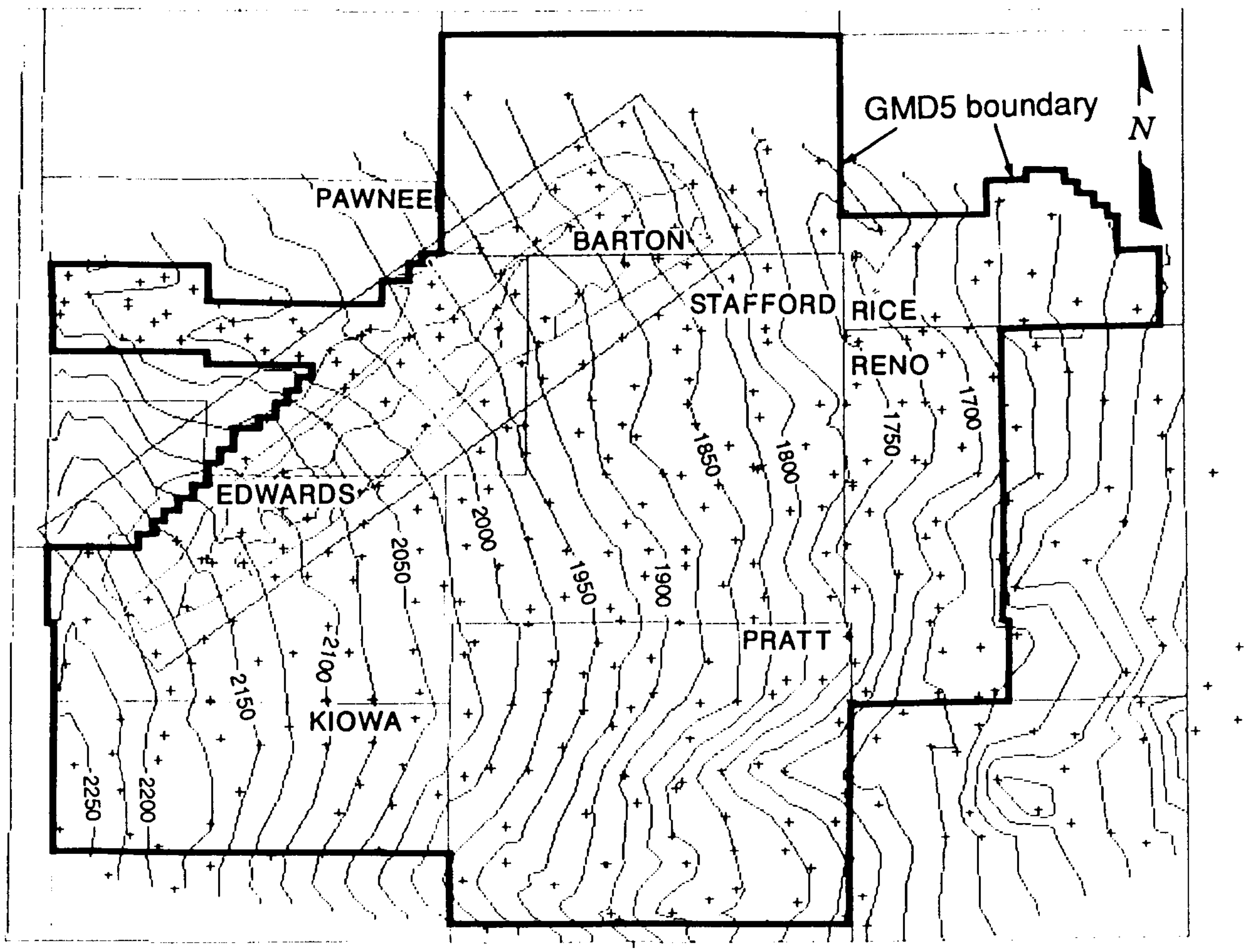
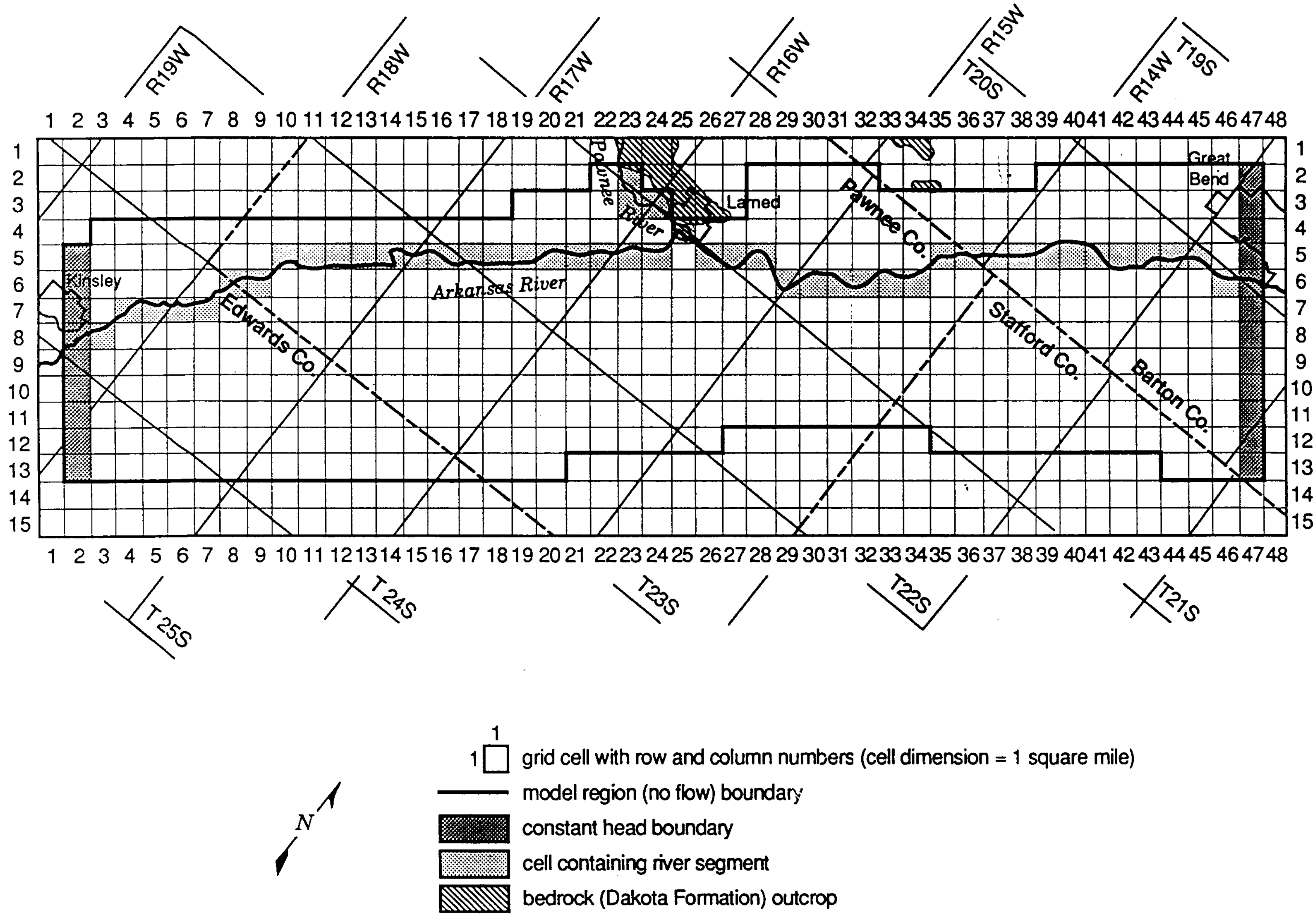


Figure 1


R19W
R18W
R17W
R16W
R15W
T20S
R14W
T19S
Kinsley
Arkansas River
Pawnee River
Larned
Great Bend
Edwards Co.
Pawnee Co.
Stafford Co.
Barton Co.
T 25S
T 24S
T23S
T22S
T21S
N
grid cell with row and column numbers (cell dimension = 1 square mile)
model region (no flow) boundary
constant head boundary
cell containing river segment
bedrock (Dakota Formation) outcrop

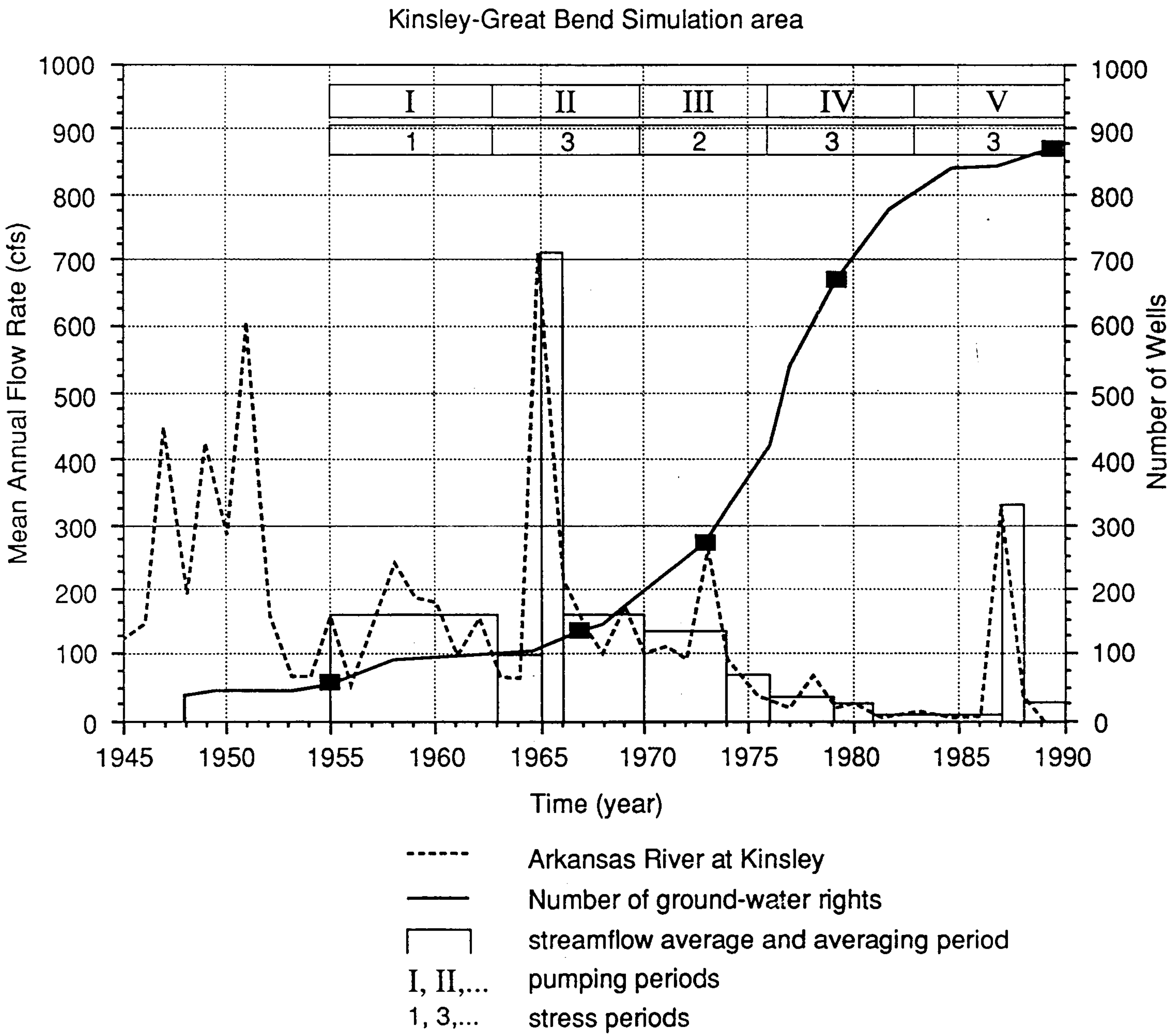
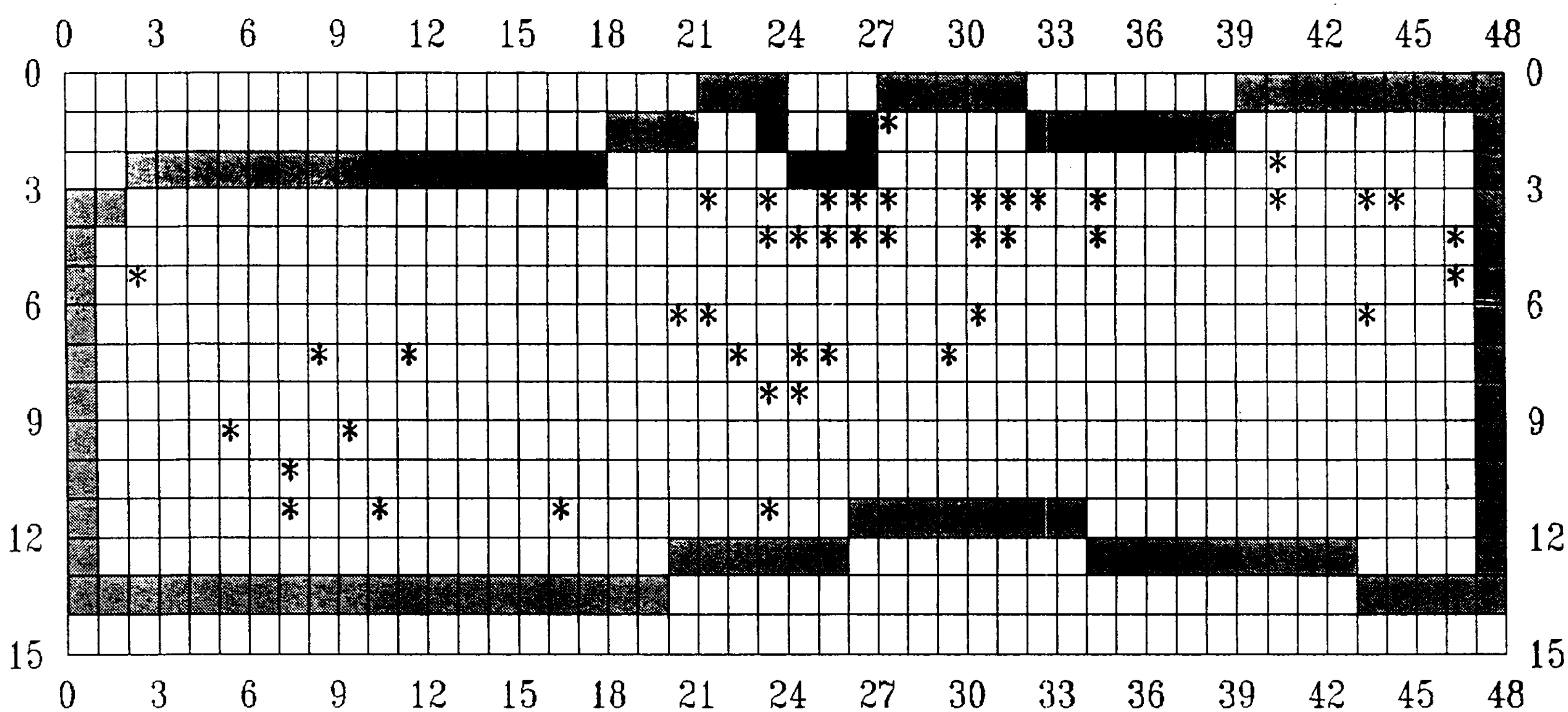
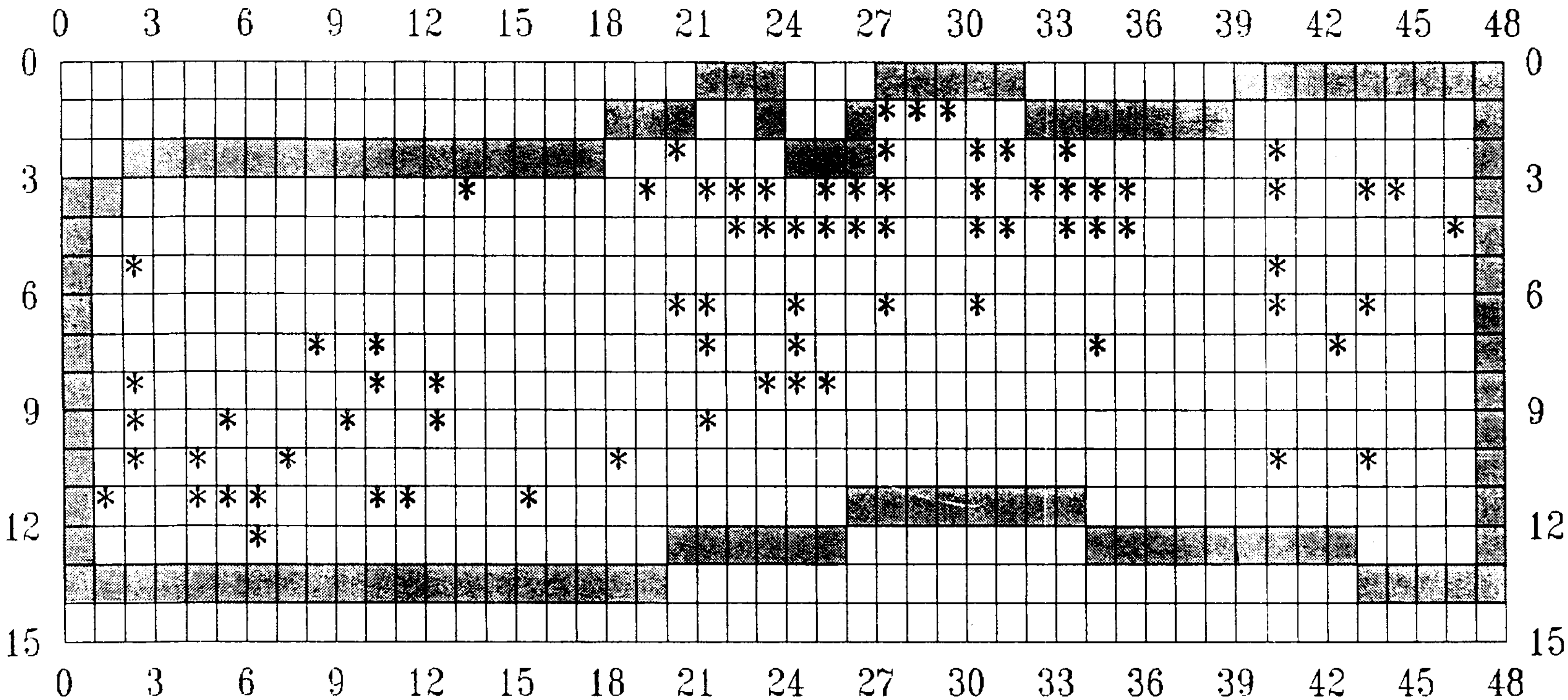

Kinsley-Great Bend Simulation area
I
II
III
IV
V
1
3
2
3
3
Mean Annual Flow Rate (cfs)
Number of Wells
Time (year)
------ Arkansas River at Kinsley
—— Number of ground-water rights
streamflow average and averaging period
I, II,... pumping periods
1, 3,... stress periods

Figure 1



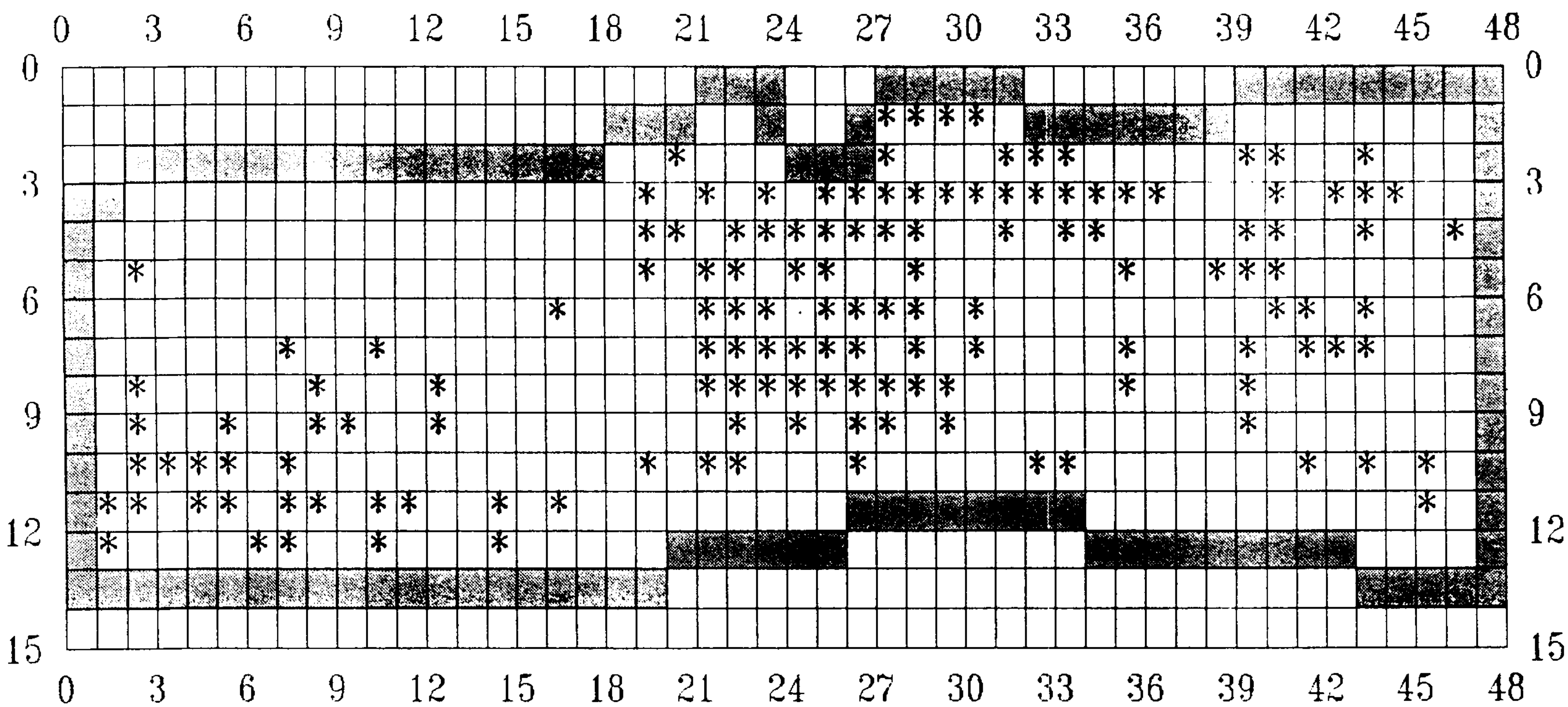
1955 IRRIGATION WELLS

Figure 1



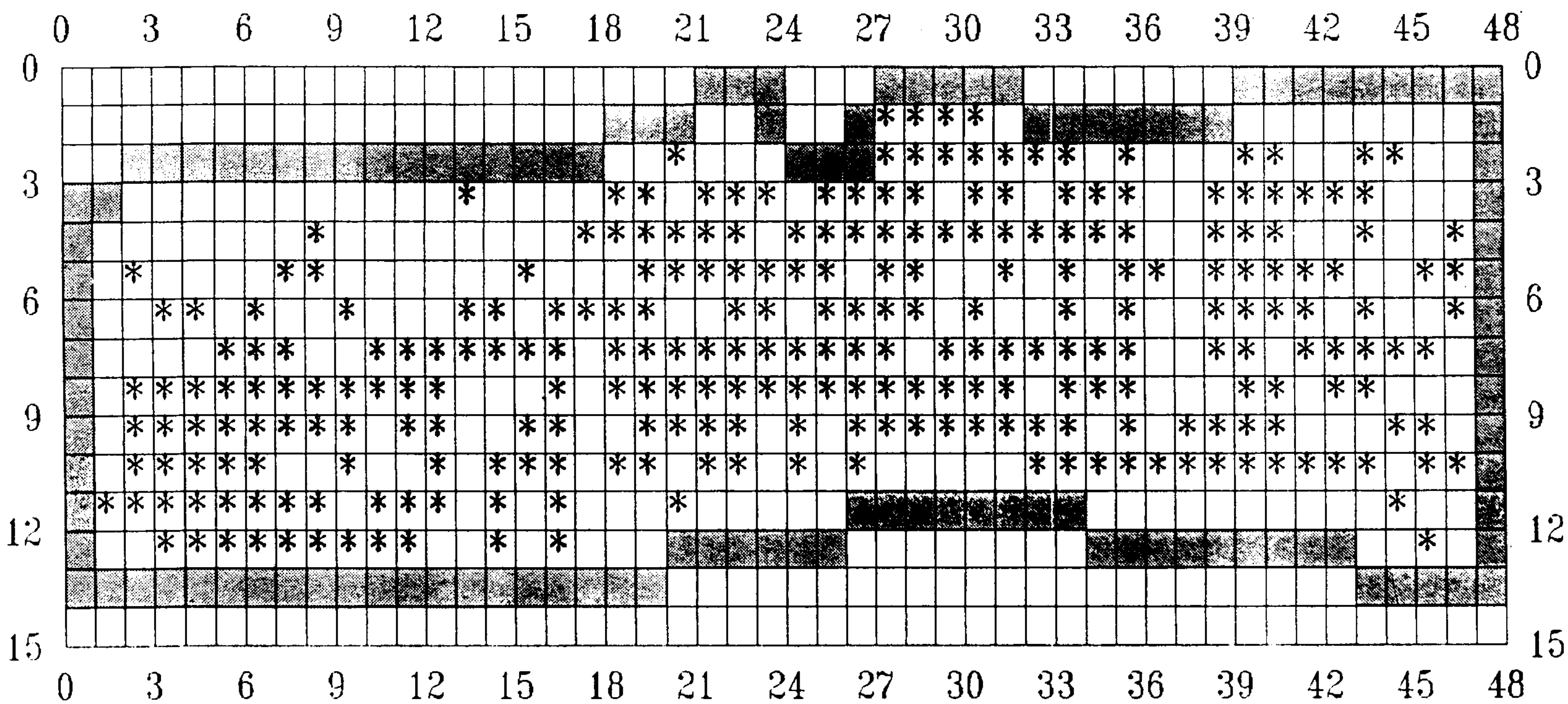
1967 IRRIGATION WELLS

Figure 1



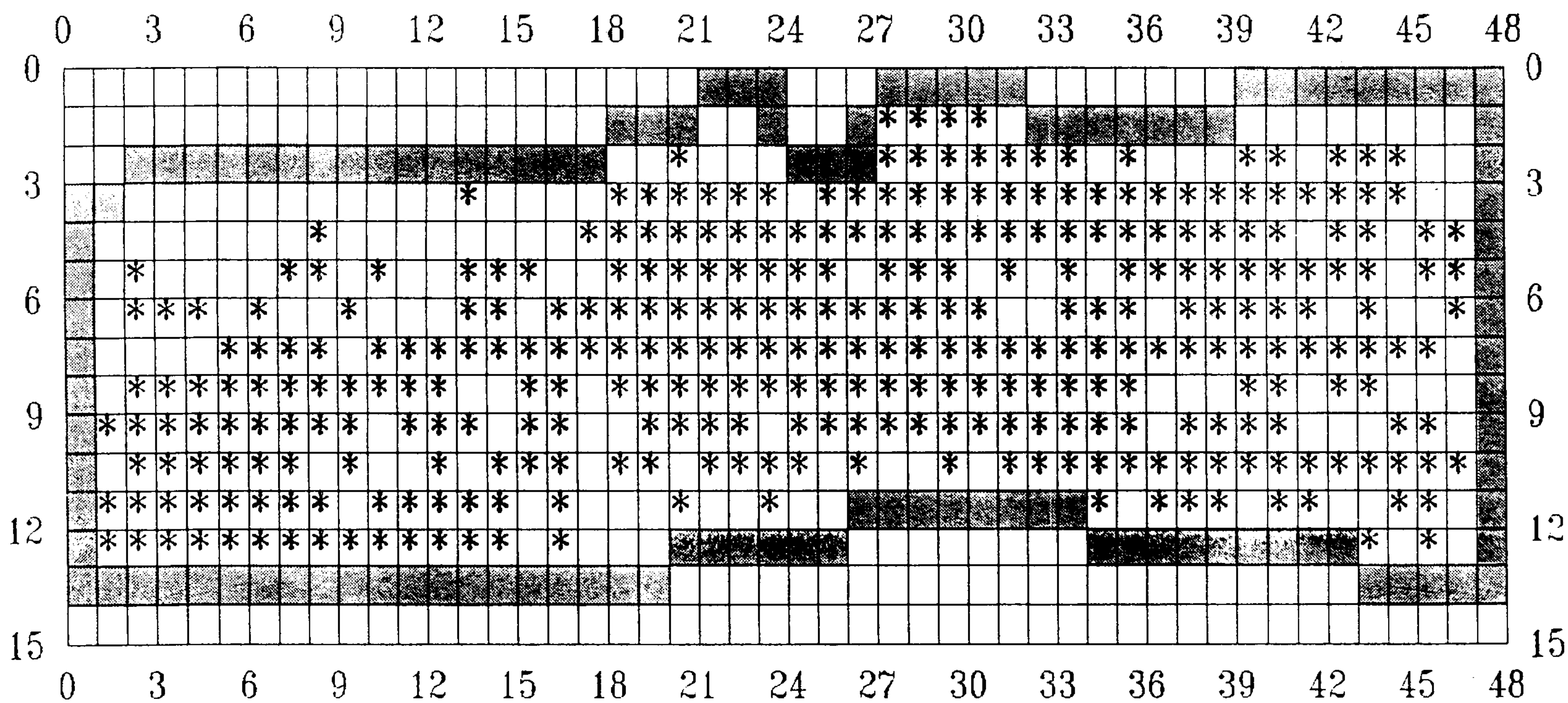
1973 IRRIGATION WELLS

Figure 1



1979 IRRIGATION WELLS

Figure 2



1990 IRRIGATION WELLS

Hydraulic Conductivity zonation

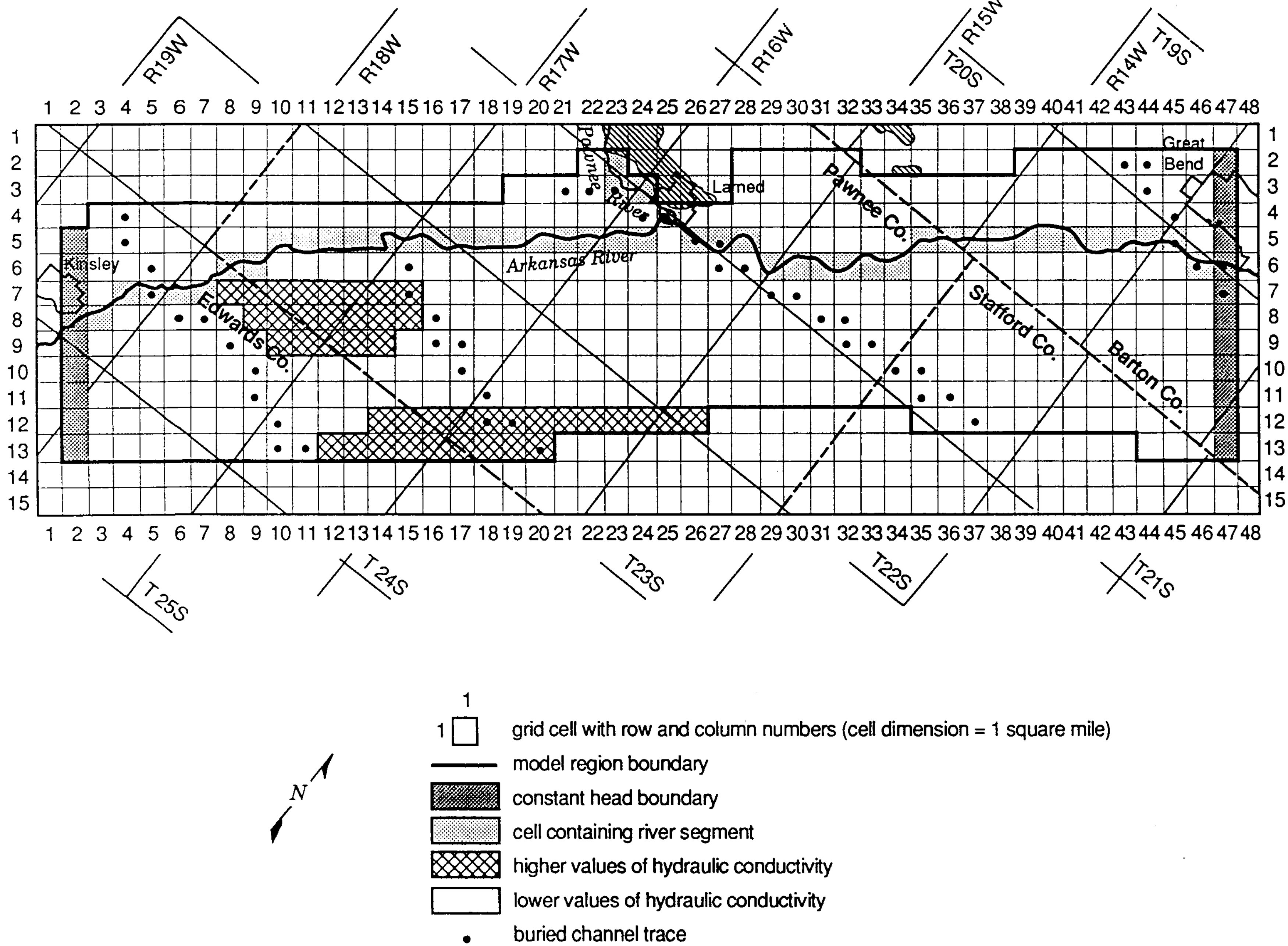


Figure 22

Storativity zonation

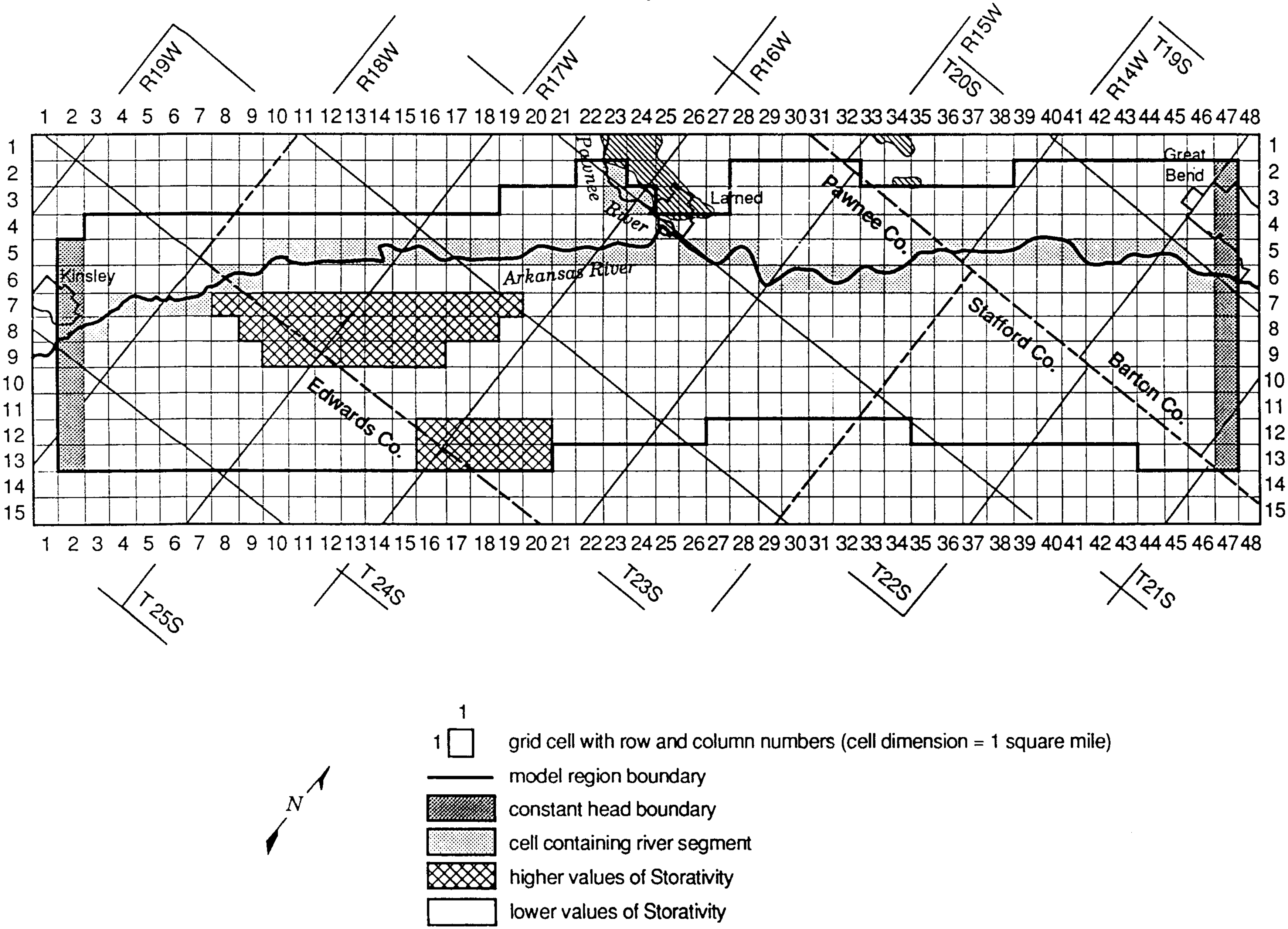
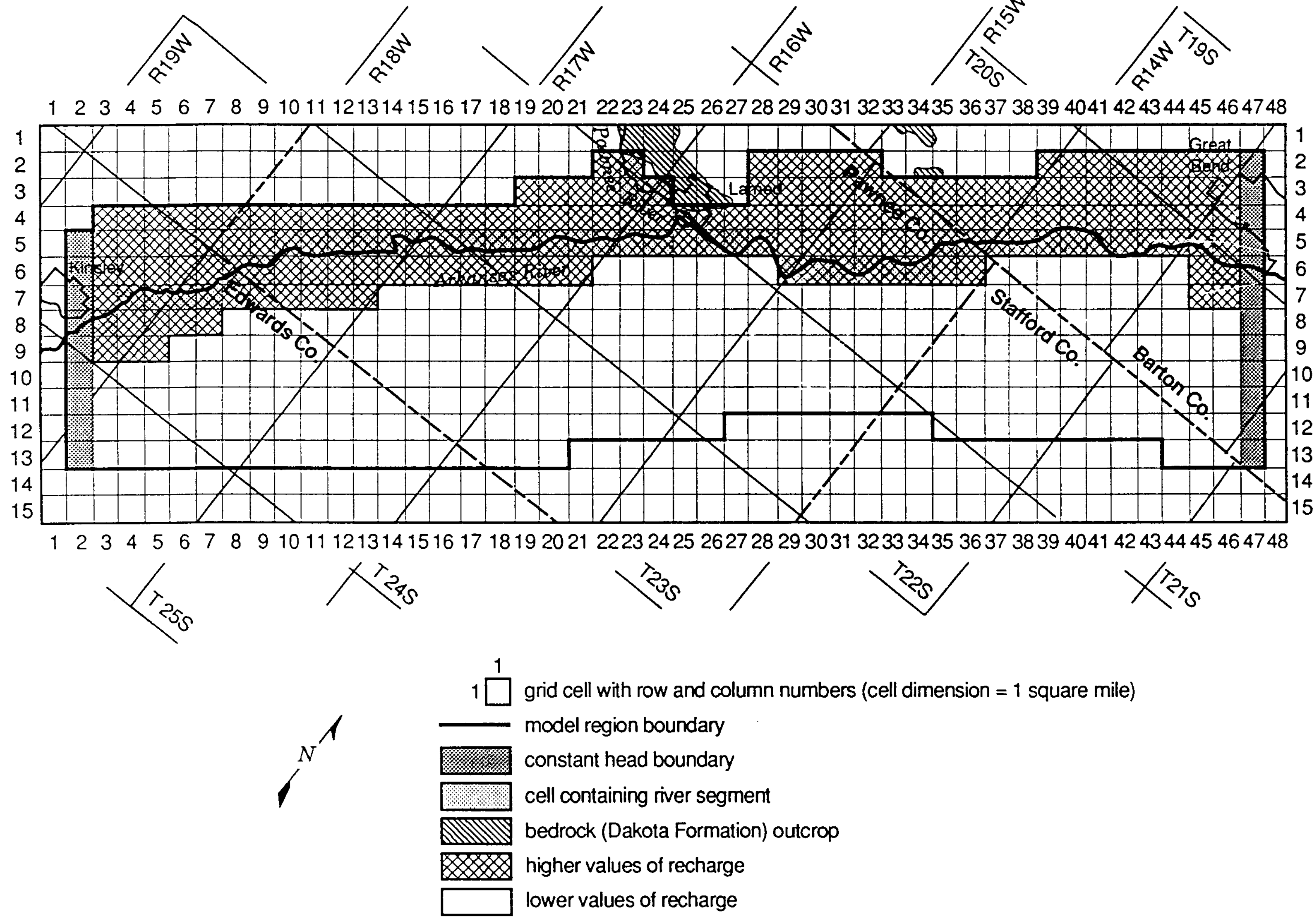
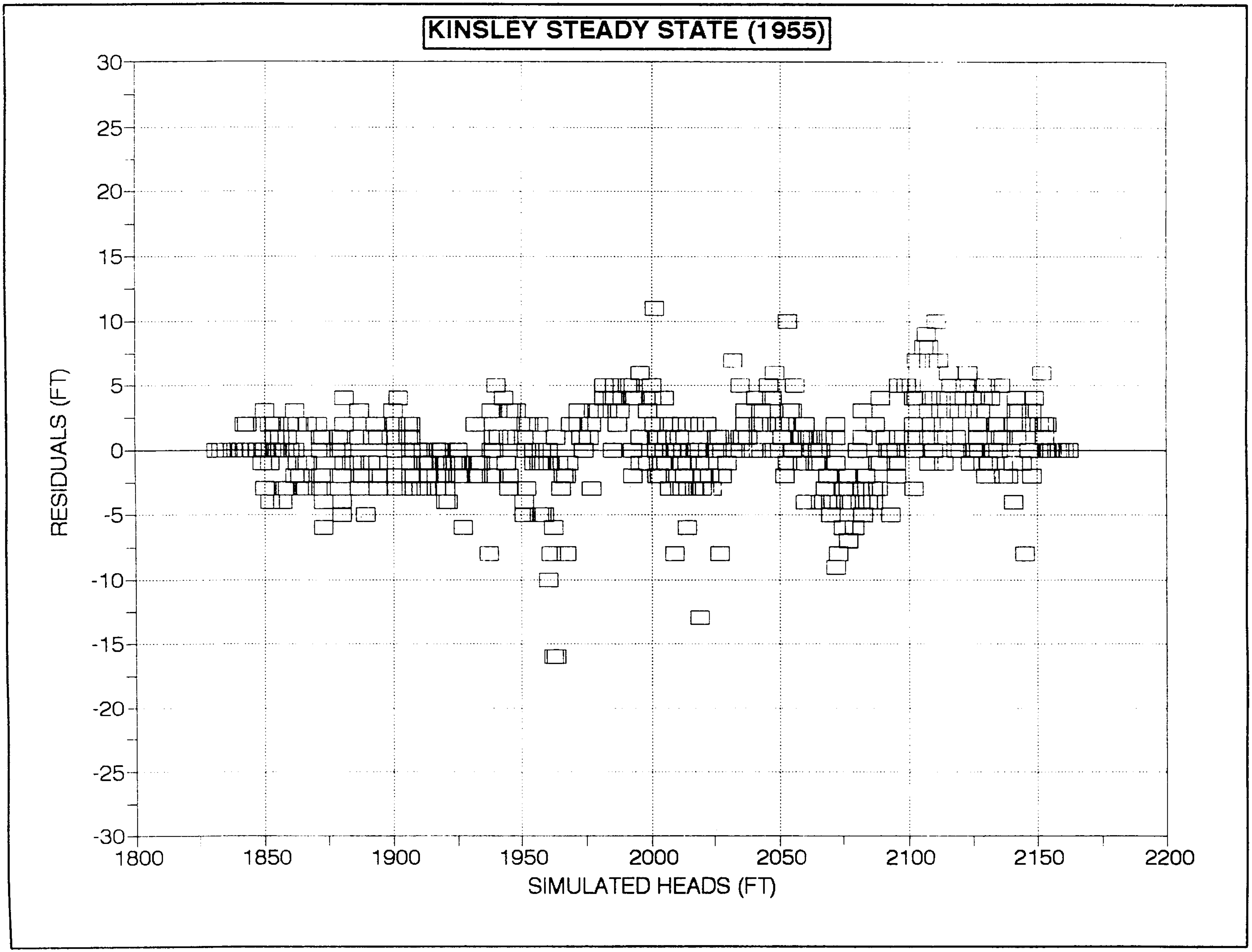


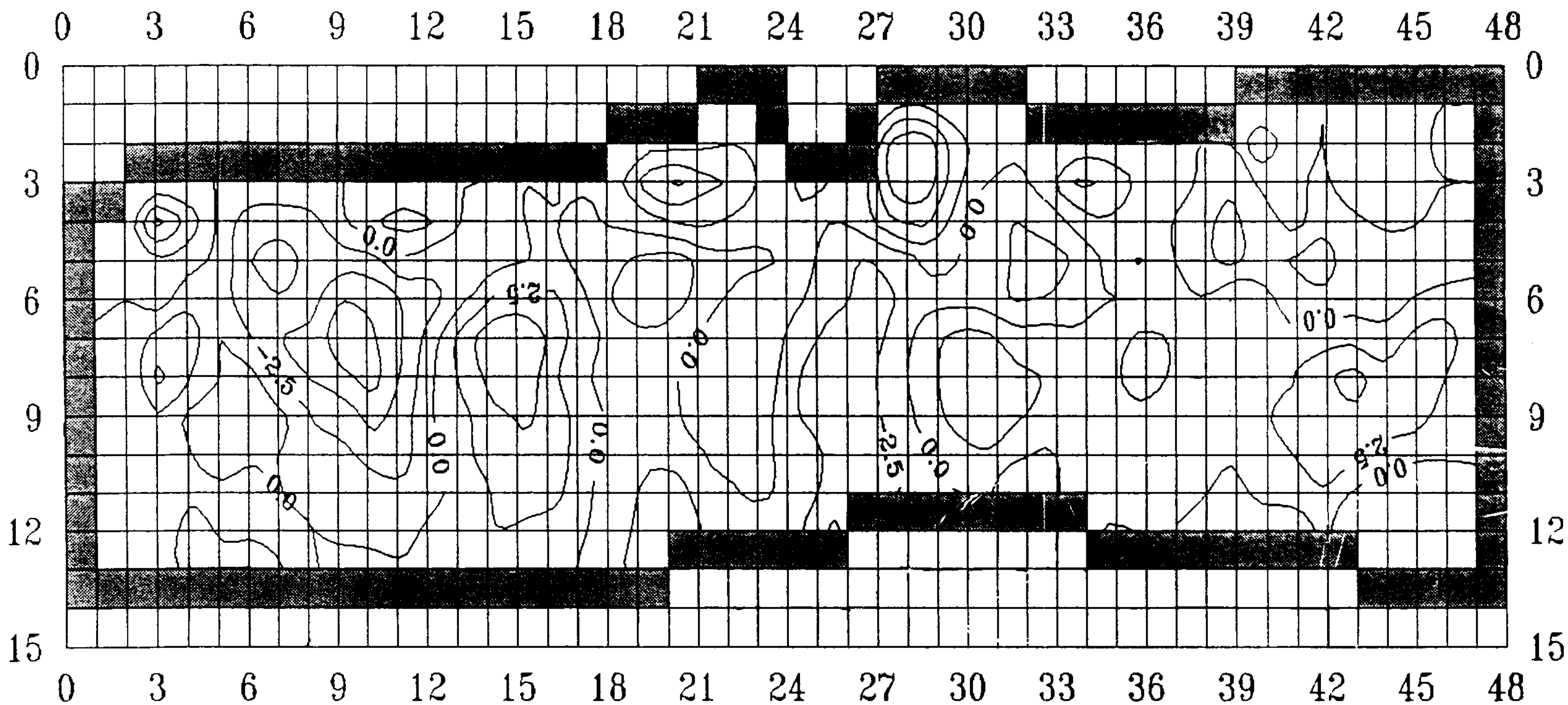
Figure 2

Recharge zonation





KINSLEY STEADY STATE (1955)
RESIDUALS (FT)
30
25
20
15
10
5
0
-5
-10
-15
-20
-25
-30
1800
1850
1900
1950
2000
2050
2100
2150
2200
SIMULATED HEADS (FT)



KINSLEY STEADY STATE SIMULATION

Figure 2

Residual probability plot (predevelopment, 1955)

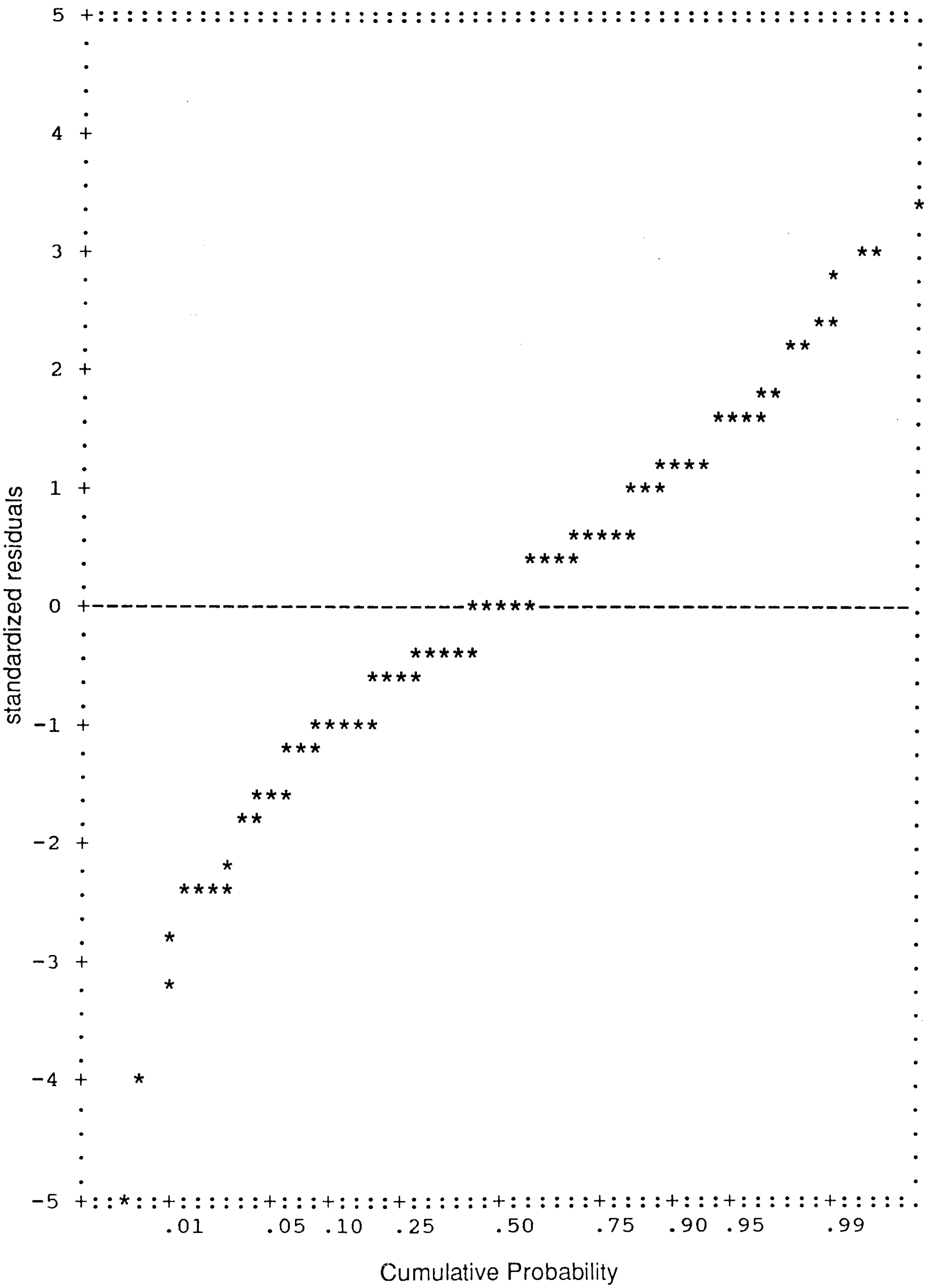


Figure 2

1985 Simulated and Measured water levels

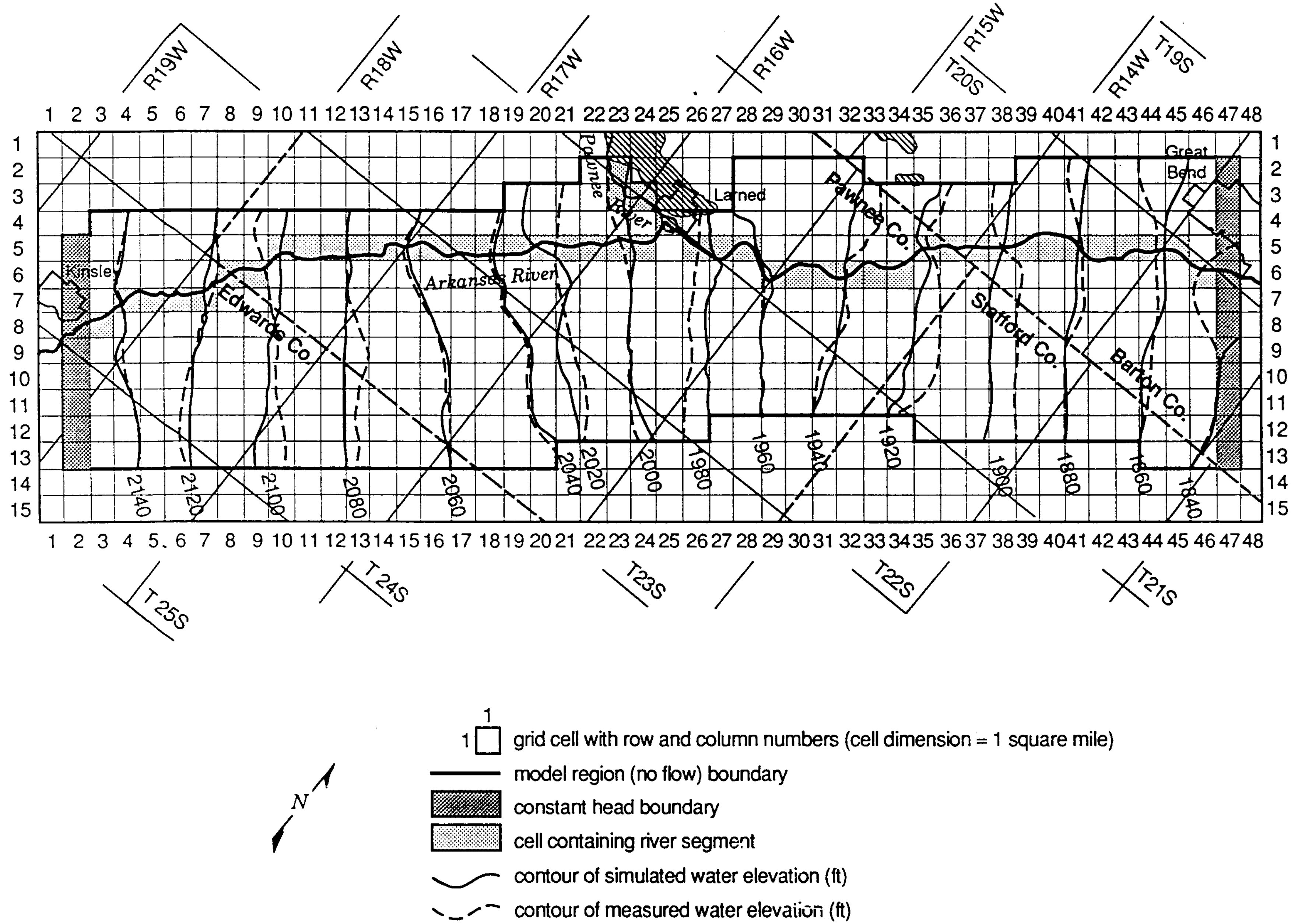


Figure 26

1990 Simulated and Measured water levels

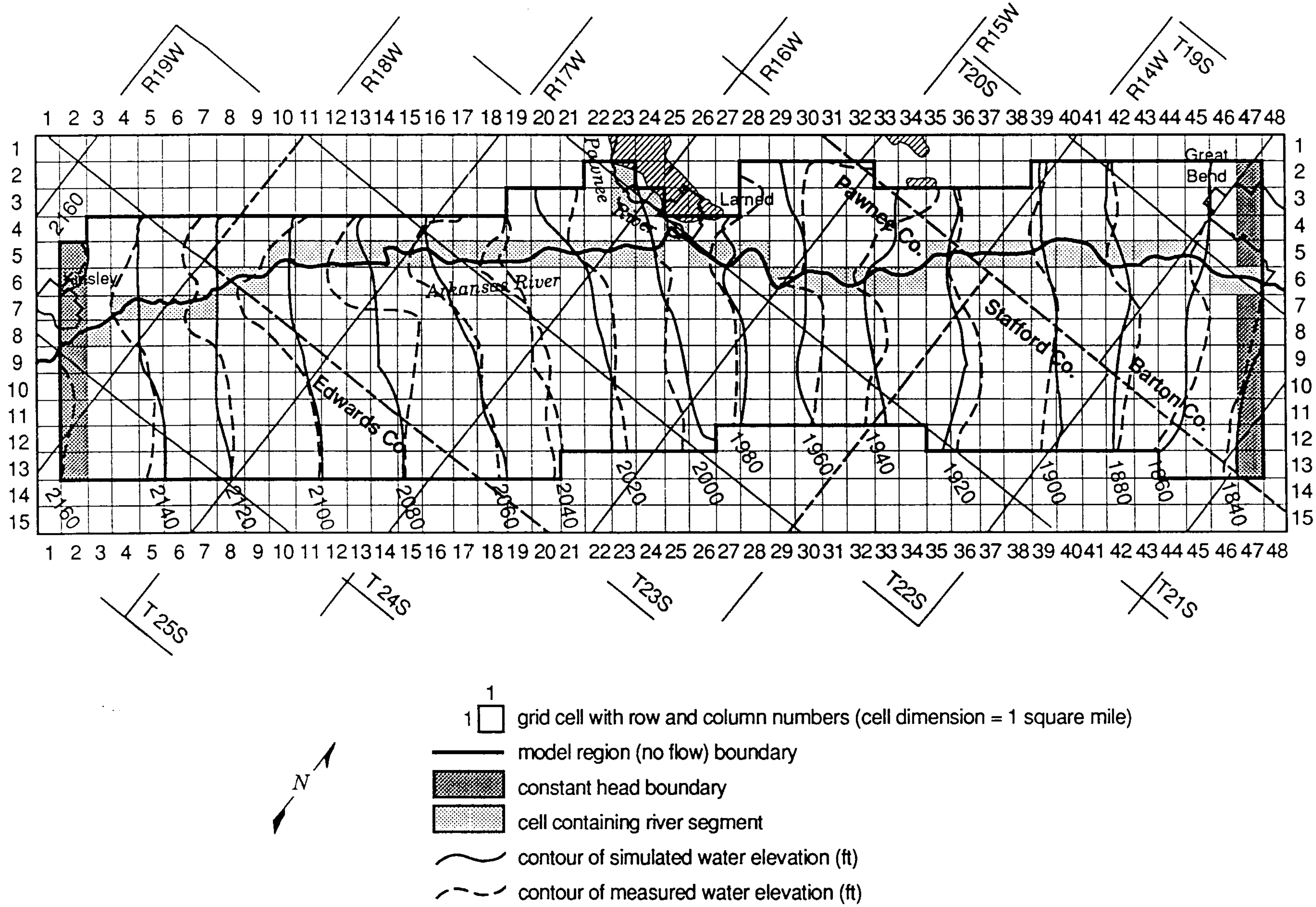
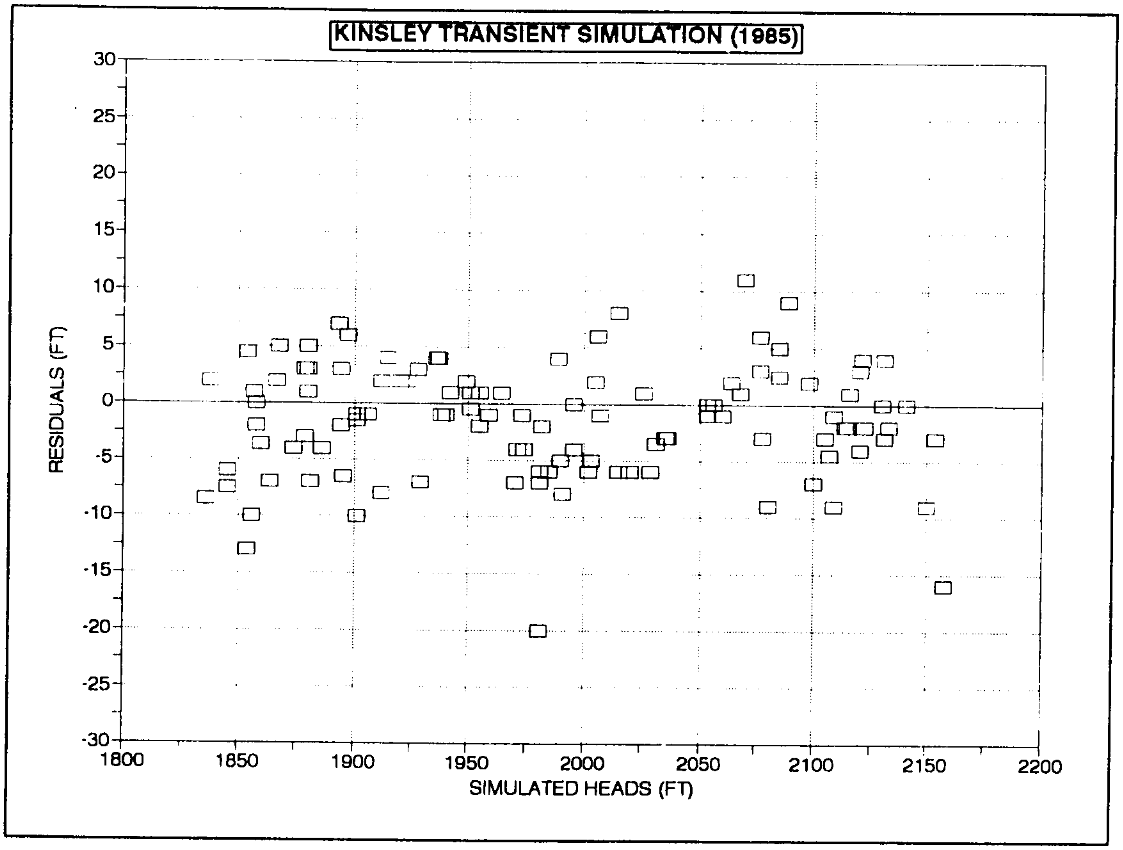


Figure 29


KINSLEY TRANSIENT SIMULATION (1985)
RESIDUALS (FT)
SIMULATED HEADS (FT)
30
25
20
15
10
5
0
-5
-10
-15
-20
-25
-30
1800
1850
1900
1950
2000
2050
2100
2150
2200

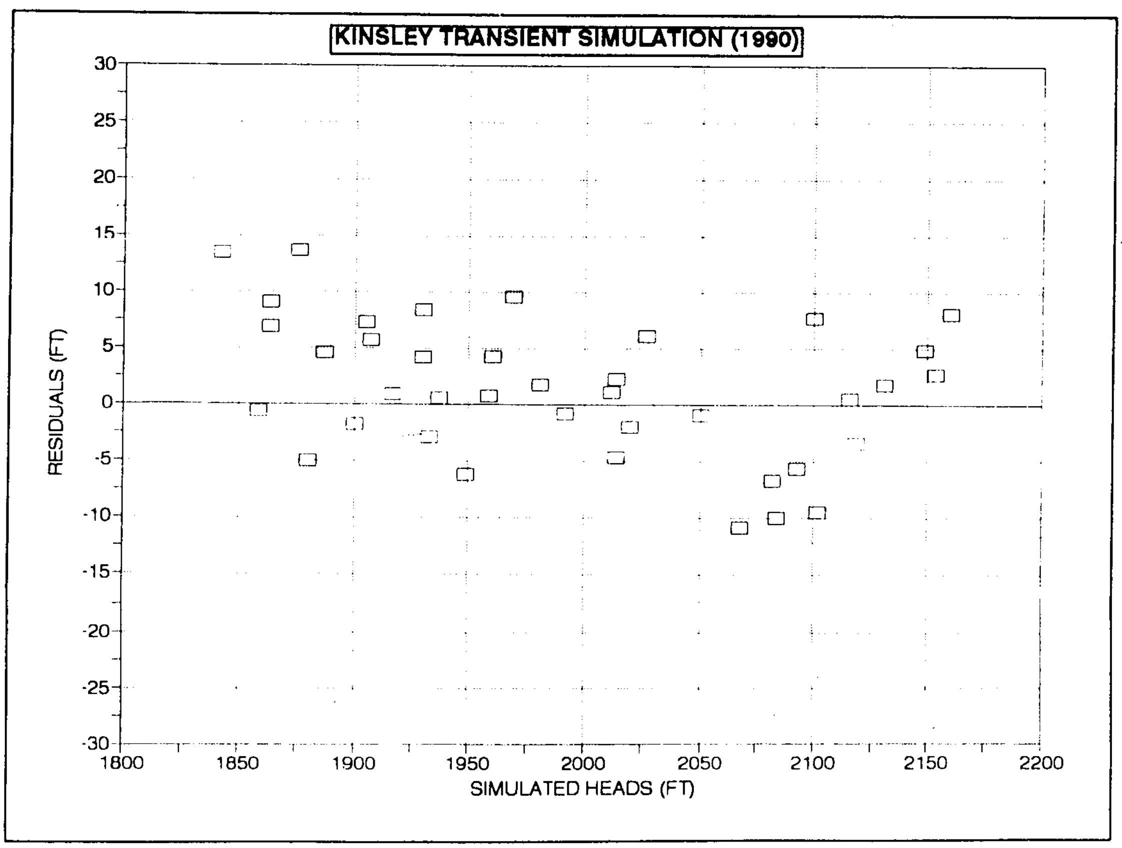
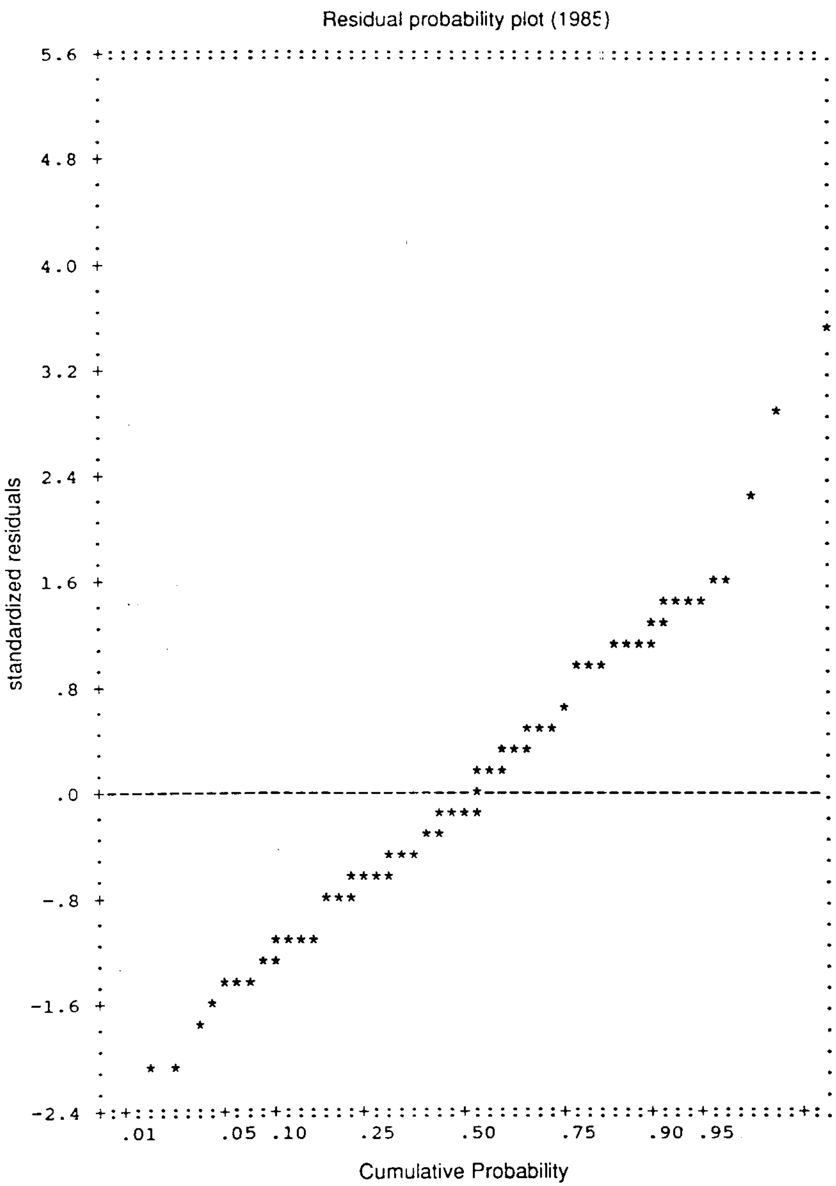

KINSLEY TRANSIENT SIMULATION (1990)
RESIDUALS (FT)
SIMULATED HEADS (FT)
30
25
20
15
10
5
0
-5
-10
-15
-20
-25
-30
1800
1850
1900
1950
2000
2050
2100
2150
2200

Figure 30


Residual probability plot (1985)
5.6
4.8
4.0
3.2
2.4
1.6
.8
.0
-.8
-1.6
-2.4
standardized residuals
.01
.05
.10
.25
.50
.75
.90
.95
Cumulative Probability

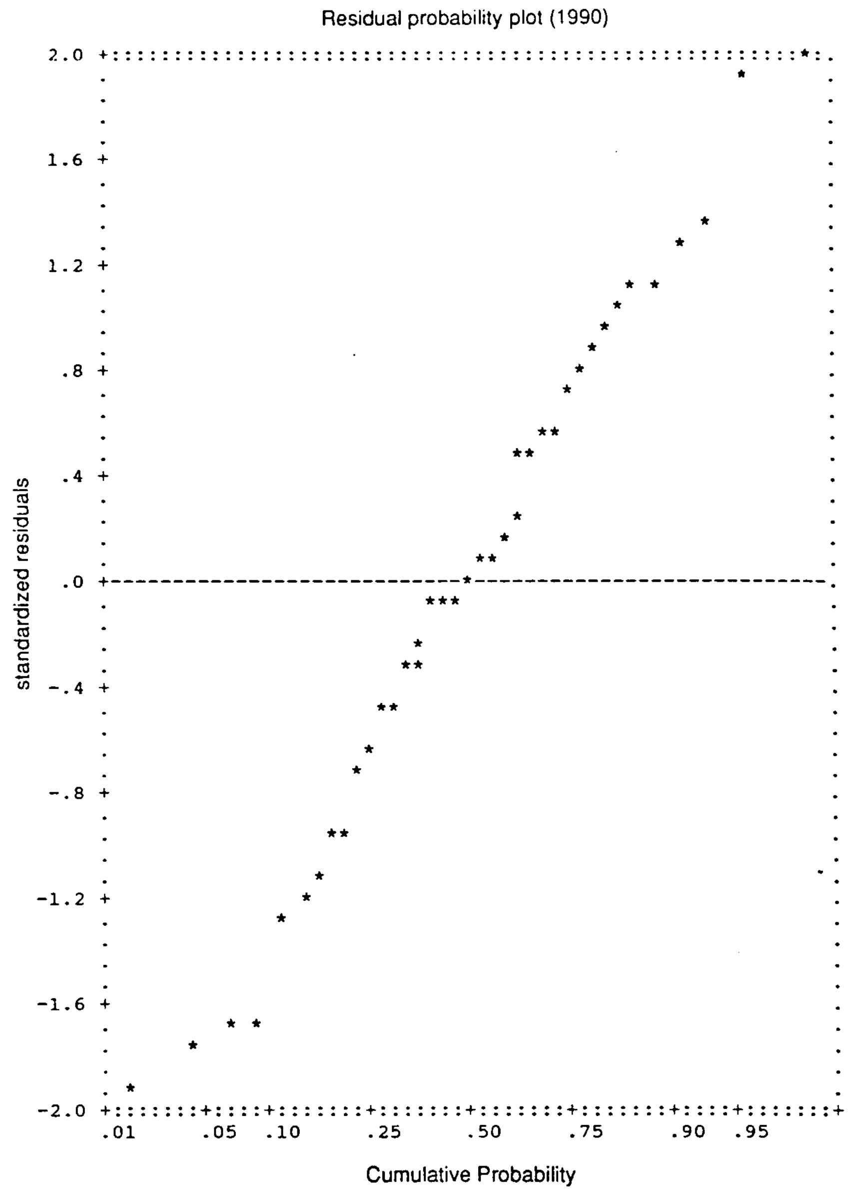
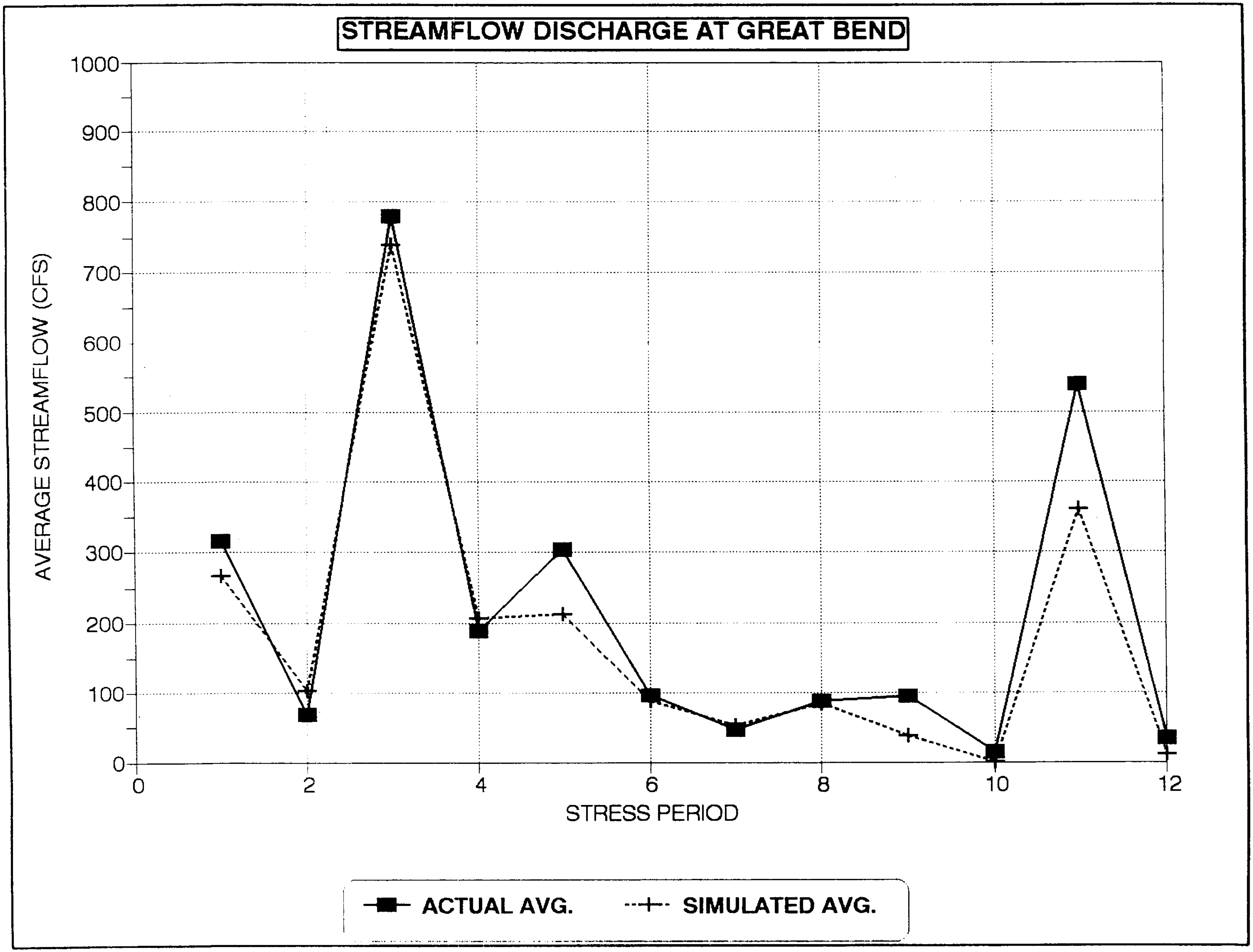

Residual probability plot (1990)
2.0
1.6
1.2
.8
.4
.0
-.4
-.8
-1.2
-1.6
-2.0
standardized residuals
.01
.05
.10
.25
.50
.75
.90
.95
Cumulative Probability

Figure 31



STREAMFLOW DISCHARGE AT GREAT BEND
1000
900
800
700
600
500
400
300
200
100
0
AVERAGE STREAMFLOW (CFS)
0
2
4
6
8
10
12
STRESS PERIOD
ACTUAL AVG.
SIMULATED AVG.

Figure 32

Gaining and loosing reaches of the Arkansas River 1955

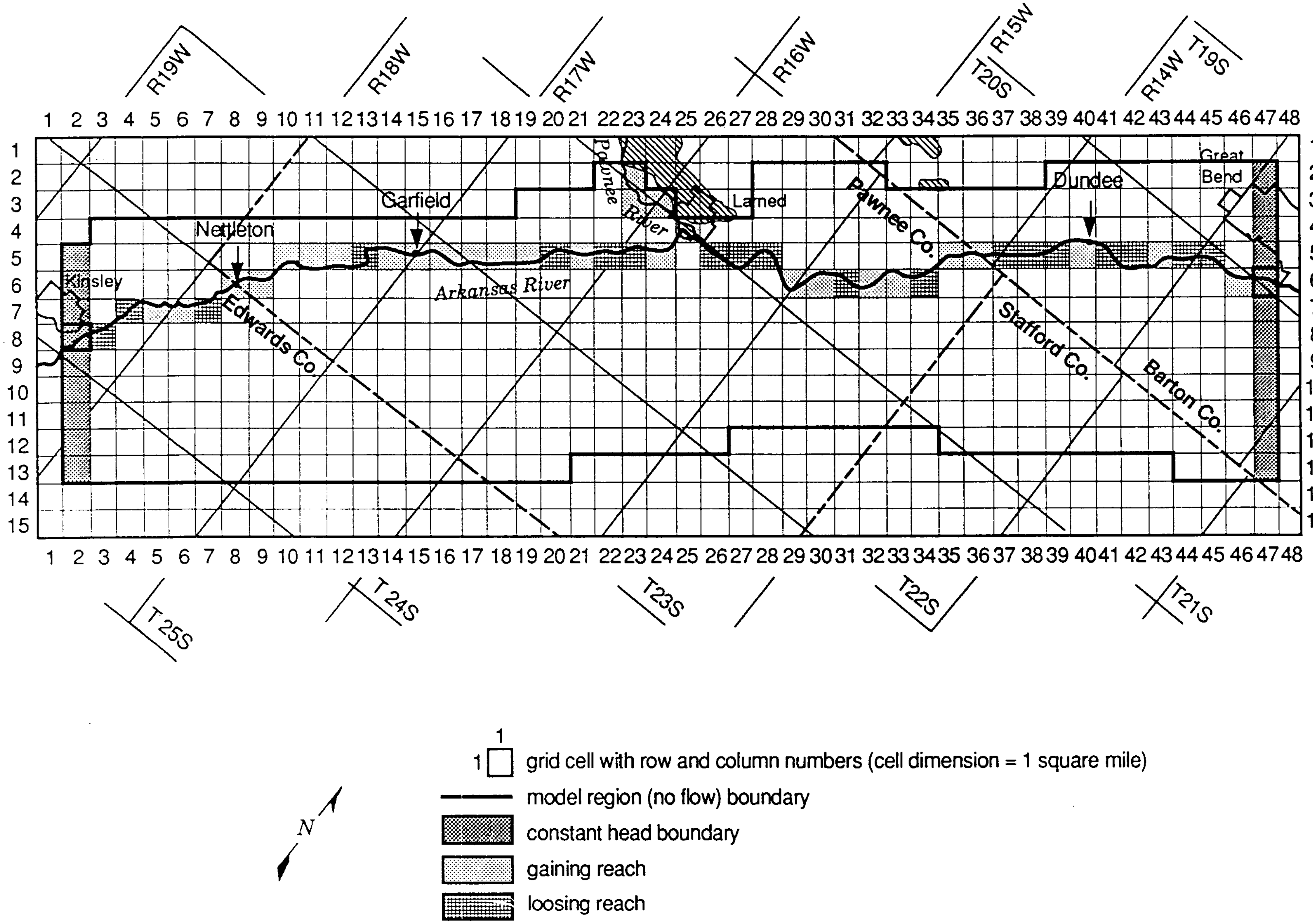


Figure 33

Gaining and loosing reaches of the Arkansas River 1985

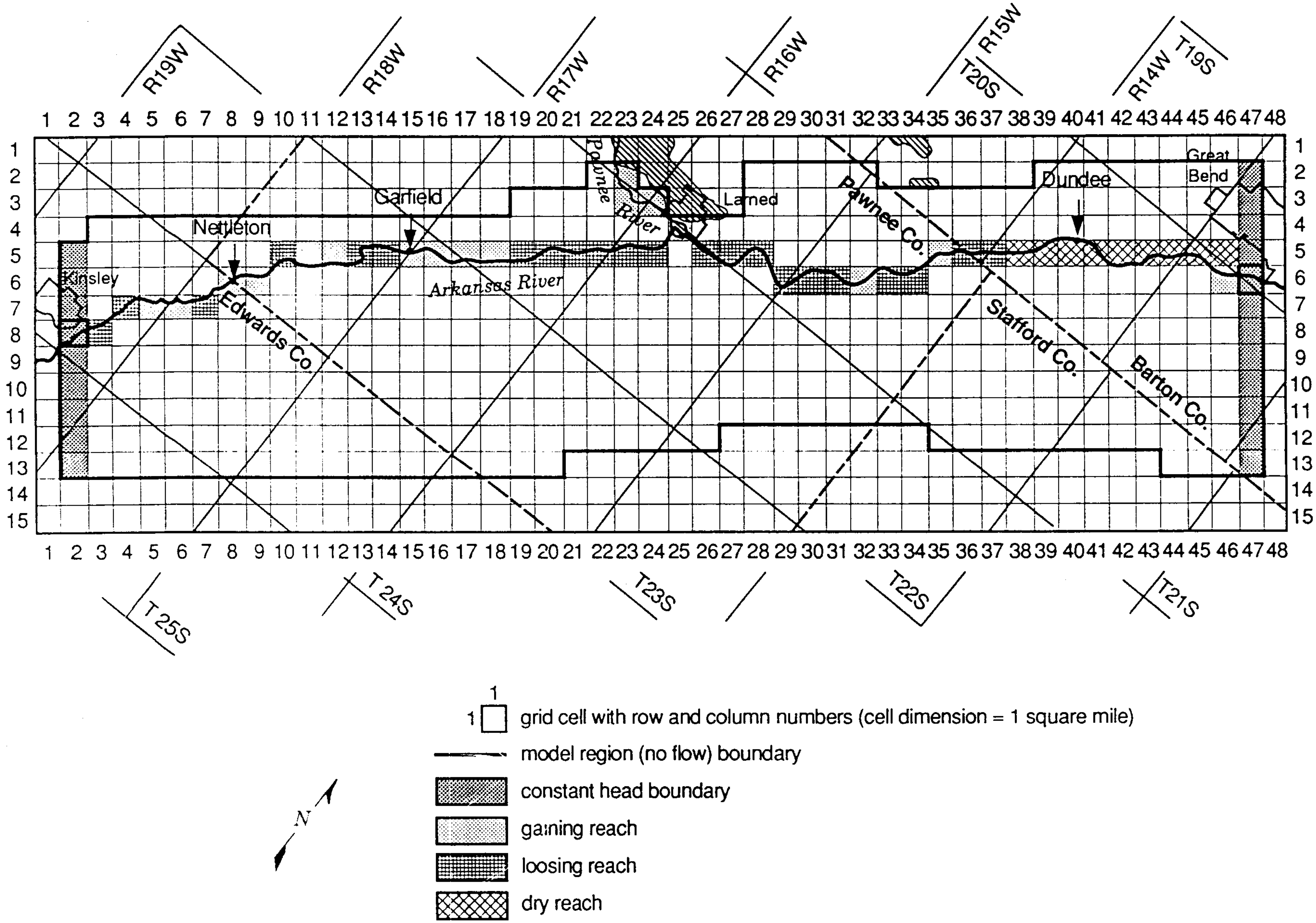
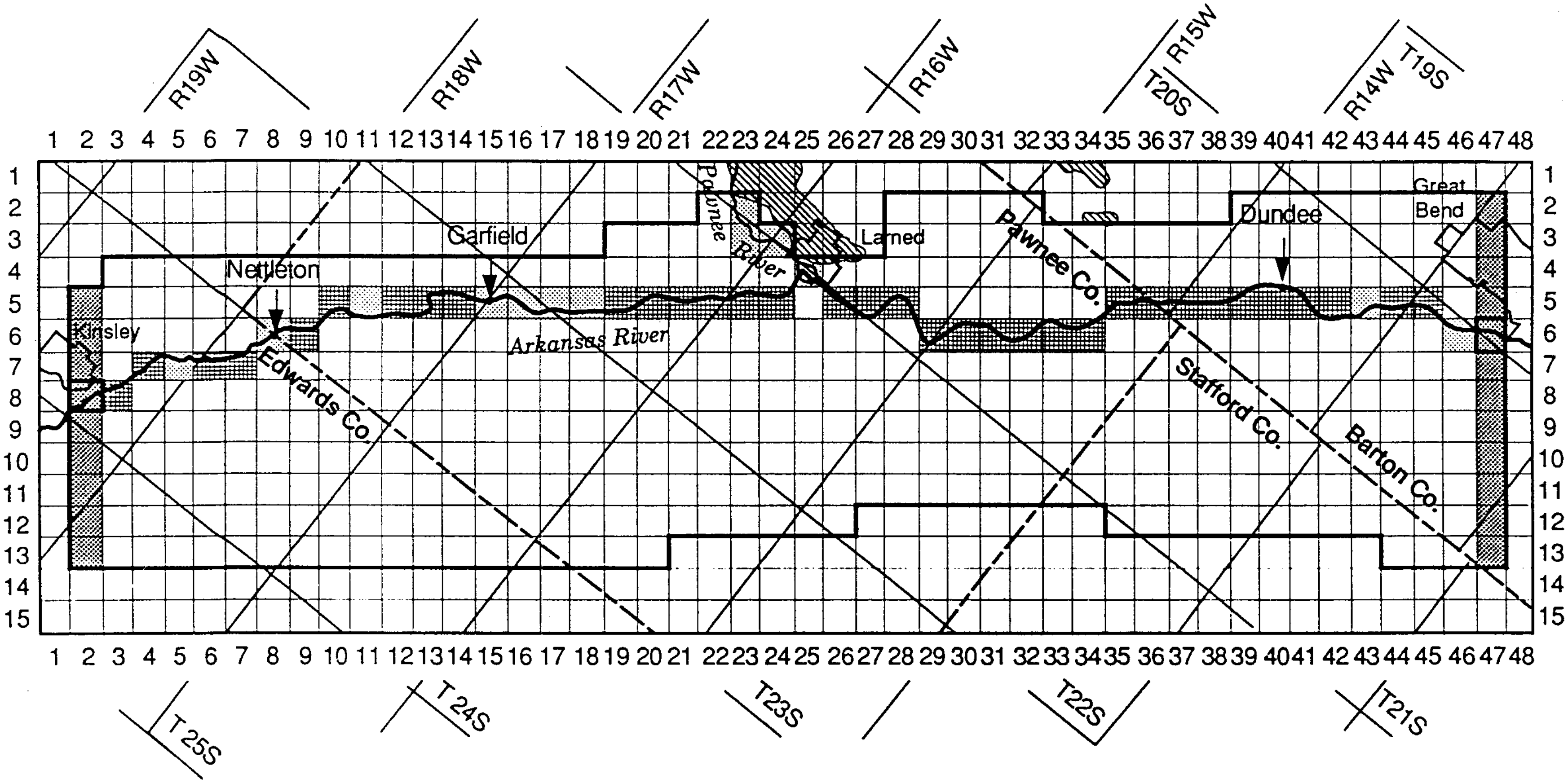
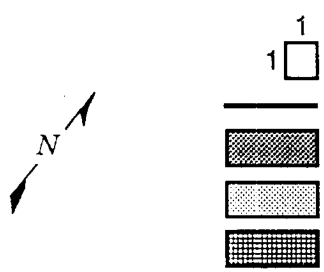


Figure 3.

Gaining and loosing reaches of the Arkansas River 1990





grid cell with row and column numbers (cell dimension = 1 square mile)

model region (no flow) boundary

constant head boundary

gaining reach

loosing reach

Breakdown of Figures 36–42

a. Changing pumpage by 50%

b. Changing recharge by 50%

c. Changing hydraulic conductivity by 50%

d. Changing storativity by 50%

e. Changing streambed conductance by 50%

f. Changing Manning's roughness coefficient by 50%

g. Changing stream channel slope by 50%

h. Changing channel width by 50%

■ Base conditions (1990 simulation)

\+ Increase 50%

* Decrease 50%

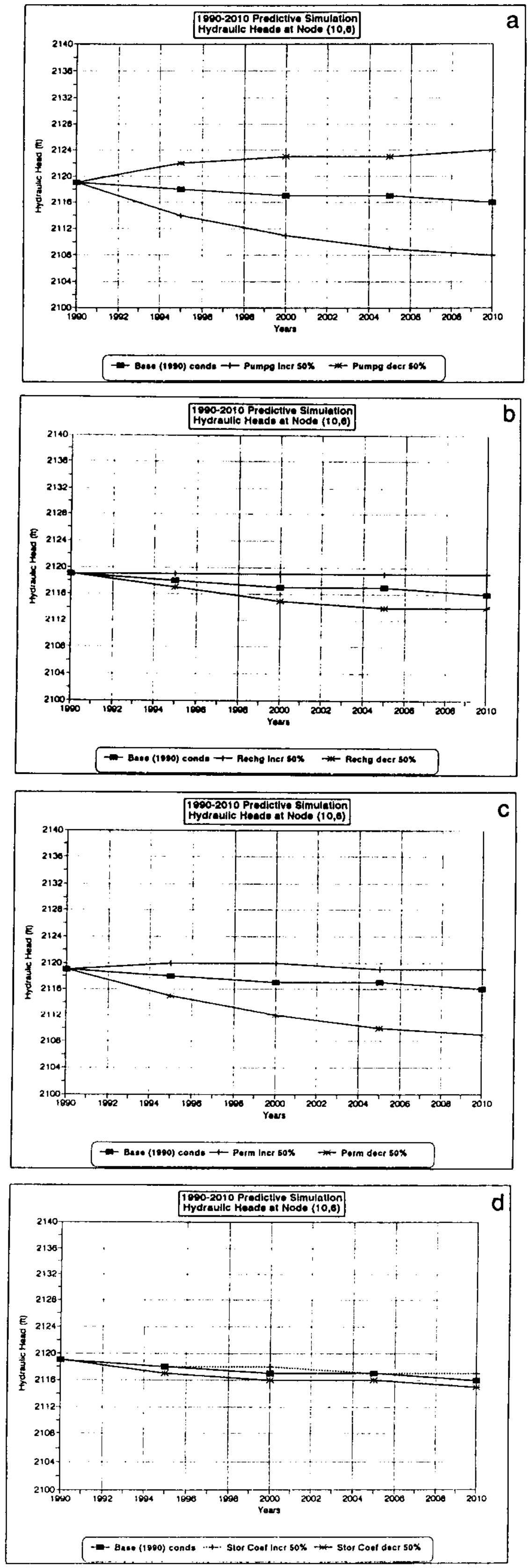


Figure 36. Sensitivity analysis of hydraulic head values near Kinsley.

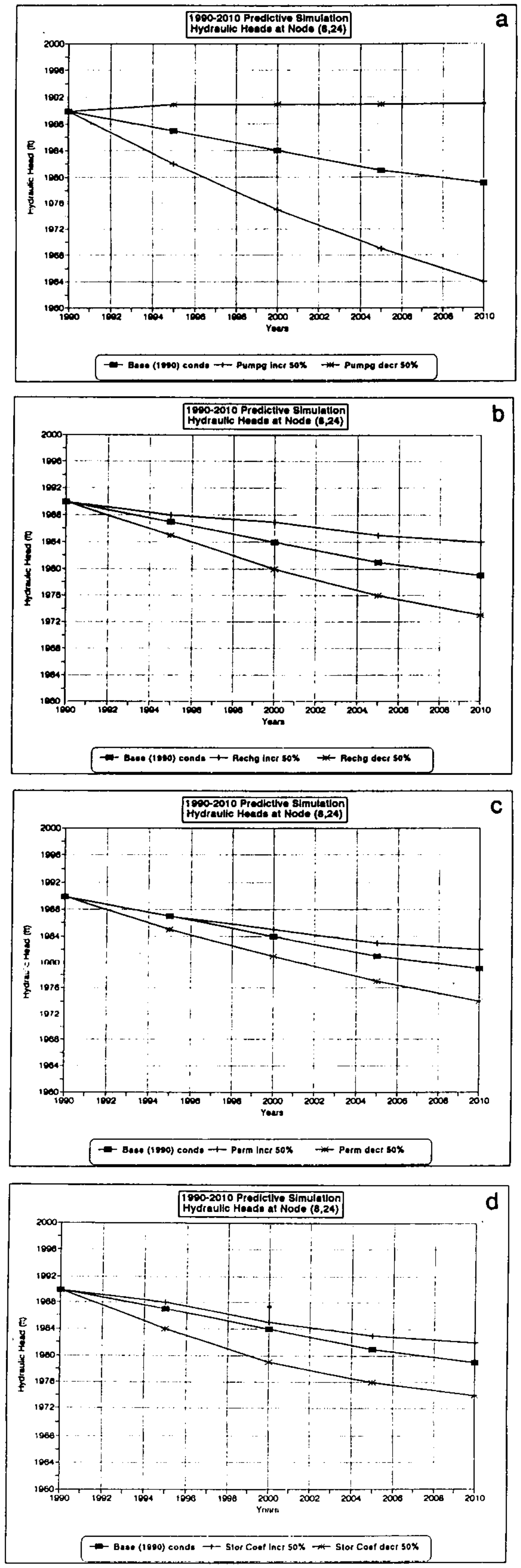


Figure 37. Sensitivity analysis of hydraulic head values near Larned.

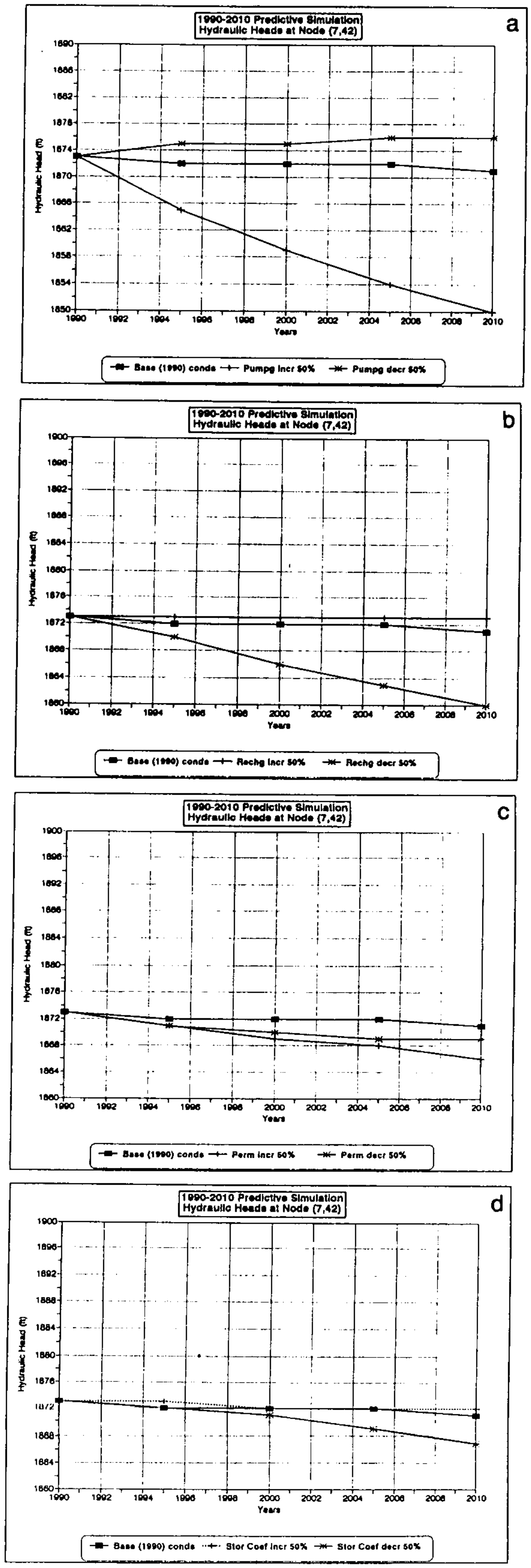


Figure 38. Sensitivity analysis of hydraulic head values near Great Bend.

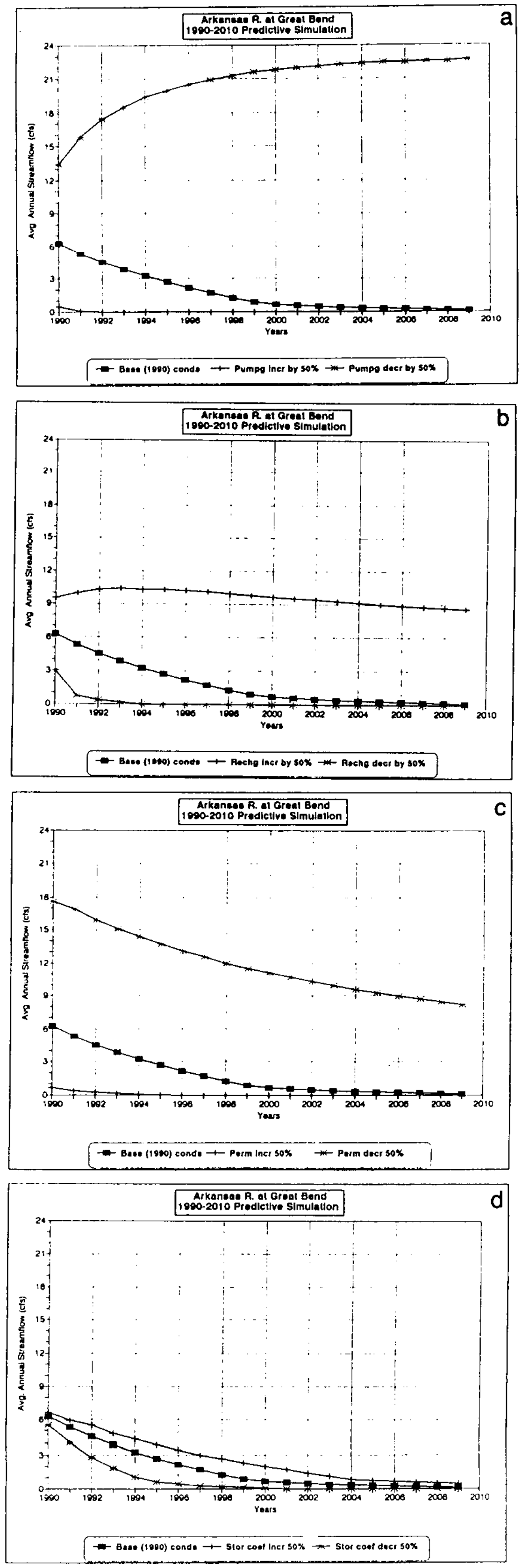


Figure 39. Sensitivity analysis of Arkansas River streamflows near Great Bend to aquifer parameters/inputs.

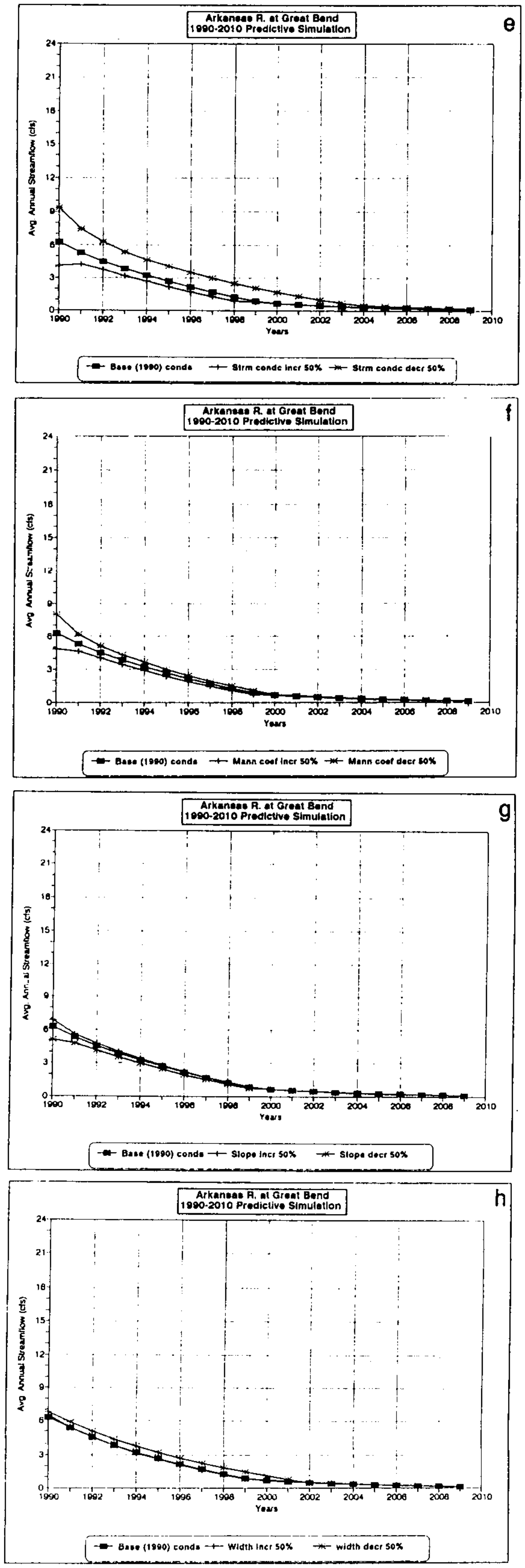


Figure 40. Sensitivity analysis of Arkansas River streamflows near Great Bend to stream parameters.

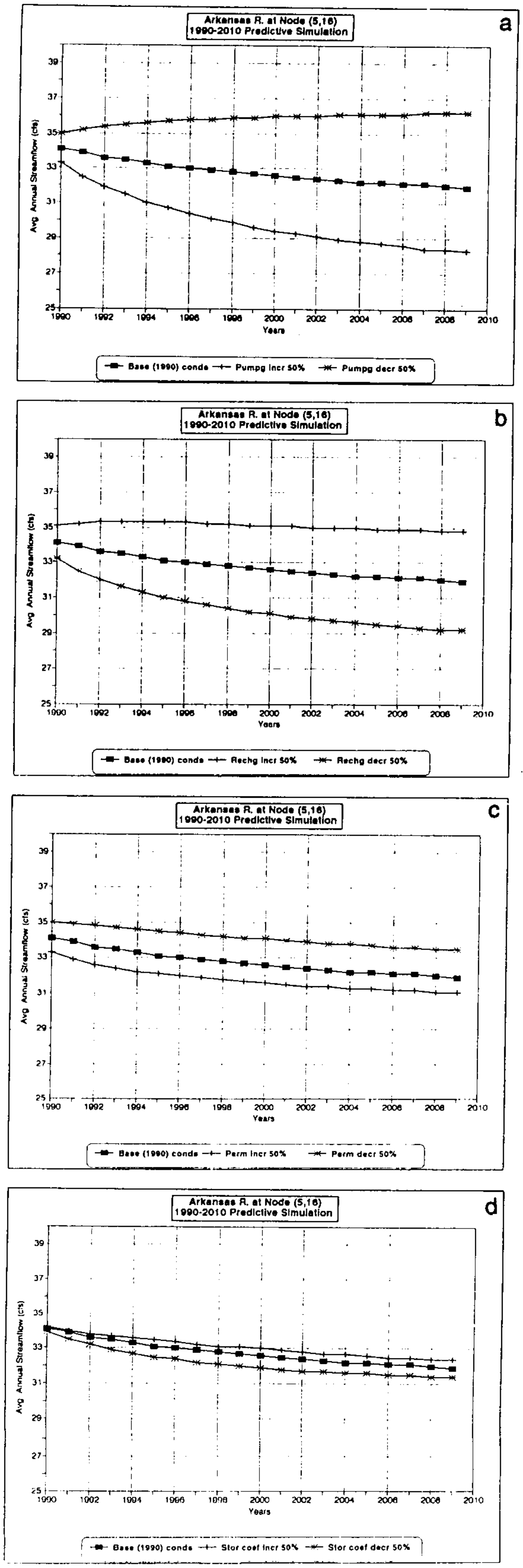


Figure 41. Sensitivity analysis of Arkansas River streamflows near Garfield to aquifer parameters/inputs.

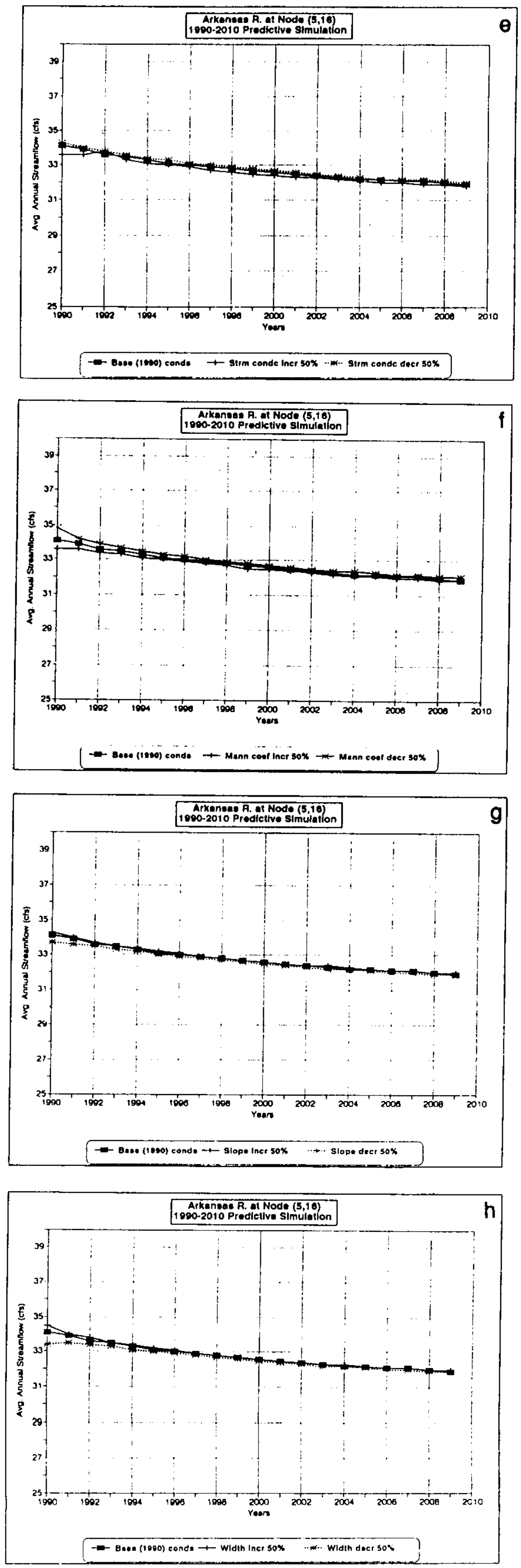


Figure 42. Sensitivity analysis of Arkansas River streamflows near Garfield to aquifer parameters/inputs.