EFFECTS OF PARTIAL PENETRATION, ANISOTROPY, BOUNDARIES AND WELL SKIN ON SLUG TESTS

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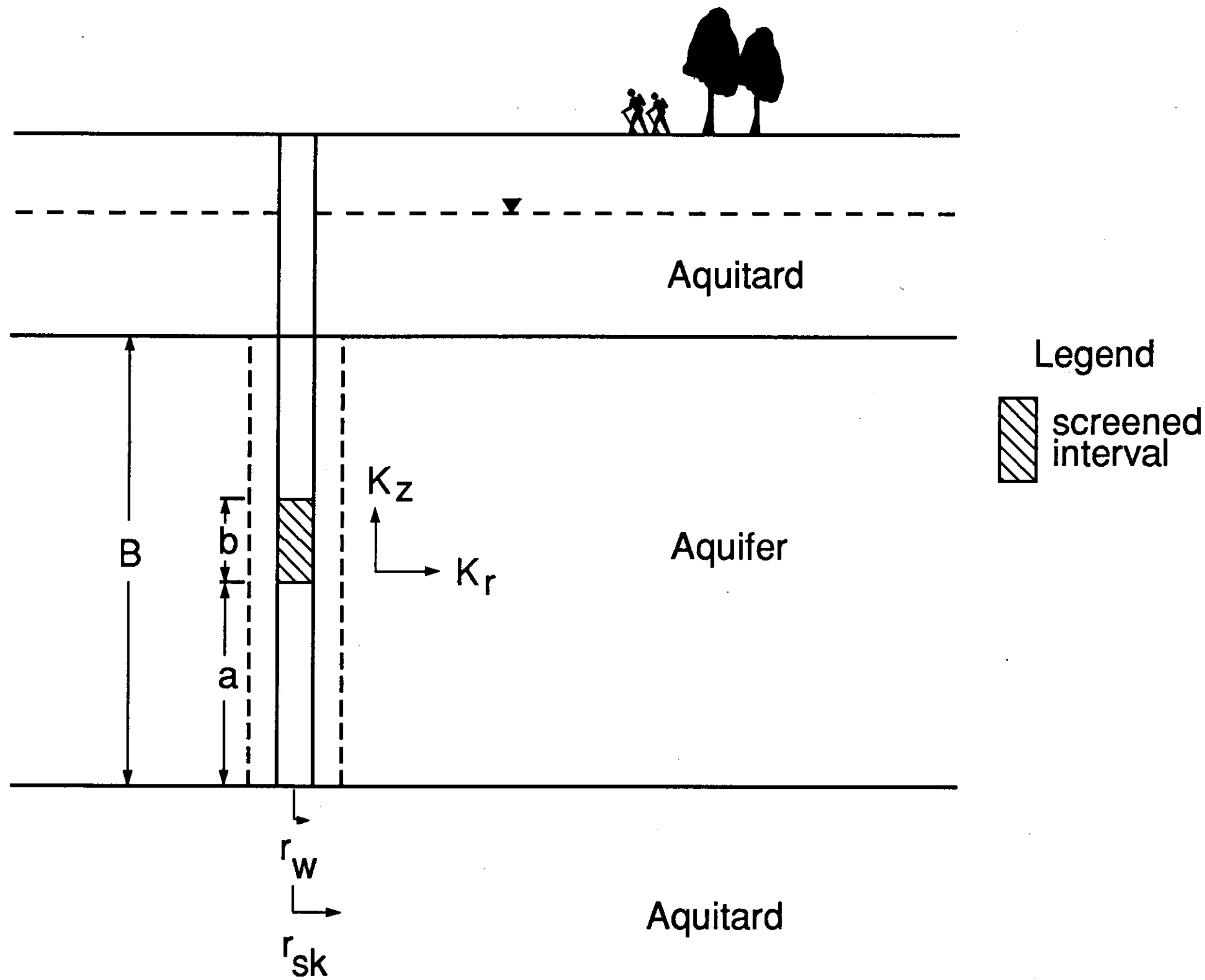
INTRODUCTION

The slug test is an important technique for site evaluation investigations. Analyses using the CBP (Cooper-Bredehoeft-Papadopulos), Bower and Rice, or Hvorslev techniques are widely reported and used. However, the reliability of these estimates of hydraulic conductivity is unknown in the presence of complications such as partial penetration, anisotropy, impermeable horizontal boundaries, and a near-well zone of disturbance (well skin). We have developed a single model for slug tests that incorporates all these effects. An analytical solution is obtained in Fourier-Laplace space. Numerical inversion techniques are used to obtain the solution in the space-time domain. This solution has been interfaced with a program that we are developing for automated slug-test design and analysis. This allows systematic generation and analysis of data under a variety of conditions with relative ease.

This figure (1) depicts the configuration of interest here. A slug test is being performed in a well that is screened for a portion (b) of the full width of the aquifer (B). In the immediate vicinity of the well, there is a well skin of radius r_{sk}. The well skin extends through the full width of the aquifer. The skin has transmissive and storage properties that may be different from those of the aquifer. In both the skin and the aquifer, the vertical (K_z) and horizontal (K_r) components of hydraulic conductivity may be different. On both the top and bottom, the aquifer is bounded by aquitards, which are represented here by impermeable horizontal boundaries.

Figure 1.


Aquitard
Legend
screened
interval
K z
B
b
K r
Aquifer
a
r w
r sk
Aquitard

BASIC MODEL EQUATION

The partial differential equation representing the flow of ground-water in response to an instantaneous slug of water is the same for both the skin and the aquifer and can be represented in the following manner:

$$\frac{\partial^2 h_i}{\partial r^2} + \frac{1}{r}\frac{\partial h_i}{\partial r} + \left(\frac{K_{z_i}}{K_{r_i}}\right)\frac{\partial^2 h_i}{\partial z^2} = \left(\frac{S_{s_i}}{K_{r_i}}\right)\frac{\partial h_i}{\partial t} \qquad (1)$$

where

h_i = head in zone i, [L]

S_i = specific storage of zone i, [1/L]

K_{z_i}, K_{r_i} = vertical and horizontal components, of hydraulic conductivity of zone i, [L/T]

t = time, [T]

r = radial direction, [L]

z = vertical direction, [L]

For $r_w \le r \le r_{sk}$, $i = 1$, and for $r_{sk} < r$, $i = 2$.

INITIAL AND BOUNDARY CONDITIONS

In order to obtain a solution to (1) for the configuration of interest, initial and boundary conditions must be defined.

Initial Conditions:

$$h_1(r,z,0) = h_2(r,z,0) = 0, \;\; r_w \le r < \infty, \;\; 0 \le z \le B \qquad (2)$$

$$h_1(r_w,z,0) = \begin{cases} H_0, & a \le z \le a+b \\ 0, & \text{elsewhere} \end{cases} \qquad (3)$$

Boundary Conditions:

$$h_2(\infty,z,t) = 0, \;\; t \ge 0, \;\; 0 \le z \le B \qquad (4)$$

$$\frac{\partial h_i(r,0,t)}{\partial z} = \frac{\partial h_i(r,B,t)}{\partial z} = 0, \;\; r_w \le r < \infty, \;\; t \ge 0 \qquad (5)$$

$$h_1(r_w,z,t) = H(t), \;\; t > 0, \;\; a \le z \le a+b \qquad (6)$$

$$2\pi r_w K_{r_1} \frac{\partial h_1(r_w,z,t)}{\partial r} = \frac{\pi r_c^2}{b} \frac{\partial H(t)}{\partial t} \square(z) \;\; t > 0, \;\; 0 \le z \le B \qquad (7)$$

where

H(t) = level of water in well, [L]

r_c = radius of well casing, casing and screen do not have to be of equal radius, [L]

$\square(z)$ = boxcar function = $\begin{cases} 1, & a \le z \le b+a. \\ 0, & \text{elsewhere} \end{cases}$

At the boundary between the skin and the aquifer additional conditions must be prescribed in order to ensure continuity of flow across that boundary:

$$h_1(r_{sk}, z,t) = h_2(r_{sk}, z,t), \;\; 0 \le z \le B, \;\; t > 0 \qquad (8)$$

$$K_{r_1} \frac{\partial h_1(r_{sk}, z,t)}{\partial r} = K_{r_2} \frac{\partial h_2(r_{sk}, z,t)}{\partial r}, \;\; 0 \le z \le B, \quad t > 0 \qquad (9)$$

FOURIER - LAPLACE SOLUTION

Equations (1) - (9) describe the flow conditions. The procedure for finding h_1 and h_2 is to apply a pair of integral transforms (Tranter, 1962). A finite Fourier cosine transform is applied in the z direction, and a Laplace transform in time is used to obtain the transformed equivalent of (1).

$$\frac{\partial^2 \bar{h}_i}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{h}_i}{\partial_r} - \left[\frac{K_{z_i}}{K_{r_i}}\omega^2 + \frac{S_{s_i}}{K_{r_i}}p\right]\bar{h}_i = 0, \; r_w \le r \le \infty \qquad (10)$$

where

$\bar{h}_i$ = the Fourier and Laplace transform of h_i, $\bar{h}_i(r, \omega, p)$;
ω = the Fourier transform variable;
p = the Laplace transform variable.

Equation (10) is a form of the modified Bessel equation (Haberman, 1987). A solution can therefore be written in the form:

$$\bar{h}_i(r,\omega,p) = A_i K_0(\nu_i r) + B_i I_0(\nu_i r) \qquad (11)$$

where

$$\nu_i = \sqrt{\frac{K_{z_i}}{K_{r_i}}\omega^2 + \frac{S_{s_i}}{K_{r_i}}p};$$

A_i, B_i = constants;

K_i = modified Bessel function of the second kind of order i;

I_i = modified Bessel function of the first kind of order i.

EVALUATION OF CONSTANTS IN FINAL SOLUTION

Transforming the initial and boundary conditions (2) - (9), the constants in (11) can be evaluated. In a slug test, only the solution for the head at the well is needed:

$$\bar{h}_1(r_w, \omega, p) = E[H_0 - pH(p)]F(\omega)f(r_w) \tag{12}$$

where

H(p) = Laplace transform of H(t);
F(ω) = Fourier transform of □ (z);

$$f(r_w) = \frac{\Delta_3 K_0(\nu_1 r_w) - \Delta_2 I_0(\nu_1 r_w)}{\nu_1[I_1(\nu_1 r_w)\Delta_2 + K_1(\nu_1 r_w)\Delta_3]};$$

$$E = \frac{r_c^2}{2 r_w K_{r_1} b};$$

$$\Delta_2 = K_0(\nu_1 r_w)K_1(\nu_2 r_w) - \left(\frac{K_{r_1}\nu_1}{K_{r_2}\nu_2}\right)K_0(\nu_2 r_w)K_1(\nu_1 r_w)$$

$$\Delta_3 = I_0(\nu_1 r_w)K_1(\nu_2 r_w) + \left(\frac{K_{r_1}\nu_1}{K_{r_2}\nu_2}\right)K_0(\nu_2 r_w)I_1(\nu_1 r_w).$$

This represents the final solution in Fourier-Laplace space.

INVERSE FOURIER AND LAPLACE TRANSFORMATIONS

Once a transform-space solution has been obtained, the expression must be inverted back to untransformed space. This involves the use of the inverse Fourier transform and the inverse Laplace transform. The analytical back transformation for the finite Fourier cosine transform can be performed for (12), resulting in the Laplace-space analogue to (12) which is:

$$\bar{h}_1(r_{w,z,p}) = H(p) = \frac{E\Delta H_0}{1 + E\Delta p} \tag{13}$$

where

$\bar{h}_1(r_w, z, p) =$ the Laplace transform of h_1;

$\Delta = \Phi_1 + \Phi_2$

$\Phi_1 = \frac{bf(r_w)}{B}$;

$$\Phi_2 = \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{f(r_w)}{n}\sin\frac{n\pi b}{2B}\cos\frac{n\pi(b+2a)}{2B}\cos\frac{n\pi\left(a+\frac{b}{2}\right)}{B}$$

The analytical inverse Laplace transform, however, is not so readily performed. Therefore, a numerical approximation of the inversion is employed, the algorithm of Stehfest (1970) has been found to be of great use in hydrologic applications (Moench and Ogata, 1984). A program to perform the Stehfest inversion of (13) was written and the program was included in the automated slug-test analysis package currently under development at the KGS (Bohling and McElwee, 1989). If b=B and skin effects are negligible, equation (13) reduces to the Laplace-space solution of Cooper, Bredehoeft, and Papadopulos (1967). If skin effects are important and the system is isotropic, (13) reduces to the Laplace-space solution given by Moench and Hsieh (1985) for the presence of a well skin.

BASE DATA FOR SIMULATION

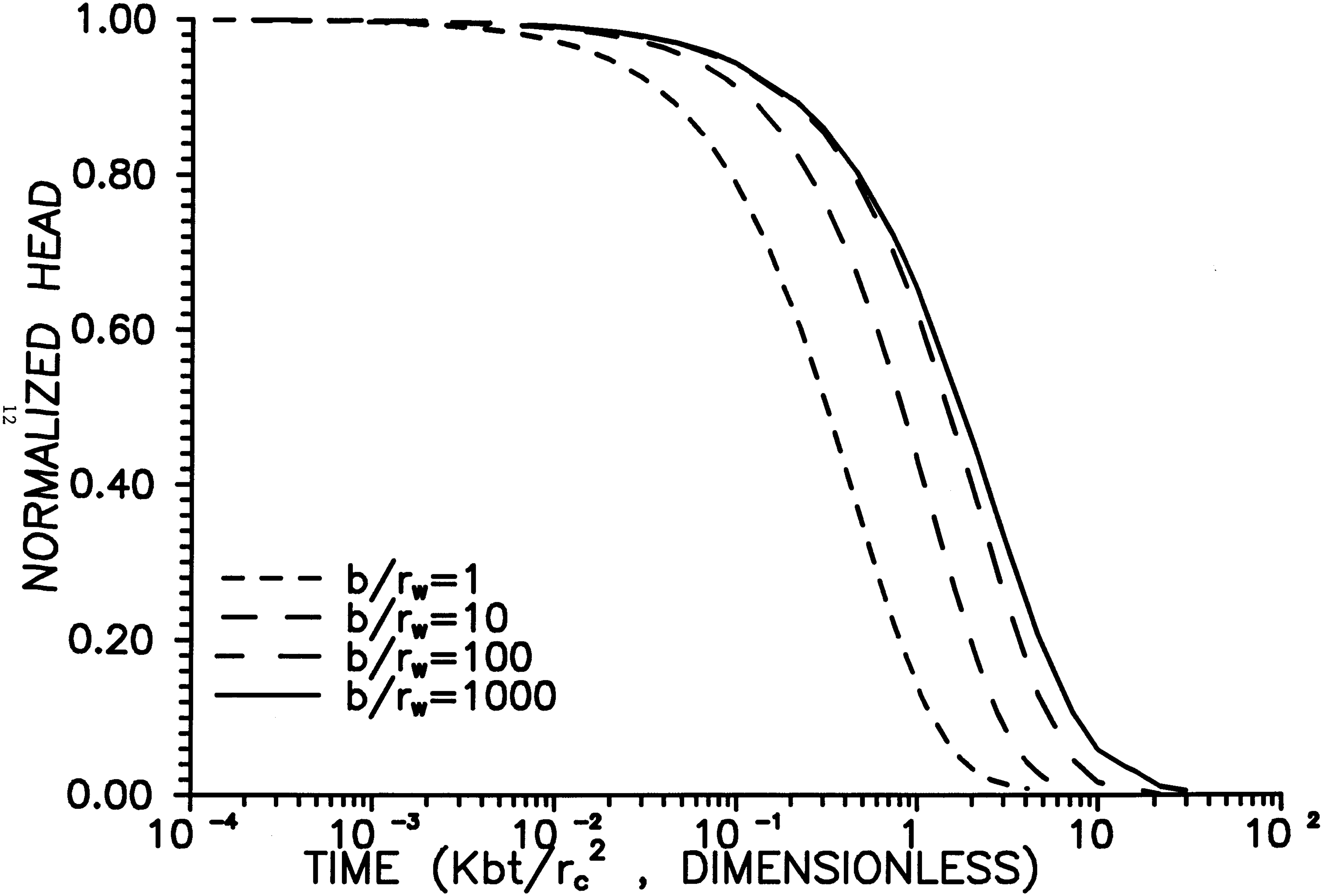
Table 1 - Base data set used in the theoretical analysis of slug tests in complex geologic settings, unless otherwise stated.

$r_w = r_c = .1$, [L];
$S_s = .0001$, [L];
$K_r = K_z = 1.$, [L/T];
$B = 10$, [L];
$b = 1.$, [L];
$a = 4.5$, [L];
$H_0 = 1.$, [L].

ASPECT RATIO EFFECTS

Figure 2 displays the dependence of slug-test responses on aspect ratio for a B/b ratio of 10. An aspect ratio of 1000 is indistinguishable from the CBP solution, i.e. partial penetration effects are negligible. As seen from this plot, partial penetration effects are only of significance for aspect ratios less than 100. This result is to be expected, the effects of partial penetration are dampened as the percentage of flow through the well screen with a vertical component decreases with respect to the total flow.

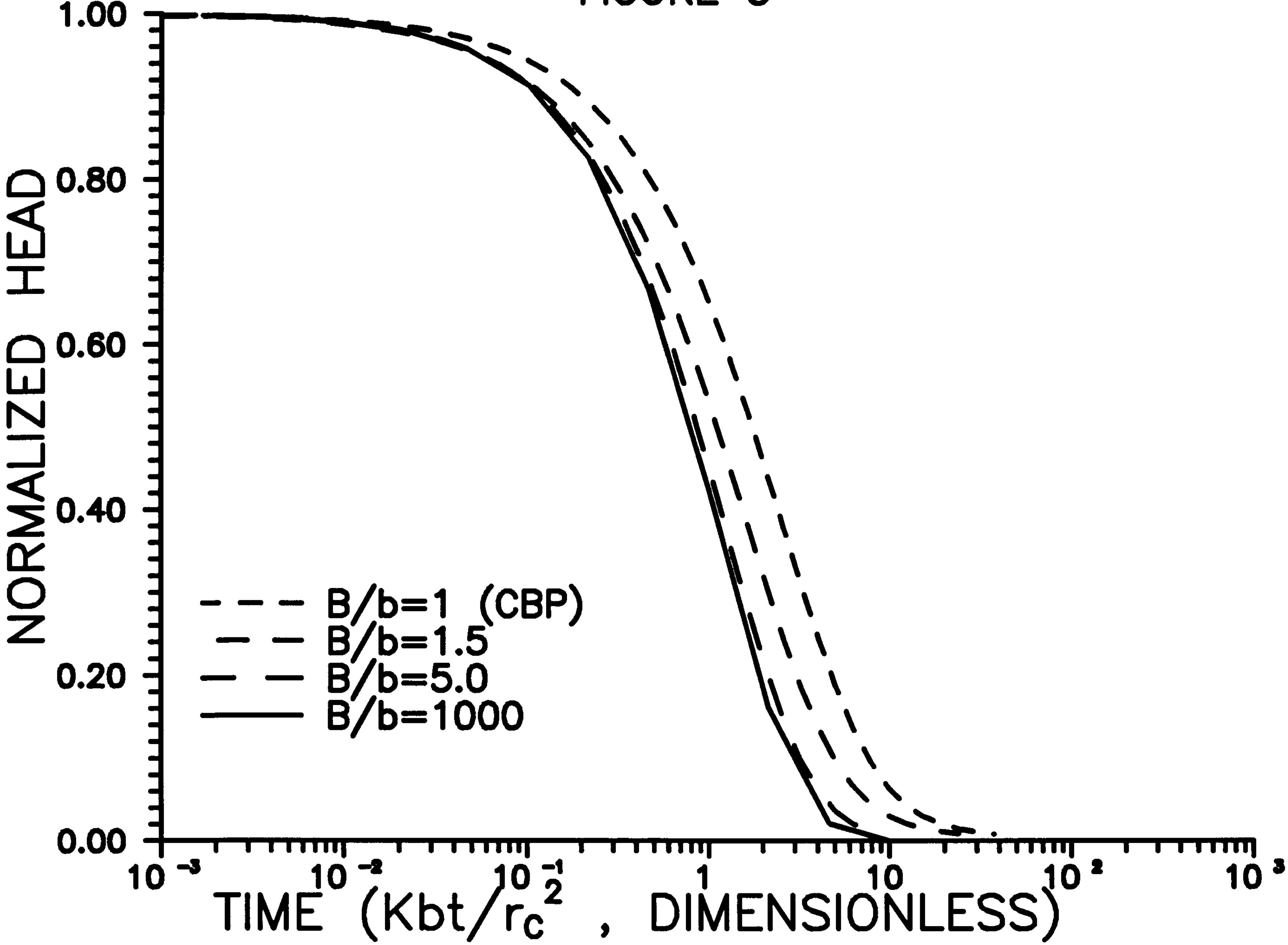
FIGURE 2


1.00
0.80
0.60
0.40
0.20
0.00
NORMALIZED HEAD
$b/r_w=1$
$b/r_w=10$
$b/r_w=100$
$b/r_w=1000$
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
10
10^{2}
TIME (Kbt/r_c^2 , DIMENSIONLESS)

EFFECT OF AQUIFER THICKNESS

The results displayed in Figure 2 are limited to a B/b ratio of 10. Figure 3 shows the dependence of slug-test responses on the ratio of aquifer thickness to screened interval (B/b) for an aspect ratio of 10. In this case, a B/b ratio of 1. is the same as the CBP solution. Note that for ratios above 5., the effects of the horizontal impermeable boundaries can be ignored. Although this figure is for the case of the screened interval located in the center of the aquifer, additional work shows that these results are not strongly dependent on the location of the screen. If a is greater than .1B, slug-test responses are essentially independent of screen location.

FIGURE 3

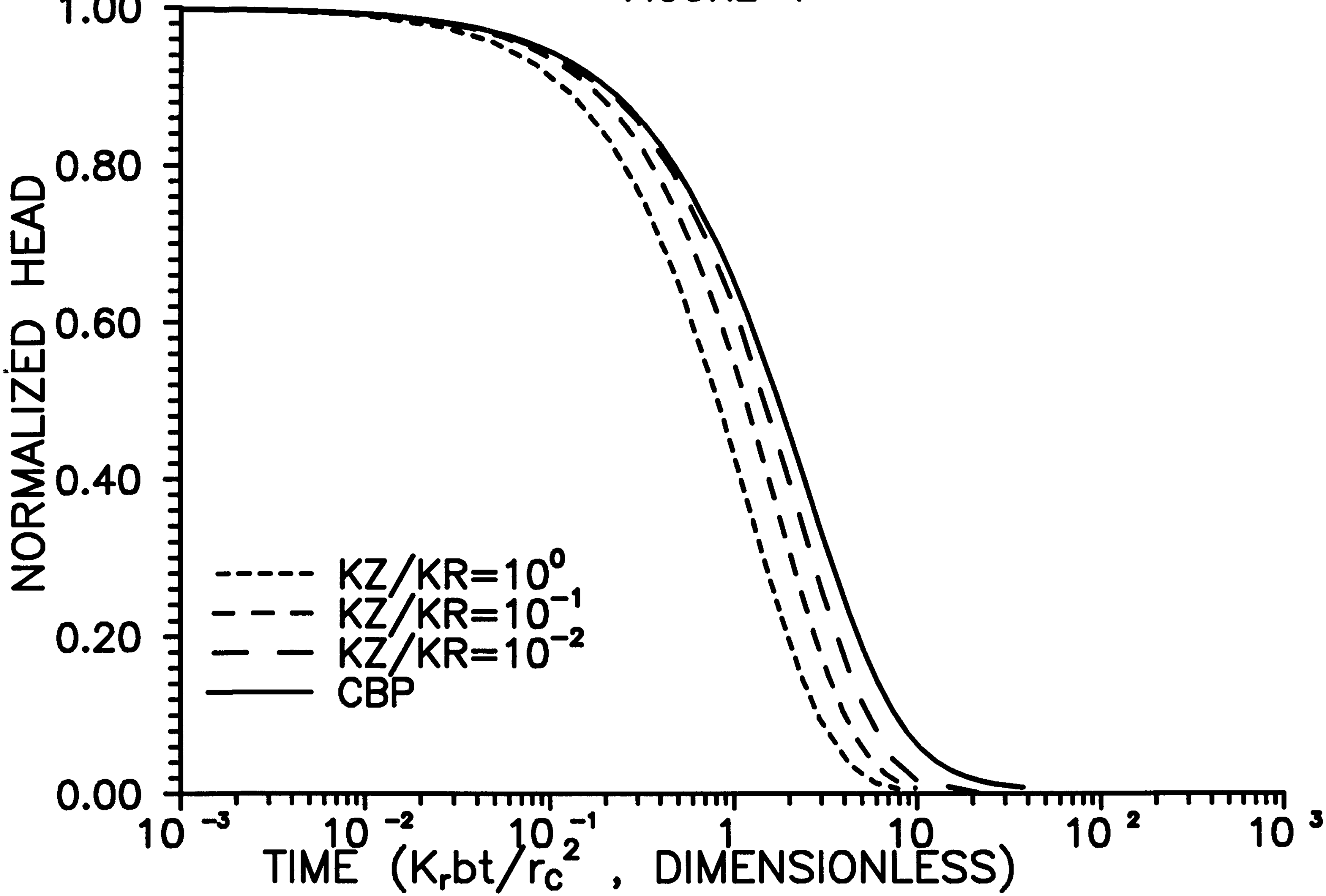


NORMALIZED HEAD
1.00
0.80
0.60
0.40
0.20
0.00
B/b=1 (CBP)
B/b=1.5
B/b=5.0
B/b=1000
10⁻³
10⁻²
10⁻¹
1
10
10²
10³
TIME (Kbt/r_c², DIMENSIONLESS)

ANISOTROPIC EFFECTS

Figures 2 - 3 display results for a slug test in an isotropic aquifer. In most natural systems, one is usually confronted with a situation where hydraulic conductivity in the horizontal direction is larger than its vertical component. An important issue to consider is how this anisotropy impacts slug tests performed in partially penetrating wells. Figure 4 displays the results caused by anisotropy for the case of B/b and aspect ratio equal to 10. This plot shows that an increase in horizontal conductivity with respect to that in the vertical results in the effects of partial penetration being progressively dampened. For anisotropy ratios of .01 or smaller, the effect of partial penetration can be essentially ignored. This result has important implications for slug tests in many aquifers where depositional processes commonly produced horizontal conductivities greater than vertical conductivities.

FIGURE 4


NORMALIZED HEAD
1.00
0.80
0.60
0.40
0.20
0.00
10 -3
10 -2
10 -1
1
10
10 2
10 3
TIME ($K_r bt/r_c^2$, DIMENSIONLESS)
KZ/KR=10^0
KZ/KR=10^-1
KZ/KR=10^-2
CBP

INFLUENCE OF WELL SKIN

Usually, the process of well drilling or development produces a near-well zone of disturbance (well skin) that may have an important influence on slug-test responses. Therefore, the impact of a well skin on slug-test responses must be considered in some detail.

Figure 5 displays the type curves for a slug test in a well with a skin whose radius is ten times that of the well. This figure depicts the results for a well that fully penetrates the aquifer. Note that the impact of a low-permeability skin is considerably larger than that of a high-permeability one. Figure 6 displays the analogous results for the case of a well skin in association with a partially penetrating well (B/b and b/r_w equal to 10.). Note how partial penetration accentuates the effect of a high-permeability skin while the response for a low-permeability skin is very similar to the fully penetrating well case. High-permeability skin along the unscreened portion of the partially penetrating well provides a conduit for fluid flow in the vertical direction along the well bore. This allows the slug-test responses to decrease much more rapidly than in the fully penetrating case.

FIGURE 5 — FULLY PENETRATING WELL

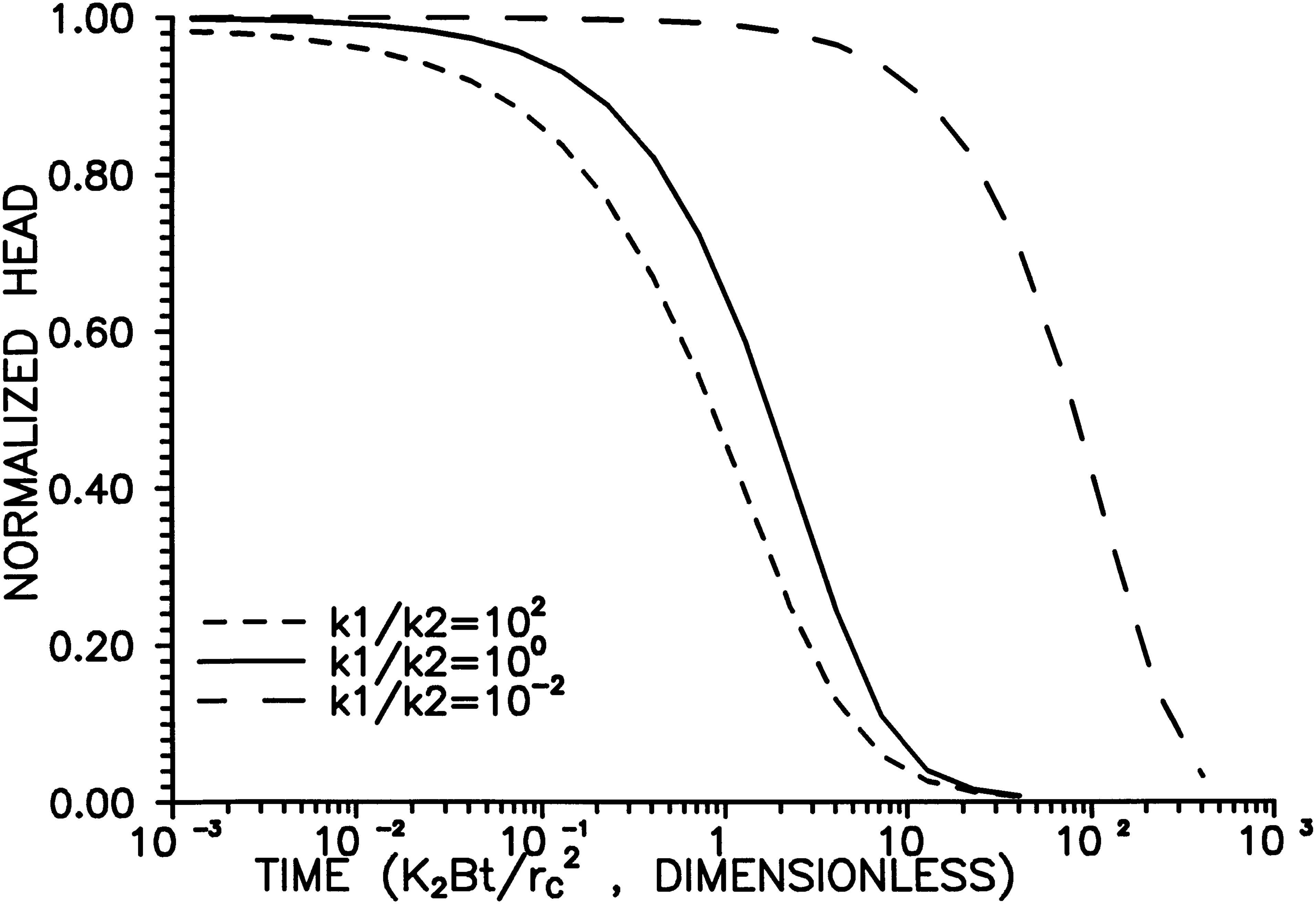
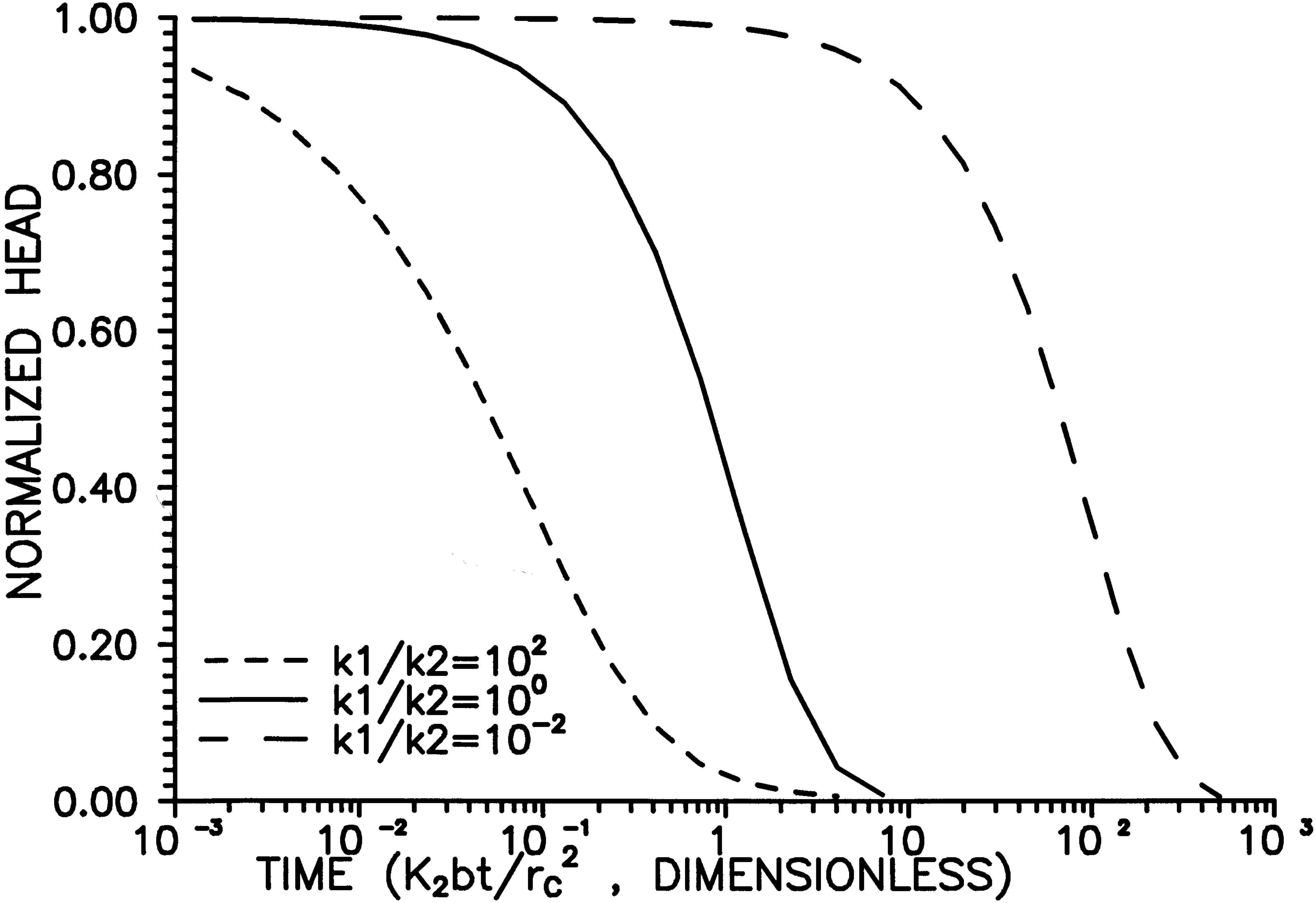


FIGURE 6 — PARTIALLY PENETRATING WELL



ANISOTROPIC SKIN EFFECTS

In Figure 6, the impact of an isotropic well skin on a slug test in a partially penetrating well was considered. In some cases, however, the hydraulic conductivity of this disturbed zone may be decidedly anisotropic. Figure 7 summarizes the results of an analysis of the impact of anisotropy in the well skin. As can be seen from this plot, anisotropy is of most concern when the vertical conductivity in the skin is considerably larger than its horizontal counterpart. This is not surprising as a relatively large conductivity in the vertical direction again provides a conduit for flow along the well bore, thereby exacerbating the effects of partial penetration. The situation of a higher horizontal component of conductivity is of little importance for the conditions of this analysis (b/r_w equal to 10.). If the aspect ratio was considerably smaller, the higher horizontal conductivity would have a considerably more important role as partial penetration effects would be dampened.

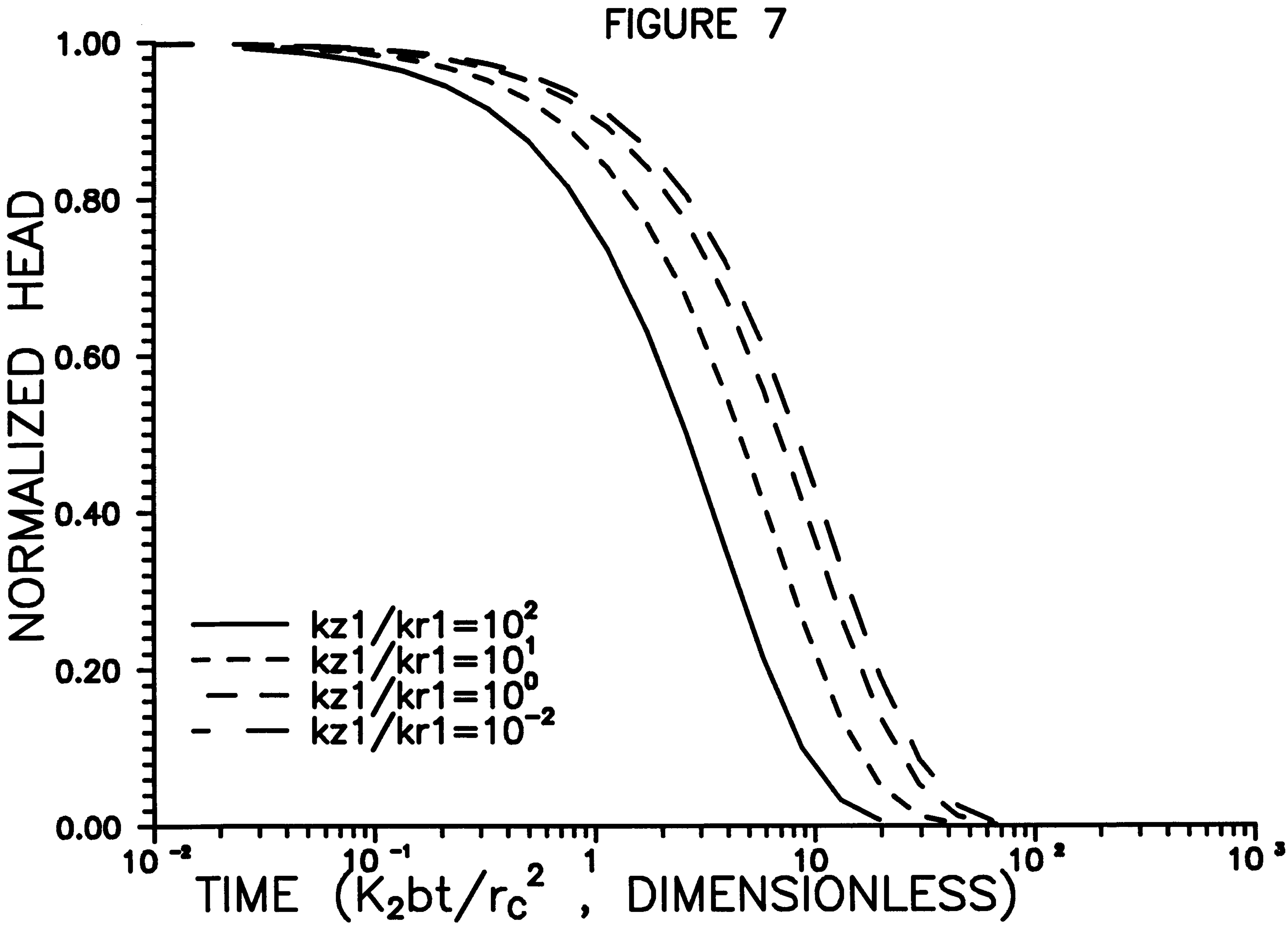
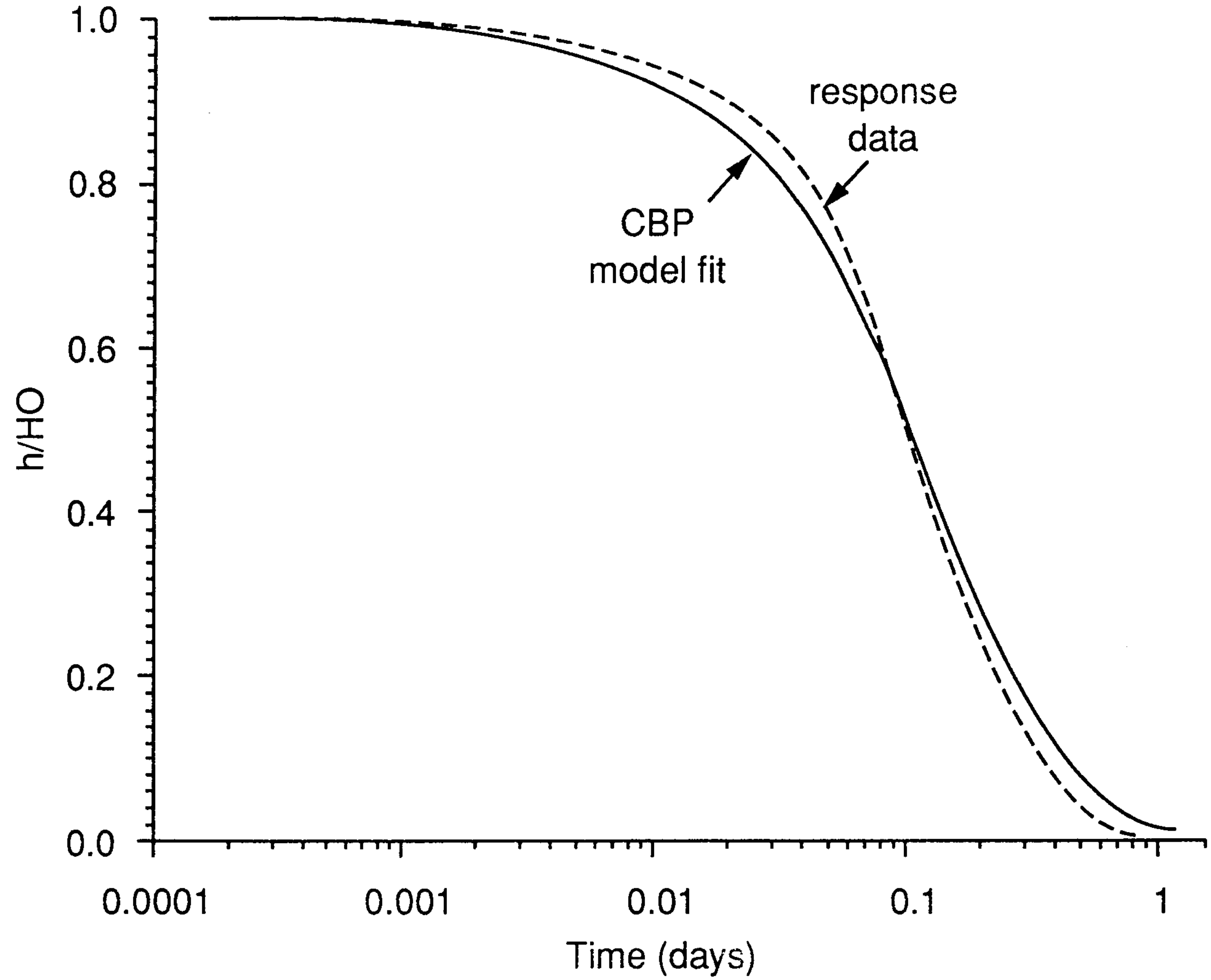


FIGURE 7
NORMALIZED HEAD
1.00
0.80
0.60
0.40
0.20
0.00
10^-2
10^-1
1
10
10^2
10^3
TIME (K2bt/rc2 , DIMENSIONLESS)
kz1/kr1=10^2
kz1/kr1=10^1
kz1/kr1=10^0
kz1/kr1=10^-2

DEVIATION FROM CBP MODEL

In general, if the CBP technique is used to analyze data where partial penetration and horizontal boundaries are important, a larger hydraulic conductivity will be obtained than warranted. Increasing the anisotropy ratio aggravates this condition while decreasing the ratio lessens the effect. The existence of the above complications will be recognized by a systematic deviation of the data from the conventional CBP or Hvorslev models. A systematic deviation of the data from one or both of these models is a clue that more complex models need to be considered. Slug test data from an experimental field site in the Kansas River alluvium illustrates some of the points considered here.

Figure 8.



1.0
0.8
0.6
0.4
0.2
0.0
h/HO
response
data
CBP
model fit
0.0001
0.001
0.01
0.1
1
Time (days)

APPLICATION AT THE GEMS SITE

We have an experimental site located in the Kansas River alluvium. It is called the Geohydrologic Experimental and Monitoring Site (GEMS). Several slug tests have been performed there and they typically show systematic deviation from the CBP model. The question is whether the more flexible model presented here can explain the observed data. The focus of this analysis of the response data at GEMS well #4 was on whether the inclusion of a low-permeability skin into the model could explain the observed behavior. Figures 9 and 10 show the results of the fitting of the Hvorslev and CBP models to GEMS #4 data. Note the good fit of the Hvorslev model and the systematic deviations seen with the CBP model. Figure 11 displays the results when a model with a low-permeability skin was employed in the fitting process. Note the dramatic improvement in model fit. Table 2 displays the parameter estimates obtained with the various fitted models, indicating that the Hvorslev estimate of transmissivity is less than 30% of the value determined using the low-permeability skin model (T_2). The Hvorslev transmissivity appears to be heavily weighted towards that of the skin (T_1). The skin radius used here was based on the outer diameter of the auger flights employed to drill GEMS #4. Results of additional theoretical work indicate that if the skin is two orders of magnitude less permeable than that of the aquifer, the Hvorslev analysis will yield a parameter that is well over one order of magnitude less than the actual aquifer transmissivity (T_2). Additional theoretical studies show that response data from a slug test in a well with a low-permeability skin will always show the behavior observed here, i.e. the Hvorslev model fits the data well but the CBP model shows systematic deviation. The assumptions of the Hvorslev analysis are actually more reasonable in a slug test in a well with a low-permeability skin, where the majority of the head drop takes place in the skin itself, so assuming zero drawdown at the radius of the skin is quite reasonable. Also, since the thickness of the skin is small, the storage of the skin can be reasonably ignored. Thus, the Hvorslev analysis yields a transmissivity very similar to that of the skin itself.

Clearly, the widespread use of the Hvorslev model that has been the conventional practice in hydrogeology should be discouraged. It appears from the preliminary results of this work that the Hvorslev and CBP models can be used together to help screen test data and suggest when a more complex model of the flow system needs to be considered. The Hvorslev model should never be used alone.

Figure 9.

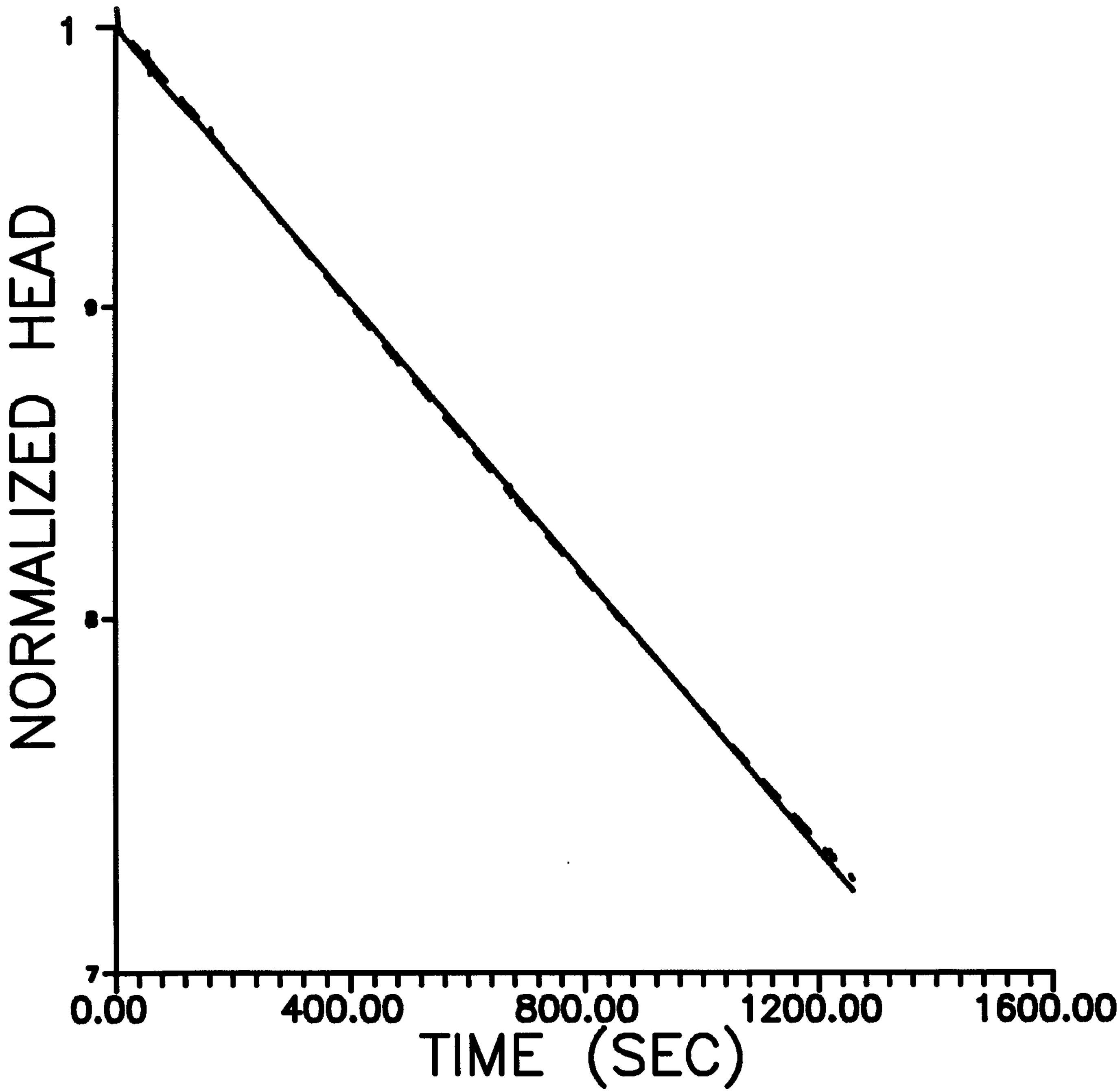
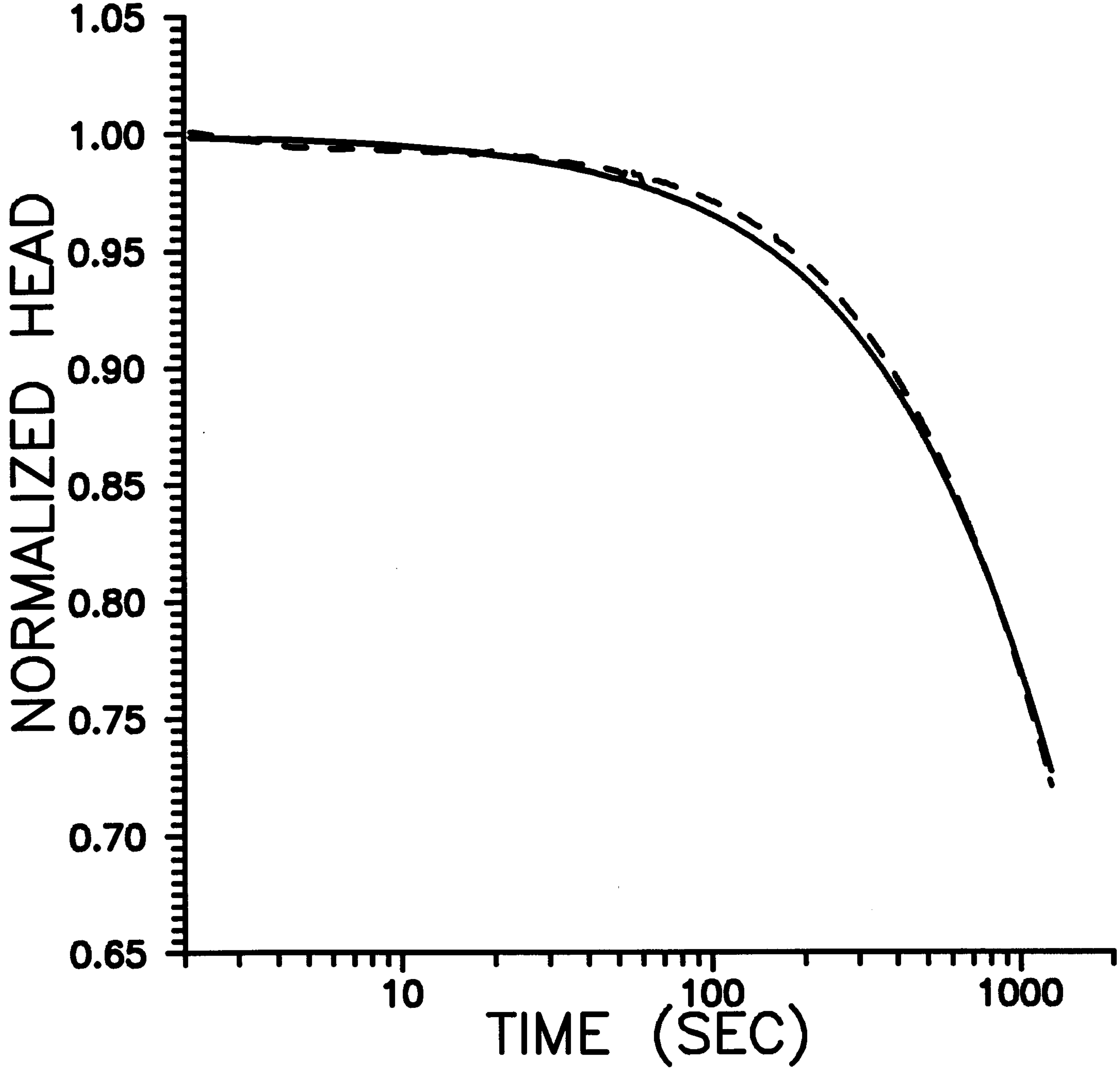
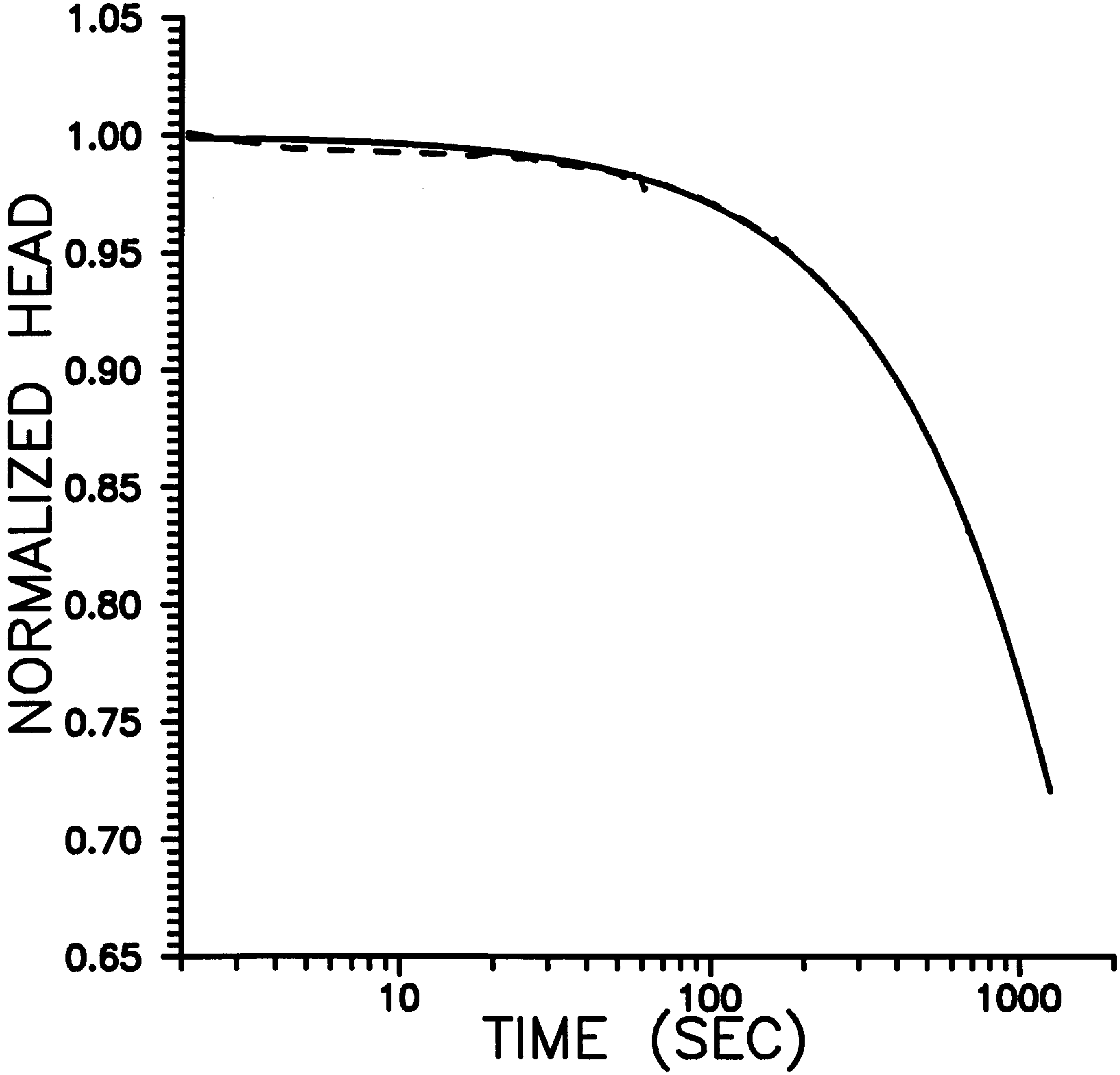


Figure 10.



1.05
1.00
0.95
0.90
0.85
0.80
0.75
0.70
0.65
NORMALIZED HEAD
10
100
1000
TIME (SEC)

Figure 11.



1.05
1.00
0.95
0.90
0.85
0.80
0.75
0.70
0.65
NORMALIZED HEAD
10
100
1000
TIME (SEC)

MODEL FITTING
TO GEMS #4 DATA

Table 2 - Parameter estimates obtained from the application of three different models to the slug-test response data at GEMS #4. The low-permeability skin is assumed to have a radius of 6.5 in (.54 ft). The subscripts a and s designate aquifer and skin, respectively.

Hvorslev Model

$T = 3.92\ ft^2/day$

CBP Model

$T = 5.46\ ft^2/day$

Skin Model

$T_a = 13.44\ ft^2/day$

$T_s = 3.14\ ft^2/day$

SUMMARY

The major conclusions of this work to date are as follows. When horizontal confining boundaries are more than about five screen lengths away from the screen, they have a negligible effect on the slug test. The ratio of screen length to casing radius (aspect ratio) is important for values below about fifty. When the aspect ratio is considerably above fifty the solution converges to the CBP solution. An aspect ratio of one gives a dramatic deviation from the classical CBP solution. Anisotropy ratio is defined as the vertical hydraulic conductivity divided by the radial hydraulic conductivity. The partially penetrating solution converges on the CBP solution as the anisotropy ratio decreases. When the anisotropy ratio is less than 10^{-3} we have essentially the CBP solution. When the anisotropy ratio is in the range of 1-10 or larger there is substantial deviation from the CBP solution when partial penetration effects are important. Even when the anisotropy ratio in the formation is quite low, a zone of disturbance created during drilling or development may serve as a vertical conduit for a considerable amount of fluid flow. The impact of this zone on slug-test results depends on its thickness, the contrast with formation properties, and the degree of anisotropy within the zone.

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