

ANALYTICAL SOLUTIONS FOR FLOW TO A PUMPING WELL
IN NONUNIFORM AQUIFERS

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ABSTRACT

Three analytical solutions for transient, pumping-induced drawdown in laterally nonuniform aquifers are described. Nonuniform configurations consisting of concentric rings of differing properties centered on the pumping well, a circular disk arbitrarily located in a matrix of differing properties, and an infinite strip located in a dissimilar matrix are examined. Although the employed configurations are idealized representations of nonuniform aquifers, they can provide much insight concerning pumping-induced drawdown in natural systems. Given the complexity of the derived solutions, large-time expressions must be employed to clarify the controls on drawdown in nonuniform systems. These expressions show that changes in drawdown at an observation well sited in a zone of anomalous properties will be impacted by the properties of that zone for a duration of quite limited extent. In addition, drawdown at an observation well is generally very weakly dependent on the properties of material between the observation and pumping wells. Considering the three solutions together, understanding of pumping-induced drawdown in complex, nonuniform systems can be greatly improved. An improved understanding should lead to more appropriate observation-well placement and better pumping-test design.

FIGURE 1

In the last decade, hydrogeologists have become increasingly aware of the importance of spatial variations in the physical properties that control flow and transport in the subsurface. Clearly an issue of some interest is that of the development of efficient techniques for the characterization of the spatial variations at any particular site. Well testing has been somewhat successful for providing information concerning the vertical variations in permeability in the vicinity of a pumping well - we are interested in the potential well testing might have for describing large-scale lateral variations in flow properties.

FIGURE 2

In this presentation, we will examine three simple models, shown here in areal view, that we feel represent the range of conditions that might be met in laterally heterogeneous units. Although we have used simplified representations for ease of mathematical analysis, we think that we have captured several general features that have a great deal of applicability for natural systems. We examine these representations using analytical techniques because we feel better understanding can result from the derived functional relationships.

FIGURE 3

Let's begin by looking at the simplest system - a series of concentric rings centered on the pumping well. The symmetric

nature of this system allows us to use the radial flow equation, written here in terms of drawdown, neglecting the angular component. This class of problem, which has been considered by many workers, can readily be solved by using Laplace transforms. The aspect of the solution that I would like to discuss is the insight that it can provide concerning systems in which pumping is occurring in a unit that is completely enclosed laterally by other units. For our purposes, we can capture the essence of such systems using a three-ring configuration. Rather than looking at the full solution, let's consider an approximate form that is valid at large dimensionless times of pumping for an obs. well at the boundary between the first and second zones (such as the point marked A in this figure).

FIGURE 4

The interesting point to note is that the large-time drawdown at A depends on the properties of the material lying radially outward of that point. The large-time drawdown is independent of the properties of material between the obs. and pumping wells. The complete independence of drawdown from the properties between the obs. and pumping wells is only seen in purely radial flow systems. As the non-radial flow component increases, the dependence on interwell properties will increase. Note the delta s expression shows that the change in drawdown at a large time is independent of the properties of the zone in which the well is located. We will explore that point further when considering the

second configuration. The large-time contribution of the second zone to drawdown, the script s term, is simply the Thiem equation using the radii of the boundaries of the zone and the transmissivity of that zone. Using the Thiem equation in this matter demonstrates that a zone of anomalous properties at a distance from the pumping well must be very large in order to have an significant impact on observation-well drawdown.

FIGURE 5

Let's now look at a more complex system consisting of an isolated body of material that is of differing properties than the material in which the pumping well is located. Again, we will use a circle in order to make the problem more tractable analytically. The asymmetry of the flow system makes the polar coordinate flow equation the operable one. A solution is considerably more difficult in this case, but it can be obtained following the approach suggested by Jaeger in 1944 for a related problem in heat conduction. I have brought copies of an article describing this approach that I would be happy to provide to those interested. What I would like to concentrate on here is a large-time approximation of this solution.

FIGURE 6

Here is the large-time form of the solution - the diagram helps clarify the terms in the equation. The properties of the disk are labelled with a subscript 1 and those of the matrix with a

subscript 2. r and θ are the radial and angular positions of the wells with respect to the center of the disk and the dashed line, respectively - the subscript pw is used to indicate the pumping well, while the observation well position variables are without subscripts. This figure only shows an expression for drawdown within the disk (s_1), as the expression for drawdown within the matrix is similar. The first term in the equation is simply the infinite-series truncation form of the well function of Theis using properties of the matrix, while the second term considers the impact of the disk. The first point I would like to address is a follow up to that of the first solution - that is, it is clear from this equation that changes in drawdown at large time are independent of the disk properties. I think one can assert that in most systems the large-time change in drawdown at a point is independent of the properties of the material in the vicinity of that point. This has some interesting ramifications for use of well testing for characterizing nonuniform systems. Carl McElwee and I discuss some of these ramifications in an upcoming Water Resources Research paper concerning variable-rate pumping procedures. Now, remember that we are talking here of changes in drawdown, the total drawdown may still be a function of disk properties - this depends on the position of the observation well with respect to the center of the disk and the distance between obs. and pumping wells. Let's look at two bounding cases for drawdown in the disk - these being when the obs. well is at the center of the disk and when it

is on the disk boundary. When an observation well is at the center of the disk, the total drawdown at the well is only dependent on the properties of the disk for a time of quite limited duration. Let me show you an example using a disk, 5 meters in radius, with a transmissivity an order of magnitude less than that of the matrix - the storativities being kept constant. Now let's vary the distance between the observation and pumping wells and look at the resulting time-drawdown plots.

FIGURE 7

Here we have a dimensionless log-log plot in which we have plotted the uniform aquifer solution of Theis along with the curves for different distances between the observation well at the center of the disk and the pumping well. Note that the observation well drawdown becomes indistinguishable from the uniform-aquifer drawdown at large times. As the distance between disk and pumping well increases, the time at which the uniform-aquifer curve and observation-well curve become indistinguishable decreases quickly. Thus, when the observation well is located at or near the center of an isolated pod of material, the properties of that material will have little impact on the total drawdown at the well. Only the very-early time data will be impacted.

FIGURE 8

Now let's look at what happens when the observation well is located on the edge of the disk - in this case let's look at

behavior for a well located at point B in this figure. In this case, as shown here, the drawdown curve converges on the uniform-aquifer case but does not become indistinguishable except at very large dimensionless times and at large distances from the pumping well. These results just reinforce one of the conclusions arising from the first solution - that is, a pumping test can only detect very large aquifer heterogeneities at a distance from the pumping well. One final point to make concerning this solution is the impact that a disk has on its surroundings.

FIGURE 9

Here we have the same 5 meter disk with a transmissivity this time an order of magnitude greater than that of the surrounding matrix. I have refined the drawdown contour interval in the vicinity of the disk in order to illustrate the impact of the disk on the flow field. Note that there is only a region of quite limited extent in the vicinity of the disk in which drawdown permanently reflects the existence of the disk. The size of this region will decrease as the distance between the disk and the pumping well increases.

FIGURE 10

Now let's turn to the last of the solutions to be discussed. In this case, we are looking at a series of infinite strips - for our purposes let's consider the case of a middle strip of a relatively narrow width set in a matrix of differing properties-

a representation of a buried stream channel. We'll write the flow equation in its Cartesian-coordinate form, using Dirac delta function notation to incorporate the pumpage directly into the equation. The solution approach is a bit involved as it requires several integral transforms. The analytical inversion of the image space solution is considerably more difficult than that in the previous solution, and, unfortunately, this meeting came upon us before we had completed the inversion. However, we have some partial results based on the analytical inversion as well as results from the numerical inversion that can be used as the basis of some discussion.

FIGURE 11

Let's begin by considering an approximate expression for the large-time drawdown. Although we haven't completely finished the analytical inversion, we do know that the equation will have a form where the first term is the infinite-series truncation of the well function of Theis using the properties of the matrix, and the second term is a function of the property contrast, pumpage, channel width, and position of pumping and observation wells. The interesting point to note is that at large times, this second term is independent of time. Thus, the changes in drawdown at large time are independent of channel properties. Let's now go to the results of the numerical inversion in order to examine some examples. Let's consider the case where we have a channel, five meters in width, and pumping and observation

wells located in the surrounding matrix. Now let's vary the contrast of the transmissivity between the channel and matrix.

FIGURE 12

Here we have a dimensionless semilog time-drawdown plot displaying the results of a number of runs. The solid line in the middle of the plot is the uniform aquifer solution. The uppermost line is the solution for a single linear boundary given by Bixel et al. in 1963. For the case of a single linear boundary, we have employed the rightmost side of the channel as our boundary and consider the material to the left to have a transmissivity an order of magnitude less than the material to the right. The remaining lines are for the solution discussed here. The line denoted 1 is for a low permeability channel with a transmissivity an order of magnitude lower than that of the matrix. Note how this plot initially takes on the behavior of the single boundary case before flattening somewhat and taking on the same slope as the uniform aquifer case. Similar behavior is seen for lines 2 and 3 which represent higher permeability channels with transmissivity one and two orders of magnitude, respectively, larger than the surrounding matrix. In all three cases, we are looking at classical double porosity behavior—similar to what would be expected in a fractured or layered system. There is only a limited duration of time during which the properties of the channel influence changes in drawdown at an observation well. This period of time is shown on this plot as a

period of transition between the two straight-line segments of equal slope. The transition behavior is rather simple for the case considered here as the plot will initially take on the slope of the single linear boundary solution before returning to the slope controlled by matrix transmissivity. If we had varied the storativity in this example, we would see considerably more complex behavior. From this example, we can see that for the higher permeability cases, the buried channel is essentially a conduit with the matrix serving as the source of most waters.

FIGURE 13

Let me bring this presentation to a close by reviewing what I have discussed here. I have used analytical solutions to look at drawdown in three models of large-scale lateral heterogeneities that I think bound the conditions that might be met in natural systems. For all three solutions, we saw that the changes in drawdown at a large time are independent of the properties in the vicinity of the observation well. For purely radial flow situations, the total drawdown at an observation well will be independent of material between the observation and pumping wells. For more complex flow systems, the strength of this dependence will depend on the size of the heterogeneity, the property contrast, and the distance from the pumping well. Thank you for your attention.

APPENDIX

Details concerning this research can be found in the following references:

- Bixel, H.C., Larkin, B.K., and Van Poolen, H.K., Effect of linear discontinuities on pressure build-up and drawdown behavior, *J. Pet. Tech.*, 885-895, 1963.
- Butler, J.J., Jr., Pumping tests in nonuniform aquifers - the radially symmetric case, *J. of Hydrology*, 101, 15-30, 1988.
- Butler, J.J., Jr., The role of pumping tests in site characterization: some theoretical considerations, *Ground Water*, in press, 1990.
- Butler, J.J., Jr., and Liu, W-Z., An analytical solution for flow to a well in radially asymmetric nonuniform media, *Water Resour. Res.*, in review.
- Butler, J.J., Jr., and McElwee, C.D., Variable-rate pumping tests for radially symmetric nonuniform aquifers, *Water Resour. Res.*, in press, 1990.
- Jaeger, J.C., Some problems involving line sources in conduction of heat, *Philosophical Mag. and J. of Science*, 35, 169-179, 1944.

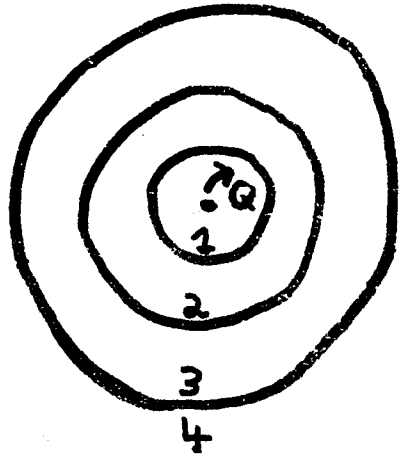
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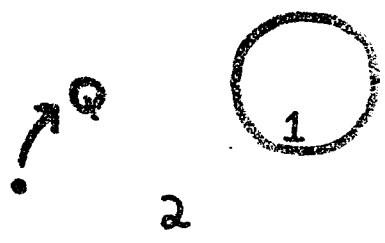
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Models of Lateral Heterogeneities

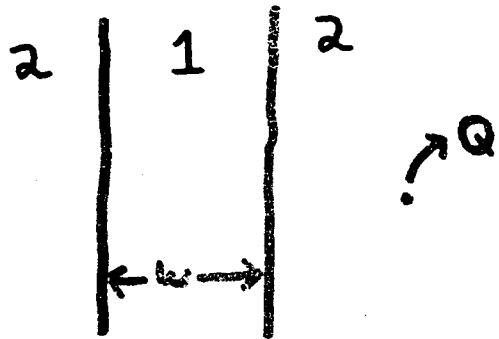
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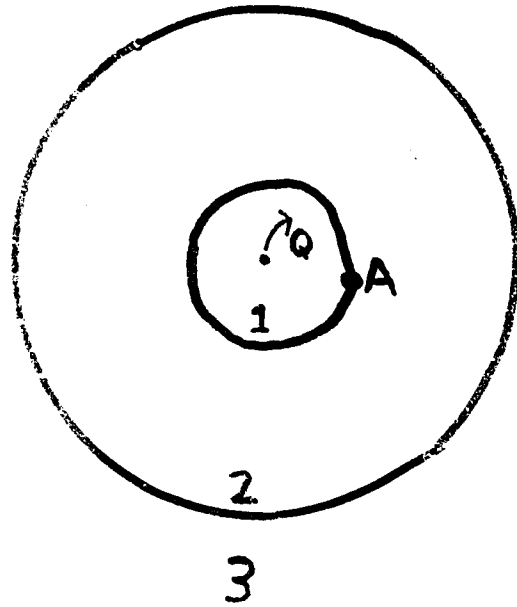


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Governing Equation:

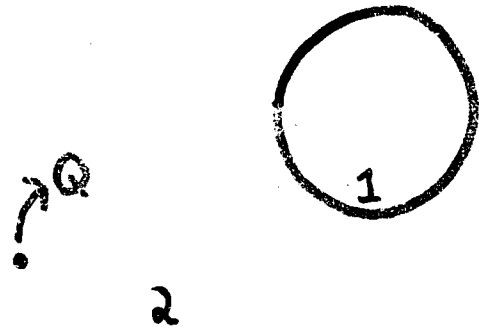
$$\frac{\partial^2 s_i}{\partial r^2} + \frac{1}{r} \frac{\partial s_i}{\partial r} = \frac{s_i}{T_i} \frac{\partial s_i}{\partial t}$$

$$s_A = \frac{s_2}{\text{independent of time}} + \frac{Q}{4\pi T_3} \ln \left(\frac{4T_3 t}{C r_2^2 S_3} \right)$$

$$s_2 = \frac{Q}{2\pi T_2} \ln \left(\frac{r_2}{r_1} \right) = \text{Thiem Equation}$$

r_i = radius of boundary between zone i and zone $i+1$.

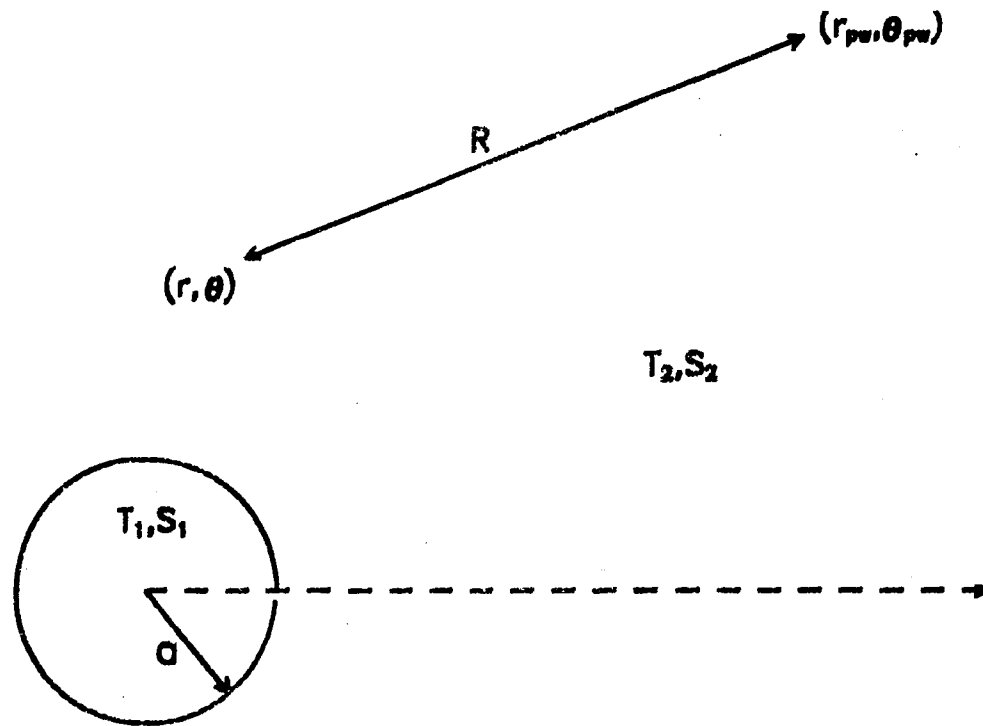
$$\Delta s_A = \frac{2.3 Q}{4\pi T_3} = \text{change in drawdown with time}$$



Governing Equation:

$$\frac{\partial^2 s_i}{\partial r^2} + \frac{1}{r} \frac{\partial s_i}{\partial r} + \frac{1}{r^2} \frac{\partial^2 s_i}{\partial \theta^2} = \frac{s_i}{T_i} \frac{\partial s_i}{\partial t}$$

Fig. 6



$$S_1 = \frac{Q}{4\pi T_2} \ln \left(\frac{4T_2 t}{C r_{pw}^2 S_2} \right) + \frac{Q}{\pi(T_1 + T_2)} \sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{r}{r_{pw}} \right)^n \cos n(\theta - \theta_{pw})$$

independent of time

$$\Delta S_i = \frac{2.3Q}{4\pi T_2}$$

Fig. 7

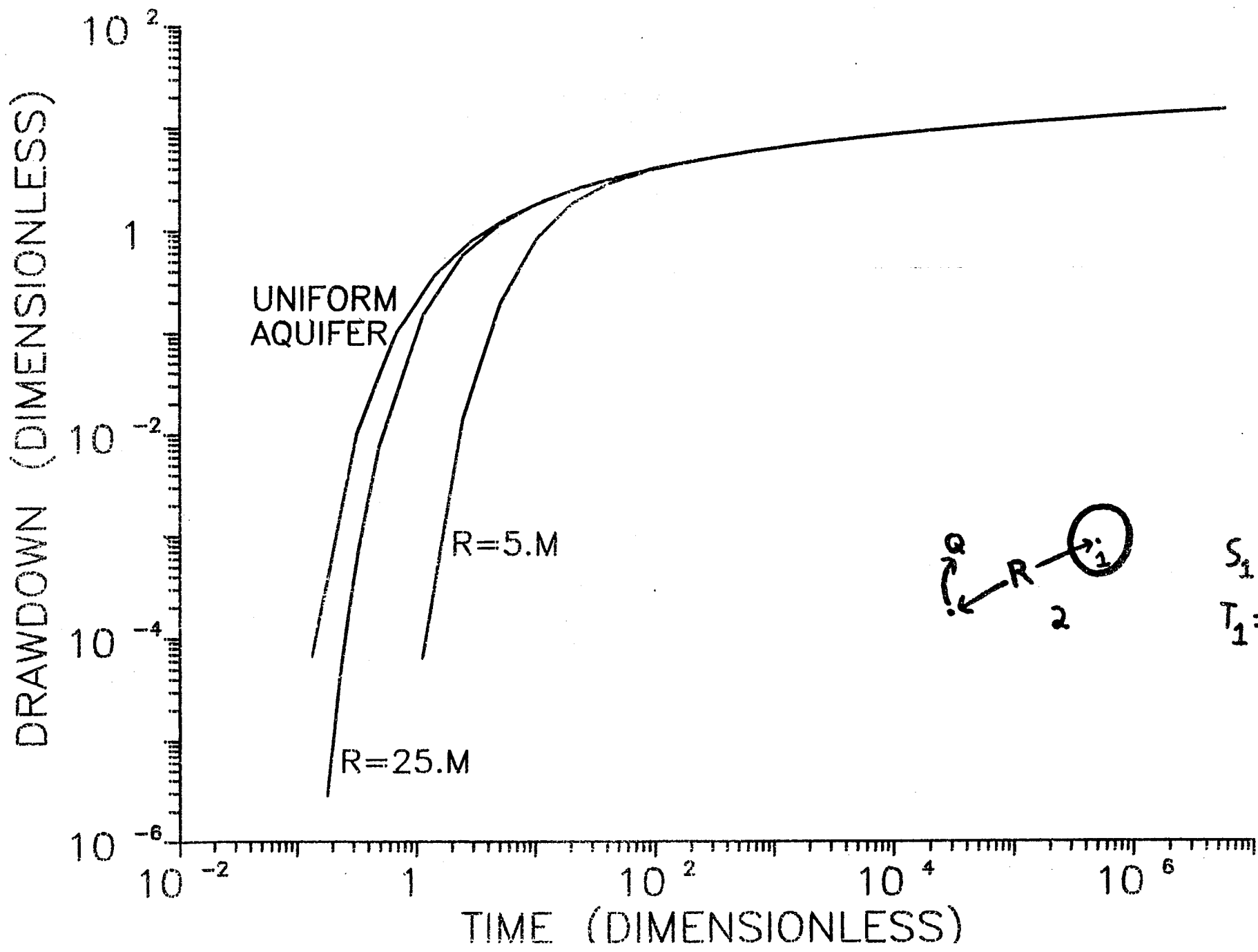


Fig. 8

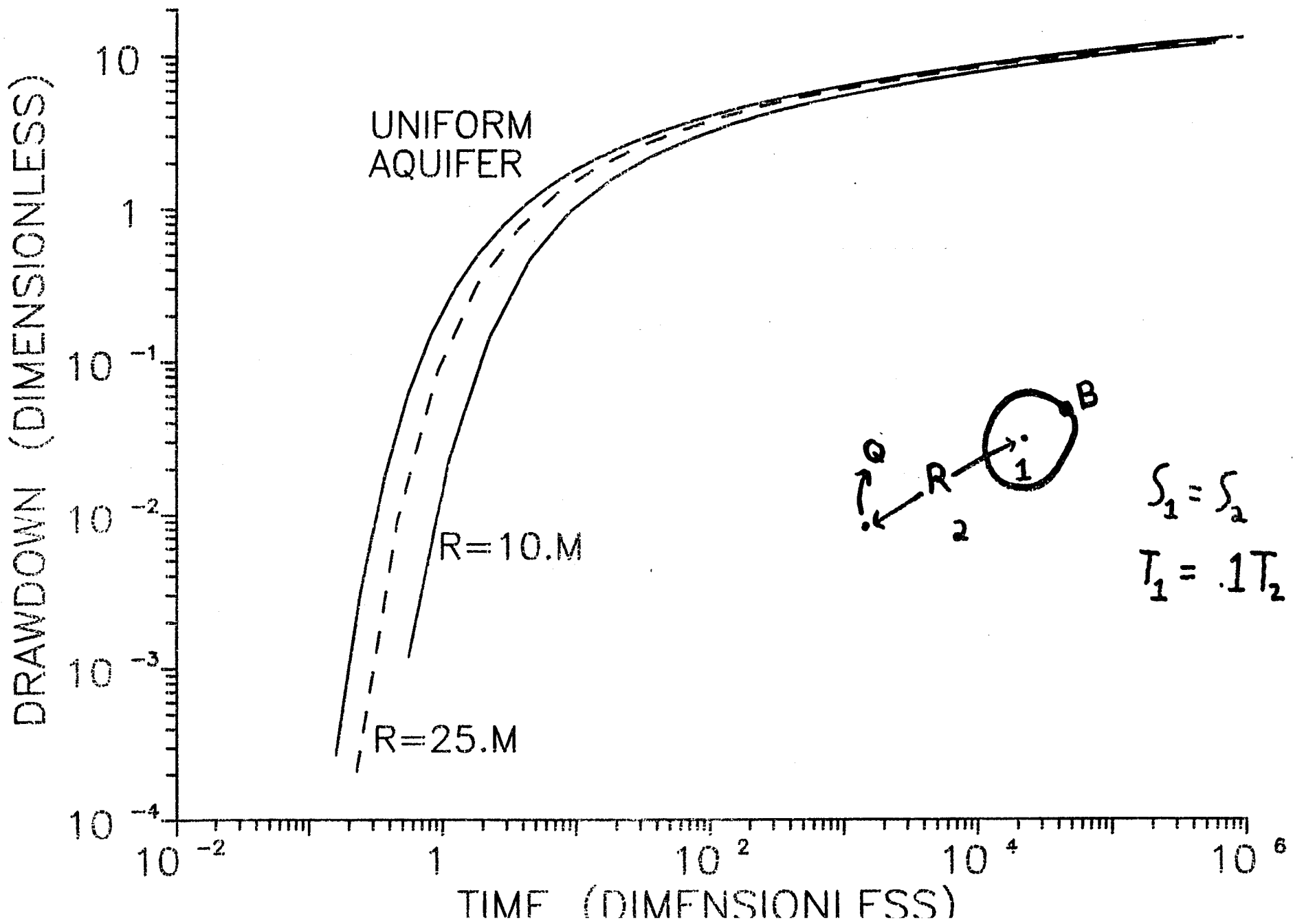
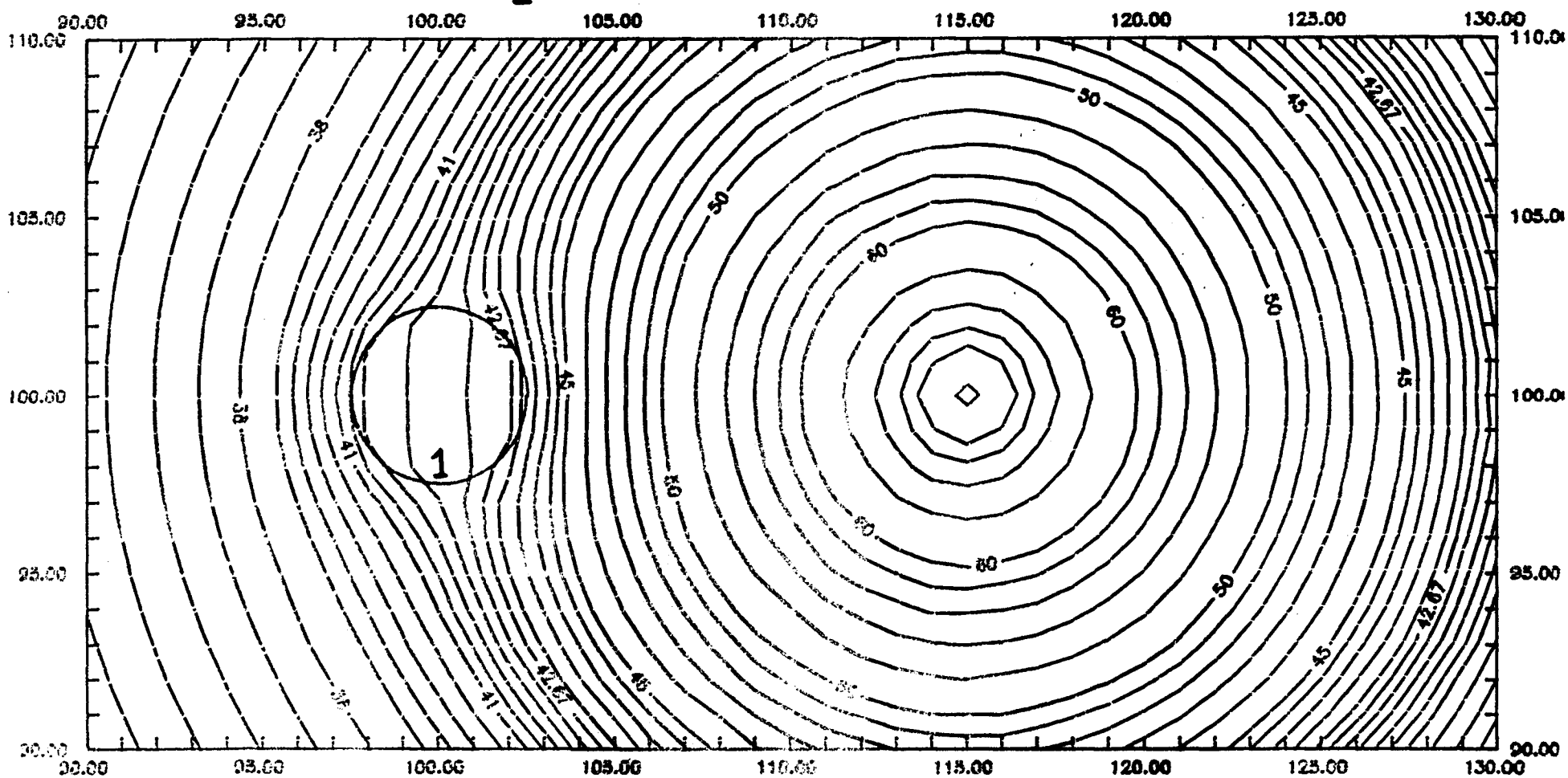


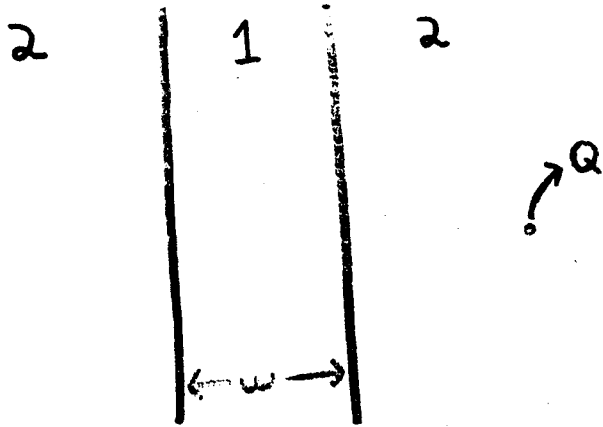
Fig. 9

$$S_1 = S_2$$
$$T_1 = 10T_2$$



(Meters)

Fig. 10



Governing Equation:

$$\frac{\partial^2 s_i}{\partial x^2} + \frac{\partial^2 s_i}{\partial y^2} + \frac{Q}{T_i} \delta(x - x_{pw}) \delta(y - y_{pw}) = \frac{S_i}{T_i} \frac{\partial s_i}{\partial t}$$

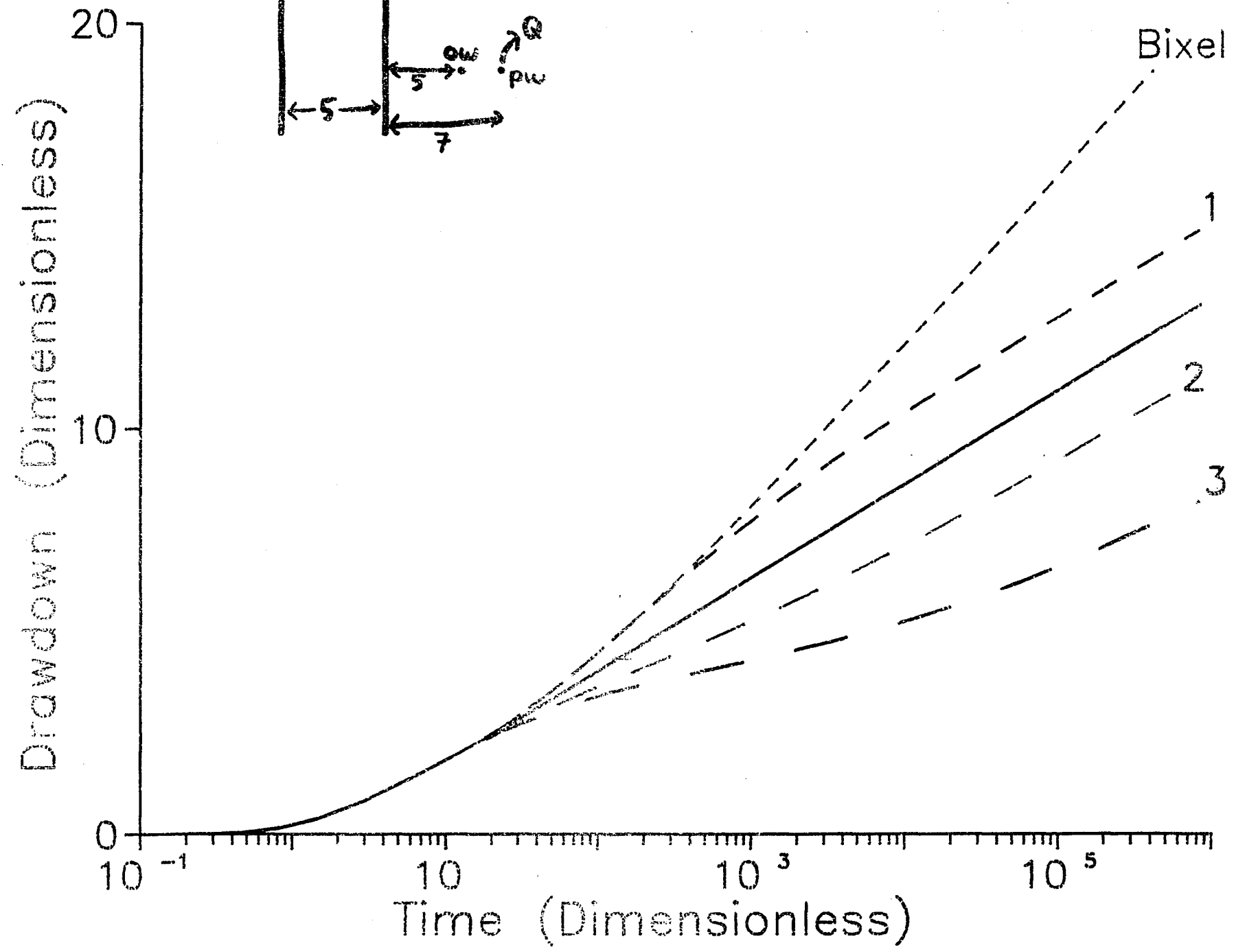
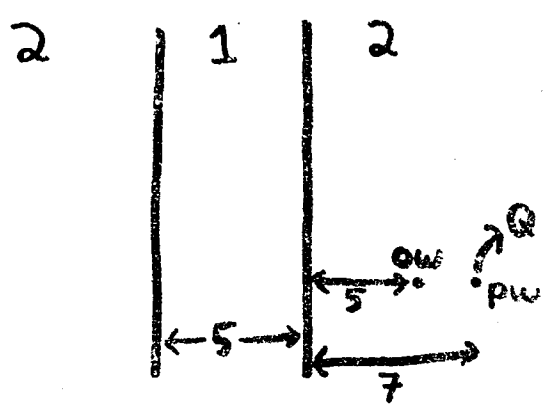
Large-time approximation

$$S_1 = \frac{Q}{4\pi T_2} \ln \left(\frac{4T_2 t}{C(x^2 + y^2) S_2} \right) + \underbrace{F \left(\frac{T_2}{T_1}, \frac{S_2}{S_1}, Q, w, x_{pw}, y_{pw}, x, y \right)}_{\text{independent of time}}$$

$$\Delta S_1 = \frac{2.3 Q}{4\pi T_2}$$

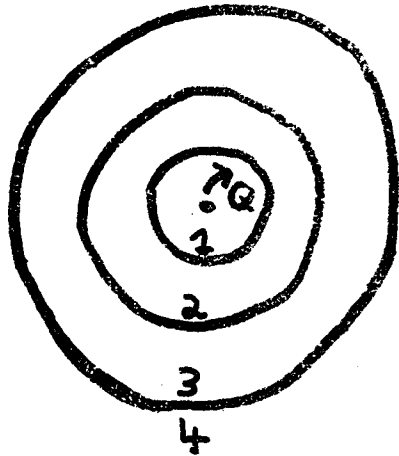
Fig. 12

$$S_1 = S_2$$

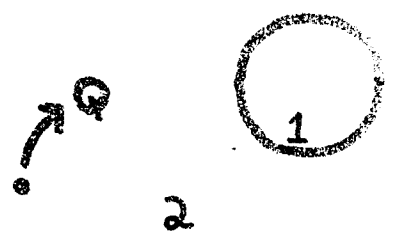


Models of Lateral Heterogeneities

1.



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