A
NEW APPROACH
FOR PUMPING TESTS
IN NONUNIFORM AQUIFERS

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ABSTRACT

The traditional approaches for analysis of pumping-test drawdowns yield single-valued parameters that are considered to be some sort of spatial average of the properties of material affected by the pumping. Though recent work has begun to clarify the nature of such averages, the traditional approaches are of limited effectiveness in situations where the variations in aquifer properties are of concern. Numerical models have been used to some extent for drawdown analysis in such situations, but have also been found to be of limited effectiveness due to uniqueness problems arising from the relative insensitivity of drawdown to flow properties at the scale of interest. A new approach for pumping tests in nonuniform aquifers will be described here. This approach, based on a better understanding of drawdown behavior during a pumping test, enables numerical models to be significantly more effective in characterizing large-scale variations in aquifer properties. This improved effectiveness is due to the increased sensitivity of drawdown to flow properties as a result of controlled variations in pumping rates. A hypothetical case study will be employed to demonstrate the decreases in estimation error that can be realized using the proposed approach.

INTRODUCTION

Figure 1

This morning I would like to briefly discuss a few aspects of the research we have been pursuing in the area of pumping tests in nonuniform aquifers. Traditionally, analyses of pumping test drawdowns have yielded single-valued parameters, which are considered to be some sort of average of the spatially-varying flow properties. Today, I would like to outline a simple pumping-test approach that will allow us to better describe the large-scale variations in aquifer properties.

Figure 2

The presentation will be organized in the following manner. I will begin by examining the underlying assumption of this work, that is that changes in drawdown at different times during a pumping test reflect conditions in different portions of the aquifer. I will quantify this relationship using sensitivity coefficients. I will then discuss the use of numerical models in pumping-test analyses, and emphasize the nonuniqueness problems that plague this procedure. Following this, I will introduce a variable-rate pumping test approach that will, among other things, significantly increase the sensitivity of drawdown to flow properties, and thus allow us to obtain a better estimate of the large-scale variations in aquifer properties.

UNDERLYING ASSUMPTION

Figure 3

As I said, the basic tenet of this approach is that the changes in drawdown at an observation well at different times during a pumping test reflect conditions in different portions of the aquifer. To examine the relationship between drawdown and flow properties in specific portions of the aquifer, we have employed sensitivity coefficients, which are defined as shown on this slide. In order to calculate the sensitivity coefficients characterizing behavior during a pumping test, we numerically solved a partial differential equation for the sensitivity coefficients.

I should point out that the results presented this morning pertain to a one-dimensional analysis, which assumes confined, radial flow to a pumping well. Ongoing work is presently extending the analysis to higher dimensions.

Figure 4

In order to examine the sensitivity coefficients characterizing behavior during a pumping test, we simulated a pumping test in the simple configuration shown in this figure. This configuration can be conceptualized as a patch of material two embedded in a matrix, represented by zone six, of differing properties. Zones three through five represent the transition from zone two to zone six. A pumping well will be assumed at the center with zone one being considered to represent a thin skin around the well. Let's first consider the case of an essentially uniform aquifer, and see how drawdown at the pumping well is influenced by different portions of the aquifer.

Figure 5

In this figure, the sensitivity coefficients for a constant-rate pumping test in a uniform aquifer are plotted. Each curve represents the sensitivity of drawdown to transmissivity in one of the zones of the previous figure. Sensitivity coefficient plots may not be familiar to many in the audience, so think of this plot as simply indicating the relative contribution of each zone to total drawdown. What I would like to draw your attention to is the following: changes in drawdown at the pumping well are only influenced by the properties in zones two and four for a finite interval of time. After that interval, further changes in drawdown are virtually independent of transmissivity in those zones, as reflected by the constant sensitivity coefficients. Thus, at large times, further measurements for the purpose of characterizing zone two transmissivity, for example, are of little additional assistance. Note that the sensitivity to zone six transmissivity is continually increasing due to the infinite size assumed for that zone.

Since this simulation was performed in a uniform aquifer, the forms of these curves are simply a function of the radial position and size of a given zone. However, we also examined the case of a nonuniform aquifer in which transmissivity varied by two orders of magnitude from zones two to six. Though the relative magnitudes of the sensitivity coefficients were different, the same feature was seen in both cases: that is a specific portion of the aquifer only influenced changes in drawdown for a limited duration of time. The length of that time interval was simply a function of the speed with which the front of the cone of depression passed through the given zone.

Let me say as an aside that, although I am only presenting the results of an analysis of the sensitivity of drawdown to transmissivity, the conclusions arising from the analysis of the sensitivity to storativity are the same.

PUMPING TEST ANALYSES USING NUMERICAL MODELS

Figure 6

The question is whether or not we can take advantage of this behavior to better describe the large-scale variations in aquifer properties. In order to examine this issue, we performed a series of pumping-test simulations. Simulated drawdown data from the pumping well then were employed in an automatic inversing analysis based on a nonlinear least-squares approach. The estimated standard error, which is essentially the standard deviation of the parameter estimates, was used to characterize the uncertainty of our inversed estimates.

Now, pumping-test analyses that attempt to describe the spatial variations in flow properties usually have problems with nonuniqueness of the inversed parameters. This nonuniqueness is a result of a host of factors including measurement error, parameter insensitivity, and correlation between parameters. For our purposes, we will primarily concentrate on the latter two factors, insensitivity and correlation.

Figure 7

This plot of the sensitivity coefficients allows us to make a few points concerning when we should measure drawdown in order to minimize the impact of insensitivity and correlation. Clearly, we should take measurements at times when the sensitivity coefficients are a maximum. In order to decrease correlation between parameters, we should take measurements during the interval of changing sensitivities. Now regardless of whether or not these recommendations are implemented, nonuniqueness is still a problem simply because the magnitude of the sensitivity coefficients and the interval during which these coefficients are changing are both quite small. Therefore, if we want information about the spatial distribution of flow properties, we need to somehow increase both the total sensitivity and the duration over which it is changing.

PROPOSED APPROACH

Figure 8

One simple approach for doing this is to vary the pumping rate. Each time the pumping rate is changed, a new cone of depression, superimposed on the original one, propagates out from the pumping well, thus producing a change in sensitivity and a new interval of time during which a given zone is influencing changes in drawdown. One could envision an approach that involves continual increases in pumping rates, producing a sensitivity plot such as this, where the sensitivity of drawdown to zone two transmissivity has been plotted on an arithmetic scale.

Figure 9

Now obviously we cannot continually increase pumping rates. However, we can approximately duplicate the effect of continually increasing pumping by alternately operating the pump at a high rate for a limited period of time and cutting the pump off. The sensitivity plot in this case displays fluctuations between a certain level and zero and may not appear to be providing much information. However, periods of changing sensitivities, regardless of the direction of the changes, are periods in which we are gaining information about the particular zone of interest. As long as the drawdown measurements are taken at an appropriate interval, one can demonstrate that the information gained about the aquifer properties using this approach is approximately the same as if pumpage was continually increased. This effect suggested to us a variable-rate approach for pumping tests in nonuniform aquifers. We then evaluated the applicability of this approach using further numerical simulation.

Figure 10

For this further series of simulations, we used the same zonal pattern as before, allowing the transmissivity to decrease two orders of magnitude from zone two to zone six. Let's quickly examine the results from these simulations to see how the uncertainty in our parameter estimates decreases by employing this variable-rate approach.

Figure 11

A key question is: At what frequency should we be turning on and off the pump? In this figure, the estimated standard error (here as a percent of the estimated parameter) for the transmissivity of each zone is plotted as a function of the frequency of rate changes. This plot, which is based on a total test length of one day, shows that increasing the frequency of rate changes decreases the estimated standard error for the times and zones of this example. I should note, however, that there is clearly a practical limit beyond which one cannot continue to increase the frequency of rate changes. Also, there do seem to be some undesirable effects that accompany higher frequencies of rate changes.

Figure 12

Now the previous plot depicts conditions for a test of one day in duration. We can also examine the effect of increasing the total length of the test. In this case, we are looking at zone two transmissivity and varying pumping rates every 3.2 hours. Significant gains can be realized by increasing the duration of the test as would be expected. Although the estimated standard errors for this simulation are small, even for the constant-pumpage case, the same effect would be seen in the presence of a greater standard error. In this figure, the constant-rate pumpage curve is flat for all the times plotted. I should note, however, that the time at which the constant-rate curve flattens depends upon the particular zone in question. Essentially, the curve will flatten as soon as the front of the cone of depression has passed out of that zone.

I think that these last two plots demonstrate that when the large-scale variations in flow properties are of interest, significant gains can be realized by a variable-rate approach to pumping tests.

CONCLUSIONS

Figure 13

In conclusion, I would like to summarize the major points of this presentation. First, through sensitivity analysis, we have been able to examine how changes in drawdown are influenced by properties in different portions of the aquifer. Second, we have shown that inversing efforts can greatly benefit from a technique that takes advantage of this relationship between drawdown and flow properties. Third, we have shown that a variable-rate pumping procedure is just such a technique, allowing us to significantly improve our estimate of the large-scale variations in flow properties. Finally, I must emphasize that these conclusions are only strictly applicable for the radial conditions for which they were derived. Preliminary results, however, do indicate that the same approach will be of much use in systems of higher dimensions.

Thank you for your attention.

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THE KANSAS GEOLOGICAL SURVEY

Figure One

OUTLINE

UNDERLYING ASSUMPTION
- Drawdown During A Pumping Test
- Sensitivity Coefficients

PUMPING-TEST ANALYSIS USING NUMERICAL MODELS
- Nonuniqueness
- Insensitivity Of Drawdown To Flow Properties

PROPOSED APPROACH
- Variable-Rate Pumping Test
- Numerical Examples

CONCLUSIONS

Figure Two

SENSITIVITY COEFFICIENTS

THE SENSITIVITY OF DRAWDOWN (s) AT POINT X AND TIME t DUE TO AN INCREMENTAL CHANGE IN PROPERTY P AT POINT Y CAN BE DEFINED AS:

$$U_P(X,t;Y) = \lim_{\Delta P \to 0} \frac{\Delta s(X,t;Y)}{\Delta P(Y)}$$

Figure Three

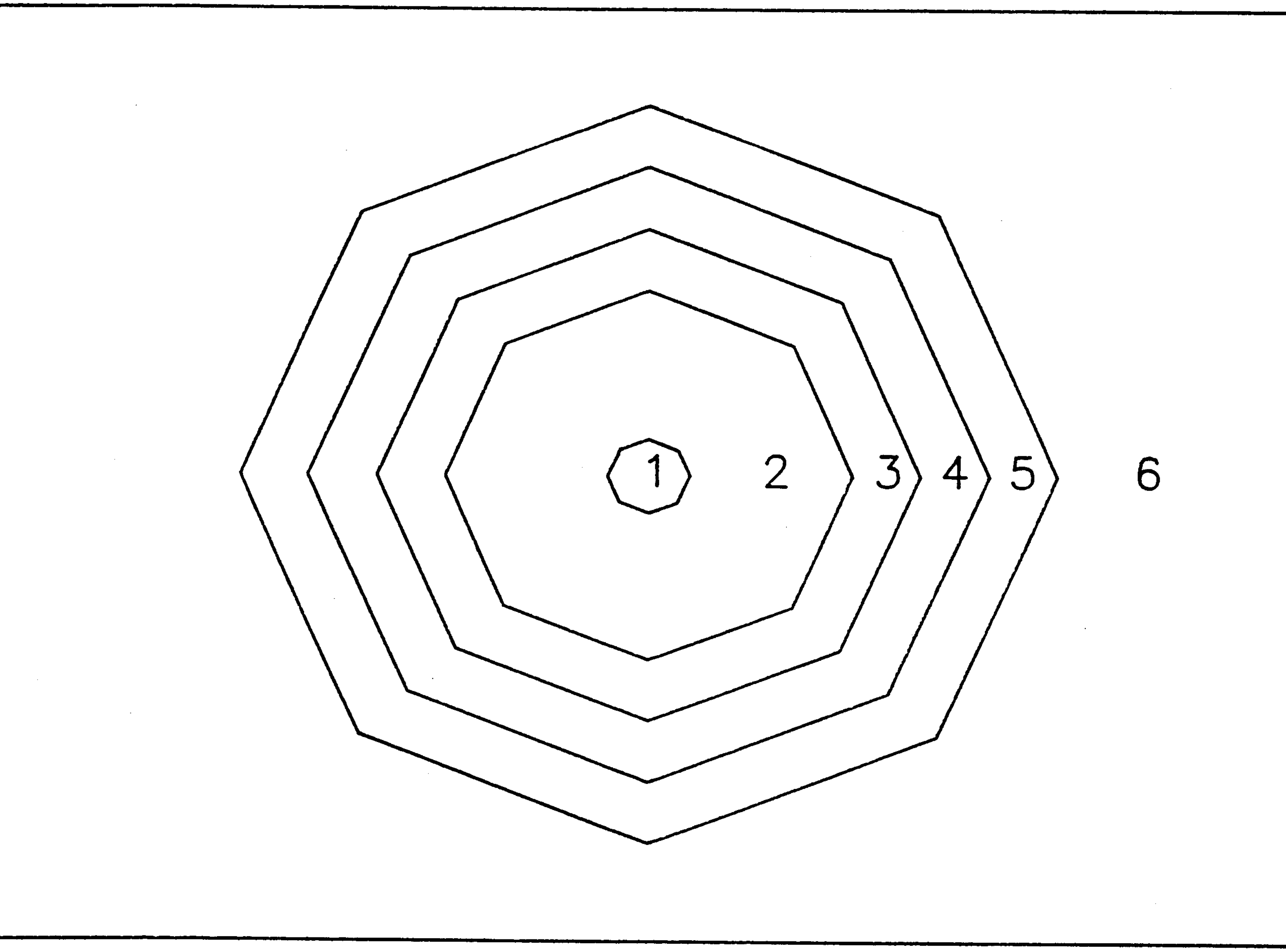

Figure Four

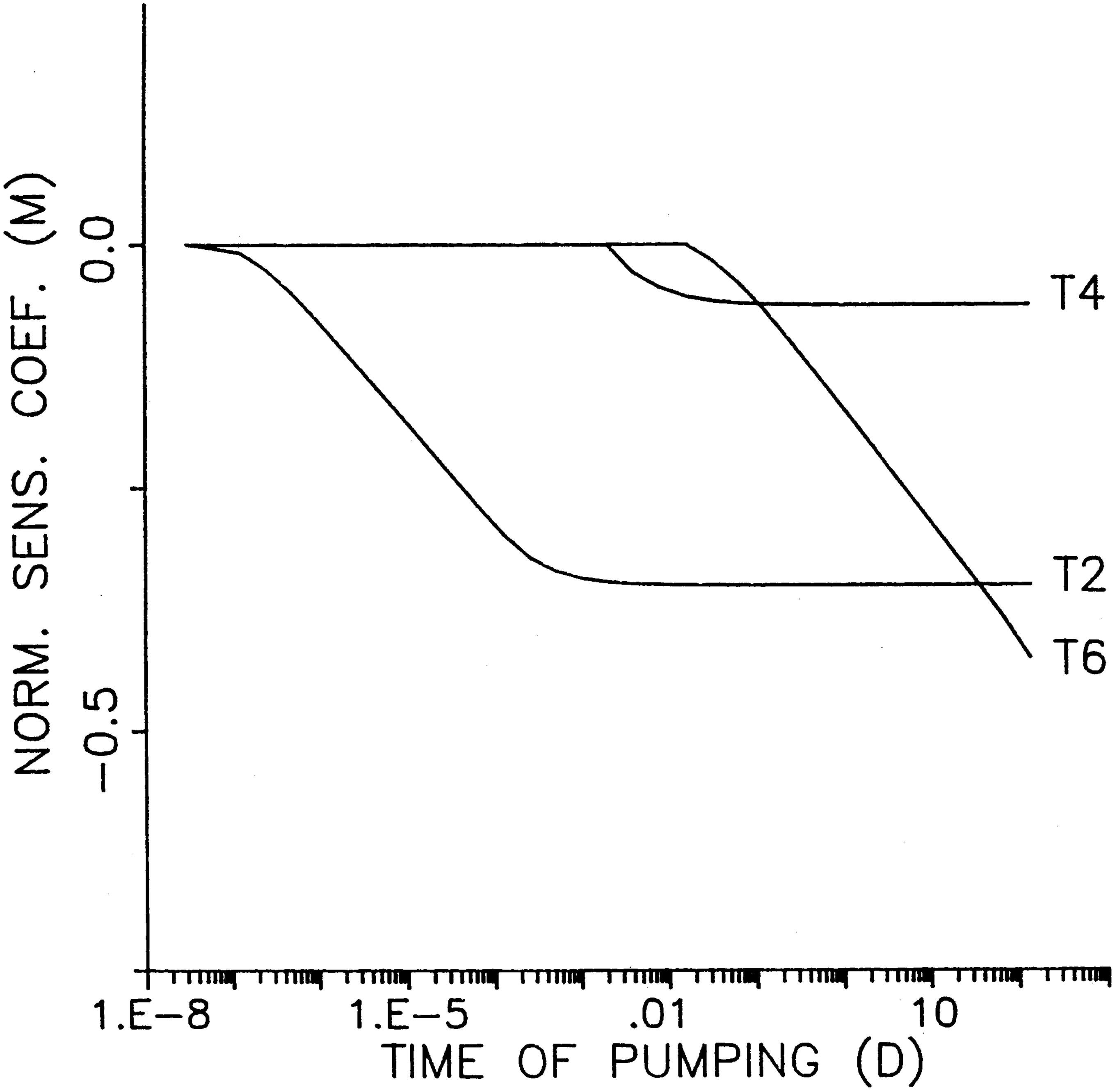

Figure Five

SOURCES OF NONUNIQUENESS

- MEASUREMENT ERROR
- INSENSITIVITY OF DRAWDOWN TO PARAMETERS
- CORRELATION BETWEEN PARAMETERS

⋮ ⋮

Figure Six

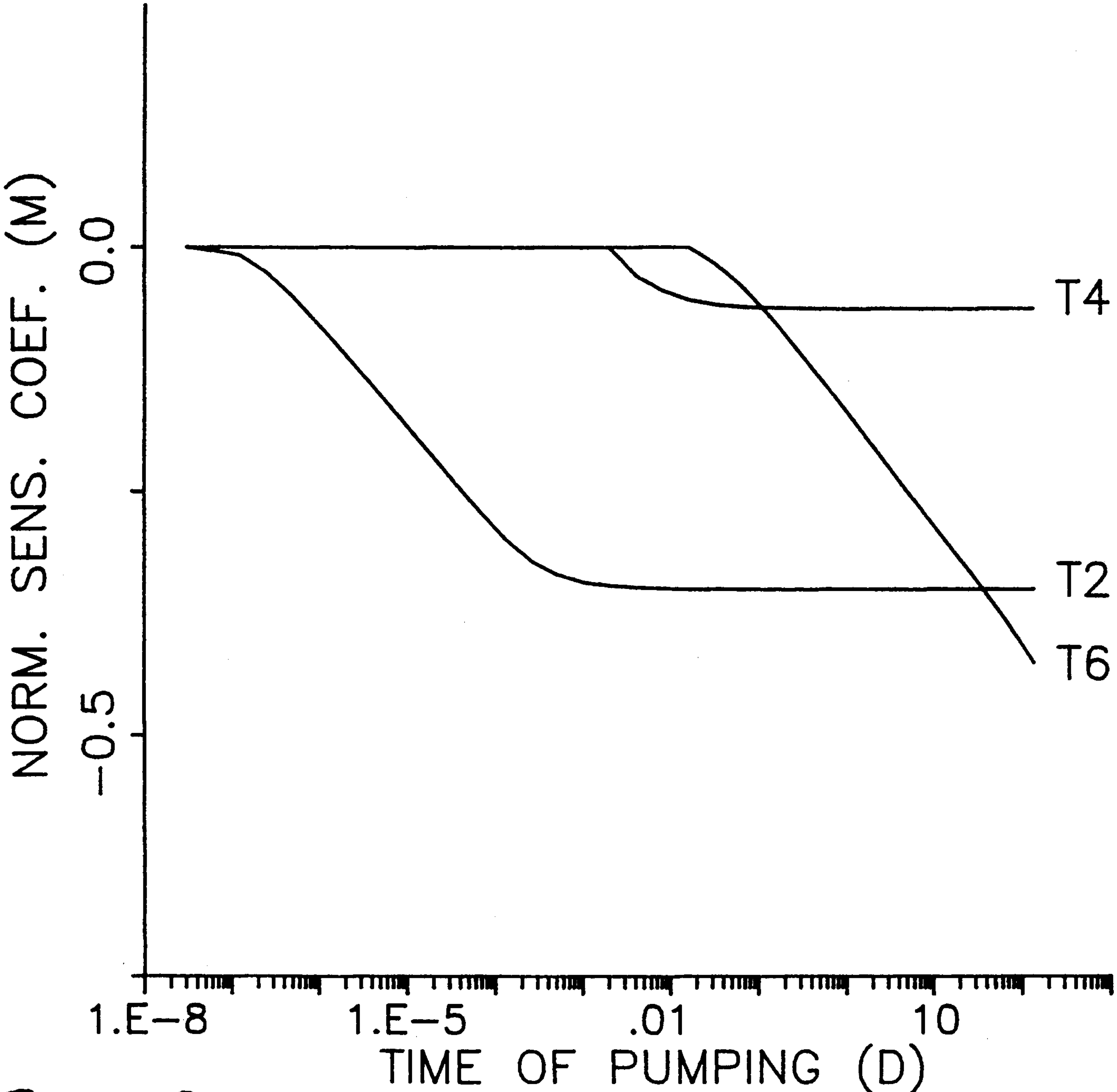

Figure Seven

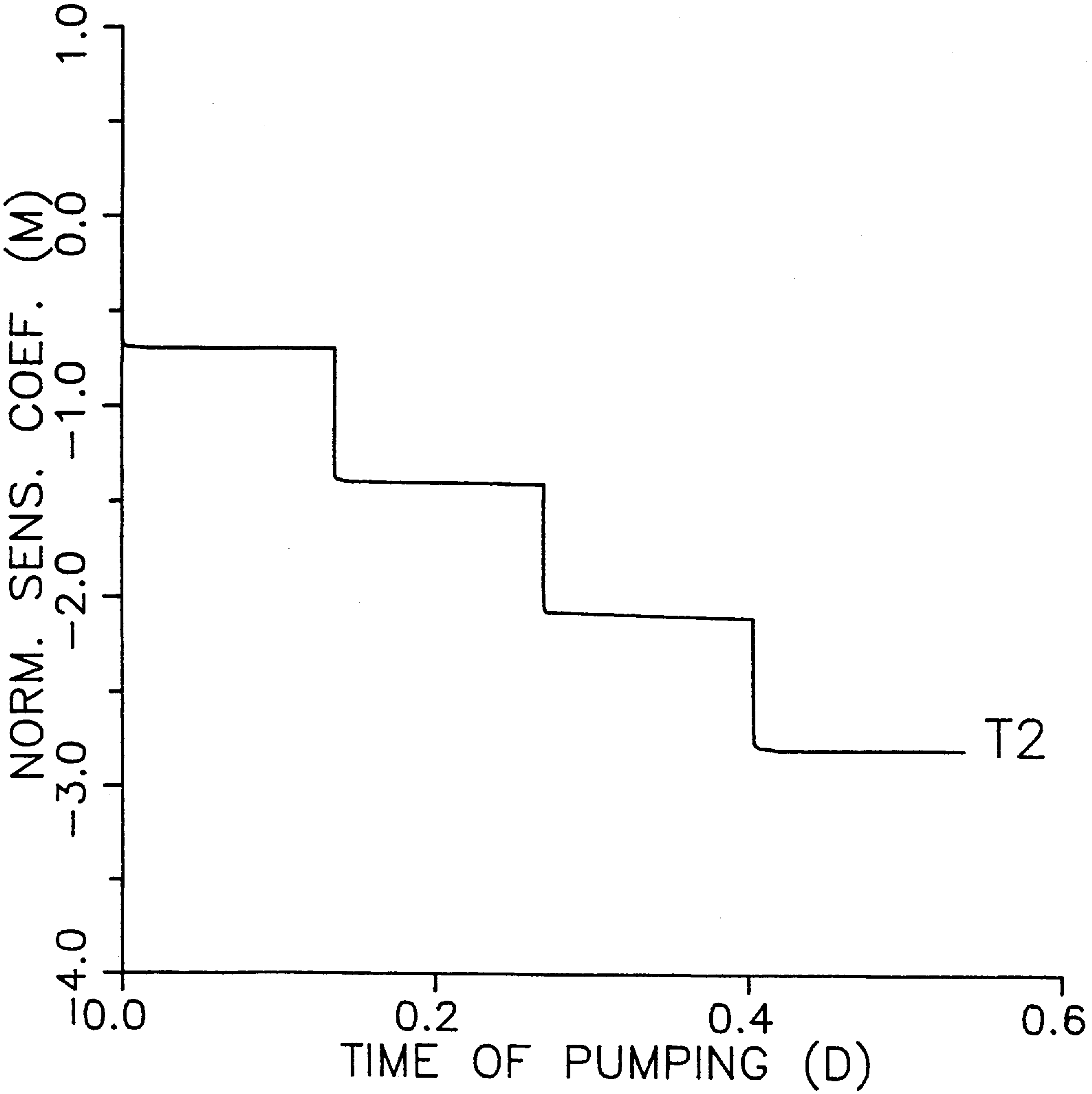

Figure Eight

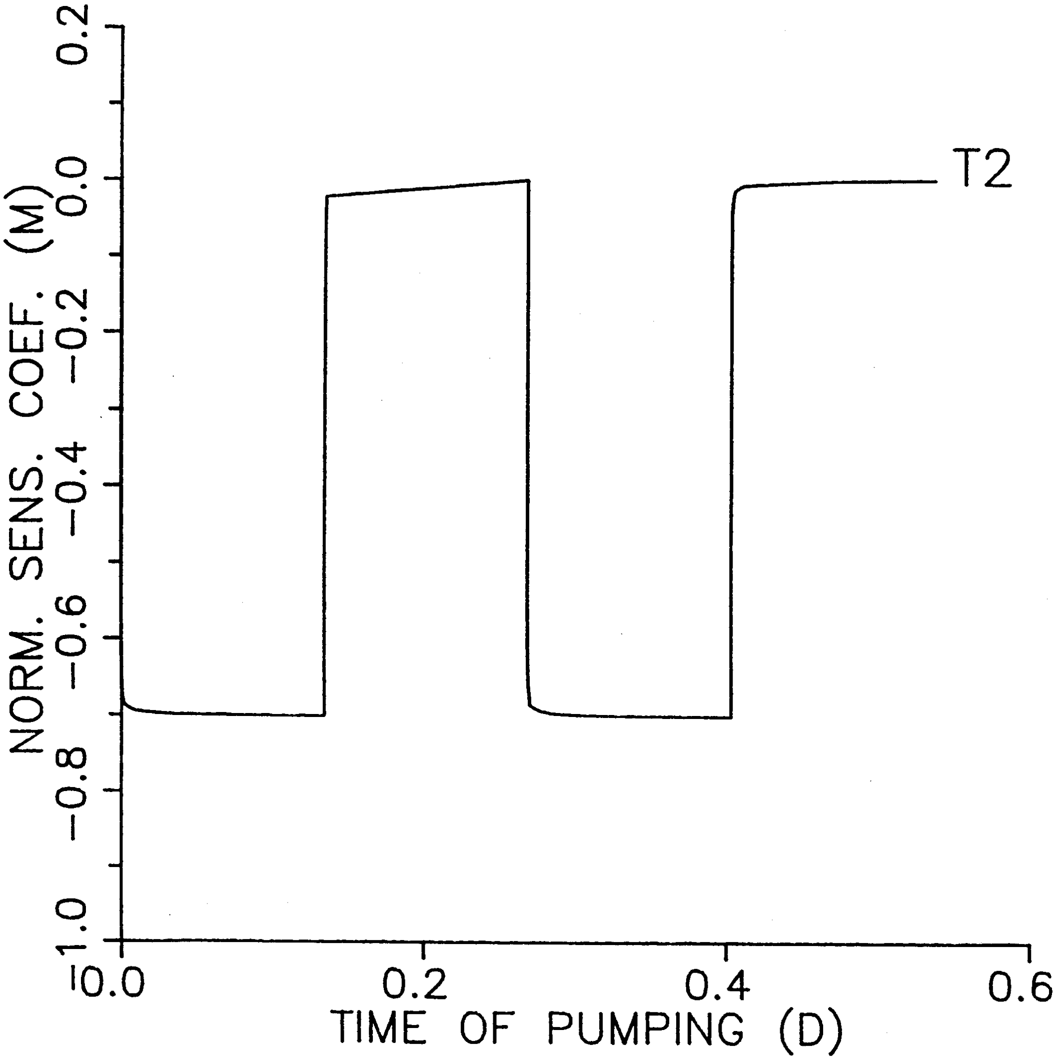
T2
NORM. SENS. COEF. (M)
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
0.0
0.2
0.4
0.6
TIME OF PUMPING (D)

Figure Nine

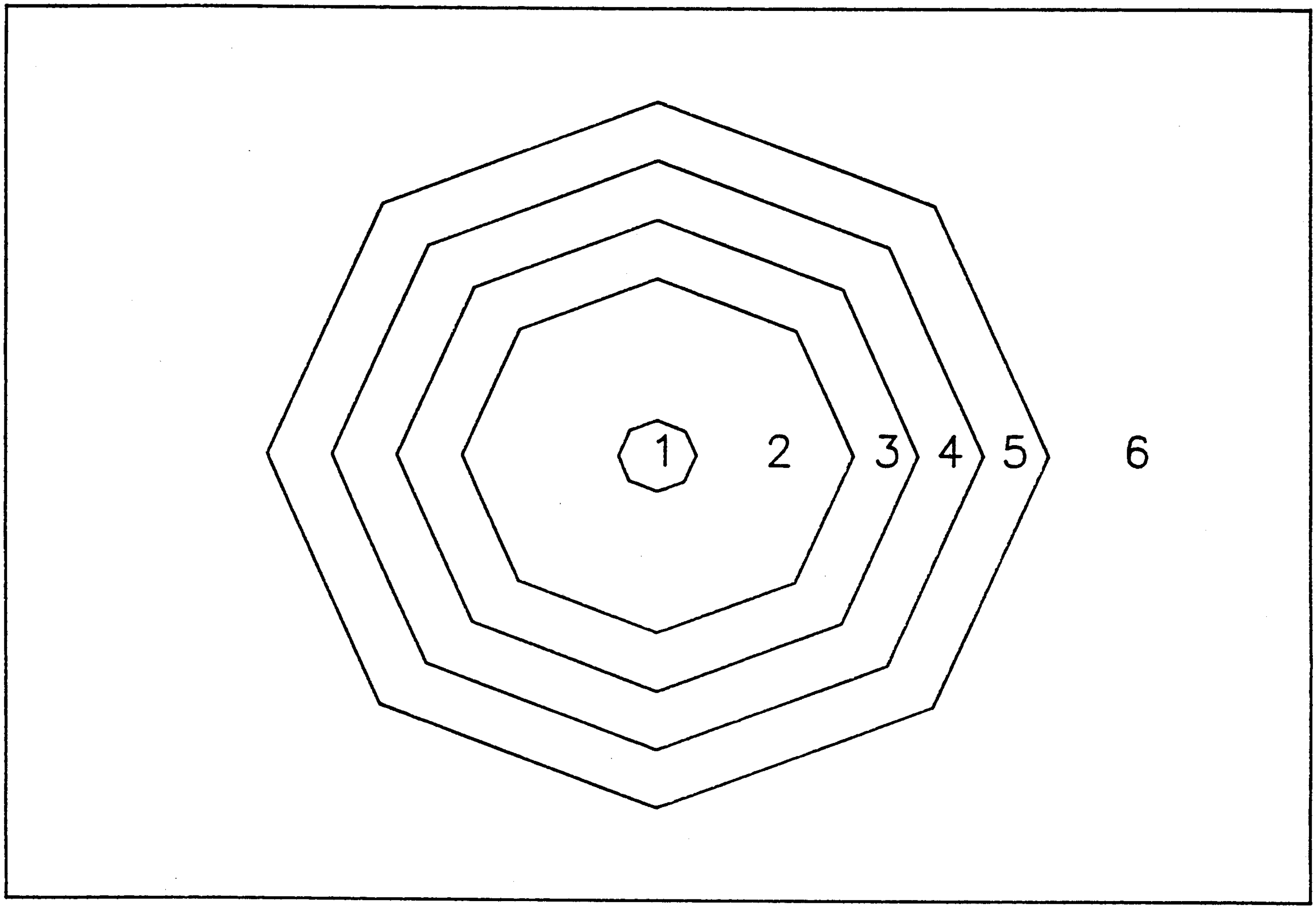

Figure Ten

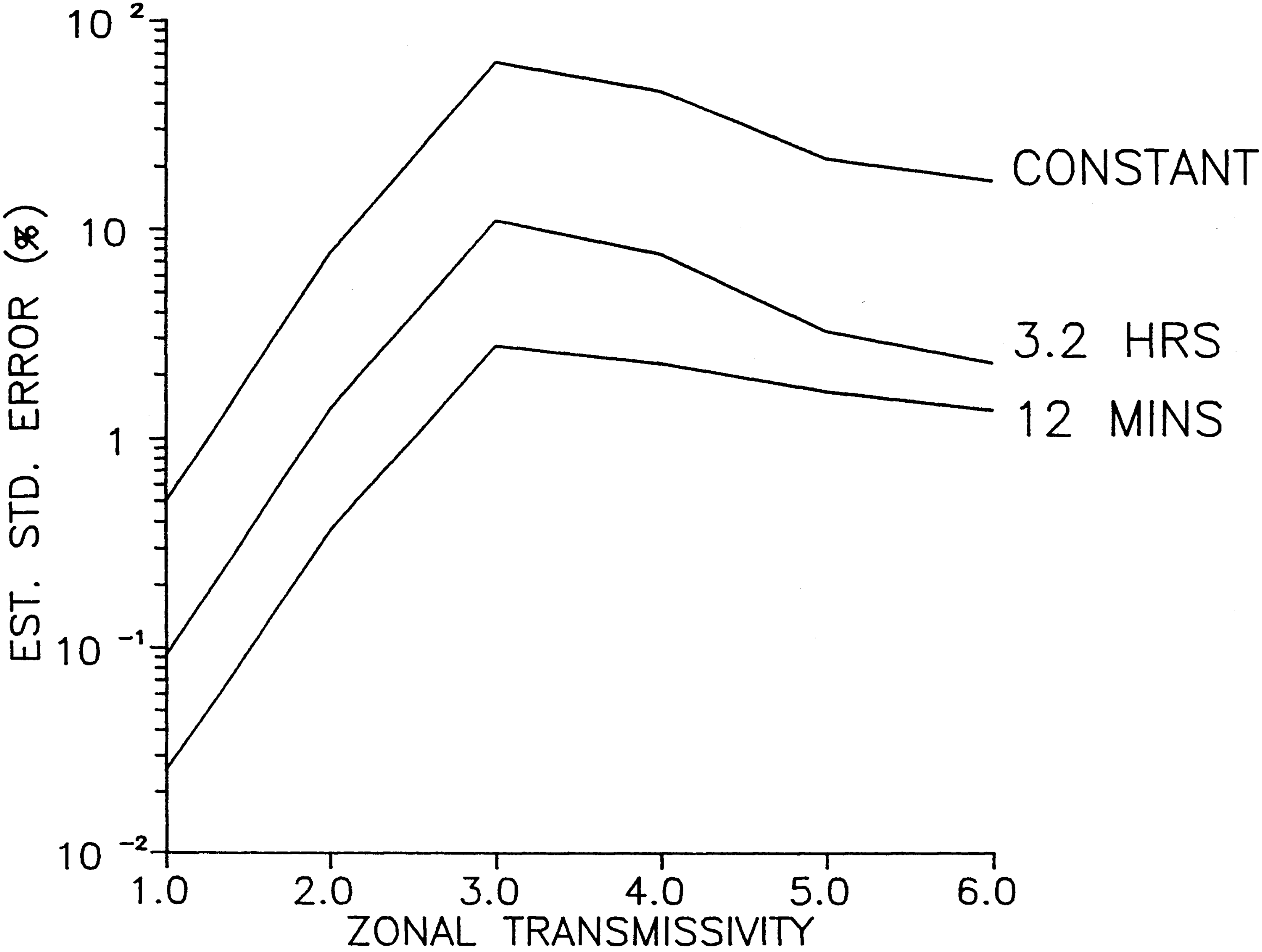

Figure Eleven

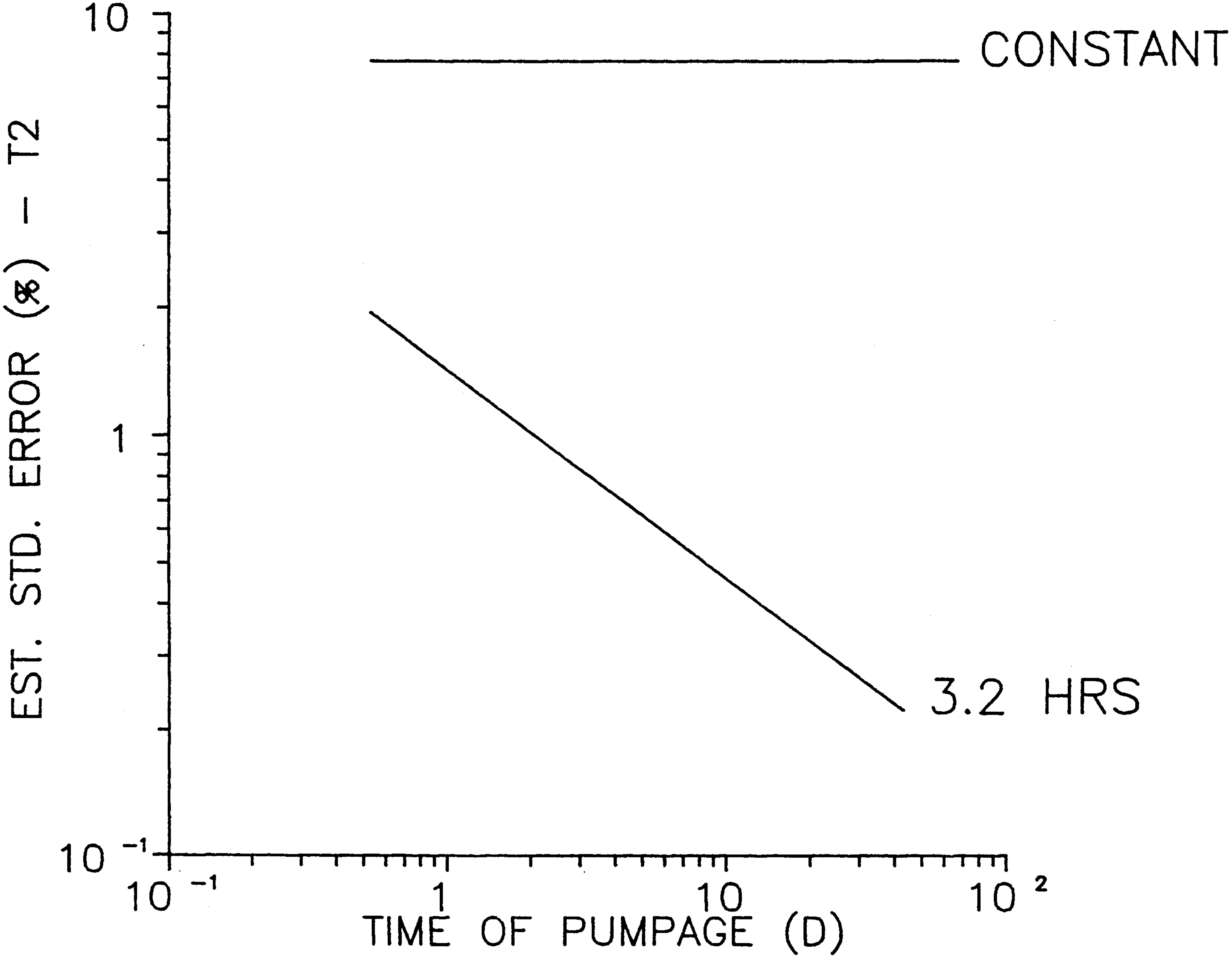

Figure Twelve

CONCLUSIONS

- INFLUENCES ON DRAWDOWN
- BENEFIT TO INVERSING EFFORTS
- VARIABLE-RATE PUMPING

Figure Thirteen