PUMPING TESTS IN NONUNIFORM AQUIFERS

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ABSTRACT

Pumping-test analyses are based on a series of assumptions required to reduce complex natural systems to mathematically tractable ones. This work examines the assumption that aquifer properties are uniform in space. The focus of the investigation is on the dependence of pumping-test transmissivity on spatial and temporal location of observations, and mode of drawdown analysis. A two-dimensional, nonuniform structure described by a stationary stochastic process is assumed to represent horizontal variations in hydraulic conductivity. The influence of vertical variations in hydraulic conductivity on pumping-test analyses is assumed negligible. Monte Carlo simulation is utilized to investigate flow behavior in this configuration. The results show that angular and temporal locations of observations have little effect on transmissivity, although radial location can often be critical. The results also demonstrate that modes of analysis that yield identical estimates in uniform units yield differing estimates in nonuniform units. This divergence is a function of the different methods emphasizing dissimilar aspects of the drawdown record. An analytical solution for a simple nonuniform system is employed to demonstrate the impact of these different emphases. The general conclusion is that pumping-test analyses can be a feasible approach in many nonuniform aquifers, providing both local and regional estimates of aquifer properties.

INTRODUCTION

Slide 1

This afternoon I would like to briefly discuss some research that I have been pursuing in the area of pumping tests in nonuniform aquifers. In this work, I subdivided nonuniform aquifers into homogeneous and heterogeneous units, where homogeneous and heterogeneous are defined in a geological field classification sense. This can also be viewed as a subdivision into systems of single and multiple components, respectively. The majority of my comments will concern homogeneous units, and, specifically, the applicability of techniques developed for uniform aquifers to homogeneous, nonuniform systems.

Slide 2

The presentation will be organized in the following manner. I will begin by quickly summarizing the research approach. Two of the more important assumptions will be examined. I will then present the results of this work, looking at the dependence of pumping-test transmissivity on observation location and mode of drawdown analysis. I will then examine the dependence of pumping-test results on mode of analysis in more detail and will also make a few comments concerning applications to heterogeneous systems.

RESEARCH APPROACH

Slide 3

I will use this slide to introduce the basic research approach. Consider this to be an areal view of an aquifer with stars representing observation wells along the circumference of three circles of radii A, B, and C. A pumping well is at the center of these circles. I was chiefly interested in assessing how transmissivity, calculated from drawdown at a single observation well using techniques for uniform aquifers, varied with angular and radial position of the observation well, as well as the duration of the analysis. In this work, I examined the effect of hydraulic-conductivity variations on pumping tests in confined aquifers. Storage and aquifer thickness were assumed constant. Based on earlier work, I also assumed that vertical variations in hydraulic conductivity could be neglected.

Slide 4

In terms of procedural details, I used a second-order stationary stochastic process with the usual normality assumptions to represent variations in the natural logarithm of hydraulic conductivity. A regularized process was employed to reduce the required computations. Correlation was represented by a double-exponential variogram model. The horizontal range and sill values are as shown for the pre-regularized process; values within parentheses represent intrabed variability as will be explained shortly. Regularization dampened the variability by over an order of magnitude. Monte Carlo simulation was employed to estimate mean behavior over the stochastic process. The turning bands method was used to generate realizations, and a stratified sampling scheme was employed to reduce the number of required realizations.

Slide 5

A finite element flow model was employed for the pumping-test simulations. Drawdowns were analyzed using automated routines; one being a nonlinear least squares approach employing the solution of Theis, the other a slope calculation using the Cooper-Jacob approximation. Unless noted otherwise, all the drawdown analyses were performed employing the nonlinear least-squares approach.

Slide 6

I would like to discuss two points concerning the procedural details. First, as has been demonstrated in several field studies, data from a vertical sampling traverse will display much greater variability than data from a traverse of equal length in the horizontal plane. Essentially, as shown here, this is due to the vertical traverse sampling a significantly greater number of beds. In this work, I assumed that the variability seen in the vertical would be reproduced in the horizontal at a large distance. I thus assumed the horizontal traverse to be providing information concerning intrabed variability, while the vertical traverse was characterizing interbed variability.

Slide 7

A second point concerning procedural details can be considered by looking at this areal view of a hypothetical aquifer. Pumping tests were simulated in a confined aquifer of large enough size so that boundary effects could be neglected. However, I assumed that the conductivity variability playing the primary role in controlling system behavior was that in the vicinity of the pumping and observation wells. The stratified sampling scheme was therefore based on sampling the variability within a small portion of the aquifer represented here by the small square centered on the pumping well. The distance R was approximately twice the distance to the most distant circle of observation wells.

RESULTS

Slide 8

With that background, let us now examine the results from the Monte Carlo simulation work. First, let us consider the variability in pumping-test transmissivity seen in the angular direction. Displayed is a plot of the mean coefficient of variation; the standard deviation of transmissivity values calculated at individual wells along the circumference of a circle at a given radial distance from the pumping well over the mean of these values, versus the radial distance to the circle of observation wells normalized by the range of the process. Vertical bars are 95% confidence intervals. Two major points concerning this plot are 1) the mean coefficient of variation increased with distance from the pumping well and 2) the magnitude of variability in the angular direction was not large enough to be of concern in most applications.

Slide 9

This slide displays the calculated dependence of pumping test transmissivity on radial location. Plotted here are the mean absolute transmissivity difference, which is the normalized difference between a transmissivity calculated at the pumping well and one at a distance from the pumping well, versus the normalized separation distance. Again, the variability increases with distance. The variability in this case was much greater than that seen in the angular direction. Considerable uncertainty may therefore arise when using a transmissivity calculated at one observation well to characterize behavior at another well at a large distance in the radial direction. Now I would also like to use these results to make a point concerning anisotropy. I found that the combined effect of both angular and radial variations in pumping-test transmissivity could easily be mistaken for some sort of anisotropy in transmissivity, if conventional techniques for analysis of uniform, anisotropic aquifers were employed. Therefore, only in the presence of additional evidence should the concept of anisotropy be resorted to as an explanation for variations in pumping-test transmissivity.

Slide 10

The examination of the temporal dependence of pumping-test transmissivity was somewhat different in that the dependence on mode of drawdown analysis was also assessed. Once again, a normalized difference was used with the normalizing quantity being the small-time transmissivity. Note that the temporal dependence was very small with the nonlinear least squares approach, and reasonably small using a Cooper-Jacob analysis. The most important point is that these two methods, which produce identical estimates in uniform units, produce differing estimates in nonuniform ones. These differing estimates are due to the two methodologies emphasizing different aspects of the drawdown record. Before demonstrating this fact, let me briefly summarize the presentation to this point. Essentially, the results have shown that angular and temporal dependence are of little concern in most cases. Radial dependence may be of concern, but only at large separation distances.

MODE OF ANALYSIS

Slide 11

In order to better understand this dependence on mode of analysis, I examined pumping-test behavior in a simple heterogeneous configuration consisting of a radial disk of one uniform material embedded in a uniform matrix of differing properties. For the case of pumping from the center of the disk, the LaPlace transform approach can be applied to calculate drawdown at any point in the system. It is instructive to consider drawdown at a point within the embedded disk, for which the large-time approximation is given by this equation.

Slide 12

The terms here need little explanation - the big R is the radius of the embedded disk, while the small r is the radial distance to the observation well. This expression was produced by truncating an infinite series in a manner analogous to the Cooper-Jacob truncation. In fact, the dimensionless times of truncation are the same except that, in this case, the radius in the dimensionless time term is that of the embedded disk, and not the observation well. In other words, the transients produced by the disk boundary, and not just the material in the vicinity of the observation well, must be dissipated before truncation. The important point to note concerning this equation is that the rate of drawdown is controlled solely by the properties of the matrix. A Cooper-Jacob analysis over time will yield the matrix transmissivity. The Cooper-Jacob approach is thus providing an estimate of the transmissivity of the material through which the front of the cone of depression passed during the duration of the analysis, and thus this approach is viable in some heterogeneous systems. An interesting result is that it can be shown, using this configuration, that the infinite series truncation of Cooper and Jacob removes all further contributions of near-well material to total drawdown.

Slide 13

With that understanding of the Cooper-Jacob method, let us quickly look at a simple simulation exercise to demonstrate the difference due to analytical methodology. The basic configuration is as before, a radial disk embedded in a matrix of differing properties. Let us also consider two other cases: 1) a uniform aquifer with properties of the radial disk and 2) a uniform aquifer with properties of the matrix.

Slide 14

A plot of the simulation results is shown here. The transmissivity of the embedded disk is 60, that of the matrix 600. Storage is assumed constant. The middle curve depicts drawdown for the embedded disk case. At a very early time, the upper two curves diverge as the drawdown cone passes out of the disk.

Slide 15

After approximately a tenth of a day, the two lower curves are parallel. If a Cooper-Jacob analysis is performed for the embedded disk case at any later time interval, the transmissivity of the matrix is computed. A nonlinear least squares analysis, however, will produce a vastly different estimate. The reason for this is that the least-squares analysis is an equation-fitting procedure. Unlike the Cooper-Jacob method, which calculates the slope of a fitted straight line, a procedure based on fitting the Theis solution to the drawdown record will be controlled by the total drawdown, and, as graphically shown here, that total drawdown is often dominated by early time behavior. A least-squares analysis performed for the embedded disk case between 1 and 100 days produced a transmissivity estimate only 15% of the matrix transmissivity. Transmissivity estimates computed by a least-squares analysis are clearly heavily dependent on near-well properties. A pumping-test analysis in a nonuniform aquifer thus produces two estimates: one that is heavily-weighted towards near-well properties, and one that is a function of the material through which the front of the cone of depression passed during the analyzed interval.

CONCLUSIONS

Slide 16

In conclusion, I would like to say that this presentation has shown that the conventional approach for analysis of pumping tests in homogeneous units under confined conditions appears viable. Transmissivity estimates display little dependence on angular and temporal location. Radial dependence is the most serious. However, I must qualify that result by saying that this was a bounding-case analysis based on a data set with one of the largest sill values reported in the literature. I have also pointed out here that the concept of anisotropy in pumping-test transmissivity may be overused. The dependence of parameter estimates on mode of drawdown analysis was demonstrated with different techniques being shown to characterize different portions of the aquifer. The viability of the Cooper-Jacob approach in some heterogeneous configurations was also shown. In addition, I briefly discussed the physical significance of the Cooper-Jacob truncation. Finally, I would like to say that, although this work employs many assumptions and is clearly incomplete, the major results reported here are independent of these shortcomings and should be considered in that light.

Thank you for your attention.

APPENDIX

Details concerning this research can be found in the following references.

Butler, J. J., Jr., Pumping tests in nonuniform aquifers: a deterministic/stochastic analysis (Ph.D. thesis), 220 pp. Stanford Univ., 1986.

Butler, J. J., Jr., Pumping tests in nonuniform aquifers: the radially symmetric case, in review.

Butler, J. J., Jr., Pumping tests in homogeneous, nonuniform aquifers, in review.

PUMPING TESTS
IN NONUNIFORM AQUIFERS

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KANSAS GEOLOGICAL SURVEY
LAWRENCE, KANSAS USA

Slide #1

OUTLINE

RESEARCH APPROACH

- HOMOGENEOUS, NONUNIFORM CASE
- TWO RELEVANT ASSUMPTIONS

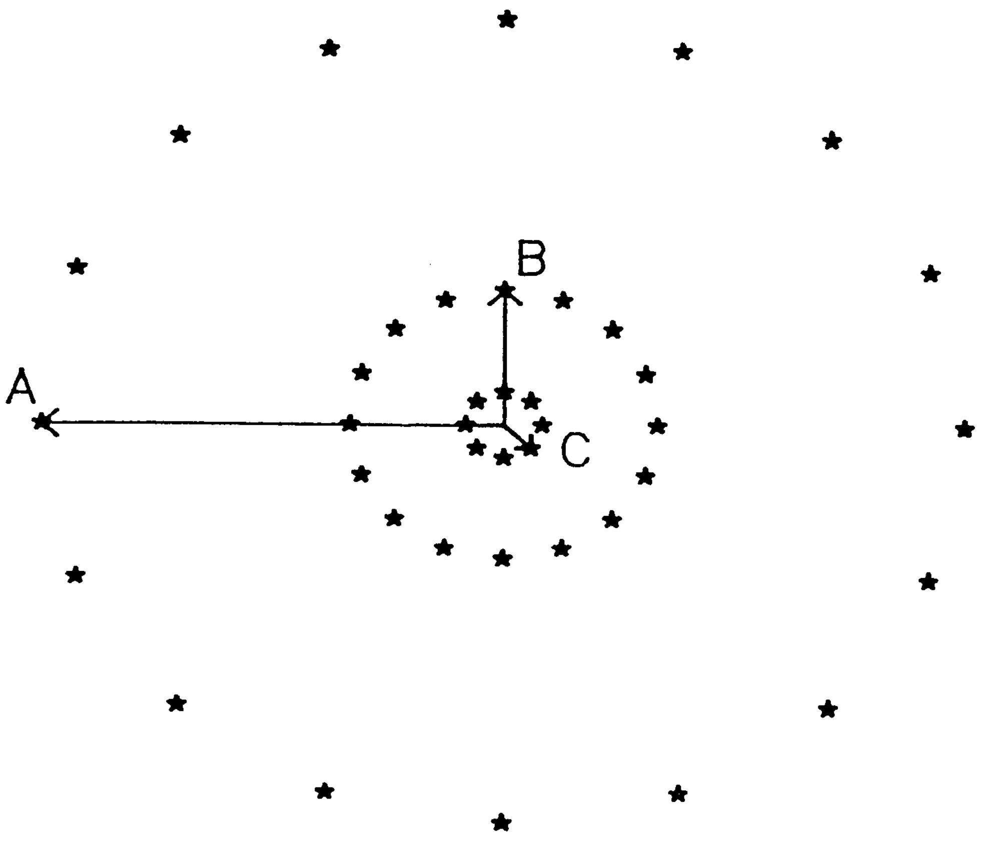
RESULTS

- DEPENDENCE
  - OBSERVATION LOCATION
  - MODE OF ANALYSIS
- EFFECTIVE PARAMETER

MODE OF ANALYSIS

- HETEROGENEOUS, NONUNIFORM CASE

CONCLUSIONS



Slide #2

PROCEDURAL DETAILS

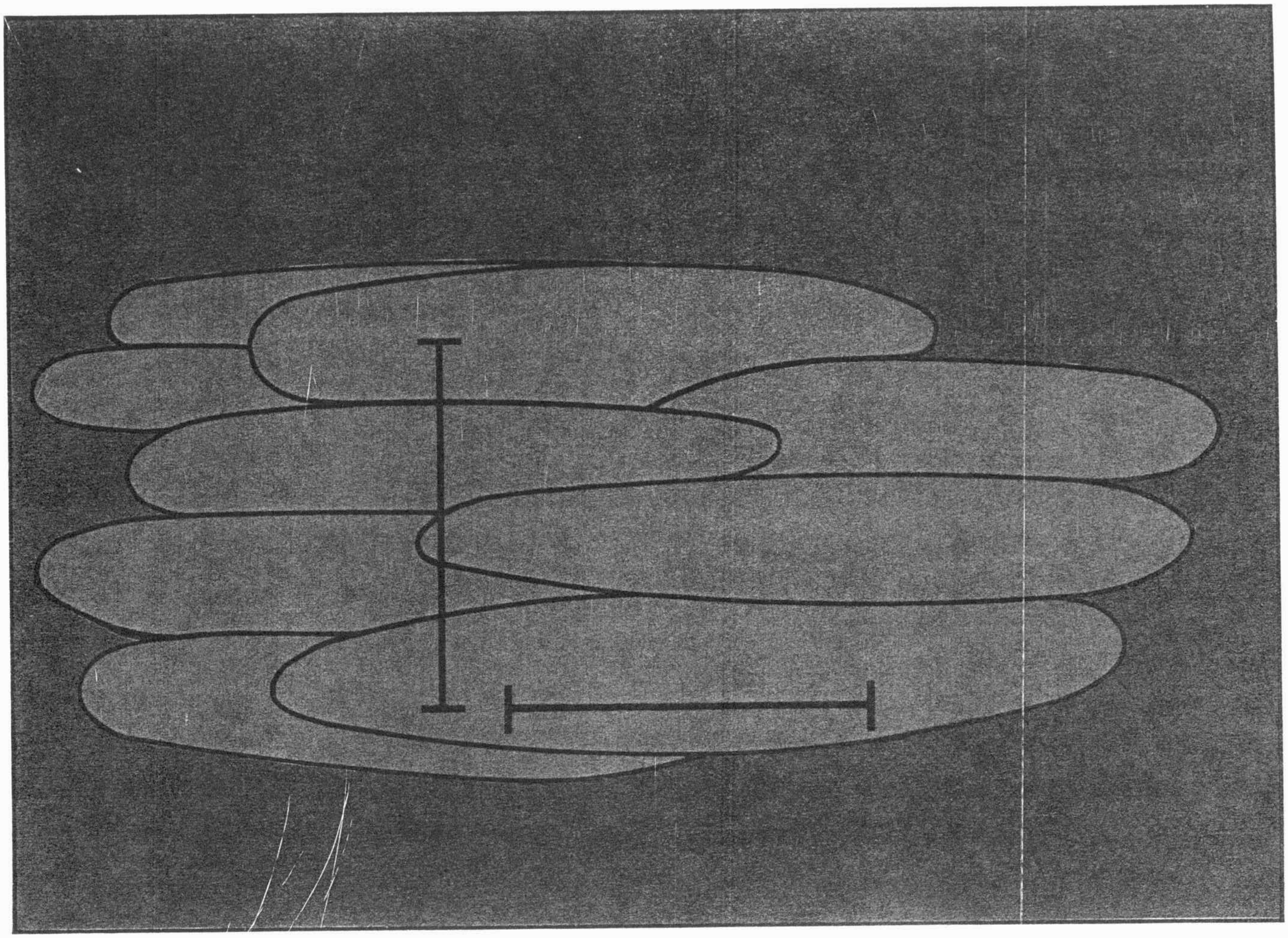
-STOCHASTIC PROCESS

- SECOND-ORDER STATIONARY
- REGULARIZED
- CORRELATION - DBLE EXP MODEL
  RANGE - 155 M (3 M)
  SILL (IN LN K (CM/S)) - 5.30 (.30)

-MONTE CARLO SIMULATION

- TURNING BANDS METHOD
- STRATIFIED SAMPLING (20 REALIZATIONS)

- FINITE ELEMENT FLOW MODEL
- PUMPING TEST ANALYSES
  - NONLINEAR LEAST SQUARES USING THEIS
  - C-J SEMILOG APPROACH



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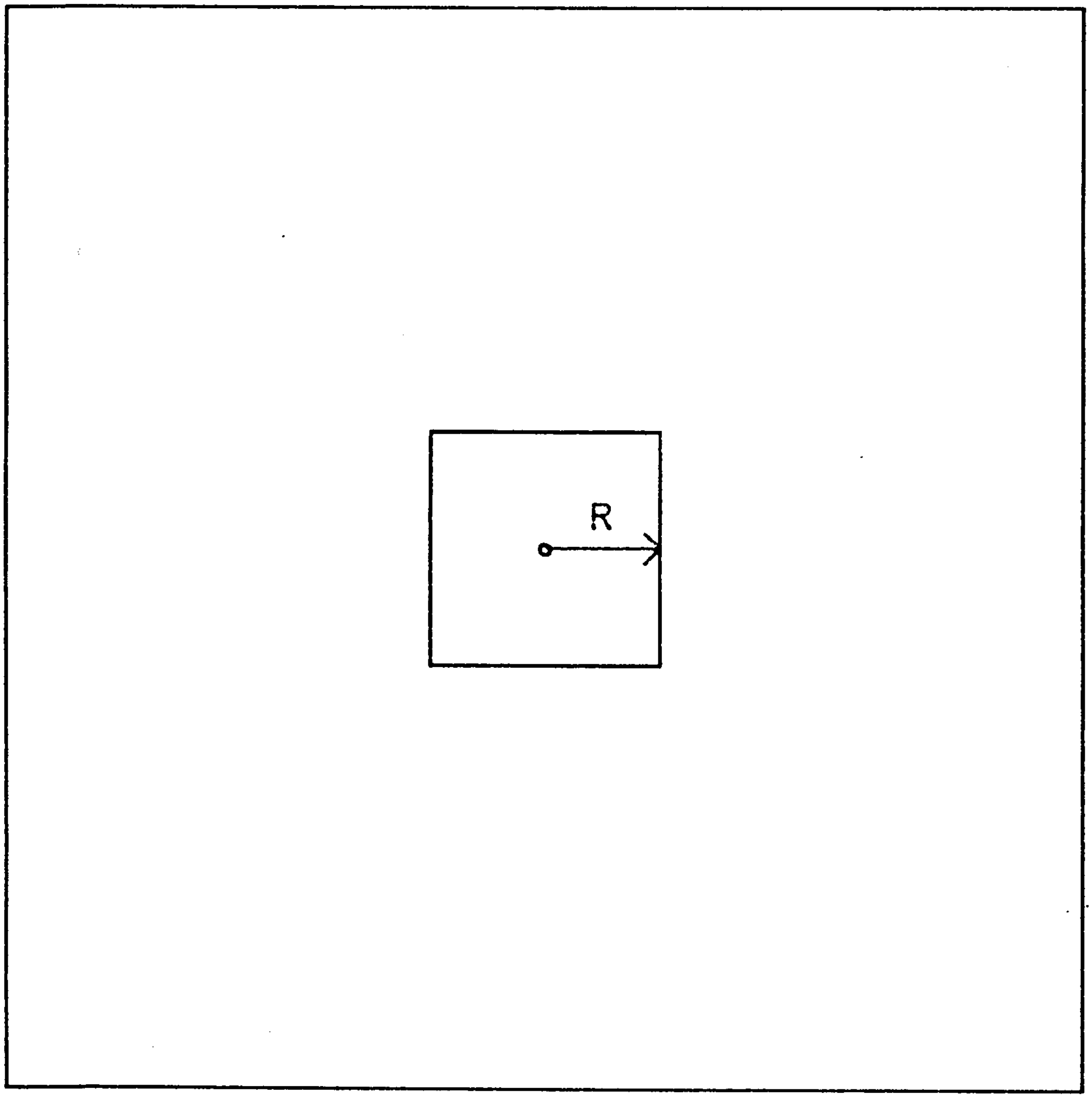

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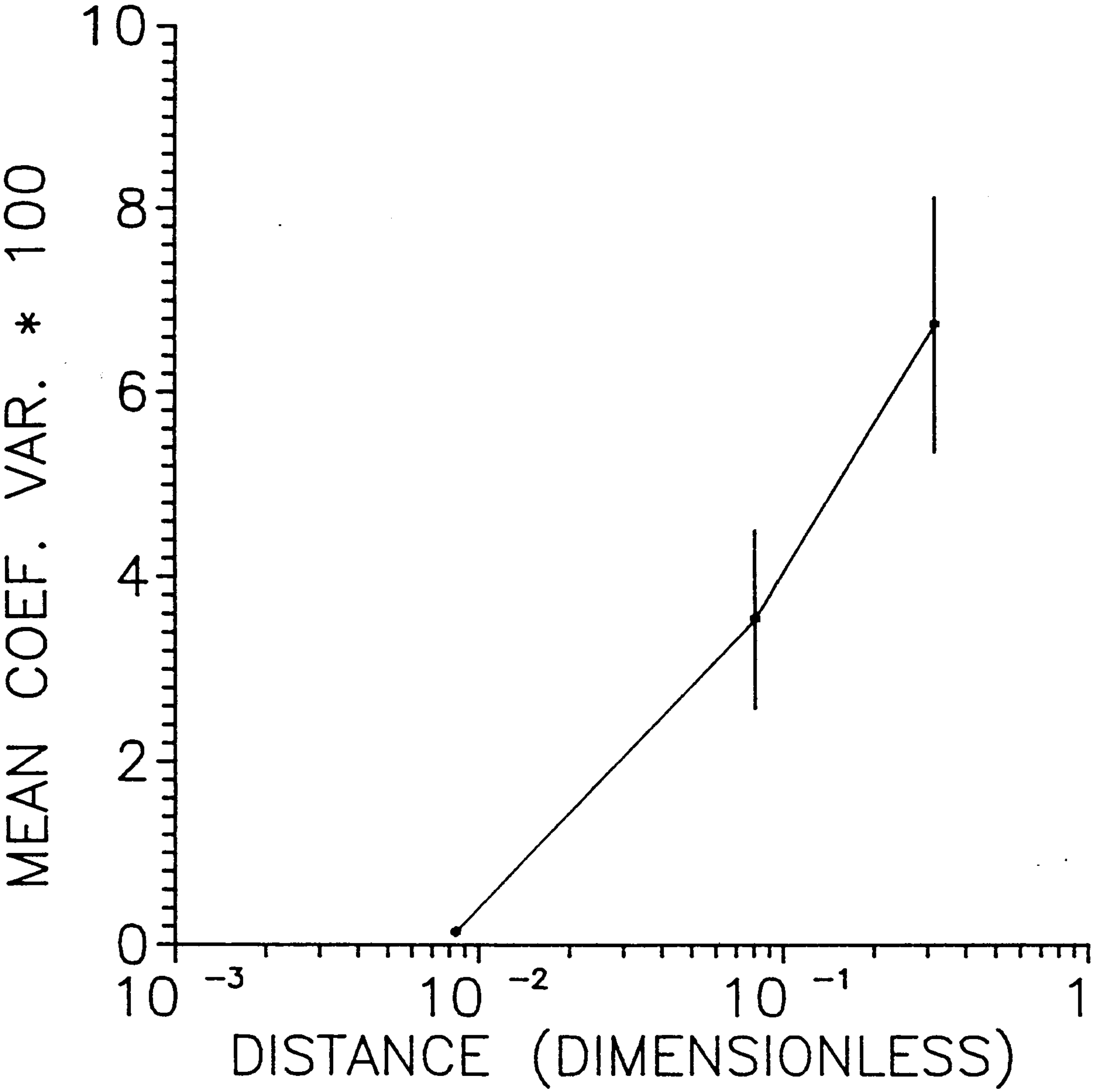
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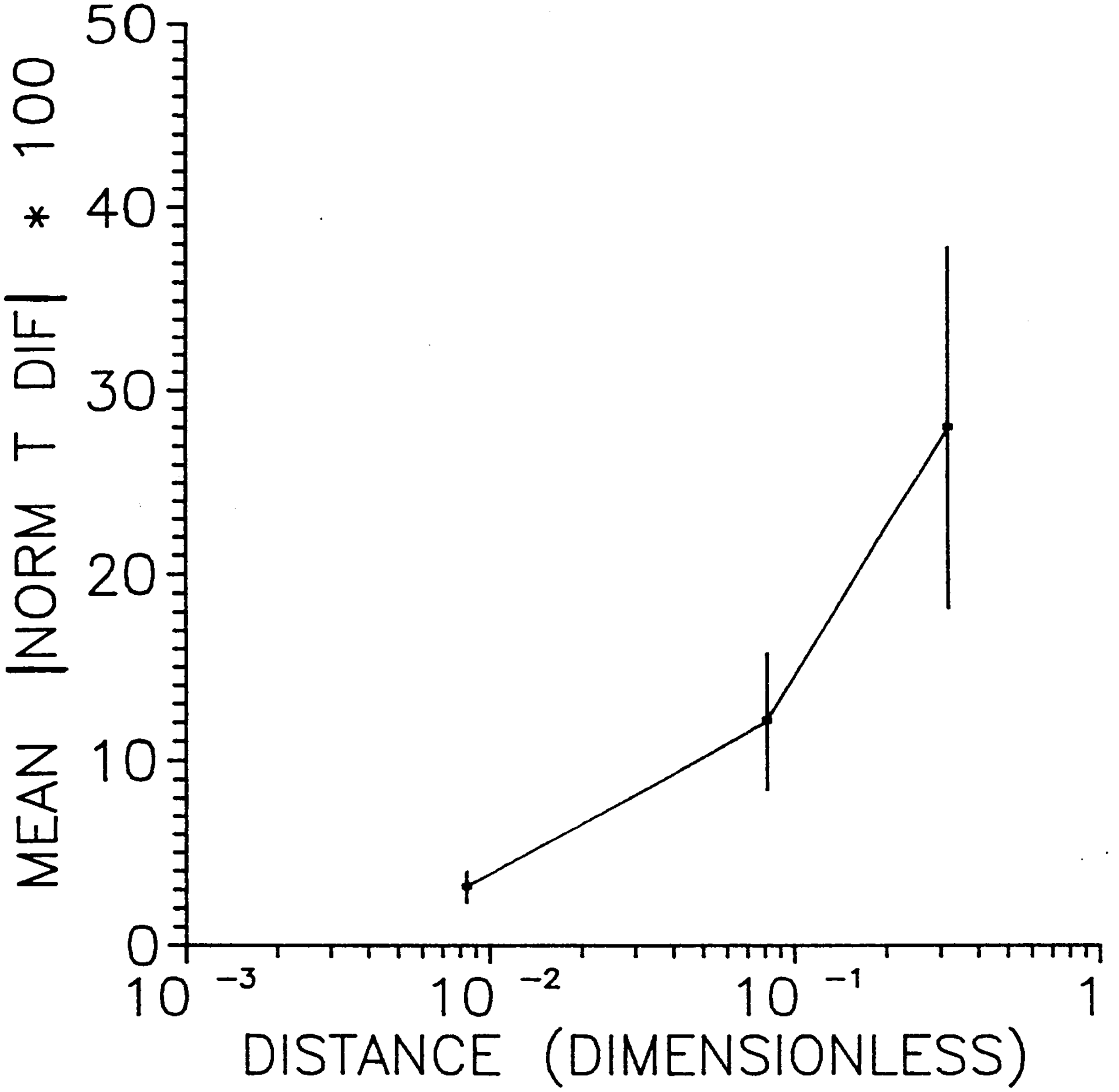
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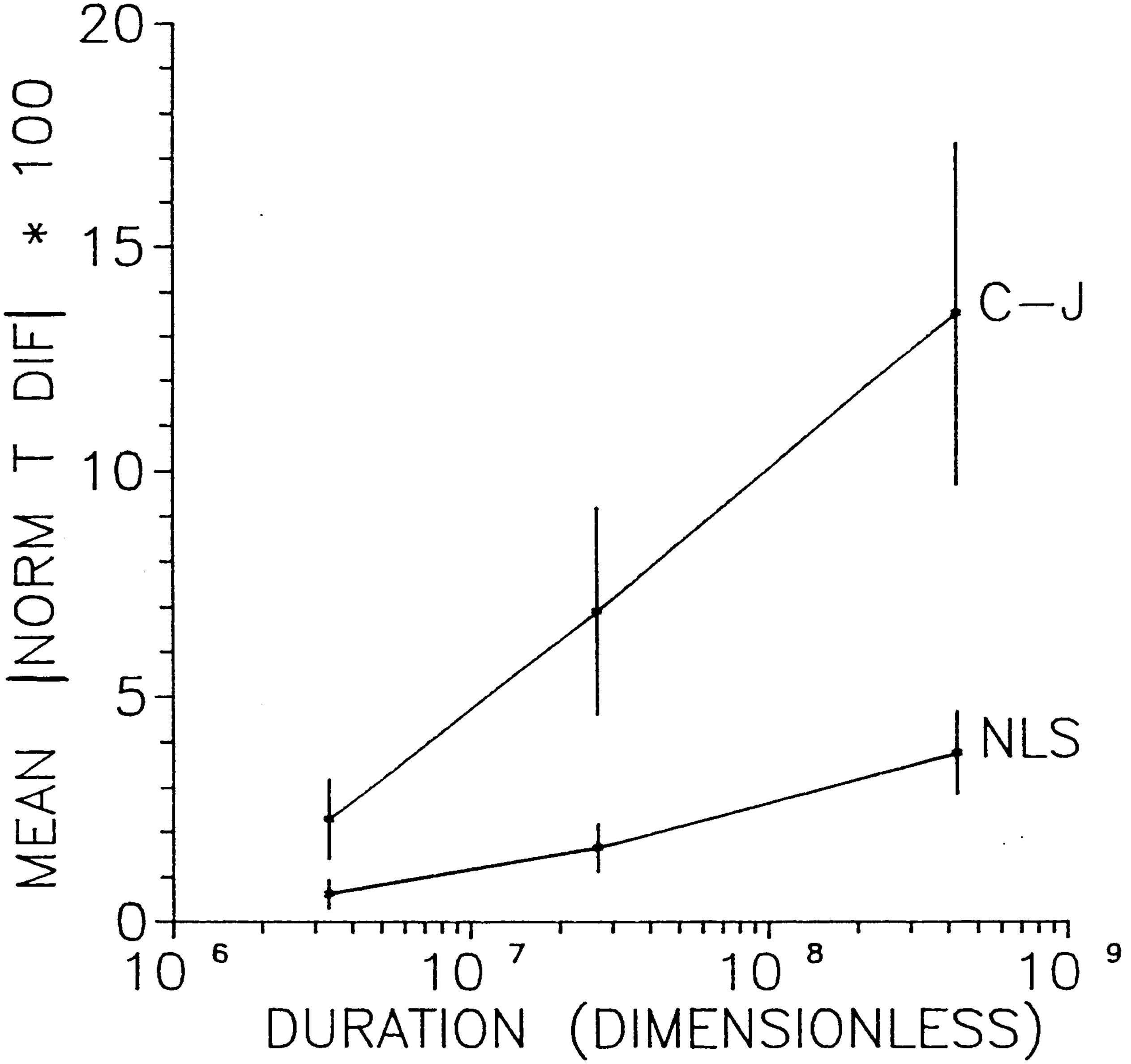
DISTANCE (DIMENSIONLESS)

Slide #9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15
10
5
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10 6
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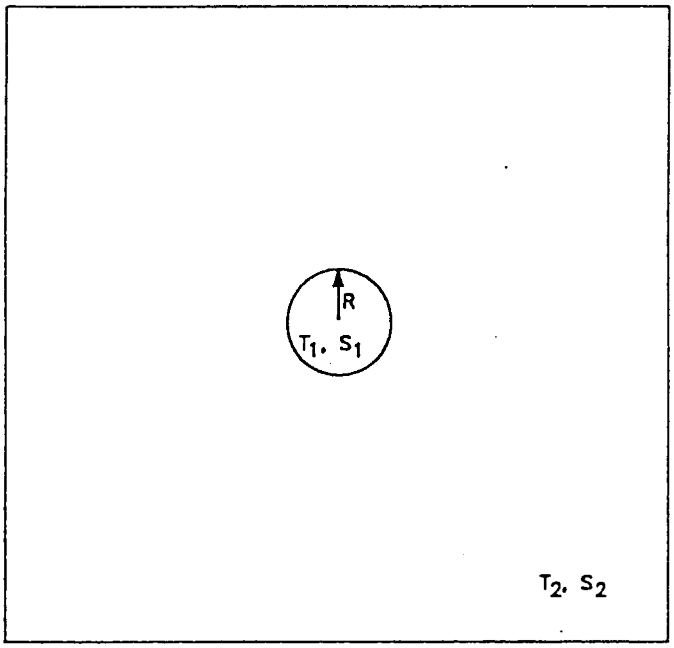
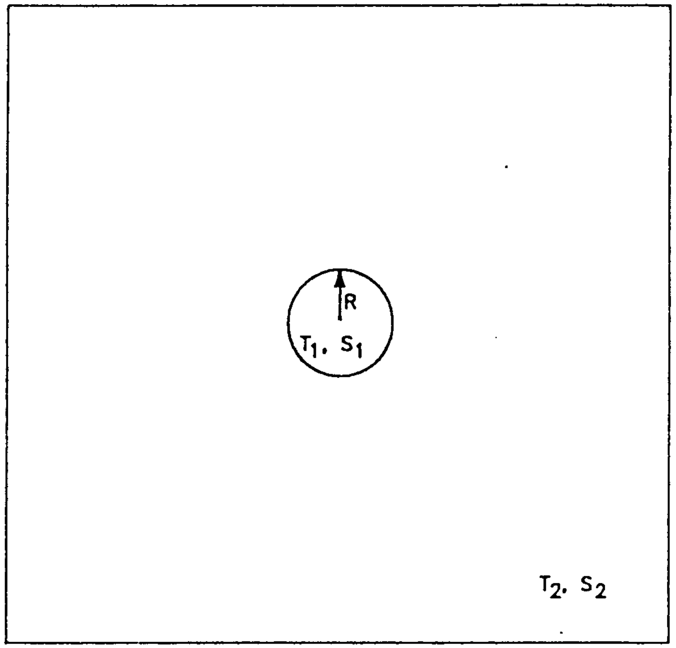
10 9
DURATION (DIMENSIONLESS)

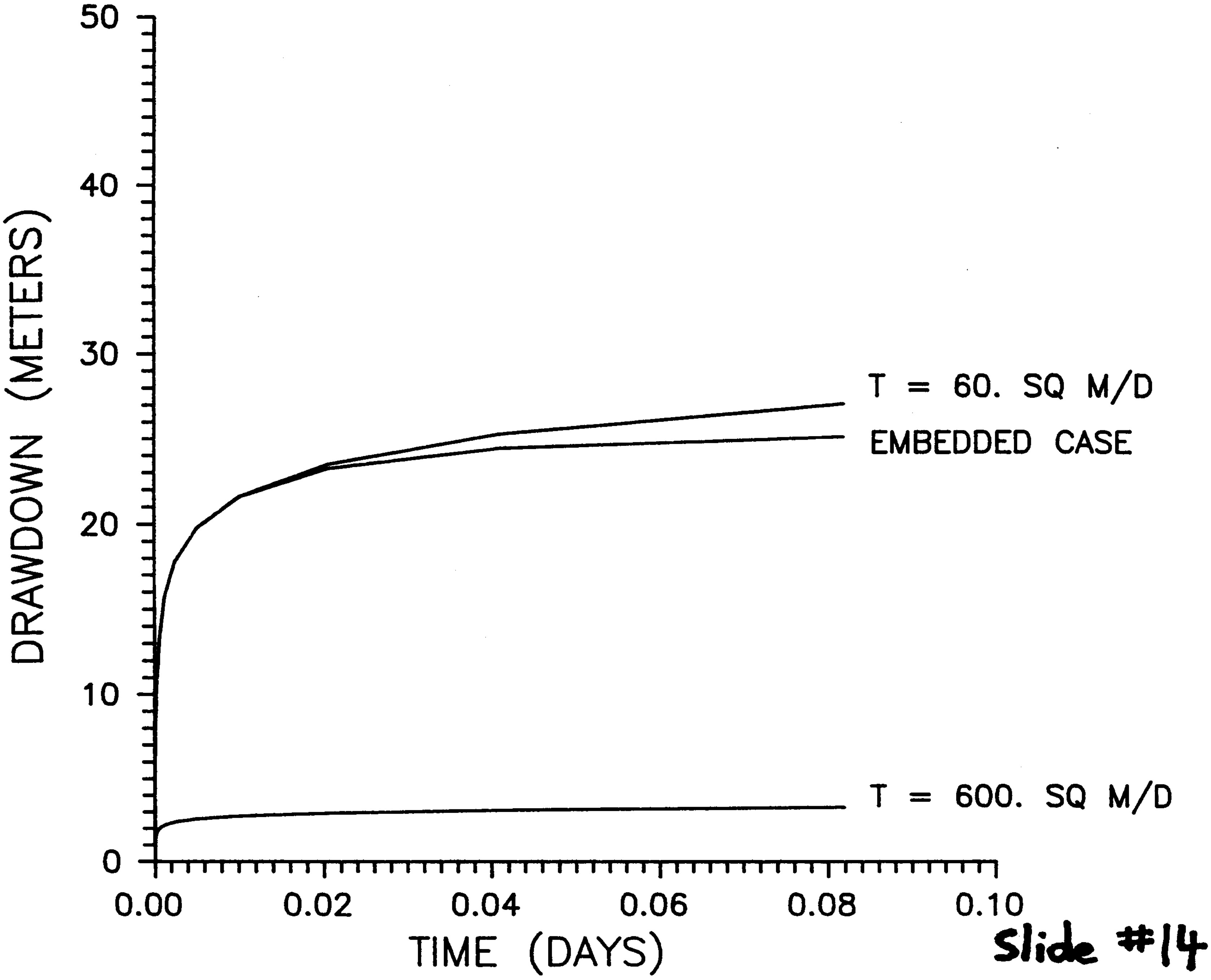
Slide #10



Slide #1

$$s_1 = \frac{Q}{4\pi T_2} \ln \left(\frac{4T_2 t}{CS_2R^2}\right) + \frac{Q}{2\pi T_1} \ln \frac{R}{r}$$


R
T_1, S_1
T_2, S_2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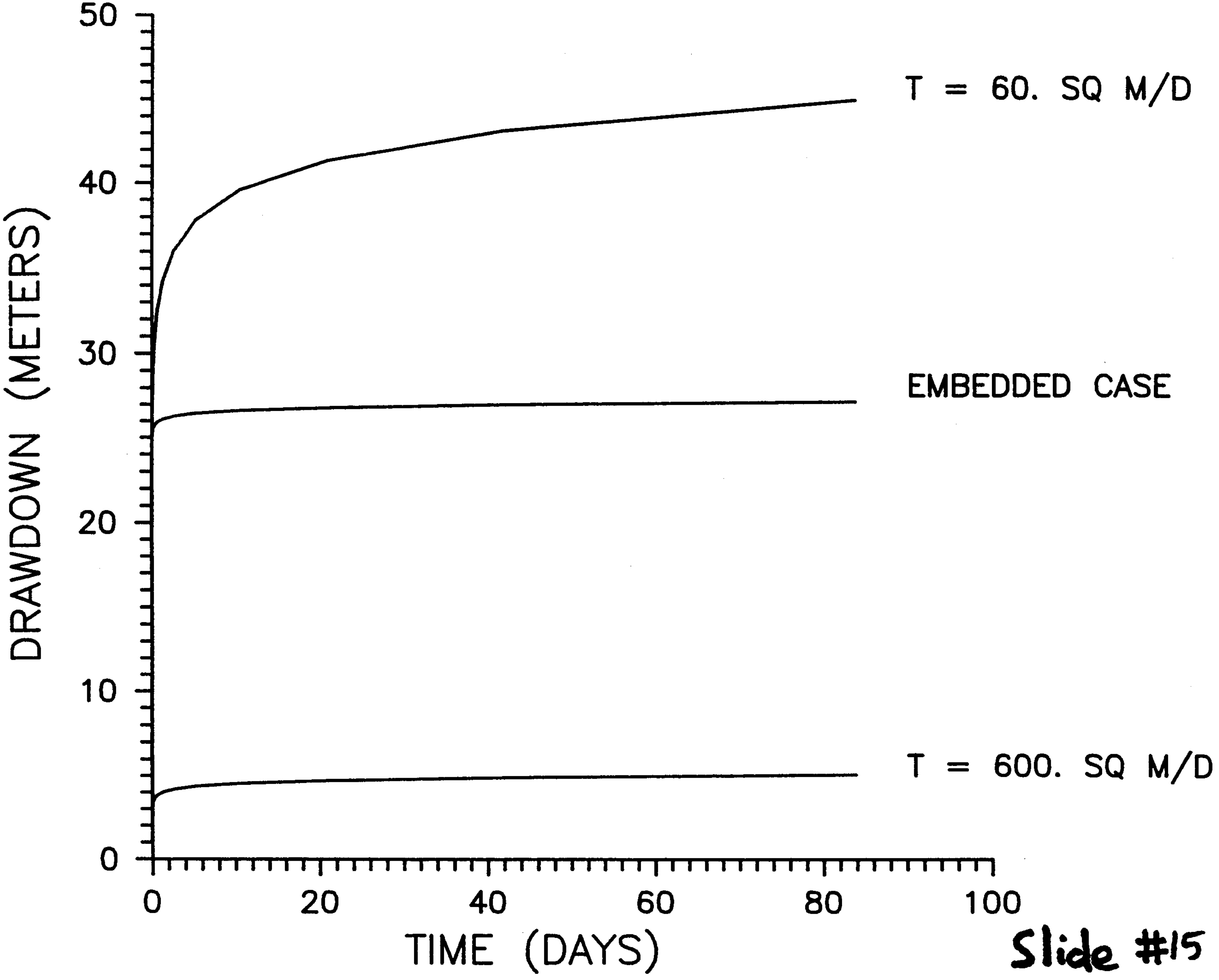
40
30
20
10
0
DRAWDOWN (METERS)
0.00
0.02
0.04
0.06
0.08
0.10
TIME (DAYS)
T = 60. SQ M/D
EMBEDDED CASE
T = 600. SQ M/D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TIME (DAYS)
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20
30
40
50
0
20
40
60
80
100
T = 60. SQ M/D
EMBEDDED CASE
T = 600. SQ M/D

CONCLUSIONS

CONVENTIONAL APPROACH VIABLE

- RADIAL DEPENDENCE MOST SERIOUS
  - BOUNDING CASE ANALYSIS
- ANISOTROPY CONCEPT OVERUSED

DEPENDENCE ON MODE OF ANALYSIS

- THEIS APPROACH PROVIDES LOCAL ESTIMATE
- C-J APPROACH PROVIDES LARGE-SCALE ESTIMATE
  - HETEROGENEOUS, NONUNIFORM UNITS
  - SIGNIFICANCE OF TRUNCATION

INCOMPLETE, YET . . .