VARIABLE-RATE PUMPING TEST: AN AUTOMATED NUMERICAL EVALUATION

by

M. A. Butt
and
C. D. McElwee

Kansas Geological Survey
1930 Constant Avenue
The University of Kansas 66046

Kansas Geological Survey
Open File Report 84-5

ABSTRACT

The reliability of predictive and management models for ground water would be improved by better aquifer-parameter estimation. As progress continues in the use of computers to simulate ground-water systems, parallel progress must occur in data collection and analysis. Among the various methods available for the determination of aquifer parameters, pumping tests occupy a prominent position. The maximum advantage is gained from an in-situ pumping test when geological knowledge of the aquifer and the analysis of aquifer test data complement each other. Relying on one piece of evidence, such as a single pumping test, might lead to unwarranted conclusions but when used with a combination of other pumping tests, borehole data, geological information, and experience, reasonably accurate conclusions about the aquifer parameters can be obtained. The purpose of this paper is to present a technique involving the use of convolution and sensitivity analysis to obtain the "best" fit of aquifer parameters in a least squares sense from a pumping test with variable pumping rate. The method also can be used to analyze the residual drawdown data obtained during the recovery period. In addition, this method can also analyze drawdown and recovery data conjunctively. Constant drawdown and variable discharge data of artesian flowing wells can also be analyzed by this method. The method is straight forward, quick, inexpensive, and is always objective. No graphical plots or graphical interpretations are needed. As a measure of error, the rms (root-mean-square) error in drawdown is calculated along with the correlation coefficient between pumping-test data and the theoretically generated data, using the converged values of transmissivity and storage coefficient.

INTRODUCTION

Accurate determination of aquifer parameters, notably the coefficients of transmissivity (T) and storage (S), is necessary for a quantitative understanding of most problems in hydrogeology. For over four decades considerable research has been done to improve the reliability of ground-water predictive and management models through better aquifer-parameter estimation, and the efforts continue. Among the methods available for the determination of aquifer parameters, pumping tests have the advantage of analyzing a significant part of the aquifer as a whole, in an in-situ condition. In addition, the test data may at times give a clue as to the existence of certain geohydrologic conditions which may otherwise be unknown.

Equations describing the drawdown distribution around pumped wells operating at constant discharge from an elastic artesian aquifer of uniform thickness have been known since Theis (1935) and have been used for a variety of aquifers. The Theis equation for non leaky aquifer pumping-test data has proved quite useful in practical applications. However, once the condition of constant pumping rate imposed by the theory is not met in the field, the Theis equation is no longer applicable to describe the drawdown distribution.

In various instances, the limitation of constant pumpage rate is not obtainable in the field. In some field installations pumping at a constant rate may not be possible due to the self adjustment of a constant-speed pump to the declining head in the well, in accordance with the head-discharge characteristics of the pump and the hydraulic properties of the aquifer. In determining the specific capacity of a production well, the pump rate often is varied in step fashion. Situations such as a sudden break-down of pumping equipment during the course of a pumping test and the resumption of pumping at the same or a different rate would result in a variable-rate pumping test.

The drawdowns observed under these conditions cannot be analyzed reliably by the common conventional methods of pumping-test analysis, because these methods presuppose a constant discharge rate during the course of the test.

Various investigators (Stallman, 1962; Abu-Zeid and Scott, 1963; Hantush, 1964; Abu-Zeid, Scott and Aron, 1964; Aron and Scott, 1965; and Sternberg, 1967, 1968) realized the limitation of conventional methods and outlined analytical and graphical methods for analysis of variable-rate pumping-test data. Stallman (1962) devised a graphical method, which is essentially based on the Theis equation. In this method, a continuously varying discharge rate is divided into a number of small segments, from which a family of type curves is constructed. A graphical fit of logarithmic drawdown and log 1/t to the family of type curves determines the aquifer parameters. Abu-Zeid and Scott (1963) studied nonsteady flow for wells with decreasing discharge in nonleaky confined aquifers; Hantush (1964) rigorously analyzed an exponentially varying discharge rate in leaky and nonleaky aquifers. Both the studies of Abu-Zeid and Hantush are based on the assumption that the discharge variation with time follows a certain mathematical relationship (exponential type). The results of these investigations led to the evaluation of variable-rate well functions. The graphical solution of aquifer parameters, proposed by Abu-Zeid and Scott (1963) and Hantush (1964), is based on empirically determined discharge parameters for an exponential discharge-time relationship. Abu-Zeid, Scott, and Aron (1964) presented modified solutions for decreasing-discharge wells and proposed adjustment of the observed drawdown by an adjustment factor that depends on the time of observation and on empirically determined discharge-variation parameters. The discharge-time relationship, as proposed by the authors, did not prove satisfactory and they suggested "approximate solutions" be sought for various time-discharge relationships. Aron and Scott (1965) proposed a graphical method, based on Jacob's approximation of the well function, W(u), that required the steepest decrease in discharge to occur at early time after pumping began. For accurate prediction of drawdowns in observation wells, where radial distances may be large, the method requires t, the time of observation, to be very large, so Jacob's approximation is valid. Sternberg (1967) gave a graphical technique for the determination of aquifer transmissivity, based on the record of recovery data generated from variable pump rates at an observation well after the pump shut down. Sternberg later (1968), using Jacob's approximation of the well function, presented a graphical method based on the principle of superposition.

All the above mentioned methods involve the visual comparison of pump-test data to theoretical type curves and involve laborious evaluations of certain expressions even where simplified graphical solutions are proposed. The purpose of this paper is to present a technique and an algorithm to automatically fit field-observed, variable-rate, pumping-test data to the uniform, horizontal, homogeneous, infinite confined aquifer equation. The method of convolution is used in handling the variable pumping rates, and sensitivity analysis is used in obtaining the "best" fit of aquifer parameters, in the least-squares sense. After the Theis equation has been evaluated for all the variable rates of pumping and the resulting drawdown obtained by convolution, the sensitivity coefficients can be obtained with little additional effort. The well function and sensitivity coefficients are calculated numerically for arbitrary space and time variables; thus, the Jacob approximation is not necessary in this work.

The automated analysis of pumping tests for nonleaky and leaky confined aquifers with constant pumpage has been published previously (McElwee, 1980a; Cobb, McElwee and Butt, 1982). For a more detailed discussion of sensitivity coefficients and their uses, see Yukler (1976) and McElwee and Yukler (1978). Sensitivity coefficients are used to predict the change in drawdown due to changes in aquifer parameters. The aquifer parameters are adjusted in a systematic way until convergence occurs. The root-mean-square error for the drawdown indicates the goodness of fit between pumping-test data and theoretical drawdown. Computation of the correlation coefficient, based on the "best" values of transmissivity and storage coefficient, also aids in evaluating the goodness of fit. A high correlation coefficient (low rms error) would indicate a good fit, whereas a low correlation coefficient (high rms error) would indicate a poor fit. Automated fitting procedures have been suggested before (Saleem, 1970; Labadie and Helwig, 1975; Holzchuh, 1976) but they have not used variable pumpages and sensitivity analysis.

THE THEIS EQUATION AND ITS NUMERICAL EVALUATION

Theis (1935), working on the concept of two-dimensional flow of water, described an analytical solution for horizontal, radial confined ground-water flow as

$$s(r,t) = \frac{Q}{4\pi T} \int_{\frac{r^2 S}{4Tt}}^{\infty} \frac{e^{-u}}{u}\, du \qquad (1)$$

Theoretically the above nonequilibrium equation applies rigorously only to aquifers which are of uniform saturated thickness, homogeneous and infinite in areal extent. Presumably, transmissivity is constant at all times and places, the well penetrates the entire aquifer, and the pumped well has an infinitesimal diameter. The derivation and solution is documented in many places and will not be discussed further here (Theis, 1935; Jacob, 1940). In

Eq. (1) s is drawdown [L] as a result of constant pumpage Q $[L^3/T]$, T is the transmissivity (hydraulic conductivity x saturated thickness) $[L^2/T]$, t is the time of observation [T], S is the dimensionless storage coefficient (volume release of water from storage per unit area per unit decline in head), and r is the radial distance from the pumped well [L].

The Theis equation in exponential integral form can be written as

$$s = \frac{Q}{4\pi T} W(u), \tag{2}$$

where $W(u) = \int_u^\infty \frac{e^{-x}}{x} dx$,

$u = r^2 S/4Tt$,

and x is a dummy variable of integration

Values of W(u) (sometimes referred to as the well function) in terms of the practical range of u are tabulated in many places (Abramowitz and Stegun, 1968). The numerical solution of the Theis equation in the range of $0 \leq u \leq 1$ and $1 \leq u \leq \infty$ is described by McElwee (1980b). The absolute error in the numerical approximation for the well function is small for all values of u and is not a real limitation.

CONVOLUTION USING THE THEIS EQUATION

For a linear time-invariant system, such as described by the Theis equation, the principle of superposition holds. This means that the system output response (drawdown) can be calculated for an arbitrary input (pumpage change) by convolution of the input with the impulse response function (Kanasewich, 1981).

The convolution integral has been actively used for ground-water flow problems in the past 15 years. Moench and Kisiel (1970) used the convolution equation in estimating recharge from an ephemeral stream using water-level fluctuations in an observation well. For nonleaky and leaky aquifers a solution for the drawdown using the convolution integral has been presented by Moench (1971) for an arbitrary discharge function. Hall and Moench (1972) applied the convolution equation to study the stream-aquifer relationship. Morel-Seytoux and Daly (1975) used the convolution equation in developing a discrete kernel generator for stream-aquifer studies whereas various other investigators (Maddock, 1972; Bhatalla and others, 1977) have used the convolution integral in its discrete form in the application of linear systems analysis of multiwell systems.

Mathematically, convolution can be expressed as

$$s(t) = \int_{o}^{t} \Delta Q(\tau)\ U(t-\tau)\ d\tau \tag{3}$$

where s(t) is the system response or drawdown at time t, τ is a dummy variable of integration, U is the system impulse response function, and ΔQ is the system input or change in pumpage. Many authors show the convolution integral with integration limits of negative and positive infinity. Those limits are not necessary here since $\Delta Q(\tau)$ and $U(t-\tau)$ will be defined as zero for negative arguments. In the proper use of convolution, the system must be linear so that the principle of superposition and proportionality are applicable. Also, stationarity is assumed to exist, which means that the aquifer model is time invariant.

The Theis equation predicts the drawdown in response to a step change in pumping rate (impulse) from zero to Q at t = 0 ($\Delta Q(\tau) = Q\delta(\tau)$, where $\delta(\tau)$ is the Dirac-delta function; Kanasewich, 1981), discharge remaining constant thereafter. From this argument, the system impulse response function (ΔQ=Q=1) is clearly given by

$$U(t) = \frac{1}{4\pi T} W\left(\frac{r^2 S}{4Tt}\right)$$

The integration in Eq. (3) is assumed to contribute nothing to the units, i.e., the product of ΔQ and U has units of drawdown. Therefore, the system response or drawdown at any time due to a series of pumpage changes or impulses (ΔQ) should be given by the convolution

$$s(r,t) = \int_0^t \Delta Q(\tau) \frac{1}{4\pi T} W\left[\frac{r^2 S}{4T(t-\tau)}\right] d\tau$$

Suppose the pumpage changes only at a finite number of discrete time points as shown in Figure 1. In this case, ΔQ can be represented by a series of delta functions

$$\Delta Q(\tau) = \sum_{i=1}^{n+1} \Delta Q_i \, \delta(\tau - t_{i-1}), \qquad \Delta Q_i = Q_i - Q_{i-1}$$

and the integral in the convolution equation can be replaced by a summation

$$s(r,t) = \sum_{i=1}^{k} \Delta Q_i \frac{1}{4\pi T} W\left[\frac{r^2 S}{4T(t-t_{i-1})}\right] \qquad t_{k-1} < t \leq t_k \tag{4}$$

If W is defined as zero for negative arguments, this equation simplifies somewhat

$$s(r,t) = \sum_{i=1}^{n+1} \Delta Q_i \frac{1}{4\pi T} W\left[\frac{r^2 S}{4T(t-t_{i-1})}\right] \tag{5}$$

Pumpage is assumed to stop at time t_n; however the drawdown may be calculated at any time by the above equation. Therefore, drawdown during the recovery period also may be calculated by equation (5).

As an example, consider a well pumping at a rate Q_1 from $t = 0$ to $t = t_1$ and then the rate Q_1 is changed to Q_2. The drawdown at every instant is given by

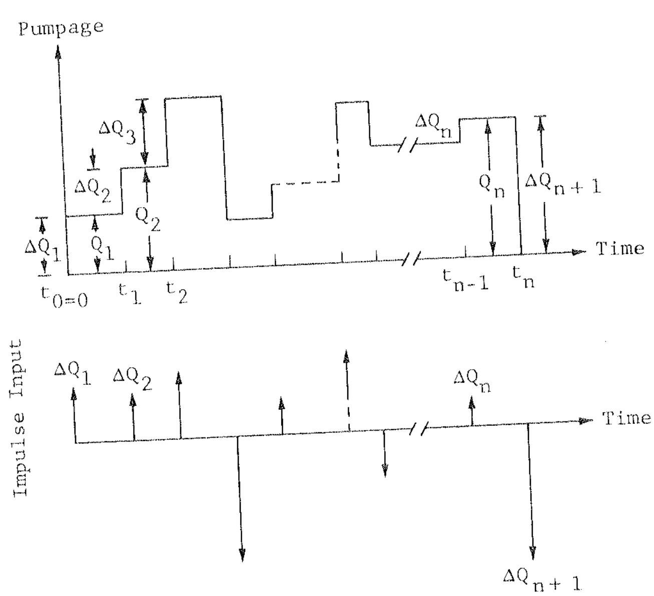


Figure 1. Varying pumping rates (impulse input).

$$s(r,t) = \frac{Q_1}{4\pi T} W(U_1) \quad ; \quad t \leq t_1 \tag{6}$$

and

$$s(r,t) = \frac{Q_1}{4\pi T} W(U_1) + \frac{(Q_2-Q_1)}{4\pi T} W(U_2) \quad ; \quad t > t_1 \tag{7}$$

where $Q_1 = \Delta Q_1$

$U_1 = \frac{r^2 S}{4Tt}$,

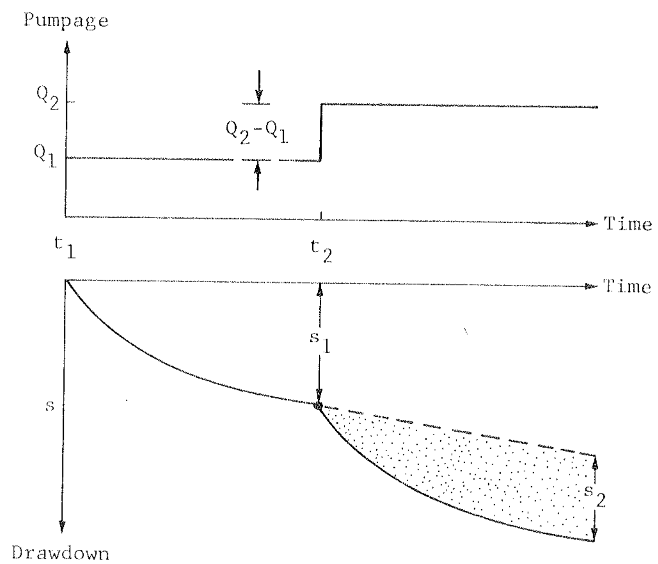
$Q_2 - Q_1 = \Delta Q_2$

and $U_2 = \frac{r^2 S}{4T(t-t_1)}$.

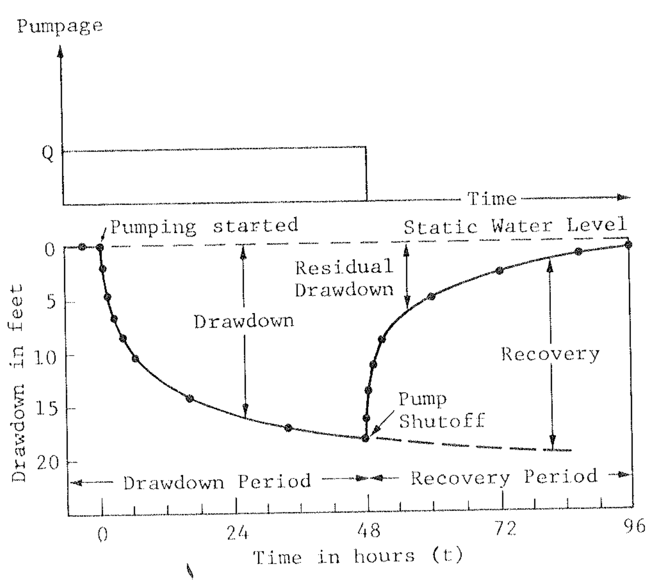
The above computation is equivalent to calculating the drawdown due to pumping at rate Q_1 beginning at $t = t_o$ from one well and adding to it the drawdown due to pumping at rate (Q_2-Q_1), beginning at $t = t_1$ from a second well at the same location (Figure 2a). The case of well shutdown and recovery is handled by considering $Q_2 = 0$ (Figure 2b). The drawdown measurements during the pumping period and residual drawdown (or recovery) measurements during the recovery period (after pump shutdown) provide two distinct sets of information from a single aquifer test. The theory of superposition treats the water-level changes during the recovery period as being the result of an imaginary recharge well in addition to the pumping well. The theoretical residual drawdown is computed by assuming the imaginary recharge well(s) injects water into the aquifer at the same rate and location as the discharging well(s). This gives a net zero withdrawal after pump shutdown.

When a well pumps Q_1 from $t = t_0 = 0$ to $t = t_1$, then Q_2 from t_1 to t_2 and in general Q_i from t_{i-1} to t_i $(i = 1,2,3,\ldots n)$ (Figure 1), then $\Delta Q_i = Q_i - Q_{i-1}$. From Eq. 4, the drawdown at any time t is given by

$$s(r,t) = \frac{1}{4\pi T} \sum_{i=1}^{k} (Q_i-Q_{i-1})\, W\left[\frac{r^2 S}{4T(t-t_{i-1})}\right] \quad ; \quad t_{k-1} < t \leq t_k \tag{8}$$



(a)



(b)

Figure 2. (a) Drawdown due to step change in pumping rate. (b) Case of well shutdown and recovery

An identical equation is derived by Bear (1979) by a slightly different approach.

Application of Eq. 5 to calculate the drawdown may seem complicated at first glance. However, it is very easy to do on a computer. The impulse response function, $W(U)/4\pi T$, can be calculated numerically by approximations mentioned earlier (McElwee, 1980b). The resulting drawdown is simply a sum of these impulse response functions multiplied by the appropriate ΔQ_i and time shifted to a new origin at t_{i-1}.

SENSITIVITY ANALYSIS

When treating ground-water flow mathematically, specifying physical parameters with high precision is possible. However, in real world situations, one has some uncertainty in these parameters. We must have some way to quantify the change in calculated head caused by a small change in the aquifer parameters. The theoretical basis of this technique is called sensitivity analysis and is outlined by Tomovic (1962), while the application to hydrologic problems has been examined by Vemuri and others (1969), McCuen (1973), Yukler (1976), McElwee and Yukler (1978), McElwee (1980a, 1980b), and Cobb, McElwee, and Butt (1982).

In formulating the sensitivity analysis of a confined aquifer, it is assumed that the solution of the flow equation may be written as

$$h = h(x,y,t;\ S,T,Q). \tag{9}$$

Perturbing the parameters S and T by small increments ΔS and ΔT produces a corresponding perturbed head, h^*. Assuming ΔT and ΔS are small enough such that higher powers can be ignored, a Taylor's series expansion gives

$$h^* = h + U_T \Delta T + U_S \Delta S \quad , \tag{10}$$

$$\text{and} \qquad U_T = \frac{\partial h}{\partial T} \quad , \qquad U_S = \frac{\partial h}{\partial S} \tag{11}$$

U_T and U_S are called sensitivity coefficients with respect to transmissivity and storage coefficient respectively. Applying Leibintz's rule for differentiating an integral (Hildebrand, 1962), McElwee (1980b) showed that the sensitivity coefficients U_T and U_S for constant pumpage rate Q are

$$U_T(r,t) = \frac{-s}{T} + \frac{Q}{4\pi T^2} \, \text{Exp}\left[\frac{-r^2 S}{4Tt}\right] \tag{12}$$

$$U_S(r,t) = -\left[\frac{Q}{4\pi TS}\right] \, \text{Exp}\left[\frac{-r^2 S}{4Tt}\right] \tag{13}$$

By analogy U_T and U_S for variable pumpage rate are obtained by

differentiating Eq. 4.

$$U_T(r,t) = \frac{\partial s(r,t)}{\partial T} = \frac{-s(r,t)}{T} + \sum_{i=1}^{k} \frac{\Delta Q_i}{4\pi T^2} \, \text{Exp}\left[\frac{-r^2 S}{4T(t-t_{i-1})}\right] \qquad t_{k-1} < t \leq t_k \tag{14}$$

$$U_S(r,t) = \frac{\partial s(r,t)}{\partial S} = -\sum_{i=1}^{k} \frac{\Delta Q_i}{4\pi TS} \, \text{Exp}\left[-\frac{r^2 S}{4T(t-t_{i-1})}\right] \qquad t_{k-1} < t \leq t_k \tag{15}$$

Coefficients U_T and U_S for variable rate Q_i may be easily evaluated by using standard exponential functions available on any computer. Values of U_T and U_S so obtained may be used in Eq. (10) to calculate the new head values caused by changes ΔT and ΔS. McElwee and Yukler (1978) indicate Eq. (10) is valid for ΔS and ΔT less than or roughly equal to 20% of S and T respectively.

LEAST-SQUARES FITTING PROCEDURE

The objective of any curve-fitting technique, whether visual or automated, is to obtain as close a fit as possible between the theoretical type curve and the experimental data set, thereby obtaining good values for the aquifer parameters. Manual curve fitting relies mainly on the best "eye ball" fit and the quality of the data plot, which is likely to vary from person to person. In the automated fitting technique no human judgement or bias of the data points is involved (unless variable weighting of data points is used). Sensitivity equations developed earlier are used to obtain a least squares fit of experimental test data to the appropriate equation. To apply this procedure, a squared error function shall be defined as:

$$\text{ERROR} = \sum_i \left[s_e(t_i) - s^*(t_i) \right]^2 \tag{16}$$

ERROR is the sum over all measurements of the squared difference between experimental drawdown s_e and the updated or new drawdown s^*. t_i is the discrete time at which an experimental measurement of the drawdown is made. The drawdown is defined as $s(t_i) = h(t_i) - h_o$; h_o represents the undisturbed or prepumping water level and $h(t_i)$ is the water level at time $t = t_i$.

The updated drawdown computed from a truncated Taylor's series after changing T and S by ΔT and ΔS is

$$s^* = s + U_T \Delta T + U_S \Delta S. \tag{17}$$

McElwee (1980b) has explicitly solved for ΔT and ΔS, which when used in eq. (16) minimizes ERROR. The updated values of T and S are given by

$$T_{i+1} = T_i + \Delta T_i$$

and

$$S_{i+1} = S_i + \Delta S_i$$

The updated values of T and S produce a new set of calculated drawdown values. These values are then employed in the least-squares procedure to obtain new values of ΔT and ΔS. This process continues until the ΔT and ΔS values simultaneously become so small as to be insignificant and satisfy a specified convergence criteria.

The goodness of fit obtained at the termination of the last iteration is indicated by the value of the root-mean-squared error, given by

$$\text{rms error} = \sqrt{\frac{\text{ERROR}}{N}}$$

where N is the total number of observations. rms error is a measure of the accuracy of fitting the measured drawdowns s_e to the theoretically generated drawdowns s^*. A low rms error indicates a good fit for the converged parameters T and S. Frequently, expressing the degree of statistical relationship by a descriptive measure, such as the correlation coefficient, is useful. The degree of relationship between the pumping-test data and the theoretically generated data would lie in the range 0 to 1. For a close relationship, the correlation coefficient would be near to 1; for poor or no correlation, it would be close to 0.

In order to realize good convergence properties, maintaining physical reality in the system, by preventing S and T from becoming negative at any iteration, is helpful. Due to the first-order sensitivity formalism, initial guess values considerably larger than the correct value may result in over-reaction and negative T or S values during that iteration. For this reason, T and S are not allowed to change by more than a certain amount in one iteration. Also, an additional requirement is that S cannot be greater than 1.

APPLICATION TO TYPICAL DATA

To check the accuracy of the proposed method, the well drawdown and discharge history synthesized by Aron and Scott (1965) was used and is presented in Table 1. Their model used a continuously varying Q, starting at 113.83 ft^3/min. Column 2 represents the average discharge during that period of pumping since the last time entry. Columns 3 and 4 show synthesized drawdowns at two piezometers 25 feet and 50 feet from the pumping well, at various times between 2 minutes and 4096 minutes. The synthesized data used a transmissivity (T) of 1 ft^2/min and a storage coefficient (S) of 0.001.

Table 1

Col 1 t (minutes)	Col 2 Q (ft^3/min)	Col 3 DRAWDOWN (ft) r = 25 ft	Col 4 DRAWDOWN (ft) r = 50 ft
2	106.47	17.12	7.49
4	94.73	20.78	11.29
8	85.97	23.46	14.60
16	77.82	25.41	17.27
32	71.11	27.04	19.51
64	65.88	28.62	21.58
128	61.94	30.06	23.46
256	58.60	31.58	25.30
512	55.88	33.10	27.09
1024	53.47	34.54	28.78
2048	51.06	36.16	30.59
4096	48.55	37.45	32.08

(T = 1 ft^2/min; S = 0.001)

Initial estimates of T and S for the automated analysis were chosen 2 to 3 orders of magnitude above and below the true value. Convergence occurred in 10-18 iterations. The convergence criterion required the change in transmissivity and storage since the previous iteration to be less than or equal to 0.01% of the previous values.

The best fit of aquifer parameters is shown in Table 2. Each data set for piezometer 1 and 2 was independently analyzed by the automated procedure. The automated rms error is only a few tenths of a foot and the correlation coefficient is close to 1, which indicates a good fit. The values for aquifer parameters determined by the automated procedure are compared with the true synthetic values for the evaluation of percent error. The percent error for parameters obtained graphically by Aron and Scott (1965) are much higher (from 5 to 13%) than those obtained by the automated fit (1-3.5%). Also, the data sets of piezometers 1 and 2 were analyzed simultaneously by the automated routine. The numerical agreement of the true synthetic values and automated fit values of S and T is very close, with the percent error being between 1.26 and 1.77. The "best fit" drawdowns for this case (1 and 2 analyzed together) are shown in Table 3. The root-mean-square error deviation in drawdown and correlation coefficient (between field-observed and theoretically generated drawdowns) varies from 0.188 to 0.231 ft and from .99968 to 0.99926 for the three cases respectively. The rms error is a measure of the magnitude of error at an "average" data point. Several other variable-rate pumping tests that were available also have been analyzed. Numerical convergence was rapid and aquifer parameter values as measured by rms and correlation coefficient indicated accurate determinations. However, the analysis can only be as good as the data. One cannot obtain satisfactory results from poor data or data that violate the assumptions inherent in the variable-rate Theis model.

Table 2

Well No.	Radial Distance r (ft)	Parameters Obtained by Graphical Fit T*	S	Parameters Obtained by Automated Fit T	S	Percent Error Graphical Fit T	S	Percent Error Automated Fit T	S	Automated rms Error (ft)	Correlation Coefficient
1	25	1.05	.00087	1.0100	.001036	5	-13	1.05	3.5	0.231	.99926
2	50	1.07	.00088	1.0128	.001006	7	-12	1.26	.65	0.188	.99968
1&2	25,50			1.0125	.001017			1.26	1.77	0.217	.99960

*T = ft^2/min

Table 3

Best Fit Time Drawdown Pairs
(Wells 1 and 2 analyzed simultaneously)

Time (minutes)	Drawdown (ft) Well No. 1	Drawdown (ft) Well No. 2
2	17.11	7.29
4	20.69	11.12
8	23.51	14.48
16	25.56	17.23
32	27.21	19.53
64	28.77	21.62
128	30.31	23.57
256	31.81	25.43
512	33.32	27.23
1024	34.76	28.94
2048	35.97	30.41
4096	36.85	31.56

PROGRAM THEIS.VARQ

Program THEIS.VARQ is written in user-friendly (time sharing) mode; a listing appears in Appendix I. All the variables and variable types (e.g., Integer, Real, Character, and Logical) are listed at the beginning of the program along with the units of measurement. Frequent comments are provided to help understand the program flow and its logic.

The program THEIS.VARQ is written in FORTRAN 77 and was used on a Data General MV8000 series computer. In general, the algorithm is not in machine-dependent code. (In order to make program THEIS.VARQ compatible and adaptable to FORTRAN IV compilers or to other foreign machines, the only statements that may need to be readjusted are PARAMETER, OPEN, and free format (read and write) statements). There are some preassigned parameter statements (i.e., array dimensions) in the algorithm that the user may need to redefine from time to time. Following is a list of the array sizes that may need to be re-dimensioned, provided the preset dimension size is insufficient.

NDM	(number of drawdown, recovery, or drawdown and recovery measurements) set presently to 200;
NOB	(number of observation wells) set currently equal to 8; and
NVQ	(number of variable pumpages) set currently to 30 in the program.

Should any of the above listed array sizes be lower than needed by the user, the appropriate array would need to be redimensioned accordingly. The program is designed to check internally for each of these array sizes and if insufficient array size is found, an appropriate message is written onto the user's terminal and program execution stops.

There are some preassigned parameter values (NITER, TOL) in the algorithm which the user may not have any need to change in normal usage. The maximum number of iterations (NITER) has been set to 25, since our experience from the experiments that we performed shows that all the data sets tested converge in fewer than 25 iterations. However, if convergence does not occur in 25 iterations, this statement may be changed and the number increased. The value TOL (abbreviation for tolerance) is used to check for convergence. Making it larger reduces the number of iterations while decreasing the accuracy of the fit. Reducing its value has the opposite effect. In the program it has been assigned to at 0.001 (i.e., 0.1 percent).

In the proper usage and execution of the THEIS.VARQ program, the user should be aware of the preassigned value of parameter NCONV. NCONV is an on-off switch which allows the user to make use of the program for two distinctly different purposes. Setting NCONV=1 (i.e., switch is on) lets the user obtain the best fit of aquifer parameters for field observed drawdown, recovery, or drawdown and recovery data. However, in instances where the aquifer parameters (i.e., transmissivity and storage coefficient) are known, setting NCONV=0 (i.e., switch is off) would allow the user to compute the drawdown, recovery, or drawdown and recovery at selected time intervals for known variable (or constant) pumpages.

The THEIS.VARQ algorithm is flexible and is designed to analyse drawdown and recovery data either separately or conjunctively, depending upon the choice of the user. Both constant or variable pumpage can be handled by the program. Thus a set of drawdown and/or recovery data from an observation well(s), as a result of variable (or constant) pumpage, could be analysed in the following three different ways:

1. DRAWDOWN data analysis alone; or

2. RECOVERY data analysis alone; or

3. DRAWDOWN AND RECOVERY data analysis conjunctively.

Data input to the program is straight forward and flexible. All the data entries are in FREE format. Most of the user-oriented responses are provided with some hints. These hints are helpful, particularly for the first-time user, since these reduce the user response time.

Listings 1, 2, and 3 are printouts of typical runs. Listing 1 refers to conjunctive data analysis of DRAWDOWN AND RECOVERY (i.e., DRAW-REC), listing 2 refers to RECOVERY data analysis, whereas listing 3 refers to analysis of DRAWDOWN data. The user's response is underlined on each printout.

The program execution on the MV8000 Data General computer initiates a series of questions after the XEQ THEIS.VARQ command is typed and a carriage return given. Each response is required to end with a carriage return which is done by pressing the RETURN key.

The first inquiry asks about the input-output printing option. This option is especially useful to CRT (screen) terminal users who may like to retain a hard copy of the entire session of user input and program output. An affirmative response (i.e., 'yes') activates the next inquiry about the filename (i.e., the filename where the entire session is going to be recorded). The user enters filename, a maximum of eight characters. Users of this option should make sure that file 'filename' has been created prior to the start of the program. (Some computer installations may not require pre-existing 'filename' files.)

The next question the program asks is - if the user wants to provide a TITLE for the job. This is helpful in job identification, particularly if a series of computer runs are done consecutively. An appropriate response to this inquiry is either 'YES' or 'NO' which is typed in capital letters (i.e., upper case) only. Any response other than 'YES' or 'NO' would not be accepted by the program. An affirmative response from the user would be followed by the job title, whatever the user may wish to assign. The user may enter a maximum of 100 characters (inclusive blanks).

The next inquiry is about the system of units. The user may use the gallon-day-foot system or any consistent set of units. A 'YES' response means the gallon-day-foot system, whereas a 'NO' response means any other consistent system of units.

The program lists the acceptable units for each parameter as the user is asked to enter the value. L is any arbitrary length unit and T is any arbitrary time unit. The consistent units of some important variables used in the program are:

Transmissivity ($L^3/T/L$) or ($^2/T$)

Pumpage (L^3/T)

Drawdown (or Recovery), Distance, rms Error (L)

Time (T)

(Storage coefficient is a positive dimensionless fraction.)

Next, the program asks the user about the type of data to be analysed. Typing 'DRAWDOWN' or 'RECOVERY' indicates that the data are to be analysed for either drawdown or recovery only, whereas typing 'DRAW-REC' indicates conjunctive data analysis of drawdown and recovery data. Any other response (such as misspellings, etc.) will not be accepted by the program and an appropriate message will be written on the user's terminal until a correct response is given.

The remaining prompting statements query the user for values of the storage coefficient, transmissivity, number of pumping wells, number of variable pumping rates of each well, time for which each pumping rate is operative, total number of observation wells, radial distance of each observation well with respect to pumping well(s), and number of time-drawdown (or recovery or drawdown and recovery) pairs.

Initial guess estimates of the storage coefficient and transmissivity within two to three orders of magnitude above or below the true value usually converge. (If no prior geohydrologic information about the aquifer is available, a guess value of 0.01 to 0.001 for the storage coefficient and a transmissivity of 1 ft^2/min to 10 ft^2/min are good starting median values. However, these values are based purely on experience and should be taken as rule-of-thumb only.) For a constant pumpage rate, the number of variable pumpages is entered as 1.

After entering all the above mentioned responses, the program checks to see if the preassigned dimensions of NDM, NOB, and NVQ are sufficient. If array sizes are found to be sufficient, all the previous user responses are echo printed and the user is given an opportunity to examine all the responses for correct entry. If any error is found, the user may restart the program by answering in the affirmative ('YES'). At this stage all the steps, as explained up to now, are repeated once again.

If the user's response is 'NO' (if no error is found) then the algorithm asks for time-drawdown (or recovery or drawdown and recovery) pairs. Only NDM pairs may be entered. After NDM time-data pairs are entered, all pairs are echo printed. The program lets the user examine for possible errors. The correction may be accomplished by entering the sequential number (I) and observation well number (J) of the erroneous time-drawdown (or recovery or drawdown and recovery) pair and the correct values. After entering a correction, the program continues to ask for more corrections. As long as affirmative responses are given by the user, the program will continue to ask for data corrections. As soon as a negative response is given, the program proceeds to execution.

At this point, starting with the initial extimates of the aquifer parameters, the program begins an iteration loop to find the 'best fit' aquifer parameters. For each iteration, the 'best fit' aquifer parameters are printed at the user's terminal. The value of TOL checks for convergence of the parameters. If convergence has not occurred, T and S are updated by the quantities WT and WS. At some iteration of the fitting algorithm if the convergence criteria set by TOL is met, the final convergence status is announced. The successful 'best fit' time-drawdown (or recovery or drawdown-recovery) pairs are printed at the user's terminal along with the converged values of transmissivity and storage coefficient. The algorithm then prints the rms (root mean square) error and finally, as a statistical measure of the fit, prints the correlation coefficient between the field (input) data and theoretical (computer generated) data.

Instances where the initial guess values of the storage coefficient and transmissivity are too far off, may result in non-convergence with very small or zero sensitivity coefficients. Should such a situation occur, the program prints the sensitivity coefficient values, indicating thereby that convergence has not occurred because the initial estimates of the aquifer parameters are too far off or the data is too poor. At this point, the program lets the user decide if other guess values are to be tried. An affirmative response requires new values for the aquifer parameters, and the program proceeds immediately to execution. Should the user decide not to try new guess values, the negative response stops the program execution.

In situations where 'DRAWDOWN' data are analysed, the algorithm asks the user if specific capacity of the production well needs to be computed. The answer to this query should be affirmative only when:

(a) the observed drawdown is from the pumped well itself, and

(b) the number of observation wells is one and it is located at a radial distance equal to the radius of the pumping well.

If all the above mentioned conditions are met, answering in the affirmative would cause computation of the specific capacity of the production well. (No corrections for well losses due to screens, friction, velocity, etc. have been applied in the specific capacity computations.) If the answer to the specific capacity query is negative, the next question that the program asks is regarding constant drawdown conditions. If the user's data is from a flowing well (where discharge may be variable and drawdown constant), then an affirmative response will allow the user to analyse this type of data.

When recovery data are being analysed, there may be situations where a storage coefficient value can not be computed. (This situation generally occurs when the radial distance of observation is too small and/or the time of observation is too large. As a result, the argument of the exponential function approaches zero.) When this occurs, an appropriate message is printed at the user's terminal, and only the value of transmissivity is printed.

SUBROUTINE THEISVQ

Appendix I contains a listing of SUBROUTINE THEISVQ. It is a FORTRAN 77 subroutine and is called by the mainline program. To employ subroutine THEISVQ the arguments (SC, KB, Q, TT, R, SE, DSDTE, DSDSCE, UNIT, WOUT) must be assigned a value and/or character designation prior to calling THEISVQ. UNIT must be declared a character type variable in the mainline program. If the user wishes to use the gallon-day-foot system, UNIT must be 'YES' in the calling mainline. SC is the storage coefficient (dimensionless); KB is the transmissivity (gal/day/ft or L^2/T); Q is the variable (or constant) pumpage (gallon/day or L^3/T); TT is the elapsed time of pumpage (day or T); R is the radial distance of observation (ft or L); SE is the theoretical drawdown computed in the subroutine THEISVQ. (ft or L) for the current aquifer parameters; DSDTE and DSDSCE are the sensitivity coefficients with respect to transmissivity and storage coefficient, respectively. These sensitivity coefficients are assigned values in the subroutine before returning to the mainline; WOUT is the unit number for writing the results to the output file.

DISCUSSION AND SUMMARY

The automated fit for variable rate pumping-test data using the method of convolution and sensitivity analysis should prove quite useful. The alternative is a laborious and time-consuming visual curve-fitting technique. The algorithm presented in this paper is quite simple. No empirical relations for an assumed exponentially decreasing or increasing discharge rate are needed. The variable discharge rate can be of any form: continuously increasing or decreasing, combination of increasing and decreasing, or intermittent. One simply needs to select a set of discrete pumpages to adequately describe the given pumpage. The automated fit technique is quick and inexpensive, and does not involve any graphical plot or graphical interpretation for obtaining T and S. The algorithm can handle both time-drawdown and distance-drawdown data. Generally, very little computer time is used. The same "best" fit values for good data usually are achieved when initial estimates of T and S range from 2 to 3 orders of magnitude above or below the converged values. As a measure of error, the rms error in drawdown is calculated for the 'best' transmissivity and storage. The rms error in drawdown should be only a few tenths of a foot for good data that satisfies the model assumptions. Subsequently, the correlation coefficient for the 'best' fit in this case should be close to 1.

Realizing that convergence may occur in certain situations with high rms error and a low correlation coefficients is important. This indicates the importance of the hydrologist in identifying the aquifer type or hydrologic situation which may or may not be represented by the variable-pumpage Theis model. The user cannot blindly select a data set, apply a fitting technique, and obtain the aquifer parameters.

The procedure set forth in this paper has been tested on hypothetically generated data and real field data with excellent results. The program listing and user manual may be obtained by writing to the authors (Butt and McElwee, 1984). A similar automated fitting routine could be easily developed for variable-rate pumping-test analysis for leaky artesian aquifers. The methods of convolution and sensitivity analysis outlined here are rather general and could be applied to many more situations in geohydrology.

REFERENCES

Abramowitz, M. and I.A. Stegun. 1968. Handbook of Mathematical Functions. Dover Publications, Inc., New York. p. 231.

Abu Zeid, M.A. and V. H. Scott. 1963. Nonsteady flow for wells with decreasing discharge. Proc. of the American Society of Civil Engineers. v. 89, no. HY3, p. 119-132.

Abu Zeid, M. A., V. H. Scott, and G. Aron. 1964. Modified solutions for decreasing discharge wells. Proc. of the American Society of Civil Engineers. v. 90, no. HY4, p. 145-160.

Aron, G. and V. H. Scott. 1965. Simplified solution for decreasing flow in wells. Proc. of the American Society of Civil Engineers. v. 91, no. HY5, p. 1-11.

Bathala, C. T., Ramachandra Rao, and J. A. Spooner. 1977. Application of linear systems analysis to groundwater evaluation studies. Purdue University Water Resources Research Center Technical Report 91, 128 p.

Bear, J. 1979. Hydraulics of Groundwater. McGraw-Hill, Inc. 324 p.

Butt, M. A. and C. D. McElwee. 1984. Variable-rate pumping test: an automated numerical evaluation. Kansas Geological Survey Open-File Report No. 84-5.

Cobb, P. M., C. D. McElwee, and M. A. Butt. 1982. Analysis of leaky aquifer pumping-test data: an automated numerical solution using sensitivity analysis. Ground Water. v. 20, no. 3, p. 325-333.

Hall, F. R., and A. F. Moench. 1972. Application of the convolution equation to stream-aquifer relationships. Water Resources Research, v. 8, no. 2, p. 487-493.

Hantush, M. S. 1964. Drawdown around wells of variable discharge. Journal of Geophysical Research. v. 69, no. 20, p. 4221-4235.

Hildebrand, F. B. 1962. Advanced calculus for applications. Englewood Cliffs, Prentice-Hall. 360 p.

Holzschuh, J. C. 1976. A simple computer program for the determination of aquifer characteristics from pump test data. Ground Water, v. 14, no. 5, p. 283-285.

Jacob, C. E. 1940. The flow of water in elastic artesian aquifer. Transactions of the American Geophysical Union. v. 21, p. 574-586.

Kanasewich, E. R. 1981. Time sequence analysis in geophysics. University of Alberta Press, Edmonton, Alberta, Canada, p. 76.

Labadie, J. W. and O. J. Helweg. 1975. Step-drawdown test analysis by computer. Ground Water. v. 13, no. 5, p. 438-444.

Maddock, T., III. 1972. Algebraic technological function from a simulated model. Water Resources Research. v. 8, no. 1, p. 129-134.

McCuen, R. H. 1973. Component sensitivity: a tool for the analysis of complex water resource systems. Water Resources Research. v. 9, no. 1, p. 243-246.

McElwee, C. D. 1980a. Theis parameter evaluation from pumping tests by sensitivity analysis. Ground Water. v. 18, no. 1, p. 56-60.

McElwee, C. D. 1980b. The Theis equation: evaluation, sensitivity to storage and transmissivity, and automated fit of pump test data. Kansas Geological Survey, Ground-water Series 3, 39 p.

McElwee, C. D. and M. A. Yukler. 1978. Sensitivity of ground-water models with respect to variations in transmissivity and storage. Water Resources Research. v. 14, no. 13, p. 451.

Moench, A. F. 1971. Ground-water fluctuations in response to arbitrary pumpage. Ground Water. v. 9, no. 2, p. 4-8.

Moench, A. F. and C. C. Kiesel. 1970. Applications of the convolution relation to estimating recharge from an ephemeral stream. Water Resources Research. v. 6, no. 4, p. 1087-1094.

Morel-Seytoux, H. R. and C. J. Daly. 1975. A discrete kernel generator for stream-aquifer studies. Water Resources Research. v. 11, no. 2, p. 253-260.

Saleem, Z. A. 1970. A computer method for pumping-test analysis. Ground Water. v. 8, no. 5, p. 21-24.

Stallman, R. W. 1962. Variable discharge without vertical leakage: Theory of Aquifer Tests. U. S. Geological Survey, Water-Supply Paper 1536-E, p. 119-122.

Sternberg, Y. M. 1967. Transmissibility determination from variable discharge pumping tests. Ground Water. v. 5, no. 4, p. 27-29.

Sternberg, Y. M. 1968. Simplified solution for variable-rate pumping test. Proc. of the American Society of Civil Engineers. v. 94, no. HY1, p. 177-180.

Theis, C. V. 1935. The relation between the lowering of the piezometric surface and the rate and duration of a well using ground-water storage. Transactions of the American Geophysical Union. v. 16, p. 519-524.

Tomovic, R. 1962. Sensitivity analysis of dynamic systems. McGraw-Hill. New York. 141 p.

Vemuri, V., J. A. Dracup, R. C. Erdmann, and N. Vemuri. 1969. Sensitivity analysis method of system identification and its poteontial in hydrologic research. Water Resources Research. v. 5, no. 2, p. 341-349.

Yukler, M. A. 1976. Analysis of error in ground-water modelling. Ph.D. dissertation, University of Kansas. 182 p.

ACKNOWLEDGMENT

The authors wish to thank various members of the Kansas Geological Survey and Don Jorgensen of the U.S. Geological Survey (Lawrence, Kansas) for their suggestions and review of the manuscript.

LISTING 1. DRAWDOWN-RECOVERY RUN

```
DO YOU WANT TO HAVE A HARD COPY OF INPUT-OUTPUT?
( Answer YES or NO ) =  YES


INPUT FILE NAME (UP TO 8 CHARACTERS) IN WHICH THE
OUTPUT IS TO BE ROUTED. MAKE SURE THE FILE EXISTS
Input file name = BBJ3.OUT


DO YOU WANT TO PROVIDE A TITLE FOR THIS JOB?
( Answer YES or NO )  = YES

INPUT JOB TITLE -UP TO 100 CHARACTERS-
=DRAW-REC RUN (SOURCE: COOPER AND JACOB, TRANS. AMER. GEOPHYS UNION V.27 PP 526-534, 1946


DO YOU WANT TO USE GAL, DAY, FT SYSTEM ?
(Answer YES or NO )  =   NO


DO YOU WANT TO ANALYZE DRAWDOWN OR RECOVERY OR
DRAWDOWN AND RECOVERY DATA ?
( Enter "DRAWDOWN" or "RECOVERY" or "DRAW-REC" )  =   DRAW-REC


ESTIMATE FOR STORAGE COEFFICIENT ?
( Enter a reasonable positive fraction )  =   0.01


ESTIMATE FOR TRANSMISSIVITY ? GAL/DAY/FT OR L**2/T
(Enter a reasonable positive value )  =   100


NUMBER OF PUMPING WELLS ?  =   3

     STARTING TIME OF PUMP WELL #  1  ? DAY OR T = 0.0

     STARTING TIME OF PUMP WELL #  2  ? DAY OR T = 30

     STARTING TIME OF PUMP WELL #  3  ? DAY OR T = 60

     NO. OF VARIABLE PUMPING RATES OF PUMP WELL #   1 ?
     (Enter 1 if Constant Pumping Rate)            =    1

     NO. OF VARIABLE PUMPING RATES OF PUMP WELL #   2 ?
     (Enter 1 if Constant Pumping Rate)            =    1

     NO. OF VARIABLE PUMPING RATES OF PUMP WELL #   3 ?
     (Enter 1 if Constant Pumping Rate)            =    1
    THE CONSTANT PUMPING RATE OF PUMP WELL #   1  IS  =   136.2

    TIME FOR WHICH PUMPING RATE Q( 1, 1) IS OPERATIVE (DAY OR T) = 525

    THE CONSTANT PUMPING RATE OF PUMP WELL #   2  IS  =   167.4

    TIME FOR WHICH PUMPING RATE Q( 2, 1) IS OPERATIVE (DAY OR T) = 605

    THE CONSTANT PUMPING RATE OF PUMP WELL #   3  IS  =   213.6

    TIME FOR WHICH PUMPING RATE Q( 3, 1) IS OPERATIVE (DAY OR T) = 695
```

```
TOTAL NUMBER OF OBSERVATION WELLS ?          =    1

    RADIAL DISTANCE BETWEEN PUMP WELL #  1 AND OBSERVATION WELL #  1 IS =   850
    RADIAL DISTANCE BETWEEN PUMP WELL #  2 AND OBSERVATION WELL #  1 IS =   780
    RADIAL DISTANCE BETWEEN PUMP WELL #  3 AND OBSERVATION WELL #  1 IS =   1060


NUMBER OF TIME-DRAW-REC PAIRS TO BE  READ ?   =   13


ECHO THE INITIAL DATA
SC =      .1000000E-01     KB =  .1000000E+03
THESE ARE THE INITIAL GUESSES FOR STORAGE COEFFICIENT AND TRANSMISSIVITY


CONSTANT PUMPAGE RATE OF P.W. #  1 IS  =      136.2000000
PUMPING TIME                           =      525.0000000

CONSTANT PUMPAGE RATE OF P.W. #  2 IS  =      167.4000000
PUMPING TIME                           =      605.0000000

CONSTANT PUMPAGE RATE OF P.W. #  3 IS  =      213.6000000
PUMPING TIME                           =      695.0000000

** RADIAL DISTANCE(S) OF OBSERVATION WELL(S) IS/ARE :


RADIAL DISTANCE FROM P.W. #  1 TO O.W. #  1 IS  =      850.0000000


RADIAL DISTANCE FROM P.W. #  2 TO O.W. #  1 IS  =      780.0000000


RADIAL DISTANCE FROM P.W. #  3 TO O.W. #  1 IS  =     1060.0000000


NUMBER OF TIME-DRAW-REC PAIRS ARE =    13
 TSTOP( 1) =        525.0000
 TSTOP( 2) =        605.0000
 TSTOP( 3) =        695.0000

TIME AT WHICH LAST RECOVERY IS MEASURED ?  DAY OR T
(Enter the LAST TIME of RECOVERY measurement)  =   695


ARE THERE ANY ERRORS IN THE ABOVE ENTRIES?
(Answer YES or NO )  =   NO

TYPE IN TIME-DRAW-REC PAIRS IN ORDER OF INCREASING TIME
(Enter TIME and DRAW-REC in free format )
 =   30  0.96

 =   60  3.20

 =   120  6.21

 =   180  7.77

 =   240  8.76

 =   305  9.50

 =   365  10.00

 =   425  10.37

 =   488  10.67

 =   525  10.84

 =   605  9.45

 =   660  7.16

 =   695  6.51
```

*** PUMP TEST DATA IN TIME DRAW-REC PAIRS ***

TIME	DRAW-REC
30.0000000	.9600
60.0000000	3.2000
120.0000000	6.2100
180.0000000	7.7700
240.0000000	8.7600
305.0000000	9.5000
365.0000000	10.0000
425.0000000	10.3700
488.0000000	10.6700
525.0000000	10.8400
605.0000000	9.4500
660.0000000	7.1600
695.0000000	6.5100

ARE THERE ANY ERRORS IN TIME - DRAW-REC PAIRS?
(Answer YES or NO) = NO

```
BEST FIT KB AND SC THIS ITERATION ARE      5.00000119     .000500000
BEST FIT KB AND SC THIS ITERATION ARE      8.72974982     .000537777
BEST FIT KB AND SC THIS ITERATION ARE     13.15962938     .000390191
BEST FIT KB AND SC THIS ITERATION ARE     16.06266945     .000288783
BEST FIT KB AND SC THIS ITERATION ARE     16.74136867     .000277557
BEST FIT KB AND SC THIS ITERATION ARE     16.76704360     .000277270
BEST FIT KB AND SC THIS ITERATION ARE     16.76699151     .000277274
```

DRAW-REC RUN (SOURCE: COOPER AND JACOB, TRANS. AMER. GEOPHYS UNION V.27 PP 526-534, 19

*** THE BEST FIT TIME_DRAW-REC PAIRS ARE ***

30.0000000	1.1809
60.0000000	3.1740
120.0000000	6.5215
180.0000000	7.8399
240.0000000	8.6849
305.0000000	9.3558
365.0000000	9.8447
425.0000000	10.2519
488.0000000	10.6171
525.0000000	10.8086
605.0000000	9.4017
660.0000000	7.2697
695.0000000	6.8793

****

```
"BEST" FIT TRANSMISSIVITY (L**2/T) IS  =      16.76699151
"BEST" FIT  STORAGE COEFFICIENT  IS    =        .000277274

 THE rms ERROR  FOR DRAW-REC (L) IS    =        .16881
 THE CORRELATION COEFFICIENT IS        =        .99887
```

****

```
DO YOU WANT TO TRY OTHER GUESS VALUES FOR SC AND KB. ?
(Answer YES or NO )  =   YES

NEW ESTIMATE FOR SC ?  =    .035

NEW ESTIMATE FOR KB ?  =    .1

SENSITIVITY COEFFICIENTS TOO SMALL (or zero)
THE SENSITIVITY COEFFICIENTS ARE :

SSUS   =         .4248111812E-62
SSUT   =         .5076297971E-63
SUTUS  =        -.1468491790E-62


DO YOU WANT TO TRY OTHER GUESS VALUES FOR SC AND KB. ?
(Answer YES or NO )  =   NO

STOP
)
```

LISTING 2. RECOVERY RUN

```
DO YOU WANT TO HAVE A HARD COPY OF INPUT-OUTPUT?
( Answer YES or NO ) =  YES


INPUT FILE NAME (UP TO 8 CHARACTERS) IN WHICH THE
OUTPUT IS TO BE ROUTED. MAKE SURE THE FILE EXISTS
Input file name = THEISOUT


DO YOU WANT TO PROVIDE A TITLE FOR THIS JOB?
( Answer YES or NO )  = YES

INPUT JOB TITLE -UP TO 100 CHARACTERS-
=RECOVERY RUN (SOURCE:"GROUNDWATER HYDRAULICS", MCWHORTER & SUNANDA, FORT COLLINS)


DO YOU WANT TO USE GAL, DAY, FT SYSTEM ?
(Answer YES or NO )  =   NO


DO YOU WANT TO ANALYZE DRAWDOWN OR RECOVERY OR
DRAWDOWN AND RECOVERY DATA ?
( Enter "DRAWDOWN" or "RECOVERY" or "DRAW-REC" )  =   RECOVERY


ESTIMATE FOR STORAGE COEFFICIENT ?
( Enter a reasonable positive fraction )  =   0.001


ESTIMATE FOR TRANSMISSIVITY ? GAL/DAY/FT OR L**2/T
(Enter a reasonable positive value )  =   1.0


NUMBER OF PUMPING WELLS ?  =   1

     NO. OF VARIABLE PUMPING RATES OF PUMP WELL #   1 ?
     (Enter 1 if Constant Pumping Rate)           =    1

    THE CONSTANT PUMPING RATE OF PUMP WELL #   1  IS  =   1.79

    TIME FOR WHICH PUMPING RATE Q( 1, 1) IS OPERATIVE (DAY OR T) = 443



TOTAL NUMBER OF OBSERVATION WELLS ?          =   1

    RADIAL DISTANCE BETWEEN PUMP WELL #  1 AND OBSERVATION WELL #  1 IS =   4.6


NUMBER OF TIME-RECOVERY PAIRS TO BE  READ ?  =   18


ECHO THE INITIAL DATA
SC =     .1000000E-02    KB =  .1000000E+01
THESE ARE THE INITIAL GUESSES FOR STORAGE COEFFICIENT AND TRANSMISSIVITY


CONSTANT PUMPAGE RATE OF P.W. #  1 IS  =       1.7900000
PUMPING TIME                           =     443.0000000

** RADIAL DISTANCE(S) OF OBSERVATION WELL(S) IS/ARE :


RADIAL DISTANCE FROM P.W. #  1 TO O.W. #  1 IS  =        4.6000000
```

```
NUMBER OF TIME-RECOVERY PAIRS ARE =     18
 TSTOP( 1) =        443.0000

TIME AT WHICH LAST RECOVERY IS MEASURED ?  DAY OR T
(Enter the LAST TIME of RECOVERY measurement)  =   443

ARE THERE ANY ERRORS IN THE ABOVE ENTRIES?
(Answer YES or NO )  =   NO

TYPE IN TIME-RECOVERY PAIRS IN ORDER OF INCREASING TIME
(Enter TIME and RECOVERY in free format )
 =   .5,1.64

 =   1,1.595

 =   1.5,1.535

 =   2,1.49

 =   2.5,1.445

 =   3,1.4

 =   4,1.305

 =   4.5,1.235

 =   5.5,1.2

 =   8,,1.06

 =   12,.93

 =   16,.845

 =   21,.755

 =   26,.7

 =   36,.59

 =   46,.521

 =   56,.451

 =   71,.384


***  PUMP TEST DATA IN TIME RECOVERY PAIRS  ***


       TIME            RECOVERY

     .5000000           1.6400
    1.0000000           1.5950
    1.5000000           1.5350
    2.0000000           1.4900
    2.5000000           1.4450
    3.0000000           1.4000
    4.0000000           1.3050
    4.5000000           1.2350
    5.5000000           1.2000
    8.0000000           1.0600
   12.0000000            .9300
   16.0000000            .8450
   21.0000000            .7550
   26.0000000            .7000
   36.0000000            .5900
   46.0000000            .5210
   56.0000000            .4510
   71.0000000            .3840
```

```
ARE THERE ANY ERRORS IN TIME - RECOVERY PAIRS?
 (Answer YES or NO )  =   NO

BEST FIT KB AND SC THIS ITERATION ARE       .16334962      .003000000
BEST FIT KB AND SC THIS ITERATION ARE       .27706219      .005529340
BEST FIT KB AND SC THIS ITERATION ARE       .41134512      .009217288
BEST FIT KB AND SC THIS ITERATION ARE       .50803974      .012710490
BEST FIT KB AND SC THIS ITERATION ARE       .53623016      .013960627
BEST FIT KB AND SC THIS ITERATION ARE       .53793383      .013963732
BEST FIT KB AND SC THIS ITERATION ARE       .53793523      .013971632
BEST FIT KB AND SC THIS ITERATION ARE       .53793600      .013970105
BEST FIT KB AND SC THIS ITERATION ARE       .53793585      .013970406

  RECOVERY RUN (SOURCE:'GROUNDWATER HYDRAULICS', MCHORTER & SUNANDA, FORT COLLINS)



*** THE  BEST  FIT   TIME_RECOVERY  PAIRS  ARE  ***

      .5000000          1.7294
     1.0000000          1.5791
     1.5000000          1.4835
     2.0000000          1.4134
     2.5000000          1.3581
     3.0000000          1.3125
     4.0000000          1.2399
     4.5000000          1.2100
     5.5000000          1.1589
     8.0000000          1.0632
    12.0000000           .9597
    16.0000000           .8866
    21.0000000           .8180
    26.0000000           .7646
    36.0000000           .6844
    46.0000000           .6252
    56.0000000           .5786
    71.0000000           .5237
**********************************************************

'BEST' FIT TRANSMISSIVITY (L**2/T) IS  =         .53793585
'BEST' FIT  STORAGE COEFFICIENT  IS    =         .013970406

 THE rms ERROR  FOR RECOVERY (L) IS    =         .07632
 THE CORRELATION COEFFICIENT IS        =         .99154

**********************************************************

DO YOU WANT TO TRY OTHER GUESS VALUES FOR SC AND KB. ?
(Answer YES or NO )  =   YES

NEW ESTIMATE FOR SC ?  =   .1

NEW ESTIMATE FOR KB ?  =   100

BEST FIT KB AND SC THIS ITERATION ARE      5.00000119      .300000000
BEST FIT KB AND SC THIS ITERATION ARE       .25000012      .015000004
BEST FIT KB AND SC THIS ITERATION ARE       .38431593      .017828576
BEST FIT KB AND SC THIS ITERATION ARE       .49471271      .015816719
BEST FIT KB AND SC THIS ITERATION ARE       .53474394      .013768584
BEST FIT KB AND SC THIS ITERATION ARE       .53790541      .013991317
BEST FIT KB AND SC THIS ITERATION ARE       .53793800      .013966130
BEST FIT KB AND SC THIS ITERATION ARE       .53793545      .013971188
BEST FIT KB AND SC THIS ITERATION ARE       .53793595      .013970192
```

RECOVERY RUN (SOURCE:'GROUNDWATER HYDRAULICS', MCHORTER & SUNANDA, FORT COLLINS)

*** THE BEST FIT TIME_RECOVERY PAIRS ARE ***

.5000000	1.7294
1.0000000	1.5791
1.5000000	1.4835
2.0000000	1.4134
2.5000000	1.3581
3.0000000	1.3125
4.0000000	1.2399
4.5000000	1.2100
5.5000000	1.1589
8.0000000	1.0632
12.0000000	.9597
16.0000000	.8866
21.0000000	.8180
26.0000000	.7646
36.0000000	.6844
46.0000000	.6252
56.0000000	.5786
71.0000000	.5237

****

'BEST' FIT TRANSMISSIVITY (L**2/T) IS = .53793595
'BEST' FIT STORAGE COEFFICIENT IS = .013970192

THE rms ERROR FOR RECOVERY (L) IS = .07632
THE CORRELATION COEFFICIENT IS = .99154

****

DO YOU WANT TO TRY OTHER GUESS VALUES FOR SC AND KB, ?
(Answer YES or NO) = YES

NEW ESTIMATE FOR SC ? = .5

NEW ESTIMATE FOR KB ? = 100

BEST FIT KB AND SC THIS ITERATION ARE	5.00000119	**********
BEST FIT KB AND SC THIS ITERATION ARE	.25000012	.050000012
BEST FIT KB AND SC THIS ITERATION ARE	.38820032	.040811973
BEST FIT KB AND SC THIS ITERATION ARE	.49983077	.019499068
BEST FIT KB AND SC THIS ITERATION ARE	.53589701	.013366145
BEST FIT KB AND SC THIS ITERATION ARE	.53787500	.014070657
BEST FIT KB AND SC THIS ITERATION ARE	.53794583	.013950520
BEST FIT KB AND SC THIS ITERATION ARE	.53793391	.013974258
BEST FIT KB AND SC THIS ITERATION ARE	.53793626	.013969588
BEST FIT KB AND SC THIS ITERATION ARE	.53793580	.013970507

RECOVERY RUN (SOURCE:"GROUNDWATER HYDRAULICS", MCHORTER & SUNANDA, FORT COLLINS)

```
*** THE  BEST  FIT   TIME_RECOVERY  PAIRS  ARE  ***

      .5000000          1.7294
     1.0000000          1.5791
     1.5000000          1.4835
     2.0000000          1.4134
     2.5000000          1.3581
     3.0000000          1.3125
     4.0000000          1.2399
     4.5000000          1.2100
     5.5000000          1.1589
     8.0000000          1.0632
    12.0000000           .9597
    16.0000000           .8866
    21.0000000           .8180
    26.0000000           .7646
    36.0000000           .6844
    46.0000000           .6252
    56.0000000           .5786
    71.0000000           .5237

************************************************************

'BEST' FIT TRANSMISSIVITY (L**2/T) IS  =          .53793580
'BEST' FIT  STORAGE COEFFICIENT  IS    =          .013970507

 THE rms ERROR  FOR RECOVERY (L) IS    =          .07632
 THE CORRELATION COEFFICIENT IS        =          .99154

************************************************************

DO YOU WANT TO TRY OTHER GUESS VALUES FOR SC AND KB. ?
(Answer YES or NO )  =   YES

NEW ESTIMATE FOR SC ?  =   .000001

NEW ESTIMATE FOR KB ?  =   .00001

BEST FIT KB AND SC THIS ITERATION ARE       .00002000     .000002000
BEST FIT KB AND SC THIS ITERATION ARE       .00004000     .000004000
BEST FIT KB AND SC THIS ITERATION ARE       .00007999     .000007999
BEST FIT KB AND SC THIS ITERATION ARE       .00015998     .000015995
BEST FIT KB AND SC THIS ITERATION ARE       .00031991     .000031981
BEST FIT KB AND SC THIS ITERATION ARE       .00063963     .000063922
BEST FIT KB AND SC THIS ITERATION ARE       .00127851     .000127686
BEST FIT KB AND SC THIS ITERATION ARE       .00255401     .000254743
BEST FIT KB AND SC THIS ITERATION ARE       .00509604     .000506975
BEST FIT KB AND SC THIS ITERATION ARE       .01014436     .001003987
BEST FIT KB AND SC THIS ITERATION ARE       .02009961     .001968763
BEST FIT KB AND SC THIS ITERATION ARE       .03945664     .003785658
BEST FIT KB AND SC THIS ITERATION ARE       .07605050     .007001704
BEST FIT KB AND SC THIS ITERATION ARE       .14145717     .011998653
BEST FIT KB AND SC THIS ITERATION ARE       .24603646     .017771052
BEST FIT KB AND SC THIS ITERATION ARE       .38025323     .020342013
BEST FIT KB AND SC THIS ITERATION ARE       .49261232     .016744245
BEST FIT KB AND SC THIS ITERATION ARE       .53449595     .013730884
BEST FIT KB AND SC THIS ITERATION ARE       .53789934     .013996696
BEST FIT KB AND SC THIS ITERATION ARE       .53793854     .013965054
BEST FIT KB AND SC THIS ITERATION ARE       .53793535     .013971399
BEST FIT KB AND SC THIS ITERATION ARE       .53793597     .013970151
```

```
RECOVERY RUN (SOURCE:"GROUNDWATER HYDRAULICS", MCHORTER & SUNANDA, FORT COLLINS)

*** THE  BEST  FIT   TIME..RECOVERY  PAIRS  ARE  ***

      .5000000          1.7294
     1.0000000          1.5791
     1.5000000          1.4835
     2.0000000          1.4134
     2.5000000          1.3581
     3.0000000          1.3125
     4.0000000          1.2399
     4.5000000          1.2100
     5.5000000          1.1589
     8.0000000          1.0632
    12.0000000           .9597
    16.0000000           .8866
    21.0000000           .8180
    26.0000000           .7646
    36.0000000           .6844
    46.0000000           .6252
    56.0000000           .5786
    71.0000000           .5237
************************************************************

'BEST' FIT TRANSMISSIVITY (L**2/T) IS  =        .53793597
'BEST' FIT  STORAGE COEFFICIENT  IS    =        .013970151

 THE rms ERROR  FOR RECOVERY (L) IS    =        .07632
 THE CORRELATION COEFFICIENT IS        =        .99154

************************************************************

DO YOU WANT TO TRY OTHER GUESS VALUES FOR SC AND KB. ?
(Answer YES or NO )  =   NO

STOP
```

```
DO YOU WANT TO HAVE A HARD COPY OF INPUT-OUTPUT?
( Answer YES or NO ) =  YES


INPUT FILE NAME (UP TO 8 CHARACTERS) IN WHICH THE
OUTPUT IS TO BE ROUTED. MAKE SURE THE FILE EXISTS
Input file name = THEISOUT


DO YOU WANT TO PROVIDE A TITLE FOR THIS JOB?
( Answer YES or NO )  = YES

INPUT JOB TITLE -UP TO 100 CHARACTERS-
=DRAWDOWN RUN (SOURCE:"GROUNDWATER RESOURCE EVALUATION BY WALTON. PP 348  TABLE 5.12)


DO YOU WANT TO USE GAL, DAY, FT SYSTEM ?
(Answer YES or NO )  =   NO


DO YOU WANT TO ANALYZE DRAWDOWN OR RECOVERY OR
DRAWDOWN AND RECOVERY DATA ?
( Enter "DRAWDOWN" or "RECOVERY" or "DRAW-REC" )  =   DRAWDOWN


DO YOU  WANT SPECIFIC-CAPACITY OF THE PRODUCTION
WELL TO BE COMPUTED ?
(Answer YES or NO)  =   YES


IS THE DATA OF CONSTANT DRAWDOWN TYPE ?
(Answer YES or NO)  =  NO


ESTIMATE FOR STORAGE COEFFICIENT ?
( Enter a reasonable positive fraction )  =   .001


ESTIMATE FOR TRANSMISSIVITY ? GAL/DAY/FT OR L**2/T
(Enter a reasonable positive value )  =   100.


NUMBER OF PUMPING WELLS ?  =   1

      NO. OF VARIABLE PUMPING RATES OF PUMP WELL #   1 ?
      (Enter 1 if Constant Pumping Rate)             =    4


     THE VARIABLE PUMPING RATES OF PUMP WELL #   1 ARE:

     VARIABLE PUMPING RATE Q( 1, 1)  IS   =   .735294
     TIME FOR WHICH PUMPING RATE Q( 1, 1) IS OPERATIVE (DAY OR T) = 9
     VARIABLE PUMPING RATE Q( 1, 2)  IS   =   .7219251
     TIME FOR WHICH PUMPING RATE Q( 1, 2) IS OPERATIVE (DAY OR T) = 100
     VARIABLE PUMPING RATE Q( 1, 3)  IS   =   1.2433155
     TIME FOR WHICH PUMPING RATE Q( 1, 3) IS OPERATIVE (DAY OR T) = 180
     VARIABLE PUMPING RATE Q( 1, 4)  IS   =   1.6978609
     TIME FOR WHICH PUMPING RATE Q( 1, 4) IS OPERATIVE (DAY OR T) = 250
```

```
EFFECTIVE RADIUS (Rw) OF THE PUMPING WELL ? FT OR L  =   .5


NUMBER OF TIME-DRAWDOWN PAIRS TO BE  READ ?   =   35

ECHO THE INITIAL DATA
SC =      .1000000E-02    KB =  .1000000E+03
THESE ARE THE INITIAL GUESSES FOR STORAGE COEFFICIENT AND TRANSMISSIVITY


THE  4 VARYING PUMP RATES OF P. W. #  1 ARE :

Q( 1, 1)  =        .7353
Q( 1, 2)  =        .7219
Q( 1, 3)  =       1.2433
Q( 1, 4)  =       1.6979

THE  4 PUMP RATES OF P. W. #  1 CHANGE AFTER
FOLLOWING PUMPING TIMES :

TP( 1, 1 )  =      9.0000000
TP( 1, 2 )  =    100.0000000
TP( 1, 3 )  =    180.0000000
TP( 1, 4 )  =    250.0000000

NUMBER OF TIME-DRAWDOWN PAIRS ARE =    35

ARE THERE ANY ERRORS IN THE ABOVE ENTRIES?
(Answer YES or NO )  =   NO

TYPE IN TIME-DRAWDOWN PAIRS IN ORDER OF INCREASING TIME
(Enter TIME and DRAWDOWN in free format )
 =   1,2.1

 =   2,4.42

 =   3,5.84

 =   4,7.63

 =   9,10.0

 =   10,10.23

 =   13,10.84

 =   15,11.09

 =   17,11.37

 =   20,11.82

 =   23,11.88

 =   26,12.09

 =   30,12.33

 =   40,12.68

 =   45,12.79

 =   50,12.89

 =   55,13.04

 =   60,13.15
```

= 70,13.34

= 80,13.54

= 90,13.79

= 100,13.93

= 121,16.27

= 130,25.84

= 140,28.99

= 150,30.13

= 160,30.85

= 170,31.48

= 180,31.84

= 193,36.82

= 203,45.89

= 220,49.82

= 230,51.00

= 240,51.92

= 250,52.28

*** PUMP TEST DATA IN TIME DRAWDOWN PAIRS ***

TIME	DRAWDOWN
1.0000000	2.1000
2.0000000	4.4200
3.0000000	5.8400
4.0000000	7.6300
9.0000000	10.0000
10.0000000	10.2300
13.0000000	10.8400
15.0000000	11.0900
17.0000000	11.3700
20.0000000	11.8200
23.0000000	11.8800
26.0000000	12.0900
30.0000000	12.3300
40.0000000	12.6800
45.0000000	12.7900
50.0000000	12.8900
55.0000000	13.0400
60.0000000	13.1500
70.0000000	13.3400
80.0000000	13.5400
90.0000000	13.7900
100.0000000	13.9300
121.0000000	16.2700
130.0000000	25.8400
140.0000000	28.9900
150.0000000	30.1300
160.0000000	30.8500
170.0000000	31.4800
180.0000000	31.8400
193.0000000	36.8200
203.0000000	45.8900
220.0000000	49.8200
230.0000000	51.0000
240.0000000	51.9200
250.0000000	52.2800

```
ARE THERE ANY ERRORS IN TIME - DRAWDOWN PAIRS?
 (Answer YES or NO )  =    NO

BEST FIT KB AND SC THIS ITERATION ARE      5.00000119      .003000000
BEST FIT KB AND SC THIS ITERATION ARE       .25000012      .009000000
BEST FIT KB AND SC THIS ITERATION ARE       .01250001      .027000000
BEST FIT KB AND SC THIS ITERATION ARE       .01341776      .072452819
BEST FIT KB AND SC THIS ITERATION ARE       .01298768      .143146071
BEST FIT KB AND SC THIS ITERATION ARE       .01249559      .199815857
BEST FIT KB AND SC THIS ITERATION ARE       .01223912      .222467516
BEST FIT KB AND SC THIS ITERATION ARE       .01215092      .228106966
BEST FIT KB AND SC THIS ITERATION ARE       .01212821      .229368503
BEST FIT KB AND SC THIS ITERATION ARE       .01212292      .229652766
BEST FIT KB AND SC THIS ITERATION ARE       .01212171      .229717108
BEST FIT KB AND SC THIS ITERATION ARE       .01212144      .229731689

  DRAWDOWN RUN (SOURCE:"GROUNDWATER RESOURCE EVALUATION BY WALTON, PP 348  TABLE 5.12


*** THE  BEST  FIT   TIME..DRAWDOWN  PAIRS  ARE  ***

     1.0000000          .7836
     2.0000000         2.2279
     3.0000000         3.4326
     4.0000000         4.4184
     9.0000000         7.6177
    10.0000000         8.0523
    13.0000000         9.1274
    15.0000000         9.7341
    17.0000000        10.2738
    20.0000000        10.9848
    23.0000000        11.6040
    26.0000000        12.1520
    30.0000000        12.7967
    40.0000000        14.1059
    45.0000000        14.6462
    50.0000000        15.1311
    55.0000000        15.5711
    60.0000000        15.9737
    70.0000000        16.6890
    80.0000000        17.3103
    90.0000000        17.8596
   100.0000000        18.3518
   121.0000000        27.3003
   130.0000000        28.8010
   140.0000000        30.1002
   150.0000000        31.1679
   160.0000000        32.0818
   170.0000000        32.8849
   180.0000000        33.6036
   193.0000000        40.1321
   203.0000000        42.3037
   220.0000000        44.7871
   230.0000000        45.9164
   240.0000000        46.9008
   250.0000000        47.7791
*********************************************************

"BEST" FIT TRANSMISSIVITY (L**2/T) IS  =        .01212144
"BEST" FIT  STORAGE COEFFICIENT  IS    =        .229731689

 THE rms ERROR  FOR DRAWDOWN (L) IS    =       3.32206
 THE CORRELATION COEFFICIENT IS        =        .97434

*********************************************************
```

SPECIFIC CAPACITY COMPUTATIONS FOR PUMPING WELL No: 1

****

TIME (T)	DRAWDOWN (L)	PUMPAGE (L**3/T)	SPECIFIC CAPACITY (L**2/T)
1.00000000	.78364322	.73529400	.93830200
2.00000000	2.22789523	.73529400	.33003976
3.00000000	3.43263470	.73529400	.21420689
4.00000000	4.41838911	.73529400	.16641676
9.00000000	7.61768896	.73529400	.09652455
10.00000000	8.05231744	.72192510	.08965433
13.00000000	9.12744501	.72192510	.07909389
15.00000000	9.73409462	.72192510	.07416459
17.00000000	10.27379327	.72192510	.07026860
20.00000000	10.98482405	.72192510	.06572022
23.00000000	11.60397307	.72192510	.06221361
26.00000000	12.15202174	.72192510	.05940782
30.00000000	12.79667588	.72192510	.05641505
40.00000000	14.10592256	.72192510	.05117886
45.00000000	14.64615786	.72192510	.04929109
50.00000000	15.13113983	.72192510	.04771122
55.00000000	15.57111617	.72192510	.04636309
60.00000000	15.97372732	.72192510	.04519453
70.00000000	16.68897280	.72192510	.04325761
80.00000000	17.31034272	.72192510	.04170484
90.00000000	17.85963561	.72192510	.04042216
100.00000000	18.35184355	.72192510	.03933802
121.00000000	27.30029150	1.24331550	.04554221
130.00000000	28.80096501	1.24331550	.04316923
140.00000000	30.10017604	1.24331550	.04130592
150.00000000	31.16790982	1.24331550	.03989088
160.00000000	32.08181929	1.24331550	.03875452
170.00000000	32.88487049	1.24331550	.03780813
180.00000000	33.60356021	1.24331550	.03699952
193.00000000	40.13210508	1.69786090	.04230680
203.00000000	42.30371931	1.69786090	.04013503
220.00000000	44.78713078	1.69786090	.03790957
230.00000000	45.91637769	1.69786090	.03697724
240.00000000	46.90078494	1.69786090	.03620112
250.00000000	47.77908763	1.69786090	.03553565

****

DO YOU WANT TO TRY OTHER GUESS VALUES FOR SC AND KB. ?
(Answer YES or NO) = NO

STOP

```
C
C                *********************************
C                *                               *
C                *     PROGRAM     THEIS.VARQ    *
C                *                               *
C                *    INTERACTIVE TIME-SHARING   *
C                *      FORTRAN-77 VERSION       *
C                *                               *
C                *********************************
C
C
C   WRITTEN BY:          MUNIR A. BUTT          DATE: 10-01-84
C                                &
C                        CARL D. McELWEE
C
C
C   LAST VERIFIED BY:    RUBEN ARTEAGA          DATE: 15-07-85
C
C
C
C   PURPOSE:
C
C           CALCULATE THE BEST FIT STORAGE AND TRANSMISSIVITY
C           BY FITTING THE THEIS EQUATION TO  FIELD  OBSERVED
C           PUMPTEST DATA, i.e.(drawdown or recovery  or both
C           drawdown and recovery data together)  USING  THE
C           METHOD OF CONVOLUTION FOR VARYING  PUMPING  RATES
C           AND SENSITIVITY ANALYSIS IN A LEAST-SQUARE SENSE.
C           CONSTANT DRAWDOWN AND VARIABLE DISCHARGE DATA  OF
C           ARTESIAN FLOWING WELLS CAN ALSO BE ANALYZED.
C
C*******************************************************************
C                                                                  *
C          DESCRIPTION OF MAIN VARIABLES AND PARAMETERS            *
C                                                                  *
C                                                                  *
C   VARIABLE        TYPE                DESCRIPTION                *
C   --------        ----    -------------------------------------- *
C                                                                  *
C   UNIT            CHAR -> SYSTEM OF UNITS; YES FOR GAL, DAY, FT; *
C                           OTHERWISE  ANY  CONSISTENT  SYSTEM  OF *
C                           UNITS ARE ASSUMED.                     *
C   CONSTORA        CHAR -> SWITCH  FOR  ANALYZING  THE   CONSTANT *
C                           DRAWDOWN DATA; 'YES' OR 'NO'.          *
C   CS              REAL -> FIELD OBSERVED CONSTANT DRAWDOWN DATA; *
C                           (FT) OR (L).                           *
C   DATYP           CHAR -> DATA TYPE TO BE ANALYZED;  'DRAWDOWN', *
C                           'RECOVERY', OR 'DRAW-REC'.             *
C   FILENM          CHAR -> CHARACTER VARIABLE CONTAINING THE NAME *
C                           OF A SEQUENTIAL FILE TO BE USED TO     *
C                           ROUT INPUT-OUTPUT WHEN THE CHARACTER   *
C                           VARIABLE PTR = 'YES'.                  *
C   KB              REAL -> AQUIFER TRANSMISSIVITY (GAL/DAY/FT) OR *
C                           (L**2/T).                              *
C   NDM,NOB,NVQ     INT  -> PARAMETERS TO SET DIMENSIONS:          *
C                           * NDM - NO. OF DRAWDOWN  (OR RECOVERY) *
C                                   MEASUREMENTS.                  *
C                           * NOB - NO. OF OBSERVATION WELLS.      *
C                           * NVQ - NO. OF VARIABLE PUMPAGES.      *
C   NTDP            INT  -> NO. OF TIME-DRAWDOWN DATA PAIRS TO  BE *
C                           READ.                                  *
C   NITER           INT  -> LIMITING  VALUE  FOR  TOTAL  NUMBER OF *
C                           ITERATIONS.                            *
```

```
C    NOW             INT  -> NO. OF OBSERVATION WELLS.                   *
C    NPR             INT  -> NO. OF VARIABLE PUMPING RATES.              *
C    PRFL            LOG  -> LOGICAL VARIABLE SET = .TRUE. WHEN          *
C                            HARDCOPY OF INPUT-OUTPUT IS TO BE           *
C                            ROUTED TO FILE 'FILENM'.                    *
C    PTR             INT  -> LOGICAL OUTPUT DEVICE NUMBER.               *
C    Q(II)           REAL -> VARIABLE PUMPING RATE (GAL/DAY) OR (        *
C                            L**3/T).                                    *
C    R(J)            REAL -> RADIAL  DISTANCE(S)  OF   OBVSERVATION *
C                            WELL(S) FROM PUMPING WELL (FT) OR (L). *
C    RIN             INT  -> LOGICAL INPUT DEVICE NUMBER                 *
C    S(I)            REAL -> FIELD OBSERVED DRAWDOWN AT TIME I (FT) *
C                            OR (L).                                     *
C    SCAPCT          REAL -> SPECIFIC  CAPACITY  OF   PUMPED   WELL *
C                            (PRODUCTION WELL) (GAL/FT) OR (L**2/T) *
C    SIGMA           REAL -> THE rms (ROOT-MEAN-SQUARE)  ERROR   IN *
C                            DRAWDOWN AFTER THE BEST FIT  HAS  BEEN *
C                            OBTAINED (FT) OR (L).                       *
C    SP(I)           REAL -> THEORETICAL  (COMPUTED)  DRAWDOWN   AT *
C                            TIME I (FT) OR (L).                         *
C    SPCAP           CHAR -> SWITCH  FOR  COMPUTING   (THEORETICAL) *
C                            SPECIFIC CAPACITY OF PUMPED  (PRODUCT- *
C                            ION) WELL; 'YES' OR 'NO'.                   *
C    T(I)            REAL -> THE ITH TIME AT WHICH  FIELD  MEASURE- *
C                            MENT FOR THE DRAWDOWNS IS MADE (DAYS)  *
C                            OR (T).                                     *
C    TITLE           CHAR -> CHARACTER VARIABLE CONTAINING THE           *
C                            TITLE FOR THE JOB (UP TO 100 CHAR LONG)*
C    TLAST           REAL -> TIME AT WHICH LAST RECOVERY  OBSERVED  *
C                            (DAYS) OR (T).                              *
C    TOL             REAL -> TOLERANCE FOR CONVERGENCE.                  *
C    TP(I)           REAL -> TIME WHILE A PARTICULAR  PUMPING  RATE *
C                            IS OPERATIVE (DAYS) OR (T).                 *
C    WOUT            INT  -> LOGICAL OUTPUT DEVICE NUMBER.               *
C                                                                        *
C***********************************************************************
C
      PARAMETER ( NDM =200, NOB =8, NVQ =30, NOPW =5 )
      DIMENSION S(NDM,NOB),T(NDM),R(NOB),SP(NDM,NOB),
     &          DSDT(NDM,NOB),DSDSC(NDM,NOB),Q(NOPW,NVQ+1),
     &          TT(NDM),TP(NOPW,NVQ+1),SCAPCT(NDM),TSTOP(NOPW),
     &          RPWOW(NOPW,NOB),NPRT(NVQ+1),STPW(NOPW),DELT(NOPW)
      REAL*8  KB, KBINIT
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      CHARACTER*3 UNIT,CHEKDATA,TRIAL,SPCAP,CONSTDRA,PTR,TITL
      CHARACTER*8 DATYP,FILENM
      CHARACTER*100 TITLE
      LOGICAL DATATYP, PRFL, CHCKFL
      INTEGER RIN,WOUT,IPTR,TLIM(NVQ)
C
      DATA BLANK/4H    /
      DATA RIN,WOUT,IPTR/5,6,15/
C
C       PRE-SET THE VARIABLES FOR ERROR TOLERANCE (TOL) AND FOR
C       MAXIMUM NUMBER OF ITERATIONS (NITER).
C
C
      PRFL      = .FALSE.
      TOL       = 0.0001
      NITER     = 25
C
    4 WRITE(WOUT,5)
    5 FORMAT(//' DO YOU WANT TO HAVE A HARD COPY OF INPUT-OUTPUT?'/
```

```
  &         ' ( Answer YES or NO ) =  ')
      READ(RIN,14) PTR
      IF (PTR .EQ. 'YES' .OR. PTR .EQ. 'NO') GO TO 6
      GO TO 4
    6 IF ( PTR .EQ. 'NO' ) GO TO 10
      PRFL = .TRUE.
      WRITE(WOUT,7)
    7 FORMAT(//' INPUT FILE NAME (UP TO 8 CHARACTERS) IN WHICH THE'/
  &         ' OUTPUT IS TO BE ROUTED. MAKE SURE THE FILE EXISTS'/
  &         ' Input file name = ')
      READ(RIN,8) FILENM
    8 FORMAT(A8)
C
C===========================================================================
C THE FOLLOWING STATEMENT OPENS A FILE IN THE ECLIPSE MV800 COMPUTER =
C CHANGE THIS STATEMENT ACCORDINGLY.                                  =
C                                                                     =
C    IPTR = DEVICE NUMBER          FILENM = NAME OF FILE              =
C                                                                     =
      OPEN (IPTR,FILE=FILENM,IOINTENT='OUTPUT',CARRIAGECONTROL=
     & 'FORTRAN')
C                                                                     =
C===========================================================================
C
C
C
   10 CONTINUE
   12 WRITE (WOUT,13)
   13 FORMAT (//' DO YOU WANT TO PROVIDE A TITLE FOR THIS JOB? ',/,
   &         ' ( Answer YES or NO )  = ')
      READ (RIN,14) TITL
   14 FORMAT (A3)
      IF(TITL .EQ. 'YES' .OR. TITL .EQ. 'NO') GO TO 15
      GO TO 12
   15 IF(TITL .EQ. 'NO') GO TO 20
      WRITE(WOUT,16)
   16 FORMAT(' INPUT JOB TITLE -UP TO 100 CHARACTERS- '/' =')
   17 READ(RIN,18) TITLE
   18 FORMAT(A100)

C
C                READ IN THE INITIAL DATA
C
   20 WRITE (WOUT,30)
   30 FORMAT (//' DO YOU WANT TO USE GAL, DAY, FT SYSTEM ?  ',/,
   & ' (Answer YES or NO )  =   ')
      READ(RIN,40) UNIT
   40 FORMAT(A3)
      IF ( UNIT .EQ. 'YES' .OR. UNIT .EQ. 'NO') GO TO 50
      WRITE  (WOUT,590)
      GO TO 20
   50 WRITE (WOUT,60)
   60 FORMAT(//' DO YOU WANT TO ANALYZE DRAWDOWN OR RECOVERY OR ',/,
   & ' DRAWDOWN AND RECOVERY DATA ?  ',/,
   & ' ( Enter "DRAWDOWN" or "RECOVERY" or "DRAW-REC" )  =    ')
      READ(RIN ,70) DATYP
   70 FORMAT (A8)
      DATATYP = ( DATYP .EQ. 'DRAWDOWN')
      SPCAP = 'NO'
      IF(DATYP.EQ.'RECOVERY'.OR.DATYP.EQ.'DRAWDOWN'.OR.
   & DATYP.EQ.'DRAW-REC')GO TO 100
   80 WRITE (WOUT,90)
   90 FORMAT (// ' *** YOU HAVE AN ERROR IN DATA ENTRY. ***',/,
```

```
   & ' ENTER EITHER "RECOVERY" OR "DRAWDOWN"  OR "DRAW-REC"   '/)
 99   FORMAT(2H  )
      GO TO 50
100   CONTINUE
      IF( .NOT. DATATYP) GO TO 150
110   WRITE(WOUT,120)
120   FORMAT (//' DO YOU  WANT SPECIFIC-CAPACITY OF THE PRODUCTION ',/,
    & ' WELL TO BE COMPUTED ? ',/,
    &  ' (Answer YES or NO)   =    ')
      READ(RIN,40)SPCAP
      IF( SPCAP .EQ. 'YES' .OR. SPCAP .EQ. 'NO' ) GO TO 130
      IF( SPCAP .NE. 'YES' .OR. SPCAP .NE. 'NO' ) WRITE(WOUT,590)
      GO TO 110
130   WRITE(WOUT,140)
140   FORMAT(//' IS THE DATA OF CONSTANT DRAWDOWN TYPE ? ',/,
    &  ' (Answer YES or NO)   =   ')
      READ(RIN,40)CONSTDRA
      IF(CONSTDRA .EQ. 'YES' .OR. CONSTDRA .EQ. 'NO') GO TO 150
      IF(CONSTDRA .NE. 'YES' .OR. CONSTDRA .NE. 'NO') WRITE(WOUT,590)
      GO TO 130
150   CONTINUE
      WRITE(WOUT,160)
160   FORMAT(//' ESTIMATE FOR STORAGE COEFFICIENT ?  ',/,
    &  ' ( Enter a reasonable positive fraction )  =    ')
      READ(RIN,*) SC
      WRITE(WOUT,170)
170   FORMAT(//' ESTIMATE FOR TRANSMISSIVITY ? GAL/DAY/FT '
    & 'OR L**2/T'/ ' (Enter a reasonable positive value )  =    ')
      READ(RIN,*) KB
      WRITE(WOUT,171)
      SCINIT = SC
      KBINIT = KB
171   FORMAT(//' NUMBER OF PUMPING WELLS ?  =    ')
      READ(RIN,*) NPW
      IF (NPW .EQ. 1)GO TO 174
      DO 175 I = 1,NPW
      WRITE(WOUT,177)I
177   FORMAT(/5X,' STARTING TIME OF PUMP WELL # ',I2,
    & '   ? DAY OR T = ')
      READ(RIN,*)STPW(I)
176   CONTINUE
174   CONTINUE
      DO 173 I = 1,NPW
      WRITE(WOUT,172) I
172   FORMAT(/5X,' NO. OF VARIABLE PUMPING RATES OF PUMP WELL #  ',
    & I2,' ?',/5X,
    &  ' (Enter 1 if Constant Pumping Rate)             =    ')
      READ(RIN,*) NPRT(I)
173   CONTINUE
180   CONTINUE
      DO 190 I =1,NPW
      IF(NPRT(I) .GT. 1)WRITE(WOUT,181)I
181   FORMAT( //5X,'THE VARIABLE PUMPING RATES OF PUMP WELL #  ',I2,' ARE:  '/
      IF(NPRT(I) .EQ. 1)WRITE(WOUT,182)I
182   FORMAT(5X,'THE CONSTANT PUMPING RATE OF PUMP WELL #  ',I2,'  IS  =   ')
      DO 184 J = 1,NPRT(I)
      IF(NPRT(I).EQ.1)GO TO 183
      WRITE(WOUT,185)I,J
185   FORMAT(5X,'VARIABLE PUMPING RATE Q(',I2,',',I2,')  IS   =   ')
183   READ(RIN,*)Q(I,J)
      WRITE(WOUT,186)I,J
186   FORMAT(5X,'TIME FOR WHICH PUMPING RATE Q(',I2,',',I2,') IS ',
    & 'OPERATIVE (DAY OR T) = ')
```

```
      READ(RIN,*) TP(I,J)
  184 CONTINUE
  190 CONTINUE
C
C          IF MORE THAN ONE PUMPING WELL AND STARTING TIME OF EACH
C          PUMPING WELL IS DIFFERENT,  THEN RE-ADJUST THE  PUMPING
C          TIME OF EACH WELL WITH TIME OF OBSERVATION WELL AS BASE
C          TIME ( BASE TIME OF OBSERVATION WELL ASSUMED  = 0.0 ).
C
      STOW=0.0
      DO 195 I=1,NPW
      DELT(I) = 0.0
      DO 194 J=1,NPRT(I)
      IF(STPW(I) .GT. STOW)DELT(I)=STPW(I)-STOW
  194 CONTINUE
  195 CONTINUE
  240 CONTINUE
      IF( SPCAP .EQ. 'YES' .AND. NPW .EQ. 1 .OR. CONSTDRA .EQ.
     & 'YES' .AND. NPW .EQ. 1 ) GO TO 270
      WRITE(WOUT,250)
  250 FORMAT(///' TOTAL NUMBER OF OBSERVATION WELLS ?          =    ')
      READ(RIN,*) NOW
      DO 260 I =1,NPW
      DO 259 J =1,NOW
      WRITE(WOUT,251)I,J
  251 FORMAT(5X,'RADIAL DISTANCE BETWEEN PUMP WELL # ',I2,' AND OBSERVATION WEL
 # '
     & ,I2,' IS =     ')
      READ(RIN,*) RPWOW(I,J)
  259 CONTINUE
  260 CONTINUE
      GO TO 290
  270 CONTINUE
      NOW=1
      WRITE(WOUT,280)
  280 FORMAT(//' EFFECTIVE RADIUS (Rw) OF THE PUMPING WELL ? FT OR L  =    ')
      READ(RIN,*)R(NOW)
  290 CONTINUE
      WRITE(WOUT,300)DATYP
  300 FORMAT(//' NUMBER OF TIME-',A8,' PAIRS TO BE  READ ?    =    ')
      READ(RIN,*) NTDP
C
C         CHECK IF PRE-ASSIGNED DIMENSIONS (NDM,NOB,NVQ) ARE SUFFICIENT.
C         IF NOT, PRINT A MESSAGE AND STOP PROGRAM EXECUTION.
C
      DO 301 I =1,NPW
      IF(NTDP.GT.NDM.OR.NOW.GT.NOB.OR.NPRT(I).GT.NVQ)GO TO 1160
  301 CONTINUE
C
C                     ECHO PRINT THE INITIAL DATA
C
C
      INQUIRE(UNIT=IPTR,OPENED=CHCKFL)
      IF(CHCKFL) GOTO 304
      WRITE(WOUT,303) IPTR,FILENM,PRFL
  303 FORMAT(//'    IPTR =',I5,5X,'FILENM =',A8,5X,'PRFL =',L1,/)
      OPEN(IPTR,FILE=FILENM,IOINTENT='OUTPUT',CARRIAGECONTROL=
     & 'FORTRAN')
C
C
  304 CONTINUE
      IF(PRFL) WRITE(IPTR,305) TITLE
  305 FORMAT(//3X,A100,//)
```

```
      WRITE(WOUT,99)
 310  WRITE ( WOUT, 320 ) SC, KB
      IF(PRFL) WRITE(IPTR,320) SC,KB
 320  FORMAT(' ECHO THE INITIAL DATA'/5H SC =,3X,E14.7,3X,5H KB =,E14.7,
     &       /1X,'THESE ARE THE INITIAL GUESSES FOR STORAGE ',
     & 'COEFFICIENT AND TRANSMISSIVITY'//)

      DO 334 I = 1,NPW
      IF(PRFL) WRITE(IPTR,321) I
 321  FORMAT(//'              ***********************'/10X,
     & '*  PUMP WELL # ',I2,'   *'/10X,'***********************'/)
      IF( NPRT(I) .EQ. 1) GO TO 328
      WRITE(WOUT , 322) NPRT(I),I
      IF(PRFL) WRITE(IPTR,322) NPRT(I),I
 322  FORMAT(  / ' THE ',I2,' VARYING PUMP RATES OF P. W. # ',I2,' ARE : ',//)
      DO 324 J = 1,NPRT(I)
      WRITE(WOUT,323) I,J,Q(I,J)
      IF(PRFL) WRITE(IPTR,323) I,J,Q(I,J)
 323  FORMAT(' Q(',I2,',',I2,')  =    ',F11.4)
 324  CONTINUE
      WRITE(WOUT,325) NPRT(I),I
      IF(PRFL) WRITE(IPTR,325) NPRT(I),I
 325  FORMAT(  / ' THE ',I2,' PUMP RATES OF P. W. # ',I2,' CHANGE AFTER
     & '/' FOLLOWING PUMPING TIMES : ',//)
      DO 327 J =1,NPRT(I)
      WRITE(WOUT,326) I,J,TP(I,J)
      IF(PRFL) WRITE(IPTR,326) I,J,TP(I,J)
 326  FORMAT(' TP(',I2,',',I2,' )  =    ',F14.7)
 327  CONTINUE
      GO TO 330
 328  CONTINUE
      J = NPRT(I)
      WRITE(WOUT,329) I,Q(I,J),TP(I,J)
      IF(PRFL) WRITE(IPTR,329) I,Q(I,J),TP(I,J)
 329  FORMAT(/' CONSTANT PUMPAGE RATE OF P.W. # ',I2,' IS  =    ',F14.7,/,
     & ' PUMPING TIME                        =    ',F14.7)
 330  CONTINUE
 334  CONTINUE
 400  CONTINUE
      WRITE(WOUT,99)
 410  CONTINUE
      IF( SPCAP .EQ. 'YES' )GO TO 450
      WRITE(WOUT,420)
      IF(PRFL) WRITE(IPTR,420)

 420  FORMAT(   / ' ** RADIAL DISTANCE(S) OF OBSERVATION WELL(S) IS/ARE : '//)
      DO 441 I =1,NPW
      DO 440 J = 1,NOW
      WRITE(WOUT,430) I,J,RPWOW(I,J)
      IF(PRFL) WRITE(IPTR,430) I,J,RPWOW(I,J)
 430  FORMAT(  / ' RADIAL DISTANCE FROM P.W. # ',I2,' TO O.W. # ',I2,' IS  =
',F14.7/)
 440  CONTINUE
 441  CONTINUE
      WRITE(WOUT,99)
 450  WRITE(WOUT,460)DATYP,NTDP
      IF(PRFL) WRITE(IPTR,460)DATYP,NTDP
 460  FORMAT(/' NUMBER OF TIME-',A8,' PAIRS ARE = ',I5)
      DO 462 I =1,NPW
      TSTOP(I) = 0.0
 462  CONTINUE
      IF (DATATYP) GO TO 480
C
```

```
C         IF RECOVERY OR DRAW-REC, THEN PRESET LAST PUMPING
C                TIME AS TIME SINCE PUMPING STOPPED.
C
      DO 465 I =1,NPW
      DO 463 J =1,NPRT(I)
      TSTOP(I) = TP(I,J)
  463 CONTINUE
      WRITE(WOUT,464) I,TSTOP(I)
      IF(PRFL) WRITE(IPTR,464) I,TSTOP(I)
  464 FORMAT('  TSTOP(',I2,') = ',F14.4)
  465 CONTINUE
      WRITE(WOUT,470)
  470 FORMAT(/' TIME AT WHICH LAST RECOVERY IS MEASURED ?  DAY OR T    ',/,
     & ' (Enter the LAST TIME of RECOVERY measurement)  =    ')
      READ(RIN,*)TLAST
  480 CONTINUE
C
C         CHECK FOR ERROR IN THE INPUT DATA. IF THERE IS AN ERROR
C                    THEN RE-ENTER THE DATA.
C
      WRITE(WOUT,99)
      WRITE(WOUT,490)
  490 FORMAT(' ARE THERE ANY ERRORS IN THE ABOVE ENTRIES? ',/,
     & ' (Answer YES or NO )  =    ')
      READ(RIN,40) CHEKDATA
      IF(CHEKDATA .NE. 'NO')GO TO 10
C
C         TYPE IN THE TIME-DRAWDOWN PAIRS IN ORDER OF INCREASING TIME.
C
      IF(DATYP.EQ.'RECOVERY'.OR.DATYP.EQ.'DRAW-REC'.OR.
     & CONSTDRA.EQ.'NO')GO TO 495
      WRITE(WOUT,491)
  491 FORMAT( ' CONSTANT DRAWDOWN  ? FT OR L   =     ')
      READ(RIN,*)CS
      DO 493 I =1,NPW
      DO 492 J =1,NPRT(I)
      T(I)      =TP(I,J)
      S(I,1) = CS
  492 CONTINUE
  493 CONTINUE
      GO TO 520
  495 CONTINUE
      WRITE(WOUT,500)DATYP,DATYP
  500 FORMAT(' TYPE IN TIME-',A8,' PAIRS IN ORDER OF INCREASING TIME      ',/,
     & ' (Enter TIME and ',A8,' in free format ) ')
     DO 510  I = 1,NTDP
     WRITE(WOUT,502)
  502 FORMAT( '   =    ')
     READ(RIN,*)T(I),(S(I,J),J=1,NOW)
  510 CONTINUE
C
C         ECHO PRINT THE TIME-DRAWDOWN PAIRS.
C
  520 WRITE(WOUT,530)DATYP
      IF(PRFL) WRITE(IPTR,530) DATYP
  530 FORMAT(/ ' ***  PUMP TEST DATA IN TIME ',A8,' PAIRS  *** ' ,//)
      WRITE(WOUT,540)DATYP
      IF(PRFL) WRITE(IPTR,540)DATYP
  540 FORMAT ( '          TIME            ',A8,'          '/)
      DO 550 I=1,NTDP
      WRITE(WOUT,560)T(I),(S(I,J),J=1,NOW)
      IF(PRFL) WRITE(IPTR,560)T(I),(S(I,J),J=1,NOW)
  550 CONTINUE
```

```
      WRITE(WOUT,554)
  554 FORMAT(5(/))
      IF(PRFL) WRITE(IPTR,555)
  555 FORMAT('1')
  560 FORMAT(F14.7,8X,8(F9.4))
C
C          CHECK THE TIME-DRAWDOWN PAIRS. IF ANY ERROR, RE-ENTER THAT
C          LINE.
C
      NEWDATA=0
      WRITE(WOUT,99)
  570 CONTINUE
      WRITE (WOUT,580)DATYP
  580 FORMAT (' ARE THERE ANY ERRORS IN TIME - ',A8,' PAIRS?',/,
     & '  (Answer YES or NO )   =    ')
      READ(RIN,40)CHEKDATA
      IF(CHEKDATA .EQ. 'NO' .AND. NEWDATA .EQ.0)GO TO 640
      IF(CHEKDATA .EQ. 'NO' .AND. NEWDATA .GE.1)GO TO 620
      IF(CHEKDATA .EQ. 'YES')GO TO 600
      IF(CHEKDATA.NE.'YES'.OR.CHEKDATA.NE.'NO') WRITE(WOUT,590)
  590 FORMAT(  / ' YOU HAVE AN ERROR IN DATA ENTRY    '//)
      GO TO 570
  600 WRITE(WOUT,610)
  610 FORMAT(' ENTER THE LOCATION ie. LINE NUMBER "I" AND
     & OBSERVATION WELL NO. "J" AND CORRECT TIME AND DRAWDOWN. ',/,
     & ' (Enter I,J,T(I),S(I,J))  =     ')
      READ ( RIN,*)I,J, T(I), S(I,J)
      NEWDATA=NEWDATA+1
      GO TO 570
C
C              ECHO PRINT THE CORRECT TIME-DRAWDOWN PAIRS.
C
  620  CONTINUE
       DO 630 I = 1,NTDP
  630  WRITE(WOUT,560)T(I),(S(I,J),J=1,NOW)
       IF(PRFL) WRITE(IPTR,560)T(I),(S(I,J),J=1,NOW)
       WRITE( WOUT,631)
  631  FORMAT(5(/))
       IF(PRFL) WRITE(IPTR,632)
  632  FORMAT('1')
  640  CONTINUE
C
C             IF RECOVERY OR DRAW-REC DATA, RE-ADJUST THE TIME
C                  SCALE AND ACCOUNT FOR 'ZERO' PUMPAGE.
C
       IF(DATATYP)GO TO 660
       DO 645 I =1,NPW
       NPRT(I) = NPRT(I)+1
       DO 644 J =1,NPRT(I)
  644  CONTINUE
       Q(I,J) = 0.0
       TP(I,J) = TLAST-TSTOP(I)
       IF( NPW .EQ. 1 )TP(I,J) = TLAST + TSTOP(I)
  645  CONTINUE
       DO 651 IP = 1,NPW
       DO 650 I  = 1,NTDP
       IF ( DATYP .EQ. 'DRAW-REC' .AND. NPW .EQ. 1 .AND.
     &  T(I) .GT. T(I+1))T(I+1) = TSTOP(IP) + T(I+1)
       IF ( DATYP .EQ. 'RECOVERY' .AND. NPW .EQ. 1) T(I) =
     &  TSTOP(IP) + T(I)
  650  CONTINUE
  651  CONTINUE
  660  CONTINUE
```

```
  670  ICOUNT    = 0
       NCONV     = 0
       NKB       = 0
  680  TEMPKB    = KB
       TEMPSC    = SC
  690  CONTINUE
C
C                INITIALIZE THE ARRAY'S.
C
       DO 710 I = 1 , NTDP
       DO 700 J = 1 , NOW
       SP(I,J) =     0.0
       DSDT(I,J) =   0.0
       DSDSC(I,J) =  0.0
  700  CONTINUE
       TT(I)=0.0
  710  CONTINUE
C
C         USE THE METHOD OF CONVOLUTION FOR CALCULATING DRAWDOWN'S FOR
C         THE VARAIBLE PUMPAGES AND FOR THE ASSUMED (GUESS) VALUES
C         OF SC AND KB. ALSO USE SENSITIVITY ANALYSIS AND LEAST SQUARES
C         TO FIND A BETTER SC AND KB.
C
  720  CONTINUE
       DO 781 IP=1,NPW
       NPR = NPRT(IP)
       DO 780 II = 1,NPR
       DO 770 I  = 1,NTDP
       IF(II .GT. 1)GO TO 730
       TT(I) = T(I)
       QQ =     Q(IP,II)
       IF( DELT(IP) .GE. STOW)TT(I)=TT(I)-DELT(IP)
       GO TO 740
  730  CONTINUE
       TT(I) = T(I) - TP(IP,II-1)
       QQ =     Q(IP,II) - Q(IP,II-1)
  740  CONTINUE
       DO 760 J = 1,NOW
       IF(SPCAP .NE. 'YES') R(J) = RPWOW(IP,J)
       SIJ =      0.0
       DSDTIJ =   0.0
       DSDSCIJ =  0.0
       IF(TT(I) .LE. 0.0)GO TO 750
C
C           CALL SUB-ROUTINE  THEISVQ AND COMPUTE DRAWDOWN(S) AND
C                SENSITIVITY COEFFICIENTS DUE TO PUMPAGE(S).
C
       CALL THEISVQ(SC,KB,QQ,TT(I),R(J),SIJ,DSDTIJ,DSDSCIJ,UNIT,WOUT)
C
C             SUM THE DRAWDOWN'S AND THE SENSITIVITY COEFFICIENTS.
C
  750  CONTINUE
       SP(I,J) =     SP(I,J)+SIJ
       DSDT(I,J) =   DSDT(I,J)+DSDTIJ
       DSDSC(I,J) = DSDSC(I,J)+DSDSCIJ
  760  CONTINUE
  770  CONTINUE
  780  CONTINUE
  781  CONTINUE
  790  CONTINUE
       IF(NCONV .GE. 1)GO TO 850
       SSUS =     0.0
       SSUT =     0.0
```

```
      SUTUS =    0.0
      SUSDIF  = 0.0
      SUTDIF =  0.0
C
C        CALCULATE THE DELTA VALUES FOR SC AND KB (USING SENSITIVITY
C        COEFFICIENTS) AND UPDATE THE INITIAL GUESS VALUES.
C
      DO 810 I =1,NTDP
      DO 800 J=1,NOW
      SSUS =    DSDSC(I,J)**2 + SSUS
      SSUT =    DSDT(I,J)**2 + SSUT
      SUTUS =   DSDSC(I,J)*DSDT(I,J) + SUTUS
      SUSDIF =  DSDSC(I,J)*(S(I,J)-SP(I,J))  + SUSDIF
      SUTDIF =  DSDT(I,J)*(S(I,J)-SP(I,J)) + SUTDIF
  800 CONTINUE
  810 CONTINUE
  820 CONTINUE
C
C        PRINT THE MESSAGE IF SENSITIVITY COEFFICIENTS ARE TOO SMALL
C        (OR ZERO).
C
      IF(SSUS .LE. 0.0 .OR. SSUT .LE. 0.0  .OR. SUTUS .LE. 0.0)GO TO 1040
      DELTSC = (SSUT*SUSDIF-SUTUS*SUTDIF)/(SSUS*SSUT-SUTUS**2)
      IF (DELTSC .LT. -.95*SC ) DELTSC = -.95*SC
      IF (DELTSC .GT. 2.0*SC ) DELTSC = 2.0*SC
      SC = SC + DELTSC
      IF (SC .GT. 1.0 ) SC = 1.0
      DELTKB = (SUTDIF- DELTSC*SUTUS)/SSUT
      IF (DELTKB .LT. -.95*KB ) DELTKB = -.95*KB
      IF (DELTKB .GT. 2.0*KB ) DELTKB = 2.0*KB
      KB = KB + DELTKB
      WRITE ( WOUT,830) KB, SC
      IF(PRFL) WRITE(IPTR,830) KB,SC
  830 FORMAT( ' BEST FIT KB AND SC THIS ITERATION ARE ', F15.8,5X,F10.9 )
      ICOUNT = ICOUNT + 1
C
C           IF IT DID NOT CONVERGE FOR THE GIVEN DATA SET,
C                      PRINT  THE  MESSAGE
C
       IF ( ICOUNT .GT. NITER) GO TO 1020
C
C          IF THERE IS NO CONVERGENCE AND ITERATION LIMIT
C               NOT EXCEEDED THEN DO OTHER ITERATION.
C
       IF(ABS((TEMPKB-KB)/KB) .LT. TOL .AND. SC .EQ. 1.0 .OR.
     & ABS((TEMPKB-KB)/KB) .LT. TOL .AND. SC .LT. 1.0E-15)
     & NKB=NKB+1
       IF(NKB .GE. 1) GO TO 1060
       IF(ABS((TEMPKB-KB)/KB).GT.TOL.OR.ABS((TEMPSC-SC)/SC).GT.TOL)
     & GO TO 680
C
C          IF CONVERGENCE SUCCESSFUL THEN PRINT THE "BEST FIT"
C                     TIME-DRAWDOWN DATA PAIRS.
C
     IF (TITL .NE. 'YES') GOTO 837
     WRITE(WOUT,305) TITLE
     IF(PRFL) WRITE(IPTR,305) TITLE
  837 CONTINUE
     WRITE(WOUT,99)
     WRITE ( WOUT,840)DATYP
     IF(PRFL) WRITE(IPTR,840) DATYP
  840 FORMAT (//' *** THE  BEST  FIT   TIME_',A8,'  PAIRS  ARE  ***  ',/)
C
```

```
C          RE-INITIALIZE  THE  ARRAYS  AND  COMPUTE   BEST-FIT
C          TIME-DRAWDOWN PAIRS FOR CONVERGED VALUES OF SC & KB.
C
      NCONV=NCONV+1
      IF(NCONV.GE.1)GO TO 690
  850 CONTINUE
      DO 860 I = 1,NTDP
      DO 859 IP= 1,NPW
      TC=T(I)
      IF ( NPW .GT. 1 ) GO TO 859
      IF(TC .GT.TSTOP(IP))TC=TC-TSTOP(IP)
  859 CONTINUE
      WRITE(WOUT,560)TC,(SP(I,J),J=1,NOW)
      IF(PRFL) WRITE(IPTR,560) TC,(SP(I,J),J=1,NOW)
  860 CONTINUE
  870 CONTINUE
      WRITE(WOUT,872)
  872 FORMAT(/60(1H*)/)
      WRITE(WOUT,880)KB,SC
      IF(PRFL) WRITE(IPTR,880) KB,SC
  880 FORMAT(/' "BEST" FIT TRANSMISSIVITY (L**2/T) IS  =    ',F15.8,/,
     & ' "BEST" FIT  STORAGE COEFFICIENT  IS    =          ',F10.9)
C
C          CALCUATE THE rms ERROR IN THE "BEST FIT" DRAWDOWN.
C
      SUM =      0.0
      SUMSP =    0.0
      SUMS =     0.0
      SUMSPSQ =  0.0
      SUMSSQ  =  0.0
      SUMSPS  =  0.0
      DO 900 I = 1, NTDP
      DO 890 J =1,NOW
      SUM =        SUM + (SP(I,J)-S(I,J))**2
      SUMSP =      SUMSP+SP(I,J)
      SUMS  =      SUMS+S(I,J)
      SUMSPSQ =    SUMSPSQ+SP(I,J)**2
      SUMSSQ =     SUMSSQ+S(I,J)**2
      SUMSPS =     SUMSPS+SP(I,J)*S(I,J)
  890 CONTINUE
  900 CONTINUE
  910 CONTINUE
      SIGMA = DSQRT(SUM/DBLE(FLOAT(NTDP*NOW)))
      IF ( CONSTDRA .EQ. 'YES' ) GO TO 930
C
C        COMPUTE CORRELATION BETWEEN FIELD OBSERVED DATA AND "BEST FIT"
C        DATA.
C         (IF THE DATA IS OF CONSTANT-DRAWDOWN TYPE THEN SKIP THIS STEP)
C
C
      COR=(SUMSPS-((SUMSP*SUMS)/DBLE(FLOAT(NTDP*NOW))))/(DSQRT(SUMSPSQ-(SUMSP*
/
     & DBLE(FLOAT(NTDP*NOW))))*DSQRT(SUMSSQ-(SUMS**2/DBLE(FLOAT(NTDP*NOW)))))
      WRITE(WOUT ,920) DATYP,SIGMA,COR
      IF(PRFL) WRITE(IPTR,920) DATYP,SIGMA,COR
  920 FORMAT(/'  THE rms ERROR  FOR ',A8,' (L) IS    =     ', F10.5 ,/,
     & '  THE CORRELATION COEFFICIENT IS         =         ',F7.5/)
      WRITE(WOUT,872)
      IF(PRFL) WRITE(IPTR,872)
      GO TO 950
  930 CONTINUE
      WRITE(WOUT,940)DATYP,SIGMA
      IF(PRFL) WRITE(IPTR,940) DATYP,SIGMA
```

```
  940   FORMAT(' THE rms ERROR FOR ',A8,' (L) IS   =     ',F10.5//)
  950   CONTINUE
        IF(SPCAP .NE. 'YES')GO TO 1100
C
C          SPECIFIC CAPACITY CALCULATIONS ONLY PERFORMED WHEN:
C          a) the observed drawdown is from the pumping
C             well itself; and
C          b) the number of observation wells is one and
C             is located at a radial distance equal to
C             the radius of the pumping well.
C
        KSP = 1
        CALL SPECAP(WOUT,PRFL,IPTR,KSP,NPR,NTDP,T,TP,Q,SP,NOW,
     &              NOPW,NVQ,NDM,TLIM,SCAPCT,NOB)
        GO TO 1100
 1020   WRITE ( WOUT,1030)NITER
        IF(PRFL) WRITE(IPTR,1030)NITER
 1030   FORMAT(' DID NOT CONVERGE IN'  ,I2,  'ITERATIONS    ')
        IF(ICOUNT .GT. NITER)GO TO 1100
 1040   CONTINUE
        WRITE(WOUT,1050)
        IF(PRFL) WRITE(IPTR,1050)
 1050   FORMAT(' SENSITIVITY COEFFICIENTS TOO SMALL (or zero) ')
        WRITE(WOUT,1055)SSUS,SSUT,SUTUS
        IF(PRFL) WRITE(IPTR,1055)SSUS,SSUT,SUTUS
 1055   FORMAT(' THE SENSITIVITY COEFFICIENTS ARE : ',//,
     &  ' SSUS  =    ',E20.10,/' SSUT  =    ',E20.10,/' SUTUS  =    ',E20.10,//)
        GO TO 1100
 1060   WRITE(WOUT,1070)DATYP
        IF(PRFL) WRITE(IPTR,1070)
 1070   FORMAT(  / ' STORAGE COEFFICIENT (SC) CANNOT BE COMPUTED FOR GIVEN ',A8,'
DATA')
        WRITE(WOUT,1080)DATYP
        IF(PRFL) WRITE(IPTR,1080)DATYP
 1080   FORMAT(  / '   **** DIAGNOSTICS *** '//,
     &  ' Radial distance (r) too small and/or time of observation (t) ',/
     &  ' too large. The argument of exponential approaches zero. Hence ',/
     &  ' the value of transmissivity is only computed for the ',A8,' data ')
C
C
        WRITE(WOUT,1090)KB
        IF(PRFL) WRITE(IPTR,1090)KB
 1090   FORMAT(//' "BEST" FIT TRANSMISSIVITY  (L**2/T) IS   =   ',F15.8/)
        WRITE(WOUT,1094)
        IF(PRFL) WRITE(IPTR,1094)
 1094   FORMAT(56(1H*)/)
 1100   WRITE(WOUT,1110)
 1110   FORMAT(' DO YOU WANT TO TRY OTHER GUESS VALUES FOR SC AND KB. ? ',/,
     &  ' (Answer YES or NO )  =    ')
        READ(RIN,40)TRIAL
        IF(TRIAL.EQ.'YES')GO TO 1120
        GO TO 1150
 1120   CONTINUE
        WRITE(WOUT,1130)
 1130   FORMAT(' NEW ESTIMATE FOR SC ?  =    ')
        READ(RIN,*)SC
        WRITE(WOUT,1140)
 1140   FORMAT(' NEW ESTIMATE FOR KB ?  =    ')
        READ(RIN,*)KB
        NEWTRIAL=NEWTRIAL+1
        GO TO 670
 1150   STOP
 1160   WRITE(WOUT,1152)
```

```
 1152  FORMAT(45H    ***  RESET THE PARAMETERS NDM,NOB,NVQ  ***)
       STOP
       END
C
C
C               ******************************
C               *                            *
C               *     SUBROUTINE  THEISVQ    *
C               *                            *
C               ******************************
C
C
C
        SUBROUTINE THEISVQ(SC,KB,Q,TT,R,SE,DSDTE,DSDSCE,UNIT,WOUT)
C
C
C
C         PURPOSE:
C
C                 CALCULATE THE DRAWDOWN FOR A WELL PUMPING
C                 AT A CONSTANT OR VARIABLE DISCHRAGE  FROM
C                 AN  INFINITE  AQUIFER  USING  THE  THEIS
C                 EQUATION FROM 'INTRODUCTION TO HYDROLOGY'
C                 BY VIESSMAN, KNAPP, LEWIS AND HARBAUGH,
C                 (PAGE 328) IEP, 1977.
C                 THE EQUATION  IS  SOLVED  USING  RATIONAL
C                 FROM 'HANDBOOK OF MATHEMATICAL FUNCTIONS'
C                 BY ABRAMOWITZ AND STEGUN, PAGE 231.
C
C           ARGUMENTS (L IS ARBITRARY LENGTH, T IS ARBITRARY TIME)
C
C         SC  -> STORAGE COEFFICIENT FOR AQUIFER (UNITLESS)
C         KB  -> TRANSMISSIVITY OF AQUIFER (GAL/DAY/FT) OF (L**2/T)
C          Q  -> VARIABLE (OR CONSTANT) PUMPAGE OF WELL (GAL/DAY) OR (L**3/T)
C         TT  -> OBSERVATION TIME (DAYS) OR (T)
C          R  -> OBSERVATION DISTANCE FROM WELL (FT) OR (L)
C         SE  -> DRAWDOWN (FT) OR (L)
C      DSDTE  -> SENSITIVITY W.R.T. TRANSMISSIVITY (FT**2*DAY/GAL) OR (T/L)
C     DSDSCE  -> SENSITIVITY W.R.T. STORAGE (FT) OR (L)
C       UNIT  -> SYSTEM OF UNITS; YES FOR GAL, DAY, FT; OTHERWISE
C                CONSISTENT UNITS ARE ASSUMED
C ***********************************************************************
C
C          CALCULATE THEIS SOLUTION
C
       REAL*8 KB
       IMPLICIT DOUBLE PRECISION(A-H,O-Z)
       INTEGER WOUT
       CHARACTER*3 UNIT
       U=(R**2*SC)/(4.*TT*KB)
       IF(UNIT .EQ. 'YES')U = 7.48*U
       IF(U .LE.0.D0)GO TO 3
       IF(U .GT. 1.0)GO TO 1
       W=-DLOG(U)-.57721566+.99999193*U-.24991055*U*U+
     & .05519968*U*U*U-.00976004*U**4+.00107857*U**5
       GO TO 2
    1  W=(EXP(-U)/U)*(U*U+2.334733*U+.250621)/
     & (U*U+3.330657*U + 1.681534)
    2  CONTINUE
       SE = (Q/(4.0*3.14159*KB))*W
       DSDTE = (Q/(4.0*3.14159*KB**2))*(-W + EXP(-U))
       DSDSCE=-(Q/(4.0*3.14159*KB*SC))*EXP(-U)
       RETURN
```

```
  3   WRITE(WOUT,4)R,SC,TT,KB
  4   FORMAT(' DRAWDOWN UNDEFINED U TOO SMALL. TERMINATE EXECUTION',
   &        /' SOME VARIABLES OF POSSIBLE INTEREST ARE:'/
   &         '       R = P.W. RADIUS     =',E12.6/
   &         '       SC= STORAGE FACTOR =',E12.6/
   &         '       TT= TIME             =',E12.6/
   &         '       KB= TRANSMISSIVITY =',E12.6)
      STOP
      END
C
C                  ******************************
C                  *                            *
C                  *    SUBROUTINE    SPECAP    *
C                  *                            *
C                  ******************************
C
C
     SUBROUTINE SPECAP (WOUT,PRFL,IPTR,NPW,NPR,NTDP,T,TP,Q,SP,
    &                  NOW,NOPW,NVQ,NDM,TLIM,SCAPCT,NOB)
C
C
C    NOTE: IN THIS SUBROUTINE NPW IS THE CURRENT PUMPING WELL
C
C
C     PURPOSE:
C
C     PERFORM SPECIFIC CAPACITY COMPUTATIONS FOR PUMPING WELLS
C
C
     IMPLICIT DOUBLE PRECISION(A-H,O-Z)
     INTEGER WOUT,TLIM(NVQ)
     DIMENSION Q(NOPW,NVQ+1),T(NDM),SCAPCT(NDM),SP(NDM,NOB),
    & TP(NOPW,NVQ+1)
     LOGICAL PRFL
C
C            PRINT HEADINGS
C
     WRITE(WOUT,10) NPW
  10 FORMAT(//11X,'SPECIFIC CAPACITY COMPUTATIONS FOR '
    & ,'PUMPING WELL No: ',I3,//
    & 79('*')/9X,'TIME',14X,'DRAWDOWN',11X,'PUMPAGE',11X,
    & 'SPECIFIC'/64X,'CAPACITY'/10X,'(T)',17X,'(L)',13X,
    & '(L**3/T)',10X,'(L**2/T)'/)
     IF(PRFL) WRITE(IPTR,10) NPW
C
C          DETERMINE NUMBER OF TIME-DRAWDOWN DATA PAIRS
C            BETWEEN PUMPAGE TIME INTERVALS FOR EACH
C                        PUMPING WELL
C
     KK = 0
     TLIM(1) = 1
     DO 40 I = 1,NPR
     DO 30 J = 1,NTDP
     IF (T(J) .GT. TP(NPW,I)) GOTO 30
     KK = KK + 1
  30 CONTINUE
     TLIM(I+1) = KK
     KK = 0
  40 CONTINUE
C
C           SPECIFIC CAPACITY COMPUTATIONS:
C
     DO 80 II = 1,NPR
```

```
      DO 70 I  = TLIM(II) , TLIM(II+1)
      SCAPCT(I) = Q(NPW,II)/SP(I,NOW)
      WRITE(WOUT,60) T(I),SP(I,NOW),Q(NPW,II),SCAPCT(I)
      IF(PRFL) WRITE(IPTR,60) T(I),SP(I,NOW),Q(NPW,II),SCAPCT(I)
   60 FORMAT(4F18.8)
   70 CONTINUE
      TLIM(II+1) = TLIM(II+1) + 1
   80 CONTINUE
   90 CONTINUE
      WRITE(WOUT,100)
  100 FORMAT(/79('*'))
C
      RETURN
      END
```