Automated Pumping-Test Analysis for Some Typical Models

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Abstract

Estimation of aquifer parameters usually involves fitting experimental pumping-test data to a standard theoretical curve(s) by visual comparison to obtain a match point for solving the appropriate groundwater formula. A disadvantage to the visual method of curve fitting is the inability to obtain a unique fit and an estimate of the error. By use of an automated curve fitting procedure, the pumping-test data may be fitted to a standard curve and the 'best' aquifer parameters obtained, based on some fitting criteria. The purpose of this paper is to present a generalized procedure for fitting the experimental test data. The aquifer models considered are the classical non-leaky confined aquifer, the semi-confined leaky aquifer, confined aquifer with one boundary, and anisotropic confined aquifer. In the fitting procedure, sensitivity analysis is used to predict the change in drawdown (or recovery) due to the changes in aquifer parameters and/or boundary condition. After the drawdown has been evaluated by analytical or numerical means, the sensitivity coefficients for aquifer parameters can be obtained with little extra effort. These sensitivity coefficients are combined with a least squares fitting technique to produce an algorithm for fitting the time-drawdown data to the theoretical aquifer type curve. The method of images and superposition is used in dealing with boundary situations. The method is not complex and is easily programmed for computer implementation. The automated analysis has the advantage of always being objective. The method also gives an indication of how well it has analyzed the data by calculating the root-mean-square error. Many sets of field data have been analyzed and the convergence properties appear to be good if the data set conforms to the model assumptions.

Introduction

Groundwater is a valuable resource and much money and effort are expended each year to collect various kinds of data. The data base is massive and it is clear that computer storage and manipulation are necessary to make full use of the data.

Theories and equations governing the flow of groundwater in porous media have been established and aquifer flow models are being employed to an increasing degree in the management of groundwater resources. The use of computers in the management of groundwater resources has been actively developed in the past 15 years. Numerical techniques have been developed in a format suitable for solution using computers.

Many pumping tests have been made through the years to help determine the aquifer parameters. Undoubtly many more pumping tests will be made in the future. The Kansas Geological Survey is pursuing an effort to automate some of the more common type-curve solutions for aquifer tests.

Solution of the inverse problem, that is the determination of aquifer parameters, usually involves fitting experimental pumping test data to a standard theoretical curve(s) **(slide #1)** by visual comparison to obtain a match point for solving the appropriate groundwater formula. Various visual curve fitting techniques are available and the disadvantage to the visual method of curve fitting is the inability to obtain a unique fit (and corresponding match point) and an estimate of the error. Some means of automating the analyses of these pumping tests would be desirable. By use of an automated curve fitting procedure, the pumping test data may be fitted to a standard curve and the 'best' aquifer parameters obtained, based on the goodness of fit.

The purpose of this paper is to present a generalized procedure for fitting the experimental test data. The aquifer models considered are:

i) classical non-leaky confined aquifer;

ii) confined aquifer with one boundary;

iii) semi-confined leaky aquifer; and

iv) anisotropic confined aquifer.

In the fitting procedure, sensitivity analysis is used to predict the change in drawdown (or recovery) due to the changes in aquifer parameters and/or boundary conditions.

The Groundwater Flow Equation

The groundwater flow equation for homogeneous, horizontal isotropic, non-leaky confined aquifer, infinite in areal extent and of uniform thickness throughout is expressed as:

$$\frac{\partial}{\partial x}\left(\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial h}{\partial y}\right) = \frac{S}{T}\frac{\partial h}{\partial t}$$

where T and S are aquifer parameters, namely transmissivity and storage coefficients;

h is the hydraulic head;

t is the time.

Sensitivity Analysis

When treating groundwater flow mathematically, it is permissible to speak of precise values of physical parameters. However, when dealing with real world situations, one is immediately faced with the uncertainty of these parameters. Tolerances for parameter uncertainty must be established such that model results are insensitive to these errors. Parametric sensitivity analysis can be used to establish these tolerances.

In studying the sensitivity of a groundwater flow system to parameter variations, the mathematical model used is (**slide #2**)

$$F(h_{xx}, h_{yy}, h_t;\ T,\ S,\ Q) = 0$$

The solution to this equation may be expressed as

$$h = h(x,y,t;\ T,\ S,\ Q) \qquad 1$$

Suppose that we choose to determine the sensitivity with respect to transmissivity T. Varying this parameter by a small increment ΔT produces a new model

$$F(h^*_{xx}, h^*_{yy}, h_t;\ T+\Delta T,\ S,\ Q) = 0$$

and the new solution h* is

$$h^* = h^*(s,y,t;\ T+\Delta T,\ S,\ Q) \qquad 2$$

Comparing the solutions of these two equations (eq. 1 and eq. 2) the stability of the system can be written as the numerical difference between perturbed and unperturbed head over the variation ΔT. If the quantity $\Delta h/\Delta T$ approaches a limiting value as ΔT approaches zero (**slide #3**), we may write $\frac{\partial h}{\partial T}$ and describe this result as the sensitivity coefficieint with respect to transmissivity U_T. Similar arguments apply for the determination of the sensitivity coeficient with respect to storage coefficient. We may expand perturbed head h* (eq. 2) as a Taylor's series in each parameter for which we have determined the sensitivity. If we assume that the parameter variation is small, second and higher orders may be neglected in each expansion. This results in a linear equation which will allow computation of a new head resulting from a small perturbation of the aquifer parameters.

The two equations (eqs. 3 and 4) indicate that if it is possible to evaluate U_T and U_S for a given model, then the response of the model to perturbations of the aquifer parameters can be obtained from the Taylor expansions, without having to evaluate the model equations again.

Least Squares Fit

The object is to obtain the 'best' values for aquifer parameters by using the sensitivity formalism and the experimental pumping test data. In order to apply this procedure, we define the squared error function (**slide #4**) where h_e is experimental or observed drawdown and h* is updated or theoretical drawdown computed from the truncated Taylor series. Expanding the error function, taking partial derivatives with respect to the perturbed parameters and setting the partials to zero (for minimization) yields a set of simultaneous equations which must be solved in order to obtain the best fit. The values obtained for ΔT and ΔS are used to update the previous values of T and S. If the delta values are less than some predetermined error parameter, the procedure terminates. If the delta values are larger than some predetermined error parameter, the updated values of T and S are used to generate a new set of sensitivity coefficients and theoretical drawdowns h*. The system of equations is again solved for delta parameters and the procedure is repeated until the delta values satisfy the error parameter. Root Mean Square ereror is a measure of the accuracy in fitting the measured drawdowns. The iteration procedure is diagrammed in the following slide (**slide #5**).

Convergence may not occur if initial guess values of aquifer parameters are especially bad. However numerical experiments indicate that good convergence may be obtained even if the initial values are under estimated or over estimated by about two orders of magnitude, from their true values.

Application

Confined Aquifer:

Let us first consider the analysis of time-drawdown data for non-leaky confined aquifer. The groundwater flow equation and its analytical solution by the classical Theis equation in the radial coordinates is given by (**slide #6**) the first two equations on this slide.

where Q is the constant pumping rate;

r is the radial distance of the observation well;

s is the drawdown at time 't' and the remaining parameters are the same as described earlier

Sensitivity coefficients U_T and U_S for non-leaky aquifer were obtained in analytical form, by simply differentiating the Theis Equation with respect to T or S.

Several sets of data have been analyzed successfully. A particular example is the evaluation of an aquifer test made in Gridley, Illinois **(slide #7)** which was taken from Walton. Shown here is a comparison of the results of aquifer parameters obtained by visual matching and computer regression. The aquifer parameters shown here were used along with the time values from Walton to compute the drawdown curves which were compared to experimental curves. The root mean square error was computed for each data set relative to experimental data. Visually matched values had higher rms error than that of the regressed values.

Distance-Drawdown Data

Aquifer parameters may also be evaluated from observations of more than one well, by use of distance-drawdown data **(slide #8)**. The second example shown here gives the regressed and visual curve fitting values of S and T. This particular example was also taken from Walton.

Time-Recovery Data

The aquifer parameters T and S can also be analyzed from pump test recovery data **(slide #9)**. The principle of superposition is used in the recovery of a well after pumping has stopped. The residual drawdown 's' at a radial distance 'r' after the pump shut down is given by the equation where t is the time since start of pumping and t' is the time since pump shut down.

(Slide #10:) Analysis of a 7-day aquifer test drawdown and recovery data of a group of wells in Texas by the regression technique yielded aquifer parameters which appear in close agreement with one another. The low RMS error indicates a good fit. This example is typical of several analyses we have performed.

Confined Aquifer with One Boundary

(slide #11:) For the analysis of pump test data operating in the vicinity of constant head or barrier boundaries, the method of images along with the principle of superposition is used. Field data for analysis of pumping test data for wells near barrier **(slide #12)** and constant head (or flow boundaries) **(slide #13)** was taken from Heath and Trainer. The converged values show smaller RMS error than those obtained by graphical analysis.

Analysis of Boundary Distances

(slide #14:) Impermeable or semi-permeable boundaries, may at times be completely hidden from view and their presence may not be suspected until the aquifer test data are plotted. Such hidden boundaries exist, for example, along the sides of buried valleys and along buried faults. The nature of the aquifer boundaries is readily apparent from the data plots. The sensitivity formalism has been successfully extended to determine the position of the boundary. In addition to guess values for S and T, the model requires some reasonable guess values for distance between pumping well and its image well. **(slide #15:)** As an example for the determination of aquifer parameters and the distance to the constant head boundary, drawdown data for three observation wells were obtained from Heath and Trainer. **(slide #16:)** Different guess values of T, S, and distance to constant head boundary yielded almost identical final values with low RMS error which compared very well to those found by graphical type curve methods.

The method works very well provided at least three observation wells at different distances and different directions from the pumped well are available. The requirement of a minimum of three observation wells is to define the boundary direction.

Semi-Confined Leaky Aquifer

The leaky artesian aquifer equation as originally stated **(slide #17)** defines the specific leakage as the ratio of the vertical permeability of the aquitard p' to its saturated thickness m. The variable B is defined as the square root of the ratio of the aquifer transmissivity T to the specific leakage. The remaining parameters are identical to the Theis equation. For convenience the parameter 1/B is replaced by L which we defined as the modified coefficient of leakage. This modification allows the leakage to be set to zero easily. Otherwise, B would have to be set to infinity. For L equal to zero the equation reduces to the Theis Equation.

Sensitivity coefficients U_T and U_S were obtained in analytical form, as described earlier. U_L was obtained by using a finite difference scheme.

(**slide #18:**) Several sets of data have been analyzed. The graphical analysis values are compared with corresponding automated-analysis values of the same set. RMS errors were computed for each data set relative to the experimental data. The automated RMS error indicates the goodness of fit between the experimental data and the theoretical type curve.

Anisotropic Confined Aquifer

Anisotropy (i.e., the dependence of hydraulic conductivity on the direction of measurement at a point in geologic formation) can be analyzed by use of sensitivity analysis. Anisotropy in aquifers may be due to different lithology of the geologic deposits and formations. Presence of fractures, stream channels, weathered and unweathered zones, and variable amounts of silt and clay greatly effect the movement and distribution of aquifer water. The coefficients of transmissibility and storage coefficient of non-leaky anisotropic aquifers with fully penetrating wells may be determined from aquifer test data by use of type-curve graphical method devised by Papadopulos, provided three observation wells at different distances and directions from the pumped well are available.

The equation, as given by Papadopulos (**slide #19**) can be transformed in terms of field coordinates. Here T_{XX} and T_{YY} are the principal axis transmissivities and Θ is the angle between the principal axis and the field coordinate system. The sensitivity coefficients U_{TXX}, U_{TYY}, U_S and U_θ can be obtained analytically from this expression. (**slide #20**) In addition to the aquifer test data and location of observation wells, the model needs initial guess values for principal transmissivities T_{XX} and T_{YY} and a guess value for θ (i.e., angle between field coordinate system and principal axis system). A convenient rectangular coordinate system is chosen with the origin at the pumped well.

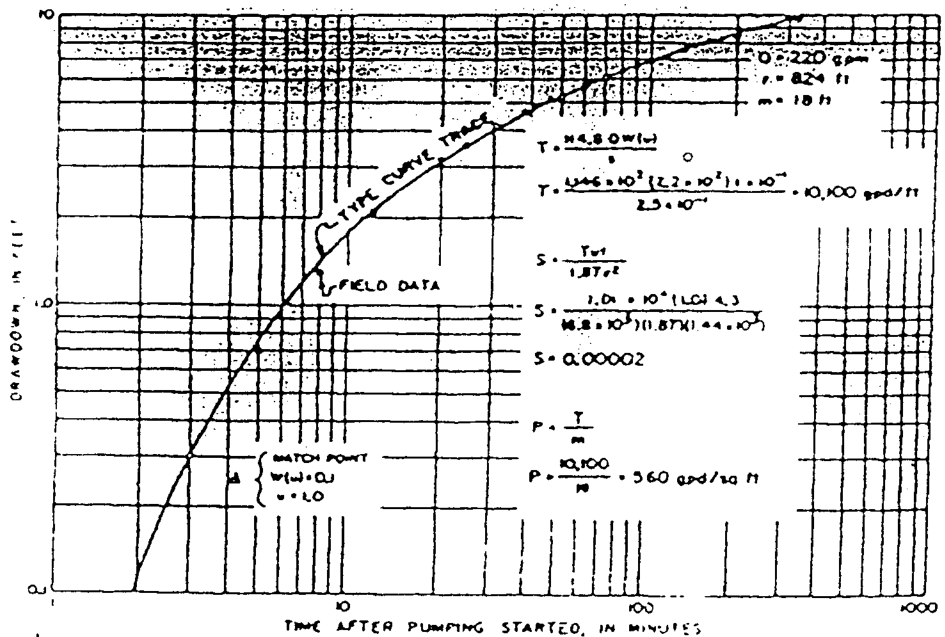
The procedure as explained earlier is repeated, until convergence is achieved. This model has been successfully tested on many hypothetical data sets with good results. At present, we are in search of some real anisotropic aquifer test data.

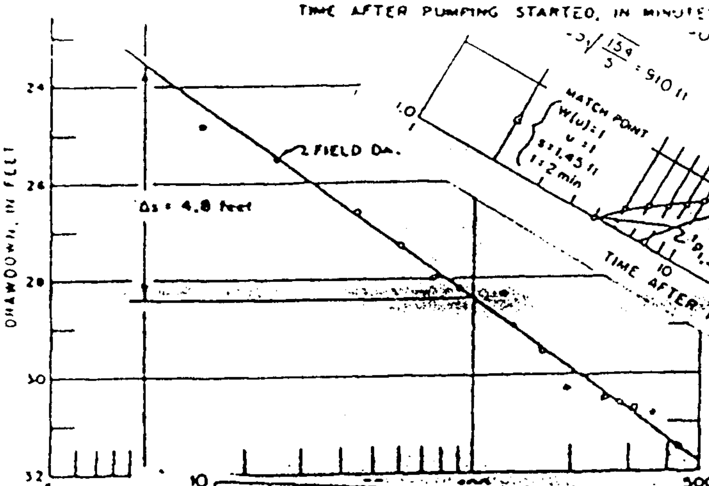
Summary

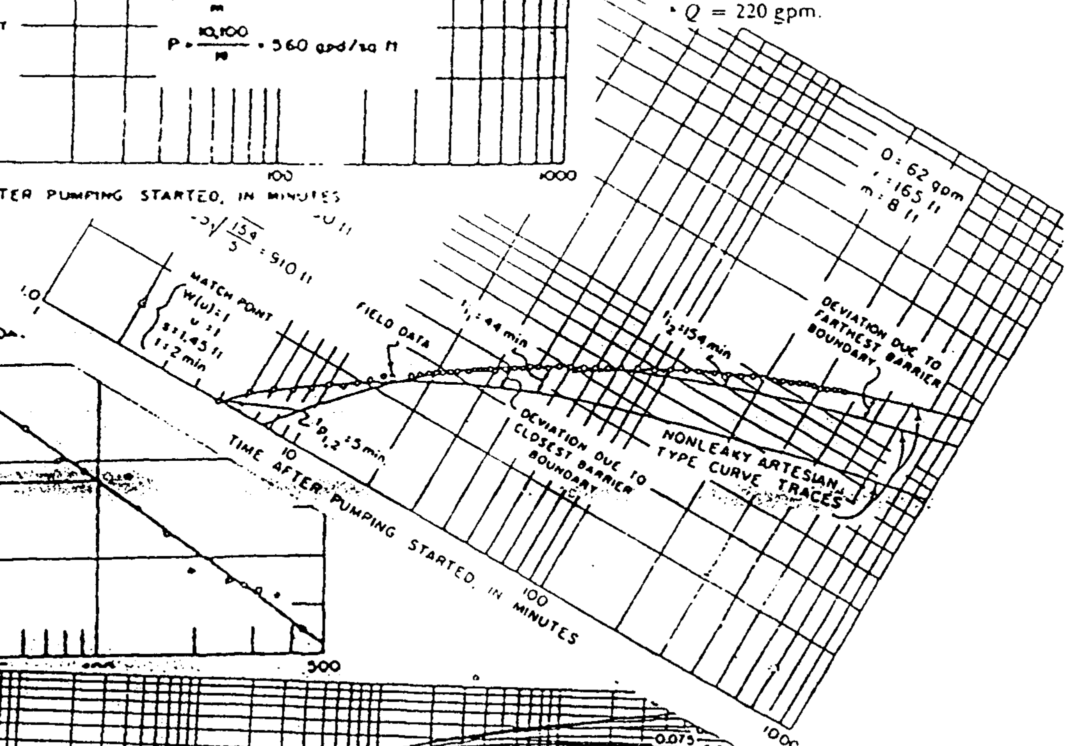
The algorithm presented here for the classical non-leaky confined aquifer, semiconfined leaky aquifer, confined aquifer with one boundary, and anisotropic confined aquifer is simple to use, inexpensive and yields accurate results quickly. The convergence properties are good and the initial guess values may vary two orders of magnitude above or below the correct values. Here, as in the solution of other hydrologic problems, considerable knowledge of the geology is necessary. The accuracy of the fit may be judged from RMS error. For high RMS error, the data probably represent a different hydrologic situation. The automatd curve fitting procedure presented here should be a useful addition to the aquifer analysis techniques available to hydrologists. Although such solutions only approximate field conditions, when applied with discretion they provide practical answers to well-hydraulics problems. This method of pumping-test analysis does not remove the requirement of having an experienced hydrologist evaluate the local hydrogeology and pumping-test data to identify the aquifer type. However, once it is decided what aquifer configuration is being observed, this method will, in quick and unbiased fashion, give an accurate assessment of the aquifer parameters within the limits of theoretical approximation.

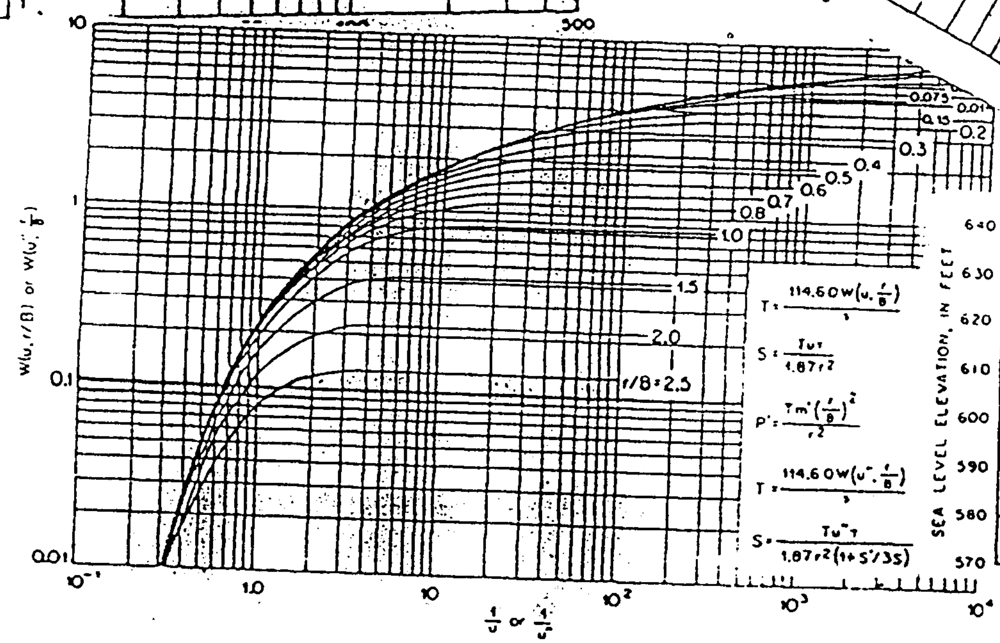
Time after pumping started, min	Drawdown, ft
15	24.8
25	25.5
45	26.6
60	27.3
76	28.0
90	28.2
132	29.0
166	29.5
195	30.3
256	30.5
283	30.6
314	30.7
360	30.8
430	31.5

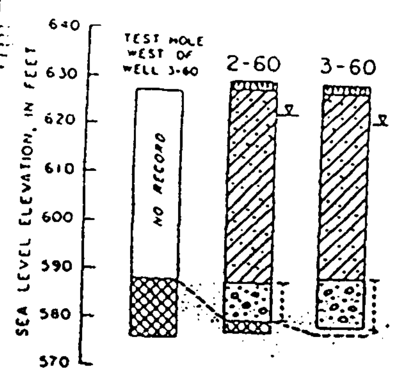
* Q = 220 gpm.











Slide #1

$$F(h_{xx}, h_{yy}, h_t; T, S, Q) = 0$$

$$h = h(x, y, t; T, S, Q) \qquad 1$$

$$F(h^*_{xx}, h^*_{yy}, h^*_t; T + \Delta T, S, Q) = 0$$

$$h^* = h^*(x, y, t; T + \Delta T, S, Q) \qquad 2$$

Slide #2

$$\frac{\Delta h}{\Delta T} = \frac{(h^{*} - h)}{\Delta T}$$

$$\lim_{\Delta T \to 0} \frac{\Delta h}{\Delta T} = U_T(x, y, t; T, S, Q)$$

$$h^{*} = h + U_T \Delta T \qquad 3$$

$$h^{*} = h + U_S \Delta S \qquad 4$$

$$h^{*} = h + U_T \Delta T + U_S \Delta S$$

Slide #3

$$\text{ERROR} = \sum_i [h_{e(ti)} - h^*_{(ti)}]^2$$

$$\frac{\partial\, \text{ERROR}}{\partial\, \Delta T} = 0 \quad ; \quad \frac{\partial\, \text{ERROR}}{\partial\, \Delta S} = 0$$

$$T^{i+1} = T^i + \Delta T^i$$

$$S^{i+1} = S^i + \Delta S^i$$

$$\text{R.M.S. ERROR} = \sqrt{\frac{\text{ERROR}}{N}}$$

Slide #4

Iteration Procedure

1) Initial guess for T and S
2) Calculate expected drawdown values
3) Calculate sensitivity coefficients U_T and U_S
4) Determine ΔT, ΔS
5) Upgrade values for T, S
6) If ΔT, ΔS are less then ε, stop
7) If maximum number of iterations is not exceeded, go to step 2

Slide #5

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} = \frac{S}{T}\frac{\partial h}{\partial t}$$

$$s = \frac{Q}{4 \pi T} \int_{u}^{\infty} \frac{e^{-u}}{u} \, du$$

$$U_T = \frac{\partial s}{\partial T} \; ; \quad U_S = \frac{\partial s}{\partial S}$$

$$u = \frac{r^2 S}{4 T t}$$

Slide #6

Q = 220 gpm | r = 824 ft

	Type Curve Values (Walton)	Regressed Values
T (gpd/ft)	10100	9910
S	2×10^{-5}	2×10^{-5}
R.M.S. (ft) ERROR	0.15	0.09

Slide #7

$Q = 500$ gpm $r_1 = 100$ ft

$r_2 = 1000$ ft $r_3 = 10000$ ft

	Type Curve Values (Walton)	Regressed Values
T (gpd/ft)	93000	94863
S	6×10^{-4}	5×10^{-4}
R. M. S. (ft) ERROR	0.041	0.025

Slide #8

$$s = \frac{Q}{4\pi T}\left[\int_{u}^{\infty} \frac{e^{-u}}{u}\,du - \int_{u'}^{\infty} \frac{e^{u'}}{u'}\,du'\right]$$

$$u' = \frac{r^2 S}{4 T t'}$$

t is the time since start of pumping

t' is the time since shut down

Slide #9

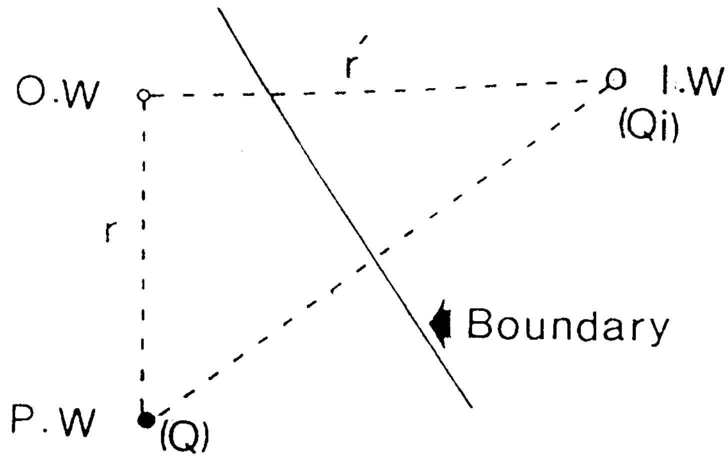
Well No.	r (ft)	T (gpd/ft)	S (10^{-3})	R.M.S (ft)
1	3800	33700	.63	.03
		* 35800	.56	.027
2	4700	36500	.57	.029
		* 37900	.56	.018
3	5250	47800	.55	.019
		* 47600	.56	.017

(* Recovery)

Well No.	r (ft)	T (gpd/ft)	S (10^{-3})	R.M.S (ft)
4	5750	48800	.54	.014
		* 46800	.58	.013
5	8200	32200	.35	.047
		* 35500	.34	.053

(* Recovery)

$$s = \frac{Q}{4\pi T} W(u) + \frac{\pm Q_i}{4\pi T} W(u')$$



Slide #11

BOUNDARY TYPE: BARRIER

$Q = 113.6\ ft^3/min$

Distance To Boundary = 422 ft

	Type Curve Values	Regressed Values
T (ft^2/min)	4.89	5.40
S (10^{-3})	0.50	0.37
R.M.S. (ft) ERROR	0.34	0.19

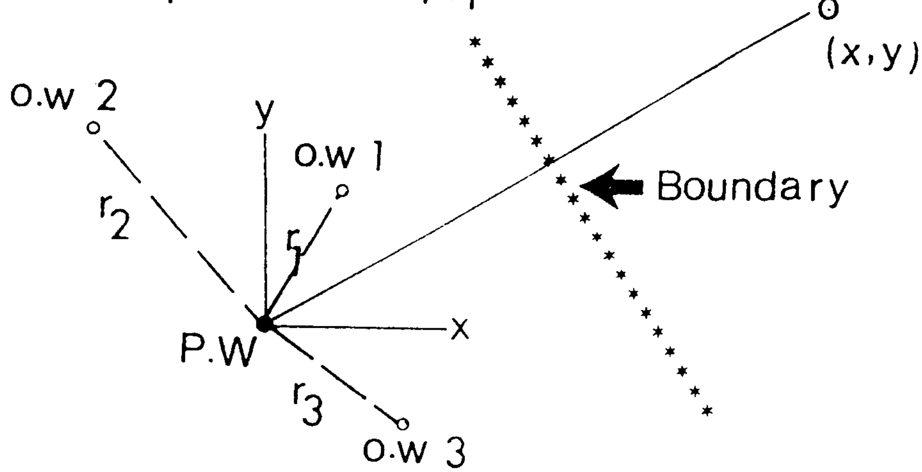
Slide #12

BOUNDARY TYPE : CONSTANT HEAD

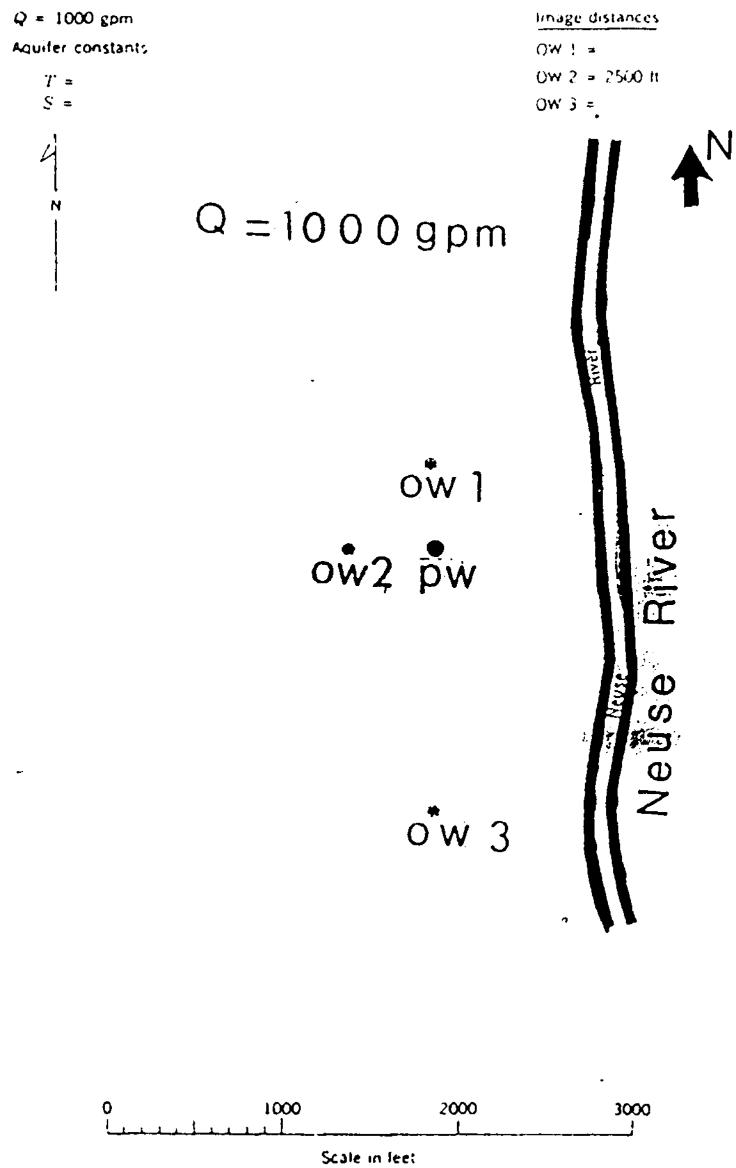
Type Curve	Q=1000 gpm	Regressed
OW#1		
T (gpd)	95500	97600
S (10^{-2})	.1	.09
RMS	.03	.01
OW# 2		
T		94900
S		.09
RMS		.004

Slide #13

$$s = \frac{Q}{4\pi T} W(u) + \pm \frac{Qi}{4\pi T} W(u')$$



Slide #14



Locations of pumping Well PW 1 and observation Wells OW 1, OW 2, and OW 3.

Slide #15

Graphical Analysis	Automated Analysis	Automated rms error
T = 9.41 ft^2/min	9.53	.036 ft
S = .00095	.00095	
R = 2084 ft	2055	

(R = normal distance between pumping well and boundary)

Slide #16

$$s = \frac{Q}{4\pi T} \int_{u}^{\infty} \frac{1}{y} e^{-\left[y + \frac{r^2}{4B^2 y}\right]} dy$$

$$s = \frac{Q}{4\pi T} \int_{u}^{\infty} \frac{1}{y} e^{-\left[y + \frac{L^2 r^2}{4y}\right]} dy$$

$$L = \frac{1}{B}$$

$$B = \sqrt{\frac{T}{p/m}}$$

U_T ; U_S ; U_L

Slide #17

Graphical Analysis	Automated Analysis	Automated rms error
T = 182000	202000[gpd/ft]	
S = .002	.002	.007 ft
B = 2500	3300 [ft]	
T = 99400	99026	
S = .0001	.000099	.046
B = 2000	1967	
T = 41000	46000	
S = .00008	.000084	.30
B = 4000	4800	

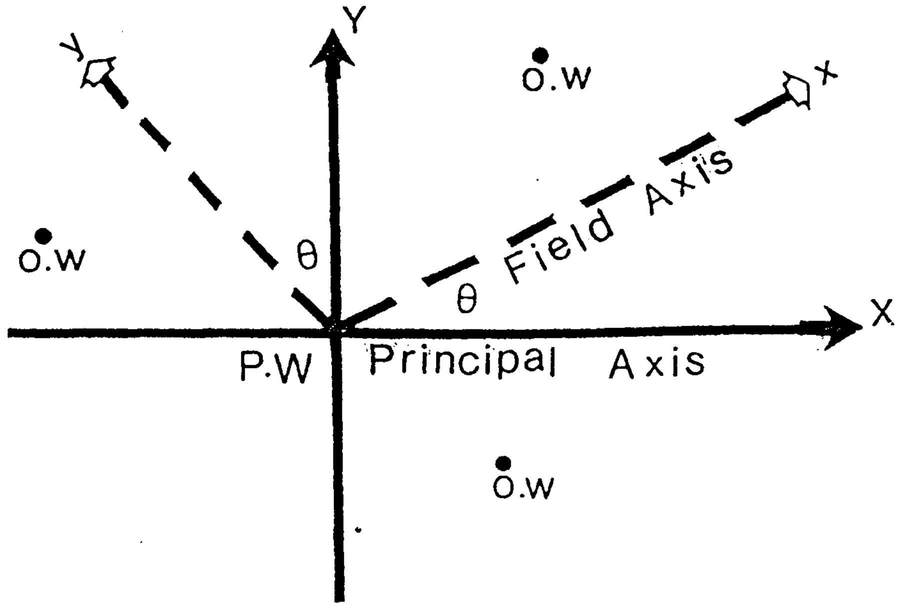
Slide #18

$$s = \frac{Q}{4\pi\sqrt{T_{XX}\,T_{YY}}}\int_{u_{XY}}^{\infty}\frac{e^{-u}}{u}\,du$$

$$u_{XY} = \frac{S}{4t}\,\frac{T_{XX}\,Y^2 + T_{YY}\,X^2}{T_{XX}\,T_{YY}}$$

$$U_{T_{XX}}\ ;\ U_{T_{YY}}\ ;\ U_S\ ;\ U_{\Theta}$$

Slide #19



Slide #20