THE THEIS EQUATION: EVALUATION, SENSITIVITY TO STORAGE AND TRANSMISSIVITY, AND AUTOMATED FIT OF PUMPTEST DATA

Carl D. McElwee

Kansas Geological Survey, University of Kansas,
Lawrence, Kansas 66044

Prepared for publication by
Kansas Geological Survey

Prepared by the Kansas Geological Survey
in cooperation with the U.S. Geological Survey

1977
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td>3</td>
</tr>
<tr>
<td>Introduction</td>
<td>4</td>
</tr>
<tr>
<td>The Theis Equation</td>
<td>4</td>
</tr>
<tr>
<td>Numerical Approximation</td>
<td>5</td>
</tr>
<tr>
<td>Sensitivity Analysis</td>
<td>6</td>
</tr>
<tr>
<td>Least Squares Fit</td>
<td>9</td>
</tr>
<tr>
<td>Program THEIS</td>
<td>15</td>
</tr>
<tr>
<td>Subroutine THEIS</td>
<td>17</td>
</tr>
<tr>
<td>Program THEISFIT</td>
<td>18</td>
</tr>
<tr>
<td>Discussion and Summary</td>
<td>25</td>
</tr>
<tr>
<td>References</td>
<td>28</td>
</tr>
<tr>
<td>Appendices</td>
<td>29</td>
</tr>
</tbody>
</table>
ABSTRACT

Traditionally, the Theis equation has played an important role in groundwater hydrology since its introduction. Comparison of experimental pump test data with this theoretical curve by graphical means has been a standard method of determining aquifer transmissivity and storage. The purpose of this paper is to present techniques and computer programs to evaluate the Theis equation, to evaluate the sensitivity with respect to transmissivity and storage, and to automatically fit experimental pump test data to the Theis equation obtaining the "best" transmissivity and storage in the least squares sense. The automated fit for pump test data developed in this work should be a useful tool for the groundwater hydrologist. It is simple to use, quick, and inexpensive. The automated fit has the advantage that it is always objective. As a measure of the error in fitting, the standard deviation in drawdown is calculated for the "best" transmissivity and storage.
INTRODUCTION

Traditionally, the Theis equation has played an important role in groundwater hydrology since its introduction (Theis, 1935). Comparison of experimental pump test data with this theoretical curve by graphical means has been a standard method of determining aquifer transmissivity and storage (Jacob, 1940). The purpose of this paper is to present techniques and computer programs to evaluate the Theis equation, to evaluate the sensitivity with respect to transmissivity and storage, and to automatically fit experimental pump test data to the Theis equation obtaining the "best" transmissivity and storage in the least squares sense. For a more detailed discussion of sensitivity coefficients and their uses see McElwee and Yukler (1977).

The Theis equation involves an integral whose upper limit is infinity. Evaluation of this integral is considered in the section on numerical approximation. After the Theis equation has been evaluated, the sensitivity coefficients can be obtained with little additional work. These sensitivity coefficients are used in the section on least squares fitting to develop an algorithm for fitting the Theis equation to experimental pump test data. The automated method is simple, quick, and inexpensive. The automated method has the advantage of always being objective and always indicating its error by calculating the standard deviation in drawdown.

THE THEIS EQUATION

The Theis equation (Theis, 1935) describes radial confined groundwater flow in a uniformly thick horizontal, homogeneous, isotropic
aquifer of infinite areal extent.

\[ \theta = \frac{Q}{4\pi T} \int_{\frac{r^2S}{4\pi T}}^{\infty} \frac{e^{-u}}{u} du \]  \hspace{1cm} (1)

The radius of the pumped well is assumed negligible (line source or sink approximation). The derivation and solution is documented many places and will not be discussed further here (Jacob, 1940). In the above equation \( s \) is drawdown (L), \( Q \) is the discharge (L\(^3\)/T), \( T \) is the transmissivity (L\(^2\)/T), \( t \) is the time (T), \( S \) is the dimensionless storage coefficient, and \( r \) is the radial observation distance from the pumped well (L).

Usually, the Theis equation is fitted graphically to experimental pump test data to obtain approximations for the storage coefficient (S) and the transmissivity (T). In this paper an algorithm will be presented for a computer automated least squares fit to the experimental data yielding approximations for \( S \) and \( T \) and giving the standard deviation for drawdown.

NUMERICAL APPROXIMATION

Many times the integral in equation (1) is symbolically represented by \( W(u) \). The drawdown can then be written as

\[ \theta = \frac{Q}{4\pi T} W(u) = \frac{Q}{4\pi T} W\left(\frac{r^2S}{4\pi T}\right) \]  \hspace{1cm} (2)

\( W(u) \) is the exponential integral and is tabulated in many places (Abramowitz and Segun, 1968). For specific values of \( u \) table interpolation
may be used to obtain the drawdown.

In order to evaluate equation (2) easily in an algorithm one needs an explicit expression for $W(u)$ involving only simple arithmetic operations. For $0 \leq u \leq 1$ (Abramowitz and Segun, 1968)

$$W(u) = -\ln u + a_0 + a_1 u + a_2 u^2 + a_3 u^3 + a_4 u^4 + a_5 u^5 + E(u)$$

where $a_0 = -0.57721566$ $\quad a_3 = .05519968$

$a_1 = .99999193$ $\quad a_4 = -.00976004$

$a_2 = -.24991055$ $\quad a_5 = .00107857.$

$E(u)$ is the error in the approximation.

For values of $u$ larger than one we use a rational approximation (Abramowitz and Segun, 1968).

$$W(u) = \frac{e^{-u}}{u} \left[ \frac{u^2 + a_1 u + a_2}{u^2 + b_1 u + b_2} + E(u) \right]$$

where $\left| E(u) \right| < 5 \times 10^{-5}$ for $1 \leq u < \infty$

$a_1 = 2.334733$ $\quad b_1 = 3.330657$

$a_2 = .250621$ $\quad b_2 = 1.681534$

The maximum error in $W(u)$ occurs for $u = 1$.

$$\left| \frac{E(1)}{e} \right| < 1.859 \times 10^{-5}$$

Therefore, we should always have at least four significant digits with these approximations.

**SENSITIVITY ANALYSIS**

In the mathematical treatment of dynamic systems it is permissible to speak of the precise values of the physical parameters. However,
in the practical simulation of real dynamic systems we are immediately faced with uncertainty as to the exact physical parameters. The investigator must establish tolerances within which the parameters of the physical system may vary without appreciably affecting the model results. These tolerances are often obtained by introducing parameter perturbations in the system and observing the changes in the system's performance. However, the application of sensitivity analysis makes it possible to obtain these tolerances more efficiently (Tomovic, 1962; Vemuri et al., 1969; McCuen, 1973; Yukler, 1976).

In studying the sensitivity of a groundwater flow system to parameter variations, the following mathematical model is used:

\[ F \left( h_{xx}, h_{yy}, h_t; T, S, Q \right) = 0 \]  \hspace{1cm} (5)

where

\[ \frac{\partial^2 h}{\partial x^2}, \quad \frac{\partial^2 h}{\partial y^2}, \quad \frac{\partial h}{\partial t} \]

\[ h = \text{hydraulic head}, \]
\[ T = \text{transmissivity}, \]
\[ S = \text{storage coefficient}, \]
\[ Q = \text{discharge} \]

The solution of equation (5) may be written in the form \( h = h(x, y, t; T, S, Q) \). Consider the variation of one of the parameters, \( T \) for example. Varying this parameter by a small amount, \( \Delta T \), the equation becomes

\[ F(h^*, h_{xx}^*, h_{yy}^*, h_t^*; T + \Delta T, S, Q) = 0, \]  \hspace{1cm} (6)

where \( h^* \) is the perturbed head. The solution to equation (6) may be written in the form \( h^* = h^*(x, y, t; T + \Delta T, S, Q) \). Comparing the solutions of equations (5) and (6), one immediately obtains an indication of the stability of the system, which is expressed by means of the fraction
\[
\frac{\Delta h}{\Delta T} = \frac{h^*(x,y,t;T+\Delta T,S,Q) - h(x,y,t;T,S,Q)}{\Delta T}
\] (7)

If expression (7) has a limiting value as \( \Delta T \) approaches zero, it may be written as

\[
U_T(x,y,t;T,S,Q) = \frac{\partial h}{\partial T} = \lim_{\Delta T \to 0} \frac{\Delta h}{\Delta T}
\] (8)

The function \( U_T(x,y,t;T,S,Q) \) will be called the sensitivity coefficient (Tomovic, 1962) for variations in the \( T \) value of a groundwater flow system. By applying similar arguments for a variation in storage coefficient \( \Delta S \) one obtains,

\[
\frac{\Delta h}{\Delta S} = \frac{h^*(x,y,t;T,S+\Delta S,Q) - h(x,y,t;T,S,Q)}{\Delta S}
\] (9)

and

\[
U_S(x,y,t;T,S,Q) = \frac{\partial h}{\partial S} = \lim_{\Delta S \to 0} \frac{\Delta h}{\Delta S}.
\] (10)

\( U_S \) is the sensitivity coefficient for variations in the storage coefficient of a groundwater flow system.

It is assumed that the solution of the flow equation (5) depends analytically upon the parameters \( T \) and \( S \); and, that \( T, S \), and \( Q \) are independent of each other. Now consider a perturbation of the transmissivity, \( \Delta T \). Since it has been assumed that the solutions depend analytically on the parameters, the function \( h^*(x,y,t;T+\Delta T,S,Q) \) may be expanded into a Taylor series (Tomovic, 1962). If \( \Delta T \) is small the second and higher order terms may be neglected,

\[
h^*(x,y,t;T+\Delta T,S,Q) = h(x,y,t;T,S,Q) + \frac{\partial h}{\partial T} \Delta T
\] (11)

Thus, the new head produced by a perturbation in transmissivity \( \Delta T \) may be calculated from equation (11) if the sensitivity coefficient and the unperturbed head are known. Similarly, if a perturbation in
storage coefficient ($\Delta S$) occurs the perturbed head is given by
\[
h^*(x,y,t;T,S+\Delta S,Q) = h(x,y,t;T,S,Q) + \frac{\partial h}{\partial S} \Delta S
\]

\[
= h(x,y,t;T,S,Q) + U_S \Delta S
\]

to first order in $\Delta S$.

Equations (11) and (12) show that it would be desirable to calculate $U_T$ and $U_S$ for a given model, if possible. Then the response of the model to various perturbations could be calculated simply from equation (11) or (12) without actually evaluating the model equations again.

The sensitivity coefficients may be obtained from equations (1) by applying the definitions given in equations (8) and (10). After applying Leibnitz's rule for differentiating an integral (Hildebrand, 1962) to equation (1) one obtains
\[
U_T = \frac{\partial \alpha}{\partial T} = -\frac{\alpha}{T} + \frac{Q}{4\pi T^2} \exp\left[-\frac{r^2}{4Tt}\right]
\]
and
\[
U_S = \frac{\partial \alpha}{\partial S} = -\frac{Q}{4\pi T S} \exp\left[-\frac{r^2}{4Tt}\right]
\]

These equations for the sensitivity coefficients may be evaluated quite easily. $U_T$ and $U_S$ calculated from equations (13) and (14) may be used in equations (11) and (12) to calculate what the drawdown would be if $S$ and $T$ were changed by $\Delta S$ and $\Delta T$ respectively. Other work (McElwee and Yukler, 1977) indicates that equations (11) and (12) are valid for $\Delta S$ and $\Delta T$ less than or roughly equal to twenty percent of $S$ or $T$ respectively.

LEAST SQUARES FIT

The objective is to use the sensitivity formalism to obtain a least squares fit of experimental pump test data to the Theis equation.
and thus obtain the "best" estimate for S and T. For a review of the least squares technique the reader is referred to Carnahan et al. (1969).

The new drawdown, after changing T and S by ΔT and ΔS respectively, is given by

$$\Delta^* = \Delta \omega + U_T \Delta T + U_S \Delta S$$  \hspace{1cm} (15)

Equation (15) is obtained from equations (11) and (12) by observing that

$$s = h_o - h$$, where \(h_o\) is the original head before pumping starts and is a constant independent of T and S.

Let \(s_e(t)\) represent the experimentally measured drawdowns. Suppose it is possible to guess a reasonable \(S\) and \(T\) and let \(s(t)\) denote the drawdowns calculated from the Theis equation with these parameters. One would like to change the original guess by ΔS and ΔT in such a way that a better fit of the experimental data results. This is done by minimizing the following error function.

$$ERROR = \sum_i \left[ \omega_e(t_i) - \omega^*(t_i) \right]^2 = \sum_i \left[ \omega_e(t_i) - \omega(t_i) - U_T(t_i) \Delta T \right.$$

$$\left. - U_S(t_i) \Delta S \right]^2$$

$$= \sum_i \left[ \omega_e(t_i) - \omega(t_i) \right]^2 - 2 \Delta T \sum_i U_T(t_i) \left[ \omega_e(t_i) - \omega(t_i) \right]$$

$$- 2 \Delta S \sum_i U_S(t_i) \left[ \omega_e(t_i) - \omega(t_i) \right]$$

$$+ \sum_i \left[ U_T^2(t_i) \Delta S^2 + 2 U_T(t_i) U_S(t_i) \Delta S \Delta T + U_T^2(t_i) \Delta T^2 \right]$$  \hspace{1cm} (16)

The \(t_i\) represents a discrete time at which an experimental measurement is made for the drawdown. The error is defined as the sum over all measurements of the squared difference in \(s_e\) and \(s\). Notice that the sensitivity coefficients \(U_T\) and \(U_S\) depend upon the time \(t_i\).
The error is minimized by taking the first derivatives with respect to $\Delta T$ and $\Delta S$, setting them equal to zero, and finally by solving the resulting equations for $\Delta T$ and $\Delta S$.

\[
\frac{\partial (\text{ERROR})}{\partial \Delta T} = -2 \sum_i U_T(t_i) \left[ \alpha_e(t_i) - \alpha(t_i) \right] + 2 \Delta S \sum_i U_S(t_i) U_T(t_i) \\
+ 2 \Delta T \sum_i U_T^2(t_i) = 0 \quad (17)
\]

\[
\frac{\partial (\text{ERROR})}{\partial \Delta S} = -2 \sum_i U_S(t_i) \left[ \alpha_e(t_i) - \alpha(t_i) \right] + 2 \Delta S \sum_i U_S^2(t_i) \\
+ 2 \Delta T \sum_i U_S(t_i) U_T(t_i) = 0 \quad (18)
\]

For ease of writing define the variables

\[
SSUS = \sum_i U_S^2(t_i) \quad (19)
\]

\[
SSUT = \sum_i U_T^2(t_i) \quad (20)
\]

\[
SUUS = \sum_i U_S(t_i) U_T(t_i) \quad (21)
\]

\[
SUUSDIF = \sum_i U_S(t_i) \left[ \alpha_e(t_i) - \alpha(t_i) \right] \quad (22)
\]

\[
SUUTDIF = \sum_i U_T(t_i) \left[ \alpha_e(t_i) - \alpha(t_i) \right] \quad (23)
\]

Notice that all quantities in equations (19) through (23) are known from experimental data or can be calculated from equations (1), (13), or (14). Solving equation (17) for $\Delta T$ yields

\[
\Delta T = \frac{SUUTDIF - (SUUS)\Delta S}{SSUS} \quad (24)
\]

Substituting for $\Delta T$ in equation (18) and solving for $\Delta S$ results in the following equation.

\[
\Delta S = \frac{(SSUT)(SUUSDIF) - (SUUS)(SUUTDIF)}{(SSUS)(SSUT) - (SUUS)^2} \quad (25)
\]
Thus the $\Delta S$ which results in the "best" fit can be found from equation (25) which involves only known quantities. This $\Delta S$ may be substituted into equation (24) to find the "best" fit value for $\Delta T$.

The values for $\Delta T$ and $\Delta S$ can be used to update the first guess for $T$ and $S$. This better estimate for $T$ and $S$ is then used in the least squares procedure again to obtain new values of $\Delta T$ and $\Delta S$. In general this can be continued until $\Delta T$ and $\Delta S$ become so small as to be insignificant, at which time the iteration is terminated. The "best" fit after the $i$th iteration is obtained by using the following equations.

$$
T^{i+1} = T^i + \Delta T^i
$$

$$
S^{i+1} = S^i + \Delta S^i
$$

The procedure may not converge if the initial guess for $T$ and $S$ is especially bad. However, numerical experiments indicate that good convergence may be obtained even if the initial guess is off considerably.

These numerical experiments show that the initial guesses for $T$ and $S$

overestimated or

may be underestimated by about three orders of magnitude while still obtaining convergence. These experiments covered the normal range of $T$ and $S$ encountered in groundwater work ($10^{-8} \leq S \leq 10^{-5}$)

$\left(10^6 \text{gal/day}/A_t \geq T \geq 10^2 \text{gal/day}/A_t \right)$.

It is possible to obtain an initial guess for $T$ and $S$ from the data if reasonable values are not known. The Theis equation may be expanded as (Jacob, 1950)

$$
Q = \frac{C \pi T}{4\pi T} \left[ -0.5772 - \ln\left(\frac{r^2 S}{4\pi T t}\right) + \frac{1}{2 \cdot 2!} \left(\frac{r^2 S}{4\pi T t}\right)^2 - \frac{1}{3 \cdot 3!} \left(\frac{r^2 S}{4\pi T t}\right)^3 - \cdots \right]
$$

For large values of $t$ and small values of $r$ such that

$$
\frac{r^2 S}{4\pi T t} \ll 1
$$
only the constant and ln terms need to be considered. Differentiating
the truncated expansion for $s$ with respect to $\ln t$ one obtains
\[ \frac{d\alpha}{d(\ln t)} \approx \frac{\epsilon}{4\pi T}. \]  \hspace{1cm} (29)
Solving for $T$ yields an estimate for this parameter.
\[ T \approx \frac{\epsilon}{4\pi \frac{d\alpha}{d(\ln t)}}. \]  \hspace{1cm} (30)
Equation (29) indicates that there is a linear relationship between
$s$ and $\ln t$ when $\frac{r^2 s}{4Tt} << 1$.
\[ \alpha = \left[ \frac{d\alpha}{d(\ln t)} \right] \ln t + C \]  \hspace{1cm} (31)
$C$ and $\frac{ds}{d(\ln t)}$ must be determined from the data. At some time $t_o$ the draw-
down predicted by equation (31) will be zero.
\[ \alpha = \left[ \frac{d\alpha}{d(\ln t)} \right] \ln t_o + C \]  \hspace{1cm} (32)
Solving for $\ln t_o$ gives
\[ \ln t_o = -\left[ \frac{C}{\frac{d\alpha}{d(\ln t)}} \right] \]  \hspace{1cm} (33)
The definition of $t_o$ implies that the first two terms of equation (28)
must equal zero when evaluated at $t_o$ if $\frac{r^2 s}{4Tt} << 1$.
\[ \alpha = \frac{\epsilon}{4\pi T} \left[ -0.5772 - \ln \left( \frac{r^2 s}{4Tt_o} \right) \right] \]  \hspace{1cm} (34)
Solving this equation for $S$ gives the following result.
\[ S = \frac{4T}{r^2} \exp \left[ \ln t_o - 0.5772 \right] \]
\[ S = \frac{4T}{r^2} \exp \left[ -\frac{C}{\frac{d\alpha}{d(\ln t)}} - 0.5772 \right] \]  \hspace{1cm} (35)
Equations (30) and (35) show that one can estimate $T$ and $S$ by
fitting equation (31) to the data. In fitting equation (31) to the
data one attempts to find the "best" value for $\frac{ds}{d(\ln t)}$ and $C$. The method
of least squares is ideally suited to this problem. Obviously, at least
two drawdown-time pairs are needed to define a straight line. If more
than two pairs are given the least squares technique finds the "best"
fit. In this work the last four drawdown-time pairs are used to make
an estimate of $T$ and $S$ if reasonable values are not known. It is
assumed that \( r \) is small enough and \( t \) is large enough so the \( \frac{r^2 S}{4 T^t} \ll 1 \) for these last four pairs. If this condition is not met then this procedure will probably not provide an acceptable guess for \( T \) and \( S \).

Let \( s_e(t) \) represent the experimental data. The squared error in fitting the last four points is

\[
\text{ERROR} = \sum_{i=N-3}^{N} \left[ e_i(t_i) - \frac{d_\alpha}{d(ln t)} \ln t_i - C \right]^2
\]

\[
= \sum_{i=N-3}^{N} \left[ e_i(t_i) + \left( \frac{d_\alpha}{d(ln t)} \right)^2 \ln t_i^2 + C - 2 e_i(t_i) \frac{d_\alpha}{d(ln t)} \ln t_i - 2 e_i(t_i) C + 2 \frac{d_\alpha}{d(ln t)} \ln t_i C \right] \quad (36)
\]

A necessary condition for the error to be minimized is that:

\[
\frac{\partial (\text{ERROR})}{\partial C} = \sum_{i=N-3}^{N} \left[ 2 C - 2 e_i(t_i) + 2 \frac{d_\alpha}{d(ln t)} \ln t_i \right] = 0 \quad (37)
\]

and

\[
\frac{\partial (\text{ERROR})}{\partial (\frac{d_\alpha}{d(ln t)})} = \sum_{i=N-3}^{N} \left[ 2 \frac{d_\alpha}{d(ln t)} \ln t_i^2 - 2 e_i(t_i) \ln t_i + C \ln t_i C \right] = 0 \quad (38)
\]

Solving these two equations simultaneously gives

\[
\frac{d_\alpha}{d(ln t)} = \frac{4 \sum_{i=N-3}^{N} e_i(t_i) \ln t_i - \frac{N}{2} \sum_{i=N-3}^{N} e_i(t_i) \sum_{i=N-3}^{N} \ln t_i}{4 \sum_{i=N-3}^{N} (\ln t_i)^2 - \left( \sum_{i=N-3}^{N} \ln t_i \right)^2} \quad (39)
\]

\[
C = -\frac{1}{4} \frac{\partial_\alpha}{d(ln t)} \sum_{i=N-3}^{N} \ln t_i + \frac{1}{4} \sum_{i=N-3}^{N} e_i(t_i) \quad (40)
\]

The results expressed by equations (39) and (40) can now be used in equations (30) and (35) to calculate an initial guess for \( T \) and \( S \) when needed. This procedure works well if the restrictions on \( r \) and \( t \) are observed.
Appendix I contains a listing of program THEIS. It is a time sharing fortran program for evaluation of equation (1). In addition to drawdown (s) one may have the quantity \( u = \frac{r^2 S}{4Tt} \), the well function \( W(u) \), and the sensitivity coefficients given by equations (13) and (14) printed out. The user may utilize the gal-day-foot system or any consistent set of units.

Figure 1 contains a printout of a typical run. The user's response is underlined. The program initiates a series of questions after the RUN command is typed and a carriage return is given. A carriage return is required after each response. The first question asks if the user wants \( u \), \( W \), and the sensitivity coefficients printed. Any response other than YES with no leading or imbedded blanks is interpreted as NO. All other questions requiring a yes or no answer are similar. In figure 1 the extra printout was requested. The second question defines the system of units. The user can use the gal-day-foot system or any consistent set of units. A YES response, as in figure 1, means the gal-day-foot system is being used.

The units for each parameter the user must specify are stated as the program asks for the user to type in the value. \( L \) is any arbitrary length unit and \( T \) is any arbitrary time unit. A storage coefficient of \( .001 \) which is unitless has been used in figure 1. Transmissivity in units of \( \text{gal/day/ft} \) or \( L^2/T \) must be specified. A value of 24,000 \( \text{gal/day/ft} \) has been used in the example. A constant pumpage of 240,000 \( \text{gal/day} \) has been used in figure 1. In consistent units pumpage must be specified in units of \( L^3/T \).
DO YOU WANT U, W, AND SENSITIVITY VALUES PRINTED?
=YES
DO YOU WANT TO USE THE GAL, DAY, FT SYSTEM?
=YES
STORAGE COEFFICIENT (UNITLESS)
= 0.001
TRANSMISSIVITY (GAL/DAY/FT) OR (L**2/T)
= 24000
CONSTANT PUMPAGE (GAL/DAY) OR (L**3/T)
= 240000
CONSTANT OBSERVATION POINT?
= YES
CONSTANT OBSERVATION TIME?
= NO
OBSERVATION POINT (FT) OR (L)
= 100
OBSERVATION TIME (DAYS) OR (T)
= 0.001

THE DRAWDOWN IS 0.25669954
U = 0.77916666E 00
W = 0.32257789E 00
SENSITIVITY WITH RESPECT TO TRANSMISSIVITY = 0.45163665E-05
SENSITIVITY WITH RESPECT TO STORAGE = -0.36509233E 03

OBSERVATION TIME (DAYS) OR (T)
= 0.01

THE DRAWDOWN IS 1.63239339
U = 0.77916667E-01
W = 0.20513243E 01
SENSITIVITY WITH RESPECT TO TRANSMISSIVITY = -0.37344505E-04
SENSITIVITY WITH RESPECT TO STORAGE = -0.73612525E 03

OBSERVATION TIME (DAYS) OR (T)
= 0.1

THE DRAWDOWN IS 3.41010541
U = 0.77916666E-02
W = 0.42852612E 01
SENSITIVITY WITH RESPECT TO TRANSMISSIVITY = -0.10918776E-03
SENSITIVITY WITH RESPECT TO STORAGE = -0.78959905E 03

Figure 1. Typical run of program THEIS.
At this point two more yes-no questions are asked. The first asks if the user wants a constant observation point. In other words, r in equation (1) will be constant for this run. This is the normal situation when one observation well is used to measure water levels as a function of time. The next question concerns time. Is t in equation (1) to be held constant for this run? This would be the situation if one measured the drawdown at the same time in a number of observation wells. If the answer to both of these questions is yes the program accepts only one value of r and t, calculates the desired quantities, and then terminates. If only one of the questions is answered yes the program continues to ask for updated values of the parameter that is not constant. In figure 1 time is not constant so the program continually asks for new times. To terminate the program simply hit the BREAK button. If both questions are not YES then the program asks for updated values of both r and t until the BREAK button is pushed. In figure 1 a constant r of 100 feet has been specified while several times have been given ranging from .001 days to .1 days.

SUBROUTINE THEIS

Appendix II contains a listing of subroutine THEIS. It is a fortran IV subroutine for the evaluation of equations (1), (13), and (14) that can be incorporated into any user deck and called by the mainline program. To utilize subroutine THEIS the user simply includes the following call statement at the appropriate place in the mainline program.

CALL THEIS(SC, KB, Q, R, T, S, DSDT, DSDSC, UNIT)
The parameters SC, KB, Q, R, T, and UNIT must be assigned a value or character designation prior to calling THEIS. SC is the storage coefficient which is unitless. UNIT must be declared a character type of variable in the mainline program. If the user wishes to use the gal-day-ft system, UNIT must be set to JHYES in the mainline. Any other character string is interpreted as NO and the subroutine assumes a consistent set of units. KB is the transmissivity (gal/day/ft or \( \text{L}^2/\text{T} \)), Q is the constant pumpage (gal/day or \( \text{L}^3/\text{T} \)), R is the distance of the observation well from the pumped well (feet or \( \text{L} \)), and T is the elapsed time since pumping started (day or \( \text{T} \)). The transmissivity KB should be declared REAL in the mainline since it would be implicitly typed INTEGER.

Upon returning to the mainline, parameters S, DSDT, and DSDSC have been assigned values by subroutine THEIS. These values represent the evaluation of equations (1), (13) and (14) respectively. S is the drawdown in units of feet or \( \text{L} \). DSDT is the sensitivity coefficient with respect to transmissivity in units of \( \text{ft}^2/\text{gal \ day} \) or \( \text{T}/\text{L} \). DSDSC is the sensitivity coefficient with respect to storage in units of feet or \( \text{L} \). Upon returning to the mainline these three parameters are available to the user for printout or further computation. This subroutine is used in the next program to be discussed; so, the reader is referred there for an example of the usage of subroutine THEIS.

PROGRAM THEISFIT

This program fits the Theis equation to experimental pumptest data to obtain the "best" values for the storage coefficient and the
transmissivity by using a least squares procedure discussed earlier in this work. The program exists in two forms: a time sharing fortran version and a fortran IV batch version. The time sharing version is listed in Appendix III. Appendix IV contains a listing of the batch version. The two programs are basically identical except for data input.

The printout of a typical time sharing run is contained in figure 2. The user's response is underlined. As with program THEIS the program initiates a series of questions after the RUN command is given with a carriage return. The first question defines the system of units. YES allows one to use the gal-day-foot system; any other response is interpreted as NO. In figure 2 NO was the response so a consistent set of units is assumed. The second question asks if the user wishes to make an initial guess for the storage and transmissivity. In figure 2 NO was given so no input for these quantities was required. If the response had been YES the program would have requested the user to type in the guesses for storage and transmissivity in the two following questions. In figure 2 the pumpage rate is given as 66.07 ft$^3$/min.

The observation well was located 545 feet from the pumped well. The next question asks how many drawdown-time pairs are to be read into the program. In this case it is eighteen. At this point the program echo prints all data typed in thus far. This allows for error checking. Since guesses for storage and transmissivity were not given they are printed as zero. SC and KB stand for storage coefficient and transmissivity respectively.

The program is now ready to accept drawdown-time pairs. They should be read in order of increasing time since the last four are used to
*RUN
DO YOU WANT TO USE GAL, DAY, FT SYSTEM ?
=NO
GUESS FOR STORAGE AND TRANSMISSIVITY ?
=NO
CONSTANT PUMPAGE RATE ? GAL/DAY OR L**3/T
=66.07
OBSERVATION DISTANCE FROM PUMPING WELL ? FT OR L
=543
NUMBER OF DRAWDOWN-TIME PAIRS TO BE READ ?
=18
THE INITIAL DATA WAS
SC = 0,
QB = 0,
Q = 0.66070000E 02 R = 0.54500000E 03 N = 18
TYPE IN DRAWDOWN-TIME PAIRS IN ORDER OF INCREASING TIME,
=0.02,50
=.05,60
=.08,70
=.13,80
=.18,90
=.22,100
=.33,120
=.43,140
=.54,160
=.64,180
=.74,200
=.94,240
=1.12,280
=1.30,320
=1.47,360
=1.66,400
=1.92,460
=2.17,535

Figure 2. Typical run of program THEISFIT (Time Sharing Version).
THE PUMP TEST DATA IN DRAWDOWN-TIME PAIRS IS

<table>
<thead>
<tr>
<th>Time (E-01)</th>
<th>DRAWDOWN (E-02)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.20000000E-01</td>
<td>0.50000000E-02</td>
</tr>
<tr>
<td>0.50000000E-01</td>
<td>0.60000000E-02</td>
</tr>
<tr>
<td>0.80000000E-01</td>
<td>0.70000000E-02</td>
</tr>
<tr>
<td>0.13000000E 00</td>
<td>0.80000000E 02</td>
</tr>
<tr>
<td>0.18000000E 00</td>
<td>0.90000000E 02</td>
</tr>
<tr>
<td>0.22000000E 00</td>
<td>0.10000000E 03</td>
</tr>
<tr>
<td>0.33000000E 00</td>
<td>0.12000000E 03</td>
</tr>
<tr>
<td>0.43000000E 00</td>
<td>0.14000000E 03</td>
</tr>
<tr>
<td>0.54000000E 00</td>
<td>0.16000000E 03</td>
</tr>
<tr>
<td>0.64000000E 00</td>
<td>0.18000000E 03</td>
</tr>
<tr>
<td>0.74000000E 00</td>
<td>0.20000000E 03</td>
</tr>
<tr>
<td>0.94000000E 00</td>
<td>0.24000000E 03</td>
</tr>
<tr>
<td>0.11200000E 01</td>
<td>0.28000000E 03</td>
</tr>
<tr>
<td>0.13000000E 01</td>
<td>0.32000000E 03</td>
</tr>
<tr>
<td>0.14700000E 01</td>
<td>0.36000000E 03</td>
</tr>
<tr>
<td>0.16600000E 01</td>
<td>0.40000000E 03</td>
</tr>
<tr>
<td>0.19200000E 01</td>
<td>0.46000000E 03</td>
</tr>
<tr>
<td>0.21700000E 01</td>
<td>0.53500000E 03</td>
</tr>
</tbody>
</table>

THE CALCULATED GUESS FOR KB AND SC IS 0.296280059E-01 0.35149625E-02

BEST FIT KB AND SC THIS ITERATION ARE 0.25232340E-01 0.46495635E-02
BEST FIT KB AND SC THIS ITERATION ARE 0.22588631E-01 0.47946684E-02
BEST FIT KB AND SC THIS ITERATION ARE 0.22523235E-01 0.47765157E-02
BEST FIT KB AND SC THIS ITERATION ARE 0.22523888E-01 0.47765840E-02
THE BEST FIT DRAWDOWN-TIME PAIRS ARE

<table>
<thead>
<tr>
<th>Time (E-01)</th>
<th>DRAWDOWN (E-02)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25206928E-01</td>
<td>0.50000000E-02</td>
</tr>
<tr>
<td>0.4936171E-01</td>
<td>0.60000000E-02</td>
</tr>
<tr>
<td>0.81189803E-01</td>
<td>0.70000000E-02</td>
</tr>
<tr>
<td>0.11926181E 00</td>
<td>0.80000000E 02</td>
</tr>
<tr>
<td>0.16227960E 00</td>
<td>0.90000000E 02</td>
</tr>
<tr>
<td>0.20906690E 00</td>
<td>0.10000000E 03</td>
</tr>
<tr>
<td>0.31022453E 00</td>
<td>0.12000000E 03</td>
</tr>
<tr>
<td>0.41701533E 00</td>
<td>0.14000000E 03</td>
</tr>
<tr>
<td>0.52587939E 00</td>
<td>0.16000000E 03</td>
</tr>
<tr>
<td>0.63457878E 00</td>
<td>0.18000000E 03</td>
</tr>
<tr>
<td>0.74101843E 00</td>
<td>0.20000000E 03</td>
</tr>
<tr>
<td>0.94917237E 00</td>
<td>0.24000000E 03</td>
</tr>
<tr>
<td>0.11451357E 01</td>
<td>0.28000000E 03</td>
</tr>
<tr>
<td>0.13292893E 01</td>
<td>0.32000000E 03</td>
</tr>
<tr>
<td>0.15021422E 01</td>
<td>0.36000000E 03</td>
</tr>
<tr>
<td>0.16645316E 01</td>
<td>0.40000000E 03</td>
</tr>
<tr>
<td>0.18904877E 01</td>
<td>0.46000000E 03</td>
</tr>
<tr>
<td>0.21471107E 01</td>
<td>0.53500000E 03</td>
</tr>
</tbody>
</table>

THE RMS ERROR FOR DRAWDOWN IS 0.17307440E-01

Figure 2. (Continued)
calculate an initial guess for storage and transmissivity. The guess calculating routine assumes these four are at large values of time. If the user supplies an initial estimate for storage and transmissivity, the drawdown-time pairs may be read in any order. Notice that the drawdown must be typed first and the time second with a separating comma. The values may be typed with or without a decimal point and may be in scientific notation (for example $1.3 \times 10^5$ would be typed 1.3E5). After the eighteen pairs have been typed the program echo prints them for error checking.

From this point on the user may not interact with the program. One of two things should now happen: the program converges to the "best" solution or the program does not converge. If it does not converge after twenty iterations the program terminates. However, if the initial guess for storage and transmissivity was bad or if the data is rather poor, unphysical values for storage and transmissivity or error messages may be generated in the iteration process. If this occurs the program may be terminated by hitting the BREAK key. In general, the storage and transmissivity guesses may be underestimated by three orders of magnitude and still achieve convergence.

In the example shown in figure 2 the calculated guess is printed first then a series of iterations is started. The current "best" fit is printed for each iteration. Convergence is achieved in four iterations in this example. The convergence criteria requires the change in storage and transmissivity since the last iteration to be less than or equal to .1%. The program proceeds to print the "best" fit drawdown-time
pairs and calculates the \( \text{rms error} \) in the drawdown. The \( \text{rms error} \) is a measure of the absolute error at an "average" data point. In this example one would expect the "average" difference between measured and calculated drawdown to be about .017 feet.

The data set for a typical run of the batch version of THEISFIT is shown in Figure 3. The first card must contain values for the two character variables GUESS and UNIT. YES or NO are the appropriate responses. The first variable is GUESS. Its value must start in column one. YES means that the user is going to supply the first guess for storage and transmissivity. The second variable, UNIT, must start in column seven. YES means the user wants to use the gal-day-foot system. Any other response indicates a consistent set of units is being used. In this particular example an initial guess for SC and KB is given; and, the gal-day-foot system is used.

The next card contains five variables: storage (SC), transmissivity (KB), pumpage (Q), observation distance (R), and the number of drawdown-time pairs to be read (N). These variables are read under a (4P10.0, I10) format. Each variable field is ten columns wide. The first four variables may be punched with or without a decimal point; however, if no decimal is punched it will be assumed to be at the extreme right of the field. If no decimal is punched the value should be right justified. The last variable, N, must be right justified (ending in column 50) and punched without a decimal point. If GUESS is given as NO the program ignores any values given for SC and KB on the second card. In this example SC is .00001, KB is 2,000 gal/day/ft, Q is 316,800 gal/day, R is 824 feet, and N is 22 pairs.
<table>
<thead>
<tr>
<th>YES</th>
<th>YES</th>
<th>2000</th>
<th>316800</th>
<th>824</th>
<th>22</th>
<th>0.00001</th>
<th>0.002083</th>
<th>0.003472</th>
<th>1.3</th>
<th>0.005556</th>
<th>2.1</th>
<th>0.008333</th>
</tr>
</thead>
<tbody>
<tr>
<td>.3</td>
<td>.013889</td>
<td>3.6</td>
<td>.016667</td>
<td>4.1</td>
<td>0.020833</td>
<td>4.7</td>
<td>0.026389</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.1</td>
<td>0.032639</td>
<td>5.3</td>
<td>.034722</td>
<td>5.7</td>
<td>0.041667</td>
<td>6.1</td>
<td>0.048611</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.3</td>
<td>0.055556</td>
<td>6.7</td>
<td>.062500</td>
<td>7.0</td>
<td>0.069444</td>
<td>7.5</td>
<td>0.090278</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.3</td>
<td>111111</td>
<td>8.5</td>
<td>.138889</td>
<td>9.2</td>
<td>1.80556</td>
<td>9.7</td>
<td>222222</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10.2</td>
<td>.263889</td>
<td>10.9</td>
<td>.347222</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Figure 3. Typical data set for batch version of THEISFIT.**
The drawdown-time pairs are punched on the third and following cards at a maximum of four pairs per card. They are read under an (8F10.0) format. Each data field is ten columns wide. Decimal consideration is the same as for SC, KB, Q, and R. They should be punched in order of increasing time if a guess for SC and KB is to be calculated by the program. In this example drawdowns are given in feet and time in days.

The output for the data set of figure 3 is shown in figure 4. The input data is printed out for error checking. Since GUESS was given as YES, no guess was calculated for SC and KB. The program converged in six iterations. The convergence criteria is the same as for the time sharing version. Comments on the convergence properties of the time sharing version also apply to the batch version. The best fit drawdown-time pairs are printed if convergence is obtained. The \( A \) for the drawdown is .091 feet for this example. Therefore, the "average" error in drawdown is .091 feet.

**DISCUSSION AND SUMMARY**

The automated fit for pump test data developed in this work should be a useful tool for the groundwater hydrologist. We have used it on many more pump tests than the two examples included here. It is simple to use, quick, and inexpensive. Typically the computer costs for running a pump test fit is less than two dollars. The automated fit has the advantage that it is always objective. As a measure of the error in fitting, the \( A \) in drawdown is calculated for the "best"
THE INITIAL DATA WAS
SC = 0.10000000E-04 KB = 0.20000000E 04
Q = 0.31680000E 06 R = 0.82400000E 03 N = 22
THE PUMP TEST DATA IN DRAWDOWN-TIME PAIRS IS

<table>
<thead>
<tr>
<th>TIME</th>
<th>DRAWDOWN</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.30000000E 00</td>
<td>0.20830000E-02</td>
</tr>
<tr>
<td>0.70000000E 00</td>
<td>0.34720000E-02</td>
</tr>
<tr>
<td>0.13000000E 01</td>
<td>0.55560000E-02</td>
</tr>
<tr>
<td>0.21000000E 01</td>
<td>0.83330000E-02</td>
</tr>
<tr>
<td>0.32000000E 01</td>
<td>0.13889000E-01</td>
</tr>
<tr>
<td>0.36000000E 01</td>
<td>0.16670000E-01</td>
</tr>
<tr>
<td>0.41000000E 01</td>
<td>0.20883000E-01</td>
</tr>
<tr>
<td>0.47000000E 01</td>
<td>0.26389000E-01</td>
</tr>
<tr>
<td>0.51000000E 01</td>
<td>0.32639000E-01</td>
</tr>
<tr>
<td>0.53000000E 01</td>
<td>0.34722000E-01</td>
</tr>
<tr>
<td>0.57000000E 01</td>
<td>0.41667000E-01</td>
</tr>
<tr>
<td>0.61000000E 01</td>
<td>0.48611000E-01</td>
</tr>
<tr>
<td>0.63000000E 01</td>
<td>0.55560000E-01</td>
</tr>
<tr>
<td>0.67000000E 01</td>
<td>0.62500000E-01</td>
</tr>
<tr>
<td>0.70000000E 01</td>
<td>0.69440000E-01</td>
</tr>
<tr>
<td>0.75000000E 01</td>
<td>0.70278000E-01</td>
</tr>
<tr>
<td>0.83000000E 01</td>
<td>0.11110000E 00</td>
</tr>
<tr>
<td>0.85000000E 01</td>
<td>0.13889000E 00</td>
</tr>
<tr>
<td>0.92000000E 01</td>
<td>0.18056000E 00</td>
</tr>
<tr>
<td>0.97000000E 01</td>
<td>0.22222000E 00</td>
</tr>
<tr>
<td>0.10200000E 02</td>
<td>0.26388000E 00</td>
</tr>
<tr>
<td>0.10900000E 02</td>
<td>0.34722000E 00</td>
</tr>
</tbody>
</table>

BEST FIT KB AND SC THIS ITERATION ARE

<table>
<thead>
<tr>
<th>KB</th>
<th>SC</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.36231568E 04</td>
<td>0.15973002E-04</td>
</tr>
</tbody>
</table>

BEST FIT KB AND SC THIS ITERATION ARE

<table>
<thead>
<tr>
<th>KB</th>
<th>SC</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.59841841E 04</td>
<td>0.21295657E-04</td>
</tr>
</tbody>
</table>

BEST FIT KB AND SC THIS ITERATION ARE

<table>
<thead>
<tr>
<th>KB</th>
<th>SC</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.84381630E 04</td>
<td>0.22418721E-04</td>
</tr>
</tbody>
</table>

BEST FIT KB AND SC THIS ITERATION ARE

<table>
<thead>
<tr>
<th>KB</th>
<th>SC</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.97266890E 04</td>
<td>0.21174726E-04</td>
</tr>
</tbody>
</table>

BEST FIT KB AND SC THIS ITERATION ARE

<table>
<thead>
<tr>
<th>KB</th>
<th>SC</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.99078571E 04</td>
<td>0.20944681E-04</td>
</tr>
</tbody>
</table>

BEST FIT KB AND SC THIS ITERATION ARE

<table>
<thead>
<tr>
<th>KB</th>
<th>SC</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.99862740E 04</td>
<td>0.20949939E-04</td>
</tr>
</tbody>
</table>

THE BEST FIT DRAWDOWN-TIME PAIRS ARE

<table>
<thead>
<tr>
<th>TIME</th>
<th>DRAWDOWN</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.35065781E 00</td>
<td>0.20830000E-02</td>
</tr>
<tr>
<td>0.2974025E 00</td>
<td>0.34720000E-02</td>
</tr>
<tr>
<td>0.14774877E 01</td>
<td>0.55560000E-02</td>
</tr>
<tr>
<td>0.21714488E 01</td>
<td>0.83330000E-02</td>
</tr>
<tr>
<td>0.31821451E 01</td>
<td>0.13889000E-01</td>
</tr>
<tr>
<td>0.3579138E 01</td>
<td>0.16667000E-01</td>
</tr>
<tr>
<td>0.40622920E 01</td>
<td>0.20830000E-01</td>
</tr>
<tr>
<td>0.45985823E 01</td>
<td>0.26389000E-01</td>
</tr>
<tr>
<td>0.50920417E 01</td>
<td>0.32639000E-01</td>
</tr>
<tr>
<td>0.52373781E 01</td>
<td>0.34722000E-01</td>
</tr>
<tr>
<td>0.56696366E 01</td>
<td>0.41667000E-01</td>
</tr>
<tr>
<td>0.60390812E 01</td>
<td>0.48611000E-01</td>
</tr>
<tr>
<td>0.63617290E 01</td>
<td>0.55556000E-01</td>
</tr>
<tr>
<td>0.66480276E 01</td>
<td>0.62500000E-01</td>
</tr>
<tr>
<td>0.69053687E 01</td>
<td>0.69444000E-01</td>
</tr>
<tr>
<td>0.75506012E 01</td>
<td>0.90278000E-01</td>
</tr>
<tr>
<td>0.80648858E 01</td>
<td>0.11111000E 00</td>
</tr>
<tr>
<td>0.86204650E 01</td>
<td>0.13889000E 00</td>
</tr>
<tr>
<td>0.92757411E 01</td>
<td>0.18055000E 00</td>
</tr>
<tr>
<td>0.97979767E 01</td>
<td>0.22222000E 00</td>
</tr>
<tr>
<td>1.0230386E 02</td>
<td>0.26388000E 00</td>
</tr>
<tr>
<td>1.0922440E 02</td>
<td>0.34722000E 00</td>
</tr>
</tbody>
</table>

THE RMS ERROR FOR DRAWDOWN IS 0.91011392E-01

Figure 4. Output for typical run of batch version of THEISFIT.
transmissivity and storage. For the cases we have run the
in drawdown is no more than a few tenths of a foot. If it
is much larger than this one has either poor data or a hydrologic
situation which can not be represented by the Theis equation.

The algorithm for fitting has good convergence properties.

Convergence is generally achieved even if the initial guess is too small by three orders of
magnitude. The procedure has been tested over a wide range of trans-
missivity and storage.

This work deals only with the Theis equation. However, sensitivity
analysis and least squares fitting could be applied to more hydrologically
complicated situations. Automated fitting routines could be developed
for anisotropic flow, partial penetration, leaky aquifers, delayed yield,
and hydrologic boundaries to name some of the more common situations.
REFERENCES


Theis, C.V., The relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using groundwater storage, Trans. AGU, 16, 519-524, 1935.


Appendix I. Listing of program THEIS.

10C PROGRAM: THEIS
20C PURPOSE: CALCULATES THE DRAWDOWN FOR A WELL PUMPING AT
30C CONSTANT DISCHARGE FROM AN INFINITE AQUIFER BY THE
40C THEIS EQUATION
50C
60C ARGUMENTS (L IS ARBITRARY LENGTH, T IS ARBITRARY TIME)
61C RS CONSTANT OBSERVATION SWITCH; YES OR NO
62C TS CONSTANT OBSERVATION TIME SWITCH; YES OR NO
63C US UNIT SWITCH; USE GAL, DAYS, FT; YES OR NO
64C PRTSW PRINT SWITCH FOR U, W, AND SENSITIVITY; YES OR NO
70C SC STORAGE COEFFICIENT FOR AQUIFER (UNITLESS)
80C KB TRANSMISSIVITY OF AQUIFER (GAL/DAY/FT) OR (L**2/T)
90C Q CONSTANT PUMPAGE OF WELL (GAL/DAY) OR (L**3/T)
100C R OBSERVATION DISTANCE FROM WELL (FT) OR (L)
110C T OBSERVATION TIME (DAYS) OR (T)
120C S DRAWDOWN (FT) OR (L)
121C DSBT SENSITIVITY W.R.T. TRANSMISSIVITY (FT**2/DAY/GAL) OR (T/L)
122C DSDSC SENSITIVITY W.R.T. STORAGE (FT) OR (L)
130C
140C READ IN ARGUMENTS
150C CHARACTER RS, TS, US, PRTSW
160C REAL KB
170C ICOUNT = 0
180C PRINT 1
181C PRINT 15
182 15 FORMAT(‘ DO YOU WANT U, W, AND SENSITIVITY VALUES PRINTED?’)
183C READ! PRTSW
190 1 FORMAT(1HO)
191C PRINT 14
192 14 FORMAT(‘ DO YOU WANT TO USE THE GAL, DAY, FT SYSTEM?’)
193C READ! US
200C PRINT 2
210 2 FORMAT(‘ STORAGE COEFFICIENT (UNITLESS)’)
220C READ! SC
230C PRINT 3
240 3 FORMAT(‘ TRANSMISSIVITY (GAL/DAY/FT) OR (L**2/T)’)
250C READ! KB
260C PRINT 4
270 4 FORMAT(‘ CONSTANT PUMPAGE (GAL/DAY) OR (L**3/T)’)
280C READ! Q
290C PRINT 10
300 10 FORMAT(‘ CONSTANT OBSERVATION POINT?’)
310C READ! RS
320C PRINT 11
330 11 FORMAT(‘ CONSTANT OBSERVATION TIME?’)
340C READ! TS
350 80 IF (ICOUNT .GT. 0 .AND. RS .EQ. 3) GOTO 70
360C PRINT 5
370 5 FORMAT(‘ OBSERVATION POINT (FT) OR (L)’)
380C READ! R
390 70 IF (ICOUNT .GT. 0 .AND. TS .EQ. 3) GOTO 95
400C PRINT 6
410 6 FORMAT(‘ OBSERVATION TIME (DAYS) OR (T)’)
420C READ! T
430C CALCULATE THEIS SOLUTION
440C THEIS EQUATION FROM ‘INTRO TO HYDROLOGY’ BY VIESSMAN PAGE 257
450C RATIONAL APPROXIMATION FROM ‘HANDBOOK OF MATH FUNCTIONS’
460C BY ABRAMOWITZ AND STEgun PAGE 231
470  95 CONTINUE
480      U = R*R*SC/(4.0*KB*T)
481      IF (US .EQ. 3HYES ) U = 7.48*U
482      IF (U .LE. 0.0) GO TO 96
483      IF (U .GT. 1.0) GO TO 86
484      W = -ALOG(U)-.57721566+.99999193*U-.24991055*U**U
485      520&       .05519968*U*U-U-.00976004*U**4+.00107857*U**5
486      530      GO TO 87
487      540      86  W = (EXP(-U)/U)*(U+2.334733*U+.250621)/
488      550&      (U+3.330657*U+1.681534)
489      560      87  S = (Q/(4.0*3.14159*KB))*W
490      570      DSQT = (Q/(4.0*3.14159*KB**2))*(W+EXP(-U))
491      580      DSQSC = -(Q/(4.0*3.14159*KB*SC))*EXP(-U)
590C OUTPUT RESULTS
560   90 PRINT 7, S
570  7 FORMAT(1HO, 'THE DRAWDOWN IS ', F20.8)
580  8 FORMAT ( ' U = ', E20.8)
590  8 FORMAT ( ' W = ', E20.8)
600   9 FORMAT ( ' SENSITIVITY WITH RESPECT TO TRANSMISSIVITY = ', E20.8)
610  10 FORMAT ( ' SENSITIVITY WITH RESPECT TO STORAGE = ', E20.8)
620  11 PRINT 13
630  13 PRINT ( ' DRAWDOWN UNDEFINED U TOO SMALL' )
640  14 PRINT 1
650  15 ICOUNT = ICOUNT +1
660  16 IF (RS .EQ. 3HYES .AND. TS .EQ. 3HYES ) GO TO 99
670  17 GO TO 80
680  18 STOP
690  19 END
SUBROUTINE THEIS(SC, KB, Q, R, T, S, DSDT, DSDSC, UNIT)
C PURPOSE: CALCULATES THE DRAWDOWN FOR A WELL PUMPING AT
C THEIS EQUATION
C ARGUMENTS (L IS ARBITRARY LENGTH, T IS ARBITRARY TIME)
C SC STORAGE COEFFICIENT FOR AQUIFER (UNITLESS)
C KB TRANSMISSIVITY OF AQUIFER (GAL/DAY/FT) OF (L**2/T)
C Q CONSTANT PUMPAGE OF WELL (GAL/DAY) OR (L**3/T)
C R OBSERVATION DISTANCE FROM WELL (FT) OR (L)
C T OBSERVATION TIME (DAYS) OR (T)
C S DRAWDOWN (FT) OR (L)
C DSDT SENSITIVITY w.r.t. TRANSMISSIVITY (FT**2*DAY/GAL) OR (T/L)
C DSDSC SENSITIVITY w.r.t. STORAGE (FT) OR (L)
C UNIT SYSTEM OF UNITS; YES FOR GAL, DAY, FT, OTHERWISE
C CONSISTENT UNITS ARE ASSUMED
C
C CALCULATE THEIS SOLUTION
C THEIS EQUATION FROM 'INTRO TO HYDROLOGY' BY VIESSMAN PAGE 257
C RATIONAL APPROXIMATION FROM 'HANDBOOK OF MATH FUNCTIONS'
C BY ABRAMOWITZ AND STEGUN PAGE 231
REAL KB
CHARACTER UNIT
U = R*R*SC/(4.0*KB*T)
IF (UNIT.EQ.3) RETURN 
U = 7.48*U
IF (U.LE.0.0) GO TO 96
IF (U.GT.1.0) GO TO 86
W = -ALOG(U)-.57721566+.9999993*U-.24991055*U*U+
   .05519968*U*U*U-.00976004*U**4+.00107857*U**5
   GO TO 86
86 W = (EXP(-U)/U)*(U*U+2.334733*U+.250621)/
   (U*U+3.330657*U+.681534)
87 S = (Q/(4.0*3.14159*KB)**W)
   DSDT = (Q/(4.0*3.14159**KB**2)**(-W+EXP(-U)))
   DSDSC = -(Q/(4.0*3.14159*KB*SC))*EXP(-U)
RETURN
96 PRINT 13
13 FORMAT (' DRAWDOWN UNDEFINED U TOO SMALL ')
99 STOP
END
Appendix III. Listing of program THEISFIT (Time Sharing Version)

10C PROGRAM THEISFIT (TIME SHARING VERSION)
20C PURPOSE: TO CALCULATE THE BEST FIT STORAGE AND TRANSMISSIVITY
30C BY FITING THE THEIS EQUATION TO EXPERIMENTAL PUMPTEST DATA IN
40C A LEAST SQUARES SENSE.
50C
60C IMPORTANT VARIABLES: (L IS ARBITRARY LENGTH, T IS ARBITRARY TIME)
70C UNIT SYSTEM OF UNITS; YES FOR GAL, DAY, FT; OTHERWISE
80C CONSISTENT UNITS ARE ASSUMED,
90C GUESS DO YOU WANT TO ENTER AN INITIAL GUESS FOR SC AND KB?
100C YES OR NO IS THE APPROPRIATE RESPONSE,
110C SC STORAGE COEFFICIENT OF AQUIFER (UNITLESS)
120C KB TRANSMISSIVITY OF AQUIFER (GAL/DAY/FT) OR (L**2/T)
130C Q CONSTANT PUMPAGE OF WELL (GAL/DAY) OR (L**3/T)
140C R OBSERVATION DISTANCE FROM WELL (FT) OR (L)
150C N NUMBER OF DRAWDOWM-TIME PAIRS TO BE READ
160C S(I) EXPERIMENTAL DRAWDOWN AT TIME I (FT) OR (L)
170C T(I) THE ITH TIME AT WHICH AN EXPERIMENTAL MEASUREMENT
180C FOR THE DRAWDOWN IS MADE (DAYS) OR (T)
190C SIGMA THE RMS ERROR IN DRAWDOWN AFTER THE BEST
200C FIT HAS BEEN OBTAINED
210C
220 DIMENSION S(100),T(100),SP(100),DSBT(100),DSDSC(100)
230 REAL KB
240 CHARACTER GUESS, UNIT
250C READ IN THE INITIAL DATA
260 PRINT 801
270 801 FORMAT ( ' DO YOU WANT TO USE GAL, DAY, FT SYSTEM? ' )
280 READ:UNIT
290 PRINT 105
300 105 FORMAT(' GUESS FOR STORAGE AND TRANSMISSIVITY? ')
310 READ: GUESS
320 IF (GUESS .NE. 3) GO TO 114
330 PRINT 112
340 112 FORMAT( ' ESTIMATE FOR STORAGE? ' )
350 READ: SC
360 PRINT 113
370 113 FORMAT(' ESTIMATE FOR TRANSMISSIVITY? GAL/DAY/FT OR L**2/T ')
380 READ: KB
390 114 PRINT 115
400 115 FORMAT( ' CONSTANT PUMPAGE RATE? GAL/DAY OR L**3/T ' )
410 READ: Q
420 PRINT 116
430 116 FORMAT(' OBSERVATION DISTANCE FROM PUMPING WELL? FT OR L ')
440 READ:R
450 PRINT 117
460 117 FORMAT(' NUMBER OF DRAWDOWN-TIME PAIRS TO BE READ? ')
470 READ: N
480C ECHO PRINT THE INITIAL DATA
490 WRITE (6, 106) SC, KB, Q, R, N
500 106 FORMAT ( ' THE INITIAL DATA WAS / SH SC = , E15.8 , SH KB = , E15.8 ' )
510% -4H Q =,$E15.8, 4H R =,$E15.8, 4H N =,$IS\)  
520C TYPE IN THE DRAWDOWN-TIME PAIRS IN ORDER OF INCREASING TIME  
530 PRINT 118  
540 118 FORMAT(' TYPE IN DRAWDOWN-TIME PAIRS IN ORDER OF INCREASING TIME, '  
550 DO 120 I = 1,N  
560 120 READ: I(S(I),T(I))  
570C ECHO PRINT THE DRAWDOWN-TIME PAIRS  
580 WRITE (6,107) (S(I),T(I)), I = 1, N  
590 107 FORMAT (' THE PUMP TEST DATA IN DRAWDOWN-TIME PAIRS IS '/(2E20.8))  
600 101 FORMAT ( 8F10.0 )  
610 100 FORMAT ( 4F10.0, I10 )  
620C CALCULATE THE INITIAL GUESS FOR KB AND SC IF NOT GIVEN  
630 NUM = 4  
640 IF (GUESS .EQ. 6HYES ) GO TO 1  
650 SUMLNT = 0.0  
660 SMLNT2 = 0.0  
670 SUMSE = 0.0  
680 SSELNT = 0.0  
690 DO 50 I = 1,NUM  
700 II = N +1-I  
710 ALOGT = ALOG(T(II))  
720 SUMLNT = SUMLNT + ALOGT  
730 SMLNT2 = SMLNT2 + ALOGT**2  
740 SUMSE = SUMSE + S(II)  
750 50 SSELNT = SSELNT + S(II)*ALOGT  
760 SLOPE = (NUM*SSELNT-SUMSE*SUMLNT)/(NUM*SMLNT2-SUMLNT**2)  
770 CON = -(SLOPE*SUMLNT - SUMSE)/NUM  
780 KB = Q/(SLOPE*4.0*3.14159)  
790 ALNTO = -CON/SLOPE  
800 SC = 4.0*KB/( R**2*EXP(- (.5772-ALNTO)))  
810 IF (UNIT .EQ. 3HYES ) SC = SC/7.48  
820C PRINT THE CALCULATED GUESS FOR KB AND SC  
830 WRITE (6,110) KB, SC  
840 110 FORMAT (' THE CALCULATED GUESS FOR KB AND SC IS ', 2E16.8)  
850 1 ICOUNT = 0  
860 5 TEMPKB = KB  
870 TEMFSC = SC  
880C USE SENSITIVITY ANALYSIS AND LEAST SQUARES TO FIND A BETTER KB AND SC  
890 DO 10 I = 1, N  
900 10 CALL THEIS(SC,KB,Q,R,T(I),SP(I),DSBT(I),DSDSC(I),UNIT)  
910 SSUS = 0.0  
920 SSUT = 0.0  
930 SUTUS=0.0  
940 SUSDIF = 0.0  
950 SUTDIF = 0.0  
960 DO 15 I = 1, N  
970 SSUS = DSDSC(I)**2+SSUS  
980 SSUT = DSBT(I)**2+SSUT  
990 SUTUS = DSDSC(I)*DSBT(I) + SUTUS  
1000 SUSDIF = DSDSC(I)*(S(I)-SP(I)) + SUSDIF  
1010 15 SUTDIF = DSBT(I)*(S(I)-SP(I)) + SUTDIF  
1020 DELTSC = (SSUT*SUSDIF-SUTUS*SUTDIF)/(SSUS*SSUT-SUTUS**2)  
1021 IF (DELTSC .LT. -.95*SC ) DELTSC = -.95*SC  
1022 IF (DELTSC .GT. 2.0*SC ) DELTSC = 2.0*SC  
1030 SC = SC + DELTSC  
1035 IF (SC .GT. 1.0 ) SC = 1.0  
1040 DELTKB = (SUDIF-DELTSC*SUTUS)/SSUT  
1041 IF (DELTKB .LT. -.95*KB ) DELTKB = -.95*KB  
1042 IF (DELTKB .GT. 2.0*KB ) DELTKB = 2.0*KB  
1050 KB = KB + DELTKB
1060 WRITE (6,104) KB, SC
1070 104 FORMAT( ' BEST FIT KB AND SC THIS ITERATION ARE ', 2E16.8 )
1080 ICOUNT = ICOUNT + 1
1090C IF IT DID NOT CONVERGE PRINT MESSAGE
1100 IF ( ICOUNT .GT. 20) GO TO 40
1110C DO ANOTHER ITERATION IF IT HASN'T CONVERGED AND ITERATION LIMIT NOT
1120C EXCEEDED.
1130 IF(ABS((TEMPKB-KB)/KB),GT,.001,OR.ABS((TEMPSC-SC)/SC),GT,.001)
11408 GO TO 5
1150C PRINT BEST FIT DRAWDOWN-TIME PAIRS IF CONVERGENCE OBTAINED
1160 108 FORMAT (' THE BEST FIT DRAWDOWN-TIME PAIRS ARE ')
1170 WRITE (6,108)
1180 DO 20 I = 1, N
1190 CALL THEIS(SC,KB,Q,R,T(I),SP(I),DSDT(I),DSDISC(I),UNIT)
1200 WRITE (6,109) SP(I), T(I)
1210 109 FORMAT(2E20.8)
1220 20 CONTINUE
1230C CALCULATE THE RMS ERROR IN DRAWDOWN AFTER BEST FIT
1240 SUM = 0.0
1250 DO 30 I = 1, N
1260 SUM = SUM + ((SP(I)-S(I))**2)
1270 SIGMA = SQRT(SUM/N)
1280 WRITE(6,111) SIGMA
1290 111 FORMAT( ' THE RMS ERROR FOR DRAWDOWN IS ',E20.8 )
1300 STOP
1310 40 WRITE (6,125)
1320 125 FORMAT( ' DID NOT CONVERGE IN 20 ITERATIONS ')
1330 STOP
1340 END
1350C SUBROUTINE THEIS(SC,KB,Q,R,T,S,DSDT,DSDISC,UNIT)
1360C PURPOSE: CALCULATES THE DRAWDOWN FOR A WELL PUMPING AT
1370C CONSTANT DISCHARGE FROM AN INFINITE AQUIFER BY THE
1380C THEIS EQUATION
1390C
1400C ARGUMENTS (L IS ARBITRARY LENGTH, T IS ARBITRARY TIME)
1410C SC STORAGE COEFFICIENT FOR AQUIFER (UNITLESS)
1420C KB TRANSMISSIVITY OF AQUIFER (GAL/DAY/FT) OF (L**2/T)
1430C Q CONSTANT PUMPAGE OF WELL (GAL/DAY) OR (L**3/T)
1440C R OBSERVATION DISTANCE FROM WELL (FT) OR (L)
1450C T OBSERVATION TIME (DAYS) OR (T)
1460C S DRAWDOWN (FT) OR (L)
1470C DSDT SENSITIVITY W.R.T. TRANSMISSIVITY (FT**2/DAY/GAL) OR (T/L)
1480C DSDSC SENSITIVITY W.R.T. STORAGE (FT) OR (L)
1490C UNIT SYSTEM OF UNITS; YES FOR GAL, DAY, FT; OTHERWISE
1500C CONSISTENT UNITS ARE ASSUMED
1510C
1520C
1530C CALCULATE THEIS SOLUTION
1540C THEIS EQUATION FROM 'INTRO TO HYDROLOGY' BY VIESSMAN PAGE 257
1550C RATIONAL APPROXIMATION FROM 'HANDBOOK OF MATH FUNCTIONS'
1560C BY ABRAMOWITZ AND STEGUN PAGE 231
1570 REAL KB
1580 CHARACTER UNIT
1590 U = R*R*SC/(4.0*KB*T)
1600 IF (UNIT .EQ. 'HYES') U = 7.48*U
1610 IF (U .LE. 0.0) GO TO 96
1620 IF (U .GT. 1.0) GO TO 86
1630 \ W = -ALOG(U) - .577215664*99999193*U - .24991055*U*U +
1640& .05519968*U*U*U - .00976004*U**4 + .00107857*U**5
1650 GO TO 87
1660 86 W = (EXP(-U)/U)*(U*U+2.334733*U+.250621)/
1670& (U*U+3.330657*U + 1.681534)
1680 87 S = (0/(4.0*3.14159*KB))*W
1690 DSDT = (0/(4.0*3.14159*KB*2))*(W + EXP(-U))
1700 DSDDSC=-(0/(4.0*3.14159*KB*SC))*EXP(-U)
1710 RETURN
1720 96 PRINT 13
1730 13 FORMAT (' DRAWDOWN UNDEFINED U TOO SMALL ')
1740 99 STOP
1750 END
Appendix IV. Listing of program THEISFIT (BATCH VERSION).

40C PROGRAM THEISFIT (FORTRAN IV BATCH VERSION)
50C PURPOSE: TO CALCULATE THE BEST FIT STORAGE AND TRANSMISSIVITY
60C BY FITTING THE THEIS EQUATION TO EXPERIMENTAL PUMPTEST DATA
70C IN A LEAST SQUARES SENSE.
80C IMPORTANT VARIABLES: (L IS ARBITRARY LENGTH, T IS ARBITRARY TIME)
90C UNIT SYSTEM OF UNITS; YES FOR GAL, DAY, FT; OTHERWISE
100C CONSISTENT UNITS ARE ASSUMED.
110C GUESS DO YOU WANT TO ENTER AN INITIAL GUESS FOR SC AND KB?
120C YES OR NO IS THE APPROPRIATE RESPONSE.
130C SC STORAGE COEFFICIENT OF AQUIFER (UNITLESS)
140C KB TRANSMISSIVITY OF AQUIFER (GAL/DAY/FT) OR (L**2/T)
150C Q CONSTANT PUMPAGE OF WELL (GAL/DAY) OR (L**3/T)
160C R OBSERVATION DISTANCE FROM WELL (FT) OR (L)
170C N NUMBER OF DRAWDOWN-TIME PAIRS TO BE READ
180C S(I) EXPERIMENTAL DRAWDOWN AT TIME I (FT) OR (L)
190C T(I) THE ITH TIME AT WHICH AN EXPERIMENTAL MEASUREMENT
200C FOR THE DRAWDOWN IS MADE (DAYS) OR (T)
210C SIGMA THE RMS ERROR IN DRAWDOWN AFTER THE BEST
220C FIT HAS BEEN OBTAINED
230C
250 DIMENSION S(100), T(100), SP(100), DSBT(100), SSDL2(100)
260 REAL KB
270 CHARACTER GUESS, UNIT
280C READ IN INITIAL DATA
290 READ (5,105) GUESS, UNIT
300 105 FORMAT ( 2A6 )
310 READ (5,100) SC, KB, Q, R, N
320C ECHO PRINT THE INITIAL DATA
330 WRITE (6, 106) SC, KB, Q, R, N
340 106 FORMAT ( 'THE INITIAL DATA WAS ' / 5H SC =, E15.8, 5H KB =, E15.8
350 + /4H Q =, E15.8, 4H R =, E15.8, 4H N =, IS )
360C READ IN DRAWDOWN-TIME PAIRS IN ORDER OF INCREASING TIME
370 READ (5, 101) (S(I), T(I)), I = 1, N
380C ECHO PRINT THE DRAWDOWN-TIME PAIRS
390 WRITE (6, 107) (S(I), T(I)), I = 1, N )
400 107 FORMAT ( 'THE PUMP TEST DATA IN DRAWDOWN-TIME PAIRS IS '/(2E20.8))
410 101 FORMAT ( 8F10.0 )
420 100 FORMAT ( 4F10.0, I10 )
430C CALCULATE THE INITIAL GUESS FOR KB AND SC IF NOT GIVEN
440 NUM = 4
450 IF ( GUESS .EQ. 'HYES' ) GO TO 1
460 SUMNT = 0.0
470 SMLNT2 = 0.0
480 SUMSE = 0.0
490 SSELNT = 0.0
500 DO 50 I = 1, NUM
510 II = N + 1 - I
520 AL0GT = AL0G(T(I))
530 SUMNT = SUMNT + AL0GT
540 SMLNT2 = SMLNT2 + AL0GT**2
550 SUMSE = SUMSE + S(I)
560  50 SSELNT = SSELNT + S(II)*ALGOT
570  SLOPE = (NUM*SSELNT-SUMSE*SMLNT)/(NUM*SMLNT2-SUMLNT**2)
580  CON = -(SLOPE*SMLNT - SUMSE)/NUM
590  KB = Q/(SLOPE*4.0*3.14159)
600  ALNTO = CON/SLOPE
610  SC = 4.0*KB/( R**2*EXP(.5772-ALNTO))
620  IF (UNIT, EQ. 3|YES ) SC = SC/7.48
630C PRINT THE CALCULATED GUESS FOR KB AND SC
640  WRITE (6,110) KB, SC
650  110 FORMAT ( ' THE CALCULATED GUESS FOR KB AND SC IS ', 2E16.8 )
660  1 ICOUNT = 0
670  5 TEMPKB = KB
680  TEMPSC = SC
690C USE SENSITIVITY ANALYSIS AND LEAST SQUARES TO FIND A BETTER KB AND SC
700  DO 10 I = 1, N
710  10 CALL THEIS(SC,KB,Q,R,T(I),SP(I),DSBT(I),DSDSC(I),UNIT)
720  SSUS = 0.0
730  SSUT = 0.0
740  SUTUS=0.0
750  SUSDIF = 0.0
760  SUTDIF = 0.0
770  DO 15 I = 1, N
780  SSUS = DSDC(I)**2+SSUS
790  SSUT = DSBT(I)**2+SSUT
800  SUTUS = DSDC(I)*DSBT(I) + SUTUS
810  SUSDIF = DSDC(I)*(S(I)-SP(I)) + SUSDIF
820  15 SUTDIF = DSBT(I)*(S(I)-SP(I)) + SUTDIF
830  DELTSC = (SSUT*SUSDIF-SUTUS*SUTDIF)/(SSUS*SSUT-SUTUS**2)
831  IF (DELTSC ,LT,-.95*SC ) DELTSC = -.95*SC
832  IF (DELTSC ,GT, 2.0*SC ) DELTSC = 2.0*SC
840  SC = SC + DELTSC
845  IF (SC ,GT, 1.0 ) SC = 1.0
850  DELTKB = (SUTDIF-DELTSC*SUTUS)/SSUT
851  IF (DELTKB ,LT,-.95*KB ) DELTKB = -.95*KB
852  IF (DELTKB ,GT, 2.0*KB ) DELTKB = 2.0*KB
860  KB = KB + DELTKB
870  WRITE (6,104) KB, SC
880  104 FORMAT( ' BEST FIT KB AND SC THIS ITERATION ARE ', 2E16.8 )
890  ICOUNT = ICOUNT + 1
900C IF IT DID NOT CONVERGE PRINT MESSAGE
910  IF ( ICOUNT ,GT, 20 ) GO TO 40
920C DO ANOTHER ITERATION IF IT HASNT CONVERGED AND ITERATION LIMIT NOT
930C EXCEEDED,
940  IF(ABS((TEMPKB-KB)/KB),GT,.001.OR,ABS((TEMPSC-SC)/SC),GT,.001)
950  U GO TO 5
960C PRINT BEST FIT DRAWDOWN-TIME PAIRS IF CONVERGENCE OBTAINED
970  108 FORMAT ( ' THE BEST FIT DRAWDOWN-TIME PAIRS ARE ' )
980  WRITE (6,108)
990  DO 20 I = 1, N
1000  CALL THEIS(SC,KB,Q,R,T(I),SP(I),DSBT(I),DSDSC(I),UNIT)
1010  WRITE (6,109) SP(I), T(I)
1020  109 FORMAT(2E20.8)
1030  20 CONTINUE
1040C CALCULATE THE RMS ERROR IN DRAWDOWN AFTER BEST FIT
1050  SUM = 0.0
1060  DO 30 I = 1, N
30 SUM = SUM + (SP(I) - S(I))**2
1080 SIGMA = SQRT(SUM/N)
1090 WRITE(6,111) SIGMA
1100 111 FORMAT( ' THE RMS ERROR FOR DRAWDOWN IS ' , E20.8 )
1110 STOP
1120 40 WRITE (6,112)
1130 112 FORMAT( ' DID NOT CONVERGE IN 20 ITERATIONS ' )
1140 STOP
1150 END

SUBROUTINE THEIS(SC,KB,Q,R,T,S,DSDT,DSDSC,UNIT)
PURPOSE CALCULATES THE DRAWDOWN FOR A WELL PUMPING AT
CONSTANT DISCHARGE FROM AN INFINITE AQUIFER BY THE
THEIS EQUATION
ARGUMENTS (L IS ARBITRARY LENGTH, T IS ARBITRARY TIME)
SC STORAGE COEFFICIENT FOR AQUIFER (UNITLESS)
KB TRANSMISSIVITY OF AQUIFER (GAL/DAY/FT) OR (L**2/T)
Q CONSTANT PUMPAGE OF WELL (GAL/DAY) OR (L**3/T)
R OBSERVATION DISTANCE FROM WELL (FT) OR (L)
T DRAWDOWN TIME (DAYS) OR (T)
S DRAWDOWN (FT) OR (L)
DSDT SENSITIVITY W.R.T. TRANSMISSIVITY (FT**2/DAY/GAL) OR (T/L)
DSDSC SENSITIVITY W.R.T. STORAGE (FT) OR (L)
UNIT SYSTEM OF UNITS: YES FOR GAL, DAY, FT; OTHERWISE
CONSISTENT UNITS ARE ASSUMED

CALCULATE THEIS SOLUTION
THEIS EQUATION FROM 'INTRO TO HYDROLOGY' BY VIESSMAN PAGE 257
RATIONAL APPROXIMATION FROM 'HANDBOOK OF MATH FUNCTIONS'
BY ABRAMOWITZ AND STEGUN PAGE 231

REAL KB

U = R**R*SC/(4.0*KB*T)
IF (UNIT .EQ. 6HYES ) U = 7.48*U
IF (U .LE. 0.0) GO TO 96
IF (U .GT. 1.0) GO TO 86
W = -ALOG(U)**.57721566+.9999993*U-.24991055*U**2+U**4+.00107857*U**5
GO TO 87
86 W = (EXP(-U)/U)*(U**2.334733**U+.250621)/
1(U**3.330657**U + 1.681534)
87 S = (Q/(4.0**3.14159*KB)**W
DSDT = (Q/(4.0**3.14159*KB)**2)**(-W + EXP(-U))
DSDSC=(-Q/(4.0**3.14159*KB**SC))*EXP(-U)
RETURN
96 PRINT 13
13 FORMAT ( ' DRAWDOWN UNDEFINED U TOO SMALL ' )
99 STOP
END