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Diurnal Drift Removal From Aeromagnetic Data Using Least Squares*

by

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Abstract

A relatively simple method of diurnal drift removal from aeromagnetic data is presented. The underlying assumption is that diurnal drift during flight is a smoothly varying low order polynomial in time. The polynomial coefficients are determined by minimizing residuals at flight line-tie line intersections using least squares. This procedure is applied to some conventional sensitivity aeromagnetic data taken in northeastern Kansas. The results of the drift determinations compare favorably with independent knowledge of actual drifts such as the magnetic K indices and other measurements. With careful attention to intersection location uncertainty and appropriate maximum polynomial order this method should also yield good results for high sensitivity surveys.

Introduction

The solar wind, a constant stream of charged particles from the sun, impinges on the earth's geomagnetic field creating a diurnal drift in magnetic field values for fixed locations on the earth's surface. Procedures for mapping the earth's magnetic field must account for distortion due to diurnal drift in addition to instrument and location errors. The care required to adequately correct for these effects depends on the intended use of the data and the inherent accuracy of the equipment. For $\pm$ 1 gamma conventional aeromagnetic surveys, with good positional control using Doppler navigation and tracking camera, the main source of error is diurnal drift. To take full advantage of high sensitivity surveys of $\pm$ 0.1 gammas or less, corrections for measurement location errors along with correction for diurnal drift must be given careful attention (Bhattacharyya, 1971).

The Kansas Geological Survey (KGS) recently undertook the task of obtaining conventional aeromagnetic coverage of the state. Due to limitations of budget, personnel and logistics of flying a large area, a base station was not used. In any event, the use of a base station for direct diurnal correction is questionable, particularly for large areas (Whitham and Niblett, 1961, and Riddihough, 1971).

Bhattacharyya (1971) discusses a drift removal approach applied to a high sensitivity $\pm$ 0.02 gamma survey. Although his end result was clearly successful because of the improved correlation of the final map with geology, the procedure is rather complex involving several data processing steps and does not make use of minimum tie line-flight line residuals in a well defined least squares sense.

The approach we adopted makes use of a least squares technique and assumes the diurnal drift varies smoothly along each flight line according to a low order polynomial in time. The central idea for this approach was presented orally in a SEG technical paper (Ackerman, 1973) but never published. Ackerman assumes the diurnal drift varies smoothly along each flight line according to a low order polynomial in distance. The distinction between distance and time is not important if the ground speed of the aircraft is fairly uniform. However, substantial variation in ground speed will result in a more complicated drift dependence on distance, making time the more desirable variable.

Theory

Assume tie lines and flight lines are flown in approximately orthogonal directions. The measured magnitude of the total magnetic field, H along tie lines and F along flight lines, can be written

$$H(x,y,s,S,h) = R(x,y,S,h) + G(x,y,h) + C(s) \quad (1)$$

$$F(x,y,t,T,h) = R(x,y,T,h) + G(x,y,h) + D(t) \quad (2)$$

where s = elapsed flying time along tie line

S = chronological time of tie line measurement

t = elapsed flying time along flight line

T = chronological time of flight line measurement

R = noncrustal field (main field)

G = Magnetic field due to crustal sources

C,D = Departure from R + G due to diurnal drift and measurement error

h = flight elevation above sea level

The goal of most aeromagnetic flying is to map the magnetic field due to crustal sources, G, which is one of three sources of the measured field F. Although the magnitude of G is normally less than 1 or 2 percent of F, it represents virtually all of the spatial variation in H related to crustal geology. G is due to permanently magnetized and inductively magnetized rocks in the crust.

R is the normal geomagnetic field due to sources below the Curie Point isotherm and is a slowly varying function of time (secular drift). Strictly speaking, G is also a slowly varying function of S and T because the induced magnetism of the crust is proportional to R. This crustal variation, however, is neglible, unless the survey flying time extends over many years. R can be readily calculated at any point x, y, h using one of several models available. The accuracy and relative merits of the various models will probably continue to be debated, but the most widely used and internationally recognized model is the International Geomagnetic Reference Field (IGRF). This model is based on a combination of world-wide ground, airborne, shipborne and satellite magnetic field measurements (Zmuda, 1971). Other models may be more desirable, in particular cases, depending on location and goal of project. If the survey is completed in a relatively short time, less than a month or two, the difference in regional fields due to the time difference S-T will be neglible. However, secular drift over the period of a year can be several tens of gammas. It is assumed the flight elevation, h, over the intersection point between tie line and flight line is approximately the same for both lines. This does not necessarily imply a constant flight elevation over the entire survey area.

The term C(or D) represents the difference between the field measured in flight, H(or F), and the quasi-static field, R + G. In a well controlled survey the major source for the "error" term C (or D) is diurnal drift. In the northern latitudes the magnetic field normally decreases during the morning hours and increases during the afternoon. The diurnal drift curve at ground level is also influenced by induction within the crust from the external time varying field. There are no satisfactory models that can accurately predict, as is the case for the main field, the diurnal drift for a specific day and location. Therefore an empirical approach must be adopted.

Let $$H'(x,y,s) = H(x,y,s,S,h) - R(x,y,S,h) \quad (3)$$

$$F'(x,y,t) = F(x,y,t,T,h) - R(x,y,T,h) \quad (4)$$

Thus H' and F' are independent of chronological times T and S since the normal field, R, has been subtracted. It is assumed that at intersections the tie line and flight line were flown at the same elevation, h(x,y) or were upward (or downward) continued to an equivalent h(x,y). Consequently, the flight altitude variable, h, has been suppressed because it does not play a role in the adjustment procedure. In most cases h(x,y) is constant.

At tie line-flight line intersection point (i,j) equations 3 and 4 become

$$H'_{ij} = G_{ij} + C(s_{ij});\ j^{th} \text{ intersection on } i^{th} \text{ tie line} \qquad (5)$$

$$F'_{ij} = G_{ij} + D(t_{ij});\ i^{th} \text{ intersection on } j^{th} \text{ flight line} \qquad (6)$$

Assume the drift terms C and D vary according to low order polynomials in time.

$$C(s_{ij}) = \sum_k c_{ik} s_{ij}^k \qquad (7)$$

$$D(t_{ij}) = \sum_k d_{jk}\ t_{ij}^k \qquad (8)$$

Normally there are substantially more flight lines than tie lines. Thus there are more intersections per unit length along a tie line which provides better control for determining the drift correction. This makes it advisable to determine $C(s_{ij})$ first. To do this, make the approximation that $D(t_{ij}) = 0$ for all flight lines. Determine the coefficients C_{ik} for each tie line, i, by fitting the tie line intersection values to the flight line intersection values. The condition for minimum residuals is

$$\frac{\partial}{\partial C_{i\ell}} [\sum_j (H'_{ij} - F'_{ij} - \sum_k c_{ik}\ s_{ij}^k)^2] = 0 \qquad (9)$$

where j is summed over all flight line intersections with tie line i and

$\ell = 0, 1, \ldots,$ maximum order.

After differentiation the result can be written

$$\sum_k \left(\sum_j s_{ij}^{k+\ell}\right) c_{ik} = \sum_j (H_{ij} - F_{ij})\, s_{ij}^{\ell}$$

which represents a set of linear equations that can be solved for the coefficients c_{ik}. For each tie line, i, this corresponds to the matrix equation

$$MC = R \tag{10}$$

$$\text{with solution } C = M^{-1}R \tag{11}$$

$$\text{where } M_{\ell k} = \sum_j s_{ij}^{k+\ell} \tag{12}$$

$$C_k = c_{ik} \tag{13}$$

$$R_\ell = \sum_j (h'_{ij} - F'_{ij})\, s_{ij}^{\ell} \tag{14}$$

$\ell = 0, 1, \ldots,$ maximum order of drift in time.

This fit procedure approximates G_{ij} at intersection points. The existence of nonzero $D(t_{ij})$'s scatter the flight line values about G_{ij} as shown in Figure 1. If the maximum order in equation 7 is much less than the total number of flight lines, then G_{ij} along tie line, i, is essentially the average surface defined by the flight lines. The zero and first order coefficients level shift and tilt the tie line, while higher order coefficients allow slight modifications in shape.

If the area was flown piecemeal over a long period of time, a year or more, it is assumed that subtraction of the regional field, R, will account for level discrepencies due to secular drift. If it is apparent that R is in poor agreement with local secular drift, then tie lines and flight lines should be leveled to each other, using zero order least squares on both tie lines and flight lines, before higher orders are determined. An indication of an areal level bias in the flight lines would be, for example, if first order drift correction to the tie lines tilted all the lines in the same direction.

Assuming the tie lines approximate the true G_{ij} at the intersections, drift coefficients d_{ij} for the flight lines can now be determined using the same approach. For each flight line j, determine the set of coefficients that best satisfies the conditions that equation 6 yields minimum residuals. The condition for minimum residuals is

$$\frac{\partial}{\partial d_{j\ell}} [\sum_i (F'_{ij} - G_{ij} - \sum_k d_{jk} t_{ij}^k)^2] = 0 \tag{15}$$

$$\text{where } G_{ij} = H'_{ij} - \sum_k c_{ik}\, s_{ij}^k \tag{16}$$

and i is summed over all tie line intersections with flight line j. For each flight line the solution in matrix form is

$$D = M^{-1}R \tag{17}$$

$$\text{where } M_{\ell k} = \sum_i t_{ij}^{k+\ell} \tag{18}$$

$$D_k = d_{jk} \tag{19}$$

$$R_\ell = \sum_i (H'_{ij} - G_{ij})\, t_{ij}^\ell \tag{20}$$

Using the correction coefficients $C(s_{ij})$ and $D(t_{ij})$ then for any measurement position

$$G(x,y) = H'(x,y) - C(s) \text{ along tie lines and} \tag{21}$$

$$G(x,y) = F'(x,y) - D(t) \text{ along flight lines.} \tag{22}$$

Application

This approach to diurnal drift correction was applied to aeromagnetic data recently acquired over northeastern Kansas. The actual flight path pattern is shown in Figure 2. The east-west flight lines are approximately 200 miles long and spaced two miles apart and flown at fixed barometric altitude of 2,500 feet above sea level. The north-south tie lines are spaced approximately 20 miles apart and flown at the same fixed barometric altitude. A representative 200 mile flight line is shown in Figure 3. The equipment consisted of a $\pm$ 1 gamma proton precession magnetometer, crystal oscillator clock, digital recording system and a 35 mm camera correlated with magnetometer. The flight lines were flown along section roads which made it unnecessary to use Doppler navigation for accurate flight path recovery.

After assignment of latitude and longitude to each measurement and removal of the IGRF, intersection residuals were calculated for all of the 363 intersections. The drift terms C(s) and D(t) were determined from least squares using equations 13 and 20. The maximum order for each line, i.e. max ℓ in equations 9 and 15, was determined empirically as follows. The rms residuals were calculated for all orders up to $\ell = 5$. The maximum order required for each line was chosen as the smallest order above which there was no significant decrease (greater than one gamma) in the rms residual for the line. The maximum orders required for individual flight lines ranged from $\ell = 1$ to $\ell = 3$ which indicates that the diurnal drift was a relatively smooth function of time as expected for magnetically quiet days.

Before drift correction the rms residual for all intersections was 25 gammas. After correcting all measurements for drift using equations 22 and 23 the rms residual was reduced to 8.4 gammas. A histogram of the residuals is shown in Figure 4a. The distribution appears to be normal and centered at zero which suggests that most of the residuals are due to random measurement error. Intersections with very large residuals greater than 15 gammas, appearing beyond the limits expected from a gaussian distribution, were examined. Most of these were found to be in areas of relatively high horizontal gradient, 2 to 8 gammas per 100 feet. Figure 3 shows representative gradients along a typical profile. Residuals at these intersections are very sensitive to errors in flight path coordinates and are therefore not reliable for determination of drift correction. New drift corrections were determined after deletion of 18 anomalous residuals resulting in a substantially improved rms residual of 4.3 gammas for the remaining intersections. As evident in the histogram in Figure 4b, deletion of the anomalous residuals considerably tightened the distribution.

In following this procedure it is assumed that tie line data is not used in the final contour map, but is used primarily for leveling and drift removal. If it is advantageous to use tie line data in the contour map, then an alternate procedure must be followed. Instead of discarding anamolous tie line-flight line mismatches, presumably due to location errors in high gradient regions, improved intersection coordinates must be found. As suggested by both Bhattacharya (1971) and Ackerman (1973), this is accomplished by identifying the minimum intersection residual within an appropriate radial distance, consistent with the accuracy of the flight path recovery procedure, from the originally calculated intersection location. For high sensitivity data it would be desirable to do this for all intersections which would further tighten the distribution of residuals about zero.

The relative effect the drift adjustment sequence has on the rms residual is shown in Figure 5. Before any adjustment the rms residual is 25.3 gammas. After tie line adjustment the residual is reduced to 11.1 gammas. Tie line fits beyond first order were examined, but did not significantly improve the residuals. This will not necessarily be true in general. The zero order flight line correction further reduced the rms residual to 6.4 gammas. It is clear that a large part of the original rms residual is due to relative level differences between lines. Zero order level adjustment of tie lines and flight lines accounts for nearly 16 gammas of the original 25 gamma rms residual. Successive higher order drift adjustment of a flight line slowly reduces the rms residual to zero as the adjustment order approaches one less than the number of intersections along the line.

An acceptable value of the final rms residual is dependent on the intended use and quality of the aeromagnetic data. For example, the residual distribution in Figure 4b is adequately narrow to permit contouring at contour intervals of ten gammas and greater.

In general, if tie line data is used for contouring, the adjustment order must be increased to avoid abrupt level changes in the vicinity of tie line-flight line intersections. For the northeastern Kansas flight lines, the third order adjustment (Figure 4a) is probably adequate for diurnal corrections, but there still remain nonzero residuals due to intersection location errors and nonuniform velocity between fiducial points. As mentioned earlier, the residuals can be further reduced by searching for a smaller residual within a reasonable circle of uncertainty about the intersection. Beyond that, the rms residual can be further reduced by going to a higher order. The residuals will ultimately go to zero when the polynomial order approaches one less than the number of intersections along the line. This last step simply distributes any intersection mismatches, remaining after correcting for normal diurnal drift and intersection location errors, along the flight line according to a smoothly varying polynomial in time.

Physical Meaning

Assuming that flying is done on relatively quiet magnetic days, it is expected that diurnal drift will vary as a low order function of time. Errors in flight path recovery will also contribute to the residuals along a line. The random mismatches at tie line-flight line intersections will appear as noise in the residuals and contribute to the higher order terms in the drift corrections C(s) and D(t).

The relationship between the drift curves determined for the flight lines (Figure 2) using the least squares procedures and true diurnal drift was investigated. The unavailability of a ground station made it impossible to directly measure diurnal drifts during flight. In one case, however, we were able to directly measure average drift. Primarily for the purpose of checking heading errors and the flight path recovery procedure one of the flight lines, located at approximately 39.1° N. latitude, was flown in one direction, then immediately reflown in the opposite direction. This made it possible to determine net drift for any point along the line. The average drift, calculated from 2,500 drift measurements along the 200 mile line, was 4.5 gammas/hour. The coefficient, d_{j1} in equation 8, determined for this line using a first order fit, was 3.6 gammas per hour. This coefficient represents the average linear drift along the entire line and is in excellent agreement with the independent determination using flights in both directions.

In general, for the northern latitudes, the earth's magnetic field at ground level decreases during the morning hours and increases during the afternoon hours (Figure 6). The precise shape of the diurnal curve, including the location of the minimum, is not predictable and varies from day to day. The amount of high frequency variation, say periods of one hour or less, depends on magnetic conditions which vary from very quiet conditions, ideal for aeromagnetic flying, to storm conditions which prevent reliable magnetic measurements altogether.

The magnetic K index for Boulder, Colorado, approximately 600 miles away, was obtained for all flights from the World Data Center A for Solar-Terrestial Physics. This three hour index is designed to measure the irregular variations in the magnetic field (Lincoln, 1967). The value of K depends on the difference between the highest and lowest deviation from the smooth diurnal variation within a three hour period. The K scale varies from zero, very quiet, to nine, storm conditions. Higher K values imply more amplitude variation or higher frequency content in the diurnal drift. The least squares drift determinations are consistent with this. The percentage of flight lines requiring more than a first order fit is roughly proportional to the K index (figure 7).

The distribution of drift magnitudes also shows some dependence on the K index. The range of first order drift magnitudes is roughly proportional to K (Figure 8). These drifts were determined from first order fits to all the flight lines which is equivalent to average linear drifts for each line. This result (Figure 8) is consistent with the fact that the magnitude of fluctuation of the earth's magnetic field increases with K.

The least squares leveling indicated that eight of the flight lines had true linear drifts. That is, the rms residuals for these lines were low and did not significantly decrease for higher order fits above first order. A plot of the resulting first order drift magnitudes, including the sign, against local time (Figure 9) is quite consistent with the known general shape of the diurnal drift (Figure 6). Four out of five morning flights had negative drifts and all three afternoon flights had positive drifts as expected.

Conclusions

This least squares approach, which makes use of the fact that the diurnal component of the earth's magnetic field normally varies as a low order polynomial series in time, provides a relatively simple unambiguous procedure for removing diurnal drift from aeromagnetic data. A drift function for each flight line is determined. The polynomial coefficients for each function are chosen to minimize the root-mean-square of the tie line-flight line intersection residuals. This procedure does not require a separate data leveling step, but automatically includes leveling as part of the drift correction. Drift determinations from some conventional sensitivity data, using this procedure, appear to be quite consistent with known characteristics of actual drift as depicted by the magnetic K index. It is not required that tie lines, or flight lines, be flown consecutively over a short period of time. This procedure should apply equally well to flight lines and tie lines flown independently on different days over periods of months or even years. This procedure could be extended to high sensitivity data by correcting intersection location errors and, if necessary, by selecting a higher order polynomial for the least squares fit.

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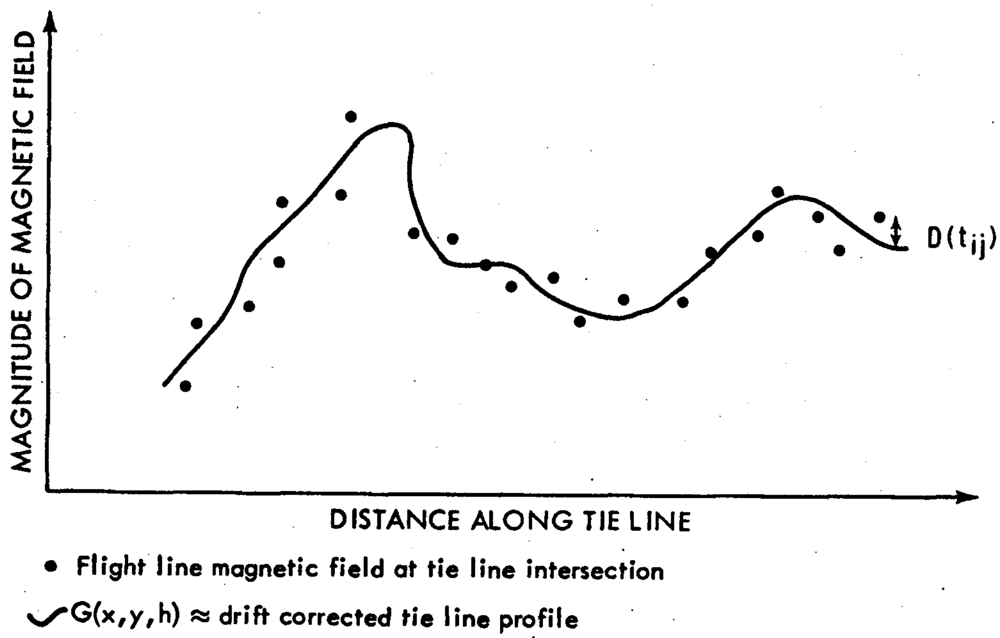

Fig. 1. Hypothetical tie line profile after least squares drift correction. The uncorrected j^{th} flight line profile is displaced from the i^{th} tie line profile at intersection (i,j) by the drift term $D(t_{ij})$.

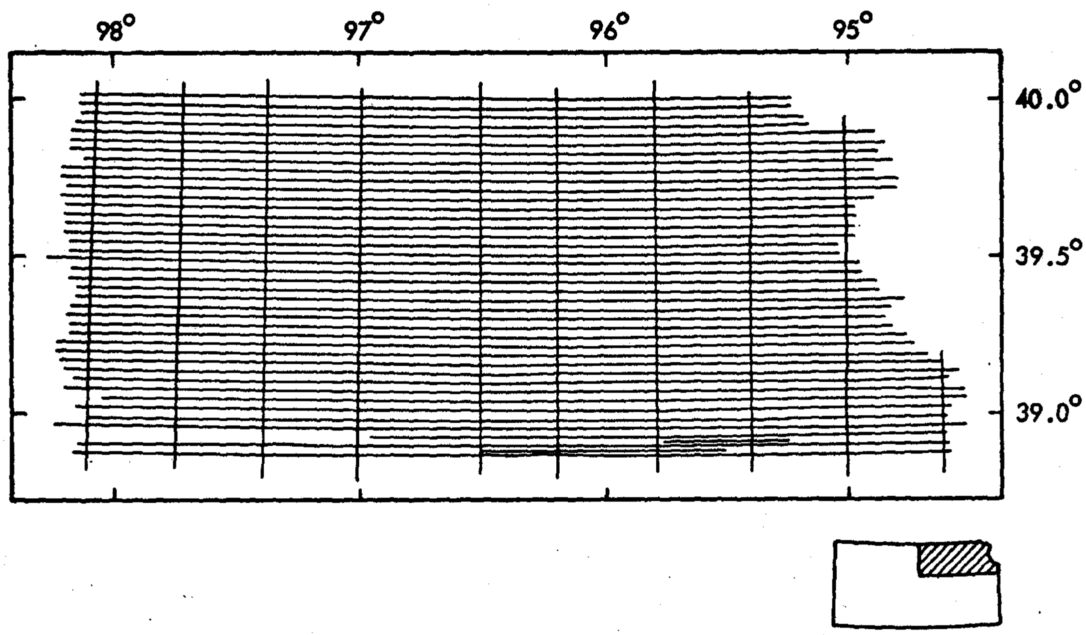

Fig. 2. East-west flight lines and north-south tie lines used in least squares adjustment procedure. All lines were flown at fixed barometric altitude of 2,500 feet above sea level which corresponds to an average ground clearance of 1,200 feet.

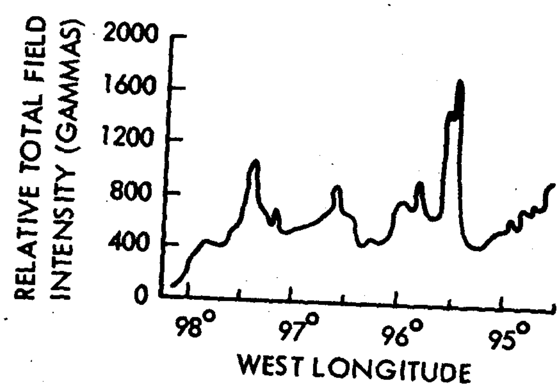

Fig. 3. Representative east-west profile in northeastern Kansas used in least squares adjustment procedure. Maximum gradients are 14 gammas per 100 feet.

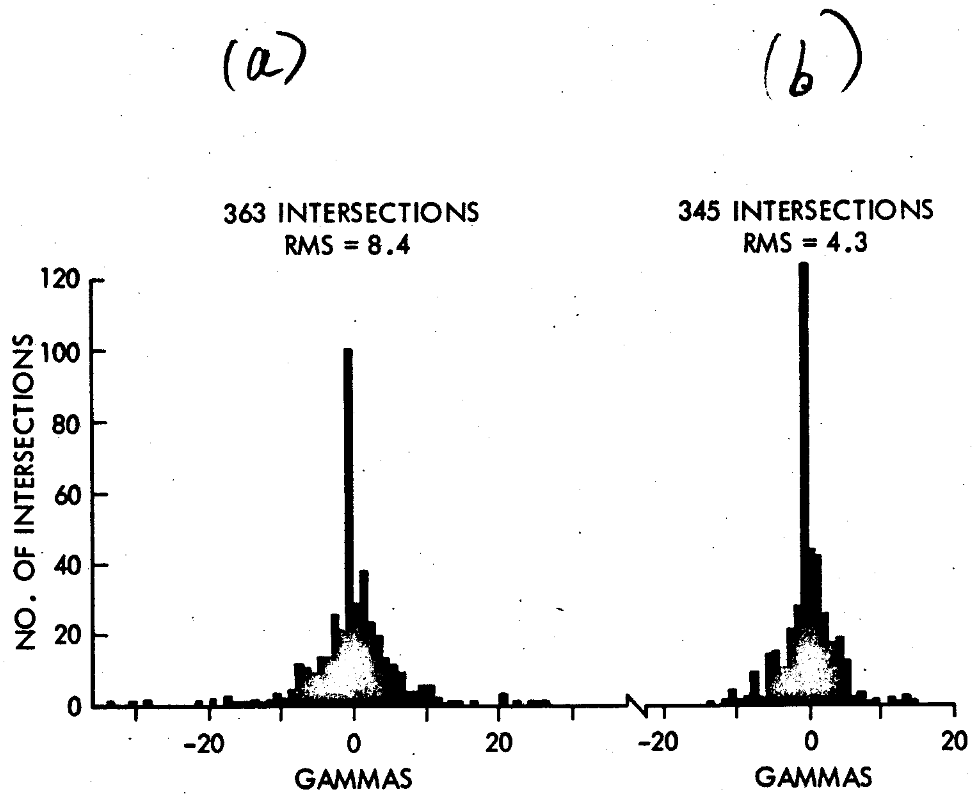

Fig. 4. a) Tie line-flight line intersection residuals after least squares adjustment of tie lines and flight lines. Maximum polynomial order used in drift correction for each line ranged from zero to third order.
b) Residuals after deleting intersections in histogram a) with high residuals occurring in high gradient regions and readjustment of remaining intersections.

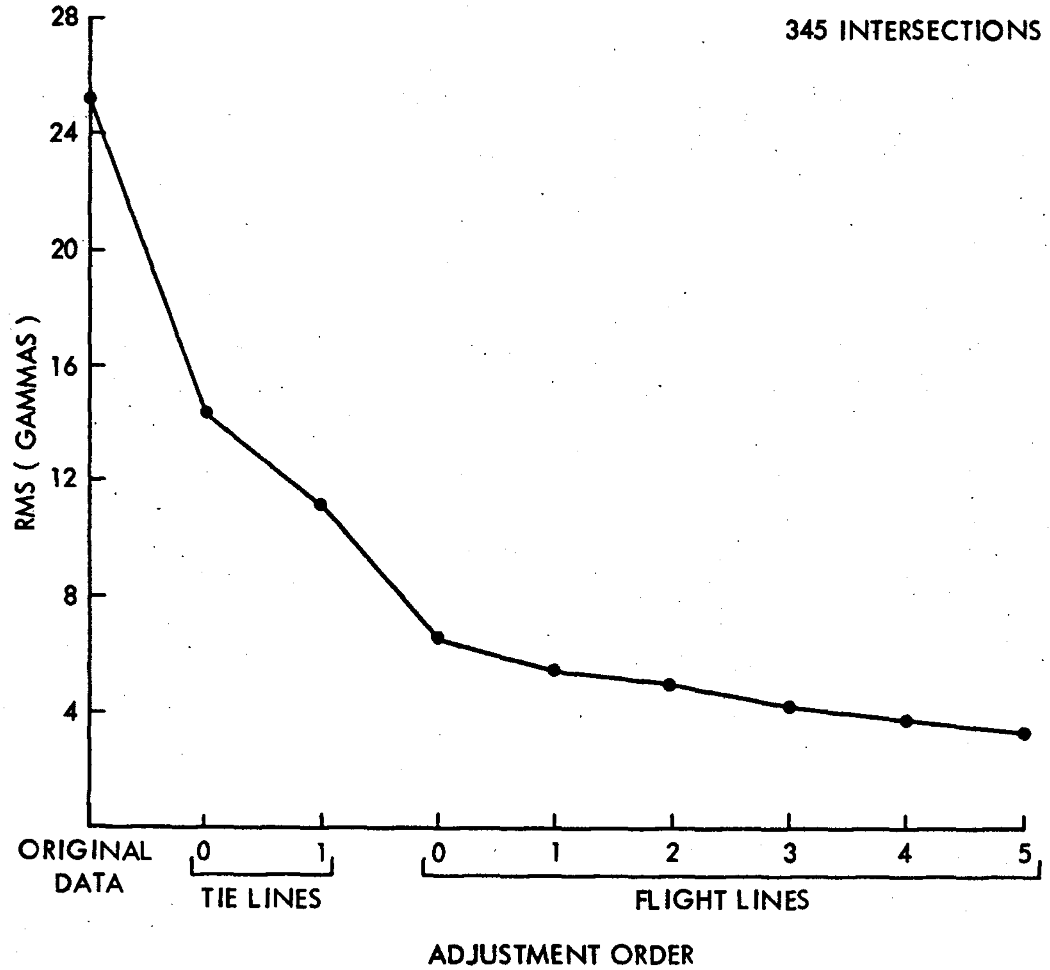

Fig. 5. Root-mean-square of the tie line-flight line intersection residuals versus the least squares drift adjustment sequence.

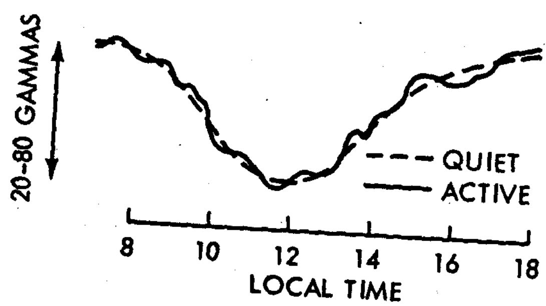

Fig. 6. Typical diurnal drift curve during daylight hours in northern latitudes.

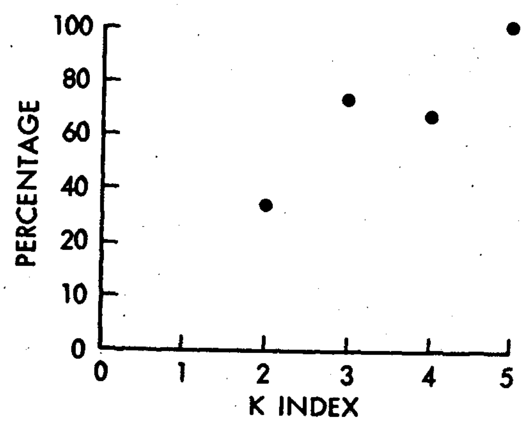

Fig. 7. Percentage of flight lines requiring more than a first order polynomial drift correction versus the magnetic K index.

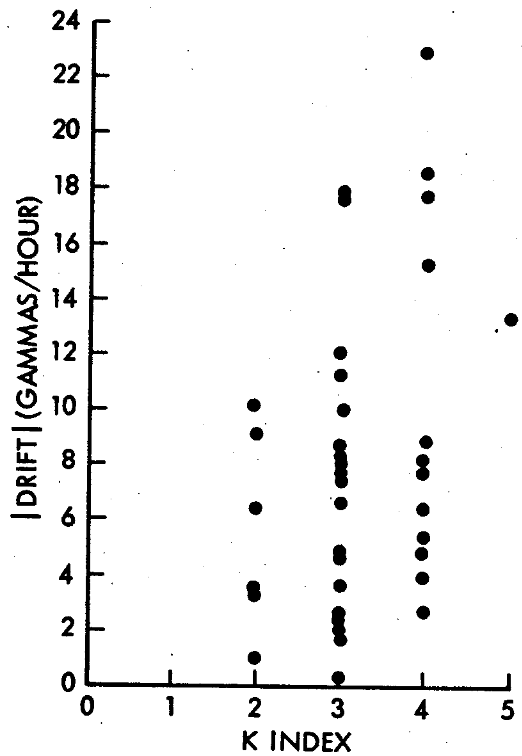

Fig. 8. Drift magnitudes resulting from first order fits to all flight lines versus the magnetic K index.

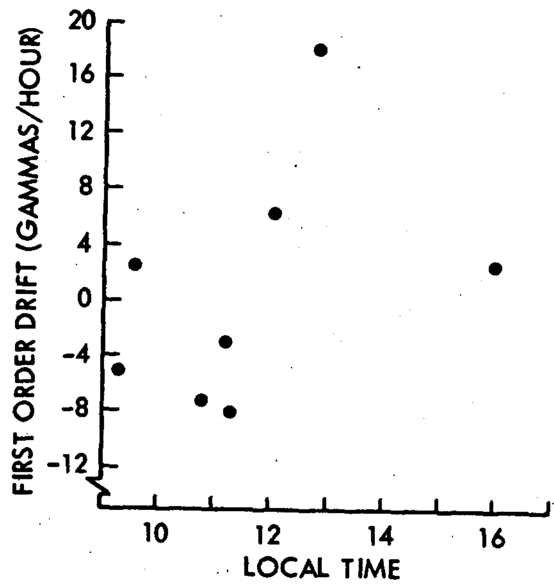

Fig. 9. First order drift versus local time. Only flight lines with apparent linear drift, i.e. lines requiring only a first order drift correction to achieve a low rms residual, are plotted.