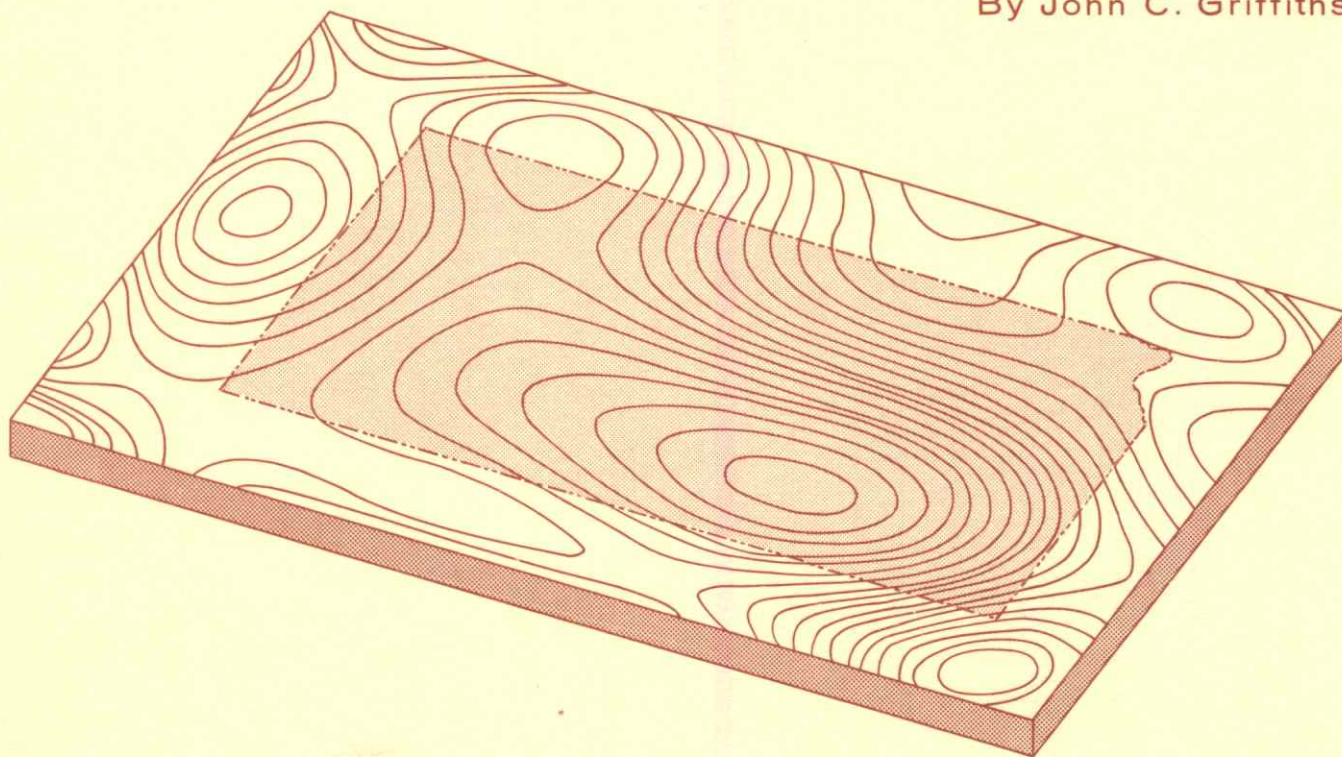


The Unit Regional-Value Concept and Its Application to Kansas

By John C. Griffiths



Special Distribution Publication 38

State Geological Survey
The University of Kansas
Lawrence, Kansas

February 1969

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CONTENTS

	<i>Page</i>
INTRODUCTION.....	5
Acknowledgments.....	5
UNIT REGIONAL VALUE OF U.S.A.....	6
UNIT REGIONAL VALUE OF U.S.A. BY STATES.....	7
FREQUENCY DISTRIBUTION OF DOLLAR VALUE PER SQUARE MILE IN U.S.A. BY STATES.....	10
UNIT REGIONAL VALUE OF KANSAS.....	13
Unit Regional Value of Kansas for the Year 1960.....	13
Nature of the Data.....	16
Frequency Distribution of the Variates.....	19
SPATIAL DISTRIBUTION OF UNIT REGIONAL VALUE.....	22
Value per Square Mile in 1960.....	24
Value per Cubic Mile in 1960.....	24
Value per Square Mile for 1952-1965.....	26
EVALUATION OF INFORMATION LEVEL PER UNIT AREA PER COUNTY.....	28
Residual Deviations from the Sixth-Degree Surfaces.....	30
Deviations from Sixth-Degree Surface of Log Dollar Value per Square and per Cubic Mile in 1960.....	31
Deviations from Sixth-Degree Surface of Log Dollar Value per Square Mile in 1952-1965.....	33
The Richest and Poorest Counties.....	34
CONCLUSIONS.....	34
REFERENCES.....	38
APPENDIX.....	40
ILLUSTRATIONS	

<i>Figure</i>	<i>Page</i>
1. Value of mineral industries products for U.S.A. by states, 1911-1964.....	12
2. Cumulative frequency distribution of log value of mineral industries products by states for U.S.A.....	13
3. Classification of states on the basis of value of mineral industries products.....	14
4. Orthogonal grid of points fitted to Kansas and two tiers of counties in adjoining states.....	17
5. Cumulative frequency curves of $\log_{10}$ dollar value for mineral industries products in Kansas for 1960 and 1952-1965.....	20

	Page
6. Coefficient of determination ($R^2\%$) as a measure of amount accounted for by trend-surfaces of degrees 1 through 6.....	23
7. Contoured surface of log dollar value per square mile for Kansas, 1960.....	24
8. Linear trend-surface of log dollar value per square mile for Kansas, 1960.....	25
9. Sixth-degree trend-surface of log dollar value per square mile for Kansas, 1960.....	25
10. Contoured surface of log dollar value per cubic mile for Kansas, 1960.....	26
11. Sixth-degree trend-surface of log dollar value per cubic mile for Kansas, 1960.....	27
12. Contoured surface of log dollar value per square mile for Kansas, 1952-1965.....	27
13. Sixth-degree trend-surface of log dollar value per square mile for Kansas, 1952-1965.....	28
14. Drilling intensity per square mile per county in Kansas, to 1965.....	29
15. Sixth-degree trend-surface of drilling intensity per square mile per county in Kansas, to 1965.....	29
16. Cumulative probability curves of observed and expected log-normal frequencies of deviations from sixth-degree trends of log value per square mile and per cubic mile in 1960 and log value per square mile in 1952-1965.....	31
17. Residual deviations of sixth-degree trend-surface from original data for log dollar value per square mile in Kansas, 1960.....	32
18. Residual deviations of sixth-degree trend-surface from original data for log dollar value per square mile in Kansas, 1952-1965.....	33
19. Richest and poorest counties in dollar value per square mile in Kansas, 1952-1965.....	35

TABLES

Table	Page
1. Total value of mineral industries products for the conterminous U.S.A., 1880-1964.....	6
2. Value of mineral industries products produced per square mile per state for the period 1911-1964.....	8
3. Comparison of states in terms of actual and expected production on the basis of area.....	9

4. Value of mineral industries products per square mile per state, 1911-1964.....	10
5. Frequency distribution of value per square mile of mineral industries products by states, 1911-1964.....	11
6. Value of mineral industries products per square mile for each state in 1960.....	15
7. Comparison of ranks of states from 1911-1964 and 1960 in terms of value per square mile.....	16
8. Number of drilled wells per county and equivalent rank.....	19
9. Frequency distribution of value of mineral industries plus agricultural products from irrigation per square mile in Kansas, 1960.....	20
10. Frequency distribution of value of mineral industries plus agricultural products from irrigation per cubic mile in Kansas, 1960.....	21
11. Frequency distribution of value of mineral industries products in Kansas, 1952-1965.....	22
12. Coefficient of determination ($R^2\%$) as a measure of amount of variation accounted for by trend-surfaces of degrees 1 through 6.....	23
13. Summary statistics for frequency distribution of deviations from sixth-degree trend-surface.....	30

APPENDIX

Tables

Page

1. Data arranged according to grid for the state of Kansas and adjoining tiers of counties.....	40
2. Analysis of variance of trend-surface fitting for log dollar value per square mile, 1960.....	44
3. Analysis of variance of trend-surface fitting for log dollar value per cubic mile, 1960.....	44
4. Analysis of variance of trend-surface fitting for log dollar value per square mile, 1952-1965.....	45
5. Analysis of variance of trend-surface fitting for drilling intensity in Kansas.....	46
6. Richest and poorest counties in terms of $\log_{10}$ dollar value per square mile in 1960.....	47
7. Richest and poorest counties in terms of $\log_{10}$ dollar value per cubic mile in 1960.....	47
8. Richest and poorest counties in terms of $\log_{10}$ dollar value per square mile, 1952-1965.....	48

THE UNIT REGIONAL-VALUE CONCEPT

AND ITS APPLICATION TO KANSAS

INTRODUCTION

The search for and development of nonrenewable natural resources is a profit-making enterprise, and the decision to explore a region rests on its potential value and the risk involved in developing its potential. The basic question underlying the decision concerns the worth of a region when developed. The importance of this information arose when modeling a global strategy for the search for nonrenewable natural resources (Griffiths, 1966, 1967) and a reconnaissance study of the feasibility of evaluating the worth of Kansas was completed in 1965 (Griffiths, Koch, Meeves, and Hornbaker).

The required information may be expressed in terms of the value of a region in dollars per square mile or dollars per cubic mile; the estimates are based on the value of mineral industries products produced before the materials are modified by manufacturing processes. The comparison of regions is, of course, a relative one in that reserves are not considered, and the dollar value of the product is not discounted or standardized; furthermore, there are no regions which may with certainty be classified as depleted in an absolute sense so the extremes of completely developed and completely undeveloped are not available as a basis for comparison. Nevertheless, the values found for various regions are comparative and may be used as a guide to what may be expected in large regions when systematic development is undertaken.

The objectives of this analysis are to formulate an estimate of the potential value of a region prior to its exploration and development and to obtain an inventory of the natural resources of large regions based on past production. The United States of America may be looked upon as a suitable "sample" of the earth's crust for this purpose, and, by successive sampling of this population, the distribution of value in subregions within the conterminous U.S.A. and Alaska may be described.

Acknowledgments

Research on the unit regional-value concept was initiated at a seminar on Economic Analysis in the Mineral Industries held under the auspices of the State Geological Survey of Kansas in June 1965 (see references under Griffiths, Koch, Meeves, and Hornbaker, 1965) and the work was continued under support from the State Geological Survey of Kansas through 1966 and 1967. Some support was also received from a research project on the applications of operations research to the mineral industries funded through the National Science Foundation under Grant Number GK 1149.

As an initial hypothesis it may be presumed that all regions are equal in value, and this hypothesis may then be tested against the value of production achieved. If the hypothesis is accepted, then the expected value of a large region would be a useful guide to the potential value of its smaller subregions.

On the other hand, if some regions are more valuable than others and a systematic arrangement emerges, the "system" may be used as a predictive tool to guide exploration and development to the more favorable areas.

The data upon which the analysis is based is contained in the U.S. Bureau of Mines' Minerals Yearbooks, Area Reports. Records of some kind exist from 1880, but more detailed information by state and county is available only for shorter periods. The comparisons are, therefore, relative and not absolute;

they are adequate to subdivide the regions into several classes based on value per unit area or volume.

UNIT REGIONAL VALUE OF U.S.A.

The first step in approaching this problem is to accumulate the value in dollars of all the mineral industries products produced in the U.S.A. Suitable records exist for this purpose in the U.S. Bureau of Mines' Minerals Yearbooks. Table 1 is a summary of values of mineral industries products accumulated over the years 1880 to 1964.

Table 1. Total value of mineral industries products for the conterminous U.S.A. 1880-1964 (in millions of dollars).

Period	Fuels	Metals	Nonmetallics other than fuels	Total
1880-1924 ^{1/}	\$ 40,792	\$24,736	\$15,054	\$ 80,582
1925-1963 ^{2/}	207,276	36,785	54,772	298,833
1964 ^{3/}	<u>13,585</u>	<u>2,265</u>	<u>4,622</u>	<u>20,472</u>
Total	\$261,653	\$63,786	\$74,448	\$399,887
Value/sq mi	\$ 86,572	\$21,105	\$24,632	\$132,308

^{1/} U.S. Bureau of Mines, Minerals Yearbook, Area Reports, 1947.

^{2/} U.S. Bureau of Mines, Minerals Yearbook, Area Reports, 1963.

^{3/} U.S. Bureau of Mines, Minerals Yearbook, Area Reports, 1964.

The most valuable products are the fuels, followed by nonmetallics other than fuels and finally metals. The cumulative value is 399,887 millions of dollars and this may be prorated over the area of the conterminous United States in square miles (3,022,387 square miles) yielding 132,308 dollars per square mile. Similarly, the value for fuels is 86,572 dollars per square mile, for the nonmetallics 24,632 dollars per square mile, and for metals 21,105 dollars per square mile.

These figures may be used as expected values per square mile for large regions which are representative samples of the earth's crust in the U.S.A.; thus if the aim is to attain a return of 132 millions of dollars, some 1,000 square miles on the average would be required. If the largest resource were fuels then 86.5 millions of dollars would be expected from 1,000 square miles. Since Kansas contains 82,264 square miles the expected value of all resources would be 10,884 millions of dollars.

It is, of course, obvious that while all areas may be equal in potential value not all areas have produced equal values, so it is a matter of some interest to evaluate the actual production per state on the same basis and compare it with the expected value for the region as a whole.

UNIT REGIONAL VALUE OF U.S.A. BY STATES

The value per state is available from 1911 to 1964 in the U.S. Bureau of Mines' Minerals Yearbooks and the accumulated data are summarized in Table 2. The data differ in terms of period and in the fact that the table contains the state of Alaska, which is not present in Table 1. Nevertheless, the overall figure of 458,101 millions of dollars for 3,608,787 square miles yields a closely comparable average figure of 126,940 dollars per square mile.

The expected value for Kansas was quoted as 10,884 millions of dollars and the actual value of production from Table 2 is 16,157 millions of dollars, which exceeds, but is close to, the expected figure. A comparison of actual production with that expected on the basis of area is summarized in Table 3, and 13 states comfortably exceed their expected value, 10 states have achieved their expected value, and 26 states are underachievers.

The average value per square mile is illustrated in Figure 1 and summarized in Table 4, in which those states that are equal achievers are grouped together. Obviously Alaska is underdeveloped with 3,483 dollars per square mile. If we use the expected value as an estimate of the potential for Alaska, then it would be expected that the total value of mineral industries natural resources potentially available is 77,352 millions of dollars. This gives some idea of the potential for profitable investment which is available in this large an area.

Oregon is certainly a strong underachiever with 9,500 dollars per square mile. Again, using the expected value as a basis, the potential value of Oregon is 13,200 millions of dollars against 922 millions of actual production. Eight states fall in the next group worth from 10 to 20 thousands of dollars per square mile. It seems clear that despite very different geological characteristics, geographical dispositions, areas, etc., these states are largely underdeveloped in terms of potential.

The richest states in terms of past production are Pennsylvania, with one million dollars per square mile, and West Virginia, worth 660,000 dollars per square mile, largely attributable to the fuel resources of coal and oil. It is remarkable that these states should possess such large poverty pockets at the present time.

Louisiana has recently profited from the development of its petroleum resources and now ranks third with 599,000 dollars per square mile, some four times the expected value on the basis of the U.S.A. as a whole. California is fourth with 404,000 dollars per square mile, about three times its expected value.

Four states--New Jersey, Ohio, Oklahoma, and Texas--all score high with from 250,000 to 350,000 dollars per square mile. Presumably petroleum forms the basis of this rich reward in Texas and Oklahoma. New Jersey and Ohio, however, are much more highly industrialized. This grouping suggests that the presence of ample resources is not the sole factor in insuring production, but a highly industrialized local economy is helpful in leading to development of many more kinds of resources. All four states have exceeded by two to four times their expected values.

There are many other possible comparisons available in these tables but it is perhaps most important to realize that the value of a state is not solely dependent on its geological environment; for example, while Indiana, Michigan, and Kansas possess a relatively thick sedimentary column and approximately similar resources and value per square mile, the states of Maryland, Minnesota, and New York have similar value per square mile but very different

Table 2. Value of mineral industries products produced per square mile per state for the period 1911-1964.

State	Area ^{1/} (incl. inland water), in sq mi	Value of min. ind. products (1911-1964) ^{2/} , in \$	Value per square mile, in \$
Alabama	51,609	5,617,647,000	108,850
Alaska	586,400	2,042,313,640	3,483
Arizona	113,909	10,812,902,000	94,926
Arkansas	53,104	5,589,856,000	110,347
California	158,693	64,190,902,000	404,497
Colorado	104,247	6,767,394,000	64,917
Connecticut	5,009	374,543,000	74,774
Delaware	2,057	31,431,000	15,280
Florida	58,560	3,298,368,000	56,325
Georgia	58,876	1,716,086,000	29,147
Idaho	83,557	2,447,407,000	29,290
Illinois	56,400	16,838,077,000	298,547
Indiana	36,291	6,033,413,000	166,251
Iowa	56,290	2,310,895,000	41,053
Kansas	82,264	16,156,824,000	196,402
Kentucky	40,395	12,112,863,000	299,860
Louisiana	48,523	29,078,970,000	599,282
Maine	33,215	362,251,000	10,906
Maryland	10,577	1,297,595,000	122,681
Massachusetts	8,257	742,048,000	89,869
Michigan	58,216	10,160,482,000	174,531
Minnesota	84,068	11,455,421,000	136,264
Mississippi	47,716	2,820,514,000	59,110
Missouri	69,686	4,876,914,000	69,984
Montana	147,138	5,346,425,000	36,336
Nebraska	77,227	1,204,981,000	15,603
Nevada	110,540	4,884,131,000	44,184
New Hampshire	9,304	135,291,000	14,541
New Jersey	7,836	2,671,461,000	340,922
New Mexico	121,666	9,792,064,000	80,483
New York	49,576	6,393,346,000	128,961
North Carolina	52,712	1,031,478,000	19,568
North Dakota	70,665	912,808,000	12,917
Ohio	41,222	12,483,777,000	302,843
Oklahoma	69,919	22,770,455,000	325,669
Oregon	96,981	922,092,000	9,508
Pennsylvania	45,333	45,519,664,000	1,004,118
Rhode Island	1,214	49,527,000	40,797
South Carolina	31,055	502,292,000	16,174
South Dakota	77,047	1,141,045,000	14,810
Tennessee	42,244	3,515,444,000	83,218
Texas	267,339	83,417,044,000	312,027
Utah	84,916	8,697,042,000	102,419
Vermont	9,609	1,142,362,000	118,885
Virginia	40,815	4,791,938,000	117,406
Washington	68,192	1,848,258,000	27,104
West Virginia	24,181	15,990,481,000	661,283
Wisconsin	56,154	1,664,074,000	29,634
Wyoming	97,914	7,367,362,000	75,182
U.S.A.	3,608,787 ^{3/}	458,101,343,000	126,940

^{1/}World Almanac, 1966, p. 245.

^{2/}U.S. Bureau of Mines, Minerals Yearbooks.

^{3/}Conterminous U.S.A. plus Alaska.

Table 3. Comparison of states in terms of actual and expected production on the basis of area (excluding Hawaii).

Area, in sq mi	State	Value in \$ x 10 ⁶		
		Actual	Expected ^{1/}	Expected ^{2/}
	Rhode Island	50		
2,000			264	146
	Delaware	31		
5,000			660	366
	Connecticut	374		
	New Jersey	2,671		
	Massachusetts	742		
	New Hampshire	135		
	Vermont	1,142		
10,000			1,320	733
	Maryland	1,298		
20,000			2,640	1,465
	West Virginia	15,990		
30,000			3,960	2,198
	South Carolina	502		
	Maine	362		
	Indiana	6,033		
40,000			5,280	2,931
	Kentucky	12,113		
	Virginia	4,792		
	Ohio	12,484		
	Tennessee	3,515		
	Pennsylvania	45,520		
	Mississippi	2,821		
	Louisiana	29,079		
	New York	6,393		
50,000			6,600	3,664
	Alabama	5,618		
	North Carolina	1,031		
	Arkansas	5,590		
	Wisconsin	1,664		
	Iowa	2,311		
	Illinois	16,838		
	Michigan	10,160		
	Florida	3,298		
	Georgia	1,716		
60,000			7,920	4,397
	Washington	1,848		
	Missouri	4,877		
	Oklahoma	22,770		
70,000			9,240	5,130
	North Dakota	913		
	South Dakota	1,141		
	Nebraska	1,205		
80,000			10,460	5,862
	Kansas	16,156		
	Idaho	2,447		
	Minnesota	11,455		
	Utah	8,697		
90,000			11,880	6,595
	Oregon	922		
	Wyoming	7,361		
100,000			13,200	7,328
	Colorado	6,767		
	Nevada	4,884		
	Arizona	10,813		
120,000			15,840	8,794
	New Mexico	9,792		
	Montana	5,346		
150,000			19,800	10,992
	California	64,191		
200,000			26,400	14,656
	Texas	83,417		
500,000			66,000	36,640
	Alaska	2,042	77,352	

^{1/}Based on expected value for U.S.A., 1880-1964. Equals 132,000 dollars per square mile.

^{2/}Based on expected value for states, log₁₀ dollar value, 1911-1964. Equals 73,280 dollars per square mile.

Table 4. Value of mineral industries products per square mile per state, 1911-1964 (excluding Hawaii).

Dollar value per sq mi	Frequency	States
less than 5,000	1	Alaska
5,000-10,000	1	Oregon
10,000-20,000	8	Delaware, Maine, Nebraska, New Hampshire, North Carolina, North Dakota, South Carolina, South Dakota
20,000-30,000	4	Georgia, Idaho, Washington, Wisconsin
30,000-40,000	1	Montana
40,000-50,000	3	Iowa, Nevada, Rhode Island
50,000-60,000	2	Florida, Mississippi
60,000-70,000	2	Colorado, Missouri
70,000-80,000	2	Connecticut, Wyoming
80,000-90,000	3	Massachusetts, New Mexico, Tennessee
90,000-100,000	1	Arizona
100,000-120,000	5	Alabama, Arkansas, Utah, Vermont, Virginia
120,000-140,000	3	Maryland, Minnesota, New York
-180,000	2	Indiana, Michigan
-200,000	1	Kansas
250,000-300,000	2	Illinois, Kentucky
-350,000	4	New Jersey, Ohio, Oklahoma, Texas
-450,000	1	California
550,000-600,000	1	Louisiana
650,000-700,000	1	West Virginia
greater than 1,000,000	1	Pennsylvania
	49	

geological environments. One may deduce, to a first approximation, that the potential of most states of equal size are almost equal in value. This, in turn, suggests that the expected value may be a useful first approximation for states with areas over 10,000 square miles.

Again there is a marked contrast in value per square mile between the neighboring states of Vermont and New Hampshire, although their areas and geology are reasonably similar. In this case, too, the fuels play no role, so that it appears possible to achieve the expected value without relying on the presence of fuel resources. In this case Vermont has clearly achieved its expected value of 1,267 millions of dollars with an actual production of 1,142 millions of dollars, whereas New Hampshire is, on the same basis, an under-achiever with an expected value of 1,228 millions of dollars and an actual production of 135 millions of dollars.

FREQUENCY DISTRIBUTION OF DOLLAR VALUE PER SQUARE MILE IN U.S.A. BY STATES

When the value per square mile per state is expressed as a frequency distribution using a logarithmic scale (base 10), as in Table 5, the summary statistics are:

$$\bar{x} = 4.865; \quad \sigma = 0.555; \quad \sqrt{b_1} = -0.015; \quad b_2 = 2.510,$$

and while the sample size of $n = 49$ is somewhat small, neither the asymmetry ($\sqrt{b_1}$) nor the peakedness (b_2) is significant (see table 4, 5, p. 95, Bennett and Franklin, 1954).

Similarly when compared with a normal distribution with the same mean and variance (Fig. 2) the value of chi square is 6.725 for 8 degrees of freedom, hence $70 > P > 50$; in other words, if these 49 values are a random sample from a log-normal population with $\mu = 4.865$ and $\sigma = 0.3085$ then the accumulated deviations would arise between 50 and 70 times in every 100. It may be concluded that the frequency distribution of log value per square mile for the 49 states is effectively log normal.

Table 5. Frequency distribution of value per square mile of mineral industries products by states, 1911-1964.

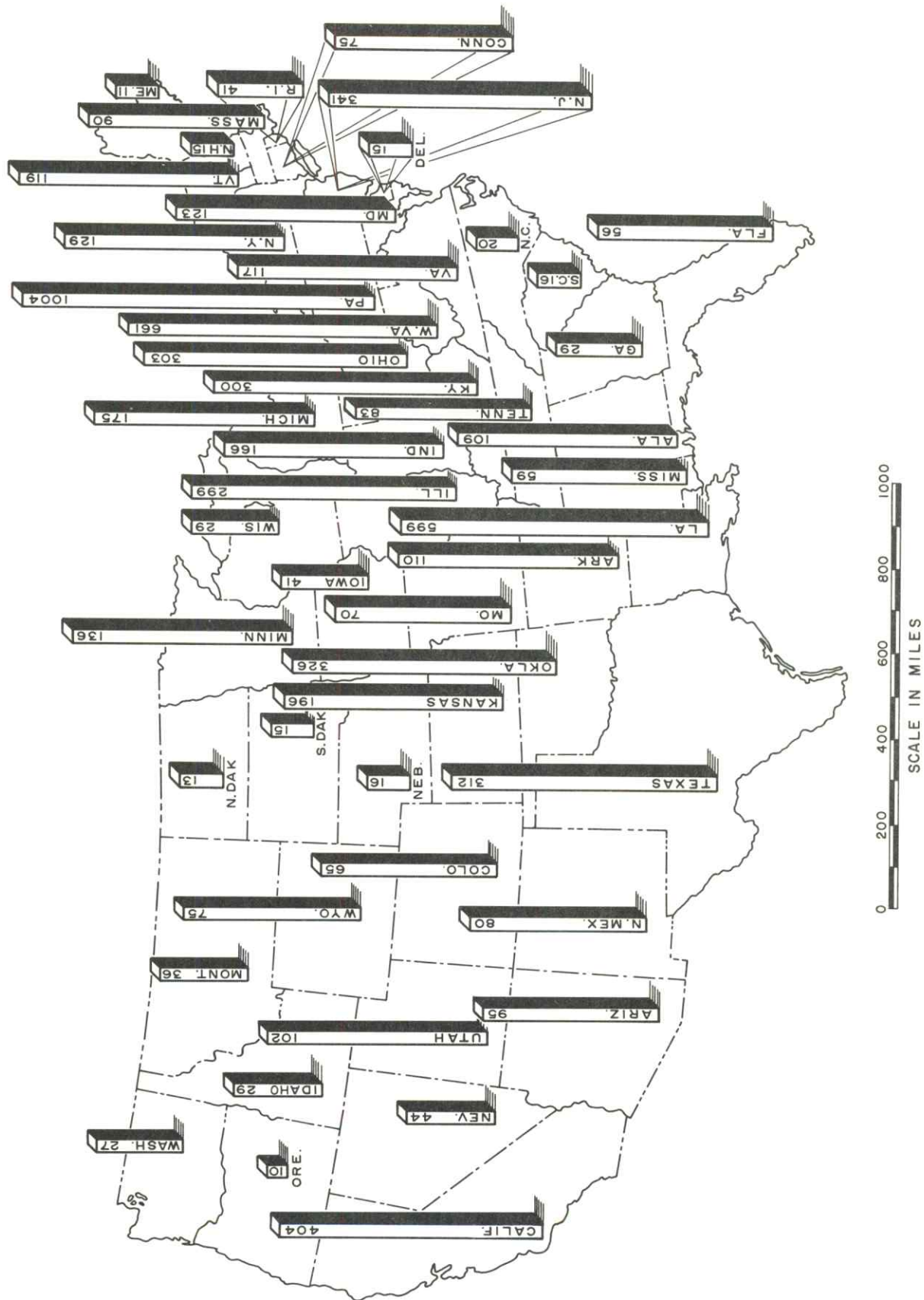
Value		$\log_{10} \$$	Frequency	Cumulative % (less than)
\$				
3,163 -	5,620	3.50 - 3.75	1	2.25
5,620 -	10,000	3.75 - 4.00	1	6.01
10,000 -	17,800	4.00 - 4.25	7	13.50
17,800 -	31,630	4.25 - 4.50	5	25.72
31,630 -	56,200	4.50 - 4.75	4	42.07
56,200 -	100,000	4.75 - 5.00	10	59.98
100,000 -	178,000	5.00 - 5.25	10	76.07
178,000 -	316,300	5.25 - 5.50	5	87.90
316,300 -	562,000	5.50 - 5.75	3	95.03
562,000 -	1,000,000	5.75 - 6.00	2	98.55
1,000,000 -	1.78×10^6	6.00 - 6.25	1	99.98
			49	

$$x = 4.865; \hat{\sigma} = 0.555; \sqrt{b_1} = 0.015; b_2 = 2.510; x^2 = 6.725; d.f. = 8; 70 > P > 50$$

It is more appropriate under these circumstances to classify the performance of the states in terms of the logarithmic mean of 4.865, or 73,280 dollars per square mile. Such a classification is carried out in Figure 3 and the general order of the states is almost identical with that of the previous tables. However, the mean of the logarithms is the geometric mean of the data in original units and is always smaller than the arithmetic mean (\$132,308), hence more states achieve their expected value under these circumstances. It is more conservative to use this geometric mean as a basis for classification. A rough comparison yields the following differences:

	Achievement Level			
	Under	Near	Over	Total
Arithmetic scale	29	4	16	49
Logarithmic scale	21	8	20	49

The main change occurs in those states near their expected value, but the outstanding contrasts remain the same. In Figure 3 only Pennsylvania exceeds two standard deviations from the mean (log scale) with Louisiana and West Virginia at or near the two standard deviation level. Similarly, Oregon and Alaska are the most marked underachievers in the terms of standard deviations, and their positions are about the same on either scale.



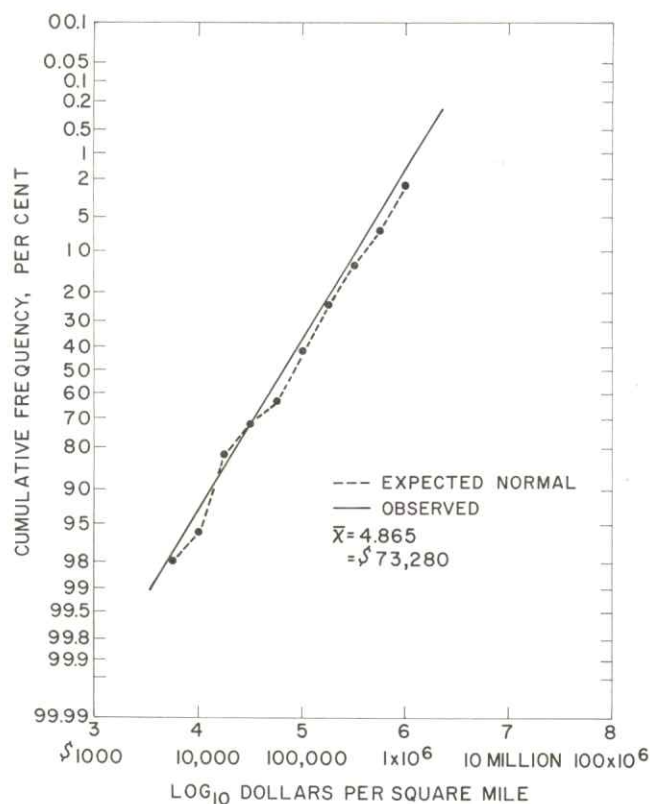


Figure 2. Cumulative frequency distribution of log value of mineral industries products by states for U.S.A. ($\log_{10}$ dollars per square mile).

UNIT REGIONAL VALUE OF KANSAS

The average value per square mile varies from state to state around the expected value for the U.S.A. as a whole. Similarly the value within a state would be expected to vary around the average value for the state and some estimate of the magnitude of this variation would be helpful in separating those areas which are over- and underdeveloped. The state of Kansas has an average value of 196,402 dollars per square mile and an area of 82,264 square miles with actual production of 16,147 millions of dollars against an expected value of 5,862 dollars on a log scale or 10,460 millions of dollars on an arithmetic scale. It would be of interest to determine how this wealth is distributed within the State and particularly if it is systematically or irregularly distributed. If it is systematically distributed some means of prediction should be feasible using the basis which leads to the systematic distribution. If, on the other hand, the distribution is irregular and with no system, then it is reasonable to suppose that areas which are not adequately developed represent potentially promising areas for future exploration. In fact, if the distribution is haphazard, then it is reasonable to expect that all areas are equally promising in terms of mineral industries resources, and those areas with a value per square mile in excess of the average value may be considered depleted, whereas those with a value per square mile below the average are potentially the most promising for future development.

Unit Regional Value of Kansas for the Year 1960

The value of mineral industries products per square mile for states for the year 1960 is summarized in Table 6, and a comparison of states for the period 1911-1964 with that for 1960 is summarized in Table 7. Each state is

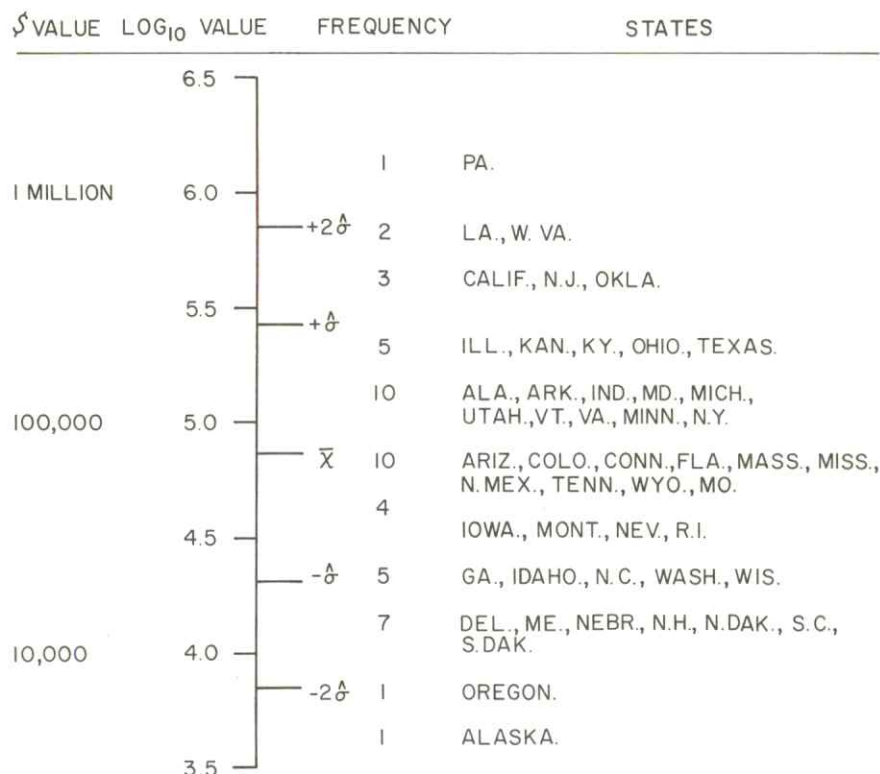


Figure 3. Classification of states on the basis of value of mineral industries products ($\log_{10}$ dollars per square mile).

ranked in terms of its value per square mile in the 1911-1964 period and independently ranked by its value for 1960. The scores are quite similar for both periods with 13 changes of rank exceeding five units as shown in Table 7. Kansas remains near the middle of the range in value and its difference in rank for the two periods is negligible. Kansas may then be taken as an example of a state with somewhat better than, but near average, production.

To achieve a more detailed breakdown of the distribution of value over the state of Kansas for 1960 it is necessary to consider the value of mineral industries products produced per square mile per county, and for 1960 the required data is contained in the U.S. Bureau of Mines' Minerals Yearbook, Area Reports for 1960. Since it is intended to test for systematic trend in the distribution of value it is convenient to use trend-surface fitting (Krumbein, 1955, 1959) to attempt to describe the distribution over Kansas.

Trend-surface fitting of orthogonal polynomials is accomplished most simply by using an orthogonal grid of points with equal spacing as the basic sampling network. Such a grid was applied to the 105 counties of Kansas (Griffiths, et al., 1965). Because Kansas is approximately rectangular, the grid needed only minor adjustment where more than one point fell in a county and also where a few counties were too small to contain a grid point. In the former case the two points were given equal values and in the latter the smaller county was added to a neighbor and the accumulated value assigned to an appropriate grid point. To remove the distortion due to abrupt discontinuities at the boundaries of the state, two tiers of grid points, about equivalent to two counties beyond Kansas on all sides, were added, resulting in a grid of 11 rows and 18 columns, a total of 198 grid points. The grid and its arrangement over the area is exhibited in Figure 4.

Table 6. Value of mineral industries products per square mile for each state in 1960.

State	Area x 10 ⁻³ (incl. inland water)	1960 value ^{1/} of \$ x 10 ⁻⁶ production	Value/sq mi \$ 10 ⁻³ 1960
Alabama	51.609	217.617	4.217 NP
Alaska	586.400	21.858	0.373 P
Arizona	113.909	415.776	3.650 NP
Arkansas	53.104	155.039	2.920 P
California	158.603	1,402.214	8.956 P
Colorado	104.247	342.223	3.294 PNG, U
Connecticut	5.009	15.255	3.046 S&G
Delaware	2.057	.989	0.481 S&G
Florida	58.560	176.920	3.021 NP
Georgia	58.876	91.203	1.549 S&G
Hawaii	6.424	9.254	1.441 S&G
Idaho	83.557	57.441	0.687 Mins.
Illinois	56.400	590.800	10.475 P
Indiana	36.291	206.882	5.701 P
Iowa	56.290	95.030	1.688 S&G
Kansas	82.264	483.958	5.883 PNG
Kentucky	40.395	413.517	10.237 PNG, coal, etc.
Louisiana	48.523	1,967.652	40.551 P
Maine	33.215	13.648	0.411 S&G
Maryland	10.577	55.527	5.250 S&G(coal)
Massachusetts	8.257	27.588	3.341 S&G
Michigan	58.216	429.055	7.370 P, Mins.
Minnesota	84.068	515.255	6.129 S&G, Mins.
Mississippi	47.716	19.862	0.416 PNG, S&G
Missouri	69.686	156.033	2.239 S&G, Mins.
Montana	147.138	178.854	1.216 P, Mins., S&G
Nebraska	77.227	103.687	1.343 P, S&G
Nevada	110.540	80.285	0.726 S&G, Mins.
New Hampshire	9.304	5.317	0.571 S&G, Mins.
New Jersey	7.836	56.409	7.199 S&G, Mg, Fe
New Mexico	121.666	652.200	5.361 PNG, U, K ₂ O
New York	49.576	254.713	5.138 S&G, Fe
North Carolina	52.712	44.968	0.853 S&G, Fsp., etc.
North Dakota	70.665	78.275	1.108 P, S&G, coal
Ohio	41.222	389.828	9.457 Coal, S&G
Oklahoma	69.919	779.116	11.431 PNG
Oregon	96.981	54.419	0.561 S&G, Ni
Pennsylvania	45.333	824.493	18.187 Coal, P, S&G
Rhode Island	1.214	5.727	4.717 S&G
South Carolina	31.055	30.001	0.966 S&G
South Dakota	77.047	46.780	0.607 S&G, Au
Tennessee	42.244	143.439	3.395 S&G, coal, Zn
Texas	267.339	4,134.901	15.467 PNG, S&G
Utah	84.916	431.396	5.080 Cu, P, coal, U
Vermont	9.609	22.879	2.381 S&G
Virginia	40.815	203.819	4.994 S&G
Washington	68.192	70.005	1.027 S&G, Zn
West Virginia	24.181	720.674	29.803 Coal, PNG
Wisconsin	56.154	77.171	1.374 S&G, Fe
Wyoming	97.914	442.738	4.522 PNG, U, etc.
Total	3,628.150 ^{2/}	17,892.000	4.931
Conterminous	(3,002.387)		

^{1/}Source: U.S. Bureau of Mines, Minerals Yearbook, 1960, p. 8-9, table 4.

^{2/}Total area of all fifty states.

NP = no petroleum; S&G = sand and gravel; Mins. = various minerals;
Fsp. = feldspar.

Table 7. Comparison of ranks of states from 1911-1964 and 1960 in terms of value per square mile.

State	Rank in terms of value/sq mi		Change in rank	State	1911-64	1960	Change in rank
	1911-1964	1960					
Pa.	1	3		Wyo.	26	21	5 ⁺
W. Va.	2	2		Conn.	27	27	
La.	3	1		Mo.	28	31	
Calif.	4	9	5*	Colo.	29	26	
N. J.	5	11	6*	Miss.	30	46	16 ⁺
Okla.	6	5		Fla.	31	28	
Texas	7	4		Nevada	32	41	9 ⁺
Ohio	8	8		Iowa	33	32	
Ky.	9	7		R. I.	34	20	14 ⁺
Ill.	10	6		Mont.	35	36	
Kan.	11	13		Wis.	36	34	
Mich.	12	10		Idaho	37	42	5 ⁺
Ind.	13	14		Ga.	38	33	5 ⁺
Minn.	14	12		Wash.	39	38	
N. Y.	15	17		N. Ca.	40	40	
Md.	16	16		S. Ca.	41	39	
Vt.	17	30	13*	Nebr.	42	35	7 ⁺
Va.	18	19		Del.	43	45	
Ark.	19	29	10*	S. Dak.	44	43	
Ala.	20	22		N. H.	45	44	
Utah	21	18		N. Dak.	46	37	9 ⁺
Ariz.	22	23		Maine	47	47	
Mass.	23	25		Oregon	48	48	
Tenn.	24	24		Alaska	49	49	
N. Mex.	25	15	10 ⁺				

*Change of rank of greater than five units: + = positive; * = negative.

Nature of the Data

The initial data were accumulated in terms of the following 11 variables and the data matrix is listed in Appendix table 1:

X₁ = county and state.

X₂ = row coordinate

X₃ = column coordinate } Arbitrary but equal units.

The coordinates commence with row 1 and column 1 in Logan County, Colorado, in the northwest and the columns increase eastward and the rows southward.

X₄ = area of county in square miles^{1/}.

X₅ = value of mineral industries products produced in 1960 per county (in dollars x 10⁻⁶).

X₆ = area of county yielding irrigated crops (square miles).

X₇ = value of agricultural production from irrigation (dollars x 10⁻⁶)^{2/}.

^{1/} Areas of states and counties from World Almanac, 1966.

^{2/} For explanation see succeeding text.

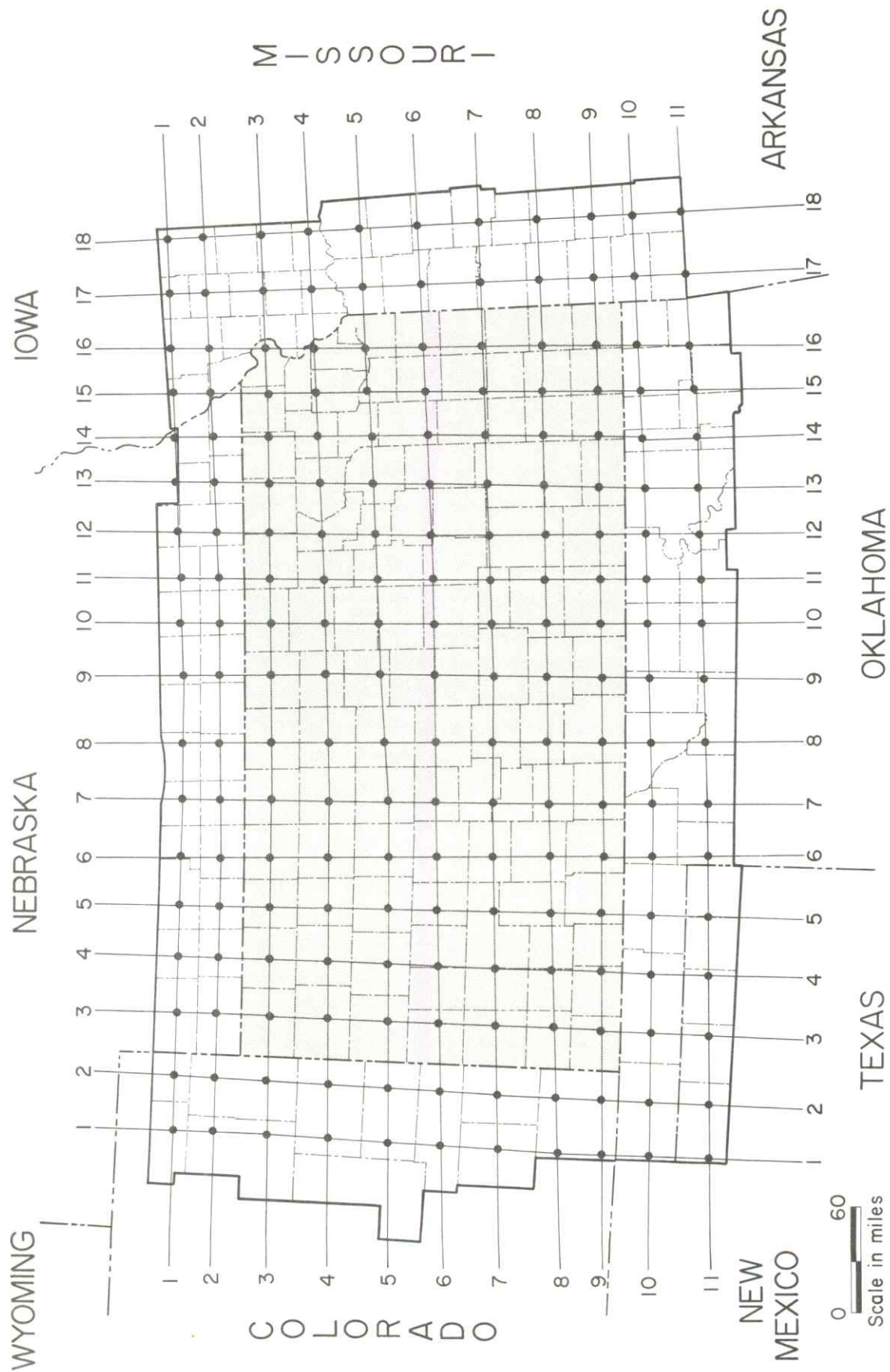


Figure 4. Orthogonal grid of points fitted to Kansas and two tiers of counties in adjoining states.

X_8 = topographic height above sea level (in feet).
 X_9 = depth below sea level to basement (in feet).
 X_{10} = number of wells drilled per county^{3/}.
 X_{11} = value of mineral industries products produced per county in 1952-65
 (dollars $\times 10^{-6}$).

Most of these variables are self-explanatory but X_6 and X_{10} need some additional explanation. It was decided, during the 1965 reconnaissance investigation, to consider the yield of crops due to irrigation as value added due to a mineral resource--water. The variable X_6 was calculated by finding the difference in yield, generally positive, between crops produced by irrigation over those produced without and multiplying the difference in yield by the number of acres irrigated and the price per unit. The crops produced by irrigation in 11 counties in Colorado that fall on the grid are chiefly corn, winter and spring wheat, and oats, and the values were compiled from Colorado Agricultural Statistics for 1960 (Colorado Crop and Livestock Rept. Serv., 1962). In Kansas, wheat and grain sorghum are the important irrigated crops, and the information was obtained from Kansas Agriculture, 44th Report for 1960-61 (Kreber, 1961). There is no record of irrigated crop yield for Missouri. There are 26 counties on the grid from Nebraska, and the crops with yields from irrigation are corn, grain sorghum, oats, barley, soybeans, and alfalfa. The values were accumulated from Nebraska Agricultural Statistics, annual report for 1960 (Finigan). There are 23 counties from Oklahoma that fall within the grid and data on irrigation values from four--Cimarron, Texas, Beaver, and Woodward--are recorded in the U.S. Census of Agriculture (1959), which is the only suitable record available for this information. Similarly, there are five counties (Dallam, Hansford, Lipscomb, Ochiltree, and Sherman) in the grid from Texas and the data on value of yield of crops from irrigation was again obtained from the U.S. Census of Agriculture (1959, Pt. 37, Texas Counties) for the crops corn, sorghum, wheat, oats, and barley.

In attempting to evaluate the potential of areas which appear underdeveloped, some measure of exploration intensity is necessary to decide whether the area is essentially untapped or is not potentially promising and the measure adopted was the total number of wells drilled per county; this choice is arbitrary and was used because there is a ready source of information in this form for the 105 counties of Kansas in the reports on oil and gas developments for the years 1947-1965 (see references, Beene, Goebel, Hilpman, Ver Weibe). The variable X_{10} is a ranking based on the following scale (Table 8).

Data for the counties in the surrounding states had to be extrapolated from the annual reports on appropriate regions published in the Bulletin of the American Association of Petroleum Geologists; the ranking is of some help in smoothing the estimates which are based on different sources of information.

These variables are combined to obtain the values used for fitting trend surfaces as follows:

X = row coordinate.
 Y = column coordinate.
 $V_1 = \log_{10} [(X_5/X_4) \times 10^6] = \log$ value of mineral industries products per square mile for 1960.
 $V_2 = \log_{10} [(X_5/X_4) + (X_7/X_6)] \times 10^6 = \log$ value of mineral industries products plus value of agricultural products added by irrigation per square mile for 1960.

^{3/} For explanation see following text.

$V_3 = \log_{10} [((X_5/X_4) + (X_7/X_6)) \times 10^6] / ((X_8 - X_9) / 5280) = \log$ value of mineral industries products plus value of agricultural products added by irrigation per cubic mile for 1960.

$V_4 =$ number of wells drilled per county in rank scores as a measure of information on the various areas.

$V_5 = \log_{10} [(X_{11}/X_4) \times 10^6] = \log$ value of mineral industries products per square mile for 1952-65.

Table 8. Number of drilled wells per county and equivalent rank.

Wells per county	X_{10} rank	Frequency
0 - 25	1	61
25 - 50	2	12
50 - 100	3	16
100 - 200	4	21
200 - 400	5	22
400 - 800	6	16
800 - 1,600	7	24
1,600 - 3,200	8	15
3,200 - 6,400	9	11
	Total	198

Frequency Distribution of the Variates

It will be assumed that the expected frequency function of the variates is a log-normal curve and this was confirmed for the distribution of log value per square mile for the states. Deviations from this model will be used as a basis for interpretation.

The frequency distribution of log dollar value per square mile for mineral industries products plus agricultural products from irrigation for Kansas and the contiguous tiers of counties in the neighboring states in 1960 is summarized in Table 9 and the distribution is graphically displayed in Figure 5 (curve B). The expected frequencies of a log-normal curve with the same mean and variance is also listed in Table 9 and plots as a straight line in Figure 5 (curve A, expected). It seems clear, both from the moment values (b_i) and the chi square, that the observed values depart widely from the expected log-normal.

The average value is 3.795, or 6,237 dollars per square mile; the spread of the curve is almost one unit in logarithms. The deviations which contribute significantly to the large chi-square value are largest at the low value end of the curve, i.e., there are too many counties with low values compared with the expected (log) normal curve; there are also too many around the mode, that is, in the center of the distribution, too few in the shoulders, and too few at the high end of the curve. On the basis of the expected normal distribution there should be eight or more counties with values exceeding 316,000 dollars per square mile, but none were observed.

The five richest counties are worth over 100,000 dollars per square mile, and the seven poorest counties are worth less than 31.6 dollars per square mile.

The frequency distribution of log dollar value per cubic mile for the same products and year, 1960, is summarized in Table 10 and graphed in Figure 5 (curve A), and includes both the observed and corresponding expected (log)

Table 9. Frequency distribution of value of mineral industries plus agricultural products from irrigation per square mile ($\log_{10}$ dollars per square mile) in Kansas, 1960.

Class limits			Observed frequency	Expected frequency	Cumulative (obs.) frequency, per cent
\$		$\log_{10}$ (\$)			
1.00 -	3.16	0. - 0.5	2	0.09	1.01
3.16 -	10.00	0.5 - 1.0	2	0.40	2.02
10.00 -	31.60	1.0 - 1.5	3	1.60	3.54
31.60 -	100.00	1.5 - 2.0	7	4.98	7.07
100.00 -	316.00	2.0 - 2.5	7	12.08	10.61
316.00 -	1,000	2.5 - 3.0	21	22.88	21.21
1,000 -	3,160	3.0 - 3.5	15	33.87	28.79
3,160 -	10,000	3.5 - 4.0	23	39.18	40.40
10,000 -	31,600	4.0 - 4.5	76	35.43	78.78
31,600 -	100,000	4.5 - 5.0	37	25.03	97.47
100,000 -	316,000	5.0 - 5.5	5	13.82	100.00
>316,000		>5.5	0	8.64	
Total			198	198.0	100.00

$$\bar{x} = 3.795 (\$6,237); \hat{\sigma} = 0.996; \sqrt{b_1} = -1.289^{**}; b_2 = 4.402^{**}$$

$$\chi^2 = 78.52, d.f. = 5, P < 0.001$$

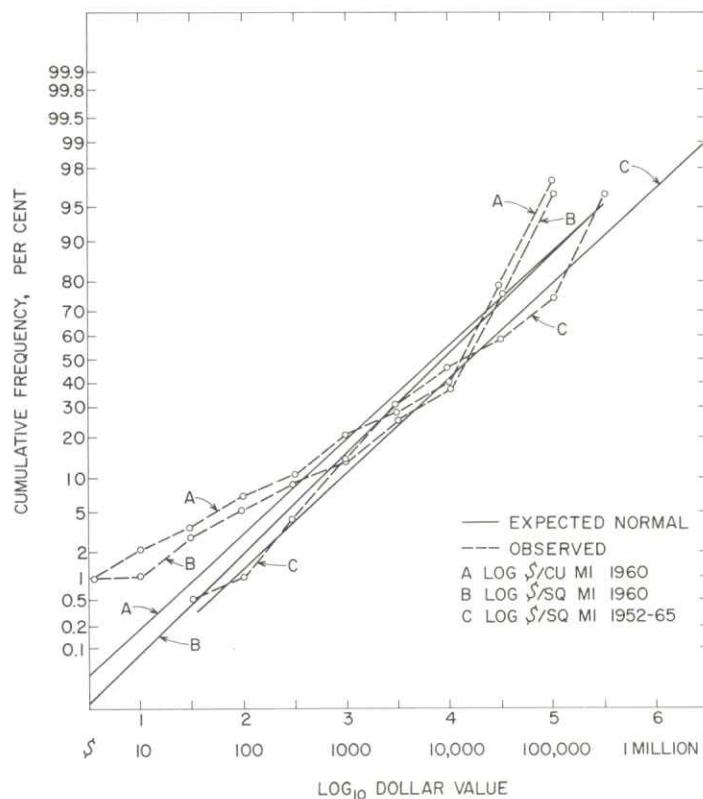


Figure 5. Cumulative frequency curves of $\log_{10}$ dollar value for mineral industries products in Kansas for 1960 and 1952-1965.

Table 10. Frequency distribution of value of mineral industries plus agricultural products from irrigation per cubic mile ($\log_{10}$ dollars per cubic mile) in Kansas, 1960.

Class limits		Observed frequency	Expected frequency	Cumulative (obs.) frequency, per cent
\$	$\log_{10}$ (\$)			
1.00 - 3.16	0.0 - 0.5	2	0.03	1.01
3.16 - 10.00	0.5 - 1.0	0	0.16	1.01
10.00 - 31.60	1.0 - 1.5	4	0.80	3.03
31.60 - 100.00	1.5 - 2.0	4	3.13	5.05
100.00 - 316.00	2.0 - 2.5	8	9.14	9.09
316.00 - 1,000	2.5 - 3.0	11	20.09	14.65
1,000 - 3,160	3.0 - 3.5	23	33.31	26.26
3,160 - 10,000	3.5 - 4.0	25	41.57	38.89
10,000 - 31,600	4.0 - 4.5	74	39.09	76.26
31,600 - 100,000	4.5 - 5.0	40	27.69	96.46
100,000 - 316,000	5.0 - 5.5	7	14.77	100.00
>316,000	>5.5	0	8.22	
Total		198	198.00	100.00

$$\bar{x} = 3.891 (\$7,780); \delta = 0.928; \sqrt{b_1} = -1.397^{**}; b_2 = 5.159^{**}$$

$$\chi^2 = 63.226, d.f. = 5; P < 0.001$$

with the same mean and variance. This curve is almost parallel to the previous one essentially because the basement rocks in Kansas are about a mile deep in most places, so that the value per square mile is close to that per cubic mile. The statistical estimators are also similar, with average value 3.891, or 7,780 dollars per cubic mile, and a similar spread and significant moment statistics. The deviations are, therefore, very similar.

The frequency distribution of dollar value per square mile for mineral industries products for the 14 years 1952-1965, in Kansas and the surrounding counties, is summarized in Table 11 and as a graph in Figure 5 (curve C, observed and expected). This curve, as may be seen from the graph, is much closer to normal than either of the other two. The average value is, in logs, 4.103, or 12,680 dollars per square mile, with the spread about the same, one log unit. The curve is symmetrical ($\sqrt{b_1}$ not significant), but it possesses low peakedness, b_2 significantly less than three. The chi-square value is highly significant but is much lower than those for the other curves.

The largest single deviation, 25 of the 38 units of chi square, is that in the class 5.0-5.5, where there are about twice as many counties observed as expected; the other contributions to chi square are quite small, six units or less per class in the shoulder of the curve. There are zero counties observed in the greater than 6 (= 1 million dollars) class although about 5 are expected.

All these deviations of observed from expected log normal in the three curves may be reflections of nonrandom sampling and an increase in the number of years would probably lead to a log-normal distribution. Similarly, the selection of these 198 counties may contribute to nonrandomness in the sampling which may be removed by adding more counties from a larger region. In other words, the log-normal model may well be an adequate representation of the distribution of value per unit area and volume over a larger region and longer time interval.

Table 11. Frequency distribution of value of mineral industries products ($\log_{10}$ dollars per square mile) in Kansas, 1952-1965.

Class limits			Observed frequency	Expected frequency	Cumulative (obs.) frequency, per cent
\$		$\log_{10}$ (\$)			
10.00-	31.60	1.0 - 1.5	1	0.77	0.50
31.60-	100.00	1.5 - 2.0	1	2.36	1.01
100.00-	316.00	2.0 - 2.5	7	6.91	4.55
316.00-	1,000	2.5 - 3.0	18	15.66	13.64
1,000-	3,160	3.0 - 3.5	36	27.52	31.82
3,160-	10,000	3.5 - 4.0	31	37.44	47.47
10,000-	31,600	4.0 - 4.5	23	39.48	59.09
31,600-	100,000	4.5 - 5.0	31	32.25	74.75
100,000-	316,000	5.0 - 5.5	43	20.40	96.46
316,000-	1,000,000	5.5 - 5.6	7	10.00	100.00
>1,000,000		>6.0	0	5.21	
Total			198	198.00	100.00

$$\bar{x} = 4.103 (\$12,680); \hat{\sigma} = 0.978; \sqrt{b_1} = -0.240; b_2 = 2.148^{**}$$

$$\chi^2 = 37.79, d.f. = 7, P < 0.001$$

The coincidence of parallelism in the value per square mile and cubic mile per county suggests that here we have a reasonably firm estimate of what the first mile of the earth's crust is worth and this may be used as an expected value for other regions. Similarly, using the log-normal frequency as a basis for comparison suggests that there is underachievement in the poorest and the richest counties or that the full potential for development has not been reached at either extreme of the distribution.

SPATIAL DISTRIBUTION OF UNIT REGIONAL VALUE

The spatial distributions of (log) value per square mile and per cubic mile for the year 1960 and the (log) value per square mile for the period 1952-1964 are of interest in attempting to locate those areas which are over- and under developed. It is possible to formulate conceptual models of expected spatial distribution; for example, the fuels, particularly petroleum and natural gas, are among the most valuable products, and two areas in Kansas are, on this basis, expected to be most valuable: the surface above the Central Kansas uplift and above the Hugoton gas field. Similarly the value of agricultural products arising from irrigation is, to a first approximation, highest in western Kansas and decreases toward the east. If the sources of variation in value were this simple, one would expect a reasonably simple surface of value to represent all of the contributions to variation; it is of some interest, therefore, to determine how much variation is accounted for by surfaces of increasing complexity. Trend-surface fitting of polynomial surfaces by least squares was used for this purpose and an appropriate computer program has been described by O'Leary, Lippert, and Spitz (1966)⁴.

⁴ This program was modified for the IBM 360-50 system at the Pennsylvania State University by Dr. C. W. Ondrick.

The amount of variation accounted for by fitting surfaces of successively higher degree (1 through 6) is summarized in Table 12 and is graphically presented in Figure 6; the coefficient of determination in per cent ($R^2\%$) is used as an estimator of amount of variation accounted for by each surface. For the variate log value per square mile in 1960 the linear surface accounts for 44.4 per cent of the total variation and the quadratic (or second-degree) surface adds about seven per cent more; the value of R^2 increases steadily to 67 per cent for the sixth-degree surface. The contributions from each surface, except that of sixth degree, are significant (see Appendix table 2).

Table 12. Coefficient of determination ($R^2\%$) as a measure of amount of variation accounted for by trend-surfaces of degrees 1 through 6.

Variable	Symbol	Degree of surface					
		1st	2nd	3rd	4th	5th	6th
Log value/sq mi 1960	V_2	44.4	51.2	55.9	61.4	64.6	66.8
Log value/cu mi 1960	V_3	30.7	39.9	46.3	53.4	57.7	60.1
Intensity of drilling	V_4	21.4	40.5	50.5	56.0	61.9	69.4
Log value/sq mi 1952-65	V_5	23.0	33.2	47.4	46.3	55.3	64.0

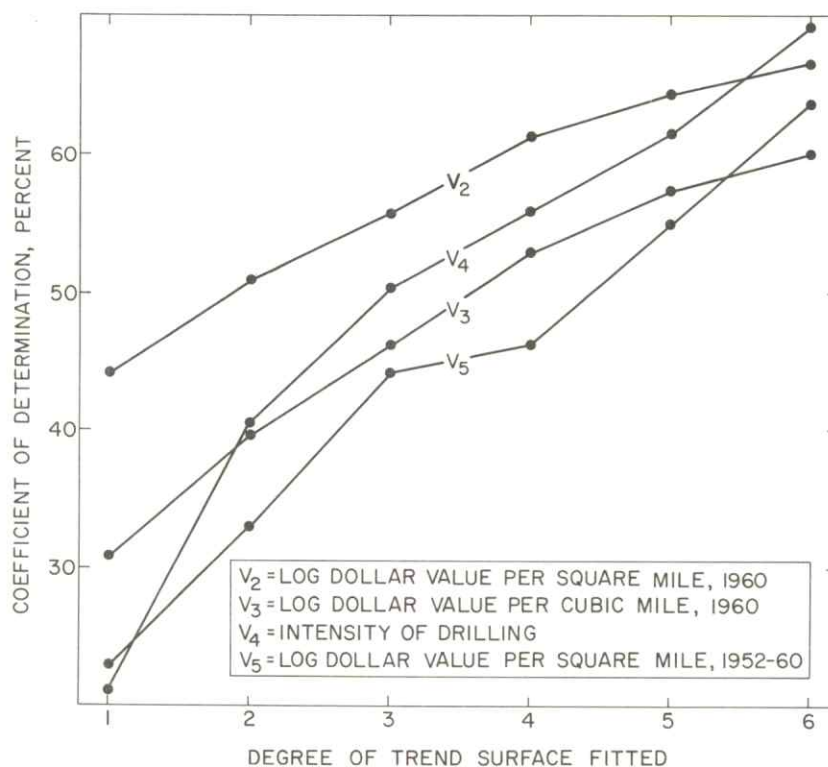


Figure 6. Coefficient of determination ($R^2\%$) as a measure of amount accounted for by trend-surfaces of degrees 1 through 6.

Somewhat similar features are shown for the other three variates (see Table 12 and Appendix tables 3, 4, 5), and the cubic surface usually accounts for 45 to 50 per cent of the variation in all cases. It is clear, therefore, that a relatively simple surface (third-degree) is representative of the main sources of variation; however, even with a sixth-degree surface, there remains unaccounted for some 30 to 40 per cent of the variation, and so there are

enough irregularities remaining to suggest that while the expected sources are dominant, there are many other sources which make an appreciable contribution. In other words, to obtain an explanation for the value per square mile of a county is not be any means simple, and there must be many interdependent features which combine to determine the value of a square mile per county.

Value per Square Mile in 1960

A contour map of the original data for the variate $\log_{10}$ dollar value per square mile per county in 1960 is illustrated in Figure 7. As expected, high



Figure 7. Contoured surface of log dollar value per square mile for Kansas, 1960 (original data in logarithms).

values are found in the west and form a ridge from northwest to southeast along the Central Kansas uplift. The surface descends towards the east, again as expected from the model; the linear surface, accounting for 44.4 per cent of the total variation, is illustrated in Figure 8 and shows the dominant trend from west to east. The sixth-degree trend-surface (Fig. 9) accounts for 66.8 per cent of the variation. By comparing Figure 7 and 9 it may be seen that the major highs and lows in the original data are represented in the sixth-degree trend. If the lows represent areas of underdevelopment, then there are three main locations for future development, one in the south-central portion surrounding Comanche County, another in the southeast, which becomes a very marked low beyond the boundaries of Kansas, and a third in the northeast corner centered on the counties of Nemaha and Brown and again becoming more of a depression beyond the borders of the state of Kansas.

Value per Cubic Mile in 1960

A contour map of the original data in terms of $\log_{10}$ dollar value per cubic mile in 1960 is shown in Figure 10 and is very nearly identical in form

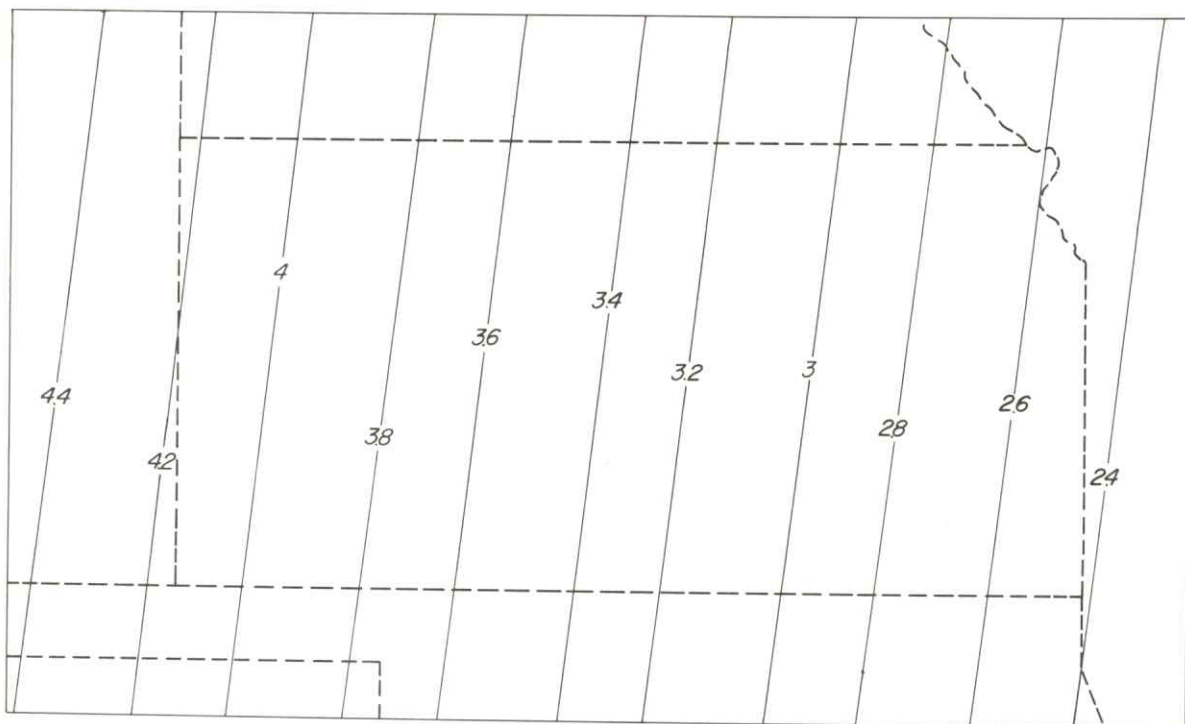


Figure 8. Linear trend-surface of log dollar value per square mile for Kansas, 1960.

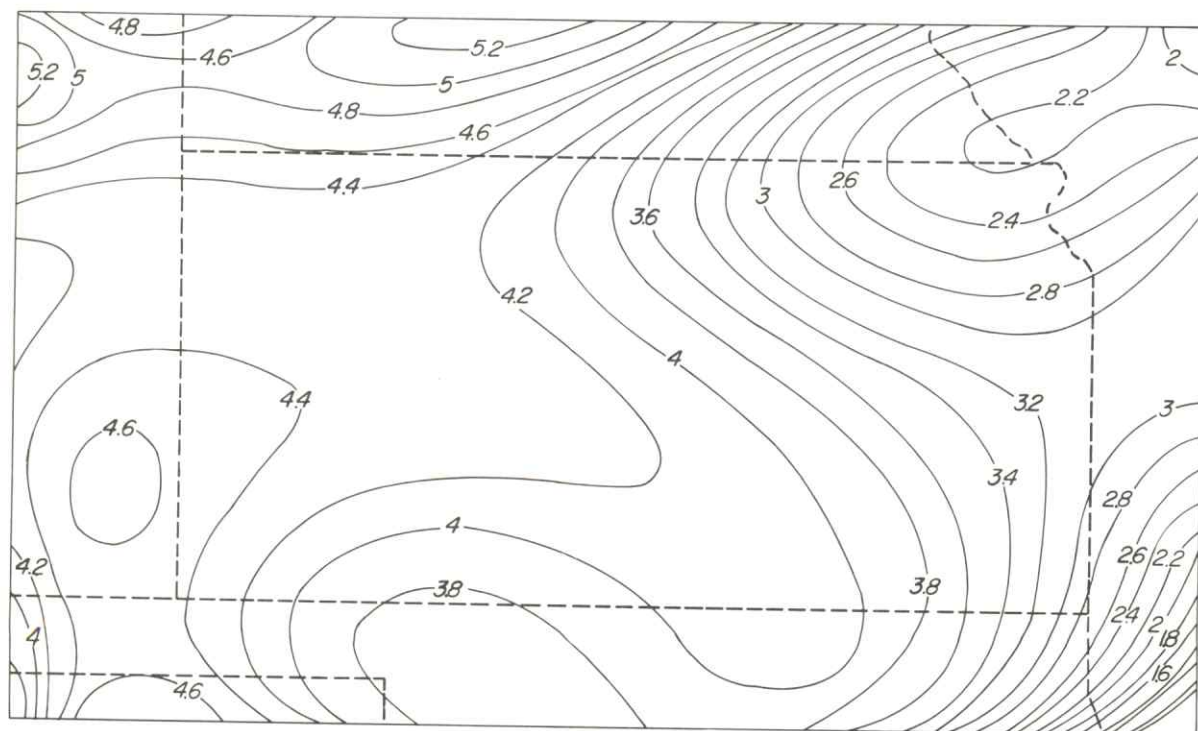


Figure 9. Sixth-degree trend-surface of log dollar value per square mile for Kansas, 1960.

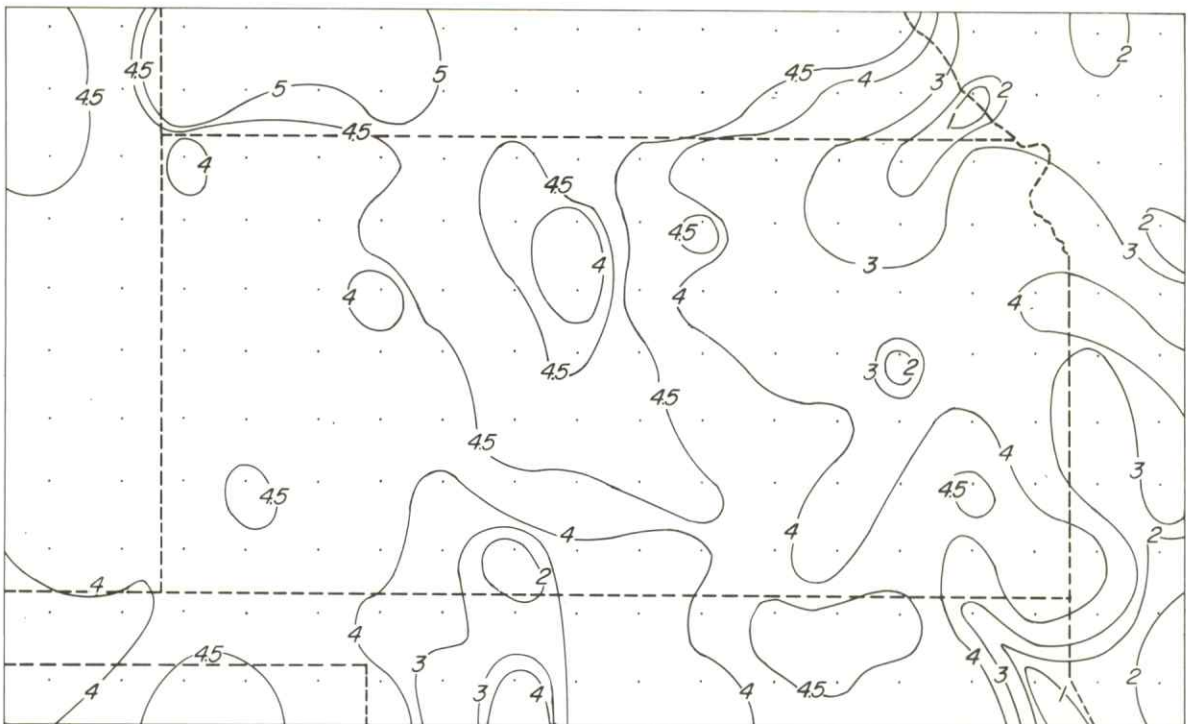


Figure 10. Contoured surface of log dollar value per cubic mile for Kansas, 1960 (original data in logarithms).

to that of the square mile data in Figure 7; the linear trend-surface, accounting for 30.7 per cent of the total variation, is also similar. The sixth-degree trend, accounting for 60.1 per cent of the total variation, is illustrated in Figure 11 and again is very similar to that of the corresponding trend for the square mile data (Fig. 9).

These variables, log dollar value per square mile and log dollar value per cubic mile for 1960, show almost identical variation both when grouped into frequency distributions, as remarked earlier, and in terms of spatial distribution. This suggests that the values per square mile and per cubic mile are equivalent and, at least to a first approximation, imply that dollar value per square mile may be used as a close estimate of dollar value per cubic mile for the first mile of the earth's crust elsewhere in the U.S.A.

Value per Square Mile for 1952-1965

The contour map of $\log_{10}$ dollar value per square mile for the period 1952 to 1965 is illustrated in Figure 12 and, while the absolute values are higher, as would be expected for a 14-year span when compared with a single year (1960), the same general features are quite marked in the map; the dominant domal high is the Central Kansas uplift lying across the center of the map in a northwest-southeast direction with a similar high area in the southwest corner covering the Hugoton gas area. The most striking lows occur in the northwest corner of the state of Kansas, in the north-central area at the Nebraska border, and in the northeast and southeast corners. It is quite evident that the bulk of the value for Kansas is tied in with fuels, particularly petroleum products, and other materials do not compare in magnitude with the return on fuels.

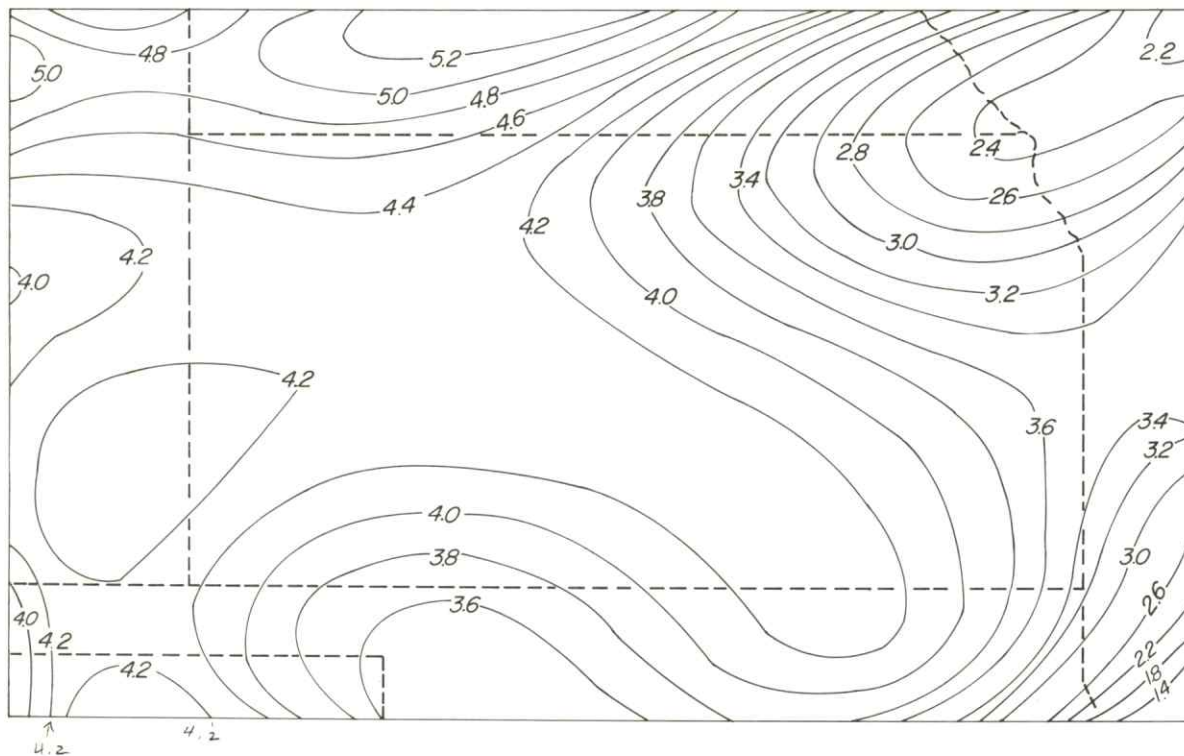


Figure 11. Sixth-degree trend-surface of log dollar value per cubic mile for Kansas, 1960

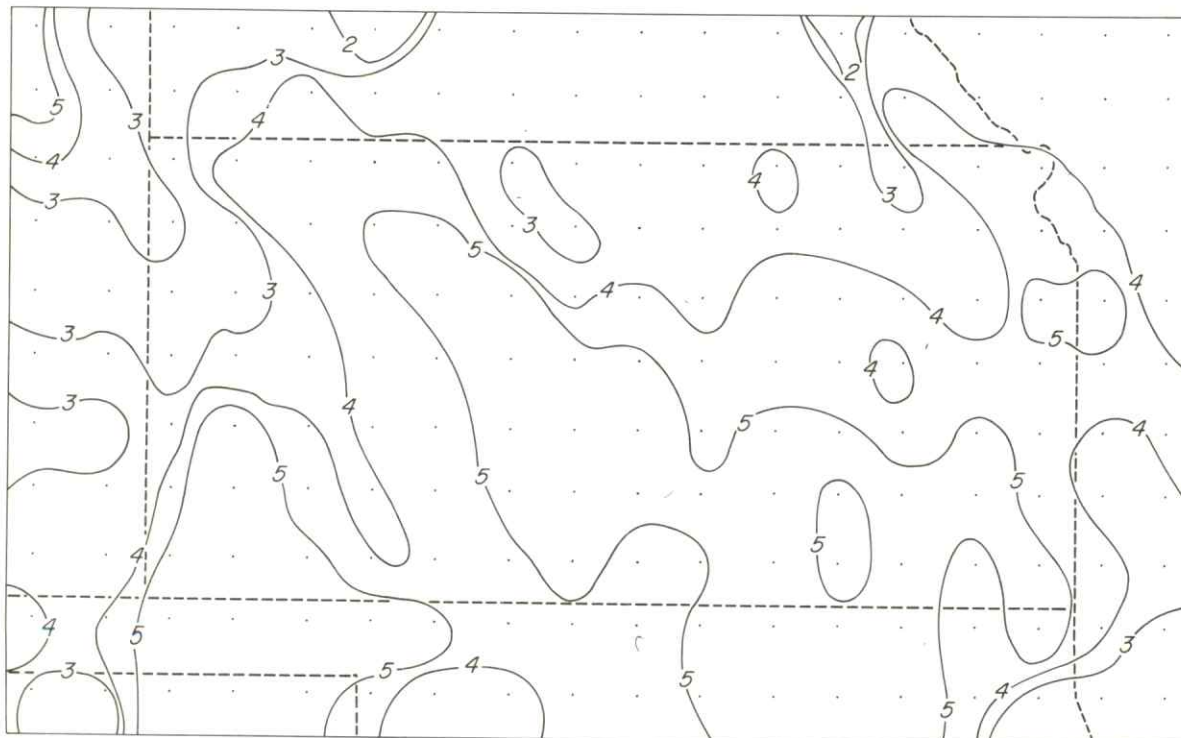


Figure 12. Contoured surface of log dollar value per square mile for Kansas, 1952-1965 (original data in logarithms).

The linear trend, accounting for 23 per cent of the total variation (see Table 12), has shifted from a west to east descending value in the 1960 case to a descent from northwest to southeast for the 1952-1965 period. The sixth-degree trend surface (Fig. 13) quite faithfully represents the features outlined above in terms of the original data but, despite this, only accounts for some 64 per cent of the total variation (Table 12), i.e., not quite two-thirds of the total variation.

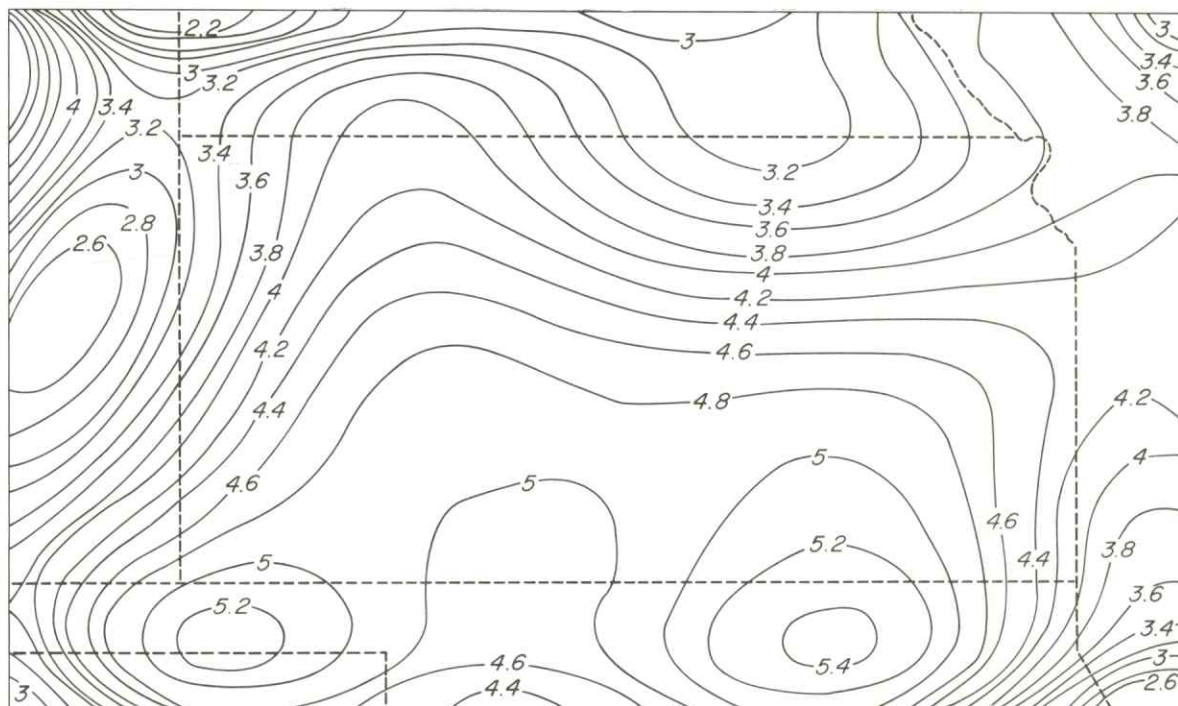


Figure 13. Sixth-degree trend-surface of log dollar value per square mile for Kansas, 1952-1965.

EVALUATION OF INFORMATION LEVEL PER UNIT AREA PER COUNTY

The next step in evaluating the potential value of a region is to attempt to estimate the amount of information which exists about its natural resources, and, while it would be advantageous to measure this in terms of exploration footage per unit area, for example, or in terms of cost of exploratory footage, such information is very difficult to collect. In the present case the drilling intensity was estimated by counting the number of wells per county and ranking this value on an arbitrary scale (Table 8).

A contour map of these data expressed in rank order per county is illustrated in Figure 14, and it is reasonably obvious that the greatest amount of information, in terms of number of wells drilled, coincides with areas of greatest production of petroleum. This reflects the prevailing philosophy of exploration and development, namely, to find a desirable target drill in an area where desirable targets have been found.

The sixth-degree surface (Fig. 15) accounts for 69.4 per cent of the total variation (see Table 12) and, apart from smoothing the steeper slopes in the original data, faithfully reproduces the surface. The dominant high extends from northwest to southeast and swings parallel to the state line from west to east. This is a very high dome with values of 13 units in the highest

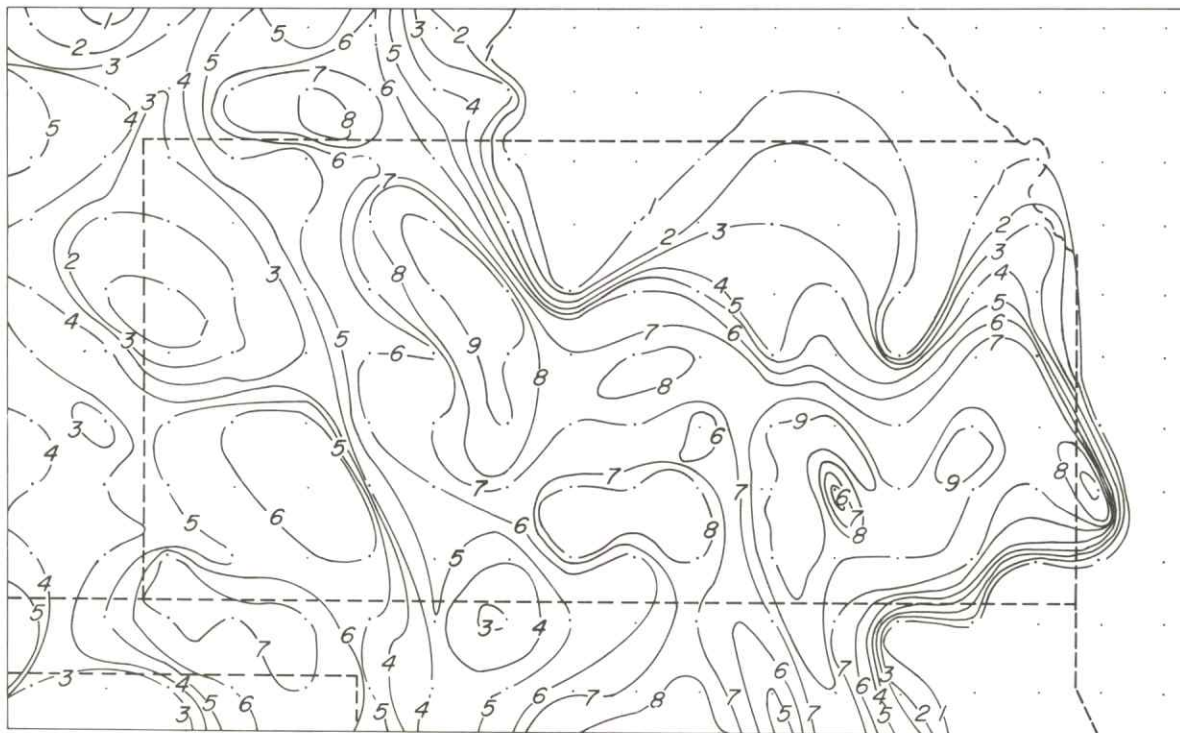


Figure 14. Drilling intensity per square mile per county in Kansas, to 1965 (original data).

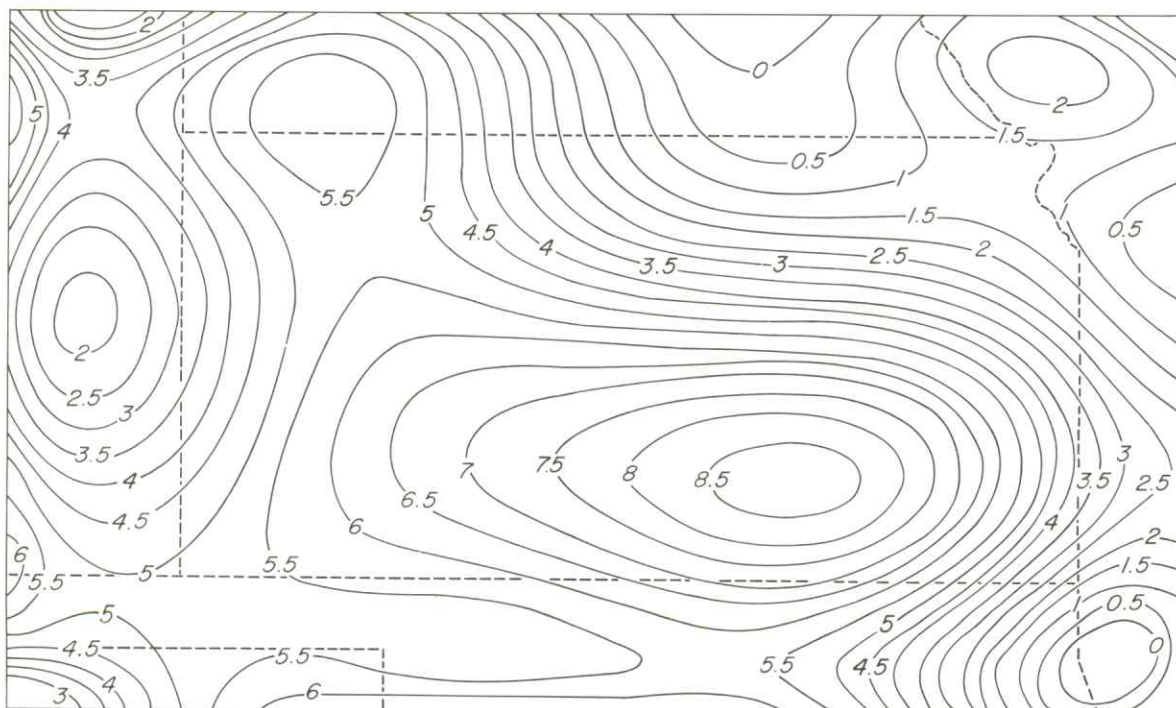


Figure 15. Sixth-degree trend-surface of drilling intensity per square mile per county in Kansas, to 1965.

part, and the lowest low goes down to the letter I, a very wide range. The lows occur in those areas where no exceptional resource value has been found, and equivalently, these are areas in which least search has been done.

It is possible, therefore, to reinterpret the philosophy of exploration and state empirically that the more valuable targets are found where the search has been the most intense. In fact, success in exploration for petroleum resources is directly proportional to the intensity of exploration drilling, and annual fluctuations are parallel (Griffiths and Drew, 1964, fig. 2, based on Lahee, F.H., 1962).

Residual Deviations from the Sixth-degree Surfaces

One aspect of the "goodness of fit" of a calculated surface to a set of observed data is contained in the nature of the deviations. If the remaining variation is random, then one could assume that the deviations represent unexplainable variation or "noise" in the system and the surface will then represent the best approximation to the true distribution of value.

The summary statistics of the frequency distributions of the residual deviations from their respective sixth-degree trend-surfaces are listed in Table 13, and the means, as would be expected, are approximately zero.

Table 13. Summary statistics for frequency distribution of deviations from sixth-degree trend-surface.

Variable statistic	\$ Value per			Intensity of drilling
	sq mi 1960	cu mi 1960	sq mi 1952-1965	
$\bar{x}$	0.0088	0.0038	-0.007 ⁵⁷⁵	0.035 ₃
$\hat{\sigma}^2$	0.332	0.334	0.332	0.23
$\hat{\sigma}$	0.576	0.578	0.576 ₆	0.472
$\sqrt{b_1}$	-1.075**	-1.168**	-0.472**	0.401*
b_2	5.393**	5.858**	3.255 ^{NS}	3.988**
n	198	198	198	198
χ^2	18.134	20.475	13.718	16.539
d.f.	9	8	8	13
prob.	05>P>02	01>P>001	10>P>05	30>P>20

* Significant at 5 per cent level.

** Significant at 1 per cent level.

NS = Not significant.

However, the beta statistics are nearly all significant, indicating that the deviations are not normally distributed around zero mean. For example, the frequency distribution of deviations of the sixth-degree surface for the variate log dollar value per square mile in 1960 is highly negatively skew and leptokurtic (b_2 significantly greater than 3); the same is true for the variate log dollar value per cubic mile in 1960, and, in both these cases, the chi-square comparisons are also significant.

For the variates log dollar value per square mile in 1952-1965 and drilling intensity (ranked value) the beta estimates for skewness ($\sqrt{b_1}$) are significant and that for peakedness (b_2) is significant for the ranked drilling intensity. The departures are, however, much less than for the 1960 variates, and the chi-square value is not significant, showing that the over-all departures are quite small.

A graphic comparison of observed and expected log-normal frequencies for the log value data is illustrated in Figure 16. The log dollar value per square (A) and per cubic (B) mile for 1960 are almost identical and their expected normal distributions cannot be separately identified in Figure 16.

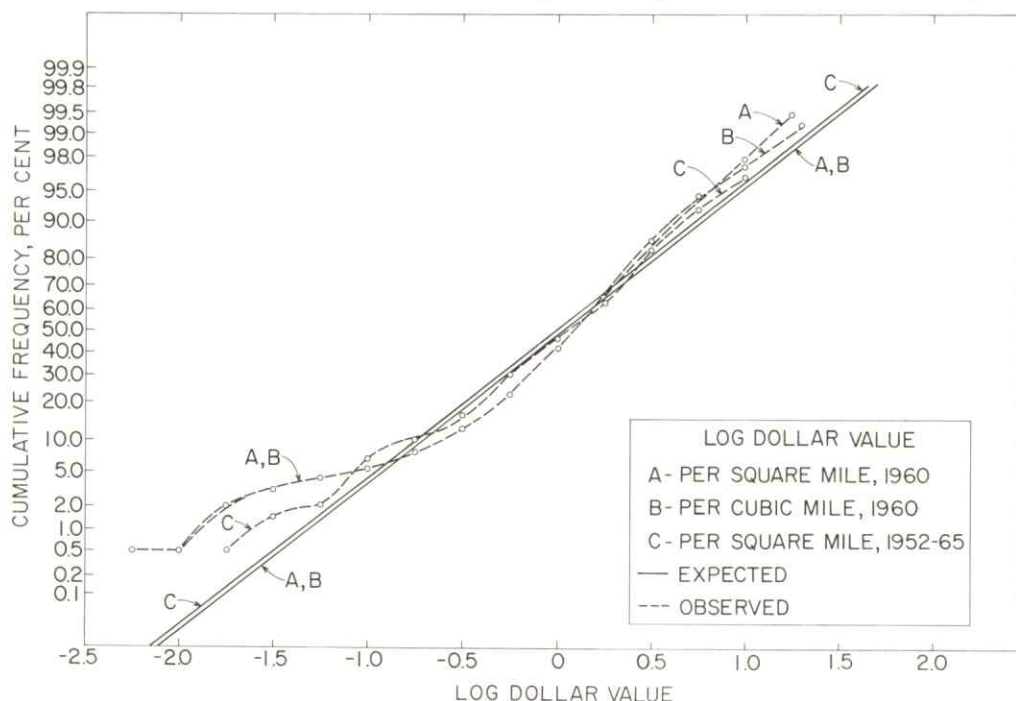


Figure 16. Cumulative probability curves of observed and expected log-normal frequencies of deviations from sixth-degree trends of log value per square mile and per cubic mile in 1960 and log value per square mile in 1952-1965.

The deviations of observed from expected frequencies are largest, both positive and negative, in the extremes, with the largest contribution occurring at the negative extreme. When a deviation (=observed-expected) is negative it implies that the observed value lies below the expected, and so these are, on a comparative basis, underachievers. The underachievers are reduced in number and magnitude for the 14-year period as compared with the single year while there is much less change at the positive extreme among the overachievers.

Deviations from Sixth-degree Surface of Log Dollar Value per Square and per Cubic Mile in 1960

Since the residual deviations are not random, but tend to be systematic, it is of interest to locate geographically the main sources of deviation. Deviations are expressed as observed minus expected, and deviations of the sixth-degree trend-surface from the original observations are displayed as a contour map in Figure 17 for the variate log dollar value per square mile in 1960. It may be seen that the fit is good over the western half of the map, and, therefore, the residuals are small in this area. The surface fails to match the original observations in the south-central part of the map, and deviations of over -1.5 units, i.e., expected greater than observed, occur at this location. This aberration occurs because of the low values of two counties, namely, Barber County in Kansas and Ellis County in Oklahoma.

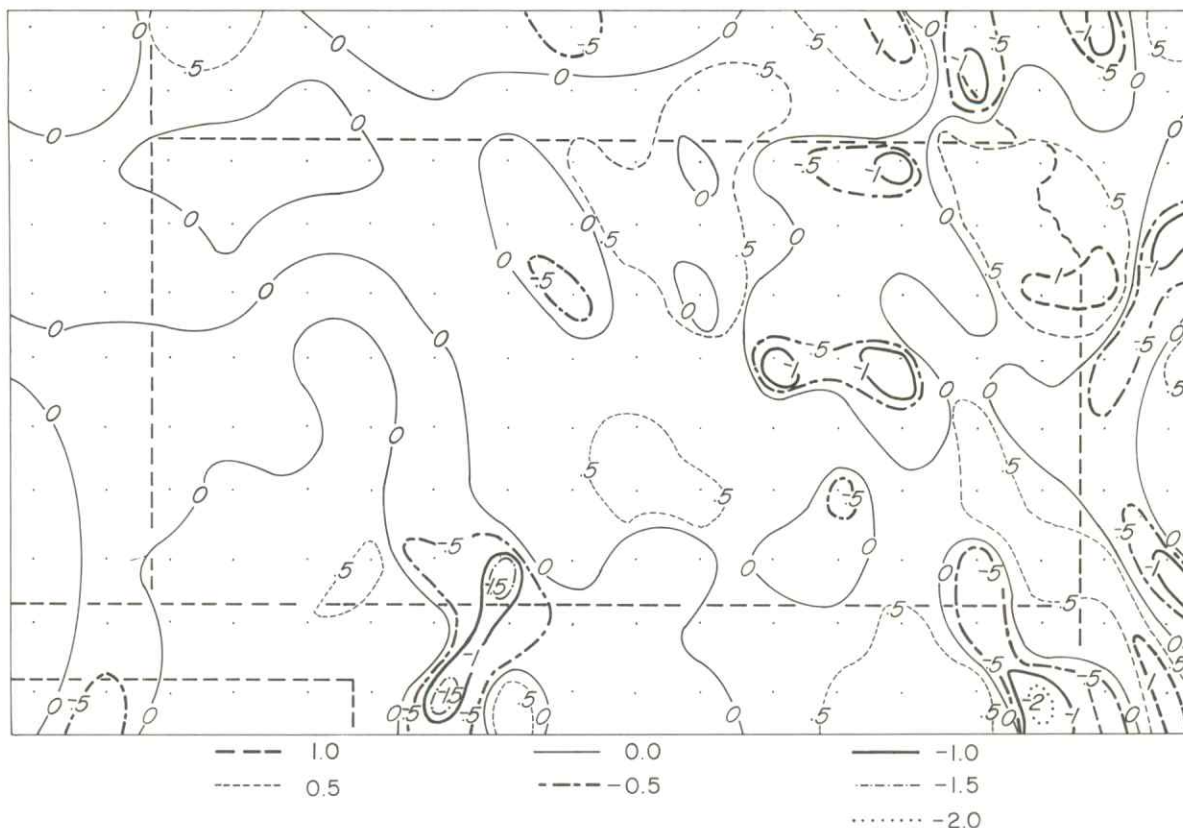


Figure 17. Residual deviations of sixth-degree trend-surface from original data for log dollar value per square mile in Kansas, 1960.

In the eastern half of the map the deviations are more common; for example, the trough extending from Labette County, Kansas, through Craig and Delaware counties, Oklahoma, and McDonald County, Missouri, represents a negative deviation of large magnitude (-0.5 to -2.0) over counties with low to very low unit regional value; the sixth-degree surface cannot bend steeply enough to match the dip in the observed figures. A similar feature occurs in the north-east corner beyond the state boundary of Kansas, both in southeastern Nebraska and northwestern Missouri.

Three other marked deviations (-1.0 or greater) occur within the confines of Kansas, namely, the two neighboring low values for Marshall and Nemaha counties, another in Washington County along the northern state boundary, and a third low in the east-central region in Osage County.

All of these deviations arise from abrupt drops in value and, if the sixth-degree surface is taken as the expected value, such deviations indicate underachievement for counties with such strong negative deviations.

Positive deviations in which the observed exceeds the expected are not so common and rarely achieve a magnitude of observed minus expected of one unit. There are two locations, both in the east, one at the counties of Johnson and Wyandotte⁵ in Kansas, and Jackson in Missouri, and the second along a ridge of positive deviations in the southeast corner of the map, where the sixth-degree surface dips below the observed values. The former represents relative overachievement but the latter may be largely a boundary effect.

⁵/Value for these two counties is assigned to a single point on the grid.

A contour map of deviations from the sixth-degree trend-surface of log dollar value per cubic mile in 1960 is almost identical with that of the value per square mile in 1960, and the description given above for the deviations in value per square mile applies equally well to the deviations in value per cubic mile.

Deviations from Sixth-degree Surface of Log Dollar Value per Square Mile in 1952-1965

The residual deviations of observed from expected for the sixth-degree trend-surface of log dollar value per square mile in 1952 to 1965 approach a normal distribution so that the deviations are approximately random; thus, when the spatial arrangement of these deviations is examined (Fig. 18) the range in magnitude is less than those in the earlier examples, and the distribution of the deviations is not nearly so systematic. The presence on the map of alternating small islands of minus one, zero, and plus one units indicates this random arrangement.

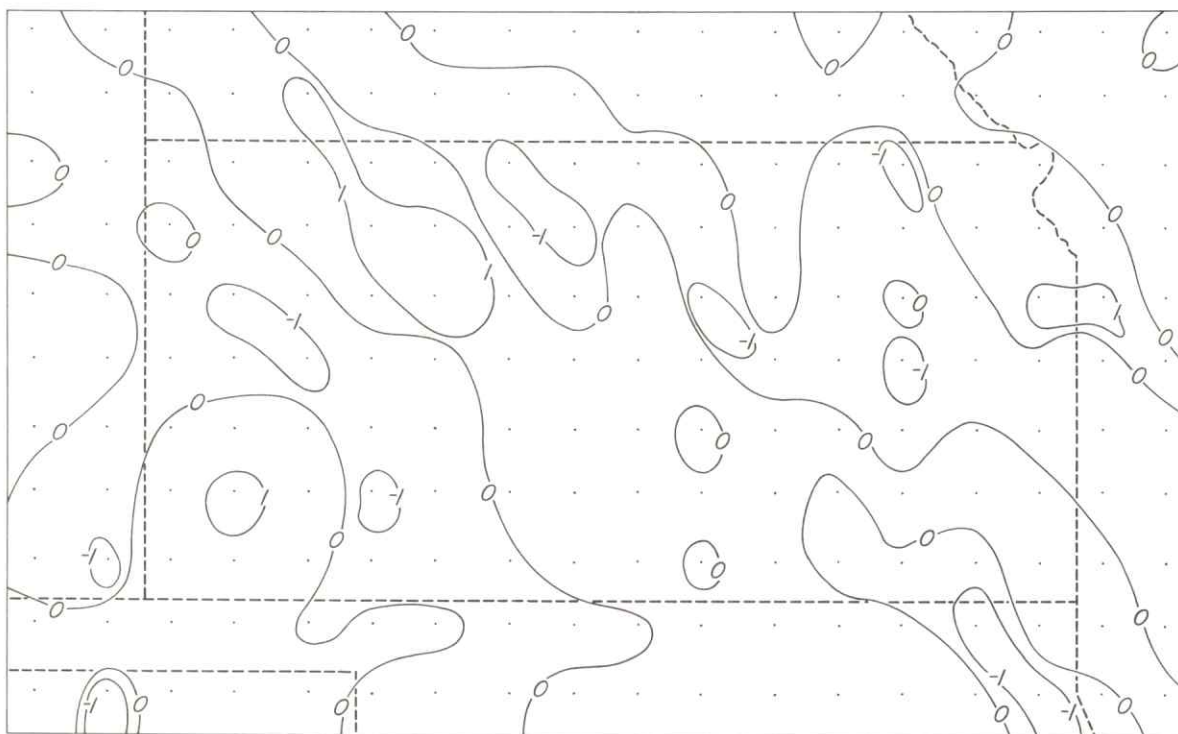


Figure 18. Residual deviations of sixth-degree trend-surface from original data for log dollar value per square mile in Kansas, 1952-1965.

The main northwest-southeast trend is still evidenced by the ridge of positive deviation ($0-E=+1$) commencing in the northwest corner; this arises because the surface fails to match the observed (Central Kansas uplift) high in this region. The negative trough in the southeast appears to lie along the same trend and implies that the expected surface exceeds the observed in this region. The entire lineation in positive and negative deviations appears to be related to the central uplift.

Negative deviations ($0-E=-1$) indicating relative underachievers compared with their expected value at the sixth-degree surface are, from northwest to southeast, Washington, Cloud, Dickinson, and Lane counties, with Nemaha at the Nebraska state line. The negative deviations in the southeast suggest that the

counties of Logan, Lane, and Ford have not achieved the relatively high potential of their neighbors.

Positive deviations ($0-E=+1$) are widely scattered and include, in the east, Johnson and Wyandotte counties, Kansas, and Jackson County, Missouri, and, in the southeast, Grant County, Kansas; these are relatively overachievers compared with their expected values.

It should perhaps be emphasized that most of these deviations are of relatively low magnitude and are widely scattered, so that variation is largely random in character; however, a deviation value of plus or minus one unit equals the difference between one-tenth and ten times the value at neighboring grid points.

The surfaces are, therefore, highly irregular and change rapidly from location to location, which is why the log scale is useful in representing the magnitude and disposition of the variation in values.

The Richest and Poorest Counties

The richest and poorest counties in terms of unit regional value are listed in Appendix tables 6, 7 and 8 for log dollar value per square mile, per cubic mile, both in 1960, and per square mile in 1952-1965, respectively.

There are 14 counties worth less than 100 dollars per square mile in terms of dollar value in 1960 and five counties worth more than 100,000 dollars per square mile. Ten of these 14 poorest counties are also poorest per cubic mile in 1960, and the same five counties worth more than 100,000 dollars per square mile plus two more are the richest per cubic mile in 1960. The values per square mile and per cubic mile per county in 1960 are, therefore, very similar.

There are two counties with less than 100 dollars per square mile for the period 1952-1965 and nine worth less than 316 dollars per square mile; seven of the 198 counties are worth over 316,000 dollars per square mile in 1952-1965 (Fig. 19).

The most striking feature is the change which takes place from the one-year (1960) to the 14-year period. The richest counties in 1960 per square and cubic mile are all in Nebraska, whereas none of these appear in the richest class for the 14-year period. Some counties, such as Gosper, Chase, and Frontier in Nebraska, are richest for 1960 and among the poorest for the period 1952-65.

The disposition of the poorest and richest counties in dollars per square mile for the 1952-1965 period are illustrated on the map in Figure 19. It would be expected that the richest and poorest counties would tend to form separate clusters since development of mineral industries resources tends to show clustering and this is, in general, borne out in the map. The richest counties occur along the Central Kansas uplift and its southeastern extension. Nevertheless, some of the poorest counties are adjacent to the richest, for example, in the center and southwestern clusters, suggesting that there is a random element in the distribution, as was shown in the deviations from the sixth-degree trend-surface. In both cases the poorest counties near the richest ones possess very low drilling intensity (lowest rank=1).

CONCLUSIONS

The exploration for nonrenewable natural resources is a high-risk venture and the decision to explore a region is made because it is considered that adequate potential resources exist to justify the investment. One basis for

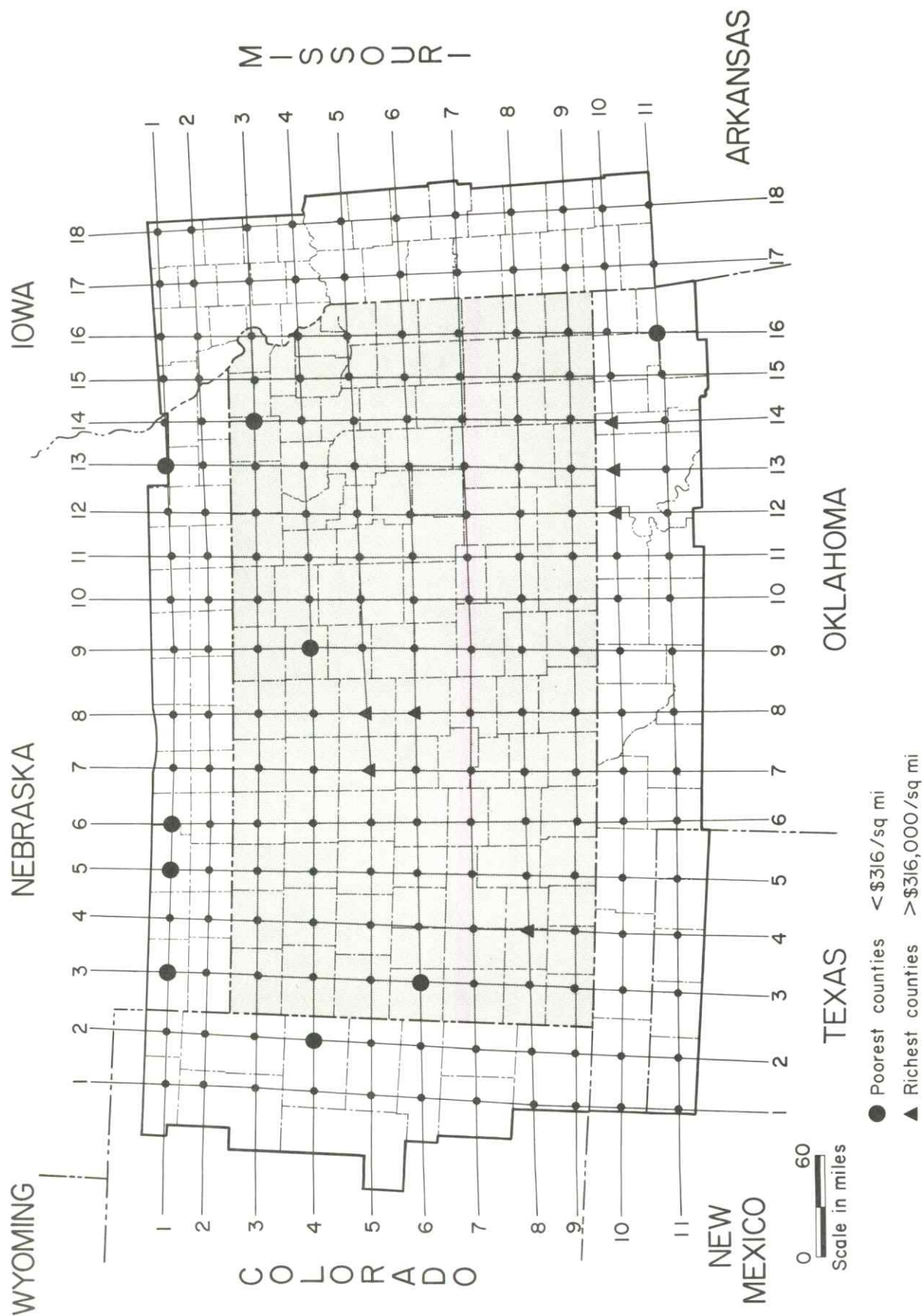


Figure 19. Richest and poorest counties in dollar value per square mile in Kansas, 1952-1965.

estimating the value of the nonrenewable resources in a region is the past production records of similar regions. Thus the U.S.A. has produced nonrenewable resources to the value of 399,887 millions of dollars in the period 1880-1964, and its area is 3,022,387 square miles; on average each square mile is worth 132,308 dollars.

Of this total, fuels account for 86,572 dollars per square mile, nonmetallics 24,632 dollars per square mile and metals 21,105 dollars per square mile; the fuel resources of coal and petroleum products are three to four times as valuable as either of the other kinds of resources.

When the dollar return per square mile is considered in terms of states (for the period 1911-1964) the over-all average is 126,785 dollars per square mile, and the states range in value from Alaska, worth 3,483 dollars per square mile, to Pennsylvania, worth one million dollars per square mile. The difference between the two estimates of average value for the U.S.A. is 5,000 dollars per square mile, which indicates that the bulk of the value has been produced after 1911.

The frequency distribution of value per square mile per state is log normal, so that the above figures tend to overestimate the value of the resources, and a more conservative estimate is obtained from the geometric mean value of 73,280 dollars per square mile. The rank order of the states is similar in dollars per square mile or log dollars per square mile (see Tables 2 and 4 and Fig. 2).

The average value may be taken as an expected value and a comparison may be made between what each state has produced and the value it may be expected to produce on the basis of its area; it is assumed that all states are equal in the potential value of their mineral industries resources per square mile. Using the logarithmic scale, 20 states exceed their expected value, about eight achieve their expected values and 21 are underachievers.

The level of achievement in dollar value is not apparently related solely to size, position, or geology and, while the fuels are the most valuable resource--the leading states are Pennsylvania, West Virginia and Louisiana--states with no outstanding fuel resources may also achieve or exceed their expected value, for example, Vermont, New Jersey, Minnesota, and Arkansas (Fig. 3 and Table 3).

To a first approximation, therefore, regions of 10,000 square miles or larger are equally valuable in terms of nonrenewable natural resources; their level of achievement depends essentially on their intensity of development.

In an attempt to study the distribution of value per square mile in greater detail the state of Kansas was analyzed in terms of value per square mile per county; Kansas has produced mineral industries resources to the value of 16,156 millions of dollars (in 1911-1964) against an expected value of 6,595 millions of dollars. Kansas has, therefore, comfortably exceeded its expected value (on both an arithmetic and logarithmic scale).

As a first step a single year, 1960, was chosen and the mineral industries products were allocated to counties using an orthogonal grid (Fig. 4); the resulting contour map of value showed the outstanding contribution of the petroleum resources of the Central Kansas uplift. Included in these figures of mineral industries products produced was the additional yield of crops arising from water used in irrigation, and this helped materially to increase the value of the western counties.

The richest counties observed are worth over 100,000 dollars per square mile while the poorest are worth less than 31.6 dollars per square mile with an overall average of 6,237 dollars per square mile.

In Kansas the basement rocks average something less than a mile below the surface so that the value may be expressed in terms of dollars per cubic mile and the distribution remains substantially the same. One may conclude therefore that in 1960 the first mile of the earth's crust is worth some 7,780 dollars per cubic mile per county on average.

When the period 1952-1965 is taken as basis, the frequency distribution of log dollar value per square mile per county in Kansas is close to log normal with an average value of 12,680 dollars per square mile; this could also be taken as a first approximation to the value of the first mile of the earth's crust for the 14-year period 1952-1965. Since the bulk of the value has been produced since 1952 then some 13,000 dollars may represent a close estimate of the value of the first cubic mile of the earth's crust.

The values per square mile and per cubic mile for 1960 and for the 14-year period (1952-1965) were plotted on maps and trend-surfaces fitted to the data to describe the nature of the surfaces. Some 40 to 50 per cent of the variation displayed by the surface may be accounted for by a surface of third degree (a cubic), indicating that a large part of the variation in value may be attributed to fairly simple and obvious sources, e.g., the domal central uplift, the Hugoton gas area, etc. About 60 to 70 per cent of the variation in log dollar value is accounted for by a sixth-degree trend-surface. In other words, it requires the complexity of a sixth-degree surface to account for the next 10 to 20 per cent of the variation in value. It is, therefore, obvious that the total variation is exceedingly complex and not entirely dependent on the dominant sources of value.

The highs expected from the petroleum resources in the Central Kansas uplift and Hugoton gas area are, of course, quite conspicuous, but there are too many other interfering sources of variation for these to account for the fluctuation in value.

When the intensity of drilling is mapped as a measure of information on resource occurrence, the surface, as would be expected, shows that drilling occurs in regions where most resource value has been produced. This is a reflection of the philosophy of exploration in which the search for new resources is concentrated in areas which already contain a major resource. It does not necessarily reflect the true potential value of a region on this smaller scale any more than on the scale of 10,000 square miles. Indeed, while this philosophy prevails and alternative approaches are not considered, it is impossible to decide whether the philosophy is successful or misleading and inefficient.

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APPENDIX

Table 1. Data arranged according to grid for the state of Kansas and adjoining tiers of counties.

- (1) = ROWS
- (2) = COLUMNS
- (3) = AREA IN SQUARE MILES
- (4) = VALUE M. I. PRODUCTS IN MILLIONS OF \$ X 10⁻⁶
- (5) = AREA IRRIGATED IN SQUARE MILES
- (6) = EXTRA VALUE ADDED BY IRRIGATION \$ X 10⁻⁶
- (7) = HEIGHT ABOVE SEA LEVEL IN FEET
- (8) = DEPTH TO BASEMENT BELOW SEA LEVEL IN FEET
- (9) = RANKED DRILLING INTENSITY
- (10) = LOG₁₀ VALUE M. I. PRODUCT PER SQUARE MILE (1952-1965)

COUNTY, STATE	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1/2LOGAN,C	1.	1.	914.	13.477	33.9	1.440	4200.	-2250.	2.	5.051
PHILLIPS,C	1.	2.	680.	0.018	1.8	0.059	3800.	-2000.	1.	2.945
CHASE,N	1.	3.	891.	0.001	14.9	1.974	3300.	-2000.	3.	2.379
HAYES,N	1.	4.	711.	0.050	7.5	1.293	3000.	-1700.	5.	2.739
FRONTIER,N	1.	5.	966.	0.026	17.6	2.007	2800.	-1000.	4.	2.298
GOSPER,N	1.	6.	462.	0.001	37.0	3.537	2500.	-1200.	6.	1.733
PHELPS,N	1.	7.	545.	0.084	134.6	9.192	2300.	-2100.	2.	3.343
KEARNEY,N	1.	8.	512.	0.096	117.1	7.182	2100.	-2250.	1.	3.284
ADAMS)CLAY,N	1.	9.	1532.	0.161	199.5	6.492	1900.	-2250.	1.	3.319
FILMORE,N	1.	10.	577.	0.056	88.4	2.159	1500.	-1800.	1.	3.086
SALINE,N	1.	11.	575.	0.084	27.4	0.664	1500.	-1600.	1.	3.426
1/2(LNCATR+GAGE),N	1.	12.	852.	0.502	10.9	0.234	1350.	-1250.	1.	3.687
JOHNSON,N	1.	13.	377.	0.001	1.0	0.031	1200.	-500.	1.	1.465
NEMAHA,N	1.	14.	399.	0.050	1.0	0.017	1000.	-1500.	1.	3.846
ATCHISON,M	1.	15.	549.	0.050	0.0	0.0	1000.	-2500.	1.	3.974
NODAWAY,M	1.	16.	877.	0.197	0.0	0.0	1000.	-3000.	1.	3.372
WORTH,M	1.	17.	267.	0.005	0.0	0.0	1000.	-2750.	1.	3.371
HARRISON,M	1.	18.	720.	0.359	0.0	0.0	1000.	-2600.	1.	3.622
1/2WASHINGTON,C	2.	1.	1263.	23.363	4.5	0.226	4500.	-3000.	5.	5.061
1/2YUMA,C	2.	2.	1185.	0.139	5.0	0.169	4000.	-2250.	4.	3.125
DUNDY,N	2.	3.	921.	0.088	19.9	3.310	3300.	-2000.	3.	2.683
HITCHCOCK,N	2.	4.	714.	0.873	30.2	3.131	2700.	-1800.	7.	3.806
RED WILLOW,N	2.	5.	716.	1.655	26.3	1.576	2500.	-1600.	8.	4.928
FURNAS,N	2.	6.	722.	0.093	34.5	2.69	2300.	-1250.	7.	3.080
HARLAN,N	2.	7.	554.	0.314	23.8	1.600	2250.	-1750.	4.	3.423
FRANK+1/2WEBST,N	2.	8.	866.	0.102	50.2	2.872	2100.	-2500.	3.	3.206
WEBSTER+NUCKOLL,N	2.	9.	867.	0.200	58.2	2.118	1950.	-2600.	1.	3.278
THAYER,N	2.	10.	577.	0.177	56.8	1.542	1500.	-2250.	1.	3.424
JEFFERSON,N	2.	11.	577.	0.227	13.8	0.339	1450.	-1900.	1.	3.597
1/2GAGE,N	2.	12.	429.	0.222	17.2	0.192	1350.	-1500.	1.	3.427
PAWNEE,N	2.	13.	433.	0.304	0.0	0.0	1400.	-1000.	1.	3.885
RICHARDSON,N	2.	14.	548.	0.573	0.0	0.0	1000.	-2500.	1.	4.074
HOLT,M	2.	15.	464.	0.001	0.0	0.0	1000.	-3000.	1.	3.611
ANDREW,M	2.	16.	435.	0.096	0.0	0.0	1000.	-2800.	1.	3.333
GENTRY,M	2.	17.	488.	0.050	0.0	0.0	950.	-2700.	1.	3.325
HARRISON,M	2.	18.	720.	0.359	0.0	0.0	1000.	-2600.	1.	3.622
1/2WASHINGTON,C	3.	1.	1263.	23.363	4.5	0.226	4500.	-3000.	5.	4.986
1/2YUMA,C	3.	2.	1185.	0.139	5.0	0.169	4100.	-2250.	4.	3.063
CHEYENNE,K	3.	3.	1008.	0.055	6.0	0.056	3400.	-2050.	3.	2.740

Table 1. Data arranged according to grid for the state of Kansas and adjoining tiers of counties (continued).

COUNTY, STATE	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
RAWLINS, K	3.	4.	1064.	1.569	2.7	0.023	3000.	-2000.	5.	4.055
DECATUR, K	3.	5.	891.	1.083	2.5	0.038	2700.	-1500.	5.	4.253
NORTON, K	3.	6.	876.	2.547	2.5	0.026	2400.	-1500.	6.	4.512
PHILLIPS, K	3.	7.	887.	5.531	1.2	0.053	2000.	-2000.	6.	4.973
SMITH, K	3.	8.	888.	0.006	1.2	0.020	1800.	-2750.	1.	2.853
JEWELL, K	3.	9.	900.	0.817	1.2	0.043	1700.	-3500.	1.	3.540
REPUBLIC, K	3.	10.	704.	0.207	2.2	0.028	1500.	-2500.	1.	3.174
WASHINGTON, K	3.	11.	902.	0.078	1.1	0.0	1400.	-1750.	1.	3.161
MARSHALL, K	3.	12.	905.	0.494	0.0	0.0	1200.	-1000.	2.	4.015
NEMAHA, K	3.	13.	716.	0.039	0.0	0.0	1200.	+ 500.	2.	3.168
BROWN, K	3.	14.	571.	0.006	0.0	0.0	1000.	-3000.	1.	2.291
DONIPHAN, K	3.	15.	378.	0.377	0.0	0.0	1000.	-2500.	1.	4.123
BUCHANAN, M	3.	16.	404.	0.375	0.0	0.0	850.	-2500.	1.	4.094
CLINTON+DEKALB, M	3.	17.	843.	0.325	0.0	0.0	1000.	-2350.	1.	3.635
DAVIESS+CALDWELL, M	3.	18.	993.	0.298	0.0	0.0	950.	-2200.	1.	3.745
1/2(LINC+K CARS), C	4.	1.	2383.	0.177	9.9	0.328	5000.	-2300.	4.	2.845
1/2KIT CARSON, C	4.	2.	1086.	0.030	8.9	0.308	4100.	-2000.	2.	2.404
SHERMAN, K	4.	3.	1049.	0.371	4.7	0.158	3700.	-2250.	2.	3.285
THOMAS, K	4.	4.	1065.	0.102	5.5	0.076	3100.	-2200.	3.	2.919
SHERIDAN, K	4.	5.	896.	1.296	6.2	0.132	2600.	-2000.	5.	4.185
GRAHAM, K	4.	6.	895.	17.595	1.5	0.044	2200.	-1800.	8.	5.349
ROOKS, K	4.	7.	890.	16.210	2.2	0.049	2000.	-1750.	9.	5.375
OSBORNE, K	4.	8.	894.	0.224	5.6	0.051	1800.	-2500.	3.	3.478
MITCHELL, K	4.	9.	713.	0.026	3.0	0.018	1600.	-3500.	1.	2.145
CLOUD, K	4.	10.	702.	0.298	4.2	0.079	1600.	-3000.	1.	3.825
CLAY, K	4.	11.	638.	0.367	1.4	0.029	1200.	-2000.	2.	3.580
RILEY, K	4.	12.	604.	0.791	0.0	0.0	1000.	-1000.	3.	3.998
POTTAWATOMIE, K	4.	13.	829.	0.220	0.0	0.0	1200.	-500.	2.	3.471
JACKSON, K	4.	14.	675.	0.090	0.0	0.0	1000.	-2500.	2.	3.404
JEFFSON+ATCHISON, K	4.	15.	955.	1.276	0.0	0.0	1000.	-2250.	1.	3.910
LEVNWORTH, K+PLATT, M	4.	16.	860.	1.002	0.0	0.0	1000.	-2000.	3.	4.070
CLAY, M	4.	17.	413.	2.037	0.0	0.0	900.	-1750.	1.	4.609
RAY, M	4.	18.	574.	0.010	0.0	0.0	1000.	-1400.	1.	3.877
1/2LINCOLN, C	5.	1.	1297.	0.146	1.0	0.029	4900.	-2250.	4.	2.947
CHEYENNE, C	5.	2.	1772.	0.142	0.6	0.012	4400.	-2000.	1.	2.608
WALLACE, K	5.	3.	921.	0.067	11.2	0.227	3400.	-2350.	1.	2.962
LOGAN, K	5.	4.	1082.	0.011	1.9	0.038	3000.	-2500.	2.	2.533
GOVE, K	5.	5.	1080.	0.034	2.7	0.046	2800.	-2500.	4.	3.455
TREGO, K	5.	6.	899.	4.838	0.0	0.0	2400.	-2500.	7.	4.823
ELLIS, K	5.	7.	901.	32.360	0.0	0.0	2100.	-2000.	9.	5.678
RUSSEL, K	5.	8.	895.	25.234	0.0	0.0	1800.	-2200.	9.	5.552
LINCOLN, K	5.	9.	721.	0.941	0.0	0.0	1400.	-3000.	1.	3.888
OTTAWA+SALINE, K	5.	10.	1432.	2.409	0.9	0.015	1300.	-3000.	6.	4.445
DICKINSON, K	5.	11.	838.	1.254	0.0	0.0	1150.	-2000.	5.	3.104
MORRIS+GEARY, K	5.	12.	1086.	1.999	0.0	0.0	1500.	-1500.	3.	4.272
WABAUNSEE, K	5.	13.	795.	0.895	0.0	0.0	1000.	-2000.	4.	4.147
SHAWNEE, K	5.	14.	544.	1.075	0.0	0.0	950.	-2000.	1.	4.353
DOUGLAS, K	5.	15.	469.	0.293	0.0	0.0	800.	-1500.	4.	3.884
JOHNSN+WYANDOTTE, K	5.	16.	629.	7.737	0.0	0.0	1000.	-1500.	3.	5.277
JACKSON, M	5.	17.	603.	11.328	0.0	0.0	1000.	-1200.	1.	5.353
LAFAYETTE, M	5.	18.	634.	0.357	0.0	0.0	950.	-1300.	1.	3.893
KIOWA, C	6.	1.	896.	0.221	2.6	0.036	4300.	-2100.	4.	3.072
KIOWA, C	6.	2.	896.	0.221	2.6	0.036	4100.	-1900.	4.	3.072
GREELEY, K	6.	3.	776.	0.011	5.6	0.079	3600.	-2500.	1.	2.435

Table 1. Data arranged according to grid for the state of Kansas and adjoining tiers of counties (continued).

COUNTY, STATE	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
WICHITA+SCOTT,K	6.	4.	1435.	0.175	115.0	2.479	3100.	-2750.	2.	3.349
LANE,K	6.	5.	715.	0.046	6.2	0.153	2800.	-2750.	3.	3.213
NESS,K	6.	6.	1079.	1.760	4.1	0.108	2250.	-2500.	6.	4.366
RUSH,K	6.	7.	719.	1.463	6.6	0.094	2050.	-2000.	6.	4.625
BARTON,K	6.	8.	892.	30.455	4.7	0.049	1800.	-2000.	9.	5.753
ELLSWORTH+RICE,K	6.	9.	1434.	21.991	3.6	0.017	1650.	-2500.	7.	5.465
MC PHERSON,K	6.	10.	900.	10.601	2.7	0.069	1500.	-2500.	8.	5.195
MARION,K	6.	11.	696.	10.515	0.0	0.0	1300.	-2000.	8.	4.952
CHASE,K	6.	12.	769.	0.324	0.0	0.0	1200.	-500.	4.	3.689
LYON,K	6.	13.	845.	0.653	0.0	0.0	1150.	-2000.	5.	4.120
OSAGE,K	6.	14.	718.	0.037	0.0	0.0	1100.	-1750.	1.	3.481
FRANKLIN,K	6.	15.	585.	1.212	0.0	0.0	900.	-1400.	7.	4.452
LINN+MIAMI,K	6.	16.	1215.	1.903	0.0	0.0	1000.	-1200.	7.	4.385
CASS+BATES,M	6.	17.	1539.	0.555	0.0	0.0	850.	-1100.	1.	4.113
JOHNSON+HENRY,M	6.	18.	1563.	6.495	0.0	0.0	900.	-800.	1.	4.657
BENT,C	7.	1.	1517.	0.114	51.8	1.305	4200.	-500.	4.	2.851
PROWERS,C	7.	2.	1626.	0.087	108.2	2.581	3700.	-2500.	3.	2.952
HAMILTON,K	7.	3.	984.	0.527	13.7	0.363	3250.	-3000.	5.	3.931
KEARNEY,K	7.	4.	853.	8.467	47.2	0.359	3000.	-3250.	6.	5.152
FINNEY,KK	7.	5.	1276.	7.075	95.8	0.548	2850.	-3500.	7.	4.878
HODGEMAN,K	7.	6.	858.	1.169	7.0	0.183	2250.	-2600.	6.	4.502
PAWNEE+EDWARDS,K	7.	7.	1343.	7.102	20.9	0.294	2100.	-2200.	7.	4.796
STAFFORD,K	7.	8.	796.	17.445	2.2	0.057	1900.	-2500.	9.	5.471
RENO,K	7.	9.	621.	6.017	3.9	0.071	1500.	-3000.	7.	5.119
RENO,K	7.	10.	621.	6.017	1.1	0.071	1500.	-3000.	7.	5.119
HARVEY,K	7.	11.	540.	2.040	2.2	0.037	1450.	-2500.	6.	4.491
BUTLER,K	7.	12.	717.	11.448	0.0	0.0	1300.	-2100.	9.	5.331
GREENWOOD,K	7.	13.	1158.	13.837	0.0	0.0	1150.	-2000.	9.	5.260
COFFEY+WOODSON,K	7.	14.	1147.	2.663	0.0	0.0	1000.	-1500.	7.	4.537
ALLEN+ANDERSON,K	7.	15.	1285.	12.736	0.0	0.0	1000.	-1250.	9.	5.230
BOURBON,K	7.	16.	656.	0.605	0.0	0.0	900.	-950.	5.	4.311
VERNON,M	7.	17.	838.	0.278	0.0	0.0	950.	-1000.	1.	3.719
ST CLAIR+CEDAR,M	7.	18.	1195.	1.000	0.0	0.0	950.	-900.	1.	4.084
1/4BACA,C	8.	1.	641.	0.061	6.3	0.187	4400.	-2000.	4.	3.207
1/4BACA,C	8.	2.	641.	0.061	6.3	0.187	4000.	-2650.	4.	3.207
STANTON,K	8.	3.	685.	2.399	66.0	1.277	3350.	-3500.	5.	4.671
GRANT,K	8.	4.	578.	16.174	51.6	1.469	3050.	-3800.	5.	5.610
GRAY+HASKELL,K	8.	5.	1434.	11.940	102.0	2.097	2800.	-4000.	4.	4.863
FORD,K	8.	6.	1086.	0.200	7.8	0.208	2500.	-3200.	4.	3.515
KIOWA,K	8.	7.	723.	2.884	5.9	0.029	2200.	-2800.	6.	4.579
PRATT,K	8.	8.	726.	5.739	3.7	0.061	1900.	-3000.	7.	5.105
KINGMAN,K	8.	9.	867.	12.910	1.4	0.021	1600.	-3800.	8.	5.207
SEDGWICK,K	8.	10.	497.	5.557	1.2	0.064	1400.	-3000.	8.	5.152
SEDGWICK,K	8.	11.	497.	5.557	1.2	0.064	1300.	-3000.	8.	5.152
BUTLER,K	8.	12.	717.	11.448	0.0	0.0	1400.	-3000.	9.	5.331
ELK,K	8.	13.	652.	1.397	0.0	0.0	1000.	-2000.	6.	4.571
WILSON,K	8.	14.	581.	4.849	0.0	0.0	900.	-1500.	8.	5.089
NEOSHO,K	8.	15.	580.	7.590	0.0	0.0	900.	-1050.	8.	5.238
CRAWFORD,K	8.	16.	605.	1.768	0.0	0.0	1000.	-900.	7.	4.643
BARTON,M	8.	17.	594.	0.200	0.0	0.0	950.	-900.	9.	3.740
DADE,M	8.	18.	504.	0.188	0.0	0.0	900.	-800.	1.	3.638
1/4BACA,C	9.	1.	641.	0.061	6.3	0.187	4500.	-2000.	4.	3.207
1/4BACA,C	9.	2.	641.	0.061	6.3	0.187	4000.	-2650.	4.	3.207
MORTON,K	9.	3.	718.	12.162	10.3	0.220	3400.	-4100.	7.	5.338

Table 1. Data arranged according to grid for the state of Kansas and adjoining tiers of counties (concluded).

COUNTY, STATE	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
STEVENS, K	9.	4.	729.	12.900	35.5	0.695	3100.	-4500.	5.	5.394
SEDWARD, K	9.	5.	643.	6.041	30.8	0.477	2800.	-5000.	6.	5.280
MEADE, K	9.	6.	984.	4.587	18.3	0.411	2400.	-5000.	6.	4.670
CLARK, K	9.	7.	974.	1.333	0.0	0.0	2000.	-4500.	5.	4.286
COMANCHE, K	9.	8.	788.	0.076	0.0	0.0	2000.	-4000.	4.	3.838
BARBER, K	9.	9.	1134.	10.924	0.0	0.0	1600.	-4000.	8.	5.038
HARPER, K	9.	10.	799.	4.217	0.0	0.0	1400.	-4200.	6.	4.694
SUMNER, K	9.	11.	1179.	9.332	0.0	0.0	1250.	-3500.	8.	4.958
COWLEY, K	9.	12.	1133.	11.557	0.0	0.0	1300.	-3000.	9.	5.170
CHATAUQUA, K	9.	13.	652.	2.567	0.0	0.0	1000.	-2100.	8.	4.747
MONTGOMERY, K	9.	14.	644.	4.806	0.0	0.0	800.	-1500.	8.	5.160
LABETTE, K	9.	15.	643.	0.449	0.0	0.0	900.	-1200.	7.	3.947
CHEROKEE, K	9.	16.	605.	3.613	0.0	0.0	900.	-950.	1.	5.209
JASPER, M	9.	17.	642.	2.301	0.0	0.0	1000.	-750.	1.	4.755
LAWRENCE, M	9.	18.	619.	0.006	0.0	0.0	1100.	-750.	1.	3.345
CIMARRON, O	10.	1.	1832.	7.078	3.0	0.023	4200.	-2000.	5.	4.616
CIMARRON, O	10.	2.	1832.	7.078	3.0	0.023	3900.	-2250.	5.	4.961
TEXAS, O	10.	3.	2056.	24.431	12.9	0.233	3500.	-4000.	7.	5.285
TEXAS, O	10.	4.	2056.	24.431	12.9	0.233	2900.	-4600.	7.	5.285
BEAVER, O	10.	5.	1793.	17.937	1.1	0.022	2400.	-7000.	7.	5.124
HARPER, O	10.	6.	1034.	11.324	0.0	0.0	2000.	-6500.	5.	5.063
HARPER, O	10.	7.	1034.	11.324	0.0	0.0	1900.	-6500.	5.	5.063
WOODS, O	10.	8.	1271.	0.772	0.0	0.0	1700.	-6000.	3.	4.061
ALFALFA, O	10.	9.	867.	3.350	0.0	0.0	1200.	-5500.	5.	4.574
GRANT, O	10.	10.	999.	5.251	0.0	0.0	1100.	-5000.	6.	4.795
KAY, O	10.	11.	944.	15.442	0.0	0.0	1000.	-4000.	7.	5.274
KAY+OSAGE, O	10.	12.	1615.	45.227	0.0	0.0	1000.	-2500.	7.	5.517
OSAGE, O	10.	13.	2286.	75.013	0.0	0.0	1000.	-2000.	7.	5.588
WASHINTON+NOWATA, O	10.	14.	1002.	29.649	0.0	0.0	1000.	-2000.	1.	5.560
CRAIG, O	10.	15.	764.	0.338	0.0	0.0	850.	-1500.	1.	3.961
OTTAWA, O	10.	16.	461.	1.673	0.0	0.0	850.	-900.	1.	5.372
NEWTON, M	10.	17.	629.	0.425	0.0	0.0	1200.	-700.	1.	4.066
BARRY, M	10.	18.	789.	0.031	0.0	0.0	1250.	-800.	1.	2.931
DALLAM, T	11.	1.	1494.	0.909	15.4	0.198	4300.	-3000.	3.	2.979
DALLAM, T	11.	2.	1494.	0.909	15.4	0.198	4000.	-2500.	3.	2.979
SHERMAN, T	11.	3.	914.	25.942	25.4	0.480	3400.	-4000.	3.	5.299
HANSFORD, T	11.	4.	907.	36.502	37.3	0.473	3000.	-5000.	6.	5.401
OCHILTREE, T	11.	5.	905.	23.488	25.7	0.394	2900.	-6000.	7.	5.262
LIPSCOMB, T	11.	6.	934.	2.114	0.4	0.007	2700.	-7000.	5.	4.514
ELLIS, O	11.	7.	1222.	0.135	0.0	0.0	2200.	-7500.	4.	3.693
WOODWARD, O	11.	8.	1232.	0.480	0.6	0.019	2000.	-7000.	5.	3.987
MAJOR, O	11.	9.	945.	3.429	0.0	0.0	1600.	-6000.	7.	4.944
MAJOR, O	11.	10.	945.	3.429	0.0	0.0	1300.	-6000.	7.	4.944
GARFIELD, O	11.	11.	1054.	6.333	0.0	0.0	1100.	-5000.	8.	5.081
NOBLE, O	11.	12.	744.	8.835	0.0	0.0	1000.	-4000.	5.	5.215
PAWNEE+OSAGE, O	11.	13.	1734.	43.603	0.0	0.0	1000.	-2500.	8.	5.481
TULSA, O	11.	14.	572.	7.420	0.0	0.0	700.	-1500.	2.	5.254
ROGERS+MAYES, O	11.	15.	1389.	6.168	0.0	0.0	700.	-1500.	1.	4.933
DELAWARE, O	11.	16.	715.	0.001	0.0	0.0	1100.	-1250.	1.	2.203
MC DONALD, M	11.	17.	540.	0.005	0.0	0.0	1100.	-750.	1.	2.588
BARRY, M	11.	18.	789.	0.031	0.0	0.0	1250.	-800.	1.	2.931

Table 2. Analysis of variance of trend-surface fitting for log dollar value per square mile, 1960.

Source of variation	Degrees of freedom	Sum of squares	Mean square	Variance ratio
Linear	2	86.524	43.262	77.80**
Residual	195	108.387	0.556	
Lin + 2	5	99.795	19.959	40.32**
Additional per Quad	3	13.271	4.424	8.94**
Residual	192	95.126	0.495	
Lin + 2 + 3	9	109.046	12.116	26.51**
Additional per cubic	4	9.251	2.313	5.06**
Residual	188	85.875	0.457	
Lin + 2 + 3 + 4	14	119.706	8.550	20.80**
Additional per 4th	5	10.661	2.132	5.19**
Residual	183	75.214	0.411	
Lin + 2 + 3 + 4 + 5	20	125.859	6.293	16.14**
Additional per 5th	6	6.153	1.025	2.63*
Residual	177	69.061	0.390	
Lin + 2 + 3 + 4 + 5 + 6	27	130.272	4.825	12.70**
Additional per 6th	7	4.412	0.635	1.67 ^{NS}
Residual	170	64.649	0.380	
Total	197	194.921	0.989	

NS = Not significant.

* = Significant at the 5 per cent probability level.

** = Significant at the 1 per cent probability level.

Table 3. Analysis of variance of trend-surface fitting for log dollar value per cubic mile, 1960.

Source of variation	Degrees of freedom	Sum of squares	Mean square	Variance ratio (F)
Linear	2	50.438	25.219	43.26**
Residual	195	113.590	0.583	
Lin + 2	5	65.379	13.076	25.44**
Additional per Quad	3	14.941	4.980	9.69**
Residual	192	98.649	0.514	
Lin + 2 + 3	9	75.936	8.437	17.99**
Additional per Cubic	4	10.556	2.639	5.63**
Residual	188	88.093	0.469	

Table 3. Analysis of variance of trend-surface fitting for log dollar value per cubic mile, 1960 (concluded).

Source of variation	Degrees of freedom	Sum of squares	Mean square	Variance ratio (F)
Lin + 2 + 3 + 4	14	87.642	6.260	15.01**
Additional per 4th	5	11.707	2.341	5.61**
Residual	183	76.386	0.417	
Lin + 2 + 3 + 4 + 5	20	94.579	4.729	12.06**
Additional per 5th	6	6.937	1.156	2.95**
Residual	177	69.449	0.392	
Lin + 2 + 3 + 4 + 5 + 6	27	98.560	3.650	9.48**
Additional per 6th	7	3.980	0.570	1.48 ^{NS}
Residual	<u>170</u>	<u>65.469</u>	<u>0.385</u>	
Total	197	164.029	0.832 ₆	

NS = Not significant.

* = Significant at the 5 per cent probability level.

** = Significant at the 1 per cent probability level.

Table 4. Analysis of variance of trend-surface fitting for log dollar value per square mile, 1952-1965.

Source of variation	Degrees of freedom	Sum of squares	Mean square	Variance ratio (F)
Linear	2	41.457	20.728	29.194**
Residual	195	138.480	0.710	
Lin + 2	5	59.805	11.961	19.107**
Additional per Quad	3	18.348	6.116	9.770**
Residual	192	120.131	0.626	
Lin + 2 + 3	9	79.821	8.869	16.639**
Additional per Cubic	4	20.016	5.004	9.388**
Residual	188	100.115	0.533	
Lin + 2 + 3 + 4	14	83.373	5.955	11.278**
Additional per 4th	5	3.552	0.710	1.345 ^{NS}
Residual	183	96.564	0.528	
Lin + 2 + 3 + 4 + 5	20	99.575	4.979	10.967**
Additional per 5th	6	16.202	2.700	5.947**
Residual	177	80.361	0.454	

Table 4. Analysis of variance of trend-surface fitting for log dollar value per square mile, 1952-1956 (concluded).

Source of variation	Degrees of freedom	Sum of squares	Mean square	Variance ratio(F)
Lin + 2 + 3 + 4 + 5 + 6	27	115.243	4.268	11.202**
Additional per 6th	7	15.668	2.238	5.874**
Residual	<u>170</u>	<u>64.693</u>	<u>0.381</u>	
Total	197	179.936	0.913	

NS = Not significant.

* = Significant at the 5 per cent probability level.

** = Significant at the 1 per cent probability level.

Table 5. Analysis of variance of trend-surface fitting for drilling intensity in Kansas.

Source of variation	Degrees of freedom	Sum of squares	Mean square	Variance ratio(F)
Linear	2	306.170	153.085	26.60**
Residual	195	1122.194	5.755	
Lin + 2	5	577.850	115.570	26.09**
Additional per Quad	3	271.681	90.560	20.44**
Residual	192	850.513	4.430	
Lin + 2 + 3	9	722.035	80.226	21.35**
Additional per Cubic	4	144.184	36.046	9.59**
Residual	188	706.329	3.757	
Lin + 2 + 3 + 4	14	799.782	57.127	16.63**
Additional per 4th	5	77.747	15.549	4.53**
Residual	183	628.582	3.435	
Lin + 2 + 3 + 4 + 5	20	883.724	44.186	14.36**
Additional per 5th	6	83.942	13.990	4.55**
Residual	177	544.640	3.077	
Lin + 2 + 3 + 4 + 5 + 6	27	990.993	36.703	14.264**
Additional per 6th	7	107.270	15.324	5.96**
Residual	<u>170</u>	<u>437.370</u>	<u>2.573</u>	
Total	197	1428.364	7.250 ₅₇	

* = Significant at the 5 per cent probability level.

** = Significant at the 1 per cent probability level.

Table 6. Richest and poorest counties in terms of $\log_{10}$ dollar value per square mile in 1960.

Class		Frequency	Grid co- ordinates	Log ₁₀		County
\$/sq mi	Log \$/sq mi			\$/sq mi	\$/sq mi	
1.0 - 3.16	0 - 0.5	2	2, 15 11, 16	0.333 0.146	2.15 1.40	Holt, Missouri Delaware, Okla.
3.16 - 10.00	0.5 - 1.0	2	9, 18 11, 17	0.986 0.967	9.68 9.27	Lawrence, Mo. McDonald, Mo.
10.00 - 31.6	1.0 - 1.5	3	1, 17 3, 14 4, 18	1.272 1.021 1.241	18.71 10.50 17.42	Worth, Mo. Brown, Kansas Ray, Missouri
31.6 - 100.0	1.5 - 2.0	7	1, 15 6, 14 3, 11 3, 13 9, 8 10, 18 11, 18	1.959 1.712 1.937 1.736 1.984 1.594 1.594	90.99 51.53 86.50 54.45 96.38 39.26 39.26	Atchison, Mo. Franklin, Kan. Washington, Kan. Nemaha, Kan. Comanche, Kan. Berry, Mo. Berry, Mo.
100,000-316,000	5.0 - 5.5	5	1, 3 1, 4 1, 5 2, 3 2, 4	5.122 5.237 5.057 5.221 5.021	132,450 172,550 114,000 166,350 104,950	Chase, Nebr. Hayes, Nebr. Frontier, Nebr. Dundy, Nebr. Hitchcock, Nebr.

Table 7. Richest and poorest counties in terms of $\log_{10}$ dollar value per cubic mile in 1960.

Class		Frequency	Grid co- ordinates	Log ₁₀		County
\$/sq mi	Log \$/sq mi			\$/sq mi	\$/sq mi	
1.0 - 3.16	0 - 0.5	2	2, 15 11, 16	0.454 0.497	2.84 3.14	Holt, Missouri Delaware, Okla.
3.16 - 10.0	0.5 - 1.0	0				
10.0 - 31.6	1.0 - 1.5	4	3, 14 9, 18 11, 17 1, 17	1.142 1.442 1.422 1.421	13.01 27.77 26.42 26.36	Brown, Kansas Lawrence, Mo. McDonald, Mo. Worth, Mo.
31.6 - 100.0	1.5 - 2.0	4	11, 7 9, 8 4, 18 6, 14	1.779 1.929 1.583 1.980	60.12 84.92 38.29 95.50	Ellis, Okla. Comanche, Kan. Ray, Missouri Franklin, Kan.

Table 7. Richest and poorest counties in terms of $\log_{10}$ dollar value per cubic mile in 1960 (concluded).

Class		Frequency	Grid		co-ordinates	Log ₁₀ \$/sq mi	County
\$/sq mi	Log \$/sq mi						
100,000-316,000	5.0 - 5.5	7	1, 3	5.121	132,100	Chase, Nebr.	
			1, 4	5.287	193,650	Hayes, Nebr.	
			1, 5	5.200	158,500	Frontier, Nebr.	
			1, 6	5.135	136,450	Gosper, Nebr.	
			2, 3	5.220	165,950	Dundy, Nebr.	
			2, 4	5.090	123,000	Hitchcock, Nebr.	
			2, 6	5.078	119,700	Furnas, Nebr.	

Table 8. Richest and poorest counties in terms of $\log_{10}$ dollar value per square mile, 1952-1965.

Class		Frequency	Grid		Log ₁₀ \$/sq mi	\$/sq mi	County
\$/sq mi	Log \$/sq mi		co- ordinates				
10.0 - 31.6	1.0 - 1.5	1	1, 13	1.465	29.18	Johnson, Nebr.	
31.6 -100.0	1.5 - 2.0	1	1, 6	1.733	54.08	Gosper, Nebr.	
100.0 -316.0	2.0 - 2.5	7	1, 3	2.378	238.8	Chase, Nebr.	
			1, 5	2.298	198.6	Frontier, Nebr.	
			11, 16	2.203	159.6	Delaware, Okla.	
			4, 9	2.145	139.6	Mitchell, Kan.	
			4, 2	2.404	253.5	Kit Carson, Col.	
			3, 14	2.291	195.4	Brown, Kan.	
			6, 3	2.435	272.3	Greeley, Kan.	
316,000-1,000,000	5.5- 6.0	7	5, 8	5.552	356,500	Russell, Kan.	
			10, 12	5.517	328,900	Kay, Osage, Okla.	
			10, 13	5.588	387,300	Osage, Okla.	
			10, 14	5.560	363,100	Wash., Nemaha, Okla.	
			6, 8	5.753	566,300	Barton, Kansas	
			5, 7	5.678	476,500	Ellis, Kansas	
			8, 4	5.610	407,400	Grant, Kansas	

