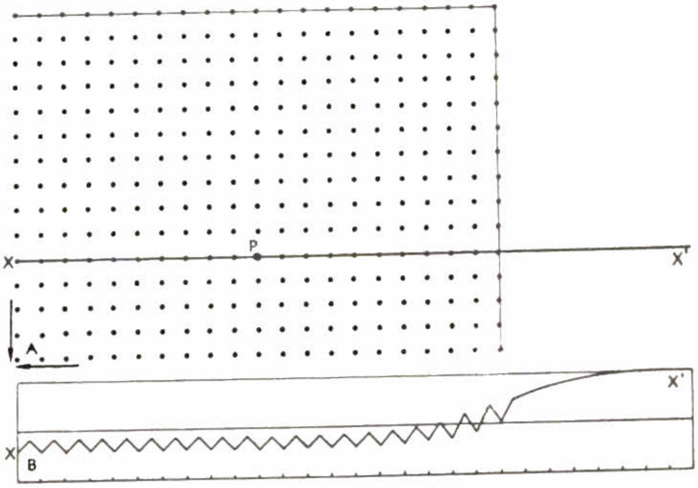
Notes on the Shape of the Truncated Cone of Depression in the Vicinity of an Infinite Well Field

By Stuart W. Fader



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ABSTRACT

The cone of depression about an infinite well field in an area of little or no recharge is not necessarily "cone-shaped." If equal amounts of water are pumped from each well and the limitations for the use of the Theis formula are allowed, the theoretical profile is "saucer-shaped."

INTRODUCTION

The purpose of this paper is to show the theoretical shape of the water surface under a well field of infinite extent after several years of pumping. An infinite well field is defined as one that has a large-capacity well on every section of land and where each well is drawing water from the same aquifer which has infinite areal extent. Well fields in aquifers in western Kansas and other western states have developed to the extent that they might be considered infinite in areal extent.

For the purpose of this paper, well fields 20 miles in diameter, or greater, are considered as infinite in areal extent. However, pumping from well fields of about 10 miles in diameter probably will result in approximately similar shapes of the water surface beneath the field.

An edge of the field is shown for demonstration purposes, but the aquifer is assumed to extend an infinite distance beyond this edge.

Withdrawals generally begin from one or two isolated wells in a well field. If economic conditions are favorable (and hydrologic conditions are assumed to be favorable in an infinite aquifer), wells will be constructed on lands adjacent to these well sites and a nucleus of a well field will be developed. This type of growth will generally continue until a nearly infinite well field (as defined above) results.

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Under each isolated well and under each cluster of wells, cones of depression develop on the water-level or piezometric surface, and a dimpled surface results. At the end of the development period and after wells have been established on nearly every section of land, the cones of depression will have coalesced, and thereafter near-equal water-level changes per unit of time can be expected over large areas. The additional drawdowns expected for this second period of pumpage (after full development) are calculated in this paper.

Most calculations were made by an IBM 7040 digital computer at the University of Kansas Computation Center. Thanks are extended to the staff of the Computer Application Section of the U.S. Geological Survey, and especially to Brent Lowell (of this Division), for their help in programming.

FORMULATION AND ASSUMPTIONS

The calculated drawdowns shown in this paper result from computations using the Theis formula, and except as stated below, the assumed requirements are the same as those stated in Water Supply Paper 887. The recharge from precipitation in western Kansas has been calculated to be less than 0.5 inch per year (Fader, *et al.,* 1964). This was less than 5 percent of the annual pumping rate in 1960. Because 0.5 inch is only 4.5 percent of the pumping rate per unit area, recharge from precipitation is considered negligible, and nearly all water is considered to be from storage.

There are enough fine-grained materials in the upper part of the Pliocene and Pleistocene aquifers in western Kansas that water levels are under artesian conditions during early periods of pumping. However, after several years of pumping in one area (Grant and Stanton Counties), an apparent storage coefficient of 0.2 was noted (Fader, *et al.,* 1964). Because there is still some doubt as to the accuracy of the total pumpage as compared with the dewatered volumes, it is believed that 0.15 is a more representative figure for this area.

The purpose of this paper is to show the shape of the water surface under a well field and the relative magnitude of the drawdown of water levels in wells; therefore, corrections needed to compensate for the reduction of saturated thickness were not made.

Irrigation wells in western Kansas are pumped for about 130 days per year, on a seasonal basis, at an average rate of about 1,000 gpm (gallons per minute) per well. If all the wells started and stopped pumping at the same time and were pumped steadily for the same 130-day period each year, the drawdown computations could be made by the cyclic method given by Theis (1963) and by Brown (1963). Both summer and winter irrigation are practiced, and pumping is discontinuous, but the total number of days averages 130 per year. Because of the discontinuous pumping, $\frac{1{,}000\ gpm \times 130\ days}{365\ days} = 356\ gpm$, an average rate, was used for this paper, and in the computations continuous rather than cyclic pumping was assumed.

A time of 10 years was chosen arbitrarily as a convenient time for a long-term computation. Also, this might be considered adequate time to amortize the cost of an irrigation system.

The formulas (Theis, 1935) used for the calculations are:

$$u = \frac{1.87\ r^2 S}{Tt} \tag{1}$$

$$W(u) = -0.577216 - Log_e\ u + u - \frac{u^2}{2 \cdot 2!} + \frac{u^3}{3 \cdot 3!} - \frac{u^4}{4 \cdot 4!} + \cdot \cdot \cdot \frac{u^n}{n \cdot n!} \tag{2}$$

$$s = \frac{114.6 Q\ W(u)}{T} \tag{3}$$

where: r is distance from the pumped well, in feet (varied as shown in tables),

S is the coefficient of storage and is nondimensional,

T is the coefficient of transmissibility, in gpd/ft,

Q is discharge rate of each well, in gpm,

t is time since pumping started, in days,

s is the drawdown, in feet, and

$W(u)$ is the well function of u, computed by digital computer and terminated when the term $\frac{u^n}{n \cdot n!}$ in formula (2) became less than 0.000001.

COMPUTATIONS

A tabulation of drawdowns, resulting from the pumping of other wells, at a point (or well) in the interior of a theoretical well field is given in Tables 1 and 2. In the heading of each table are given: the point being observed (node is at a well; one-half node is the point half the distance between two wells); the coefficients of transmissibility and storage, and the pumping rate. (Except as noted, all calculations were made by an IBM 7040 digital computer.)

Column 1 shows the distance (R), in feet, from the well effecting drawdown to the well or point of observation (P-point on the line X-X', Fig. 1,A).

Column 2 shows values of u computed by formula (1) as R was varied as shown in column 1.

Column 3 shows values of $W(u)$ computed by formula (2) as u was varied; and column 4 shows values of s computed by formula (3) as $W(u)$ was varied.

Column 5 is the number of pumping wells, at a distance R, that cause drawdown at the point being observed. For example: if a circle with a radius of 5,280 feet is drawn around the P-point on Figure 1,A, the circle will be drawn through four wells. Thus, these four wells will

TABLE 1.--COMPUTATIONS OF DRAWDOWN AT A WELL IN THE INTERIOR OF A THEORETICAL WELL FIELD.

T= 50000. S= 0.1500000 Q= 356. TM = 10 YEARS

DISTANCE R (FEET) (1)	U (2)	W(U) (3)	DRAWDOWN PER WELL (FEET) (4)	NO OF WELLS CAUSING DRAWDOWN (5)	TOTAL DRAWDOWN CAUSED BY WELLS AT R (FEET) (6)
1	153699E-08	19.7162	16.0875	1	16.0875
5280	428487E-01	2.61526	2.13392	4	8.53570
7390	839380E-01	1.98267	1.61776	4	6.47105
10560	0.171395	1.35089	1.10226	4	4.40905
11700	0.210398	1.18137	0.963939	8	7.71151
14800	0.336661	0.821781	0.670533	4	2.68213
15800	0.383693	0.730510	0.596061	4	2.38424
16700	0.428650	0.656660	0.535803	8	4.28642
19100	0.560708	0.492298	0.401692	8	3.21353
21120	0.685580	0.384180	0.313473	4	1.25389
21700	0.723751	0.357393	0.291615	8	2.33292
22300	0.764328	0.331465	0.270460	4	1.08184
23600	0.856040	0.281002	0.229284	8	1.83427
26400	1.07122	0.194945	0.159066	12	1.90879
26900	1.11218	0.182348	0.148787	8	1.19030
28510	1.24929	0.146575	0.119598	8	0.956786
29880	1.37224	0.121226	989150E-01	4	0.395660
30780	1.45615	0.106787	871329E-01	8	0.697063
31700	1.54450	936380E-01	764042E-01	4	0.305617
32100	1.58373	883890E-01	721212E-01	8	0.576969
33370	1.71152	734275E-01	599133E-01	8	0.479307
33840	1.76007	684959E-01	558894E-01	8	0.447115
35270	1.91197	552703E-01	450979E-01	8	0.360783
36960	2.09959	426381E-01	347907E-01	4	0.139163
37380	2.14758	399347E-01	325848E-01	8	0.260678
38120	2.23345	355465E-01	290043E-01	8	0.232034
38440	2.27110	337882E-01	275695E-01	8	0.220556
40230	2.48754	253273E-01	206658E-01	8	0.165327
41300	2.62162	212413E-01	173319E-01	8	0.138655
42300	2.75011	179760E-01	146675E-01	4	586702E-01
42600	2.78926	170899E-01	139446E-01	16	0.223113
43600	2.92175	144171E-01	117636E-01	8	941091E-01
45100	3.12625	111192E-01	907270E-02	20	0.181454
47520	3.47075	722706E-02	589694E-02	12	707632E-01
53000	4.31739	257831E-02	210378E-02	8	168302E-01
				TOTAL =	71.4038

cause increments of drawdown, or 4s as shown in column 6. This same process was repeated for the various radii *R*, until negligible drawdown was noted in column 6. The number of wells on each circle of radius *R* was counted manually.

The summation of the drawdowns in column 6 is the total drawdown at the P-point or observation point caused by all of the wells listed in column 5. For the conditions listed in the heading of Table 1, the total drawdown at P-point is 71 feet. Table 2 shows the results of a

TABLE 2.--COMPUTATIONS OF DRAWDOWN AT A POINT 1/2 DISTANCE BETWEEN WELLS IN THE INTERIOR OF A THEORETICAL WELL FIELD.

T= 50000. S= 0.1500000 Q= 356. TM = 10 YEARS

DISTANCE R (FEET) (1)	U (2)	W(U) (3)	DRAWDOWN PER WELL (FEET) (4)	NO OF WELLS CAUSING DRAWDOWN (5)	TOTAL DRAWDOWN CAUSED BY WELLS AT R (FEET) (6)
2640	107122E-01	3.96984	3.23920	2	6.47840
5870	529598E-01	2.41327	1.96912	4	7.87646
7920	964096E-01	1.85607	1.51446	2	3.02892
9520	0.139298	1.52852	1.24720	4	4.98879
10820	0.179939	1.31008	1.06896	4	4.27585
13210	0.268210	0.990014	0.807804	6	4.84682
14200	0.309918	0.881700	0.719425	4	2.87770
16100	0.398402	0.705065	0.575299	4	2.30120
17000	0.444189	0.633643	0.517022	4	2.06809
17600	0.476097	0.589846	0.481286	4	1.92514
18500	0.526034	0.529382	0.431950	2	0.863900
19100	0.560708	0.492298	0.401692	4	1.60677
20600	0.652236	0.409726	0.334317	4	1.33727
21200	0.690783	0.380381	0.310373	8	2.48298
22300	0.764328	0.331465	0.270460	4	1.08184
23700	0.863310	0.277422	0.226363	2	0.452726
24300	0.907575	0.256789	0.209527	4	0.838109
24900	0.952947	0.237541	0.193822	4	0.775289
25900	1.03103	0.208315	0.169975	4	0.679900
26700	1.09570	0.187297	0.152825	4	0.611300
27600	1.17081	0.165934	0.135394	4	0.541576
28000	1.20500	0.157160	0.128235	4	0.512938
28500	1.24842	0.146776	0.119762	4	0.479050
29100	1.30154	0.135129	0.110259	2	0.220518
29500	1.33756	0.127831	0.104304	4	0.417216
30900	1.46753	0.104983	856609E-01	4	0.342644
31700	1.54450	936380E-01	764042E-01	8	0.611233
32200	1.59361	871187E-01	710847E-01	4	0.284339
33000	1.67378	775318E-01	632623E-01	8	0.506098
34300	1.80825	639601E-01	521883E-01	10	0.521883
34600	1.84002	611495E-01	498951E-01	4	0.199580
35000	1.88281	575753E-01	469786E-01	4	0.187915
35600	1.94791	525663E-01	428916E-01	8	0.343133
36900	2.09278	430375E-01	351166E-01	8	0.280933
38900	2.32578	313986E-01	256198E-01	8	0.204958
39600	2.41024	280556E-01	228920E-01	20	0.457841
41400	2.63433	208919E-01	170468E-01	8	0.136374
42700	2.80237	168036E-01	137109E-01	24	0.329062
44600	3.05731	121325E-01	989950E-02	14	0.138593
46300	3.29482	899669E-02	734087E-02	16	0.117454
47700	3.49709	699501E-02	570759E-02	20	0.114152
				TOTAL =	58.3449

similar process used to calculate a total drawdown of 58 feet for a point half the distance between the wells. The same process was used to compute drawdown for different values of *T* and *S* as shown in Table 3.

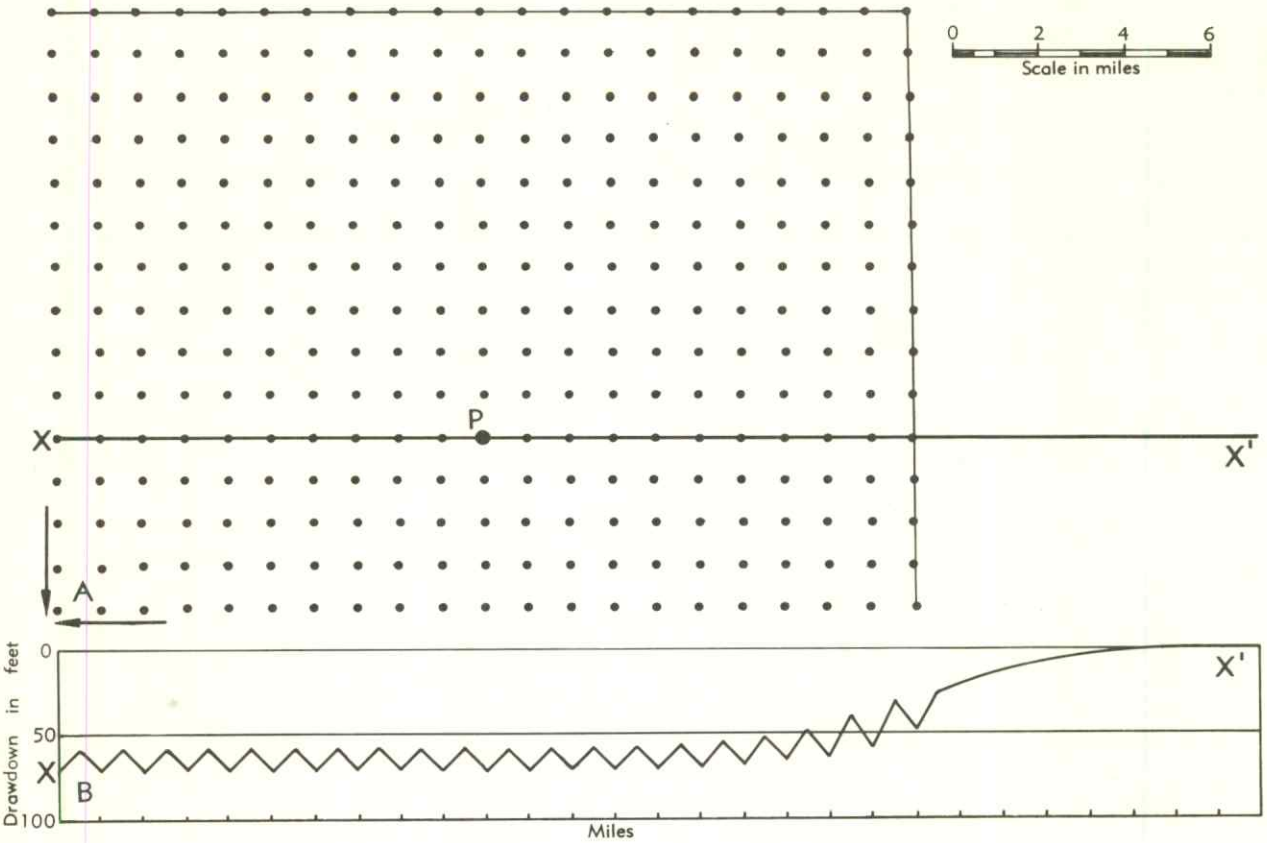

Figure 1.--A, Array of irrigation wells in an assumed infinite well field. Assumed: S = 0.15, T = 50,000 gpd/ft., wells pumped steadily for 10 years at 356 gpm, and no well loss. Arrows indicate direction in which well field assumed infinite. B, Computed drawdown profile through line X-X' (shown on array above).

Table 3.--Theoretical drawdown at points in the interior of an infinite well field. Q = 356 gpm, t = 10 years.

Transmissibility, T, gpd/ft	Storage, S	Drawdown at well, feet	Drawdown at one-half node, feet
50,000	0.05	170	156
50,000	.10	99	86
50,000	.15	71	58
50,000	.20	57	44
100,000	.05	134	128
100,000	.10	84	78
100,000	.15	62	56
100,000	.20	50	43

As shown in Table 1, column 6, the resultant drawdown is 0.016 foot, or almost negligible, at 10 miles from a pumping well. After full development of such a well field, the total drawdown at observation points 10 miles and less inside the well field will be less than the drawdown at points near the center of the nearly infinite well field. Total drawdown at points outside the well field will be progressively less as the points of observation are moved outward and will be essentially negligible at points about 8 miles outside the well field, as shown by profile X-X' on Figure 1,B. Total drawdown at each observation point was computed by counting the number of wells causing drawdown at the observation points at half-mile intervals along profile X-X' from 10 miles outside to 10 miles inside the edge of the well field. The calculations and results were tabulated (not shown in this report) in a manner similar to Tables 1 and 2, and the drawdowns are shown on the profile (Fig. 1,B).

APPLICATION AND DISCUSSION

If we assume that a well field has been increased from one or two wells to the distribution of wells shown in Figure 1,A, and if water levels in the wells were measured in the field, a profile of the water surface would be as shown in the profile in Figure 2,A. Using the principle of superimposition, the water levels shown in Figure 2,A are lowered by the amount shown in the profile in Figure 1,B, resulting in the profile in Figure 2,B which is the approximate water-level profile 10 years after full development of the well field.

It is noted that pumping from each of the wells is intermittent but averages 356 gpm for the year. If the wells are idle for part of the year, the areas of higher water levels between the wells tend to equalize with the areas of lower water levels under the wells, and the smoothed piezometric surface (solid line, Fig. 3) probably would be more nearly correct.

The water-level profile should be still flatter because most of the water pumped can originate only from storage in the interior of the well field. Consider the point at well *b* in the

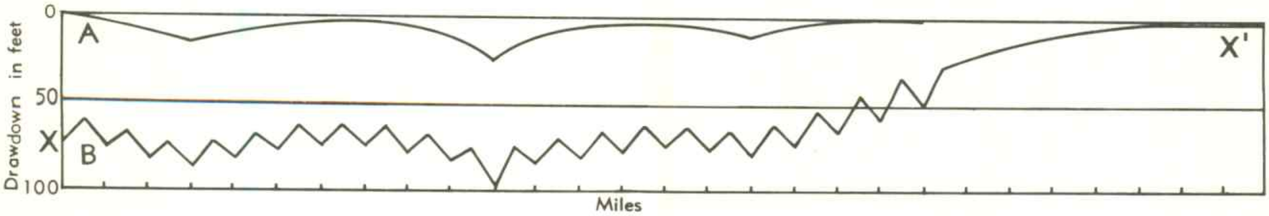

Figure 2.--A, Assumed measured profile at the end of well-field development (period from no wells to full or infinite development). B, Drawdown profile after an additional 10 years of pumping after full well-field development (summation of drawdown in Figure 1,B and Figure 2,A).

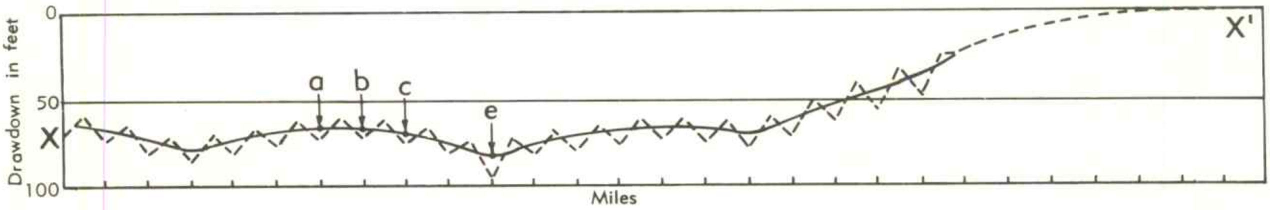

Figure 3.--Profile along X-X' depicting a theoretical water surface beneath a well field; dashed line is the profile from Figure 2,B and solid line shows adjustment due to unsteady pumping.

profile (Fig. 3). The hydraulic gradient between points *a* and *b* is zero, thus no appreciable amount of water can flow from *b* to *a* . The net hydraulic gradient between *b* and *c* (Fig. 3) is toward *c* , indicating that water is flowing toward *c* . Also, water would flow toward point *e* from both directions along the profile X-X' or from all directions if the profile is rotated 180° through space to represent three dimensions. Thus, because of the movement of water within the aquifer, it effectively would be removed from storage near well *b* at a greater rate than 356 gpm and at a lesser rate from storage near well *e*. This situation would adjust in time and produce a still flatter profile in the interior of the well field.

If the water-level profile beneath a well field is essentially flat, there is no effective hydraulic gradient from adjacent areas, and virtually all water pumped must be removed from storage in the vicinity of the well. If a well is located on every section of land as shown, and each well pumped at an equal rate, the area from which each well would draw from storage would be 1 square mile. Assuming a specific yield of 0.15 and a pumping rate of 356 gpm (averaged over the year), the drawdown should be about 6 feet per year in the interior of the well field after such a well field is fully developed.

CONCLUSIONS

The graphs shown are indications of expected drawdowns under the conditions stated. Most well fields do not develop as homogeneous units, nor do all wells pump at the same time or rate. Therefore, the familiar "cones" of depression will develop as shown in the profile in Figure 2,A, but the annual net decline of water level should be nearly the same over a large area after the well field has fully developed.

As pumping continues, the water surface beneath the well field should become progressively flatter and most of the water pumped from a well will then be derived from storage in an area near the well itself.

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