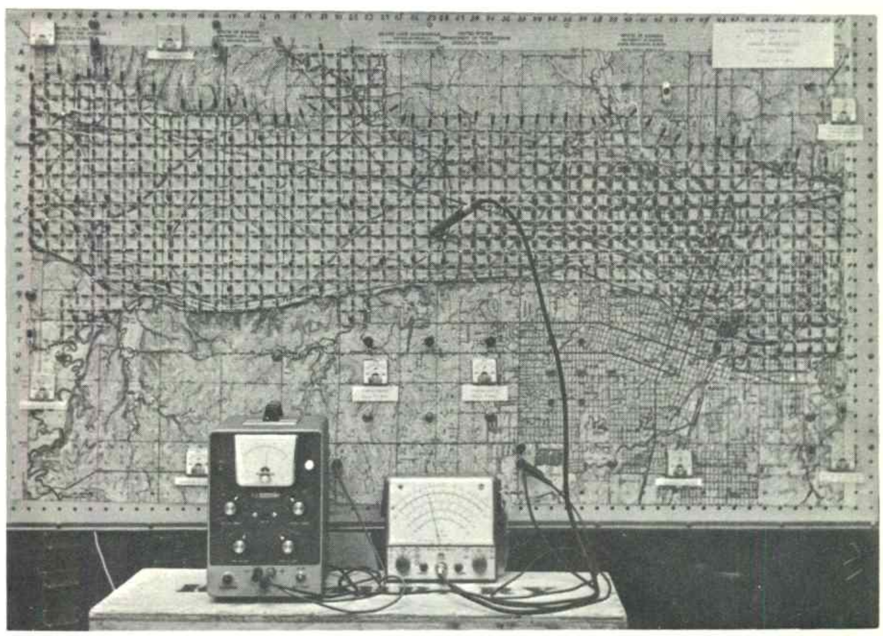
Electronic Simulation of Ground-Water Hydrology in the Kansas River Valley near Topeka, Kansas

By John D. Winslow and Carl E. Nuzman



State Geological Survey of Kansas
The University of Kansas
Lawrence, Kansas

1966

Special Distribution Publication 29

ELECTRONIC SIMULATION OF GROUND-WATER HYDROLOGY IN THE KANSAS RIVER VALLEY NEAR TOPEKA, KANSAS

By

John D. Winslow
Carl E. Nuzman

Special Distribution Publication No. 29

Prepared as part of the cooperative ground-water program in Kansas conducted by the United States Geological Survey, the State Geological Survey of Kansas, the Division of Water Resources of the Kansas State Board of Agriculture, and the Environmental Health Services of the Kansas State Department of Health.

State Geological Survey

The University of Kansas, Lawrence, Kansas

1966

CONTENTS

ILLUSTRATIONS

TABLE

ELECTRONIC SIMULATION OF GROUND-WATER HYDROLOGY IN THE KANSAS RIVER VALLEY NEAR TOPEKA, KANSAS

By

John D. Winslow* and Carl E. Nuzman+

ABSTRACT

The simulation of a physical hydrologic system by an electrical analog model is a very useful technique of investigation being applied to ground-water studies in Kansas. A model constructed on the basis of adequate data, used in conjunction with suitable analyzing equipment, can provide a useful method of quantitative analysis of the hydrology of an area.

The model of the Kansas River valley near Topeka described in this report illustrates the theoretical approach, the method of construction, and the application to ground-water investigation of steady-state electrical analog models. An electrical analogy provides an understanding of the physical system that is difficult to achieve by other means. Also it permits evaluation of changes induced in the hydrologic system of an area by man, such as pumping ground water.

The steady-state model of the Kansas River valley near Topeka consists of a network of resistors mounted on a base map in a scaled space relationship. The conductivity of the resistors comprising the network are scaled to the spatial transmissibility of the alluvium in the valley. The resistance to the electron flow across the modeled area is directly proportional to the (frictional) resistance of the flow of ground water in saturated unconsolidated materials.

Perimeter hydrologic boundary conditions in the Kansas Valley were uniquely determined by this electrical analogy. In the analysis of equilibrium conditions in 1963, inflow into the modeled area as underflow through the aquifer was 1.6 mgd (million gallons per day), inflow to the aquifer from glacial till along the north side of the valley was 2.3 mgd, infiltration into the aquifer from Soldier Creek was 1.2 mgd, and the average annual vertical infiltration (originating from precipitation) was 6.8 mgd (equivalent to 3.2 inches of water per year). Outflow from the modeled area as underflow through the aquifer was 0.8 mgd, the gain in flow of the Kansas River was 6.6 mgd, and the discharge from industrial wells was 4.5 mgd.

---

* Geologist, U.S. Geological Survey, Lawrence.
+ Engineer, Division of Water Resources, Kansas State Board of Agriculture, Topeka.

INTRODUCTION

The purpose of this report is two-fold: (1) to describe the steady-state electrical simulation of the ground-water hydrology in the Kansas River valley near Topeka, Kansas; and (2) to illustrate the application of steady-state electrical analog computer models to hydrologic investigation. Because this report is the first on the subject in the publications of the State Geological Survey of Kansas, theoretical and descriptive information are included to provide the basis for the application of electronic analog computer models to ground-water investigations in Kansas. Subsequent reports will describe only changes in the method of application or extension of the theoretical development of the subject.

Simulation of a hydrologic system by a steady-state electrical analog model (hereafter referred to as the model) is a very useful technique because it not only permits an understanding of the system not readily achieved by other means of investigation, but it also permits prediction of the effects of projected changes in the system that may be imposed by man. The model and suitable analyzing equipment comprise an analog computer which can be programed to represent an aquifer and which can provide solutions to quantitative problems. The model consists of a network of resistors superposed on a base map of the aquifer area (photograph on front cover). The resistance of individual resistors is scaled so as to impede the flow of electrical currents across any part of the model in a manner analogous to the resistance of the aquifer to the flow of water across a similar part of the system. Voltages and currents, generated by a power supply and measured by a vacuum-tube voltmeter, are applied to the model and adjusted, in various parts of the model, to represent the hydraulic head and rate of flow in the system.

The resistor network has superseded conducting paper, such as that used by Mack (1958) for models in Kansas, because the resistor network is more flexible in application and provides better definition of the system.

Much of the theoretical information and some of the methodology in this report were provided by personal communication from H. E. Skibitzke, Mathematician in Charge of the U.S. Geological Survey Water Resources Division research facilities at Phoenix, Arizona, and from B. J. Bermes and E. P. Patten, U.S. Geological Survey. Acknowledgment is due personnel of the Goodyear Tire and Rubber Company, Topeka, Kansas, for co-operation and assistance in conducting aquifer tests and installing permanent observation wells on company property.

Models are being used in Kansas as part of the co-operative program of ground-water investigations conducted by the Ground-Water Branch of the U.S. Geological Survey, the State Geological Survey of Kansas, the Division of Water Resources of the Kansas State Board of Agriculture, the Environmental Health Services of the Kansas State Department of Health, and the City of Wichita.

GEOLOGIC SITUATION

The area described in this report comprises 45 square miles of the Kansas River valley between Silver Lake and the eastern limits of Topeka, near the municipal airport (Fig. 1). A significant part of the industrial area and of the fertile agricultural lands near Topeka are within this area.

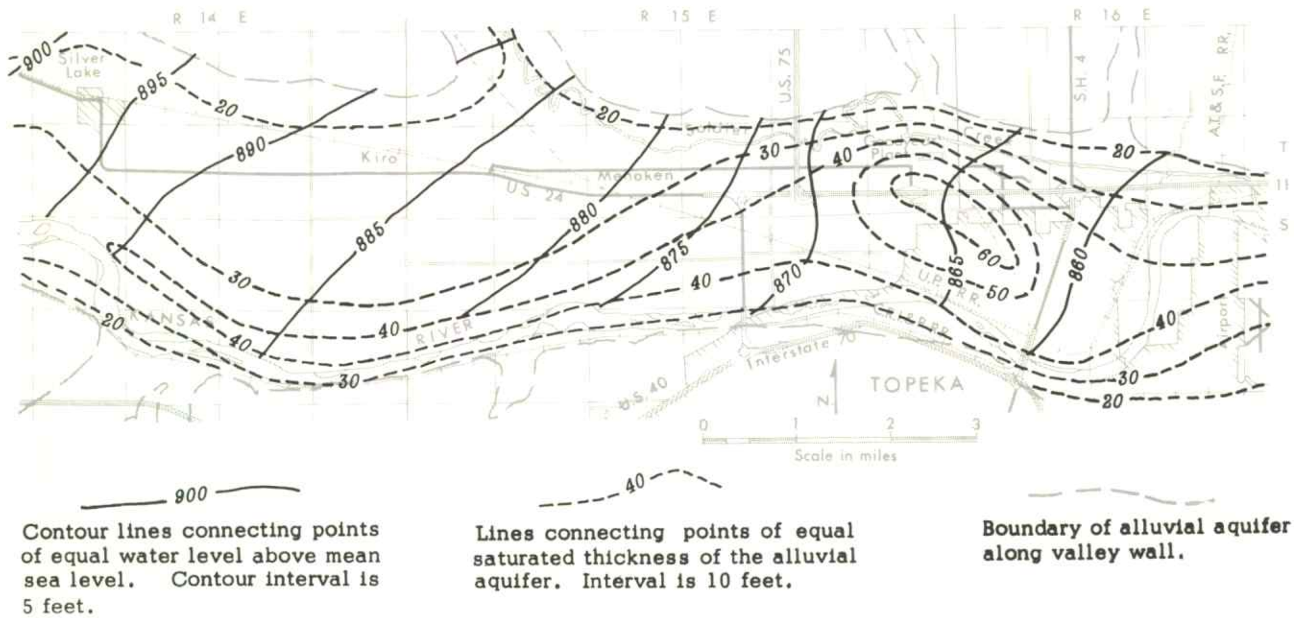


Figure 1.--Saturated thickness of the alluvial aquifer and ground-water levels in 1950.

The geology and ground-water resources of the Kansas River valley area described in this report have been studied and described by S. N. Davis and W. A. Carlson (1952) and H. V. Beck (1959). In general, these reports furnished sufficient hydrologic data to program the model network. However, some supplementary data were obtained for the present (1965) study.

The topography of the area modeled consists of two basic elements: (1) the flood plain of the Kansas River, which is about $2\frac{1}{2}$ miles wide and which trends from west to east across the area with a general eastward slope of about 3 feet per mile; and (2) the low dissected hills that bound the flood plain on the north. The present channel of the Kansas River is along the south side of the flood plain; however, scars on the surface of the flood plain mark relict channels.

The valley alluvium that comprises the aquifer consists principally of sand, but also contains lenses of both coarser and finer material. Generally, the maximum saturated thickness of this aquifer is about 40 feet (Fig. 1); however, in a linear depression in the bedrock floor of the valley near the Goodyear Company plant, the saturated thickness of the aquifer is about 60 feet. The bedrock in the area consists of limestone, shale, siltstone, and sandstone (Davis and Carlson, 1952; Beck, 1959). These rocks are relatively impermeable as compared to the alluvial material in the Kansas River valley, and they were considered to be impermeable in the hydrologic analysis of the model. The dissected hills that border the flood plain consist of bedrock mantled in most places by till and other glacial materials that are less permeable than the valley alluvium but more permeable than the bedrock. These glacial materials contribute a small amount of ground water to the alluvium in the valley and also contribute to the flow of small streams tributary to the Kansas River.

HYDROLOGIC SITUATION

Climate

The Topeka area has a humid continental climate. Normally, more than 70 percent of the annual precipitation falls during the growing season, April through September. Precipitation during this period is usually from thunderstorms (high intensity rainfall of brief duration) in the evening and early morning hours. The mean hourly wind speed is 11.2 miles per hour, and the sun shines on Topeka 60 percent of the daylight hours.

Climatic data are collected at the U.S. Weather Bureau station at the Topeka Municipal Airport on the east edge of the area modeled and are tabulated in the Climatological Data bulletins of the U.S. Weather Bureau (1964). Mean annual precipitation, for the period of record, 1931-1960, was 32.36 inches. The wettest year of record was 1951 with 48.60 inches, and the driest year was 1963 with 19.07 inches. For the period of record 1931-1960, June was the wettest month, with a mean monthly precipitation of 4.51 inches, and January was the driest month, with a mean monthly precipitation of 1.02 inches. The maximum precipitation in any month of record was 12.02 inches in July 1950, and the minimum precipitation in any month was a trace (less than 0.01 inch) in November 1954.

The mean annual temperature was 54.9°F for the period of record 1931-1960. January was the coldest month with a mean monthly temperature of 28.8°F, and July was the warmest month with a mean monthly temperature of 79.9°F.

Surface Water

The Kansas River, which flows in an easterly direction, is the principal stream in the area modeled. The average discharge of the river at Topeka for 43 years of record (1920-1962) is 5,287 cfs (cubic feet per second). The average annual discharge of the river is 3,828,000 acre-feet for the same period of record. Extremes vary considerably from the average.

Soldier Creek is hydrologically the most important of the tributary streams in the modeled area because it is a major source of recharge to the alluvial aquifer. After entering the Kansas Valley from the north, Soldier Creek flows eastward for several miles across permeable materials on the northern side of the flood plain to its confluence with the Kansas River northeast of Topeka. Soldier Creek has a drainage area of 268 square miles. The average discharge of Soldier Creek, measured at a gaging station near Topeka, is 126 cfs for 31 years of record and the average annual discharge is 91,220 acre-feet for the same period of record. There have been periods of no flow in the stream at times in 1931, 1935 to 1940, and during each of the years 1953 through 1957.

A special investigation of the seepage characteristics of Soldier Creek was made by the Surface-Water Branch of the U.S. Geological Survey in co-operation with State water agencies on November 7, 1963, during a period of below-normal runoff. The net loss from Soldier Creek at that time within the area modeled was 1.91 cfs, or 1.21 mgd, which was approximately equal to the flow of the stream at the point where it entered the area modeled. The flow below the Goodyear Company plant was virtually the same as the amount of process cooling water discharged to the stream at the plant.

Ground Water

North of the Kansas River in the area of investigation, 8 industrial wells, 2 municipal wells, 44 irrigation wells, and some domestic wells were withdrawing water in 1964 from the valley alluvium. Of these, the 8 wells of the Goodyear industrial complex, which pumped continuously at an average rate of 4.5 mgd (based on 1963 data), are the most significant hydrologically. Irrigation wells are reportedly pumped at an average rate of 1.8 mgd, based on annual use. However, irrigation pumpage is intermittent and usually is concentrated within a 6-week period beginning in the later part of July. Because of the good distribution of irrigation wells over the area and the short period of pumping, the recovery of water levels from the effect of irrigation pumpage is essentially complete by November of each year. The remaining wells north of the Kansas River pump less than 0.1 mgd, based on annual use.

On the south side of the Kansas River the few wells that pump from the alluvial material in the valley are near the bank of the river. The low volume of intermittent pumping from these wells has no apparent effect on the water level across the river. A good hydraulic connection does exist between the valley alluvium and the river. The water level in observation wells along the river responds quickly to small changes in river stage.

A water-level contour map constructed from field data is needed for the application of the model technique to the quantitative solution of a ground-water flow problem.

The 1950 water-level contour map (Fig. 1) was synthesized from water-level measurements made in 35 wells over the 3-year period 1950 through 1953. Most of the measurements were made during a period of above-normal annual precipitation and prior to the maximum flood of record on the Kansas River, in July 1951. The validity of the map suffers from the relatively long period of time over which the water-level data were collected and the abnormal hydrologic events within the period. It should be noted that the up-valley deflection of the 865-foot and 870-foot contour lines in Figure 1 are the result of pumping at the Goodyear Company plant which is estimated to have averaged about 1.5 mgd in 1951.

Additional water-level data were collected in November 1963 (a year of below-normal precipitation) when both water levels and apparent hydraulic gradients in the modeled area were lower than in the period 1950-1953. The November 1963 measurements were made in 29 wells within a 2-day period. The water-level contours plotted from the 1963 data (Fig. 2) delineate

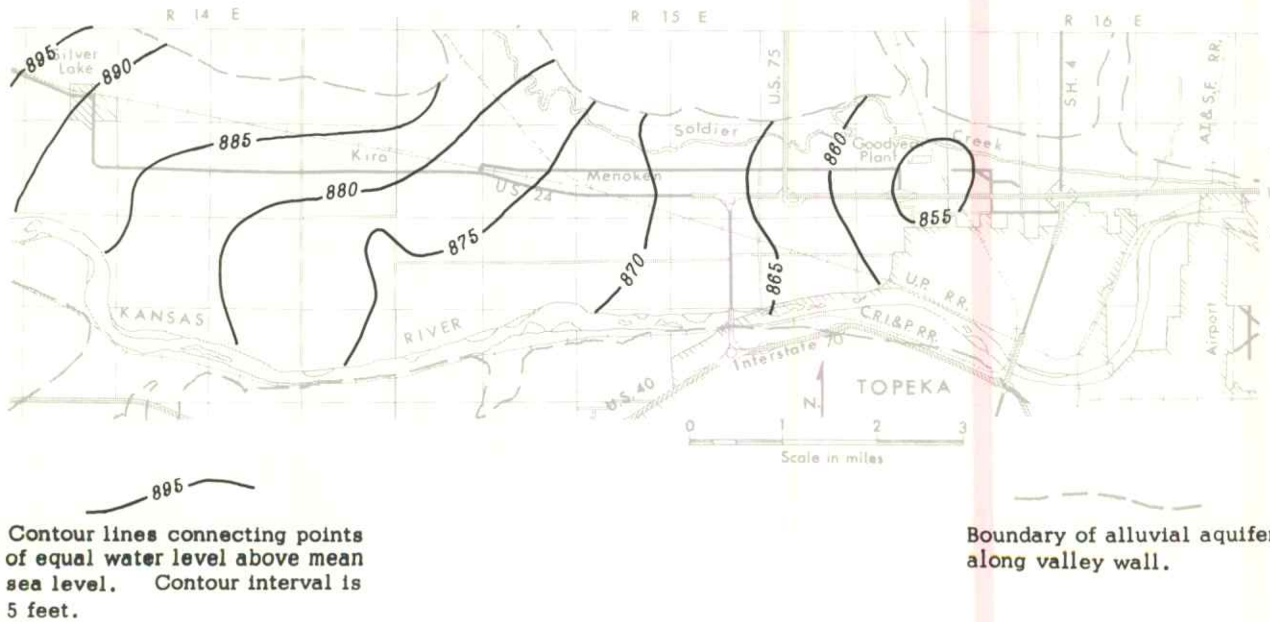


Figure 2.--Ground-water level in the alluvial aquifer in November 1963.

the hydraulic potential distribution for the area of investigation when nearly equilibrium conditions existed for the annual average pumping rate of 4.5 mgd at the Goodyear Company plant in 1963.

Annual and seasonal variations of water levels result from climatic changes. Hydrographs showing changes of water levels in wells 11-15-13dac, 11-15-16dca, and 11-15-22cac, shown in Figure 3, are representative of water-level fluctuations in the area.

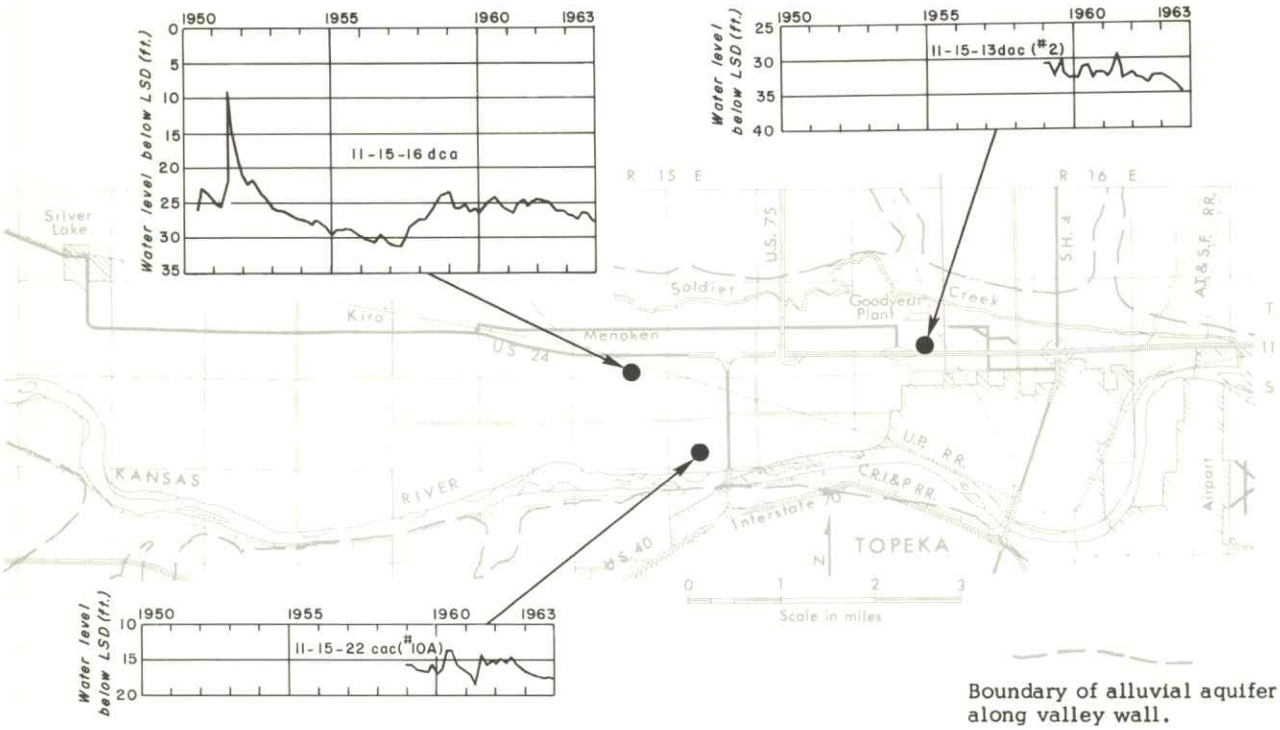


Figure 3.--Hydrographs of water levels below land-surface datum (LSD) in selected wells in the alluvial aquifer.

The T and S (coefficients of transmissibility and storage, respectively) of the alluvial aquifer were determined from aquifer tests conducted on wells at the Goodyear Company plant well field in 1958 and 1960. The average T of the alluvial aquifer was determined to be 370,000 gpd/ft (gallons per day per foot width of aquifer) except in the thicker part of the aquifer in the vicinity of the well field where the average T is about 700,000 gpd/ft. The average P (coefficient of permeability) was assumed to be about 5,000 gpd/ft^2 (gallons per day per square foot) except in the vicinity of the well field where the average P was determined to be about 10,000 gpd/ft^2. The value of S, based on test results, was about 0.04, indicative of a partially confined aquifer.

Developments in electronic analog computers in the last several years have made possible the evaluation of complex hydrologic relationships that previously were either too difficult or too time consuming by other methods. Analog computers provide a method of handling complex mathematical expressions in differential form and make unnecessary many of the simplifying assumptions used by some earlier investigators to obtain neat algebraic expressions. In this report emphasis is given to the study of steady flow of ground water by resistor networks.

In order to develop the analogy between ground-water flow and electron flow, it is essential to define the basic relationship between them. The law of conservation of energy and matter underlies theory in most areas of physics. These relationships may be identified for each system for the analogy.

The term "steady-state" implies that excitation of the resistor network or of the physical system is constant at all times. In other words, sufficient time has elapsed since any change occurred in excitation so that equilibrium conditions have become established. Thus the steady-state implies zero rate of change of all dependent variables at all points in the flow system. This is expressed mathematically for head (h) with respect to time (t) in equation 1:

$$\frac{dh}{dt} = 0. \qquad (1)$$

Darcy's law is an expression accounting for the dissipation of energy in ground-water flow, and may be expressed as:

$$Q = PIA \qquad (2)$$

where Q = flow in gallons per day,

P = the coefficient of permeability in gallons per day per square foot,

I = the hydraulic gradient in foot per foot, and

A = the cross-sectional area of aquifer through which the water moves in square feet.

Darcy's equation, rewritten in vectorial notation form is:

$$\vec{Q} = -P \; grad \; h. \qquad (3)$$

Dividing both sides by the area, equation 3 becomes:

$$\vec{V} = -P \frac{\partial h}{\partial s} (x,y,z), \qquad (4)$$

where $\vec{V}$ = velocity with direction,

P = permeability (assumed to be constant in space),

h = head, and

s = a resultant distance in the x, y, z coordinate system.

If the fluid is incompressible and with irrotational motion, the continuity equation for fluid flow is:

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = 0. \tag{5}$$

The substitution of equation 4 into equation 5 and cancelling constant terms results in:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0. \tag{6}$$

This is the Laplace equation and it applies to fluid flow when: (1) liquid particles are forced through a medium such as an aquifer containing very small pore channels, (2) the forces upon the fluid due to viscous friction are much larger than inertial forces, (3) the fluid is assumed to be incompressible with irrotational motion, and (4) the system is purely dissipative.

The energy dissipation of the electrical system is expressed by Ohm's law as:

$$I = \frac{1}{R} V, \tag{7}$$

where I = current flow in amperes,

R = electrical resistance in ohms, and

V = electrical potential in volts.

This equation rewritten in vectorial notation becomes:

$$\vec{I} = -\frac{1}{R}\frac{\partial V}{\partial s}(x,y,z). \tag{8}$$

Similarly, then for a steady-state electric current, the continuity equation is:

$$\frac{\partial I_x}{\partial x} + \frac{\partial I_y}{\partial y} + \frac{\partial I_z}{\partial z} = 0. \tag{9}$$

The substitution of equation 8 into equation 9 and cancelling constant terms results in:

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0. \tag{10}$$

This is the Laplace differential equation of continuity that underlies the mathematical formulation of potential theory and is useful in describing steady-state conditions in both systems. A term by term comparison of Darcy's equation (equation 4) with Ohm's law (equation 8) and the continuity equations (6 and 10) clearly shows the analogy is complete between the two systems.

The analogy between the flow of ground water and the flow of electricity can be continued to show the application of the general diffusion equation (a parabolic equation) to both for the transient or non-steady state in a three-dimensional flow system. The partial differential equation describing the unsteady confined flow of water in a uniform porous medium as modified from Jacob (1950, p. 333-334) is:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = \frac{S'}{P} \frac{\partial h}{\partial t}, \qquad (11)$$

where S' = the storage coefficient per unit volume of the medium and the remaining terms are as previously defined.

The equivalent equation for a three-dimensional diffusion field in electricity is given by Karplus (1958, p. 33):

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = RC \frac{\partial V}{\partial t}. \qquad (12)$$

The analogy is modified from Skibitzke (1961, p. 352) and expressed in finite-difference form for fluid flow as:

$$mP \left(\sum_{1}^{6} h_i - 6h_o\right) \approx L^3 S \frac{\partial h}{\partial t}, \qquad (13)$$

where m = the contributing thickness of the aquifer ($m = L$ in the vertical direction),

L^3 = the volume of material,

S = an effective specific yield instantaneously released from storage with a change in head,

h_o = the hydraulic head at the point of interest, and

h_i = hydraulic head at a distance L away from the point where h_o is being determined, and the other terms are as previously defined.

The finite-difference form of the differential equation for a node is:

$$Y \left(\sum_{1}^{6} V_i - 6V_o\right) = C \frac{\partial V}{\partial t}, \qquad (14)$$

where Y = electrical conductance, the equivalent of $1/R$, and directly analogous to T, transmissibility, the product of mP, and other terms are as previously defined.

Each of the terms in equations 13 and 14, or equations 11 and 12, are comparable and can be taken as directly analogous to each other. This analogy was further illustrated by Patten (1965).

Further check of the consistency of the equations can be made by substituting equation 1, the definition of steady-state flow, into equation 11, the fluid-flow non-steady state equation, which yields equation 6, the Laplace equation of continuity.

The quantitative relationship between hydraulic units and electrical units can be determined with proportionality constants. These constants (K factors) were first defined by B. J. Bermes (personal communication, 1962) as follows:

K_1 = gallons of water per coulomb of electrons;

K_2 = feet of head per volt of potential;

K_3 = gallons per day per ampere;

K_4 = days of real time per second.

By definition:

$$K_1 = K_3 \times K_4. \tag{15}$$

Initial selection of the value of K_2, potential level, is independent of the remaining scale constants. However, once K_2 is selected, it becomes part of and is incorporated in the computation of resistance and capacitor values. These constants and subsequent formulas were published by Walton and Prickett in 1963.

By combining Darcy's law and Ohm's law with the use of the scale constants and solving for resistance (R) in terms of transmissibility (T), the following relationship is derived:

$$R(ohms) = \frac{1}{T} \cdot \frac{K_3}{K_2} \cdot \frac{\Delta x}{\Delta y}, \tag{16}$$

where Δx and Δy are the junction spacings in the x and y directions, ($\Delta x \div \Delta y$ equals one for square junction spacing), and other terms are as previously defined.

Thus, determination of the values of R in terms of T with the appropriate scale factors is complete. It should be noted here that for solution of the steady-state Laplace equation, the magnitude of resistance is undefined, and only the distribution is considered. However, with scale constants, actual resistance values are determined.

Similarly, for non-steady state application where a change in potential with respect to time is significant, the value of capacitance (C) is computed from the following relationship:

$$C\ (farads) = 7.48\ S \frac{K_2}{K_1} \Delta x\ \Delta y\ , \tag{17}$$

where C = the capacitance in farads for the space area defined by the product of Δx and Δy (in square feet),

S = the storage coefficient of the aquifer, and

K_1 and K_2 = as previously defined.

If, after the model has been constructed, a change of voltage level should become necessary because of unforeseen limitations of the equipment, the scale constants can be changed and the constructed model can be used without changing the resistor values, providing that the relation $K_1 = K_3 \times K_4$ is maintained.

By use of the model the problem being solved is a boundary value problem of potential theory, and a brief discussion of the application to steady-state modeling is appropriate. The problem is one of finding a solution to a differential equation which will meet certain specified requirements for a given set of values for the variables (boundaries).

The first boundary value problem of potential theory, called the Dirichlet problem (James and James, 1960), is stated as: given a region ρ (called a field by Karplus, 1958), its boundary surface Ψ, and a function f defined and continuous over Ψ determine a solution U of Laplace's equation $\nabla^2 U = 0$ which is regular in ρ, continuous in $\rho + \Psi$, and which satisfies the equation $U = f$ on the boundary. This type of problem occurs in problems of heat flow, electron flow, and is now being used for the solution of ground-water flow. The problem restated in hydrologic units would be: given the aquifer (transmissibility distribution), its piezometric surface (water-level contour map) and the functional relation given by equation 6, there is at most one solution of the boundary points which is the amount of inflow and outflow at the perimeter of the given aquifer.

If a uniformly distributed excitation is applied over the region ρ, the functional relation is then defined by Poisson's differential equation expressed in electrical units as:

$$\nabla^2 V = - RI, \qquad (18)$$

where ∇^2 (pronounced Del squared) is the Laplacian operator $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$ and RI is the strength of the distributed excitation. This last term is taken as being analogous to recharge to the aquifer that is received directly from precipitation by infiltration through the unsaturated zone.

Thus if the aquifer transmissibility distribution is known and modeled by use of equation 18, and the piezometric distribution is known for an equilibrium condition in a period prior to significant pumping, then the boundary contributions and recharge from precipitation are uniquely determined by trial from the model when the potential distribution matches the piezometric distribution.

One major limitation in application of this analogy is that one of the conditions for validity of equation 11 was that fluid flow must be acting in a confined aquifer. So long as the system acts as an artesian aquifer, the accuracy of the analogy is good. However, should the piezometric surface be lowered so that the system is unconfined (a water-table condition) the resulting piezometric surface from pumpage simulation must be corrected for aquifer thinning to reflect accurately the expected field condition. The major effect of the change to water-table conditions is the reduction in transmissibility due to the reduction in saturated thickness as water levels are lowered into the aquifer. Therefore, no significant error will result if the contributing thickness of the aquifer is not significantly reduced.

DESIGN AND CONSTRUCTION OF THE MODEL

The design of the model of the Kansas Valley near Topeka was based on the hydrologic situation previously described. The values used for permeability *(P)*, in determining transmissibility *(T)*, were assumed to be 5,000 gpd/ft^2 where aquifer thickness was 40 feet or less, and 10,000 gpd/ft^2 where aquifer thickness exceeded 40 feet (Fig. 1). The original values computed from aquifer tests were rounded off downward, and to this extent, the results would be slightly conservative.

The selection of scale constants was based on the relationship of equation 15 with regard to the measurement capabilities of available analyzing equipment. The values used in this study were:

$K_1 = 10^{16}$ gallons per coulomb,

$K_2 = 10$ feet of head per volt,

$K_3 = 10^{10}$ gpd per ampere,

$K_4 = 10^6$ days (real time) per second.

Equation 16 was used to determine resistance values. These values were then rounded off to the nearest standard manufactured values of resistors, for reasons of economy and ease in assembly. Resistance values along continuing boundaries of any model were defined in terms of relative values by Karplus (1958, p. 174-178).

Untempered peg board, with holes on one-inch centers, was mounted in a suitable frame to hold the base resistor network. Untempered pressed board has nearly infinite resistance and is reported to contain less free carbon and to have better dielectric properties than tempered pressed board.

The base map of the area modeled for this study was modified from two $7\frac{1}{2}$-minute series quadrangle maps (Silver Lake and Topeka quadrangles, U.S. Geological Survey). These maps were photographically enlarged to a scale of 4 inches per mile and printed on mylar (a stable-base plastic sheet). The mylar map was then fastened to the peg board with plain-brass shoe eyelets which served also as solder lugs for resistors at each junction in the area modeled (Fig. 4).

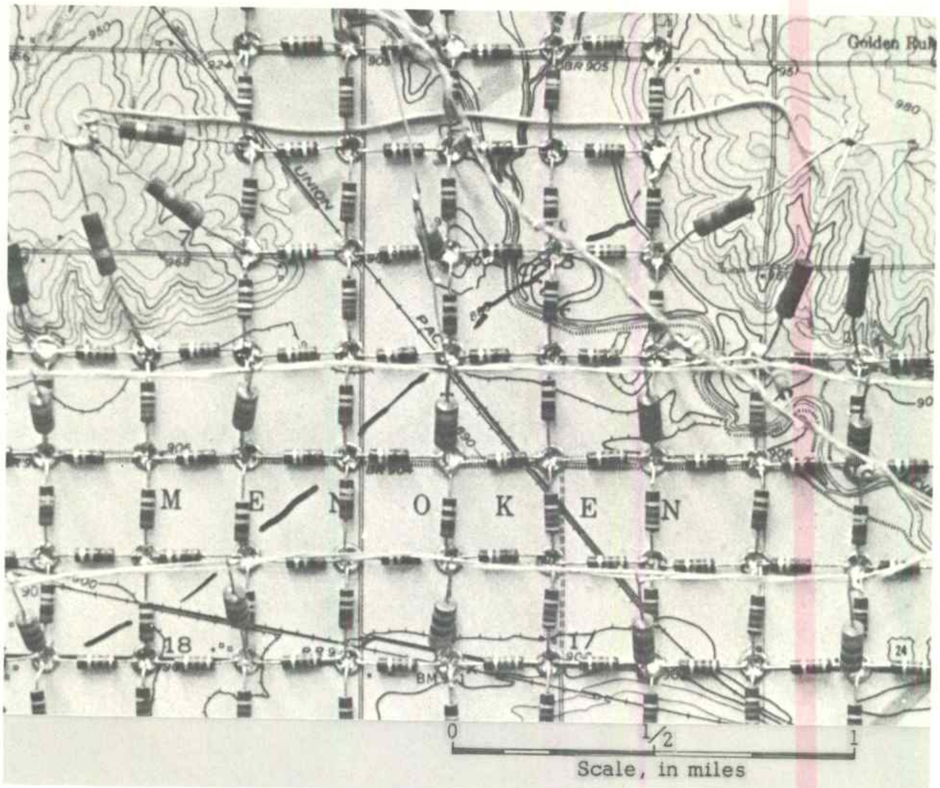


Figure 4.--Photograph of part of the model showing components and construction.

The types of recharging or discharging boundaries that can be imposed upon the model are extremely varied and can include almost any possible situation that can be observed or evaluated in nature. The effect of recharging or discharging boundaries usually is a part of the solution to be obtained by the model. To construct either a discharging or recharging boundary, one lead of a resistor of at least 10 times greater resistance than the average values of resistors in the base network is connected to each junction along the scale location of the boundary to be modeled (Fig. 5). The other lead of the resistors for each boundary is then connected by a common bus-bar, through a variable potentiometer and a milliammeter, to either the board ground or to a common power distribution point, depending upon whether the boundary is discharging or recharging. The current flow in the boundary circuits registered by the milliammeters indicates the relative amounts of recharging or discharging flow.

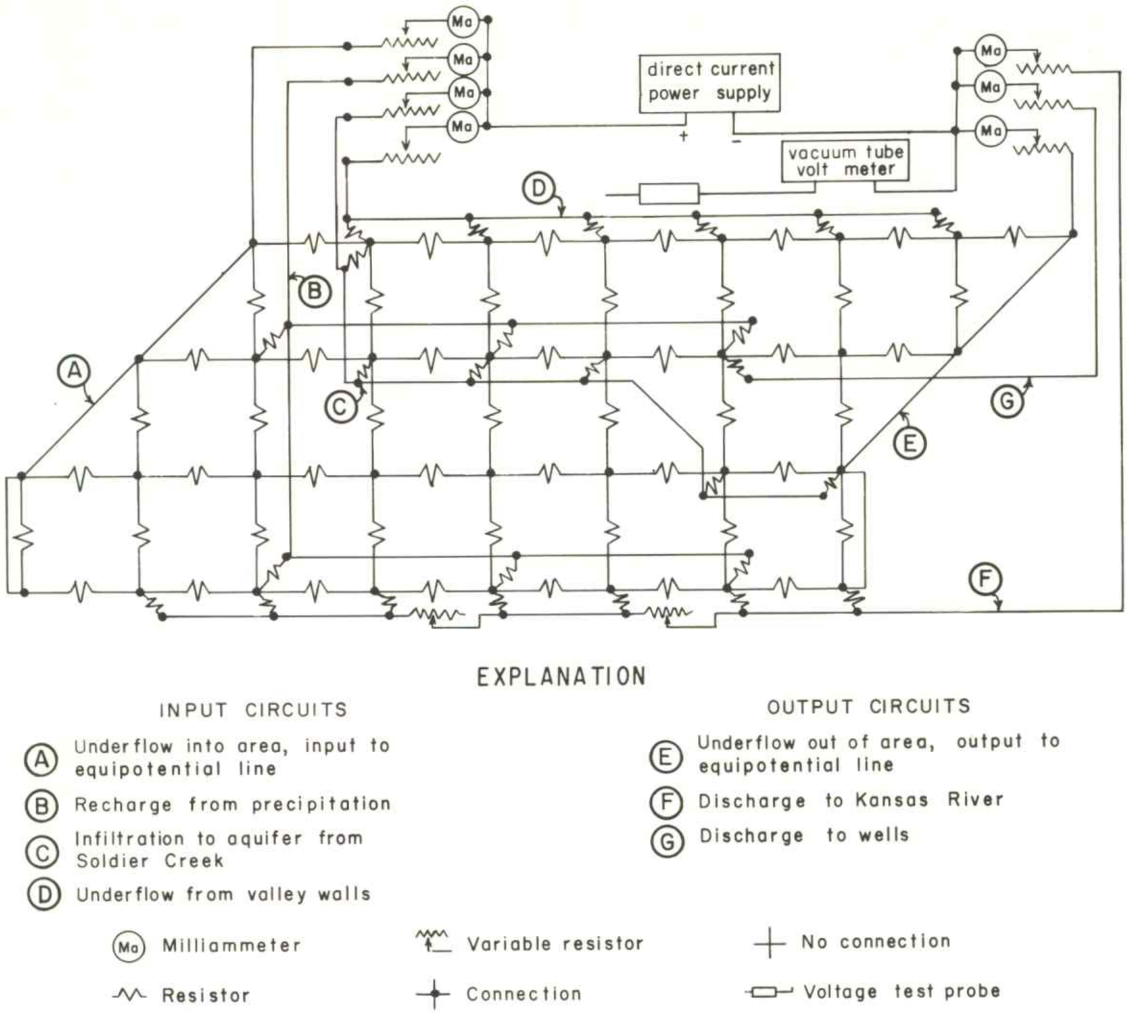


Figure 5.--Generalized wiring diagram of the resistor network simulating steady flow.

PROGRAMING THE MODEL

The program of the model was developed after the basic resistor network of the model had been constructed. The initial phase of programing consisted of working out the spatial configuration of the boundary system and the potentials to be applied to them. If a voltage were to be applied across the basic network of the model of the Kansas Valley near Topeka between the upstream and downstream margins of the model, the resulting equipotential lines would be relatively uniformly spaced and normal to the axis of the valley, unless affected by boundary potentials. The water-level contours in Figure 1 are askew to the axis of the valley, which indicates the significance of boundaries on the water level in the Kansas Valley. The effective boundaries in the physical system were identified and then simulated on the computer model.

The Kansas River is an effluent stream (or discharging boundary) that tends to drain the aquifer system. Along the course of the Kansas River, on the model, one lead of a 100,000-ohm resistor was soldered at each junction. The other leads of these resistors were then connected by a common bus-bar through a variable potentiometer to the common ground terminal (Fig. 5). Early tests with the computer model showed the effect of the river was not constant throughout the length of the model. For this reason, the river discharge network was split into several sections by insertion of variable potentiometers into the Kansas River bus-bar circuit. The sectioning was actually based on changes in alignment of the river with reference to the valley axis, but the need for this sectioning was due, in part, to relating the eastward potential gradient of the model to the eastward gradient of the water level in the valley.

Water-level contours show that Soldier Creek is an influent stream in the model area. To simulate this recharging boundary one lead of a 0.47-megohm (million ohm) resistor was soldered to each junction along the course of the creek, and the other lead was connected by a common bus-bar through a potentiometer to the positive power-distribution terminal.

An important contribution of ground water to the valley alluvium is from glacial till bordering the north side of the Kansas Valley. The common bus-bar for the one-megohm resistors, used to simulate this boundary, was connected through a potentiometer to the positive power-distribution terminal.

Pumpage from wells was simulated by connecting the junction or junctions nearest the well location through a potentiometer to the common ground terminal.

The inflow and outflow of the aquifer was determined by current flow, and was measured by milliammeters connected in series with each of the boundary circuits. Current flow, in ma. (milliamperes), was then converted to flow units by use of the proportionality constants (scale factors). For a given aquifer conductivity and potential distribution, the Laplace equation has only one solution of boundary values. Thus, the boundary values are uniquely determined.

Some recharge to the valley alluvium is derived from precipitation in the area modeled. For the purpose of this report, reference to recharge from precipitation is limited to the direct contribution to the aquifer received by vertical flow through the overlying unsaturated zone. The recharge was applied uniformly over the model. One lead of one-megohm resistors was soldered to alternate junctions in alternate rows of junctions on the model, and the other lead was connected by bus-bar systems through potentiometers to the positive power-distribution terminal (Fig. 5). This network of resistors represents recharge from precipitation and acts independently of other boundary values.

COMPUTER-MODEL PROBLEM ANALYSIS

For analysis of a problem by a model, a regulated direct-current power supply is connected across the power-distribution terminal and the common ground terminal (Fig. 5). In this project, 30 volts was applied to the model of the Kansas Valley. This potential was adequate to provide the necessary potentials and currents to the network boundary circuits. Known scaled values of potential were first imposed on the computer-model. These values included the potential difference across the model area, equivalent current flow across each of the boundaries, and pumpage. Potentiometers in each curcuit (Fig. 5) were then adjusted by trial to obtain the solution of the model problem. The solution was complete when the potential distribution matched the equivalent piezometric distribution of the area.

A piezometric map representative of natural equilibrium conditions before significant pumpage is desirable for the first trial solution. The water-level contour map of 1950 (Fig. 1) was used to approximate this condition. The electrical currents applied to the model (and their hydraulic equivalents) are shown in Table 1, and the resulting potential distribution on the model is shown in Figure 6.

Table 1.—Programed current values of model solutions, and their hydraulic equivalents, under hydrologic conditions prevailing in 1950, 1963, and under a possible future condition.

	Electrical current and hydraulic equivalents		
	1950	1963	Future
Input circuits			
Underflow into study area	0.24 ma (2.4 mgd)	0.16 ma (1.6 mgd)	0.16 ma (1.6 mgd)
Infiltration from Soldier Creek	.16 ma (1.6 mgd)	.12 ma (1.2 mdg)	.12 ma (1.2 mgd)
Recharge from Valley walls	.27 ma (2.7 mgd)	.23 ma (2.3 mgd)	.23 ma (2.3 mgd)
Recharge from precipitation	.68 ma (6.8 mgd)	.68 ma (6.8 mgd)	.68 ma (6.8 mgd)
Total input	1.35 ma (13.5 mgd)	1.19 ma (11.9 mgd)	1.19 ma (11.9 mgd)
Output circuits			
Underflow out of study area	0.14 ma (1.4 mgd)	0.08 ma (0.8 mgd)	0.05 ma (0.5 mgd)
Discharge from aquifer to Kansas River	1.21 ma (12.1 mgd)	.66 ma (6.6 mgd)	.24 ma (2.4 mgd)
Discharge from wells at Goodyear Company plant		.45 ma (4.5 mgd)	.90 ma (9.0 mgd)
Total output	1.35 ma (13.5 mgd)	1.19 ma (11.9 mgd)	1.19 ma (11.9 mgd)

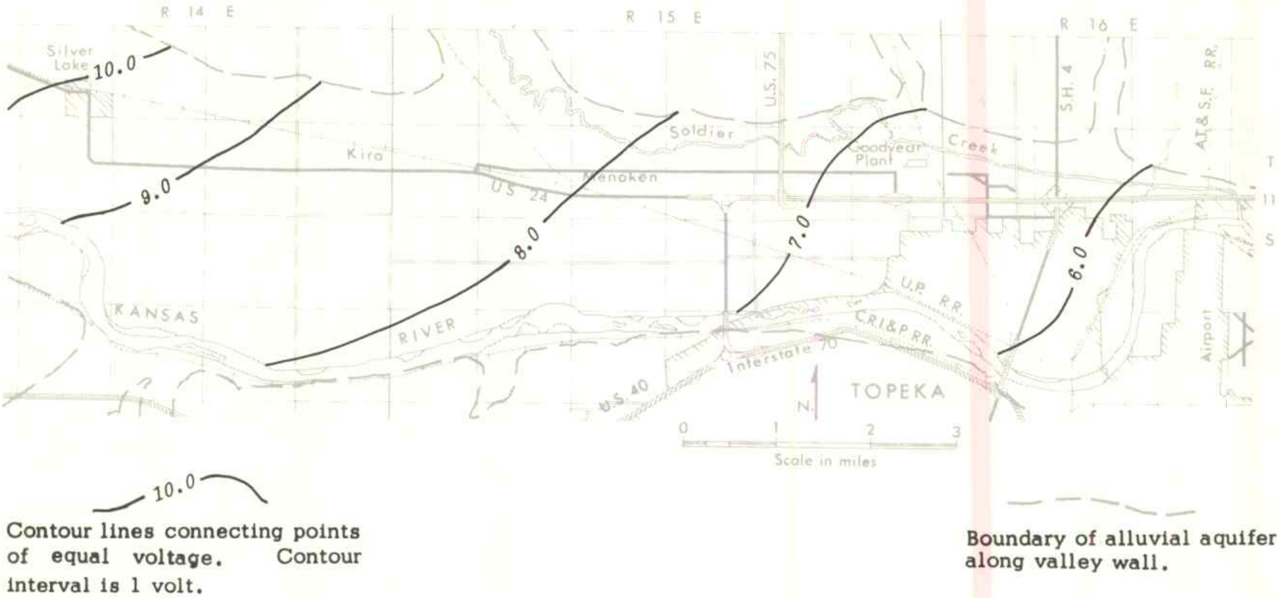


Figure 6.--Map showing lines of equal voltage rèpresenting water levels in 1950.

To verify the design and validity of the model, it must be possible to set up a subsequent hydrologic situation and obtain results consistant with the previous solution. The November 1963 piezometric map (Fig. 2) was used for this purpose. The electrical currents applied to the model (and their hydraulic equivalents) are shown in Table 1, and the resulting electrical potential distribution on the model is shown in Figure 7.

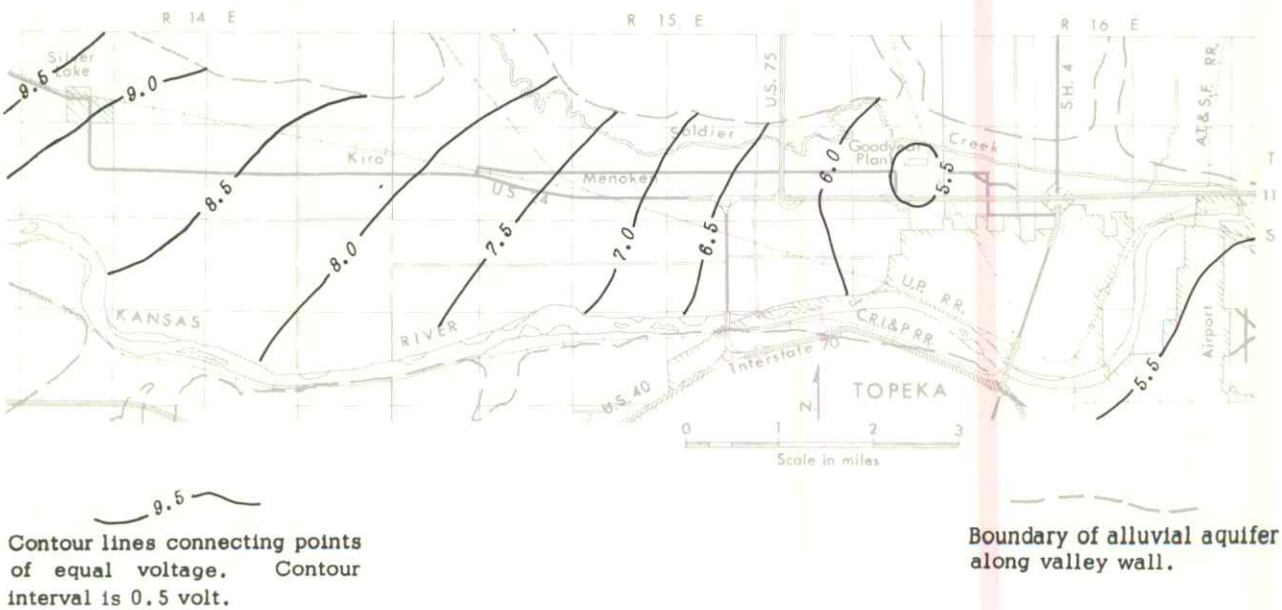
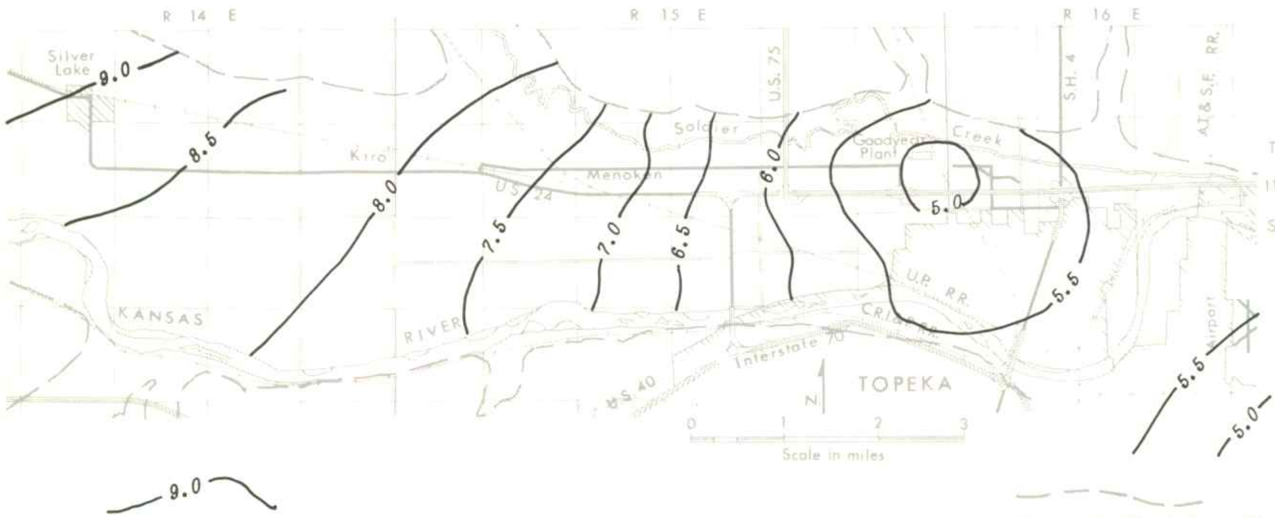


Figure 7.--Map showing lines of equal voltage representing water levels in November 1963.

The model solution for the 1950 and 1963 programs, tabulated in Table 1, show a marked difference in the relative amounts of electrical current in the input and output circuits. This difference is to be expected because of hydrologic differences between the two periods. Precipitation, water levels, and hydraulic gradients were above normal in the 1950-52 period whereas precipitation was below normal in 1963. Of course, the apparent difference in hydraulic gradients largely results from the fact that the 1950 map was simulated from actual water-level measurements, made over a three-year period, 1950-1953.

The essential agreement between the November 1963 piezometric map (Fig. 2) and the distribution of electrical potential on the model (Fig. 7) served to verify the design and validity of the model. The slight differences between water-level contours derived from field observation and those derived from the model were considered to be the result of the residual recovery effects from irrigation in the late summer and early fall of 1963.

After verification, the model was programed to determine the approximate effect on the ground-water level in the modeled area if the industrial pumpage at the Goodyear Company plant was increased to 9.0 mgd. The solution was obtained by maintaining input currents the same as for the previous (1963) solution and increasing pumpage at the plant to the specified amount. The electrical currents applied to the model and their hydraulic equivalents are shown in Table 1, and the potential distribution is shown in Figure 8.



Contour lines connecting points of equal voltage. Contour interval is 0.5 volt.

Boundary of alluvial aquifer along valley wall.

Figure 8.--Map showing lines of equal voltage representing water levels if 9 mgd were pumped in the Goodyear Company plant area.

Figure 9 shows the potential surface, converted to the hydrologic potential distribution in feet above mean sea level. The increase in pumping resulted in a reduction in underflow out of the area and in the discharge to the Kansas River.

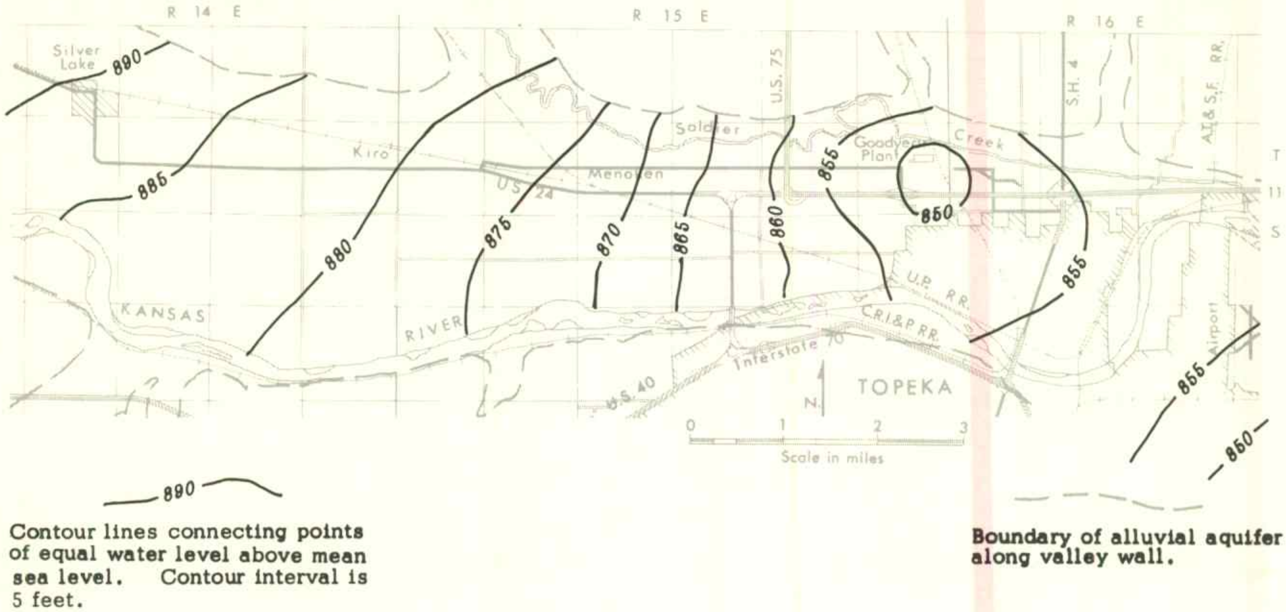


Figure 9.--Map showing the expected ground-water level in the alluvial aquifer if 9 mgd were pumped in the Goodyear Company plant area.

HYDROLOGIC SUMMARY

Recharge of the aquifer system in the modeled area of the Kansas Valley is principally from four sources. These are underflow through the valley alluvium into the area from the west, seepage from relatively impermeable glacial till along the valley walls, infiltration from tributary streams to the Kansas River where they cross the valley alluvium, and direct recharge from precipitation over the modeled area. Discharge from the aquifer system in the modeled area is by underflow through the valley alluvium out of the area to the east, ground-water discharge to the Kansas River, and pumping from wells. Evapotranspiration from the zone of saturation is not deemed significant in the area modeled.

Recharge from precipitation over the modeled area was found to be the equivalent of 3.2 inches per year. This parameter can act somewhat independently of the other boundary parameters, as is defined by Poisson's equation, and it was assumed constant for all analyses. This is the only parameter not explicitly defined, and it was determined by a "best-fit" method.

The total inflow (including recharge from precipitation) and outflow for 1950 were both 13.5 mgd for the 45 square-mile area modeled. In November 1963 the inflow and outflow were both 11.9 mgd. When industrial pumping is increased to 9 mgd, total inflow and outflow will be 11.9 mgd under hydrologic conditions prevailing in 1963; however, the increase in outflow by pumping will be offset by an equivalent reduction in the outflow as underflow through the alluvium and as discharge to the Kansas River.

On the basis of information obtained from the working model, recharge from present sources is capable of sustaining a continuous withdrawal of 9 mgd without undue interference with other wells presently in use in the modeled area. Should significant additional amounts of ground water be desired in the area, well location, spacing, and pumping rates should be planned to minimize well interference. Wells designed to induce infiltration from the Kansas River would be the most effective way to minimize well interference.

CONCLUSIONS

The development of the design, construction, and analysis procedure of a model has been presented for steady-state analysis. The development for design of a non-steady-state model has also been presented. The purpose of this report has been to show application of the technique rather than to present a rigorous mathematical treatment of the subject. The results of analysis obtained by use of the electrical-model technique can provide a valuable guide for administrative judgment in matters previously considered too complex for solution.

The steady-state form of a model is very useful in determining the source and magnitude of recharge and discharge of a ground-water aquifer and the total effect of pumpage on a long-term basis. For consumptive use to be sustained in excess of average annual recharge, either the importation of water from an outside source (induced river infiltration) or a reduction of available water in storage is required. In only the later condition is time a factor requiring a non-steady-state solution.

REFERENCES

Beck, H. V., 1959, Geology and ground-water resources of Kansas River valley between Wamego and Topeka vicinity: Kansas Geol. Survey Bull. 135, 88 p., 8 pl., 12 fig.

Davis, S. N., and Carlson, W. A., 1952, Geology and ground-water resources of the Kansas River valley between Lawrence and Topeka, Kansas: Kansas Geol. Survey Bull. 96, pt. 5, p. 201-276, 7 pl., 8 fig.

Ferris, J. G., 1949, chap. 7, Ground Water, *in* Wisler, C. O., and Brater, E. F., Hydrology: John Wiley and Sons, Inc., New York, 419 p.

Jacob, C. E., 1950, Flow of ground water, *in* Rouse, Hunter (ed.), Engineering hydraulics, chap. 5: John Wiley and Sons, New York, p. 321-386.

James, Glenn, and James, R. C., 1960, Mathematics dictionary: D. Van Nostrand Company, Inc., Princeton, N. J., 546 p.

Karplus, W. J., 1958, Analog simulation: McGraw-Hill Book Company, Inc., New York, 434 p.

Mack, L. E., 1957, Evaluation of a conducting paper analog field plotter as an aid in solving ground-water problems: Kansas Geol. Survey Bull. 127, pt. 2, p. 27-47, 1 pl., 9 fig.

Patten, E. P., Jr., 1965, Design, construction, and use of electric analog model, *in* Wood, L. A., Analog model study of ground water in the Houston district: Texas Water Comm. Bull. 6508, p. 41-103.

Skibitzke, H. E., 1961, Electronic computers as an aid in the analysis of hydrologic problems: Internat. Assoc. Sci. Hydrology pub. 52, Comm. of Subterrenean Waters, p. 347-358.

U.S. Weather Bureau, 1964, Climatological data, annual summary, Kansas, 1963: Washington, U.S. Govt. Printing Office, p. 214-225.

Walton, W. C., and Prickett, T. A., 1963, Hydrogeologic electric analog computers: Jour. Hyd. Div. ASCE, v. 89, no. HY6, p. 67-91.