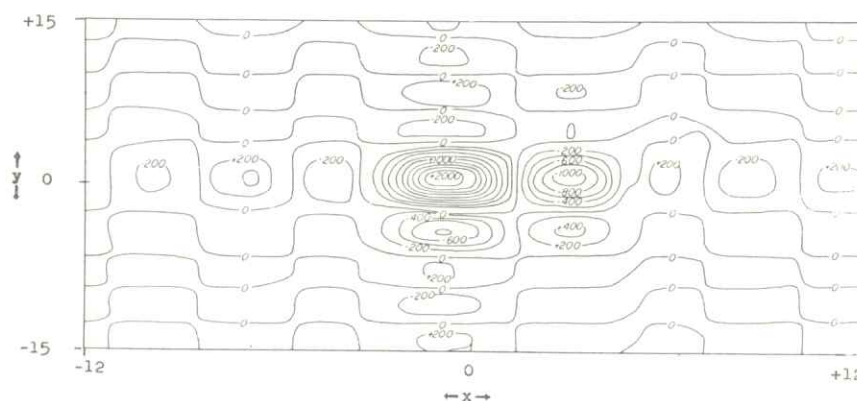


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1965



BALGOL Programs and Geologic Application for Single and Double Fourier Series Using IBM 7090/7094 Computers

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By
Floyd W. Preston
The University of Kansas
and
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BALGOL PROGRAMS AND GEOLOGIC APPLICATION FOR SINGLE AND
DOUBLE FOURIER SERIES USING IBM 7090/7094 COMPUTERS

by

Floyd W. Preston and John W. Harbaugh

INTRODUCTION

This publication describes computer programs for analysis with Fourier series and geological applications of these programs. One program deals with single Fourier series, which may be used to analyze data that may be represented by curves; the other program deals with double Fourier series, which may be used to analyze surfaces. The programs may be used with IBM 7090 or 7094 computers. The programs are written in a version of ALGOL-58 known as SUBALGOL (abbreviated as BALGOL) and require the use of the SUBALGOL compiler system. Card decks of the single Fourier series program, with appropriate test data and a test-generator program may be obtained from the Kansas Geological Survey for a limited time at a cost of \$5.00. Card decks pertaining to the double Fourier series program may be obtained from the Survey for \$15.00. The compiler system may be obtained by sending four IBM-compatible magnetic tapes of 1200-foot length (or greater) to the Computation Center, Stanford University, Stanford, California. The SUBALGOL compiler system will be recorded on the tapes at a cost of \$35.00.

Fourier series are mathematical series consisting of terms containing sines and cosines. Single Fourier series may be used to analyze data represented by curves, whereas double Fourier series may be used to analyze surfaces.

Fourier series should be useful in portraying or modeling geologic features that vary in a more or less cyclic manner. For example, a topographic profile of an upland area that has been dissected by a dendritic drainage system might be thought of as a complex wave form composed of interacting cycles or harmonics of various amplitudes and periods. This topographic profile could be approximated to any given accuracy by recording elevations along the profile at a sufficient number of data points and by fitting a single Fourier series to these points. A curve may be obtained whose configuration depends upon the number of terms in the Fourier series and the values of the coefficients of the terms. The coefficients may be thought of as providing a numerical means of description of the profile or surface being studied, and also providing a means of isolating and identifying the contribution of various underlying harmonics.

An objective in using Fourier analysis is to determine whether variations in some natural feature can be represented by an oscillatory function. If a Fourier series provides a good representation of observed data, then one would generally suspect that the variations are due to causes which vary in some periodic manner. Conversely, if Fourier series do not represent the data convincingly, other mathematical means of representation might be sought, as for example, the power series polynomials employed in conventional trend-surface analysis. Trend analysis permits interpolation between data points, and, to some extent, extrapolation. The effectiveness of both of these uses depends, however, on careful appraisal of the quality of "fit" of the data to the trend function (Wilks, 1963).

There are two principal applications of Fourier analysis. Trend-surface analysis is currently used in ore reserve calculations, geochemical prospecting, analysis of geologic structure maps, and analysis of lithofacies maps. Several of the applications reported here were described previously by Harbaugh and Preston (1965).

A growing number of investigators are applying polynomial approximation or "trend" surfaces to the description of geological and geophysical measurements. Among these are Miller (1956), Krumbein (1959, 1963), Mandelbaum (1963), Connor (1964), Whitten (1962), Merriam and Harbaugh (1964), Harbaugh (1964), and Agterberg (1964). An excellent review of trend surface methods and the general problem of mapping geological parameters is given by Miller and Kahn (1962). The predominant effort has been to use non-orthogonal polynomials. An exception is the work of Oldham and Sutherland (1955). Up to now, however, little use has been made of Fourier series for data representation in geology.

Spectral analysis is concerned with identification of the amplitudes and frequencies of the component cycles that constitute the periodic portion of an oscillatory phenomenon. Techniques for this type of analysis have been developed by Blackman and Tukey (1958), Wiener (1949) and used in geology by Anderson and Koopmans (1963) for varve analysis.

This paper is largely concerned with the trend analysis aspects of Fourier series. Both trend analysis and spectral analysis are, however, closely related.

Acknowledgments

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MATHEMATICAL DEVELOPMENT

Single Fourier Series

The single Fourier infinite series (Fourier, 1822) can be expressed in several ways. The one chosen for the present work is

$$z = \frac{a_0}{2} + \sum_{n=1}^{n=\infty} \left[a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L} \right] \quad (1)$$

This series is frequently used to approximate a known function of the form, $z = f(x)$. Under these circumstances, the coefficients a_0 , a_n , and b_n , $n = 1, 2, \dots \infty$ are obtained from the expressions:

$$a_n = \frac{1}{L} \int_{-L}^{+L} f(x) \cos \frac{n\pi x}{L} dx \quad n = 0, 1, 2, \dots \infty \quad (2)$$

$$b_n = \frac{1}{L} \int_{-L}^{+L} f(x) \sin \frac{n\pi x}{L} dx \quad n = 1, 2, \dots \infty \quad (3)$$

The general restrictions under which Equation (1) converges to $f(x)$ as n approaches infinity are well known and are documented in such texts as Gaskell (1958), Churchill (1941), and Wylie (1960), covering Fourier series. However, in many physical problems one does not know the form of the function $z = f(x)$, thereby precluding the analytical integration of Equations (2) and (3) to obtain the coefficients. However, if information concerning the function is available as tabulated data (x_i, z_i) representing experimentally obtained points on the unknown function, then the coefficients a_n and b_n may be approximated by numerical integration methods.

If k points are chosen at equal intervals of x_i in the region $-L < x_i < +L$, and if the trapezoidal rule of numerical integration is used, the expressions for a_n and b_n become:

$$a_n = \frac{2}{k} \left[\frac{z_0 + z_k}{2} + \sum_{i=1}^{i=k-1} z_i \cos \frac{n\pi x_i}{L} \right] \quad n=0, 1, 2, \dots, (k-1)/2 \quad (4)$$

$$b_n = \frac{2}{k} \sum_{i=0}^{i=k-1} z_i \sin \frac{n\pi x_i}{L} \quad n=1, 2, \dots, (k-1)/2 \quad (5)$$

The data set, (x_i, z_i) is such that $x_0 = -L$, $(x_{i+1} - x_i)$ is constant, and $x_k = +L$, the value of k being odd. The number of coefficients a_n or b_n , that can be extracted from a data set de-

depends upon the number of data points. The restriction on n is such that $0 \leq n \leq N$.

when $N = (k-1)/2$, the approximating function

$$z_i \approx P_{x_i} = \frac{a_0}{2} + \sum_{n=1}^{n=N} \left(a_n \cos \frac{n\pi x_i}{L} + b_n \sin \frac{n\pi x_i}{L} \right) \quad (6)$$

passes through each measured data point (x_i, z_i) .

Although the use of the trapezoidal rule would seem to be an unnecessary simplification for the numerical integration of Equations (2) and (3), it is shown by Whittaker and Robinson (1924) that the use of Equations (4) and (5) produces coefficients which satisfy the least-squares criterion of a "best" fit to the existing data, that is,

$$\sum_{i=0}^{i=k-1} (z_i - P_{x_i})^2 = \text{Minimum} \quad (7)$$

where P_{x_i} is the polynomial approximation to the observed value z_i corresponding to the value x_i of the independent variable.

Before the advent of digital computers, many numerical schemes were devised to generate the coefficients a_n and b_n with the least computational effort. Because of the relative ease of expressing the sines and cosines of angles that are multiples of 60° , 30° , or 15° , the computational methods involved data sets of 6, 12, 24, or 48 points. Such numerical schemes are described in detail by Scarborough (1962), Manley (1945), and Gaskell (1958). In addition, various optical and mechanical devices have been built for the determination of the coefficients (Johnson, 1941). These are described by Manley (1945).

Use of a function such as Equation (6) to represent a set of experimental data is analogous to use of a power series polynomial for trend-surface approximation. The term "trend" is used here in the context defined by Grant (1957) as a function that describes the behavior of the measured quantities, independent of any experiment. This meaning for "trend" is different from that usually implied by those working with time-series analysis.

Figure 1 shows the shape of a synthetic single Fourier series which uses the set of coefficients in Table 1. Diagrams in the right-hand column are the series truncated at 2, 3, 4, and 5 harmonics. The pure sine and cosine components of each harmonic are also given.

The availability of digital computers now makes generation of trend surfaces of even a high polynomial order a relatively routine task. This fact underscores the importance of applying statistical methods for determining confidence intervals for these surfaces. Krumbein (1963) and Agterberg (1964) each develop methods for the conventional power series polynomials. Oldham and Sutherland (1955) discuss hypothesis tests for selecting the "best" order for orthogonal polynomial

approximations of geophysical measurements. Additional tests for each term in any series, orthogonal or non-orthogonal, are developed by Brown (1955). These latter tests are applied herein to determine the desirability of including specific terms in the Fourier series.

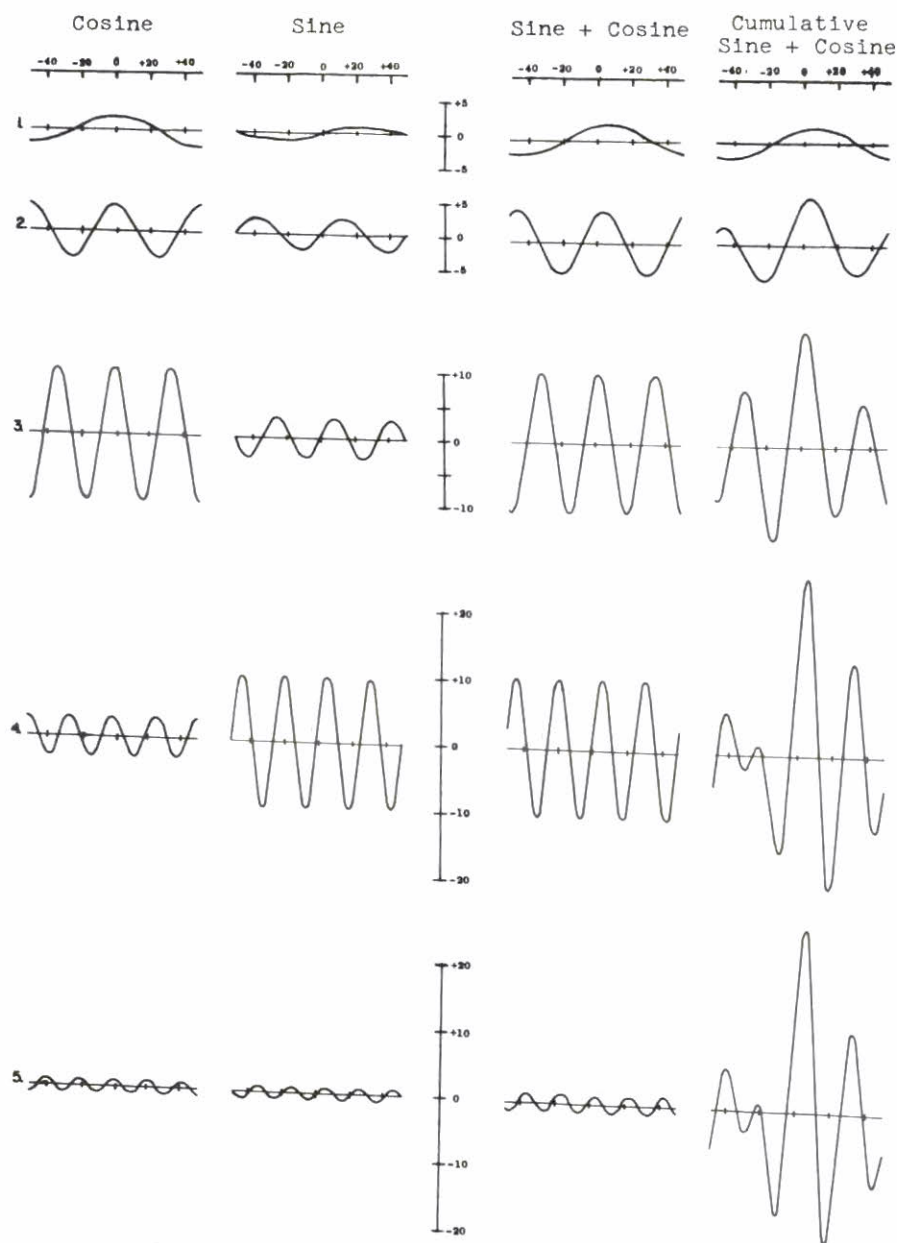


Figure 1.- Synthetic single Fourier series showing (1) wave forms of individual terms and (2) wave form of series representing the summed individual terms. Coefficients are listed in Table 1. Numbers at left indicate harmonics.

Table 1.- Coefficients used for generating synthetic single Fourier series.

n	a _n	b _n
0	0.0	---
1	2.0	1.0
2	4.0	2.5
3	10.0	3.0
4	3.0	-10.0
5	1.0	1.0

The particular test used for the coefficients depends upon the nature of the model used to describe variations in the property being measured. One can consider the i'th value of the dependent variable z to be

$$z_i = f(x_i) + \varepsilon_i \quad (8)$$

where x_i is the i'th value of the independent variable and the "trend" of z with x is given by the function

$$z = f(x) \quad (9)$$

If it is assumed that ε is a random variable, that is ε_i is not correlated with ε_{i+1} , then as shown by Brown (1955), a ranking of the polynomial terms according to the values of their associated statistic $S_{n,p} c_{n,p}^2$ will reveal a sharp change at the transition from those terms which contribute to the trend to those which do not contribute. The terms $S_{n,p}$ and $c_{n,p}$ are defined as:

$$S_{n,1} = \sum_{i=0}^{i=k-1} a_n \cos \frac{n\pi x_i}{L} \quad (10)$$

$$S_{n,2} = \sum_{i=0}^{i=k-1} b_n \sin \frac{n\pi x_i}{L} \quad (11)$$

$$S_{0,1} = ka_0; S_{0,2} = 0 \quad (12)$$

$$h_{n,1} = \sum_{i=0}^{i=k-1} z_i \left(a_n \cos \frac{n\pi x_i}{L} \right) \quad (13)$$

$$h_{n,2} = \sum_{i=0}^{i=k-1} z_i \left(b_n \sin \frac{n\pi x_i}{L} \right) \quad (14)$$

$$c_{n,p} = h_{n,p} / S_{n,p} \quad (15)$$

$$p = 1, 2$$

If, in addition, the random variable ε is considered to be normally distributed, then, by use of an F test one can determine the desirability of adding a term to the polynomial. Here F is the ratio s_1^2 / s_2^2 where s_1^2 and s_2^2 are taken to be the averaged values of $S_{n,p} c_{n,p}^2$ for v_1 terms and v_2 terms respectively. For this F test the degrees of freedom are v_1 and v_2 .

The calculated Fourier series $F(x_i)$ may be subtracted from the observed value $f(x)$ at each point, the difference being the residual value, $r(x_i)$ as follows:

$$r(x_i) = f(x_i) - F(x_i) \quad (16)$$

As the degree of the fitted Fourier series becomes higher, the quality of fit generally improves and the residual values become smaller and smaller. A convenient measure of the quality of fit is given by the percent of total sum of squares, which was explained in a previous paper by Harbaugh (1964). A value of 100 percent would indicate perfect fit of the calculated values to the observed values. The programs described in this publication compute the percent of total sum of squares and provide for automatic plotting of the observed data, calculated values, and residual values.

It is often convenient to represent the Fourier coefficients of given degree by using power spectrum values, or, alternatively, harmonic vector magnitude values. Power spectrum values are defined simply as the sums of the squared Fourier coefficient values. In turn, the harmonic vector magnitude values are obtained as the square roots of the power spectrum values.

Double Fourier Series

The Fourier series may be extended to represent a dependent variable z as a function of two independent variables, x and y .

$$z = f(x, y) \quad (17)$$

Thus, a parameter z may be considered to be oscillatory in two mutually perpendicular directions such as east-west and north-south, allowing Equation (17) to represent the areal variation in z .

Figure 2 shows a contoured surface generated by a double Fourier series using the coefficients of Table 2. The surface shows a high degree of regularity. If many more terms had been used and if the coefficients had been randomly selected, the surface would show very little regularity.

If we consider a function z to have a fundamental period of $2L$ along the x coordinate and $2H$ in the y coordinate, then the double Fourier series is:

$$z \approx P_{x,y} = \sum_{m=0}^{m=M} \sum_{n=0}^{n=N} \lambda_{m,n} \left[a_{m,n} \cos \frac{\pi m x}{L} \cos \frac{\pi n y}{H} + b_{m,n} \sin \frac{\pi m x}{L} \cos \frac{\pi n y}{H} + c_{m,n} \cos \frac{\pi m x}{L} \sin \frac{\pi n y}{H} + d_{m,n} \sin \frac{\pi m x}{L} \sin \frac{\pi n y}{H} \right] \quad (18)$$

where

$$\begin{aligned}
 \lambda_{m,n} &= 1/4 & m=n=0 \\
 \lambda_{m,n} &= 1/2 & m=0, n > 0, \text{ or } m > 0, n=0 \\
 \lambda_{m,n} &= 1 & m > 0, n > 0
 \end{aligned} \tag{19}$$

Table 2.- Coefficients used for generating synthetic double Fourier series.

$a_{m,n}$						$b_{m,n}$					
m	0	1	2	3	4	m	0	1	2	3	4
n	0	1	2	3	4	n	0	1	2	3	4
0	43	54	65	76	87	0	0	40	48	56	64
1	45	56	67	78	89	1	0	44	52	60	68
2	47	58	69	80	91	2	0	48	56	64	72
3	49	60	71	82	93	3	0	52	60	68	76
4	51	62	73	84	95	4	0	56	64	72	80

$c_{m,n}$						$d_{m,n}$					
m	0	1	2	3	4	m	0	1	2	3	4
n	0	1	2	3	4	n	0	1	2	3	4
0	0	0	0	0	0	0	0	0	0	0	0
1	27	32	37	42	47	1	0	22	24	26	28
2	33	38	43	48	53	2	0	27	29	31	33
3	39	44	49	54	59	3	0	32	34	36	38
4	45	50	55	60	65	4	0	37	39	41	43

If the relation expressed by Equation (18) can be integrated, the coefficients are determinable from the following relations:

$$a_{m,n} = \frac{1}{LH} \int_{-H}^{+H} \int_{-L}^{+L} f(x,y) \cos \frac{m\pi x}{L} \cos \frac{n\pi y}{H} dx dy \tag{20}$$

$$b_{m,n} = \frac{1}{LH} \int_{-H}^{+H} \int_{-L}^{+L} f(x,y) \sin \frac{m\pi x}{L} \cos \frac{n\pi y}{H} dx dy \tag{21}$$

$$c_{m,n} = \frac{1}{LH} \int_{-H}^{+H} \int_{-L}^{+L} f(x,y) \cos \frac{m\pi x}{L} \sin \frac{n\pi y}{H} dx dy \tag{22}$$

$$d_{m,n} = \frac{1}{LH} \int_{-H}^{+H} \int_{-L}^{+L} f(x,y) \sin \frac{m\pi x}{L} \sin \frac{n\pi y}{H} dx dy \tag{23}$$

For the instances in which the relation among x , y , and z is known only as a tabulated set of data, the equations for $a_{m,n}$, $b_{m,n}$, $c_{m,n}$, and $d_{m,n}$ analogous to Equations (4) and (5) are:

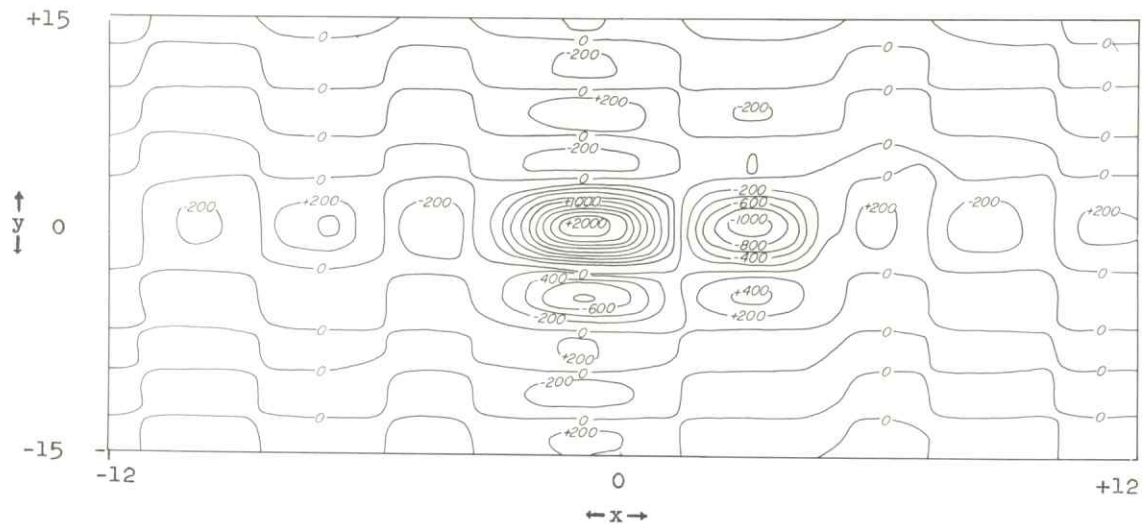


Figure 2.- Synthetic double Fourier series showing a wave form containing four harmonics in each direction. Coefficients are listed in Table 2.

$$a_{m,n} = \frac{4}{LH} \left[\left(\frac{z_{0,0} + z_{k,0} + z_{0,1} + z_{k,1}}{4} \right) \cos m\pi \cos n\pi + \sum_{j=0}^{j=1-1} \left(\frac{z_{0,j} + z_{k,j}}{2} \right) \cos m\pi \cos \frac{n\pi y_j}{H} + \sum_{i=0}^{i=k-1} \left(\frac{z_{i,0} + z_{i,1}}{2} \right) \cos \frac{m\pi x_i}{L} \cos n\pi + \sum_{j=0}^{j=1-1} \sum_{i=0}^{i=k-1} z_{i,j} \cos \frac{m\pi x_i}{L} \cos \frac{n\pi y_j}{H} \right] \quad (24)$$

$$b_{m,n} = \frac{4}{LH} \left[\sum_{i=0}^{i=k-1} \left(\frac{z_{i,0} + z_{i,1}}{2} \right) \sin \frac{m\pi x_i}{L} \cos n\pi + \sum_{j=0}^{j=1-1} \sum_{i=0}^{i=k-1} z_{i,j} \sin \frac{m\pi x_i}{L} \cos \frac{n\pi y_j}{H} \right] \quad (25)$$

$$c_{m,n} = \frac{4}{LH} \left[\sum_{j=0}^{j=1-1} \left(\frac{z_{0,j} + z_{k,j}}{2} \right) \cos m\pi \sin \frac{n\pi y_j}{H} + \sum_{j=0}^{j=1-1} \sum_{i=0}^{i=k-1} z_{i,j} \cos \frac{m\pi x_i}{L} \sin \frac{n\pi y_j}{H} \right] \quad (26)$$

$$d_{m,n} = \frac{4}{LH} \left[\sum_{j=0}^{j=1-1} \sum_{i=0}^{i=k-1} z_{i,j} \sin \frac{m\pi x_i}{L} \sin \frac{n\pi y_j}{H} \right] \quad (27)$$

The data set $(z_{i,j}, x_i, y_j)$ is such that

$$i = 0, 1, 2, \dots, k; \quad x_0 = -L, x_k = +L$$

$$j = 0, 1, 2, \dots, 1; \quad y_0 = -H, y_1 = +H$$

$$x_{i+1} - x_i = \text{constant} \quad y_{j+1} - y_j = \text{constant}$$

The limiting number of coefficients of all types $a_{m,n}$, $b_{m,n}$, $c_{m,n}$, and $d_{m,n}$ that can be extracted from a set of data is

$$4(M+1)(N+1) - 2((M+1) + (N+1)) \quad (28)$$

where:

$$N > 0; \quad M > 0$$

$$m = 0, 1, 2, \dots, M$$

$$n = 0, 1, 2, \dots, N$$

In a one dimensional series the term b_0 is always zero. Similarly, the following relation holds for a two-dimensional Fourier series

$$b_{0,n} = c_{m,0} = d_{0,n} = d_{m,0} = 0 \quad (29)$$

For the two dimensional series the equations corresponding to $s_{n,p}$, $c_{n,p}$, and $n_{n,p}$ are as follows:

$$S_{m,n,1} = \sum_{j=0}^{j=1-1} \sum_{i=0}^{i=k-1} \left[\lambda_{m,n} \cos \frac{m\pi x_i}{L} \cos \frac{n\pi y_j}{H} \right]^2 \quad (30)$$

$$S_{m,n,2} = \sum_{j=0}^{j=1-1} \sum_{i=0}^{i=k-1} \left[\lambda_{m,n} \sin \frac{m\pi x_i}{L} \cos \frac{n\pi y_j}{H} \right]^2 \quad (31)$$

$$S_{m,n,3} = \sum_{j=0}^{j=1-1} \sum_{i=0}^{i=k-1} \left[\lambda_{m,n} \cos \frac{m\pi x_i}{L} \sin \frac{n\pi y_j}{H} \right]^2 \quad (32)$$

$$S_{m,n,4} = \sum_{j=0}^{j=1-1} \sum_{i=0}^{i=k-1} \left[\lambda_{m,n} \sin \frac{m\pi x_i}{L} \sin \frac{n\pi y_j}{H} \right]^2 \quad (33)$$

$$h_{m,n,1} = \sum_{j=0}^{j=1-1} \sum_{i=0}^{i=k-1} z_{i,j} \left[\lambda_{m,n} \cos \frac{m\pi x_i}{L} \cos \frac{n\pi y_j}{H} \right] \quad (34)$$

$$h_{m,n,2} = \sum_{j=0}^{j=1-1} \sum_{i=0}^{i=k-1} z_{i,j} \left[\lambda_{m,n} \sin \frac{m\pi x_i}{L} \cos \frac{n\pi y_j}{H} \right] \quad (35)$$

$$h_{m,n,3} = \sum_{j=0}^{j=1-1} \sum_{i=0}^{i=k-1} z_{i,j} \left[\lambda_{m,n} \cos \frac{m\pi x_i}{L} \sin \frac{n\pi y_j}{H} \right] \quad (36)$$

$$h_{m,n,4} = \sum_{j=0}^{j=1-1} \sum_{i=0}^{i=k-1} z_{i,j} \left[\lambda_{m,n} \sin \frac{m\pi x_i}{L} \sin \frac{n\pi y_j}{H} \right] \quad (37)$$

$$c_{m,n,p} = h_{m,n,p} / S_{m,n,p} \quad (38)$$

The same type of ranking of the double Fourier series coefficients according to the value $S_{m,n,p}^2 c_{m,n,p}^2$ can be made to select the most appropriate terms. Likewise if normality is assumed for the random component, ε , in the expression

$$z_1 = f(x_1, y_1) + \varepsilon \quad (39)$$

then an F test can be made to determine whether a particular coefficient should be added.

BALGOL SINGLE FOURIER SERIES PROGRAM

The BALGOL single Fourier series program may be used to analyze data that can be plotted as a curve. It is necessary that the data points be regularly spaced, and that an odd number of data points be used. The program provides for automatic plotting of the original data, the value of the Fourier series at each data point, and the residual value at each data point. Any number of harmonics up to 105 may be incorporated into the Fourier series.

Major Steps in Single Fourier Series Program

Major steps in the single Fourier series program are outlined below and listed in Table 3.

- (1) Declare integers: lines 3-5
- (2) Dimension arrays: lines 6-7
- (3) Format statements: lines 9-30
- (4) Input statements: lines 32-35, 42-64
- (5) Calculate various operands used to control loops and for other purposes: lines 36-41, 65-66
- (6) If ITRGOP equals 1, calculate values of cosines and sines storing in arrays TCOSNI() and TSINNI(): lines 69-81
- (7) If JAROP equals 1, print, or print and punch cosine and sine arrays: lines 83-109
- (8) Compute a_0 term: lines 112-114
- (9) Compute a_n and b_n terms and power spectrum values: lines 116-157
- (10) Compute sum of data values, sum of squared data values: lines 162-166
- (11) Set each point to be evaluated equal to a_0 : line 167
- (12) Scale data values plotting purposes and convert to integers XS(): lines 168-170
- (13) Calculate operand ZOR used later in calculation of percent of total sum of squares: line 172
- (14) For specified numbers of terms in Fourier series, compute calculated value at data points: lines 175-211
- (15) Calculate percent of total sum of squares: lines 212-219
- (16) Compute residual values and compute spacing of asterisks used for plotting of original data, calculated values and residual values: lines 224-239.

Table 3.- Listing of BALGOL statements in single Fourier series program. Each line represents one punched card. Numbers in left column are for reference purposes in this report and in practice may be placed within columns 73-80 on punched cards. Each program card must have 2 punched in column 1.

```

1  COMMENT SINGLE FOURIER SERIES PROGRAM FOR IBM 7090 OR 7094 IN CONJUNCT-
2  ION WITH STANFORD SUBALGOL COMPILER SYSTEM , F. PRESTON AND J.HARBAUGH$
3  INTEGER I,J, LA(1), N, N1, N2, N3, N4, NT,NI, N5, I1, IQ, IR, ISTMT ,
4  LSTMT , JVALOP, ITRGOP, JAROP, NR, LINE(1), KRES,KCAL, AST, KB1,
5  KB2, K ,INTOP, MTERM(1), IB, IPCHOP, I1I, N8, NTT,NTTT, N9, N6,NTQ$
6  ARRAY X( 401), XS( 401), LINE( 401), A(105), B(105), C(105), MTERM(105)
7  , TCOSNI(105,101), TSINNI(105,101), LA(12) $
8  AST = '!' $
9  FORMAT
10 FMTA(*5$*,12A6,*$*,C3,W),
11 FMT1(*COSINE ARRAY*,W),
12 FMT2(      *ARRAY CONTAINS *,           I4,* ROWS CORRESPONDING TO *,
13      I4,* DATA POINTS AND*,W,           I3,* COLUMNS CORRESPONDING *,
14      *TO *,I3,* N TERMS*,W,W),
15 FMTB(*5*,X7.4,9X8.4,C,W),      FMTW(16X8.4,W,W),
16 FMTC(12A6,W3,W),
17 FMTD(*PLOT BASE =*,X8.1,*      PLOT TOP =*,X8.1,*      SCALE =*,X8.2,
18      *      RESIDUAL AMPLIFICATION =*, X5.2,W,W),
19 FMT3(*AZERO/2 = *,X10.3,W,W),
20 FMT4(*SINE ARRAY*,W),
21 FMT5(*5*,I6,3X14.5,C,W),      FMT10(I6,B6,3X14.5,W,W),
22 FMT6(*DEGREE NUMBER      A(N)      B(N)      POWER SPECTRUM*,W,W),
23 FMT7(3X10.3,I5,B1,
24      *I*,B$LINE(I)$,A1,B$KB1$,*I*,B$KCAL$,A1,B$KB2$,*I*,B$KRES$,
25      A1,W),
26 FMT8(*OBSERVED      CALCULATED      RESIDUAL      OBSERVED      *,
27      *      CALCULATED      RESIDUAL*,W,W),
28 FMT9(*FOURIER SERIES FITTED TO *,I3,* DATA POINTS,      CONTAINS *,I3,
29      * A(N) TERMS AND *,I3,* B(N) TERMS*,W,
30      * AND PERCENT OF TOTAL SUM OF SQUARES = *,X8.2,W,W)$
31 START..
32 INPUT ALPH(FOR I =(1,1,12)$ LA(I)) $
33 INPUT CONTROL( NR, NT, ITRGOP, JAROP, JVALOP, INTOP,      BASE, TOP,
34 SCALE, AMP, IPCHOP)$
35 READ($$ALPH)$      READ($$CONTROL)$
36 N1 = NR-1 $ N2 = N1/2 $ N3 = NR+1 $ N4 = N3/2 $ N6 = N4+1 $
37 N5 = N1/4 $ FN5 = N1/4.0 $ FN5F = N5 $ N9 = N5 + 1 $
38 IF FN5 GTR FN5F $ (N8 = N5 + 1 $ GO PT1)$ N8 = N5 $ PT1..
39 NTT = 2.(NT/2)$ IF NTT LSS NT$(NTTT = NT$ NTQ = NT-1)$
40 IF NTT EQL NT$(NTTT = NT-1$ NTQ = NT)$
41 NR2 = 2.0/N1 $
42 IF JVALOP EQL 1 $
43 BEGIN
44 INPUT ORDERS(IR, FOR IQ =(1,1,IR) $ MTERM(IQ)) $
45 READ ($$ ORDERS) $
46 END $
47 IF ITRGOP EQL 2 $
48 BEGIN
49 INPUT TRIGARRAY3(FOR I =(1,1,N9) $ FOR N=(1,1,NT )$TCOSNI(N,I)),
50 TRIGARRAY4(FOR I =(1,1,N9) $ FOR N=(1,1,NT )$TSINNI(N,I))$
51 READ ($$ TRIGARRAY3) $ READ ($$ TRIGARRAY4) $
52 END $
53 COMMENT INPUT DATA IN EITHER INTEGER OR DECIMAL POINT FORM $
54 IF INTOP EQL 1 $
55 BEGIN
56 INPUT DATI(FOR I=(1,1,NR) $ LINE(I)) $
57 READ($$ DATI)$
58 FOR I =(1,1,NR) $ X(I) = LINE(I) $
59 END $
60 IF INTOP EQL 2 $
61 BEGIN
62 INPUT DATD(FOR I=(1,1,NR)$ X(I)) $
63 READ ($$ DATD) $
64 END $
65 PL = 3.1415926/N2 $
66 DIFF = TOP - BASE $ BD = BASE/DIFF $

```

```

67 COMMENT CALCULATE VALUES OF SINE AND COSINE FUNCTIONS FOR (N-1)/2
68 POINTS AND NT TERMS IF ITRGOP EQUALS 1 $
69 IF ITRGOP EQL 1 $
70 BEGIN
71   FOR N =(1,1,NT ) $
72   BEGIN
73     XNPL = N.PL $
74     FOR I =(1,1,N9) $
75     BEGIN
76       ARGNI = XNPL.(N4-I) $
77       TCOSNI(N,I) = COS(ARGNI) $
78       TSINNI(N,I) = SIN(ARGNI) $
79     END $
80   END $
81 END $
82 OUTPUT ALPHA(FOR I =(1,1,12)$ LA(I)) $
83 IF JAROP EQL 1 $
84 BEGIN
85   COMMENT PRINT AND PUNCH COSINE AND SINE ARRAYS $
86   IF IPCHOP EQL 2 $
87     WRITE ($$ ALPHA, FMTA) $
88   IF IPCHOP EQL 1 $ WRITE($$ ALPHA, FMTC)$
89   WRITE ($$ FMT1) $
90   OUTPUT TERMS(N9,NR,NT ,NT )$
91   WRITE ($$ TERMS, FMT2) $
92   FOR I =(1,1,N9) $
93   BEGIN
94     OUTPUT T3(FOR N =(1,1,NT ) $ TCOSNI(N,I))$
95     IF IPCHOP EQL 2 $
96       WRITE ($$ T3, FMTB) $
97     IF IPCHOP EQL 1 $ WRITE($$ T3, FMTW)$
98   END $
99   IF IPCHOP EQL 2$
100   WRITE ($$ FMT4) $
101   WRITE ($$ TERMS, FMT2) $
102   FOR I =(1,1,N9) $
103   BEGIN
104     OUTPUT T4(FOR N =(1,1,NT ) $ TSINNI(N,I)) $
105     IF IPCHOP EQL 2 $
106       WRITE ($$ T4, FMTB )$
107     IF IPCHOP EQL 1$ WRITE($$ T4, FMTW)$
108   ENDS
109 END $
110 COMMENT COMPUTE, PRINT AND PUNCH FOURIER COEFFICIENTS AND POWER
111 SPECTRUM VALUES $
112 AZ =0.0 $
113 FOR I =(2,1,N1)$ AZ = AZ + X(I) $
114 AZ =((AZ + ((X(1) + X(NR)).0.5)).NR2).0.5 $
115 WRITE($$ALPHA,FMTC) $
116 OUTPUT AZERO(AZ)$ WRITE($$AZERO, FMT3)$ WRITE($$FMT6)$
117 FOR N =(1,1,NT ) $
118 BEGIN
119   A(N) = B(N) = 0.0 $
120   FOR I =(2,1,N8) $
121   BEGIN
122     II = N3 - I $
123     A(N) = A(N) + (TCOSNI(N,I) .(X(I) + X(II))) $
124     B(N) = B(N) + (TSINNI(N,I).(X(I) - X(II))) $
125   END $
126 END $
127 FOR I =(2,1,N8)$
128 BEGIN
129   II = N6 - I $ III = N2 + I $
130   FOR N = (1,2,NTII)$
131   BEGIN
132     A(N) = A(N) - (TCOSNI(N,I) .(X(II) + X(III))) $
133     B(N) = B(N) + (TSINNI(N,I) .(X(II) - X(III))) $
134   ENDS
135   FOR N=(2,2,NTQ)$
136   BEGIN
137     B(N) = B(N) - (TSINNI(N,I) .(X(II) - X(III))) $
138     A(N) = A(N) + (TCOSNI(N,I) .(X(II) + X(III))) $
139   ENDS
140 ENDS

```

```

141 IF FN5 EQL FN5F $
142 BEGIN
143   FOR N =(2,2,NTQ) $
144     A(N) = A(N) + ((TCOSNI(N,N9)).(X(N9) + X(NR-N5))) $
145   FOR N =(1,2,NTTT)$
146     B(N) = B(N) + (TSINNI(N,N9).(X(N9) - X(NR-N5))) $
147 END $
148 FOR N = (1,1,NT)$
149   BEGIN
150     A(N)=(A(N) + (0.5.(X(1) + X(NR)).TCOSNI(N,1))).NR2) + (X(N4).NR2)$
151     B(N) =(B(N) + (0.5.(X(1) + X(NR)).TSINNI(N,1))).NR2 $
152     C(N) = ((A(N).A(N)) + (B(N).B(N))) $
153     OUTPUT FC( N, A(N),B(N),C(N)) $
154     IF IPCHOP EQL 2$
155     WRITE ($$ FC, FMT5) $
156     IF IPCHOP EQL 1$ WRITE($$ FC, FMT10)$
157 END $
158 COMMENT IF JVALOP EQUALS 1, EVALUATE FOURIER SERIES FOR 1R DIFFERENT
159 NUMBERS OF TERMS AS SPECIFIED IN MTERM() ARRAY. ALSO COMPUTE SUMSQS $
160 IF JVALOP EQL 1 $
161 BEGIN
162   SUMX = SUMX2 = 0.0 $
163   FOR I =(1,1,NR) $
164     BEGIN
165       SUMX = SUMX + X(I) $
166       SUMX2= SUMX2+ (X(I).X(I))$
167       XS(I)= AZ $
168       LINE(I)=((X(I)/DIFF) - BD).SCALE $
169       IF LINE(I) GTR 30 $(LINE(I) = 30 $ GO BEYOND)$
170       IF LINE(I) LSS 0 $ LINE(I) = 0 $ BEYOND..
171     END $
172   ZOR = SUMX2 -((SUMX.SUMX)/ NR)$
173   LSTMT = ISTMT = 0 $
174   OUTPUT SCLHED(BASE, TOP, SCALE, AMP)$
175   FOR IQ =(1,1,IR) $
176     BEGIN
177       WRITE($$ ALPHA, FMTC)$
178       ISTMT = LSTMT+1 $
179       LSTMT = MTERM(IQ) $
180       FOR I =(2,1,N8) $
181         BEGIN
182           IB = I + N2 $ II = N3 - I $
183           FOR N =(ISTMT,1,LSTMT) $
184             BEGIN
185               XS(I) =XS(I) + (A(N).TCOSNI(N,I)) + (B(N).TSINNI(N,I)) $
186               NI = 2.(N/2) $
187               IF N GTR NI$
188                 BEGIN
189                   XS(N6-I)= XS(N6-I) -(A(N).TCOSNI(N,I))+(B(N).TSINNI(N,I))$
190                   XS(IB) = XS(IB) -(A(N).TCOSNI(N,I))-(B(N).TSINNI(N,I))$
191                   XS(II) = XS(II) + (A(N).TCOSNI(N,I))-(B(N).TSINNI(N,I))$
192                 END $
193               IF N EQL NI $
194                 BEGIN
195                   XS(N6-I)= XS(N6-I)+ (A(N).TCOSNI(N,I))-(B(N).TSINNI(N,I))$
196                   XS(IB) = XS(IB) + (A(N).TCOSNI(N,I))+(B(N).TSINNI(N,I))$
197                   XS(II) = XS(II) + (A(N).TCOSNI(N,I))-(B(N).TSINNI(N,I))$
198                 END $
199             END $
200           END $
201         FOR N =(ISTMT,1,LSTMT)$
202           BEGIN
203             XS(N4) = XS(N4) + A(N) $
204             IF FN5 EQL FN5F $
205               BEGIN
206                 XS(N9) = XS(N9) + (A(N).TCOSNI(N,N9)) + (B(N).TSINNI(N,N9)) $
207                 XS(NR-N5)=XS(NR-N5)+(A(N).TCOSNI(N,N9))-(B(N).TSINNI(N,N9)) $
208               END $
209             XS(1) = XS(1) + (A(N).TCOSNI(N,1)) $
210           END$
211           XS(NR) = XS(1) $
212           COMMENT CALCULATE PERCENT SUM OF SQUARES $
213           SUMVAL = SMVAL2 = 0.0 $

```

```

214     FOR I =(1,1,NR) $
215     BEGIN
216         SUMVAL = SUMVAL + XS(I) $
217         SMVAL2 = SMVAL2 + (XS(I),XS(I)) $
218     END $
219     PCTS = 100.0,((SMVAL2 -((SUMVAL,SUMVAL)/NR))/ZOR) $
220     OUTPUT HED(NR,MTERM(IQ),MTERM(IQ), PCTS)$
221     WRITE($$ HED, FMT9) $
222     WRITE($$ SCLHED,FMTD)$
223     WRITE ($$ FMT8) $
224     FOR I =(1,1,NR) $
225     BEGIN
226         K = N4 - I $
227         KB1 = 30 - LINE(I) $
228         KCAL = ((XS(I)/DIFF) - BD).SCALE $
229         IF KCAL GTR 30 $(KCAL = 30 $ GO PAST )$
230         IF KCAL LSS 0 $ KCAL = 0 $
231         PAST..
232         KB2 = 30 - KCAL $
233         RESID = X(I) -XS(I) $
234         KRES = (RESID.AMP)+ 15.0 $
235         IF KRES GTR 30 $ KRES = 30 $
236         IF KRES LSS 0 $ KRES = 0 $
237         OUTPUT GRAPH(X(I),XS(I),RESID,K, AST,AST,AST)$
238         WRITE($$ GRAPH, FMT7) $
239     END $
240 END $
241 END $
242 GO START $ FINISH $

```

Input to Single Fourier Series Program

Alphanumeric heading card.— The first card of each data set (example data set is shown in Table 4) contains alphabetical and numerical (alphanumeric) information for identification purposes. This information will be reproduced at certain places in output from the program (Table 5, Fig. 3), for identification purposes. The card should be punched as follows:

- (1) 5 in column 1.
- (2) \$ in columns 2 and 75.
- (3) Any desired combination of letters, numbers, characters and blanks in columns 3 through 74.

Program-control card.— The second card that accompanies each data set must be a program control card containing the following information listed in the sequences below. Note that on this and all subsequent data cards, each number is to be separated from adjacent numbers by at least one blank space on the card. Except for 5 in column 1, the numbers may be placed in whatever columns are convenient, providing that the described sequence of numbers is maintained:

- (1) 5 in column 1.
- (2) An integer (NR) specifying the number of observations. This number must be odd, and may not exceed 401 unless the array dimensions in the program are changed.
- (3) An integer (NT) specifying the maximum number of cosine or sine terms that the Fourier series is to contain, including the a_0 term. For example, if a_0 , a_1 , a_2 , a_3 , and a_4 terms are to be included, the value of NT is 5. Unless array dimensions are changed, the maximum number of cosine or sine terms is 105.

- (4) An integer (ITRGOP) specifying whether the arrays of cosine and sine values are to be fed in as data or are to be calculated, as follows:
- 1 calculate
 - 2 feed in on data cards
- This provision allows arrays of cosine and sine values that have been previously calculated and punched to be re-used without having to be recalculated.
- (5) An integer (JAROP) specifying whether arrays of cosine and sine values are to be part of output from program, as follows:
- 0 arrays are not part of output
 - 1 arrays are part of output.
- (6) An integer (JVALOP) specifying whether value of Fourier series and residual value is to be calculated and plotted for each data point, as follows:
- 0 do not calculate and plot
 - 1 do calculate and plot
- (7) An integer (INTOP) specifying whether observed data values are to be fed as integers or as decimal point numbers, as follows:
- 1 feed in as integers
 - 2 feed in as decimal numbers.
- (8) A decimal (BASE) specifying the algebraically lowest value which is chosen to serve as the base line when the observed and calculated data values are plotted. In general, this value would be equal to the lowest observed data value, but it can be set at any desired value; any value less than BASE is plotted adjacent to the base line.
- (9) A decimal (TOP) specifying the value for the top of the plots of observed and calculated data values.
- (10) A decimal value (SCALE) specifying the number of columns to be used for each of the three graphs. The value of SCALE, in conjunction with the difference between TOP and BASE, determines the range of plotted observed and calculated data points (see Table 5 and Figure 3 for examples of output). SCALE must be less than or equal to 30.0.
- (11) All residuals are multiplied by the decimal value AMP to place them in the range -15.0 to +15.0 for plotting. Values beyond the +15 to -15 range are plotted at the limits of the graph.
- (12) An integer (IPCHOP) specifying whether cosine and sine arrays, and Fourier coefficients and power spectrum values are to be punched or not, as follows:
- 1 do not punch
 - 2 punch.

Term-control card.- If the Fourier series is to be evaluated (JVALOP = 1 on control card), the third data card must contain information that controls the number of terms for which evaluations are made, as follows:

- (1) 5 in column 1.
- (2) An integer (IR) specifying the number of times the series is to be evaluated.
- (3) Integers MTERM() which specify the number of terms to be included in each evaluation. These numbers must be in ascending order. For example, if it is desired to evaluate the series for 3, 5, and 6 terms, the card would be punched so that IR is 3 and the values for MTERM() are 3, 5, and 6. See Table 4 for example.

If JVALOP does not equal 1, this card should not be included in the data deck.

Trigonometric arrays.— If the arrays containing values of cosines and sines have been calculated and punched previously, they may be fed in as data, provided that ITRGOP = 2 on the control card. The previously punched array cards should be inserted in the deck here, and are already in correct form. If ITRGOP does not equal 2, the arrays of trigonometric values must be omitted from the data deck.

Observational data cards.— Cards containing observational data should be arranged so that the data are read in proper sequence, from one end of the data string to the other.

- (1) 5 in column 1 of each card.
- (2) Any convenient number of data values per card. The type (integer or decimal) must be consistent and agree with the specification (INTOP) on the control card. The number of data values must be exactly equal to NR on the control card. At least one blank space must separate each data value from the next.

Sequences of data sets.— Any number of data sets may be run in succession. Each must be complete with alphanumeric heading card, control card, term control card (if required), trigonometric arrays (if these are to be read in), and observational data cards.

Table 4.— Listing of example data set used as input to BALGOL single Fourier series program. Each line represents one punched card.

5\$TEST FOR SINGLE FOURIER SERIES INVOLVING 30 TERMS AND 101 ROWS											\$	
5	101	30	1	1	1	2	-500.0	2500.0	50.0	0.3	1 *CONTROL FOR TES	
5	1	30										
5	123.0700		121.2843		116.7837		112.4418		108.1571		103.8411	99.4086
5	94.7689		89.8197		84.4359		78.4513		71.6392		63.6693	54.0321
5	41.8888		25.7551		2.6989		-33.9900		-103.8220		-297.4028	2259.1883
5	581.0975		327.1315		245.9656		205.0907		179.8516		162.2657	148.9642
5	138.2744		129.2665		121.3769		114.2394		107.5995		101.2680	95.0932
5	88.9450		82.7002		76.2311		69.3952		62.0194		53.8786	44.6640
5	33.9253		20.9676		4.6440		-17.0973		-48.3479		-98.6406	-196.5531
5	-484.9389		1879.9997		652.3506		363.8721		265.8044		215.2927	183.7574
5	161.6636		144.9166		131.4604		120.1434		110.2651		101.3676	93.1324
5	85.3235		77.7532		70.2602		62.6933		54.8984		46.7036	37.9003
5	28.2209		17.2957		4.5867		-10.7406		-30.0670		-55.8824	-93.1989
5	-153.8722		-274.3935		-649.1845		1447.1137		669.2601		372.9883	266.8839
5	211.1551		176.0176		151.2681		132.4580		117.3326		104.6196	93.5412
5	83.5895		74.4051		65.7276		57.3494		49.0887		40.7797	32.2538
5	23.3215		13.7558		3.2639							

Output From Single Fourier Series Program

Initial output from the single Fourier series program is shown in Table 5. If specified (JAROP = 1), the arrays of cosine and sine values are printed. The a_0 value is printed, and the coefficients representing the a_n and b_n terms and the power spectrum values are printed. In addition, the cosine and sine arrays, and the a_n and b_n coefficients and power spectrum values will also be punched if IPCHOP = 2. Figure 3 illustrates subsequent output in which the original data, evaluated Fourier series, and residual values are listed and also graphically plotted.

Table 5.- Initial output from single Fourier series program.

GENERATED TEST DATA , SINGLE FOURIER SERIES PROGRAM, SINE + COSINE TERMS				
COSINE ARRAY				
ARRAY CONTAINS 8 ROWS CORRESPONDING TO 31 DATA POINTS AND 5 COLUMNS CORRESPONDING TO 5 N TERMS				
-1.0000	1.0000	-1.0000	1.0000	-1.0000
-.9781	.9135	-.8090	.6691	-.5000
-.9135	.6691	-.3090	-.1045	.5000
-.8090	.3090	.3090	-.8090	1.0000
-.6691	-.1045	.8090	-.9781	.5000
-.5000	-.5000	1.0000	-.5000	-.5000
-.3090	-.8090	.8090	.3090	-1.0000
-.1045	-.9781	.3090	.9135	-.5000
SINE ARRAY				
ARRAY CONTAINS 8 ROWS CORRESPONDING TO 31 DATA POINTS AND 5 COLUMNS CORRESPONDING TO 5 N TERMS				
.0000	-.0000	.0000	-.0000	.0000
.2079	-.4067	.5878	-.7431	.8660
.4067	-.7431	.9511	-.9945	.8660
.5878	-.9511	.9511	-.5878	-.0000
.7431	-.9945	.5878	.2079	-.8660
.8660	-.8660	-.0000	.8660	-.8660
.9511	-.5878	-.5878	.9511	.0000
.9945	-.2079	-.9511	.4067	.8660
GENERATED TEST DATA , SINGLE FOURIER SERIES PROGRAM, SINE + COSINE TERMS				
AZERO/2 = 49.954				
DEGREE NUMBER	A(N)	B(N)	POWER SPECTRUM	
1	32.00552	23.96284	1598.57057	
2	33.88898	27.93007	1928.55170	
3	36.01508	31.98282	2319.98712	
4	37.82895	35.84264	2715.72385	
5	40.16360	40.13792	3224.16801	

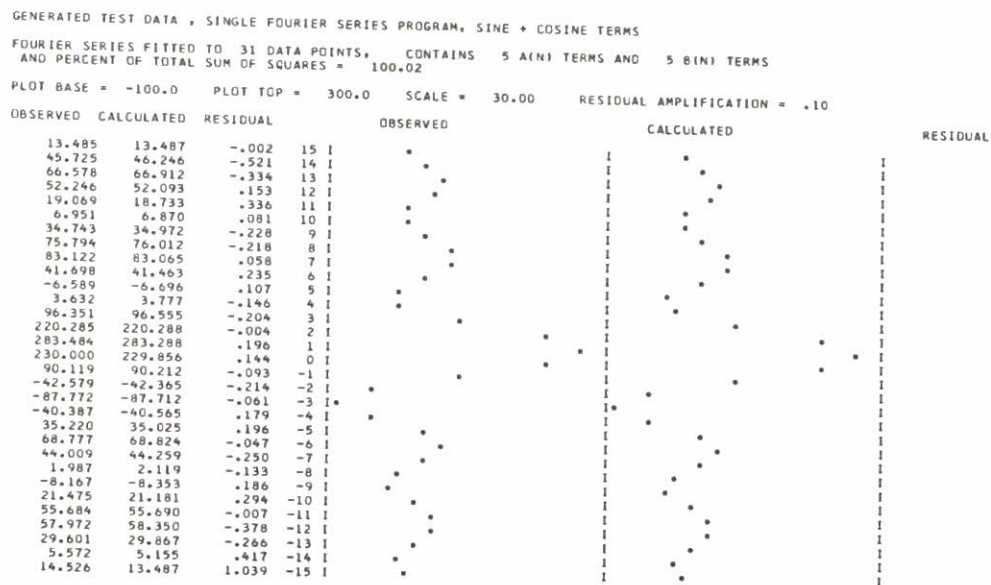
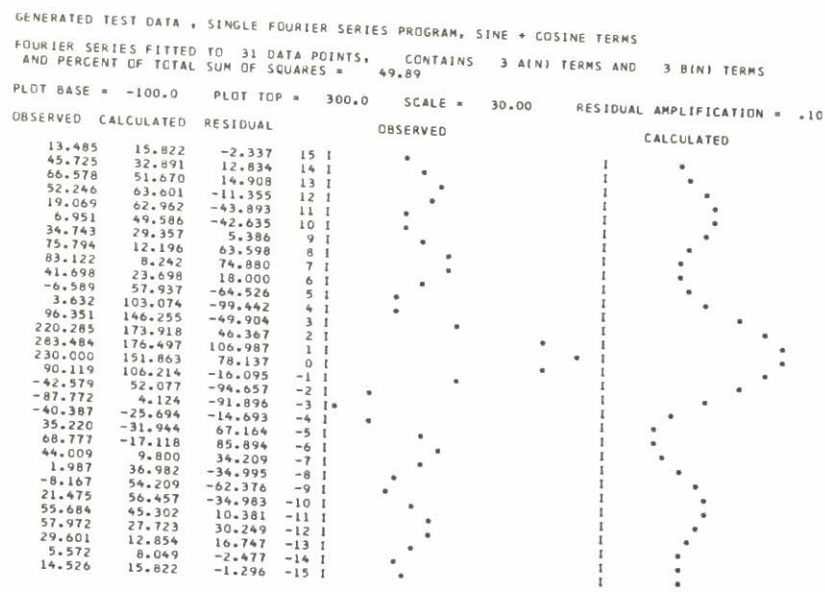


Figure 3.- Subsequent output from single Fourier series program, listing and plotting observed data, evaluated Fourier series, and residual values. Upper output data set pertains to series containing 3 a_n and 3 b_n terms, whereas lower data set pertains to series with 5 a_n and 5 b_n terms.

Single Fourier Series Program Limitations and Operating Times

The BALGOL single Fourier series program compiles in about 10 seconds on the IBM 7090. Execution time varies with the number of data points, the number of terms that are to be included in the Fourier series, and whether the arrays of sine and cosine values must be calculated, or are fed in on data cards, having been calculated previously. Execution time for the example shown in Table 4 and Figure 3 required approximately 10 seconds. This example contains 31 data points, a maximum of 5 Fourier a_n and b_n terms were used, and the series was evaluated for two different sets of terms.

The program is limited to a maximum of 401 data points in a given data set, and a maximum of 105 a_n and 105 b_n terms. By changing the array dimensions (lines 6-7, Table 3), the number of data points can be greatly increased if the maximum number of Fourier terms is reduced.

Single Fourier Series Program Testing and Verification

The single Fourier series program has been extensively tested and is believed to be free of errors. The results obtained with synthetic data generated by the short program listed in Table 6 indicate that the Fourier coefficients used in generating that data were completely recovered by the single Fourier series program (Table 3).

Table 6.- Listing of BALGOL program to generate test data for verification of single Fourier series program. The a_0 term is given on line 8, and constants of lines 12 and 13 pertain to a_n and b_n coefficients. Line 31 is single data card. Output from program is listed in Table 4. Numbers in left column are for reference purposes in this report, and in practice can be placed in columns 73-80 of punched cards. A 2 should be punched in column 1 of each card and the statement can start in any other column.

```

1  COMMENT DATA GENERATOR TO TEST  SINGLE FOURIER SERIES PROGRAM $
2  INTEGER  I, J, N , P, XX          $
3  ARRAY  T(201),A(60),B(60)        $
4  FORMAT FMT(*5*,7X11.4,C) , FMT1(13X10.4,W)$
5  START..
6  INPUT CONTROL(P,XX)$ READ($$ CONTROL)$
7  AH = (XX-1)/2.0 $ AHH= (XX+1)/2.0 $
8  AZ = 50.0$
9  FOR N =(1,1,P) $
10 BEGIN
11   FN = N $
12   A(N) =30.0 + (2.0.FN) $
13   B(N) =20.0 + (4.0.FN) $
14 END $
15 OUTPUT ARA(FOR N=(1,1,P)$A(N)), ARB(FOR N=(1,1,P)$B(N))$
16 WRITE($$ARA,FMT1)$ WRITE($$ARB,FMT1)$
17 FOR I =(1,1,XX) $
18 BEGIN
19   T(I) = AZ $
20   FI = I  $ Y = AHH - FI $
21   FOR N =(1,1,P) $
22   BEGIN
23     FNT = N $ PINL = FNT.0.2097287 $
24     ARGY = PINL.Y $
25     T(I) = T(I)+ (A(N).COS(ARGY)) + (B(N).SIN(ARGY)) $
26   END $
27 END $
28 OUTPUT DATA(FOR I =(1,1,XX) $ T(I)) $
29 WRITE ($$ DATA, FMT) $          GO START$
30 FINISH $
31 5      30      101      *CONTROL FOR SINGLE TEST GEERATOR

```

Major Steps in BALGOL Double Fourier Series Program

Steps in the program are outlined below and the program is listed in Table 7.

- (1) Integer declarations: lines 4-9
- (2) Dimension arrays: lines 10-13
- (3) Format statements: lines 15-64
- (4) Input and read statements: lines 69-73, 83-137
- (5) If specified, multiply data values by a scale factor: line 138
- (6) Calculate values of simple variables used for control and other purposes: lines 74-82
- (7) If specified, plot raw data values: lines 139-143
- (8) If specified, provide for leveling of input data by computation of matrix equation, matrix inversion and calculation of residual values after leveling: lines 144-229
- (9) If specified, fill arrays of trigonometric function values by calculation and assignment, as follows:
 - (a) Calculate values of trigonometric functions in arrays indexed on N and I: lines 231-245
 - (b) Fill remainder of arrays of trigonometric functions indexed on N and I by assignment: lines 246-265
 - (c) Calculate values of trigonometric functions indexed on M and J: lines 266-275
 - (d) Fill remainder of arrays of trigonometric functions indexed on M' and J by assignment: lines 276-297
- (10) If specified, print or print and punch, arrays of trigonometric values: lines 299-335
- (11) Compute Fourier coefficients by multiplication of observed data values by appropriate trigonometric values, followed by summation: lines 336-442
- (12) Compute harmonic vector values: lines 443-445
- (13) Print, or print and punch, arrays of Fourier coefficients, and vector magnitude values: lines 448-473
- (14) If specified, restore tilt to leveled initial data and to Fourier surface data: lines 477-491
- (15) Compute sums of squared observed data values: lines 492-502, 568-577
- (16) If specified, evaluate Fourier series for different numbers of terms in series, in ascending order: lines 503-567
- (17) If specified, print out strip maps of calculated data values for specified number of terms in Fourier series: lines 579-653
- (18) If specified, print out strip maps of residual values: lines 654-702
- (19) If specified, plot profiles along specified vertical and horizontal rows: lines 703-755
- (20) Binary external procedure INPROD used within matrix inversion method of lines 169-217: lines 757-760

Table 7.- Listing of BALGOL statements in double Fourier series program. Each line represents one punched card. Numbers in left column are for reference purposes in this report, and in practice can be placed within columns 73-80 on punched cards. Each program card (except for binary external procedure, lines 757-759) must have 2 punched in column 1.

```

1  COMMENT DOUBLE FOURIER SERIES PROGRAM FOR IBM 7090 OR 7094 IN CONJUNC-
2  TION WITH STANFORD SUBALGOL COMPILER SYSTEM. F. PRESTON AND J. HARBAUGH
3  UNIV OF KANSAS AND STANFORD UNIV $
4  INTEGER LA(), NR,MC, N1, N2, N3, N4, M1, M2, M3, M4, I, J, M,N, MT, NT,
5  KB1,KCAL,ITRGOP, II, JJ,KB2, JAROP, JVALOP, ISTMT, LSTMT, 1STNT, L,
6  LSTNT, NTERM(), MTERM(), IQ, IR, LINE(), INTOP, AST, JRESOP,KRES,
7  JPRTOP, IPCHOP, IB,K, JB, HP, VP, IT, JPROF, IVALS(),JVALS(),IH,
8  JDTOP, M5, N5, JCOFOP, P, Q, M7,N7, MTT,NTT $
9  INTEGER NTTT, MTTT, M8, N8 ,M9, N9 ,JPLAN, LVLOP,JTILT $
10 ARRAY TSINNI(25, 71), TCOSNI(25, 71), TSINMJ(25, 73), TCOSMJ(25, 73),
11 X( 71,73), A(25,25), B(25,25), C(25,25), D(25,25), PS(25,25),
12 IVALS(15), JVALS(15),
13 ALAM( 2, 2) , NTERM(25), MTERM(25), LA(12), LINE( 73),XS( 71, 73)$
14 AST = '!' $
15 FORMAT
16 FMTA(12A6,W,W), FMTC(*5$*,12A6,$$,C3,W),
17 FMTB(*5$,X7.4,9X8.4,C,W), FMTW(16X8.4,W,W),
18 FMT1(12A6,W3,W),
19 FMT2(*COSINE MJ ARRAY*,W),
20 FMT3(*SINE MJ ARRAY*,W),
21 FMT5(*COSINE NI ARRAY*,W),
22 FMT6(*SINE NI ARRAY*,W),
23 FMT7(*PLOT OF LEVELED ORIGINAL DATA VALUES, WITH APPROXIMATE ZERO MEAN*
24 ,W,W,W),
25 FMT8(*A ARRAY OF FOURIER COEFFICIENTS*,W,W),
26 FMT9(*B ARRAY OF FOURIER COEFFICIENTS*,W,W),
27 FMT10(*C ARRAY OF FOURIER COEFFICIENTS*,W,W),
28 FMT11(*D ARRAY OF FOURIER COEFFICIENTS*,W,W),
29 FMT12(*HARMONIC VECTOR MAGNITUDE ARRAY*,W,W),
30 FMT13( 12F11.5,W,W),
31 FMT14( 12X11.2,W2 ),
32 FMT15(*VALUES OF EVALUATED DOUBLE FOURIER SERIES AT SPECIFIED GRID*,
33 * POINTS, NUMBER OF COLUMNS = *,I4,W, *NUMBER OF ROWS = *,I4,
34 * , NUMBER OF M TERMS =*,I4,* , NUMBER OF N TERMS =*,I4,
35 W, *PERCENT OF TOTAL SUM OF *,
36 *SQUARES = *,X8.2,
37 * , SCALE FACTOR = *,X8.2,W ),
38 FMT16(26I5 ,W2,W),
39 FMT17(*STRIP ONE*,W),
40 FMT18(*STRIP TWO*,W),
41 FMT19(*STRIP THREE*,W),
42 FMT21(*STRIP FOUR*,W),
43 FMT22(*5*,7X11.2,C) ,
44 FMT23($MT$F11.5,W,W),
45 FMT24($MT$X11.2,W2 ),
46 FMT25(*RESIDUAL VALUES, NUMBER OF COLUMNS = *,I3,* , NUMBER OF ROWS = *,
47 I3,* , NUMBER OF M TERMS = *,I3,* , NUMBER OF N TERMS =*,I4,
48 W,*SCALE FACTOR =*,X8.3,W,W)$
49 FORMAT
50 FMT26(*HORIZONTAL PROFILE OF ROW*,I6, W ,
51 *PLOT TOP = *,X12.3,* PLOT BASE = *,X12.3,B4,*SCALE = *,X6.3,B4,
52 *RESIDUAL AMPLIFICATION FACTOR = *,X6.3,
53 * , M = *,I4,* , N = *,I4,W,W),
54 FTT8(*OBSERVED CALCULATED RESIDUAL OBSERVED
55 * CALCULATED RESIDUAL*,W,W),
56 FMT27(3X10.3,I5,B1,
57 *I*,B$IT$,A1,B$KB1$,*I*,B$KCAL$,A1,B$KB2$,*I*,B$KRES$,
58 A1,W,W,W),
59 FMT28(*VERTICAL PROFILE OF COLUMN*,I6,W ,
60 *PLOT TOP = *,X12.3,* PLOT BASE = *,X12.3,B4,*SCALE = *,X6.3,B4,
61 *RESIDUAL AMPLIFICATION FACTOR = *,X6.3,
62 * , M = *,I4,* , N = *,I4,W,W),
63 FMT29(*PLOT OF OBSERVED DATA VALUES, NUMBER OF ROWS = *,I4,
64 * , NUMBER OF COLUMNS = *,I4,* , SCALE FACTOR = *,X8.3,W,W,W)$
65 COMMENT ASSIGN LAMBDA VALUES $
66 ALAM(1,1)=0.25 $
67 ALAM(2,1) = ALAM(1,2) = 0.5 $ ALAM(2,2) = 1.0 $

```

```

68 START..
69 INPUT ALPH(FOR I =(1,1,12)$ LA(I)) $
70 READ ($$ ALPH) $
71 INPUT CONTROL(NR, MC, NT, MT, ITRGOP,JAROP,INTOP,JVALOP, JPRTOP,JRESOP,
72 JDTOP, 1PCHOP, L, SCALE, JPROF,JCOFOP, LVLOP, JTILT,JPLAN)$
73 READ($$ CONTROL)$
74 N1 = NR-1 $ N2 = N1/2 $ N3 = NR+1 $ N4 = N3/2 $ N5 = N1/4 $
75 M1 = MC-1 $ M2 = M1/2 $ M3 = MC+1 $ M4 = M3/2 $ M5 = M1/4 $
76 PH = 3.1415926/M2 $ PL = 3.1415926/N2 $ RLH=4.0/(N1,M1)$
77 M9 = M5 + 1 $ N9 = N5 + 1 $ M7 = M4 + 1 $ N7 = N4 + 1 $
78 MTT = 2.0*(MT/2) $ NTT = 2.0*(NT/2) $
79 FN5 = N1/4.0 $ FN5F = N5 $ IF FN5 GTR FN5F$(N8= N5 + 1 $GO PT1)$
80 N8 = N5 $ PT1..
81 FM5 = M1/4.0 $ FM5F = M5 $ IF FM5 GTR FM5F$(M8= M5 + 1 $GO PT2)$
82 M8 = M5 $ PT2..
83 IF JVALOP EQL 1$
84 BEGIN
85 INPUT ORD1(IR, FOR IQ =(1,1,IR)$ NTERM(IQ))$ READ($$ ORD1)$
86 INPUT ORD2( FOR IQ =(1,1,IR)$ MTERM(IQ))$ READ($$ ORD2)$
87 END $
88 IF ITRGOP EQL 2 $
89 BEGIN
90 INPUT TRIGARRAY1(FOR J =(1,1,M9)$ FOR M =(1,1,MT)$ TCOSMJ(M,J)),
91 TRIGARRAY2(FOR J =(1,1,M9)$ FOR M =(1,1,MT)$ TSINMJ(M,J)),
92 TRIGARRAY3(FOR I =(1,1,N9)$ FOR N =(1,1,NT)$ TCOSNI(N,I)),
93 TRIGARRAY4(FOR I =(1,1,N9)$ FOR N =(1,1,NT)$ TSINNI(N,I))$
94 READ ($$ TRIGARRAY1) $ READ ($$ TRIGARRAY2) $
95 READ ($$ TRIGARRAY3) $ READ ($$ TRIGARRAY4) $
96 END $
97 IF JPROF EQL 1 $
98 BEGIN
99 INPUT HORPROF(TOP,BASE,AMP,VSCALE, HP, FOR I=(1,1,HP)$IVAL$(I))$
100 READ ($$ HORPROF) $
101 INPUT VERTPROF(VP, FOR J =(1,1,VP) $ JVAL$(J)) $
102 READ ($$ VERTPROF) $
103 END $
104 IF INTOP EQL 1 $
105 BEGIN
106 FOR I =(1,1,NR) $
107 BEGIN
108 INPUT DAT1(FOR J=(1,1,MC) $ LINE(J)) $ READ ($$ DAT1) $
109 FOR J =(1,1,MC) $ X(I,J) = LINE(J) $
110 END $
111 END $
112 IF INTOP EQL 2 $
113 BEGIN
114 INPUT DATD(FOR I =(1,1,NR)$ FOR J =(1,1,MC) $ X(I,J)) $
115 READ ($$ DATD) $
116 END $
117 IF INTOP EQL 3 $
118 BEGIN
119 FOR I =(1,1,NR)$
120 BEGIN
121 INPUT DATZ1(FOR J =(1,1,MC) $ LINE(J))$ READ($$DATZ1)$
122 FOR J =(1,1,MC)$
123 BEGIN
124 IF LINE(J) EQL 0 $ (X(I,J) = -100.0 $ GO SECLAB)$
125 X(I,J) = 100.0 $ SECLAB..
126 END$
127 END$
128 SCALE = 100.0 $
129 END$
130 IF INTOP EQL 4 $
131 BEGIN
132 FOR I =(1,1,NR) $
133 BEGIN
134 INPUT DAT4(FOR J=(1,1,MC) $ LINE(J)) $ READ ($$ DAT4) $
135 FOR J =(1,1,MC) $ X(I,J) =-LINE(J) $
136 END $
137 END $
138 IF L EQL 1$(FOR I=(1,1,NR)$FOR J=(1,1,MC)$ X(I,J) =SCALE.X(I,J))$
139 IF JDTOP EQL 1 $
140 BEGIN
141 FOR I =(1,1,NR) $ FOR J =(1,1,MC) $ XS(I,J) = X(I,J) $

```

```

142 GO DATPLOT $
143 END $
144 RESUME..
145 IF LVLOP EQL 1 $
146 BEGIN
147 COMMENT THIS SECTION PROVIDES FOR LEVELING OF INPUT DATA
148 COMPUTE MATRIX WITH PIERSON'S ARITHMETIC PROGRESSION METHOD $
149 ARRAY APMAT(5,5), COVEC(5), DATVEC(5) $
150 APMAT(1,1) = 0.1666.MC.NR.(MC-1).((2.MC)-1) $
151 APMAT(1,2) = 0.25.NR.(NR-1).MC.(MC-1) $
152 APMAT(1,3) = 0.5.MC.NR.(MC-1) $
153 APMAT(2,1) = 0.25.NR.(NR-1).MC.(MC-1) $
154 APMAT(2,2) = 0.1666.NR.MC.(NR-1).((2.NR)-1) $
155 APMAT(2,3) = 0.5.NR.MC.(NR-1) $
156 APMAT(3,1) = 0.5.MC.NR.(MC-1) $
157 APMAT(3,2) = 0.5.MC.NR.(NR-1) $
158 APMAT(3,3) = NR.MC $
159 COMMENT COMPUTE DATVEC BY SUMMATION $
160 DATVEC(1)=DATVEC(2)=DATVEC(3)=0.0 $
161 FOR I =(1,1,NR) $ FOR J =(1,1,MC) $
162 BEGIN
163 DATVEC(1)= DATVEC(1) + ((J-1).X(I,J)) $
164 DATVEC(2) = DATVEC(2) + ((I-1).X(I,J)) $
165 DATVEC(3) = X(I,J) + DATVEC(3) $
166 END $
167 COMMENT OBTAIN PLANE COEFFICIENTS BY MATRIX INVERSION $
168 EXTERNAL PROCEDURE INPROD() $
169 PROCEDURE SOLV(N$A(,),B(,),Y())$SINGULAR)$
170 BEGIN
171 COMMENT THIS IS THE METHOD OF CROUT,WITH INTERCHANGES,
172 TO SOLVE AY=B FOR Y,GIVEN A AND B. $
173 COMMENT EXTERNAL PROCEDURE INPROD IS CALLED BY SOLV,
174 SO INPROD MUST BE AVAILABLE WHEN SOLV IS CALLED. $
175 COMMENT SINGULAR IS THE LABEL OF THE STATEMENT TO WHICH
176 SOLV() EXITS IF A() IS SINGULAR$
177 COMMENT REAL A(,),B(,),Y()$
178 INTEGER I,IMAX,J,K,N$
179 FOR K=(1,1,N)$
180 BEGIN
181 TEMP = 0$
182 FOR I=(K,1,N)$
183 BEGIN
184 XX = A(I,K) - INPROD(1,1,K-1,A(I,),A(K,))$
185 A(I,K) = XX $
186 IF ABS(XX) GTR TEMP $
187 3BEGIN
188 TEMP = ABS(XX) $
189 IMAX = I
190 END
191 END$
192 IF TEMP EQL 0.0$
193 GO SINGULAR$
194 COMMENT WE HAVE FOUND THAT A(IMAX,K) IS THE LARGEST PIVOT IN COL K.
195 NOW WE INTERCHANGE ROWS K AND IMAX$
196 IF IMAX NEQ K$
197 BEGIN
198 FOR J = (1,1,N)$
199 BEGIN
200 TEMP=A(K,J)$ A(K,J)=A(IMAX,J)$ A(IMAX,J)=TEMP
201 END$
202 TEMP=B(K)$B(K)=B(IMAX)$B(IMAX)=TEMP
203 END$
204 COMMENT NOW FOR THE ELIMINATIONS
205 DENOM = A(K,K) $
206 FOR I=(K+1,1,N)$
207 A(I,K) = A(I,K)/DENOM $
208 FOR J = (K+1,1,N)$
209 A(K,J) = A(K,J) - INPROD(1,1,K-1,A(K,),A(J)) $
210 B(K) = B(K) - INPROD(1,1,K-1,A(K,),B())
211 END$
212 FOR I=(1,1,N)$ Y(I) = A(I,I)$
213 COMMENT NOW FOR THE BACK SUBSTITUTION$
214 FOR K = (N,-1,1)$
215 Y(K) = (B(K) - INPROD(K+1,K+1,N-K,A(K,),Y()))/A(K,K) $

```

```

216 RETURN
217 END SOLV()$
218 SOLV(3 $ APMAT(), DATVEC(), COVEC() $ START) $
219 COMMENT CALCULATE RESIDUAL VALUES AFTER LEAST-SQUARES PLANE
220 COEFFICIENTS ARE AVAILABLE $
221 FOR I = (1,1,NR) $ FOR J = (1,1,MC) $
222 X(I,J) = X(I,J) - (COVEC(1)*(J-1)) - (COVEC(2)*(I-1)) - COVEC(3) $
223 IF (JPLAN EQL 2) AND (JDTOP EQL 2) $
224 BEGIN
225 JPLAN = 1 $
226 FOR I = (1,1,NR) $ FOR J = (1,1,MC) $ XS(I,J) = X(I,J) $
227 GO DATPLOT $
228 END $
229 END $
230 RESUME1..
231 IF ITRGOP EQL 1 $
232 BEGIN
233 COMMENT CALCULATE VALUES FOR (N-1)/2 AND (M-1)/2 ELEMENTS OF
234 TRIG ARRAYS. FILL MIDROWS, MIDCOLUMNS, AND REMAINING ELEMENTS BY
235 ASSIGNMENT $
236 FOR N = (2,1,NT) $
237 BEGIN
238 XNPL=(N-1).PL $
239 FOR I = (1,1,N9) $
240 BEGIN
241 ARGNI = XNPL.(N4-I) $
242 TCOSNI(N,I) = COS(ARGNI) $
243 TSINNI(N,I) = SIN(ARGNI) $
244 END $
245 END $
246 COMMENT FILL OTHER QUARTER OF NI TRIGONOMETRIC ARRAYS $
247 FOR N = (2,2,NTT)$
248 FOR I = (2,1,N8) $
249 BEGIN
250 TCOSNI(N,N7-I) = -TCOSNI(N,I) $
251 TSINNI(N,N7-I) = TSINNI(N,I) $
252 END $
253 IF NTT LSS NT$ NTTT= NT $
254 IF NTT EQL NT$ NTTT= NT -1 $
255 FOR N = (3,2,NTT)$ FOR I = (2,1,N8)$
256 BEGIN
257 TCOSNI(N,N7-I) = TCOSNI(N,I) $
258 TSINNI(N,N7-I) = -TSINNI(N,I) $
259 END $
260 COMMENT FILL TCOSNI AND TSINNI ROW AND COLUMN WHERE ARE ZERO ARGUMENTS$
261 FOR I = (1,1,N2) $
262 BEGIN
263 TCOSNI(1,I) = 1.0 $
264 TSINNI(1,I) = 0.0 $
265 END $
266 FOR M = (2,1,MT) $
267 BEGIN
268 XMPH=(M-1).PH $
269 FOR J = (1,1,M9) $
270 BEGIN
271 ARGMJ = XMPH.(M4-J) $
272 TCOSMJ(M,J) = COS(ARGMJ) $
273 TSINMJ(M,J) = SIN(ARGMJ) $
274 END $
275 END$
276 COMMENT FILL OTHER QUARTER OF MJ TRIGONOMETRIC ARRAYS $
277 FOR M = (2,2,MTT)$
278 FOR J = (2,1,M8) $
279 BEGIN
280 TCOSMJ(M,M7-J) = -TCOSMJ(M,J) $
281 TSINMJ(M,M7-J) = TSINMJ(M,J) $
282 END $
283 IF MTT LSS MT$ MTTT= MT $
284 IF MTT EQL MT$ MTTT= MT -1 $
285 FOR M = (3,2,MTT)$ FOR J = (2,1,M8)$
286 BEGIN
287 TCOSMJ(M,M7-J) = TCOSMJ(M,J) $
288 TSINMJ(M,M7-J) = -TSINMJ(M,J) $
289 END $

```

```

290 COMMENT FILL TCOSMJ AND TSINMJ ROW AND COLUMN WHERE      ZERO ARGUMENTS
291 ARE PRESENT $
292 FOR J =(1,1,M2) $
293 BEGIN
294     TCOSMJ(1,J) = 1.0 $
295     TSINMJ(1,J) = 0.0 $
296 END$
297 END $
298 OUTPUT ALPHA(FOR I =(1,1,12)$ LA(I)) $
299 IF JAROP EQL 1 $
300 BEGIN
301     IF IPCHOP EQL 2$ WRITE($$ALPHA,FMTC)$
302     IF IPCHOP EQL 1$
303         WRITE ($$ ALPHA,FMT1) $                WRITE($$ FMT2) $
304     FOR J =(1,1,M9) $
305     BEGIN
306         OUTPUT T11(FOR M =(1,1,MT)$ TCOSMJ(M,J))$
307         IF IPCHOP EQL 2$
308             WRITE ($$ T11, FMTB) $
309         IF IPCHOP EQL 1$ WRITE($$T11,FMTW)$
310     END $
311     WRITE ($$ ALPHA, FMTA) $ WRITE ($$ FMT3) $
312     FOR J =(1,1,M9) $
313     BEGIN
314         OUTPUT T21(FOR M =(1,1,MT)$TSINMJ(M,J))$
315         IF IPCHOP EQL 2$
316             WRITE ($$ T21,FMTB) $
317         IF IPCHOP EQL 1$ WRITE($$T21,FMTW)$
318     END $
319     WRITE ($$ ALPHA, FMTA) $ WRITE ($$ FMT5) $
320     FOR I =(1,1,N9) $
321     BEGIN
322         OUTPUT T31(FOR N =(1,1,NT)$ TCOSNI(N,I))$
323         IF IPCHOP EQL 2$
324             WRITE($$ T31, FMTB ) $
325         IF IPCHOP EQL 1$ WRITE($$ T31,FMTW)$
326     END $
327     WRITE ($$ALPHA,FMTA) $ WRITE ($$ FMT6) $
328     FOR I =(1,1,N9) $
329     BEGIN
330         OUTPUT T41(FOR N =(1,1,NT)$ TSINNI(N,I))$
331         IF IPCHOP EQL 2$
332             WRITE ($$ T41, FMTB ) $
333         IF IPCHOP EQL 1$ WRITE($$T41,FMTW)$
334     END $
335 END $
336 COMMENT COMPUTE FOURIER COEFFICIENTS $
337 FOR M =(1,1,MT) $
338 BEGIN
339     FOR N=(1,1,NT) $
340     BEGIN
341         A(M,N) = B(M,N) = C(M,N) = D(M,N) = 0.0 $
342         FOR I =(2,1,N2) $
343         BEGIN
344             II = N3 - I $
345             FOR J =(2,1,M2) $
346             BEGIN
347                 JJ = M3-J $
348                 A(M,N)=A(M,N)+ ((X(I,J)+X(II,J)+X(I,JJ)+X(II,JJ)).
349                     (TCOSMJ(M,J).TCOSNI(N,I)))$
350                 IF M EQL 1 $ GO LAB1 $
351                 B(M,N)=B(M,N)+ ((X(I,J)+X(II,J)-X(I,JJ)-X(II,JJ)).
352                     (TSINMJ(M,J).TCOSNI(N,I))) $
353                 LAB1.. IF N EQL 1 $ GO LAB2 $
354                 C(M,N)=C(M,N)+ ((X(I,J)-X(II,J)+X(I,JJ)-X(II,JJ)).
355                     (TCOSMJ(M,J).TSINNI(N,I))) $
356                 IF M EQL 1 $ GO LAB2 $
357                 D(M,N)=D(M,N)+ ((X(I,J)-X(II,J)-X(I,JJ)+X(II,JJ)).
358                     (TSINMJ(M,J).TSINNI(N,I))) $
359                 LAB2..
360             END $
361         END $
362     COMMENT CALC VALUES FOR TOP AND BOTTOM ROWS OF DATA ARRAY,
363     OMITTING THOSE IN FIRST, MID AND LAST COLUMNS $

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```

364 FOR J =(2,1,M2) $
365 BEGIN
366 JJ = M3-J $
367 A(M,N)=A(M,N) + (0.5.(X(1,J)+X(1,JJ)+X(NR,J)+X(NR,JJ)).
368 TCOSMJ(M,J).TCOSNI(N,1)) $
369 IF M EQL 1 $ GO LAB3 $
370 B(M,N)=B(M,N) + (0.5.(X(1,J)-X(1,JJ)+X(NR,J)-X(NR,JJ)).
371 TSINMJ(M,J).TCOSNI(N,1)) $
372 LAB3.. IF N EQL 1 $ GO LAB4 $
373 C(M,N)=C(M,N) + (0.5.(X(1,J)+X(1,JJ)-X(NR,J)-X(NR,JJ)).
374 TCOSMJ(M,J).TSINNI(N,1)) $
375 IF M EQL 1 $ GO LAB4 $
376 D(M,N)=D(M,N) + (0.5.(X(1,J)-X(1,JJ)-X(NR,J)+X(NR,JJ)).
377 TSINMJ(M,J).TSINNI(N,1)) $
378 LAB4..
379 END $
380 COMMENT CALC VALUES FOR FIRST AND LAST COLUMNS OF DATA ARRAY,
381 OMITTING THOSE IN TOP, MID AND BOTTOM ROWS $
382 FOR I =(2,1,N2) $
383 BEGIN
384 II = N3 - I $
385 A(M,N)=A(M,N) + (0.5.(X(I,1)+X(II,1)+X(I,MC)+X(II,MC)).
386 TCOSMJ(M,1).TCOSNI(N,I)) $
387 IF M EQL 1 $ GO LAB5 $
388 B(M,N)=B(M,N) + (0.5.(X(I,1)+X(II,1)-X(I,MC)-X(II,MC)).
389 TSINMJ(M,1).TCOSNI(N,I)) $
390 LAB5.. IF N EQL 1 $ GO LAB6 $
391 C(M,N)=C(M,N) + (0.5.(X(I,1)-X(II,1)+X(I,MC)-X(II,MC)).
392 TCOSMJ(M,1).TSINNI(N,I)) $
393 IF M EQL 1 $ GO LAB6 $
394 D(M,N)=D(M,N) + (0.5.(X(I,1)-X(II,1)-X(I,MC)+X(II,MC)).
395 TSINMJ(M,1).TSINNI(N,I)) $
396 LAB6..
397 END $
398 COMMENT CALC VALUES FOR FOUR CORNERS OF DATA ARRAY $
399 A(M,N)=A(M,N) + (0.25.(X(1,1)+X(1,MC)+X(NR,1)+X(NR,MC)).
400 TCOSMJ(M,1).TCOSNI(N,1)) $
401 IF M EQL 1 $ GO LAB7 $
402 B(M,N)=B(M,N) + (0.25.(X(1,1)-X(1,MC)+X(NR,1)-X(NR,MC)).
403 TSINMJ(M,1).TCOSNI(N,1)) $
404 LAB7.. IF N EQL 1 $ GO LAB8 $
405 C(M,N)=C(M,N) + (0.25.(X(1,1)+X(1,MC)-X(NR,1)-X(NR,MC)).
406 TCOSMJ(M,1).TSINNI(N,1)) $
407 IF M EQL 1 $ GO LAB8 $
408 D(M,N)=D(M,N) + (0.25.(X(1,1)-X(1,MC)-X(NR,1)+X(NR,MC)).
409 TSINMJ(M,1).TSINNI(N,1)) $
410 LAB8..
411 COMMENT CALC VALUES FOR MID-COLUMN AND MIDROW, OMITTING THOSE AT
412 EDGES AND CENTER OF DATA ARRAY $
413 FOR J =(2,1,M2) $
414 BEGIN
415 JJ = M3 - J $
416 A(M,N)=A(M,N) + ((X(N4,J) + X(N4,JJ)).TCOSMJ(M,J)) $
417 IF M EQL 1 $ GO LAB9 $
418 B(M,N)=B(M,N) + ((X(N4,J) - X(N4,JJ)).TSINMJ(M,J)) $
419 LAB9..
420 END $
421 FOR I =(2,1,N2) $
422 BEGIN
423 II = N3 - I $
424 A(M,N)=A(M,N) + ((X(I,M4) + X(II,M4)).TCOSNI(N,I)) $
425 IF N EQL 1 $ GO LAB10 $
426 C(M,N)=C(M,N) + ((X(I,M4) - X(II,M4)).TSINNI(N,I)) $
427 LAB10..
428 END $
429 COMMENT CALC VALUE FOR MIDPOINT $
430 A(M,N)=A(M,N) + X(N4,M4) $
431 COMMENT CALC VALUES IN MID-COLUMN AND MID-ROW ON OUTER EDGES
432 OF DATA ARRAY $
433 A(M,N)=A(M,N) + (0.5.(X(1,M4) + X(NR,M4)).TCOSNI(N,1)) $
434 A(M,N)=A(M,N) + (0.5.(X(N4,1) + X(N4,MC)).TCOSMJ(M,1)) $
435 IF M EQL 1 $ GO LAB11 $
436 B(M,N)=B(M,N) + (0.5.(X(N4,1) - X(N4,MC)).TSINMJ(M,1)) $
437 LAB11.. IF N EQL 1 $ GO LAB12 $

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438 C(M,N)=C(M,N) + (0.5*(X(1,M4) + X(NR,M4))*TSINNI(N,1)) $
439 LAB12..
440 COMMENT MULTIPLY BY RECIPROCAL OF L TIMES H $
441 A(M,N) = RLH.A(M,N)$ B(M,N) = RLH.B(M,N)$
442 C(M,N) = RLH.C(M,N)$ D(M,N) = RLH.D(M,N)$
443 COMMENT COMPUTE VECTOR MAGNITUDE VALUES$ PS(M,N) =
444 Sqrt(((A(M,N).A(M,N))+(B(M,N).B(M,N))+(C(M,N).C(M,N))
445 + (D(M,N).D(M,N))))$
446 END $
447 END $
448 COMMENT PRINT TABLES OF FOURIER COEFFICIENTS AND MEAN AND VECTOR
449 MAGNITUDE VALUES$ PS(1,1) = PS(1,1)/4.0 $
450 OUTPUT FCA(FOR N=(1,1,NT)$FOR M=(1,1,MT)$ A(M,N)),
451 FCB(FOR N=(1,1,NT)$FOR M=(1,1,MT)$ B(M,N)),
452 FCC(FOR N=(1,1,NT)$FOR M=(1,1,MT)$ C(M,N)),
453 FCD(FOR N=(1,1,NT)$FOR M=(1,1,MT)$ D(M,N)),
454 PSA(FOR N=(1,1,NT)$FOR M=(1,1,MT)$PS(M,N))$
455 IF MT GTR 12$ BEGIN
456 WRITE ($$ ALPHA,FMT1) $ WRITE ($$ FMT8)$ WRITE ($$ FCA, FMT13) $
457 WRITE ($$ FMT9)$ WRITE ($$ FCB, FMT13) $
458 WRITE ($$ FMT10)$WRITE ($$ FCC, FMT13) $
459 WRITE ($$ FMT11)$WRITE ($$ FCD, FMT13) $
460 WRITE ($$ FMT12)$WRITE ($$ PSA, FMT14) $
461 ENDS$
462 IF MT LEQ 12$ BEGIN
463 WRITE ($$ ALPHA,FMT1) $ WRITE ($$ FMT8)$ WRITE ($$ FCA, FMT23) $
464 WRITE ($$ FMT9)$ WRITE ($$ FCB, FMT23) $
465 WRITE ($$ FMT10)$WRITE ($$ FCC, FMT23) $
466 WRITE ($$ FMT11)$WRITE ($$ FCD, FMT23) $
467 WRITE ($$ FMT12)$WRITE ($$ PSA, FMT24) $
468 ENDS$
469 IF JCOFOP EQL 2$
470 BEGIN
471 WRITE($$FCA, FMT22)$ WRITE($$ FCB,FMT22)$ WRITE($$FCC,FMT22)$
472 WRITE($$ FCD,FMT22)$ WRITE($$ PSA,FMT22)$
473 ENDS$
474 COMMENT CALCULATE SUM OF DATA VALUES AND SUM OF SQUARED DATA VALUES $
475 COMMENT CLEAR DATA ARRAY,XS(1,J)TO ZERO TO USE FOR STORAGE OF
476 EVALUATED FOURIER SERIES TO NT AND MT ORDER $
477 IF JVALOP EQL 1$
478 BEGIN
479 FOR I =(1,1,NR)$ FOR J =(1,1,MC)$ XS(I,J) = 0.0 $
480 COMMENT THIS SECTION RESTORES TILT TO ORIGINAL DATA AND TO
481 FOURIER TRANSFORM VALUES AT DATA POINTS$
482 IF JTILT EQL 2$
483 BEGIN
484 FOR I = (1,1,NR)$ FOR J = (1,1,MC)$
485 BEGIN
486 X(I,J) = X(I,J) +(COVEC(1).(J-1)) + (COVEC(2).(I-1))
487 + COVEC(3) $
488 XS(I,J) = XS(I,J) +(COVEC(1).(J-1)) + (COVEC(2).(I-1))
489 + COVEC(3) $
490 ENDS$
491 ENDS$
492 SUMX = SUMX2 = 0.0 $
493 FOR I =(1,1,NR) $
494 BEGIN
495 FOR J =(1,1,MC) $
496 BEGIN
497 SUMX = SUMX + X(I,J)$
498 SUMX2= SUMX2+(X(I,J).X(I,J)) $
499 END $
500 END $
501 GRID = NR.MC $
502 ZOR = SUMX2 - ((SUMX.SUMX)/ GRID ) $
503 COMMENT EVALUATE FOURIER SERIES AT EACH GRID POINT IF JVALOP EQUALS 1 $
504 COMMENT EVALUATE FOURIER SERIES FOR IR DIFFERENT NUMBERS OF TERMS
505 AS SPECIFIED IN LSTMT() AND MTERM() ARRAYS $
506 LSTMT = LSTNT = 0 $
507 FOR IQ= (1,1,IR)$
508 BEGIN
509 ISTMT = LSTMT + 1 $ ISTNT = LSTNT + 1 $
510 LSTMT = MTERM(IQ) $ LSTNT = NTERM(IQ)$
511 K = 0 $

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512 SUMVAL = SMVAL2 = 0.0 $
513 FOR I =(1,1,N2) $
514 BEGIN
515   L = 0 $ K=K+2 $ IB = I+ N4 $ II = IB - K $
516   FOR J =(1,1,M2) $
517   BEGIN
518     L = L + 2 $ JB = J + M4 $ JJ = JB - L $
519     FOR M =(ISTMT,1,LSTMT) $
520     BEGIN
521       IF M EQL 1 $ (P = 1$ GO ALP1)$ P = 2 $ ALP1..
522       FOR N =(ISTNT,1,LSTNT) $
523       BEGIN
524         IF N EQL 1$(Q = 1$ GO ALP2)$ Q = 2 $ ALP2..
525         XS(I,J) =XS(I,J) +(ALAM(P,Q). (TCOSNI(N,I). (A(M,N).
526           TCOSMJ(M,J)) + (B(M,N).TSINMJ(M,J))) + (TSINNI(N,I).
527           ((C(M,N).TCOSMJ(M,J)) + (D(M,N).TSINMJ(M,J)))))) $
528         XS(IB,JB)=XS(IB,JB)+(ALAM(P,Q). (TCOSNI(N,II). (A(M,N).
529           TCOSMJ(M,JJ))- (B(M,N).TSINMJ(M,JJ))) - (TSINNI(N,II).
530           ((C(M,N).TCOSMJ(M,JJ)) - (D(M,N).TSINMJ(M,JJ)))))) $
531         XS(IB,J )=XS(IB,J )+(ALAM(P,Q). (TCOSNI(N,II). (A(M,N).
532           TCOSMJ(M,JJ))- (B(M,N).TSINMJ(M,JJ))) - (TSINNI(N,II).
533           ((C(M,N).TCOSMJ(M,J ) + (D(M,N).TSINMJ(M,J )))))) $
534         XS(I ,JB)=XS(I ,JB)+(ALAM(P,Q). (TCOSNI(N,I ). (A(M,N).
535           TCOSMJ(M,JJ))- (B(M,N).TSINMJ(M,JJ))) + (TSINNI(N,I ).
536           ((C(M,N).TCOSMJ(M,JJ)) - (D(M,N).TSINMJ(M,JJ)))))) $
537       END $
538     END $
539   END $
540 END $
541 FOR M=(ISTMT,1,LSTMT)$
542 BEGIN
543   IF M EQL 1 $(P = 1$ GO ALP3)$ P= 2 $ ALP3..
544   FOR N=(ISTNT,1,LSTNT)$
545   BEGIN
546     IF N EQL 1$( Q = 1 $ GO ALP4)$ Q = 2 $ ALP4..
547     K=0$
548     FOR I =(1,1,N2)$
549     BEGIN
550       K=K + 2$ IB = I + N4$ II = IB - K $
551       XS(I,M4) = XS(I,M4) +(ALAM(P,Q). ((TCOSNI(N,I).A(M,N)) +
552         (TSINNI(N,I).C(M,N)))) $
553       XS(IB,M4)=XS(IB,M4) +(ALAM(P,Q). ((TCOSNI(N,II).A(M,N)) -
554         (TSINNI(N,II).C(M,N)))) $
555     END$
556     K=0$
557     FOR J=(1,1,M2)$
558     BEGIN
559       K=K+2$ JB = J +M4$ JJ = JB - K $
560       XS(N4,J) = XS(N4,J) + (ALAM(P,Q). ((A(M,N).TCOSMJ(M,J))
561         + (B(M,N).TSINMJ(M,J)))) $
562       XS(N4,JB) =XS(N4,JB) + (ALAM(P,Q). ((A(M,N).TCOSMJ(M,JJ))
563         - (B(M,N).TSINMJ(M,JJ)))) $
564     END $
565     XS(N4,M4) = XS(N4,M4) + (ALAM(P,Q).A(M,N)) $
566   END $
567 END $
568 COMMENT COMPUTE PRECENT OF TOTAL SUM OF SQUARES$
569 FOR I =(1,1,NR)$
570 BEGIN
571   FOR J = (1,1,MC)$
572   BEGIN
573     SUMVAL = SUMVAL + XS(I,J) $
574     SMVAL2 = SMVAL2 +(XS(I,J).XS(I,J)) $
575   END $
576 END $
577 PCTS = 100.0.(( SMVAL2 -((SUMVAL.SUMVAL)/ GRID ))/ZOR) $
578 IF (JVALOP EQL 1) AND (JPRTOP NEQ 1) $ WRITE($$HED1,FMT15)$
579 IF JPRTOP EQL 1 $
580 BEGIN
581   COMMENT PRINT OUT MAPS IN STRIPS UP TO 13 INCHES WIDE $
582   OUTPUT HED1( MC, NR, LSTMT,LSTNT, PCTS, SCALE)$
583   OUTPUT HED2(MC,NR,LSTMT ,LSTNT,SCALE)$
584   DATPLOT..
585   WRITE ($$ ALPHA,FMT1) $

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586 IF JPLAN EQL 1 $ (WRITE ($$ FMT7) $ GO PAST1) $
587 IF JDTOP EQL 1 $ ( OUTPUT DT1(NR,MC,SCALE)$ WRITE ($$ DT1,FMT29)) $
588 IF JDTOP NEQ 1 $ WRITE ($$ HED1, FMT15)$
589 PAST1..
590 WRITE($$FMT17)$
591 IF MC LEQ 26 $ M = MC $
592 IF MC GTR 26 $ M = 26 $
593   FOF I =(1,1,NR) $
594   BEGIN
595     FOR J =(1,1,M ) $ LINE(J)= XS(I,J) $
596     OUTPUT STRIP1(FOR J =(1,1,M) $ LINE(J)) $
597     WRITE ($$ STRIP1, FMT16) $
598   END $
599
600 IF MC GTR 26 $
601 BEGIN
602   WRITE ($$ ALPHA,FMT1) $
603   IF JPLAN EQL 1 $ (WRITE ($$ FMT7) $ GO PAST2) $
604   IF JDTOP EQL 1 $ WRITE ($$ DT1,FMT29) $
605   IF JDTOP NEQ 1 $ WRITE ($$ HED1, FMT15) $
606   PAST2..
607   WRITE($$FMT18)$
608   IF MC LEQ 52 $ M = MC $
609   IF MC GTR 52 $ M = 52 $
610   FOR I =(1,1,NR) $
611   BEGIN
612     FOR J =(27,1,M) $ LINE(J) =XS(I,J) $
613     OUTPUT STRIP2(FOR J=(27,1,M) $ LINE(J)) $
614     WRITE ($$ STRIP2, FMT16) $
615   END $
616 END $
617 IF MC GTR 52 $
618 BEGIN
619   WRITE ($$ ALPHA,FMT1) $
620   IF JPLAN EQL 1 $ (WRITE ($$ FMT7) $ GO PAST3) $
621   IF JDTOP EQL 1 $ WRITE ($$ DT1,FMT29) $
622   IF JDTOP NEQ 1 $ WRITE ($$ HED1, FMT15) $
623   PAST3..
624   WRITE($$FMT19)$
625   IF MC LEQ 78 $ M = MC $
626   IF MC GTR 78 $ M = 78 $
627   FOR I =(1,1,NR) $
628   BEGIN
629     FOR J =(53,1,M ) $ LINE(J) =XS(I,J) $
630     OUTPUT STRIP3(FOR J=(53,1,M )$ LINE(J))$
631     WRITE ($$ STRIP3, FMT16) $
632   END $
633 END $
634 IF MC GTR 78 $
635 BEGIN
636   WRITE ($$ ALPHA,FMT1) $
637   IF JPLAN EQL 1 $ (WRITE ($$ FMT7) $ GO PAST4) $
638   IF JDTOP EQL 1 $ WRITE ($$ DT1,FMT29) $
639   IF JDTOP NEQ 1 $ WRITE ($$ HED1, FMT15) $
640   PAST4..
641   WRITE($$FMT21)$
642   IF MC LEQ 104 $ M = MC $
643   IF MC GTR 104 $ M =104 $
644   FOR I =(1,1,NR) $
645   BEGIN
646     FOR J =(79,1,M) $ LINE(J) = X(I,J) $
647     OUTPUT STRIP4(FOR J =(79,1,M) $ LINE(J))$
648     WRITE ($$ STRIP4, FMT16) $
649   END $
650 END $
651 IF JDTOP EQL 1 $ (JDTOP = 2 $ GO RESUME) $
652 IF JPLAN EQL 1 $ (JPLAN = JPLAN - 1 $ GO RESUME1 )$
653 END $
654 COMMENT IF JRESOP EQUALS 1, CALCULATE AND PRINT RESIDUALS $
655 IF JRESOP EQL 1 $
656 BEGIN
657   WRITE($$ ALPHA,FMT1)$ WRITE($$ HED2, FMT25) $
658   WRITE($$ FMT17) $
659   IF MC LEQ 26 $ M= MC $ IF MC GTR 26 $ M = 26 $

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660 FOR I =(1,1,NR) $
661 BEGIN
662   FOR J =(1,1,M) $ LINE(J) = X(I,J) - XS(I,J) $
663   OUTPUT RMAP1(FOR J =(1,1,M) $ LINE(J)) $
664   WRITE ($$ RMAP1, FMT16) $
665 END $
666 IF MC GTR 26 $
667 BEGIN
668   WRITE ($$ ALPHA, FMT1) $ WRITE ($$ HED2, FMT25) $
669   WRITE ($$ FMT18) $
670   IF MC LEQ 52 $ M = MC $ IF MC GTR 52 $ M = 52 $
671   FOR I =(1,1,NR) $
672   BEGIN
673     FOR J =(27,1,M) $ LINE(J) = X(I,J) - XS(I,J) $
674     OUTPUT RMAP2(FOR J =(27,1,M) $ LINE(J)) $
675     WRITE ($$ RMAP2, FMT16) $
676   END $
677 END $
678 IF MC GTR 52 $
679 BEGIN
680   WRITE($$ ALPHA, FMT1) $ WRITE ($$ HED2, FMT25) $
681   WRITE ($$ FMT19) $
682   IF MC LEQ 78 $ M = MC $ IF MC GTR 78 $ M = 78 $
683   FOR I =(1,1,NR) $
684   BEGIN
685     FOR J =(53,1,M) $ LINE(J) = X(I,J) - XS(I,J) $
686     OUTPUT RMAP3(FOR J =(53,1,M) $ LINE(J)) $
687     WRITE ($$ RMAP3, FMT16) $
688   END $
689 END $
690 IF MC GTR 78 $
691 BEGIN
692   WRITE($$ ALPHA, FMT1) $ WRITE ($$ HED2, FMT25) $
693   WRITE($$ FMT21) $
694   IF MC LEQ 104 $ M = MC $ IF MC GTR 104 $ M = 104 $
695   FOR I =(1,1,NR) $
696   BEGIN
697     FOR J =(79,1,M) $ LINE(J) = X(I,J) - XS(I,J) $
698     OUTPUT RMAP4(FOR J =(79,1,M) $ LINE(J)) $
699     WRITE ($$ RMAP4, FMT16) $
700   END $
701 END $
702 END $
703 IF JPROF EQL 1 $
704 BEGIN
705   DIFF = TOP - BASE $ BD = BASE/ DIFF $
706   FOR I =(1,1,HP) $
707   BEGIN
708     IH = IVALS(I) $
709     WRITE($$ ALPHA, FMT1)$
710     OUTPUT HORPROFVAL( IH ,TOP, BASE,VSCALE, AMP,LSTMT,LSTNT)$
711     WRITE ($$ HORPROFVAL, FMT26) $
712     WRITE($$FMT8)$
713     FOR J =(1,1,MC) $
714     BEGIN
715       K = M4 - J $ IT = ((X(IH,J)/DIFF) - BD).VSCALE) $
716       IF IT GTR 30 $ IT = 30 $ IF IT LSS 0 $ IT = 0 $
717       KCAL=((XS(IH,J)/DIFF) - BD).VSCALE $ KB1 = 30 - IT $
718       IF KCAL GTR 30 $ KCAL = 30 $
719       IF KCAL LSS 0 $ KCAL = 0 $
720       KB2 = 30 - KCAL $
721       RESID = X(IH,J) - XS(IH,J)$
722       KRES = (RESID.AMP)+ 15.0 $
723       IF KRES GTR 30 $ KRES = 30 $
724       IF KRES LSS 0 $ KRES = 0 $
725       OUTPUT HORPLOT( X(IH,J), XS(IH,J), RESID, K,AST,AST,AST )$
726       WRITE($$HORPLOT, FMT27)$
727     END $
728   END $
729   FOR J =(1,1,VP) $
730   BEGIN
731     WRITE($$ ALPHA, FMT1)$
732     OUTPUT VERTPROFVAL( JVALS(J), TOP, BASE,VSCALE, AMP,LSTMT,LSTNT)$
733     WRITE ($$ VERTPROFVAL, FMT28) $

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734      WRITE($$FTT8)$
735      IH = JVALS(J) $
736      FOR I =(1,1,NR) $
737      BEGIN
738          K = N4 - I $      IT = (((X(I,IH)/DIFF) - BD).VSCALE) $
739          IF IT GTR 30 $ IT = 30 $ IF IT LSS 0 $ IT = 0 $
740          KCAL=(((XS(I,IH)/ DIFF) - BD).VSCALE) $ KB1 = 30 - IT $
741          IF KCAL GTR 30 $(KCAL = 30 $ GO PST1 )$
742          IF KCAL LSS 0 $ KCAL = 0 $
743          PST1..
744          KB2 = 30 - KCAL $
745          RESID = X(I,IH) - XS(I,IH)$
746          KRES = (RESID.AMP)+ 15.0 $
747          IF KRES GTR 30 $(KRES = 30 $ GO PST2)$
748          IF KRES LSS 0 $ KRES = 0 $
749          PST2..
750          OUTPUT VERPLOT( X(I,IH), XS(I,IH), RESID, K,AST,AST,AST )$
751          WRITE($$VERPLOT, FMT27) $
752      END $
753  END $
754 END $
755 END $ END $
756 GO START $ FINISH $
757 )1 (IG 7 (IG
758 *J,X)*PE.2 9 '76'75'7447 '7X'76 7 77X'75 449' 7448' 14 '1 9' 14 '1 9'
759 *J7((P'1- 74'G 04'9 '76 75 6 '06705'-4'76 75 74 549
760 2 FINISH $

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Input to BALGOL Double Fourier Series Program

Each data set used with the BALGOL double Fourier series program must be prepared as follows:
Any number of data sets may be used in succession. See Table 8 for an example data set.

Alphanumeric heading card.— The first card of each data set (Table 8) contains alphabetical and numerical information for identification purposes, and must be prepared as follows:

- (1) 5 in column 1
- (2) \$ in columns 2 and 75
- (3) Any desired combination of letters, numbers, other symbols and blanks in columns 3 through 74.

Program control card or cards.— The second card that accompanies each data set must be a program control card. Note that each number must be separated by at least one blank from adjacent numbers. More than one card may be used if desired. The columns in which the numbers are placed is not critical (except for column 1), but the sequence of numbers must be exactly as listed below:

- (1) 5 in column 1 of each card
- (2) An odd integer (NR) specifying the number of rows in data array; 71 is maximum number unless array dimensions are changed.
- (3) An odd integer (MC) specifying the number of columns in data array; 73 is maximum number unless array dimensions are changed.
- (4) An integer (NT) specifying the maximum number of N terms pertaining to vertical or y direction of data array; 25 is the maximum number unless array dimensions are changed.

- (5) An integer (MT) specifying the maximum number of M terms pertaining to horizontal or x direction of data array; 25 is the maximum number unless array dimensions are changed.
- (6) An integer (ITRGOP) specifying whether trigonometric arrays are to be calculated, or read in as data:
- 1 calculate trigonometric values
 - 2 read in trigonometric values as data
- (7) An integer (JAROP) specifying whether trigonometric arrays are to be part of output
- 0 not part of output
 - 1 part of output
- (8) An integer (INTOP) specifying whether data values are integer or decimal-point numbers:
- 1 Integer numbers
 - 2 Decimal-point numbers
 - 3 Input data consists of 0's and 1's which will be converted to -100 and +100. This option is intended for use with optical pattern recognition studies where the original 0's and 1's are used to indicate black-white or present-absent relationships.
 - 4 Algebraic signs of input data values are to be reversed. This option is useful if subsea elevation values are to be read in, because it avoids the necessity of punching minus signs on data cards.
- (9) An integer (JVALOP) specifying whether double Fourier series is to be evaluated at data points:
- 0 do not evaluate
 - 1 evaluate
- (10) An integer (JPRTOP) specifying whether calculated double Fourier values are to be printed:
- 0 do not print
 - 1 print
- (11) An integer (JRESOP) specifying whether residual values are to be calculated and printed:
- 0 do not calculate and print
 - 1 calculate and print
- (12) An integer (JDTOP) specifying whether the original data are to be printed out as an array in strip map form:
- 0 do not print
 - 1 print
- (13) An integer (IPCHOP) specifying whether trigonometric arrays (if part of output) are to be punched:
- 1 do not punch
 - 2 punch

- (14) An integer (L) specifying whether data values are to be multiplied by a scale factor:
- 0 do not multiply
 - 1 multiply
- (15) A decimal-point number (SCALE) specifying the scale factor that data values are to be multiplied by. If data are to be used without scaling, use value of 1.0, inasmuch as a number for SCALE must be included on the card.
- (16) An integer (JPROF) specifying whether profiles (either, or both, vertical and horizontal) of the evaluated data are to be plotted:
- 0 do not plot profiles
 - 1 plot profiles
- (17) An integer (JCOFOP) specifying whether Fourier coefficient arrays and harmonic vector magnitude arrays are to be punched:
- 1 do not punch
 - 2 punch
- (18) An integer (LVLOP) specifying whether the original data are to be leveled before the Fourier coefficients are calculated and Fourier series evaluated at the data points:
- 1 level
 - 2 do not level
- (19) An integer (JTILT) to restore the original tilt to the data if they were leveled earlier:
- 1 do not restore tilt
 - 2 restore tilt
- (20) An integer (JPLAN) specifying whether the data values are to be printed out as an array in strip map form. If original data are to be printed (JDTOP = 1), then JPLAN should be punched as follows:
- 1 do not print
 - 2 print
- If, on the other hand, the original data (prior to leveling) are not to be printed, then JPLAN should be punched as follows:
- 0 do not print
 - 1 print.

Term cards.— If the Fourier series are to be evaluated (JVALOP punched as 1 on control card) two cards must be used to specify the specific number of terms for which the series are to be evaluated.

(A) First Term Card:

- (1) 5 in column 1
- (2) An integer (IR) specifying number of times that series is to be evaluated
- (3) Integers specifying the values of the N terms, pertaining to vertical or y

direction of data array, for which series is to be evaluated. The values must be in ascending order, and the last or highest value must not exceed the maximum value (NT) for the N terms as specified on the program control card.

(B) Second Term Card

- (1) 5 in column 1
- (2) Integers specifying the values of the M terms, which pertain to the horizontal or x direction of data array. Note that the values must be in ascending order, and the number of values must be the same as specified (IR) on the previous card. The values of the M terms do not have to be the same as the values of the N terms; however, care should be taken that the last or highest value does not exceed the maximum value (MT) for the M terms as specified on the program control card.

Trigonometric array value cards.— If the trigonometric array values have been calculated and punched previously, using the program with a data array of the same number of rows and columns and the same maximum number of terms, NT and MT, they can be read in if ITRGOP on the control card equals 2. The punched cards containing the trigonometric arrays are in correct form for input.

Profile plot cards.— If specified on the main control card (JPROF = 1), two cards must be used at this point in the data set to control the profiles to be plotted, as follows:

(1) First profile control card:

- (a) 5 in column 1
- (b) A decimal-point number specifying the value of the plot top (TOP)
- (c) A decimal-point number specifying the value of the plot base (BASE)
- (d) A decimal-point number specifying the amplitude of the magnification of residual values
- (e) A decimal-point number (VSCALE) specifying the maximum span of the plotted values. The maximum is 30.0.
- (f) An integer (HP) specifying the number of horizontal profiles to be plotted.
- (g) Integer numbers (IVALS()) specifying the row index values of the horizontal profiles to be plotted. The number of values listed must be equal to the number specified (HP)

(2) Second profile card:

- (a) 5 in column 1
- (b) An integer (VP) specifying the number of vertical profiles to be plotted
- (c) Integer number (JVALS()) specifying the column indexes of the vertical profiles to be plotted.

Data cards.— Data cards must be punched as follows:

- (1) 5 in column 1 of each card

- (2) If the data are in integer form (INTOP = 1 on control card) each row of the data array must begin with a new card. Any number of data values may be placed in columns 2 to 80, and any number of cards may be used for each row. The maximum number of values per row is 71, and the maximum number of rows is 73, unless the array dimensions are changed. The values are read in just as reading a book, left to right, within each row, and downward from row to row.
- (3) If the data are in decimal-point form (INTOP = 2), the data are read in sequence as above, but the provision that each new row in the data array begin with a new card does not have to be observed.
- (4) If the data consist of 0's and 1's (integers) and if it is desirable to convert the 0's to -100 and 1's to +100 (INTOP = 3), then all values (except for column 1) must be either 0 or 1.
- (5) If INTOP = 4, the algebraic signs of integer data values will be reversed.
- (6) The columns occupied on the cards are not critical (except for 5 in column 1); the sequence of numbers is critical, however. At least one blank must separate each number from adjacent numbers and information punched to the right of an asterisk will be ignored when the cards are read. This permits identifying numbers and letters to be placed on the cards.

Table 8.- Example data set used as input to double Fourier series program. Output from program, using this input data set, is shown in Tables 9 and 10 and Figures 4 to 8. Data were generated with program listed in Table 11.

```

5$DOUBLE FOURIER SERIES TRIAL DATA SET GENERATED BY PROGRAM, SIN + COS $
5 31 25 5 5 1 1 2 1 1 1 1 0 1.0 1 1* CONTROL FOR GENERATED DATA
5 1 5
5 5
5 4000.0 -2000.0 0.02 30.0 1 15
5 1 12
5 23.7500 -9.4705 -31.9506 -13.7170 28.4269 46.7963 15.7500
5 -33.8360 -38.4295 34.7267 146.4506 211.0959 169.7500 44.4083
5 -78.2570 -119.7830 -73.0705 .2659 31.7500 6.3341 -33.9269
5 -40.2267 -7.2430 26.4060 23.7500
5 1.2480 -.7639 -2.9288 -3.2206 -1.6179 -.4109 -2.0634
5 -6.6353 -11.3292 -12.7450 -9.8064 -4.6382 -.6055 .5925
5 .0588 .1449 1.8980 4.0680 4.5595 2.7298 .1382
5 -1.0090 -.0879 1.3510 1.2480
5 -22.1925 8.9446 28.5818 9.3627 -31.2123 -47.6086 -18.1457
5 25.9395 23.8191 -52.5850 -161.5206 -219.0329 -170.6375 -41.7620
5 80.9779 121.8046 75.8619 4.4156 -26.2394 -2.4344 34.8493
5 39.2833 6.8338 -25.1516 -22.1926
5 -26.6869 11.7371 36.0726 13.1337 -35.7867 -55.5265 -18.6869
5 36.5933 36.9435 -52.3067 -181.9416 -251.7014 -196.1821 -45.3782
5 97.7005 143.2368 86.3041 .6692 -34.6869 -4.4440 41.2867
5 46.6837 7.4209 -30.9398 -26.6869
5 -6.4192 4.7760 10.8076 3.7125 -9.3416 -13.1755 -1.5118
5 14.1415 14.1456 -9.3568 -41.9269 -57.3438 -40.5801 -1.9925
5 30.9132 36.8701 17.9784 -4.6212 -11.3266 -.9215 11.4907
5 11.6437 .3183 -9.1849 -6.4192
5 25.0478 -7.2053 -30.3879 -12.9878 29.8529 50.4300 21.5837
5 -26.8819 -30.5728 45.2430 161.9520 231.4734 190.7888 59.5687
5 -73.5693 -124.9065 -82.6087 -7.5306 28.5119 6.1876 -34.1830
5 -42.1808 -9.7654 25.4401 25.0478
5 43.8293 -14.9621 -56.2302 -24.6247 51.4937 86.5888 32.9231

```

5	-55.5560	-64.1876	68.1029	272.3838	393.3488	321.4016	93.0961
5	-135.1476	-218.1235	-138.5276	-5.7094	54.7354	11.8643	-60.6263
5	-73.7334	-15.2816	46.4742	43.8293			
5	33.3798	-11.4506	-43.9887	-21.5532	35.5589	62.6948	22.1725
5	-47.1590	-58.8626	35.5507	187.5093	281.7430	234.4169	69.7274
5	-97.4890	-158.6462	-99.4073	-3.3735	44.5872	11.7715	-43.8091
5	-54.8772	-11.5147	35.1206	33.3798			
5	-1.4743	2.4054	1.4767	-4.9123	-10.7442	-10.0524	-4.5474
5	-3.5144	-14.8816	-35.7492	-52.2426	-50.1616	-27.3741	3.4185
5	24.4460	26.6578	15.4891	4.1785	1.5988	5.9184	8.9810
5	5.7951	-7.7356	-3.9919	-1.4743			
5	-34.5729	17.1070	46.7254	12.3091	-53.9986	-77.8499	-26.5729
5	43.8051	34.5035	-95.1898	-271.4541	-356.5584	-267.1558	-52.1568
5	144.1233	201.8919	120.2879	3.3721	-42.5729	-4.4874	59.4986
5	63.2803	8.3139	-42.3974	-34.5729			
5	-33.8445	20.0268	49.2121	12.7026	-54.4651	-76.2448	-21.3086
5	51.3514	41.6067	-89.3796	-264.5711	-345.3073	-251.2420	-35.9313
5	154.3179	202.6134	113.7811	-4.3007	-46.3805	.0684	61.1349
5	62.8007	5.7013	-44.4574	-33.8446			
5	17.0902	3.7065	-14.3000	-13.9379	7.1654	26.3743	20.1811
5	-7.5875	-24.3011	1.2828	65.1275	124.6829	132.6960	78.5185
5	-1.2971	-53.4870	-51.9783	-15.6799	13.9994	12.3813	-9.8049
5	-23.3349	-12.8436	8.8154	17.0902			
5	105.1890	-28.3394	-128.7938	-62.2994	113.6060	202.1039	84.8783
5	-122.0178	-152.8398	144.7900	623.2529	923.7742	779.5943	258.7927
5	-282.3591	-497.8031	-330.5817	-24.3228	125.4998	34.5899	-134.4943
5	-172.5510	-42.0048	104.2296	105.1891			
5	191.8821	-62.3398	-244.8524	-112.3669	216.9064	372.6845	142.9056
5	-244.4542	-292.3239	271.5666	1157.5930	1695.0824	1401.8507	424.9572
5	-564.0973	-932.8116	-597.1132	-24.2033	240.8586	58.0780	-257.5424
5	-319.0056	-68.7478	200.1320	191.8822			
5	233.4031	-81.4939	-304.5298	-139.6027	264.1974	451.1854	163.7034
5	-315.2357	-375.6287	313.3057	1393.6456	2044.8924	1682.2596	491.8305
5	-703.8223	-1137.2124	-714.8725	-13.4901	303.1029	71.6503	-316.4934
5	-388.4723	-79.2344	248.3736	233.4032			
5	205.7500	-75.5193	-274.2735	-128.7077	228.1295	391.8551	133.7499
5	-296.0640	-357.5782	243.2851	1192.7735	1768.6156	1455.7498	417.6724
5	-622.7441	-992.7922	-613.9218	3.9916	277.7500	67.6855	-277.6294
5	-340.7851	-66.7559	221.7627	205.7500			
5	119.0905	-46.7160	-164.9073	-82.6730	123.3829	217.7153	64.9554
5	-192.5124	-241.4034	93.1666	636.2030	974.3337	809.0922	229.6447
5	-353.1657	-555.5956	-333.4799	20.6314	173.2257	46.6084	-156.2231
5	-193.9870	-35.8520	130.2979	119.0906			
5	12.6226	-8.7875	-26.2417	-21.5170	-1.0950	8.6291	-11.5902
5	-51.5895	-77.9595	-64.4627	-17.8391	26.5626	36.1515	10.0610
5	-21.5853	-26.8921	-1.5755	29.1886	36.8354	16.9241	-8.5865
5	-15.0709	-0.0216	16.0695	12.6226			
5	-67.1612	20.9015	80.6187	28.6436	-89.5645	-143.2163	-62.8505
5	64.3622	62.1127	-157.3355	-478.6878	-656.9815	-525.3435	-148.6667
5	222.4243	359.1803	236.1974	26.7472	-71.4719	-9.2778	99.4529
5	117.2638	25.6609	-70.2330	-67.1612			
5	-91.7123	30.9262	116.5713	49.6563	-110.0369	-183.5323	-71.6811
5	111.7040	126.6186	-153.4077	-582.0333	-832.8878	-677.1236	-193.4196
5	288.4385	463.4032	295.8382	16.1059	-111.7435	-22.7078	128.5727
5	155.4844	32.6049	-97.0111	-91.7123			
5	-64.1554	21.4985	83.5711	40.1947	-69.6349	-121.9320	-44.6913
5	87.1648	107.7189	-75.4977	-368.2121	-548.2941	-454.7580	-134.9530
5	189.2987	308.4892	194.8933	3.8365	-83.6195	-21.4002	84.9650
5	106.0765	22.6821	-67.1257	-64.1555			
5	-13.9271	3.0484	18.8361	14.4680	-6.3193	-19.4065	-5.9271
5	24.9243	40.8913	17.0796	-37.1075	-82.0394	-81.3443	-34.4102
5	22.5870	48.3310	31.3172	-3.7041	-21.9271	-10.8459	11.8193
5	19.8299	5.9758	-12.4009	-13.9271			
5	23.3141	-11.0355	-30.8272	-7.8207	36.6659	52.9214	18.9981
5	-27.6356	-20.7546	66.9882	185.9979	243.5536	183.2925	37.8159
5	-95.7426	-136.2649	-82.4522	-3.9136	27.6301	.0609	-39.9827
5	-42.6934	-5.9732	28.1897	23.3141			
5	27.3311	-12.6945	-37.6604	-13.6074	36.9299	56.7956	18.0621
5	-39.3629	-39.6140	52.6562	186.1119	257.2824	199.3614	44.1138
5	-102.0154	-147.1097	-87.1923	1.0697	36.6001	4.5453	-42.7572
5	-47.8818	-7.0393	32.3112	27.3311			
5	3.1429	-3.3379	-6.7946	-3.1560	3.5316	5.0697	-1.9509
5	-11.5577	-13.4442	-3.2192	12.2510	20.1830	13.3476	-3.0920
5	-15.9844	-15.8172	-4.7292	6.3466	8.2367	1.5085	-5.3989

5	-5.3175	.6921	5.2892	3.1429			
5	-26.0477	8.4197	32.0481	12.3309	-33.3170	-54.0938	-22.5836
5	27.9353	29.1086	-53.8998	-177.6121	-247.8792	-199.7887	-57.3190
5	83.7653	135.5632	88.0727	7.7304	-29.5118	-4.8527	37.6471
5	44.8376	9.5692	-27.4265	-26.0477			
5	-36.8309	13.4370	47.7496	19.8040	-45.1124	-74.0503	-27.5690
5	47.0406	52.1839	-62.4092	-236.0199	-335.9837	-270.7402	-73.9955
5	119.7762	187.9144	117.9167	4.3870	-46.0929	-8.7152	52.7046
5	62.7528	12.2777	-39.9384	-36.8310			
5	-21.8121	8.4183	29.4889	13.6434	-24.5312	-41.7299	-13.8131
5	32.1361	38.4514	-25.8035	-126.6199	-186.8963	-152.3180	-41.1888
5	69.0097	106.9861	65.3010	-1.0012	-29.8131	-6.8893	30.0312
5	36.4265	6.8689	-23.8586	-21.8131			
5	6.6091	-2.2598	-7.4358	-1.0749	11.2515	16.1259	7.6170
5	-3.9562	-3.3828	25.6463	59.9476	76.1840	58.1874	15.0016
5	-25.6464	-39.9694	-26.5390	-4.4683	5.6012	-.7972	-11.4133
5	-12.3328	-2.3707	7.3206	6.6091			
5	26.6583	-10.2275	-34.5606	-13.1034	34.6404	55.1134	20.5134
5	-33.1835	-34.3411	52.3455	180.2983	251.4190	199.7523	52.2765
5	-90.8567	-139.9514	-87.3568	-3.7203	32.8032	4.9806	-39.6619
5	-46.0963	-8.5064	29.5298	26.6583			
5	23.7500	-9.4705	-31.9506	-13.7170	28.4269	46.7963	15.7500
5	-33.8360	-38.4295	34.7267	146.4506	211.0959	169.7500	44.4083
5	-78.2570	-119.7830	-73.0705	.2659	31.7500	6.3341	-33.9269
5	-40.2267	-7.2430	26.4060	23.7500			*LAST CARD

Output From BALGOL Double Fourier Series Program

Output from the BALGOL double Fourier series program, based on input data shown in Table 8, is shown in Tables 9 and 10 and Figures 4 to 8. The following information is included in the output.

Trigonometric arrays.- If specified, trigonometric array values (Table 9) are printed, or printed and punched, in the following order.

- (1) Array containing cosine values indexed on M and J and pertaining to horizontal or x direction of data array
- (2) Array containing sine values indexed on M and J and pertaining to horizontal or x direction of data array.
- (3) Array containing cosine values indexed on N and I and pertaining to vertical or y direction of data array
- (4) Array containing sine values indexed on N and I and pertaining to vertical or y direction of data array.

Fourier coefficient and harmonic vector magnitude arrays.- Arrays containing Fourier coefficients and harmonic vector magnitudes are printed routinely as part of the output, and may be punched as well if desired (Table 10).

- (1) A array of Fourier coefficients
- (2) B array of Fourier coefficients
- (3) C array of Fourier coefficients
- (4) D array of Fourier coefficients

Harmonic vector magnitude array.- Harmonic vector magnitude values are printed routinely and may be punched if desired.

Map of initial data values.- If specified, a map containing the initial data values is printed out (Fig. 4). The algebraic signs may be reversed, depending on the input data option exercised. The data points are plotted in vertical strips up to 13 inches wide, and up to four strips may be printed. The grid points are spaced 0.5 inches apart. Maximum vertical length depends on array dimensions.

Maps of leveled data values.- If specified, maps of data values that have been modified by subtracting a plane (first-degree trend surface) are printed out (Fig. 16).

Maps of calculated values.- If the Fourier series are to be evaluated, maps contain the calculated values at grid points and are printed out in vertical strips up to 13 inches wide (Fig. 5). Up to four strips may be printed. The grid points are spaced 0.5 inches apart. Numbers printed out are integers, and it is essential that the data be scaled, so that integers do not contain more than 4 digits. Maximum vertical lengths of the maps depends on the array dimensions.

As many maps will be printed as specified on the first term card (IR). Information is printed out at the top of each map identifying the map, listing the number of rows and columns, the scale factor, and the percent of total sum of squares of the observed data represented by the calculated values at grid points.

Maps of residual values.- Maps of residual values (Fig. 6) are printed out if specified.

Profiles.- Profiles through the original data surface, the Fourier surfaces, and residual values may be plotted along specified rows and columns (Fig. 7).

Table 9.- Output from double Fourier series program consisting of values of trigonometric arrays: arrays of cosine and sine values of terms indexed on M and J and arrays of cosine and sine values of terms indexed on N and I. Output of these data is optional. Input data yielding this output are listed in Table 8.

DOUBLE FOURIER SERIES TRIAL DATA SET GENERATED BY PROGRAM, SIN + COS					DOUBLE FOURIER SERIES TRIAL DATA SET GENERATED BY PROGRAM, SIN + COS				
COSINE MJ ARRAY					COSINE NI ARRAY				
1.0000	-1.0000	1.0000	-1.0000	1.0000	1.0000	-1.0000	1.0000	-1.0000	1.0000
1.0000	-.9659	.8660	-.7071	.5000	1.0000	-.9781	.9135	-.8090	.6691
1.0000	-.8660	.5000	.0000	-.5000	1.0000	-.9135	.6691	-.3090	-.1045
1.0000	-.7071	-.0000	.7071	-1.0000	1.0000	-.8090	.3090	.3090	-.8090
1.0000	-.5000	-.5000	1.0000	-.5000	1.0000	-.6691	-.1045	.8090	-.9781
1.0000	-.2598	-.8660	.7071	.5000	1.0000	-.5000	-.5000	1.0000	-.5000
1.0000	.0000	-1.0000	-.0000	1.0000	1.0000	-.3090	-.8090	.8090	.3090
					1.0000	-.1045	-.9781	.3090	.9135
DOUBLE FOURIER SERIES TRIAL DATA SET GENERATED BY PROGRAM, SIN + COS					DOUBLE FOURIER SERIES TRIAL DATA SET GENERATED BY PROGRAM, SIN + COS				
SINE MJ ARRAY					SINE NI ARRAY				
.0000	.0000	-.0000	.0000	-.0000	.0000	.0000	-.0000	.0000	-.0000
.0000	.2598	-.5000	.7071	-.8660	.0000	.2079	-.4057	.5878	-.7431
.0000	.5000	-.8660	1.0000	-.8660	.0000	.4067	-.7431	.9511	-.9945
.0000	.7071	-1.0000	.7071	.0000	.0000	.5878	-.9511	.9511	-.5878
.0000	.8660	-.8660	-.0000	.8660	.0000	.7431	-.9945	.5878	.2079
.0000	.9659	-.5000	-.7071	.8660	.0000	.8660	-.8660	-.0000	.8660
.0000	1.0000	.0000	-1.0000	-.0000	.0000	.9511	-.5878	-.5878	.9511
					.0000	.9945	-.2079	-.9511	.4057

Table 10.- Output from double Fourier series program consisting of A, B, C, and D Fourier coefficient arrays. Harmonic vector magnitudes are also part of output but are not shown here. Input data yielding this output are listed in Table 8. Seven M terms and 10 N terms have been calculated.

DOUBLE FOURIER SERIES TRIAL DATA SET GENERATED BY PROGRAM, SIN + COS

A ARRAY OF FOURIER COEFFICIENTS

4.3000, 01 5.4000, 01 6.5000, 01 7.6000, 01 8.7000, 01
 4.5000, 01 5.6000, 01 6.7000, 01 7.8000, 01 8.9000, 01
 4.7000, 01 5.8000, 01 6.9000, 01 8.0000, 01 9.1000, 01
 4.9000, 01 6.0000, 01 7.1000, 01 8.2000, 01 9.3000, 01
 5.1000, 01 6.2000, 01 7.3000, 01 8.4000, 01 9.5000, 01

B ARRAY OF FOURIER COEFFICIENTS

0.0000, 00 4.0000, 01 4.8000, 01 5.6000, 01 6.4000, 01
 0.0000, 00 4.4000, 01 5.2000, 01 6.0000, 01 6.8000, 01
 0.0000, 00 4.8000, 01 5.6000, 01 6.4000, 01 7.2000, 01
 0.0000, 00 5.2000, 01 6.0000, 01 6.8000, 01 7.6000, 01
 0.0000, 00 5.6000, 01 6.4000, 01 7.2000, 01 8.0000, 01

C ARRAY OF FOURIER COEFFICIENTS

0.0000, 00 0.0000, 00 0.0000, 00 0.0000, 00 0.0000, 00
 2.7000, 01 3.2000, 01 3.7000, 01 4.2000, 01 4.7000, 01
 3.3000, 01 3.8000, 01 4.3000, 01 4.8000, 01 5.3000, 01
 3.9000, 01 4.4000, 01 4.9000, 01 5.4000, 01 5.9000, 01
 4.5000, 01 5.0000, 01 5.5000, 01 6.0000, 01 6.5000, 01

D ARRAY OF FOURIER COEFFICIENTS

0.0000, 00 0.0000, 00 0.0000, 00 0.0000, 00 0.0000, 00
 0.0000, 00 2.5000, 01 2.7000, 01 2.9000, 01 3.1000, 01
 0.0000, 00 3.3000, 01 3.5000, 01 3.7000, 01 3.9000, 01
 0.0000, 00 4.1000, 01 4.3000, 01 4.5000, 01 4.7000, 01
 0.0000, 00 4.9000, 01 5.1000, 01 5.3000, 01 5.5000, 01

DOUBLE FOURIER SERIES TRIAL DATA SET GENERATED BY PROGRAM, SIN + COS

PLOT OF OBSERVED DATA VALUES, NUMBER OF ROWS = 31 , NUMBER OF COLUMNS = 25 , SCALE FACTOR = 1.000

STRIP	ONE	-31	-13	28	46	15	-33	-38	34	146	211	169	44	-78	-119	-73	0	31	6	-33	-40	-7	26	23
23	-9																							
1	0	-2	-3	-1	0	-2	-6	-11	-12	-9	-4	0	0	0	0	1	4	4	2	0	-1	0	1	1
-22	8	28	9	-31	-47	-18	25	23	-52	-161	-219	-170	-41	80	121	75	4	-26	-2	34	39	6	-25	-22
-26	11	36	13	-35	-55	-18	36	36	-52	-181	-251	-196	-45	97	143	86	0	-34	-4	41	46	7	-30	-26
-6	4	10	3	-9	-13	-1	14	14	-9	-41	-57	-40	-1	30	36	17	-4	-11	0	11	11	0	-9	-6
25	-7	-30	-12	29	50	21	-26	-30	45	161	231	190	59	-73	-124	-82	-7	28	6	-34	-42	-9	25	25
43	-14	-56	-24	51	86	32	-55	-64	68	272	393	321	93	-135	-218	-138	-5	54	11	-60	-73	-15	46	43
33	-11	-43	-21	35	62	22	-47	-58	35	187	281	234	69	-97	-158	-99	0	44	11	-43	-54	-11	35	33
-1	2	1	-4	-10	-10	-4	-3	-14	-35	-52	-50	-27	3	24	26	15	4	1	5	8	5	0	-3	-1
-34	17	46	12	-53	-77	-26	43	34	-95	-271	-356	-267	-52	144	201	120	3	-42	0	59	63	8	-42	-34
-33	20	49	12	-54	-76	-21	51	41	-89	-264	-345	-251	-35	154	202	113	-4	-46	0	61	62	5	-44	-33
17	3	-14	-13	7	26	20	-7	-24	1	65	124	132	78	-1	-53	-51	-15	13	12	-9	-23	-12	8	17
105	-28	-128	-62	113	202	84	-122	-152	144	623	923	779	258	-282	-497	-330	-24	125	34	-134	-172	-42	104	105
191	-62	-244	-112	216	372	142	-244	-292	271	1157	1695	1401	424	-564	-932	-597	-24	240	58	-257	-319	-68	200	191
233	-81	-304	-139	264	451	163	-315	-375	313	1393	2044	1682	491	-703	-1137	-714	-13	303	71	-316	-388	-79	248	233
205	-75	-274	-128	228	391	133	-296	-357	243	1192	1768	1455	417	-622	-992	-613	3	277	67	-277	-340	-66	221	205
119	-46	-164	-82	123	217	64	-192	-241	93	636	974	809	229	-353	-555	-333	20	173	46	-156	-193	-35	130	119
12	-8	-26	-21	-1	8	-11	-51	-77	-64	-17	26	36	10	-21	-26	-1	29	36	16	-8	-15	0	16	12
-67	20	80	28	-89	-143	-62	64	62	-157	-478	-656	-525	-148	222	359	236	26	-71	-9	99	117	25	-70	-67
-91	30	116	49	-110	-183	-71	111	126	-153	-582	-832	-677	-193	288	463	295	16	-111	-22	128	155	32	-97	-91
-64	21	83	40	-69	-121	-44	87	107	-75	-368	-548	-454	-134	189	308	194	3	-83	-21	84	106	22	-67	-64
-13	3	18	14	-6	-19	-5	24	40	17	-37	-82	-81	-34	22	48	31	-3	-21	-10	11	19	5	-12	-13
23	-11	-30	-7	36	52	18	-27	-20	66	185	243	183	37	-95	-136	-82	-3	27	0	-39	-42	-5	28	23
27	-12	-37	-13	36	56	18	-39	-39	52	186	257	199	44	-102	-147	-87	1	36	4	-42	-47	-7	32	27
3	-3	-6	-3	3	5	-1	-11	-13	-3	12	20	13	-3	-15	-15	-4	6	8	1	-5	-5	0	5	3
-26	8	32	12	-33	-54	-22	27	29	-53	-177	-247	-199	-57	83	135	88	7	-29	-4	37	44	9	-27	-26
-36	13	47	19	-45	-74	-27	47	52	-62	-236	-335	-270	-73	119	187	117	4	-46	-8	52	62	12	-39	-36
-21	8	29	13	-24	-41	-13	32	38	-25	-126	-186	-152	-41	69	106	65	-1	-29	-6	30	36	6	-23	-21
6	-2	-7	-1	11	16	7	-3	0	25	59	76	58	15	-25	-39	-26	-4	5	0	-11	-12	-2	7	6
26	-10	-34	-13	34	55	20	-33	-34	52	180	251	199	52	-90	-139	-87	-3	32	4	-39	-46	-8	29	26
23	-9	-31	-13	28	46	15	-33	-38	34	146	211	169	44	-78	-119	-73	0	31	6	-33	-40	-7	26	23

Figure 4.- Plot of initial data values listed in Table 8. Values have been truncated to integers to save space in plotting. Values were generated artificially for purposes of testing the double Fourier series program, using the data generator program listed in Table 11.

DOUBLE FOURIER SERIES TRIAL DATA SET GENERATED BY PROGRAM, SIN + COS

VALUES OF EVALUATED DOUBLE FOURIER SERIES AT SPECIFIED GRID POINTS,
 NUMBER OF ROWS = 31, NUMBER OF M TERMS = 5, NUMBER OF N TERMS = 5
 PERCENT OF TOTAL SUM OF SQUARES = 100.00, SCALE FACTOR = 1.00

NUMBER OF COLUMNS = 25

STRIP ONE	23	-9	-31	-13	28	46	15	-33	-38	34	146	211	169	44	-78	-119	-73	0	31	6	-33	-40	-7	26	23
1	0	-2	-3	-1	0	-2	-6	-11	-12	-9	-4	0	0	0	0	1	4	4	2	0	-1	0	1	1	
-22	8	28	9	-31	-47	-18	25	23	-52	-161	-219	-170	-41	80	121	75	4	-26	-2	34	39	6	-25	-22	
-26	11	36	13	-35	-55	-18	36	36	-52	-181	-251	-196	-45	97	143	86	0	-34	-4	41	46	7	-30	-26	
-6	4	10	3	-9	-13	-1	14	14	-9	-41	-57	-40	-1	30	36	17	-4	-11	0	11	11	0	-9	-6	
25	-7	-30	-12	29	50	21	-26	-30	45	161	231	190	59	-73	-124	-82	-7	28	6	-34	-42	-9	25	25	
43	-14	-56	-24	51	86	32	-55	-64	68	272	393	321	93	-135	-218	-138	-5	54	11	-60	-73	-15	46	43	
33	-11	-43	-21	35	62	22	-47	-58	35	187	281	234	69	-97	-158	-99	0	44	11	-43	-54	-11	35	33	
-1	2	1	-4	-10	-10	-4	-3	-14	-35	-52	-50	-27	3	24	26	15	4	1	5	8	5	0	-3	-1	
-34	17	46	12	-53	-77	-26	43	34	-95	-271	-356	-267	-52	144	201	120	3	-42	0	59	63	8	-42	-34	
-33	20	49	12	-54	-76	-21	51	41	-89	-264	-345	-251	-35	154	202	113	-4	-46	0	61	62	5	-44	-33	
17	3	-14	-13	7	26	20	-7	-24	1	65	124	132	78	-1	-53	-51	-15	13	12	-9	-23	-12	8	17	
105	-28	-128	-62	113	202	84	-122	-152	144	623	923	779	258	-282	-497	-330	-24	125	34	-134	-172	-42	104	105	
191	-62	-244	-112	216	372	142	-244	-292	271	1157	1695	1401	424	-564	-932	-597	-24	240	58	-257	-319	-68	200	191	
233	-81	-304	-139	264	451	163	-315	-375	313	1393	2044	1682	491	-703	-1137	-714	-13	303	71	-316	-388	-79	248	233	
205	-75	-274	-128	228	391	133	-296	-357	243	1192	1768	1455	417	-622	-992	-613	3	277	67	-277	-340	-66	221	205	
119	-46	-164	-82	123	217	64	-192	-241	93	636	974	809	229	-353	-555	-333	20	173	46	-156	-193	-35	130	119	
12	-8	-26	-21	-1	8	-11	-51	-77	-64	-17	26	36	10	-21	-26	-1	29	36	16	-8	-15	0	16	12	
-67	20	80	28	-89	-143	-62	64	62	-157	-478	-656	-525	-148	222	359	236	26	-71	-9	99	117	25	-70	-67	
-91	30	116	49	-110	-183	-71	111	126	-153	-582	-832	-677	-193	288	463	295	16	-111	-22	128	155	32	-97	-91	
-64	21	83	40	-69	-121	-44	87	107	-75	-368	-548	-454	-134	189	308	194	3	-83	-21	84	106	22	-67	-64	
-13	3	18	14	-6	-19	-5	24	40	17	-37	-82	-81	-34	22	48	31	-3	-21	-10	11	19	5	-12	-13	
23	-11	-30	-7	36	52	18	-27	-20	66	185	243	183	37	-95	-136	-82	-3	27	0	-39	-42	-5	28	23	
27	-12	-37	-13	36	56	18	-39	-39	52	186	257	199	44	-102	-147	-87	1	36	4	-42	-47	-7	32	27	
3	-3	-6	-3	3	5	-1	-11	-13	-3	12	20	13	-3	-15	-15	-4	6	8	1	-5	-5	0	5	3	
-26	8	32	12	-33	-54	-22	27	29	-53	-177	-247	-199	-57	83	135	88	7	-29	-4	37	44	9	-27	-26	
-36	13	47	19	-45	-74	-27	47	52	-62	-236	-335	-270	-73	119	187	117	4	-46	-8	52	62	12	-39	-36	
-21	8	29	13	-24	-41	-13	32	38	-25	-126	-186	-152	-41	69	106	65	-1	-29	-6	30	36	6	-23	-21	
6	-2	-7	-1	11	16	7	-3	0	25	59	76	58	15	-25	-39	-26	-4	5	0	-11	-12	-2	7	6	
26	-10	-34	-13	34	55	20	-33	-34	52	180	251	199	52	-90	-139	-87	-3	32	4	-39	-46	-8	29	26	
23	-9	-31	-13	28	46	15	-33	-38	34	146	211	169	44	-78	-119	-73	0	31	6	-33	-40	-7	26	23	

Figure 5.- Plot of evaluated double Fourier series at data points. Values are identical with those of Figure 4, indicating complete representation of the original data by the Fourier series.

RESIDUAL VALUES, NUMBER OF COLUMNS = 25, NUMBER OF ROWS = 31, NUMBER OF M TERMS = 5 , NUMBER OF N TERMS = 5
SCALE FACTOR = 1.000

Figure 6.- Plot of residual values obtained by subtracting evaluated Fourier series (Figure 5) from initial data values (Figure 4). The fact that residual values are all zero indicates virtually complete representation of the initial data by the Fourier series.

DOUBLE FOURIER SERIES TRIAL DATA SET GENERATED BY PROGRAM, SIN + COS

VERTICAL PROFILE OF COLUMN 12
 PLOT TOP = 4000.000 PLOT BASE = -2000.000 SCALE = 30.000 RESIDUAL AMPLIFICATION FACTOR = .020 , M = 5 , N = 5

OBSERVED	CALCULATED	RESIDUAL	OBSERVED	CALCULATED	RESIDUAL
211.096	211.096	.000	15 I	*	I
-4.638	-4.638	-.000	14 I	*	I
-219.033	-219.033	-.000	13 I	*	I
-251.701	-251.701	-.000	12 I	*	I
-57.344	-57.344	-.000	11 I	*	I
231.473	231.473	.000	10 I	*	I
393.349	393.348	.000	9 I	*	I
281.743	281.743	.000	8 I	*	I
-50.162	-50.162	.000	7 I	*	I
-356.558	-356.558	-.000	6 I	*	I
-345.307	-345.307	-.000	5 I	*	I
124.683	124.683	-.000	4 I	*	I
923.774	923.774	.000	3 I	*	I
1695.082	1695.081	.001	2 I	*	I
2044.892	2044.891	.001	1 I	*	I
1768.616	1768.615	.001	0 I	*	I
974.334	974.333	.001	-1 I	*	I
26.563	26.562	.000	-2 I	*	I
-656.981	-656.981	-.000	-3 I	*	I
-832.888	-832.887	-.000	-4 I	*	I
-548.294	-548.294	-.000	-5 I	*	I
-82.039	-82.039	-.000	-6 I	*	I
243.554	243.553	.000	-7 I	*	I
257.282	257.282	.000	-8 I	*	I
20.183	20.183	.000	-9 I	*	I
-247.879	-247.879	-.000	-10 I	*	I
-335.984	-335.983	-.000	-11 I	*	I
-186.896	-186.896	-.000	-12 I	*	I
76.184	76.184	.000	-13 I	*	I
251.419	251.419	.000	-14 I	*	I
211.096	211.096	.000	-15 I	*	I

Figure 7.- Profile of surfaces shown in Figures 4, 5, and 6. Profile pertains to vertical column 12. Observed and calculated profiles are identical, indicating that observed data are virtually completely represented by Fourier series.

Double Fourier Series Program Limitations and Operating Times

Array limitations.- The following limitations on the use of the program are established by the array dimensions:

- (1) Maximum number of rows and columns in data array is 71 and 73. By reducing other array dimensions, a larger rectangular array, or square data array, as for example a 101 by 101 array, could be accommodated. Data arrays used with the program must contain an odd number of rows and of columns. If the maximum numbers of rows and columns in the data arrays is increased, some of the following arrays are necessarily reduced. $X(,)$, $XS(,)$, $LINE(,)$, $TSINNI(,)$, $TCOSNI(,)$, $TSINMJ(,)$, and $TCOSMJ(,)$.
- (2) The maximum number of M terms and N terms is set at 25, by present array dimension. These could be increased if the $X(,)$, $XS(,)$, and $LINE(,)$ arrays are reduced. The following arrays would be affected if the maximum number of M and N terms are changed.
 $A(,)$, $B(,)$, $C(,)$, $D(,)$, $PS(,)$, $NTERM(,)$, $MTERM(,)$, $TSINNI(,)$, $TCOSNI(,)$, $TSINMJ(,)$, and $TCOSMJ(,)$.
- (3) The maximum number of M and N terms is also dependent on the numbers of rows and columns in the data array. The maximum number of M terms is equal to $(\text{number of columns} - 1)/2$, and the maximum number of N terms is equal to $(\text{number of rows} - 1)/2$.

Operating times.- The program compiles from the BALGOL source deck in about 28 seconds on the IBM 7090. Operating times vary depending on the size of the data arrays, the maximum numbers of M terms and N terms, the number of different times the Fourier series is evaluated, whether residual values are calculated and plotted, and whether trigonometric array values are calculated or read in as data. As an example, about $2\frac{1}{2}$ minutes of main-frame time were required by the IBM 7090 for execution with the Lansing Group structure example data set which consists of a 25 by 37 data array which was leveled, evaluated successively for 12 and 18 M and N terms, residuals computed and profiles plotted. Printing time on the IBM 1401/1403, at 600 lines a minute, is generally longer than main-frame execution time. For evaluation of a 71 by 73 data array for 25 N terms and 25 M terms, about 40 minutes main-frame time would be required.

Double Fourier Series Program Testing and Verification

The double Fourier series program has been extensively tested and is believed to be free of errors. The results obtained with synthetic data generated by the short program listed in Table 11 indicate that the Fourier coefficients used in generating that data were completely recovered by the double Fourier series program (Table 10). When the Fourier series has been evaluated, the initial data have been found to be virtually completely represented (Figures 4 to 8).

Table 11.- Listing of BALGOL statements in program to generate synthetic data for test purposes. Left column of figures is for reference purposes in this report, and in practice, 2 should be punched in column 1. Last line lists input data used to control program to produce output data listed in Table 8.

```

1 COMMENT DATA GENERATOR TO TEST DOUBLE FOURIER SERIES PROGRAM $
2 INTEGER I,J, M,N, XS(), P,Q,XX,YY $
3 FORMAT FMT1(11X12.2,W),FMT2(*5*,7X11.4, C), FMT3(21I5,W2,W) $
4 ARRAY A(35,35), B(35,35), C(35,35), D(35,35), AMDA(35,35), T(35,35),
5 XS(35,35) $
6 AMDA(1,1) = 0.25 $
7 FOR J = (2,1,35)$ AMDA(1,J) = AMDA(J,1) = 0.5 $
8 FOR I = (2,1,35)$ FOR J = (2,1,35) $ AMDA(I,J) = 1.0 $
9 START..
10 INPUT CONTROL(XX, YY, P ,Q )$ READ($$ CONTROL)$
11 PI = 3.1415926 $ AH = (XX-1)/2.0 $ AL = (YY-1)/2.0 $ PIL = PI/AL$
12 PI4 = PI/AH $ AHH = (XX+1)/2.0 $ ALL = (YY+1)/2.0 $
13 FOR N = (1,1,P) $
14   FOR M = (1,1,Q) $
15     BEGIN
16       FN = N $ FM = M $
17       A(M,N) = 30.0 + ( 2.0.FN) + (11.0.FM) $
18       B(M,N) = 20.0 + ( 4.0.FN) + ( 8.0.FM) $
19       C(M,N) = 10.0 + ( 6.0.FN) + ( 5.0.FM) $
20       D(M,N) = 5.0 + ( 8.0.FN) + ( 2.0.FM) $
21     END $
22   FOR M = (1,1,35)$ C(M,1)=B(1,M)=D(1,M)=D(M,1)=0.0 $
23   OUTPUT ARA(FOR N=(1,1,P)$ FOR M=(1,1,Q)$ A(M,N)),
24     ARB(FOR N=(1,1,P)$ FOR M=(1,1,Q)$ B(M,N)),
25     ARC(FOR N=(1,1,P)$ FOR M=(1,1,Q)$ C(M,N)),
26     ARD(FOR N=(1,1,P)$ FOR M=(1,1,Q)$ D(M,N)) $
27   WRITE ($$ ARA,FMT1) $ WRITE ($$ ARB,FMT1) $
28   WRITE ($$ ARC,FMT1) $ WRITE ($$ ARD,FMT1) $
29   FOR J = (1,1,XX) $
30     BEGIN
31       FJ = J $ X = AHH - FJ $
32       FOR I = (1,1,YY) $
33         BEGIN
34           T(I,J) = 0.0 $
35           FI = I $ Y = ALL - FI $
36           FOR N = (1,1,P) $
37             BEGIN
38               FNT = N - 1 $ PINL = PIL.FNT $
39               FOR M = (1,1,Q) $
40                 BEGIN
41                   FMT = M - 1 $ PIMH = PIH.FMT $
42                   ARGX = PIMH.X $ CX = COS(ARGX) $ SX = SIN(ARGX) $
43                   ARGY = PINL.Y $ CY = COS(ARGY) $ SY = SIN(ARGY) $
44                   T(I,J) =
45                     T(I,J) + (AMDA(M,N). ((A(M,N).CX.CY) + (B(M,N).SX.CY) +
46                     (C(M,N).CX.SY) + (D(M,N).SX.SY)))$
47                 END $
48             END $
49           XS(I,J) = T(I,J) $
50         END $
51     END $
52   FOR I = (1,1,YY) $(OUTPUT DAT1(FOR J=(1,1,XX)$T(I,J))$
53   WRITE($$DAT1,FMT2))$
54   IF XX LEQ 25 $ BEGIN
55     FORMAT HEAD(*INTEGER ARRAY*,W3)$ WRITE($$ HEAD)$
56     OUTPUT DAT2(FOR I = (1,1,YY) $ FOR J = (1,1,XX) $XS(I,J))$
57     WRITE($$ DAT2,FMT3)$
58   END$
59   GO START$
60 FINISH $
61 5 25 31 5 5 * CONTROL FOR S DOUBLE GENERATOR

```

APPLICATIONS OF FOURIER SERIES ANALYSIS

Single Fourier Series Analysis of Shale Oil Yield

Single Fourier series analysis was used to study fluctuations in shale oil yield with respect to depth. Shale oil yield values were measured at approximately one-foot intervals in a vertical core slightly less than 400 feet long penetrating the Green River oil shale. The core was obtained from the National Farmers Union Exploration Company Corehole 9, in SW SW NW sec. 32, T. 10 S., R. 5 E., Uintah County, Utah. Shale oil yields in the core were determined by the U.S. Bureau of Mines and are tabulated in the report by Stanfield, Smith, and Trudell (1964, p. 69-71). The oil yield values, in gallons of oil per ton of shale, range from 2.2 to 77. When oil yield values versus depth are plotted (Fig. 8), the resulting curve is complex and irregular. However, casual inspection of the curve suggests that more or less regular "cycles" are present. The question arises as to a means of characterizing this oscillatory behavior and of evaluating the respective contributions of cycles of different wave length.

Results of single Fourier analysis of the data are also shown in Figure 8. Curves that were obtained by evaluating single Fourier series contain 41, 101, and 161 terms respectively. The quality of fit is given by the percentage reduction in total sum of squares, which is labeled simply as percent fit. A fit of 100 percent would represent perfect fit of the values obtained by evaluation of the Fourier series to the observed data values.

Contributions of the various terms are given in Table 12, which lists the harmonic number, the cosine coefficients a_n , the sine coefficients, b_n , and the discrete power spectrum estimates, which are obtained by squaring and adding the coefficients for each harmonic. The wave length of each harmonic is obtained by dividing the total sampling length, 388 feet, by the harmonic number. Thus, the wave length corresponding to the first harmonic is 388 feet and to the second harmonic is 194 feet, etc. Terms whose contribution is relatively important correspond to harmonics 1, 2, 3, 4, 6, 11, 14, and 15, as indicated by their power spectrum estimates. However, many of the terms with lower power spectrum values have significance. The interaction of terms, of course, provides the complexity observed in the Fourier series. Inspection of the 41-term curve reveals an important wave length of about 25 feet, which is substantiated by the relatively large value of the power spectrum of degree 15 (which represents a wave length of 25.8 feet here). Inspection of the 101-term curve reveals an important wave length of about 8 to 9 feet. This is, of course, superimposed upon the broad-period waves emphasized in the curve with 41 terms. Further study is needed to interpret the geological significance of the different harmonics in Green River oil shale.

Double Fourier Series Analysis of Calcium Carbonate Variation

A double Fourier series was fitted to observed values of percent calcium carbonate occurring in part of the magnesite deposit at Gabbs, Nevada (Fig. 9). The purpose was to see if the rather

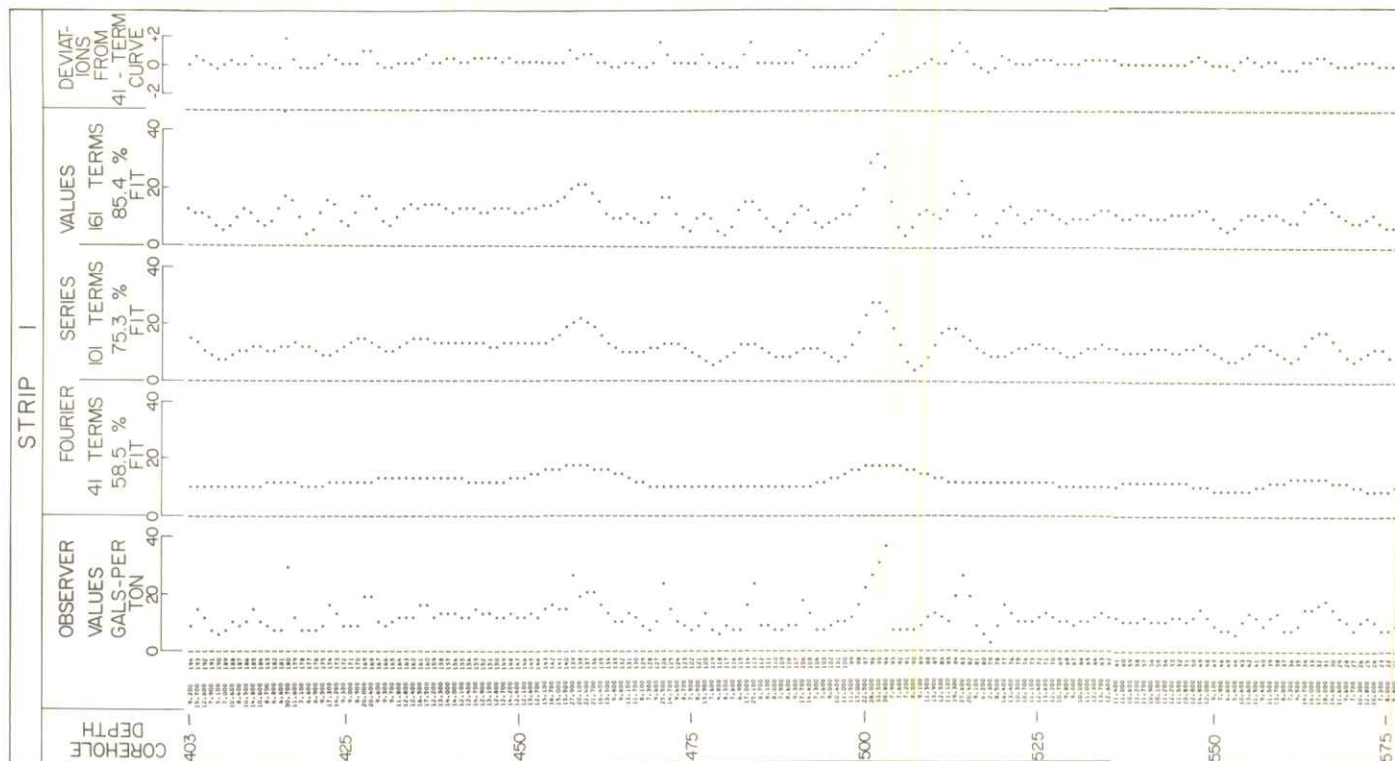
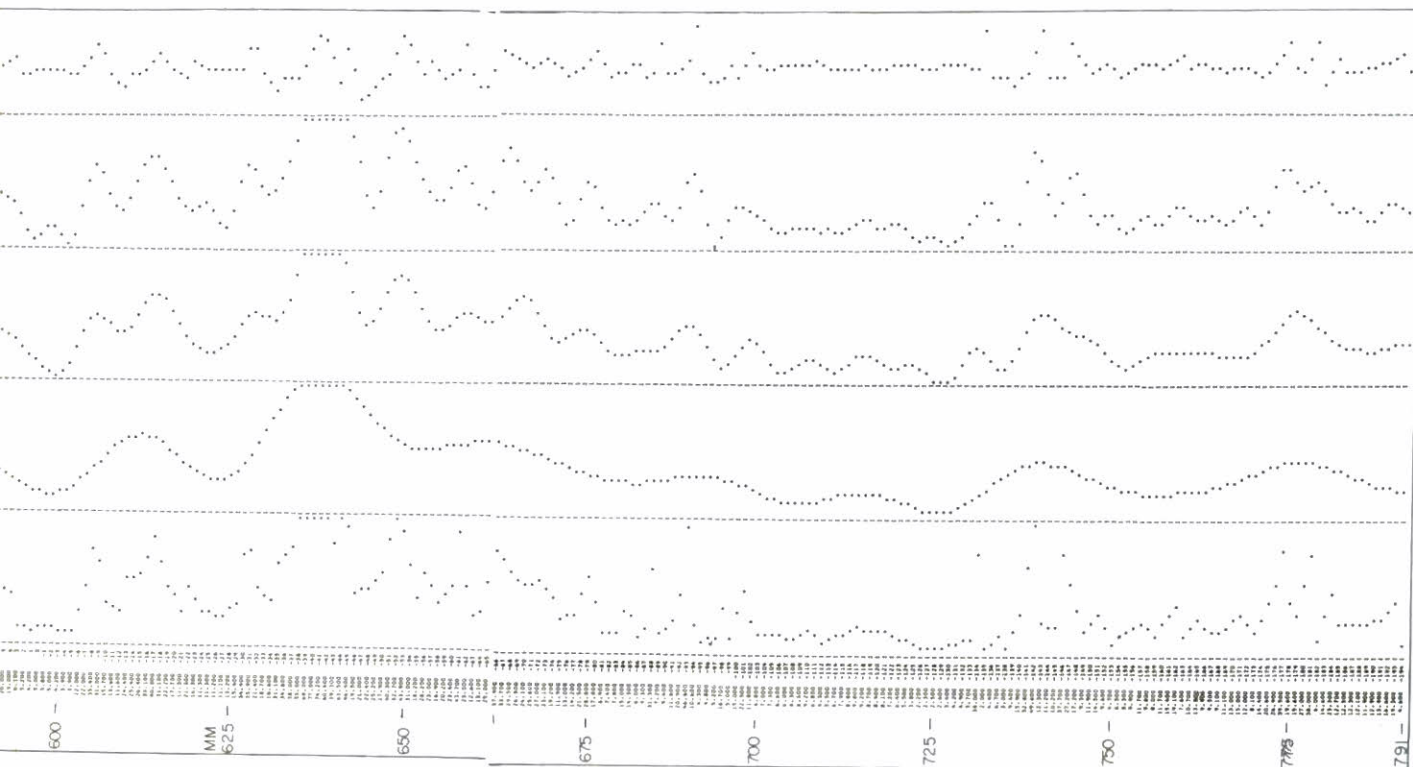


Figure 8.- Plot of observed, calculated, and residual values of shale oil yield data from National Farmers Union Exploration Company Corehole 9 in Uintah County, Utah. Figures at the bottom are depth in corehole in feet. MM indicates the position of Mahogany Marker, a widely used stratigraphic datum within the Green River Formations. Shale oil

large areal variations in calcium carbonate content can be represented as a series of interacting harmonics. The geology of magnesite deposits at Gabbs has been described by Martin (1958). Magnesite occurs as a zone about 400 feet thick within recrystallized dolomite in the Luning Formation of Late Triassic age. Impurities in the magnesite consist of silica and calcium carbonate. Distribution of calcium carbonate is controlled by a network of intersecting flexures, faults, and joints, which in turn have affected movement of fluids through the magnesite. The magnesite is mined in benches about 10 feet thick.

Data used in this study were obtained through the courtesy of Conrad Martin, recently of Stanford University, and formerly of Basic, Incorporated. The data consist of calcium carbonate percentages as determined by chemical analysis of drill cuttings within a ten-foot bench from drill holes spaced about 6 feet apart. Martin (1958) mapped these values over an irregular-shaped area with dimensions roughly 350 by 400 feet. Within this area, a smaller rectangular area (Fig. 9) about 220 by 240 feet in extent was chosen for analysis using double Fourier series.

A double Fourier series is capable of effectively representing the general variations in calcium carbonate content within the area of study (Fig. 10, 11). A series containing 6 M



yield values are shown at one-foot intervals. Interpolation was employed at several places to fill in values where observed values were missing. Original data from Stanfield, Smith, and Trudell (1964, pp. 69-71).

terms and 6 N terms has almost reached the maximum degree of complexity attainable with the grid spacing employed; addition of more terms in the series results in only very slight improvement in percent of total sum of squares.

The usefulness of fitting a double Fourier series to the magnesite data may be assessed as follows: (1) Double Fourier series provides an effective approximation of gross variations but not local fluctuations. (2) Gross variations in calcium carbonate in the part of the Gabbs deposit studied may be effectively represented by as few as six harmonics. Inspection of the power spectrum values reveals that the gross variation may be interpreted in terms of the interactions between two harmonics across the horizontal direction of the map, two harmonics across the vertical direction, and two other harmonics that represent a "cross-product" effect of the two map directions. (3) If a larger area were analyzed by double Fourier series, it might be possible to develop an effective method of predicting calcium carbonate impurities in the magnesite deposits, provided that the network of intersecting flexures, faults and joints displays a considerable degree of periodicity over a large area. A surface represented by a given Fourier series would, of course, be repeated over and over as it is extended beyond the boundaries of the initial area, permitting some extrapolation.

Table 12.- Fourier coefficients and power-spectrum values of single Fourier series fitted to shale oil yield data. The AZERO/2 value corresponds to an arithmetic mean of 15.318 gallons per ton. Columns headed A(N) are cosine term coefficients whose subscripts (N) pertain to the harmonic number. Columns headed B(N) are sine term coefficients. Power spectrum values represent the sum of the squared coefficient values of each harmonic.

AZERO/2 = 15.318

DEGREE NUMBER	A(N)	B(N)	POWER SPECTRUM	DEGREE NUMBER	A(N)	B(N)	POWER SPECTRUM
1	3.70982	-3.08208	23.26195	41	-.01045	.02262	.00062
2	1.49276	-5.41977	31.60222	42	-.95580	.39086	1.06632
3	-2.74432	-4.30078	26.02800	43	-1.06806	.44549	1.33922
4	-3.02088	.41361	9.29679	44	.16216	-.03206	.02733
5	-.72282	.81069	1.17968	45	.21260	.00888	.04528
6	-1.07028	2.03220	5.27534	46	.90643	.24978	.88401
7	-.76042	.83164	1.26986	47	.22171	-1.26873	1.65884
8	1.90315	1.27368	5.24424	48	.05032	-.27337	.07726
9	1.08351	-.53227	1.45731	49	.58201	.85081	1.06262
10	-.59251	.54815	.65153	50	1.22200	-.16010	1.51891
11	1.23910	-2.07324	5.83368	51	.43889	-.80347	.83820
12	-1.40983	-1.29937	3.67599	52	.91920	-.03203	.84595
13	-1.10858	-.45392	1.43498	53	.53515	-1.08157	1.45619
14	-2.54833	-.56472	6.81288	54	-.03756	-.65961	.43650
15	-2.83180	1.65968	10.77363	55	.32143	-1.52913	2.44156
16	-.95670	1.68273	3.74684	56	-.77963	-.83184	1.29979
17	-.02259	1.27026	1.61408	57	-.47774	1.05434	1.33987
18	-1.28623	1.02808	2.71132	58	1.18293	.13164	1.41666
19	.22600	.42276	.22980	59	-.10683	-.25972	.07887
20	.54519	-.35407	.42260	60	.08487	-.76772	.59660
21	.03201	.41202	.17078	61	.24279	-.49524	.30421
22	.53217	-1.24118	1.82373	62	.14568	.45534	.22855
23	-.93701	-.87575	1.64492	63	-.00680	1.36897	1.87414
24	.15548	-.44861	.22543	64	-.18605	-1.40958	2.02152
25	-.94770	.38886	1.04934	65	.17148	.02654	.03011
26	-.49849	1.64414	2.95169	66	.26108	.63407	.47021
27	-.15250	-.45346	.22889	67	-.44509	-.15865	.22327
28	1.24169	.94602	2.43674	68	-.93837	.40579	1.04520
29	.93558	.67975	1.33736	69	.54030	-.38915	.44337
30	.20883	-.89660	.84751	70	-.33703	-.39577	.27022
31	-.81312	-1.78467	3.84622	71	.23634	-.64182	.46779
32	-.88840	-.17066	.81837	72	-1.07989	-.24942	1.22838
33	-1.41829	.28975	2.09549	73	-1.28058	-.37979	1.78413
34	-1.30997	1.60069	4.27824	74	.01024	1.02368	1.04802
35	-1.25022	.87448	2.32776	75	.40702	-.08168	.17233
36	-.31736	.03105	.10168	76	-.17398	.10838	.04201
37	.93039	1.63092	3.52551	77	.21292	-.47893	.27471
38	.83444	1.34813	2.51373	78	-.46872	-.34888	.34142
39	.16784	-.52838	.30735	79	.11181	.33166	.12250
40	-.35685	-.75084	.69110	80	.30496	-1.19367	1.51785

PERCENT Ca CO_3

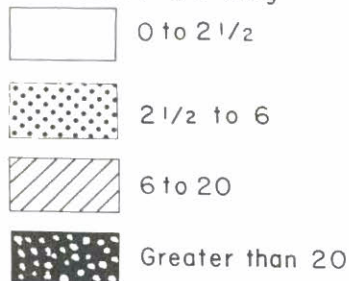
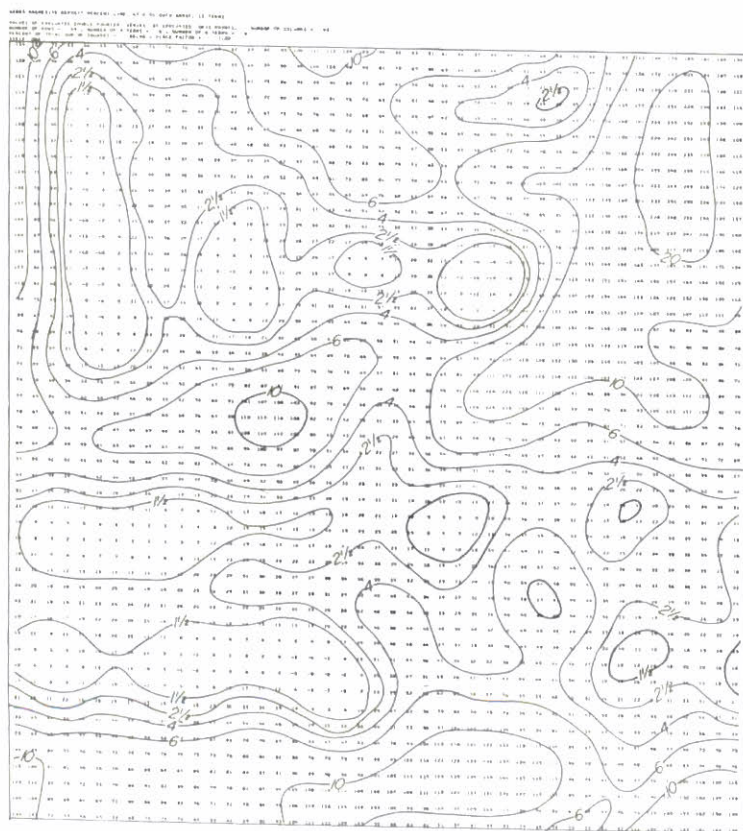
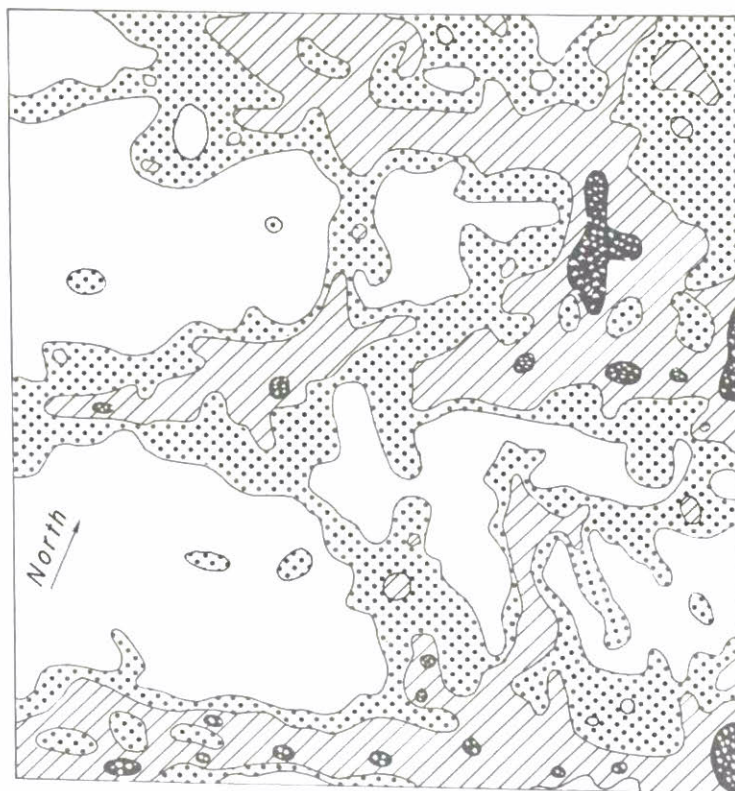


Figure 9.- Map showing distribution of percent calcium carbonate in magnesite deposits at Gabbs, Nevada. The data pertain to part of the 5800-5810 foot bench of the Gloria 5 - Betty O'Neal Pit operated by Basic, Incorporated. Patterns portray the range of values within contour bands. Data from Martin (1958).



0 20 40 60
Scale in feet

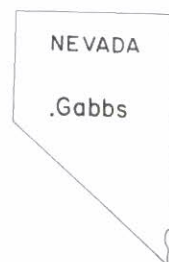


Figure 10.- Contour map of surface portraying double Fourier series containing 6 M terms and 6 N terms fitted to percent calcium carbonate in magnesite deposit (Fig. 9) at Gabbs, Nevada. Numbers on grid represent percent in tenths. Percent of total sum of squares is 66.46.

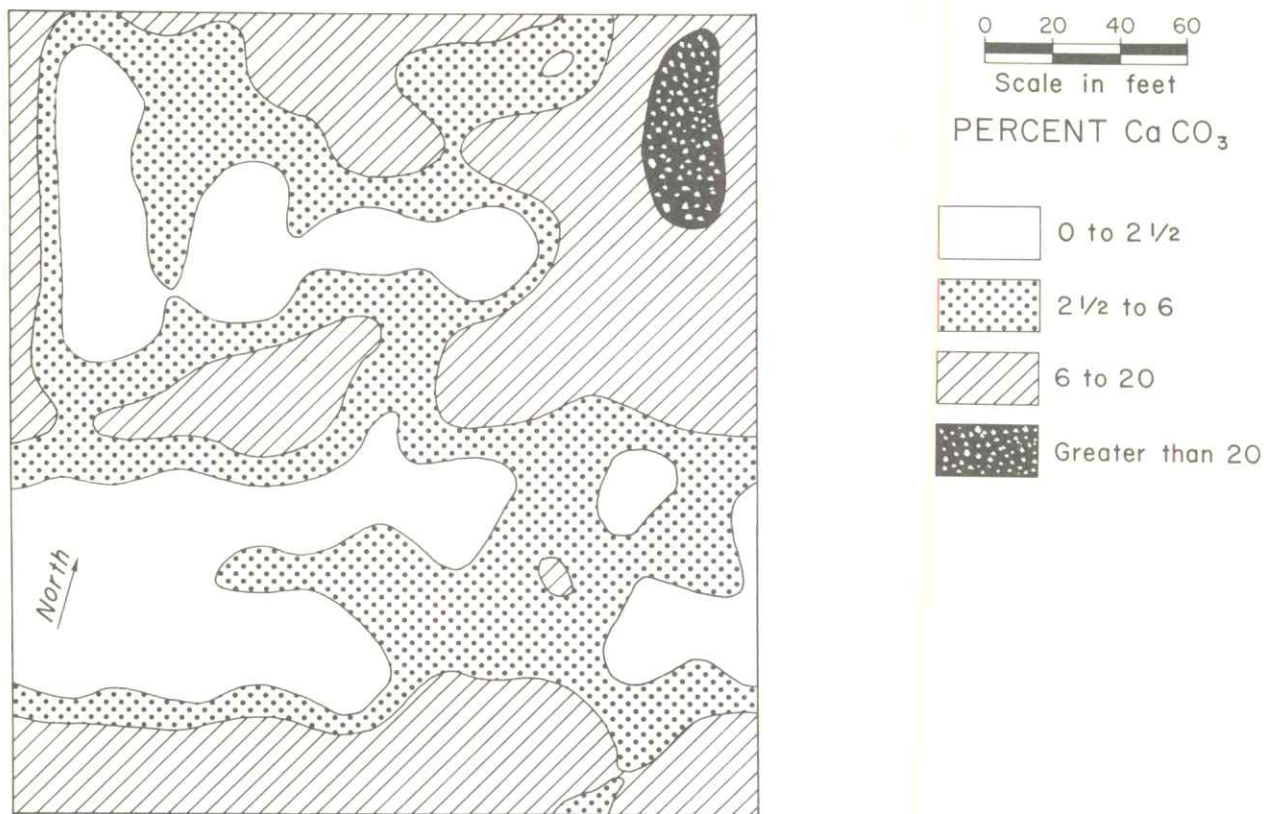


Figure 11.- Map of double Fourier surface representing percent calcium carbonate with contour bands distinguished by same patterns used in Figure 9. Comparison of Figures 9 and 11 reveals considerable similarity of broad-scale variations.

Double Fourier Series Analysis of Geologic Structure

The geologic structure of the top of the Pennsylvanian Lansing Group was analyzed in an area of about 3500 square miles in south-central Kansas (Fig. 12) to see whether double Fourier series are capable of successfully representing the rather complex shape of the surface (Fig. 13). The structure within the area of study is a southwest-dipping homocline on which are superimposed many lesser gentle folds.

A surface represented by a double Fourier series is shown in Figure 14. Except for that part near the long edges of the map, the double Fourier series surface is a fairly good approximation of the actual surface (Fig. 13). Conspicuously absent in the Fourier surface are the relatively small crenulations present in the actual surface. The limit of the ability of this Fourier surface to approximate the actual surface has been reached with 6 M terms and 6 N terms in the double Fourier series. Incorporation of additional terms does not result in an improvement. Thus, the "noise level" has already been reached, and increased complexity of the fitted surface merely interacts at random with data representing the actual surface.

The failure of the Fourier surface of Figure 13 to provide a good approximation along the long edges of the map is due to the fact that the actual surface was not "leveled" before the Fourier series was fitted. The actual surface has, of course, a pronounced slope toward the southwest (toward left side of map). The effect of this slope is to pose a rather severe constraint on the fitted surface, namely that it must be the same elevation at corresponding points on either side of the fitted surface, whereas the elevation of the actual surface is quite different on the two long sides of the map. The original surface may be leveled by fitting a least-squares plane to the observed data and subtracting the elevations of this plane at each point. Pierson (1960) demonstrates the usefulness of this technique.

The residual map, obtained by subtracting the unleveled Fourier surface from the actual surface, is shown in Figure 15. The configuration of the residual surface reflects the fact that the small crenulations in the actual surface are not represented by the Fourier surface. The strongly elongate parallel trends of the residuals reflect the interaction between the M harmonic terms (across the short dimension of the map) and data of the actual surface. Furthermore, the elongate residual trends also reflect the orientation of the sampling grid. Different orientations presumably would have an important effect on the configuration of the residuals.

When the original surface is leveled, there is dramatic improvement in the quality of fit of the Fourier surface. To demonstrate the improvement, the original data were leveled by fitting a least-squares plane (Fig. 16), which is, of course, the same process as obtaining the residuals of a first-degree power series polynomial trend surface. When a Fourier surface is fitted to the data of Figure 16, the fitted surface (Fig. 17) provides a very close approximation. Note that the fitted surface represents a double Fourier series containing the maximum possible number of terms (12 x 18) considering the dimensions of the grid (25 x 37). The percent of total sum of squares of the fitted surface is 91.3, which represents the highest attainable value utilizing leveled data in this example.

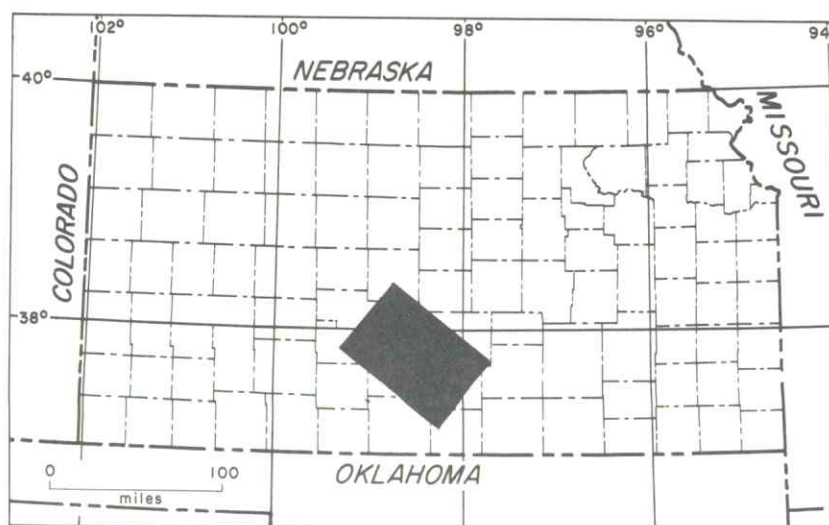


Figure 12.- Index map showing area in which top of Lansing Group was analyzed.

There is an advantage in restoring the "tilt" to the Fourier surface after it has been fitted. This may be done by simply reversing the process by which the leveling was accomplished, namely by adding to the fitted surface the least-squares plane that was subtracted earlier in the leveling process. If the least-squares plane removed in this example is added to the fitted Fourier surface of Figure 17, the resulting surface is very nearly identical to that of Figure 16, although it does fail to match in places along the edges.

Figure 18 is an example of a Fourier surface that was fitted to leveled data, and then tilted back. Although the surface is represented by a double Fourier series containing only 7 M terms (horizontal direction) and 10 N terms (vertical direction), the approximation of the original surface (Fig. 13) is fairly good. A measure of the closeness of approximation is given by the residual values (Fig. 19). The pattern of the residual values is of interest, although comparison of the residual highs of Figure 19 with the locations of oil fields reveals only minor geographic coincidence, and it is concluded that the grid spacing is too coarse to perceive the structural irregularities that are associated with structurally controlled oil pools.

Double Fourier Series Analysis of Topography

The topography of a dissected upland plateau near Lawrence, Kansas, was analyzed with the double Fourier series program to determine whether a typical ridge and valley complex could be represented as a series of harmonic terms. The area is in the northern part of the Lone Star Quadrangle (Fig. 20) and is 4.5 x 2.25 miles in extent.

Data consist of topographic elevations observed at points on a grid containing 49 rows and 25 columns. A total of 1225 data points were used. Topography in the grid area consists of a series of dendritic-shaped valleys which are incised about 200 feet below the upland surface.

To provide an effective comparison of the original data and subsequent Fourier surfaces, the original data were contoured manually (Fig. 21). There is, of course, some loss in detail in the manually contoured map compared with the actual surface because of the relatively coarse spacing of the data points.

A double Fourier series containing 0 to 10 M terms (short dimension of map) and 0 to 11 N terms (long dimension of map) was fitted to the data. The Fourier coefficients and harmonic vector magnitudes are shown in Table 13. The harmonic vector magnitude values suggest that M terms 0 through 4 (5 columns counting from left), are significant, and that the higher degree M terms (columns 6 to 11 counting from left) contribute relatively little, judging from their relatively low harmonic vector magnitude values. On the other hand, N terms 0 to 9, and 11 are significant, judging from the fact that there are relatively high values within the first 5 columns of the harmonic vector magnitude array for all rows except the eleventh row (which represents N terms of the tenth degree).

DOUBLE FOURIER ANALYSIS LANSING STRUCTURE, SOUTH-CENTRAL KANSAS

PLOT OF OBSERVED DATA VALUES, NUMBER OF ROWS = 37, NUMBER OF COLUMNS = 25, SCALE FACTOR = 1.000

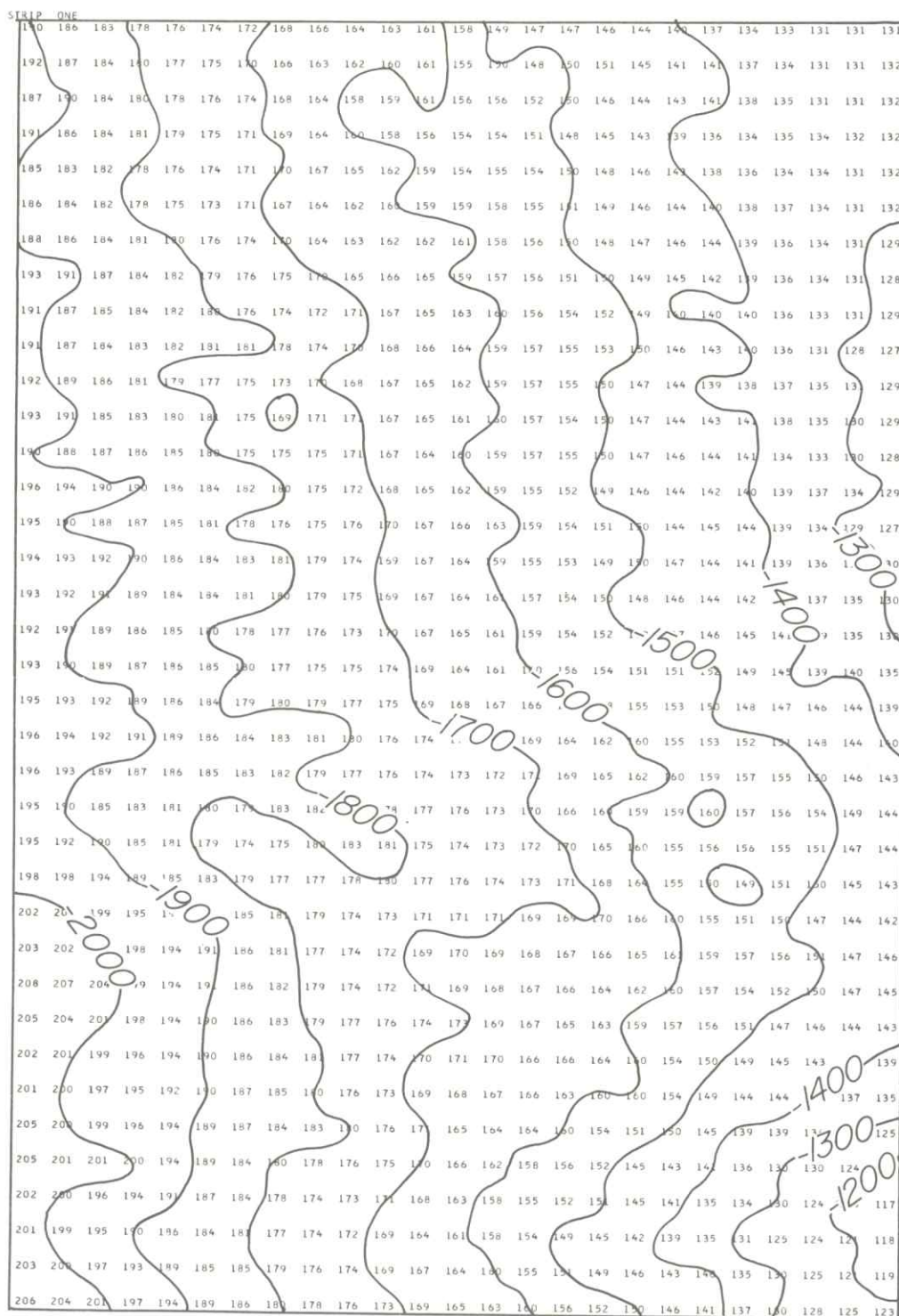


Figure 13.- Structure contour map based on observed data values of elevations of top of Pennsylvanian Lansing Group in south-central Kansas. See Figure 12 for location of area and orientation of map. Area is about 48 miles wide and 72 miles long. Data from Merriam, Winchell, and Atkinson (1958). Contour values are in feet below sea level. The numbers printed on the grid are in tens of feet and negative signs have been omitted. North is toward the upper right.

DOUBLE FOURIER ANALYSIS LANSING STRUCTURE, SOUTH-CENTRAL KANSAS

VALUES OF EVALUATED DOUBLE FOURIER SERIES AT SPECIFIED GRID POINTS, NUMBER OF COLUMNS = 25
 NUMBER OF ROWS = 37, NUMBER OF M TERMS = 6, NUMBER OF N TERMS = 9
 PERCENT OF TOTAL SUM OF SQUARES = 76.84, SCALE FACTOR = 1.00
 STRIP ONE

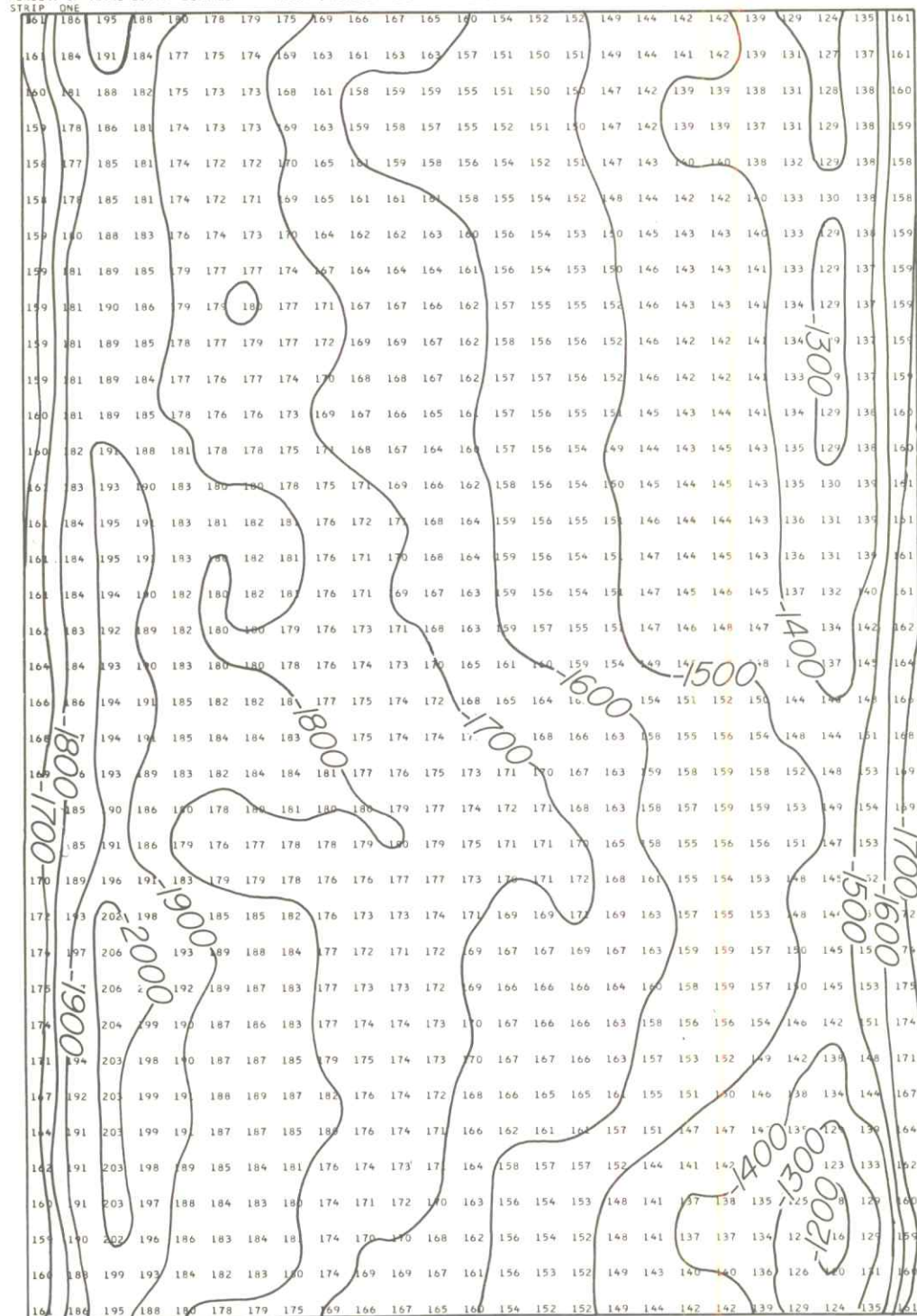


Figure 14.- Structure contour map of top of Lansing Group obtained by evaluating double Fourier series containing 6 M terms and 9 N terms. M terms pertain to short dimension of map; N terms to long dimension. Coefficients of terms have been obtained by analysis of observed data. Percent of total sum of squares is 76.84. Contour values are in feet below sea level. Numbers printed on the grid are in tens of feet and the negative signs have been omitted. North is toward the upper right (Fig. 12).

DOUBLE FOURIER ANALYSIS LANSING STRUCTURE, SOUTH-CENTRAL KANSAS

RESIDUAL VALUES, NUMBER OF COLUMNS = 25, NUMBER OF ROWS = 37, NUMBER OF M TERMS = 6, NUMBER OF N TERMS = 9
SCALE FACTOR = 1.000

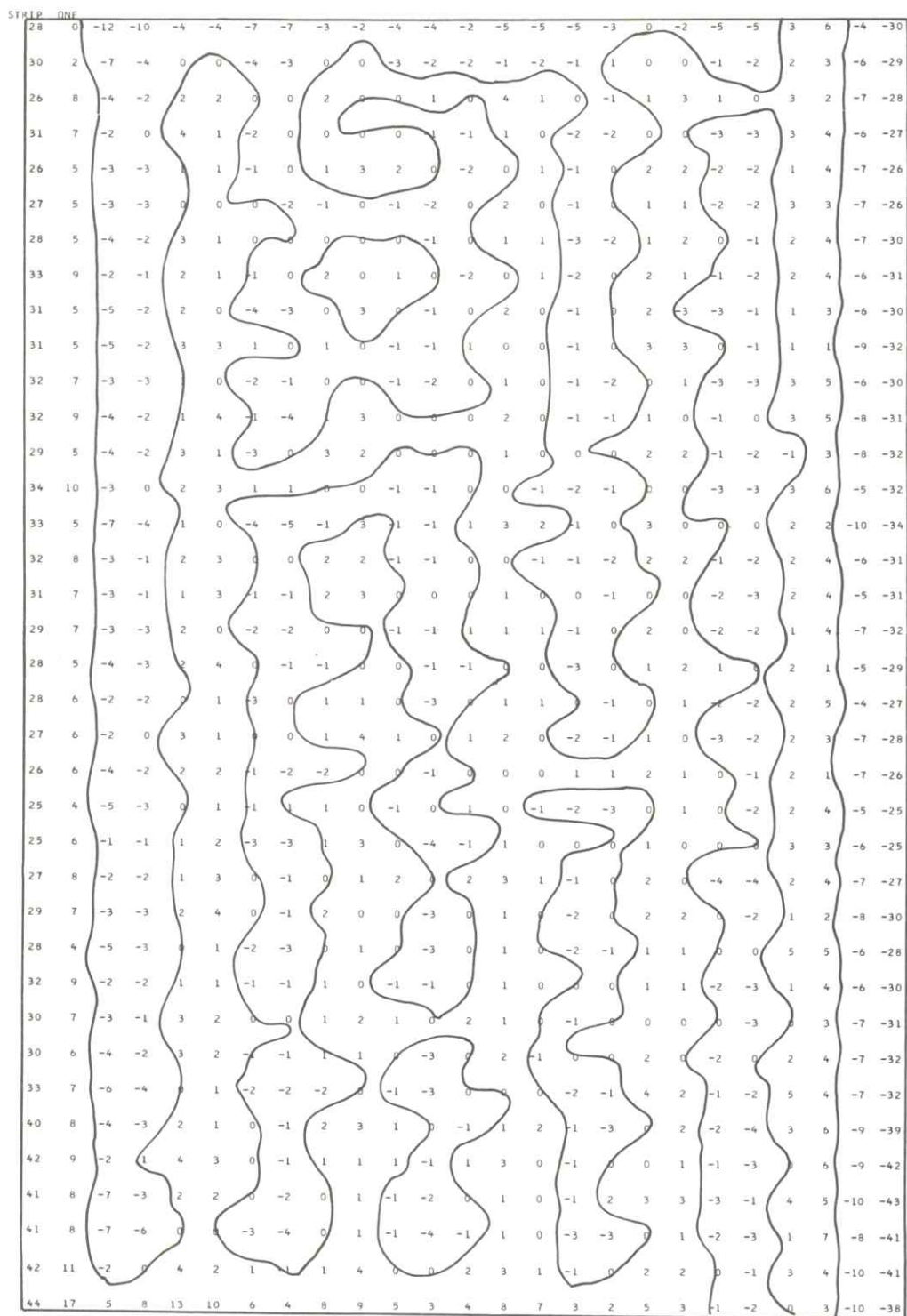


Figure 15.- Contour map of residual values of structure of top of Lansing Group in south-central Kansas. Contour lines have a value of zero. Printed values pertain to tens of feet.

DOUBLE FOURIER ANALYSIS LANSING STRUCTURE, SOUTH-CENTRAL KANSAS
 PLOT OF LEVELED ORIGINAL DATA VALUES, WITH APPROXIMATE ZERO MEAN

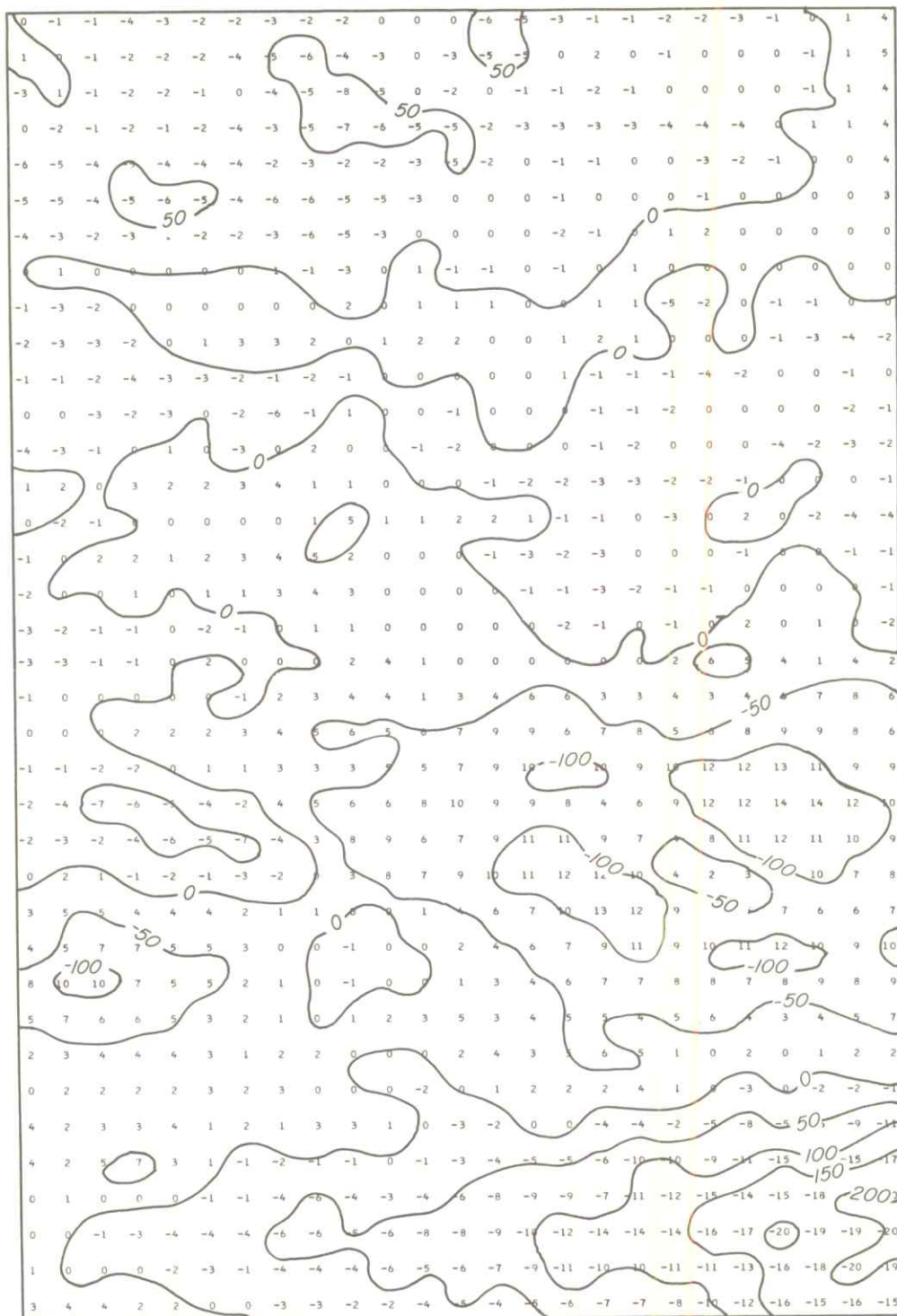


Figure 16.- Leveled original Lansing structure data of Figure 13. The signs have been reversed because the original data were fed in as positive values, although they actually represent feet below sea level. Arithmetic mean of leveled values is approximately zero. Numbers on grid are in tens of feet.

DOUBLE FOURIER ANALYSIS LANSING STRUCTURE, SOUTH-CENTRAL KANSAS

VALUES OF EVALUATED DOUBLE FOURIER SERIES AT SPECIFIED GRID POINTS, NUMBER OF COLUMNS = 25
 NUMBER OF ROWS = 17, NUMBER OF M TERMS = 12, NUMBER OF N TERMS = 18
 PERCENT OF TOTAL SUM OF SQUARES = 91.32, SCALE FACTOR = 1.00
 STIPPLED

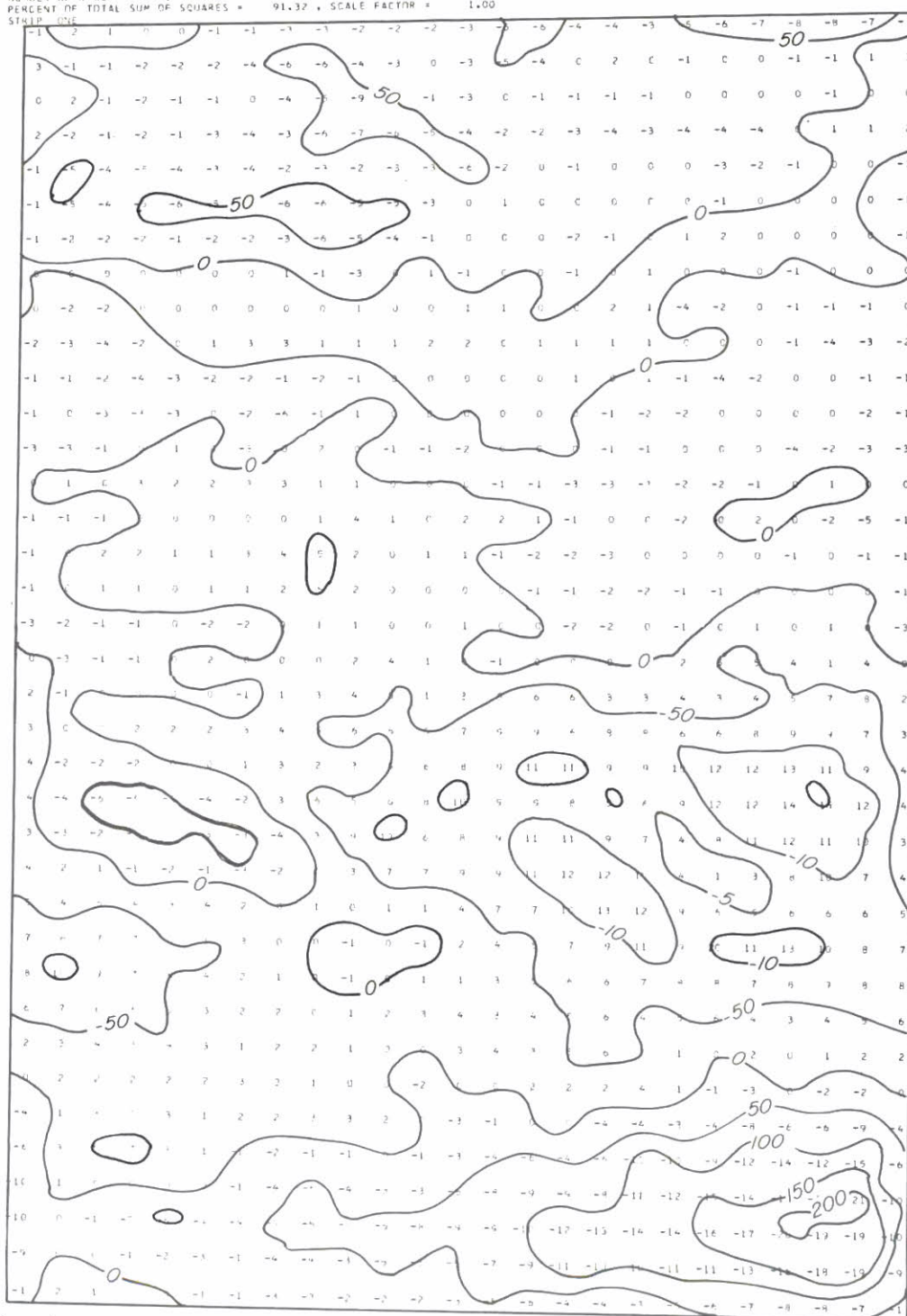


Figure 17.- Fourier surface fitted to leveled Lansing structure data of Figure 16. Note that approximation is very good, except at some places along edge of map. The signs have been reversed because the original data (Fig. 13) were fed in as positive values, whereas they represent feet below sea level. Numbers on grid represent tens of feet.

DOUBLE FOURIER ANALYSIS LANSING STRUCTURE, SOUTH-CENTRAL KANSAS

VALUES OF EVALUATED DOUBLE FOURIER SERIES AT SPECIFIED GRID POINTS, NUMBER OF COLUMNS = 25
 NUMBER OF ROWS = 37, NUMBER OF M TERMS = 7, NUMBER OF N TERMS = 10
 PERCENT OF TOTAL SUM OF SQUARES = 97.54, SCALE FACTOR = 1.00
 STRIP ONE

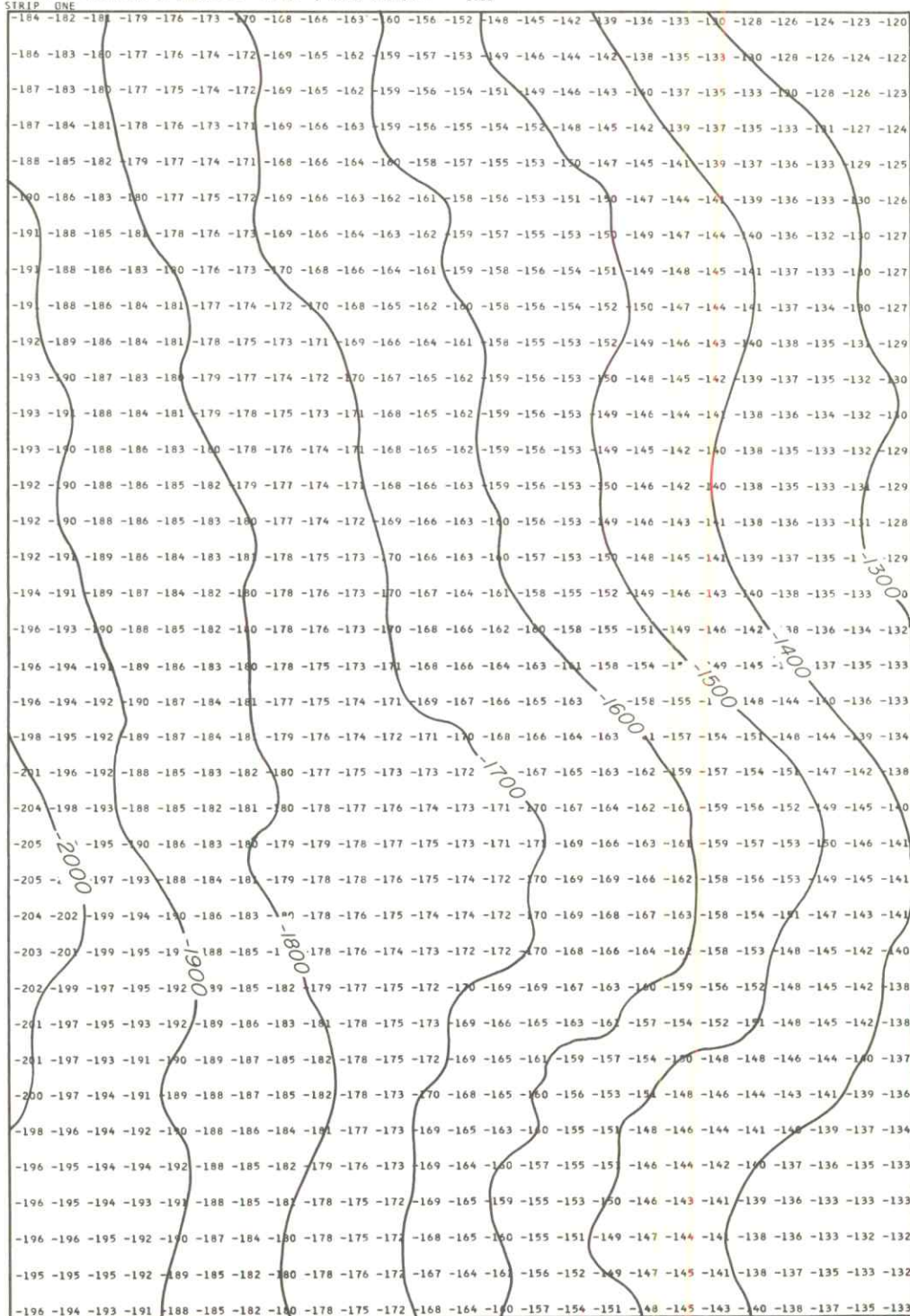


Figure 18.- Lansing structural surface obtained by tilting Fourier surface represented by double Fourier series containing 7 M terms and 10 N terms fitted to leveled original data (Fig. 16). Note that signs are all negative on grid and represent tens of feet below sea level.

DOUBLE FOURIER ANALYSIS LANSING STRUCTURE, SOUTH-CENTRAL KANSAS

RESIDUAL VALUES, NUMBER OF COLUMNS = 25, NUMBER OF ROWS = 37, NUMBER OF M TERMS = 7, NUMBER OF N TERMS = 10
SCALE FACTOR = 1.000

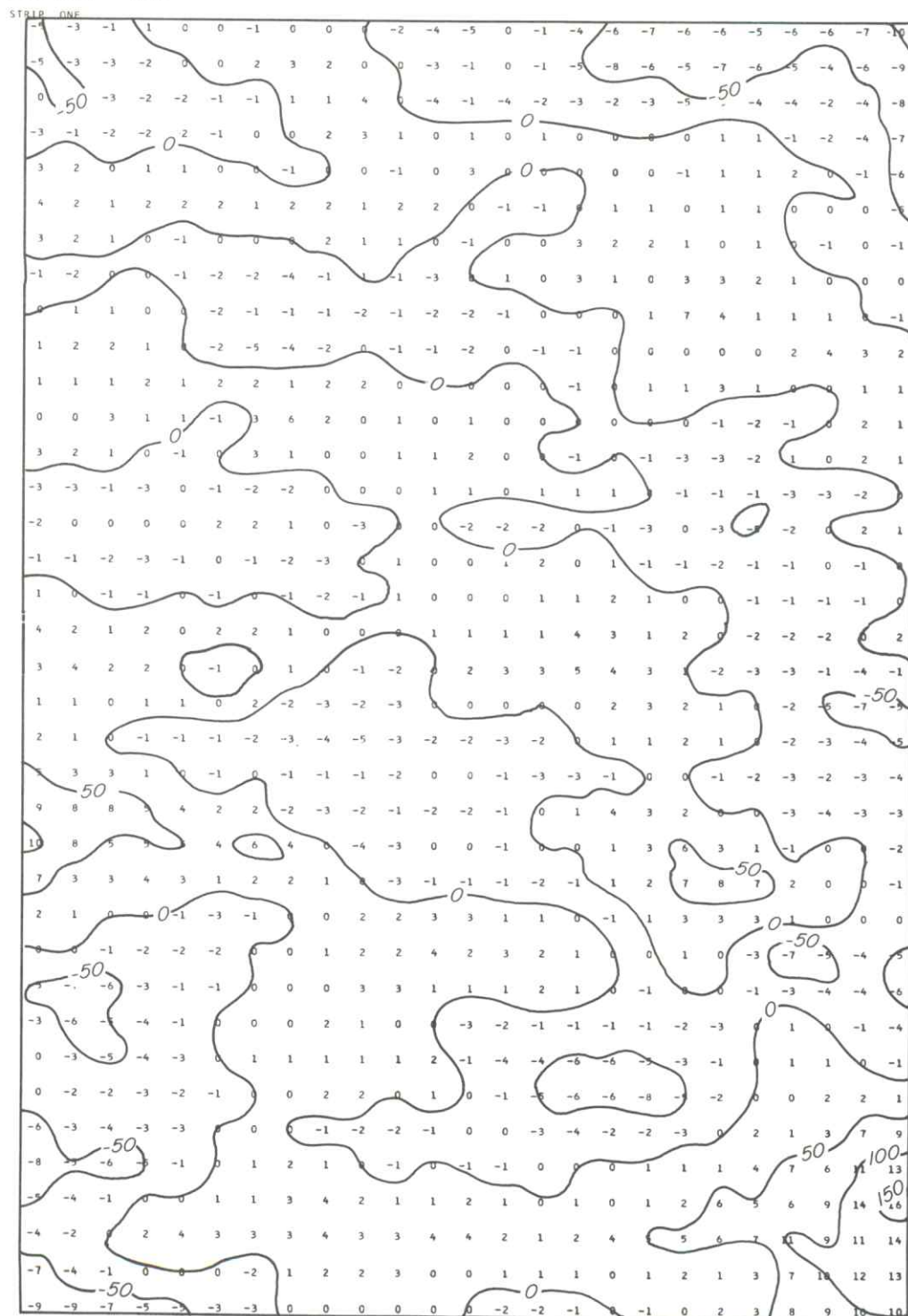


Figure 19.- Residual map of Lansing structural data obtained by subtracting fitted surface (Fig. 18) from actual surface (Fig. 13). Numbers on grid are in tens of feet above (positive) or below (negative) fitted surface.

The harmonic vector magnitude of a given degree represents the Fourier coefficient values for the terms of that degree. The harmonic vector magnitude is defined as the square root of the sum of the squared Fourier coefficient values pertaining to terms of specified M and N degree. Thus, scanning of the harmonic vector magnitude values provides a ready method of ascertaining the contribution of terms of specified degree.

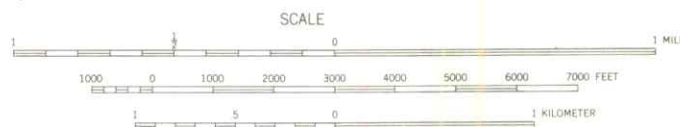
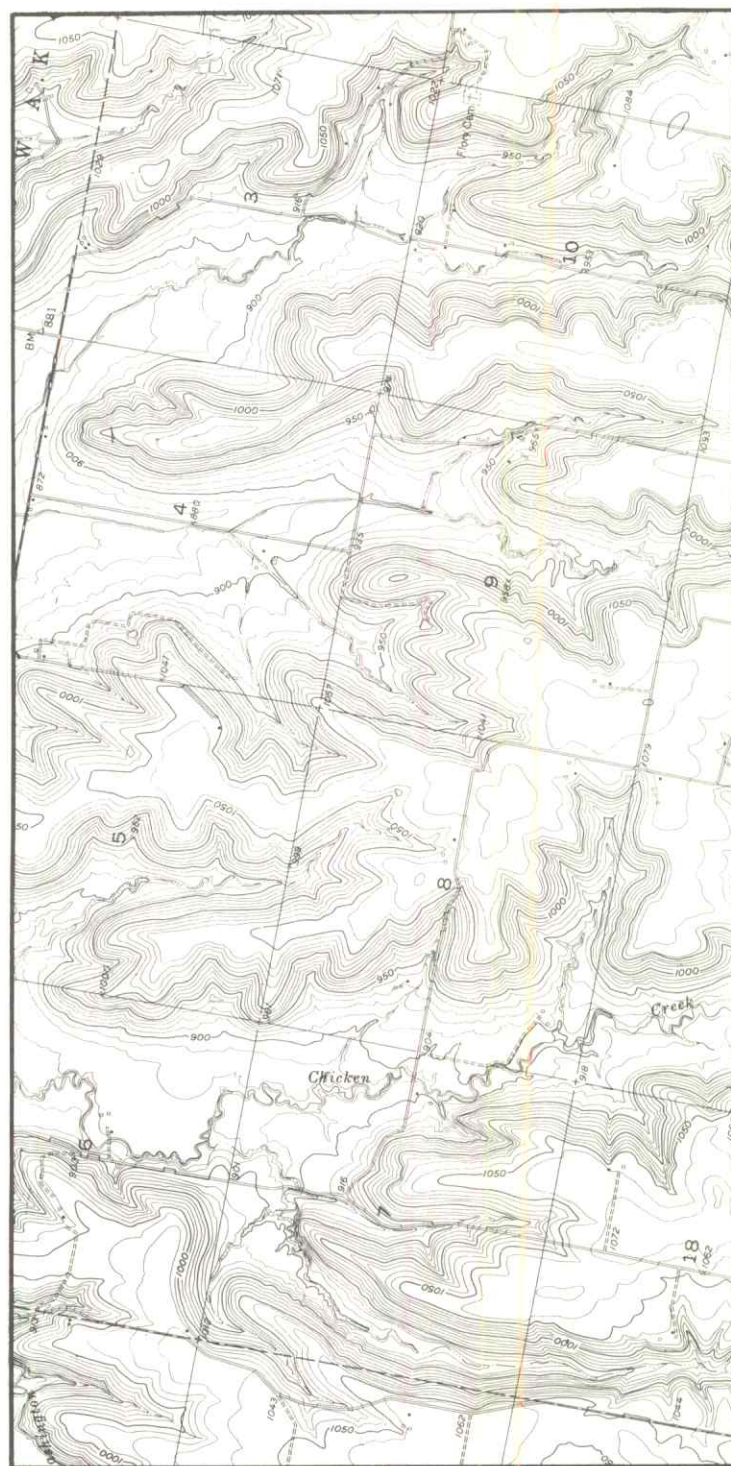
Comparison of the Fourier surface obtained by evaluating the double Fourier series (Fig. 22-24) with the original surface (Fig. 21) reveals a remarkable similarity. In spite of the extremely complex configuration of the actual surface, the Fourier surface provides such a good approximation of the actual surface, that it represents 92.58 percent of the total sum of squares of the original data. The Fourier surface, however, fails to accord with the actual surface at places along the edge of the map. This is due to the fact that the map-edge values of the Fourier surface must be the same at opposite points on any two edges. In the Fourier surface shown (Fig. 22), the original data were first leveled, the Fourier surface fitted, and the Fourier surface then retitled. This practice minimizes the "edge effect."

The values of the Fourier coefficients provide a direct measure of the configuration of the land surface. For example, the A_{20} coefficient (first column, third row of A array of Fourier coefficients, Table 13) is 86.5 (the Fourier coefficients are expressed in floating-point notations in the output). The Fourier coefficients are in the same units as the original data. Thus, this coefficient has a value of 86.5 feet, and pertains to a cosine-cosine term of infinite period length (zeroth degree term) in the horizontal direction and $4.5/2 = 2.25$ miles period length (second degree term) in the long direction.

The ability to represent a dendritically dissected upland by a relatively small number of Fourier coefficients has important implications for geomorphology. It suggests that complex land surfaces can be quantitatively and objectively compared with each other by transforming data representing the surfaces to double Fourier series coefficients, and, in turn, using the Fourier coefficients as descriptors of the surfaces. The techniques of numerical taxonomy (Sokal and Sneath, 1963) could be used to produce an objective classification system for land surfaces employing Fourier coefficients.

Table 13.- Double Fourier series coefficients and harmonic vector magnitude values pertaining to topography of dissected upland surface. Fourier coefficients pertain to A, B, C, and D arrays, and are expressed in floating-point notation.

DENDRITIC DISSECTED UPLAND, 4.5 X 2.25 MILES EXTENT										
A ARRAY OF FOURIER COEFFICIENTS										
-0.2724	00-9.1424	00-6.8323	00-2.6524	00-2.6420	00-5.5441	-01-1.8993	00-1.0043	00-1.3837	00-2.3647	-01-1.1851
-0.6658	00-4.7212	00-1.5209	00-1.3854	00-3.3222	00-2.1026	00-7.2168	-01-1.8952	00-1.4286	00-4.2949	-01-4.6680
0.6499	01-6.7020	00-5.4792	00-1.4067	00-2.6443	00-9.2842	-01-8.1146	-01-9.1345	-01-2.8023	-01-7.7736	-01-9.1358
1.8586	01-1.4622	01-4.1586	-01-1.3036	00-3.4036	00-1.6663	00-2.4717	00-8.8598	-01-7.5165	-01-1.9898	-01-1.9623
2.2073	01-9.4540	00-6.5940	-01-2.4187	-01-3.1744	00-1.0935	-01-4.0169	00-1.5779	00-1.3091	-03-1.4517	-01-7.8687
-3.6651	00-1.2697	01-1.4649	01-4.2589	00-3.8696	00-2.0448	00-2.0127	00-1.2839	00-5.9572	-01-4.1004	-01-3.4258
5.9037	00-7.7394	00-4.8679	00-5.7374	00-2.1562	00-1.1877	00-2.2176	00-1.4022	00-7.9997	-01-3.6872	-01-4.8598
1.5065	-01-8.4530	00-1.6323	00-3.8242	-01-3.5877	-01-2.2720	00-9.5846	-01-8.2513	-01-1.5160	00-9.3444	-01-5.4993
6.8585	00-3.3465	00-2.7844	00-5.5848	00-1.8351	00-1.4139	00-1.2509	00-6.8512	-01-2.6215	-01-2.0461	-01-2.0456
-1.8071	01-4.5029	00-6.6239	00-8.4616	-01-2.0004	00-2.3275	00-3.5671	-01-3.1621	00-8.3323	-01-1.0458	00-2.4544
5.7953	-01-1.7162	00-8.6935	-01-2.3851	00-9.3407	-01-3.8163	-01-3.3788	-01-2.1919	00-9.9814	-01-9.9719	-01-1.1005
1.9876	00-1.0149	00-1.5225	-01-2.2540	00-2.5376	00-2.4272	00-1.7035	00-9.5379	-01-2.2048	-01-6.8255	-01-1.1005
B ARRAY OF FOURIER COEFFICIENTS										
0.0000	00-1.5455	00-7.4335	00-7.5069	00-1.1033	01-1.8534	00-3.1008	-01-1.6812	00-6.5850	-01-1.8760	-01-1.8342
0.0000	00-6.8051	00-5.4845	-01-7.2338	00-2.4208	00-6.6691	-01-2.1697	-01-1.1351	00-1.2483	00-2.5887	-01-5.3394
0.0000	01-1.2960	01-5.1357	00-2.9831	00-1.1997	00-4.0814	00-1.7005	00-7.1533	-01-5.2939	-01-1.2356	-01-7.3935
0.0000	00-1.4252	01-4.9240	00-3.8356	00-4.1037	-01-2.5476	00-1.9654	00-4.9827	-01-2.3819	00-7.2657	-01-8.0453
0.0000	00-3.0094	00-2.3176	00-1.1775	00-1.9747	-01-1.4013	00-2.9138	00-4.5946	-01-5.2601	-02-1.4949	-01-8.8054
0.0000	00-2.0043	01-4.2064	00-6.9114	-01-7.5650	-01-1.8065	-01-6.3363	-01-5.4896	-01-7.6174	-01-1.6490	-01-4.7797
0.0000	00-4.6659	00-5.0595	00-4.7097	00-1.9294	00-1.4129	00-8.9467	-01-7.4544	-01-4.9661	-01-2.5049	-01-2.1892
0.0000	00-2.2512	00-8.0288	00-8.2586	-01-3.0607	00-5.3192	-01-3.2274	00-2.7039	00-5.6877	-03-7.0928	-01-7.5354
0.0000	00-4.4235	00-8.4789	-01-7.2545	-01-1.5080	00-3.0210	-01-1.2465	00-2.4538	-01-1.1051	00-3.9905	-01-1.0736
0.0000	00-1.1027	01-7.9280	-02-3.2356	00-1.8515	00-1.0662	00-2.2717	00-9.4798	-01-1.2717	00-3.7737	-01-3.9489
0.0000	00-1.5169	00-1.3912	00-2.0034	00-3.3143	00-1.4588	00-3.4789	-02-2.5622	00-3.5014	-01-1.1628	00-4.3274
0.0000	00-1.1914	01-7.7824	00-2.9440	00-1.1093	00-8.5693	-01-3.6518	-01-1.8165	00-5.3988	-01-1.1376	00-3.3836
C ARRAY OF FOURIER COEFFICIENTS										
0.0000	00-0.0000	00-0.0000	00-0.0000	00-0.0000	00-0.0000	00-0.0000	00-0.0000	00-0.0000	00-0.0000	00-0.0000
-5.6142	00-1.1196	00-3.0818	00-2.6493	00-6.4403	-01-2.8392	00-8.5524	-01-2.8324	00-4.4205	-01-9.2489	-01-1.1776
-5.4640	00-9.7144	00-5.2296	00-5.3088	00-4.2350	00-2.0577	00-6.7611	-01-7.6851	-01-1.1377	00-1.4968	-02-4.2684
-4.3961	01-1.3385	01-1.2806	00-1.3388	-01-1.7615	00-1.1934	00-9.2461	-01-2.0603	00-8.2751	-01-1.5070	00-2.5497
-3.4647	01-7.3651	00-7.7854	-01-2.7223	00-1.6806	00-3.3799	-02-2.6568	00-1.9549	00-1.7236	00-4.6443	-01-9.1743
3.7077	01-5.7247	00-1.0894	00-1.9329	00-4.5550	00-2.1127	00-3.9479	-01-1.7391	00-2.1869	-01-2.5788	-01-1.2328
1.1456	01-6.2369	00-7.6531	00-1.8110	00-1.8171	-01-8.2607	-01-1.9390	00-4.3088	-01-1.6942	00-7.6659	-01-1.3061
-1.3398	01-6.7621	00-8.0831	-01-5.2780	00-2.7957	00-1.2429	-01-1.4004	00-7.7753	-01-1.0885	00-1.8482	-01-7.8201
-6.1970	00-1.4124	00-1.9861	00-2.9816	00-2.6522	00-4.7177	-01-3.4265	00-7.2329	-01-1.2329	00-3.7725	-02-1.0298
-8.4006	00-1.6247	00-1.6336	00-9.9962	-01-1.1348	00-3.3861	00-1.6986	00-6.9939	-01-2.8915	-01-4.4411	-01-1.9367
-5.0415	00-5.2433	00-2.0067	00-4.4360	00-3.5278	-01-4.9717	-01-7.8857	-01-6.5520	-02-1.4363	00-5.6931	-01-2.9611
-1.4144	00-3.9777	00-7.4306	-01-3.1332	00-1.0996	00-5.9399	-01-1.1524	00-2.5729	00-6.4142	-01-3.9174	-01-5.4335
D ARRAY OF FOURIER COEFFICIENTS										
0.0000	00-0.0000	00-0.0000	00-0.0000	00-0.0000	00-0.0000	00-0.0000	00-0.0000	00-0.0000	00-0.0000	00-0.0000
0.0000	00-1.0539	01-1.1659	01-3.1112	00-2.0361	00-1.5956	00-3.3971	00-1.3548	00-3.6446	-01-3.9756	-01-3.7175
0.0000	00-1.3842	01-2.5462	00-1.7574	00-2.8657	00-2.5149	00-2.8659	00-2.6043	00-2.3304	00-1.1353	00-6.9016
0.0000	00-3.2612	00-4.4120	-01-2.5503	00-1.0256	00-1.2721	00-6.4603	-02-6.1875	-01-5.1007	-01-4.8742	-02-3.5961
0.0000	00-7.3070	00-2.9071	00-2.1025	00-1.6577	00-3.0680	00-5.4452	-01-1.1554	00-7.4089	-01-1.5001	-01-3.9592
0.0000	00-4.2940	00-8.8321	-01-4.9253	00-7.0456	00-2.8178	00-1.7753	00-2.0042	00-2.2424	00-1.0209	00-2.2315
0.0000	00-1.8089	00-4.3879	00-3.6845	00-1.7867	00-2.7761	-01-7.9970	-01-1.1925	00-1.0554	00-2.2312	-01-2.8307
0.0000	00-6.1725	00-1.7249	00-2.2886	00-3.7384	00-9.9945	-01-5.5344	-01-8.1632	-01-1.4026	00-8.3494	-01-4.9224
0.0000	00-8.1840	00-4.6724	00-7.9909	-01-9.0885	-01-1.8381	00-6.4952	-01-1.1806	00-4.9219	-01-4.7589	-01-5.2706
0.0000	00-1.3262	00-4.7532	00-6.7020	00-2.3545	00-1.2482	00-1.6105	00-2.4800	00-1.7497	00-9.9198	-01-2.0672
0.0000	00-3.4279	00-7.3694	00-2.1351	00-4.2058	00-1.2227	00-7.1118	-01-3.2846	-01-2.8029	-01-1.7787	00-6.4999
0.0000	00-1.0967	01-2.3977	00-3.2460	00-3.5412	00-2.8661	00-3.2318	-01-4.6959	-01-1.1622	00-9.6754	-02-2.5068
HARMONIC VECTOR MAGNITUDE ARRAY										
1.07	9.27	10.10	7.96	11.39	1.93	3.64	1.96	1.53	.30	2.18
11.18	13.45	12.17	8.42	4.83	3.90	3.58	3.84	1.98	1.12	1.42
86.67	22.33	10.08	6.49	5.88	5.30	3.33	2.95	2.66	1.38	1.25
47.73	24.63	9.06	4.79	3.99	3.51	3.30	2.38	2.68	1.69	.49
41.08	14.35	3.86	3.64	3.96	3.37	5.66	2.80	1.88	.53	1.50
37.26	24.78	15.58	6.83	9.27	4.08	2.79	3.00	2.45	1.14	1.30
12.89	11.13	11.27	8.48	3.41	2.04	3.18	2.03	2.21	.91	1.36
13.40	12.66	8.41	5.82	5.59	2.54	3.69	3.04	2.33	1.45	.96
9.24	9.99	6.36	6.42	3.67	2.30	3.91	1.56	1.75	.65	1.50
19.93	12.09	8.32	7.55	3.78	4.42	3.28	4.19	2.34	1.55	.38
5.07	6.67	3.51	5.83	5.45	2.00	1.11	3.39	1.81	2.42	1.22
2.44	16.70	3.98	5.39	4.48	3.97	2.73	3.61	1.72	1.23	.97



CONTOUR INTERVAL 10 FEET
DATUM IS MEAN SEA LEVEL

Figure 20.- Topographic map of northern part of Lone Star Quadrangle, Kansas. Area shown is 4.5 by 2.25 miles in extent, and lies roughly ten miles southwest of Lawrence, Kansas.

DEMORPHIC DISSECTED UPLAND, 4.5 X 2.25 MILES EXTENT
 PLOT OF OBSERVED DATA VALUES, NUMBER OF ROWS = 49, NUMBER OF COLUMNS = 25, SCALE FACTOR = 1.000

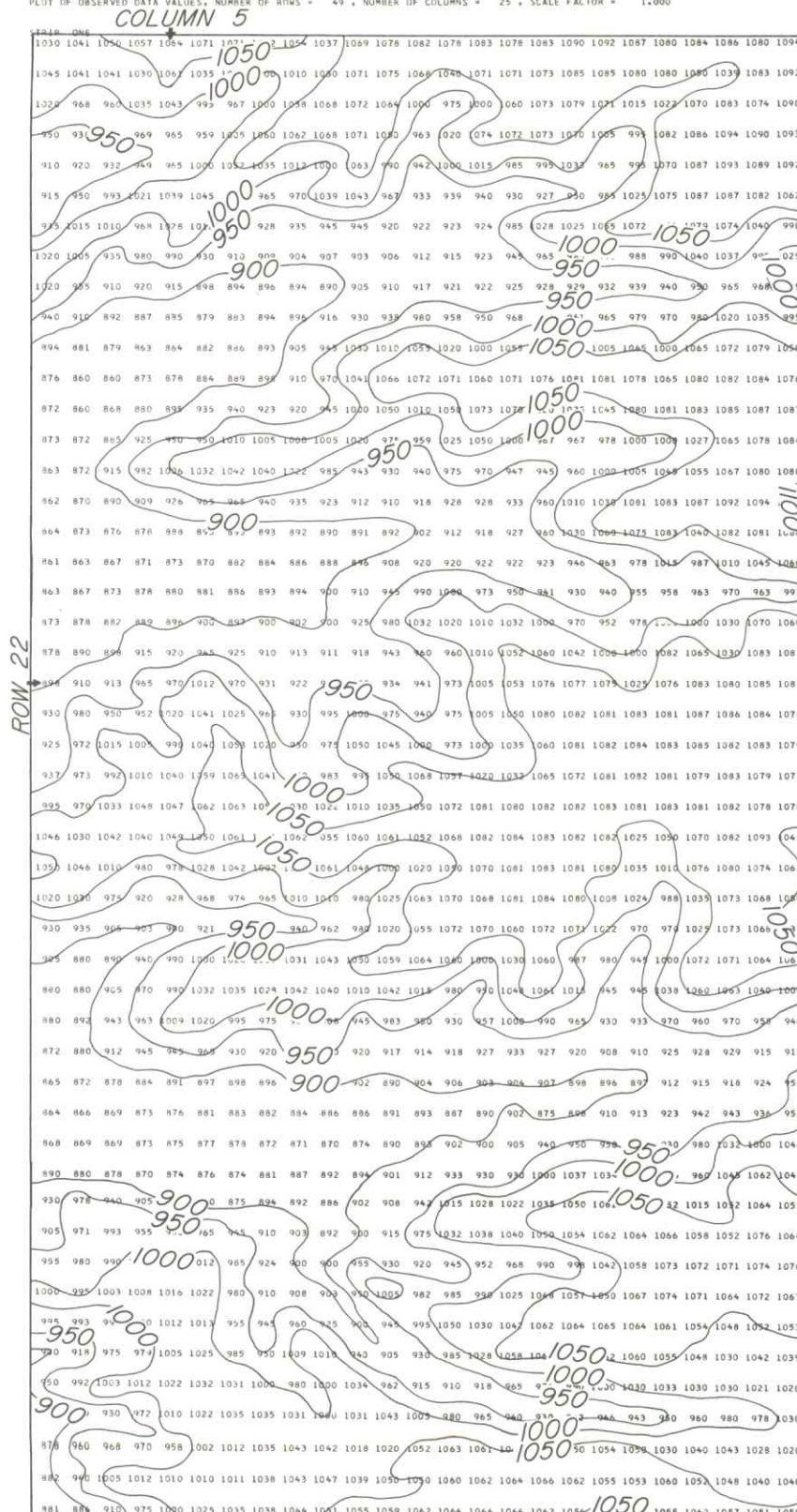


Figure 21.- Data values taken from topographic map (Fig. 20) and recontoured manually for comparison purposes. Data array contains 49 rows and 25 columns. Grid points are spaced slightly less than 1/10 mile apart.

EPYRETIC DISSECTED UPLAND, 4.5 X 2.25 MILES EXTENT

VALUES OF EVALUATED DOUBLE FOURIER SERIES AT SPECIFIED GRID POINTS: NUMBER OF COLUMNS = 25
 NUMBER OF ROWS = 49, NUMBER OF M TERMS = 11, NUMBER OF N TERMS = 12
 PERCENT OF TOTAL SUM OF SQUARES = 92.58, SCALE FACTOR = 1.00
 51412-006

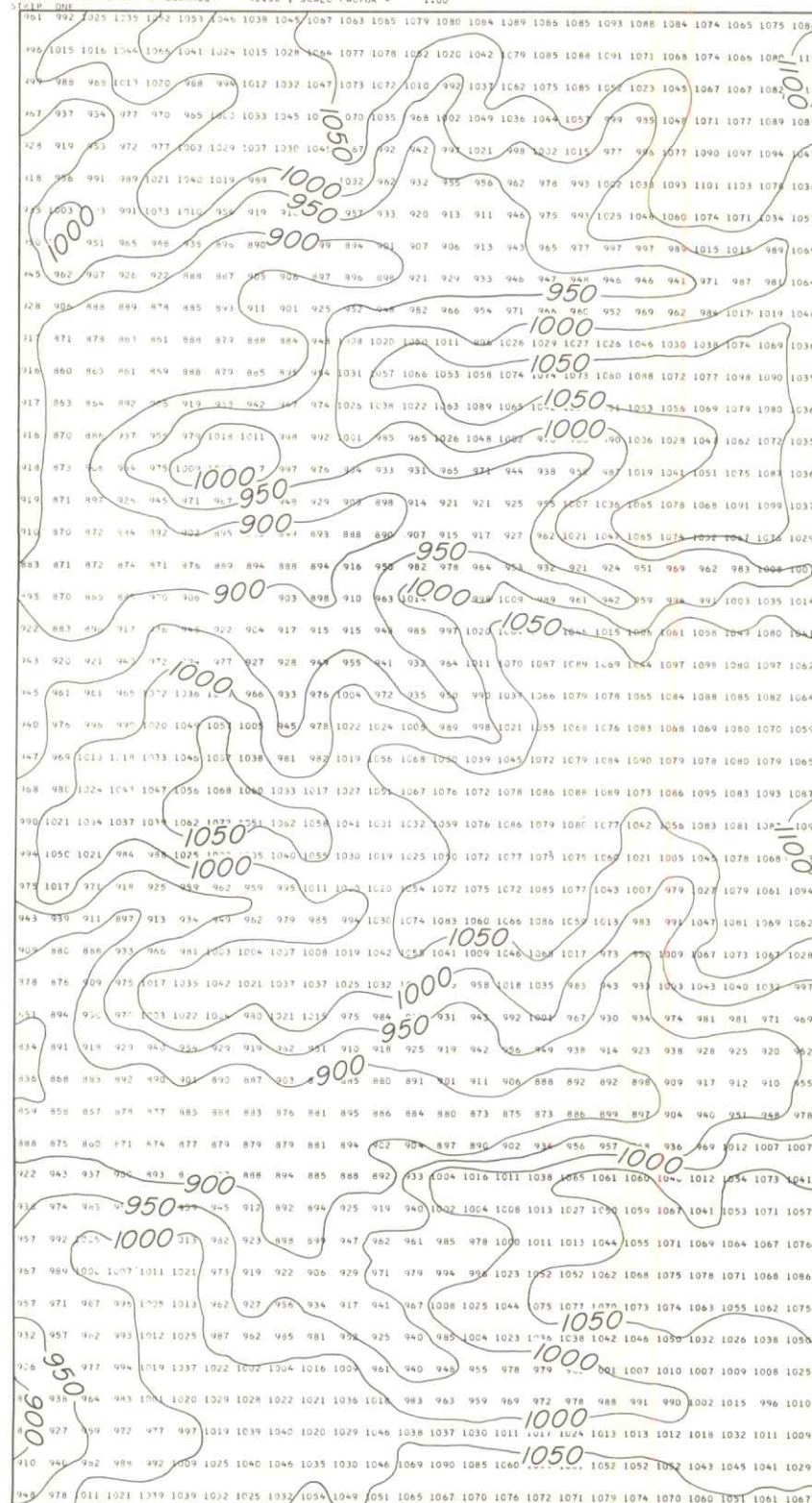


Figure 22.- Data representing evaluated double Fourier series containing 11 M terms (short dimension of map) and 12 N terms (long dimension) fitted to data shown in Figure 21.

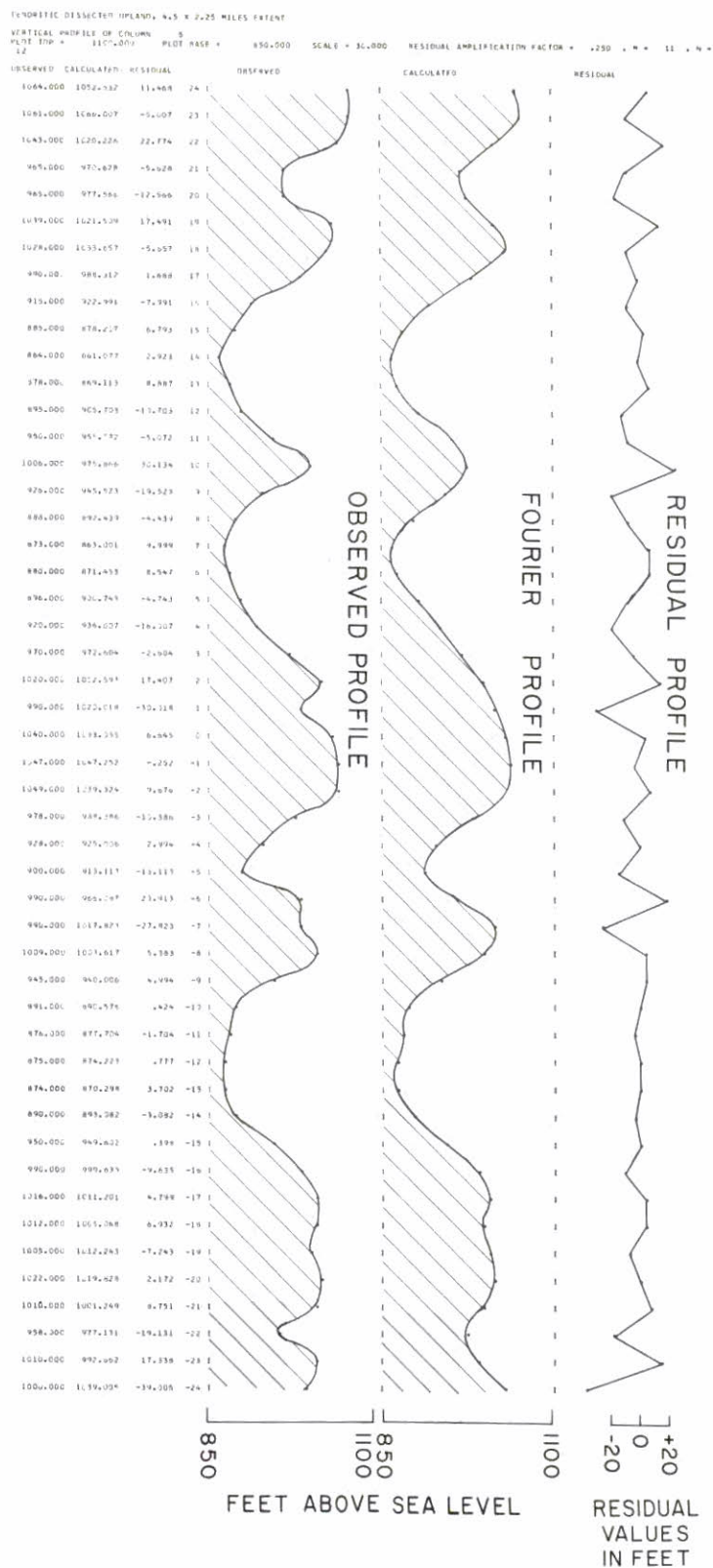


Figure 23.- Profiles of observed topographic data, calculated Fourier data, and residual values. Profile pertains to column 5 of map (see Figure 21). Double Fourier series provides very effective representation of observed surface except for small crenulations.

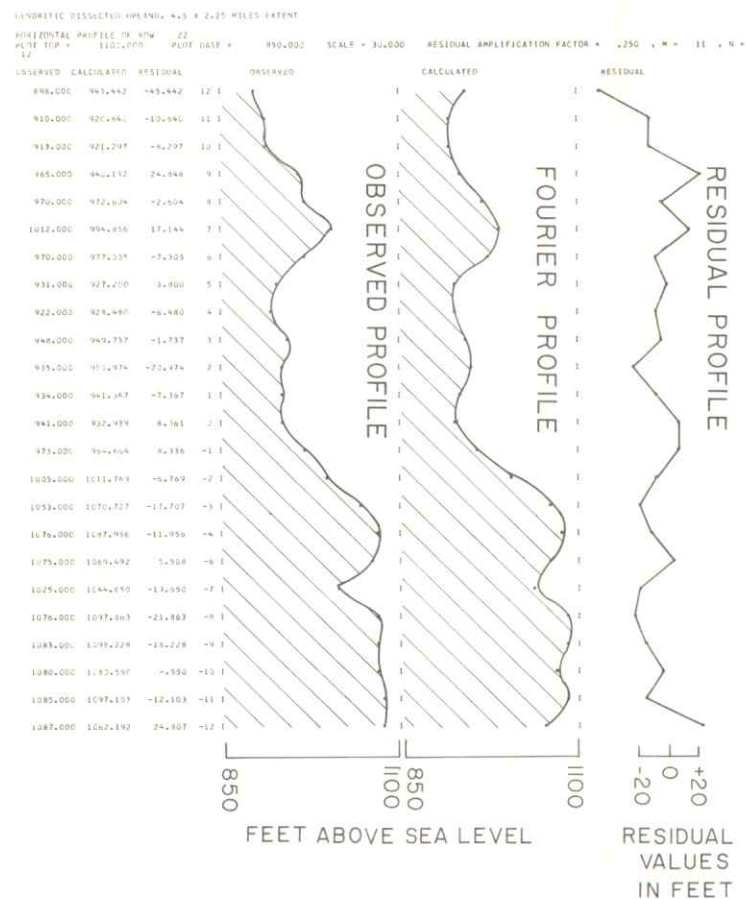


Figure 24.- Profiles pertaining to row 22 of map (see Figure 21).

SUGGESTED USES OF FOURIER ANALYSIS

Fourier series provide a convenient means for interpolation, and limited extrapolation of data that are oscillatory. The use of single Fourier series is analogous to the use of power series in curve-fitting and the use of double Fourier series is analogous to use of power series in trend-surface analysis. Power series are, of course, best applied to non-oscillatory data. Both Fourier series and power series can be fitted to satisfy the least squares criterion, permitting calculation of residuals.

A possible geologic use of single Fourier series would be analysis of rock data encountered in different bore holes. Fourier coefficients from different bore holes could be plotted on maps and contoured. This could provide a mappable index of the geographic variation in oscillatory properties of strata, ore grades, or other features encountered in bore holes.

Double Fourier series may be useful for (1) "unraveling" cross folding in layered rocks, (2) predicting laterally the intersections of faults or joints in a complex network which exhibits an underlying periodicity, (3) possibly predicting veins in ore deposits, (4) predicting oil field structures, and (5) analyzing the harmonics of topographic surfaces, including those of sand dunes, submerged oolite dunes, and dendritic drainage systems.

Finally, a possible general use of both single and double Fourier series would be to produce numerical indexes that would be useful in classification. For example, an objective numerical system of classifying the configuration of oil field structures or of landforms could be developed using double Fourier series analysis.

In conclusion, it appears that double Fourier analysis of structural data is useful. However, additional work needs to be done which includes (1) assessment of the desirability of leveling of the data prior to Fourier analysis, (2) finding means of determining how the Fourier sampling grid should be oriented with respect to the structural "grain," (3) establishing guides for grid-point spacing and for the number of columns and rows in the grid, and (4) arriving at suitable guides for determining the "cut-off" point with respect to numbers of M terms and N terms to be used in the Fourier series.

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KANSAS GEOLOGICAL SURVEY COMPUTER PROGRAM
THE UNIVERSITY OF KANSAS, LAWRENCE

PROGRAM ABSTRACT

Title (If subroutine state in title):

BALGOL Programs and Geologic Application for Single and Double Fourier Series

Using IBM 7090/7094 Computers

Computer: IBM 7090 or 7094

Date: November, 1965

Programming language: ALGOL-58 (SUBALGOL or BALGOL version)

Author, organization: Floyd W. Preston, University of Kansas, and John W. Harbaugh, Stanford
University.

Direct inquiries to: Authors or

Name: D. F. Merriam

Address: Kansas Geological Survey

Univ. of Kansas, Lawrence, Kansas

Purpose/description: Computer coefficients of Fourier series, and evaluates and plots the
function. Also computes and plots residual values.

Mathematical method: _____

Restrictions, range: _____

Storage requirements: 32,728 words high-speed memory.

Equipment specifications:

Memory 20K _____ 40K _____ 60K _____ K _____

Automatic divide: Yes _____ No _____

Indirect addressing: Yes _____ No _____

Other special features required _____

Additional remarks (include at author's discretion: fixed/float, relocatability; optional: running time, approximate
number of times run successfully, programming hours) Need Stanford University SUBALGOL compiler
system, which may be obtained for \$35.00 by sending four 1200-foot IBM-compatible
tapes to Computation Center, Stanford University, Stanford, California.

