

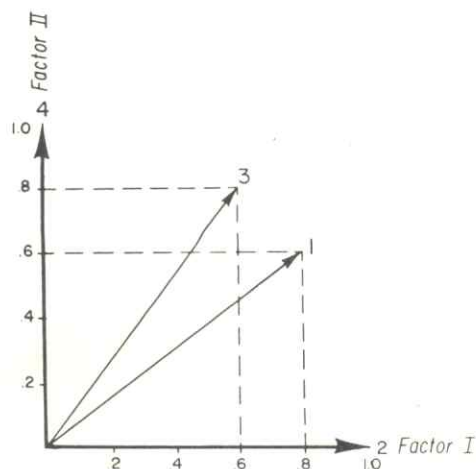
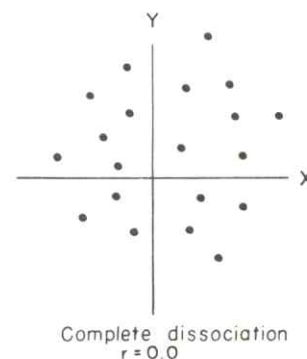
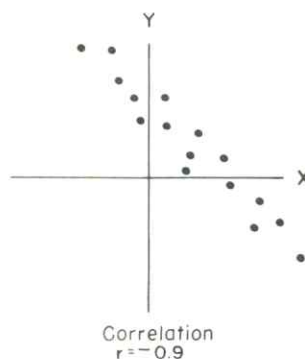
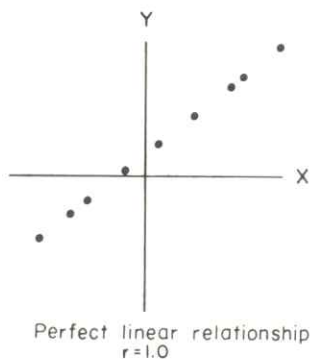
Application of Factor Analysis to Petrologic Variations of Americus Limestone (Lower Permian), Kansas and Oklahoma

By

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APPLICATION OF FACTOR ANALYSIS TO
PETROLOGIC VARIATIONS OF
AMERICUS LIMESTONE (LOWER PERMIAN), KANSAS AND OKLAHOMA

by

John W. Harbaugh and Ferruh Demirmen

ABSTRACT

The upper limestone unit or "main ledge" of the Lower Permian Americus Limestone Member was studied for 250 miles along its outcrop belt in Kansas and northern Oklahoma. The main ledge ranges from one to five feet in thickness and consists mostly of a "hash" of bryozoans, fusulinids, crinoids, and shells in a matrix of microcrystalline calcite and visibly crystalline calcite. Shells are concentrated in the lower and northern part of the main ledge, fusulinids in the upper and central-southern part, and bryozoans in the lower-middle and central part. Factor analysis was used to pick out facies based on variations in these and other constituents at 27 of the sampling localities. The results led to division of the main ledge into four depositional phases, which, from north to south along the outcrop belt are interpreted as (A) an open-marine shelf in which shelled animals thrived, (B) a "thicket" in which fenestrate bryozoans flourished, (C) a tidal flat or shoal in which intraclasts of partly-consolidated lime mud were deposited, and (D) an embayment south of the intraclast area and north of a low-lying alluvial plain.

INTRODUCTION

This report deals with the petrology of the upper limestone unit in the Americus Limestone Member of the Foraker Limestone (Lower Permian). The upper limestone is a resistant, easily identified, ledge-forming unit which we have informally termed the "main ledge." The main ledge is very persistent laterally and we have studied and sampled it at intervals for 250 miles along its outcrop belt in Kansas and Oklahoma. The main ledge ranges from less than one to about five feet in thickness. Its relatively small thickness permitted us to sample it effectively with 97 hand specimens at 29 localities (Figs. 1 and 2). The granularity of the main ledge allows it to be studied readily in thin section and was a major factor in choosing it for this study. Other Pennsylvanian and Permian limestones in Kansas and Oklahoma were considered, but none seemed as favorable for our purposes as the main ledge of the Americus Limestone.

The first purpose of this study has been to describe and interpret lithologic and chemical variations in the main ledge along its outcrop belt. We thought paleoecology would be advanced by quantitative study of a single, thin, laterally persistent, marine limestone that seemingly was deposited more or less simultaneously throughout its lateral extent. By comparing its constituents at different places, it should be possible to interpret the environment of deposition by reconstructing the distribution of sediments on the sea bottom. Terriere's (1963) quantitative study of lithologic constituents of several Pennsylvanian limestone units in central Texas is somewhat similar to our study in objectives. In our study, constituents were determined by point counting of thin sections, and by chemical and spectrographic analysis of powdered rock specimens.

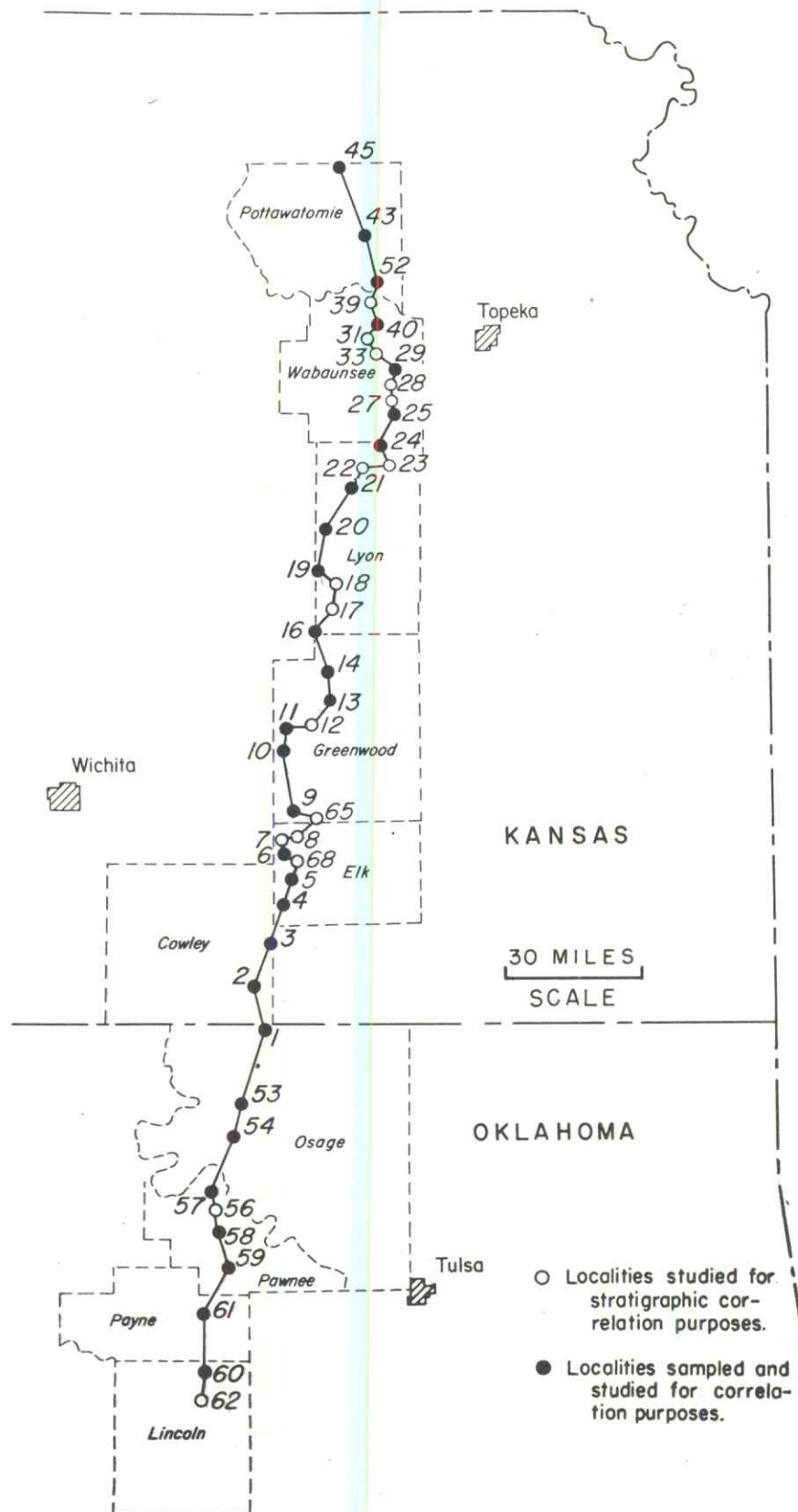


Figure 1. Map of localities where main ledge of Americus Limestone was sampled (solid circles), and where studied but not sampled (open circles).

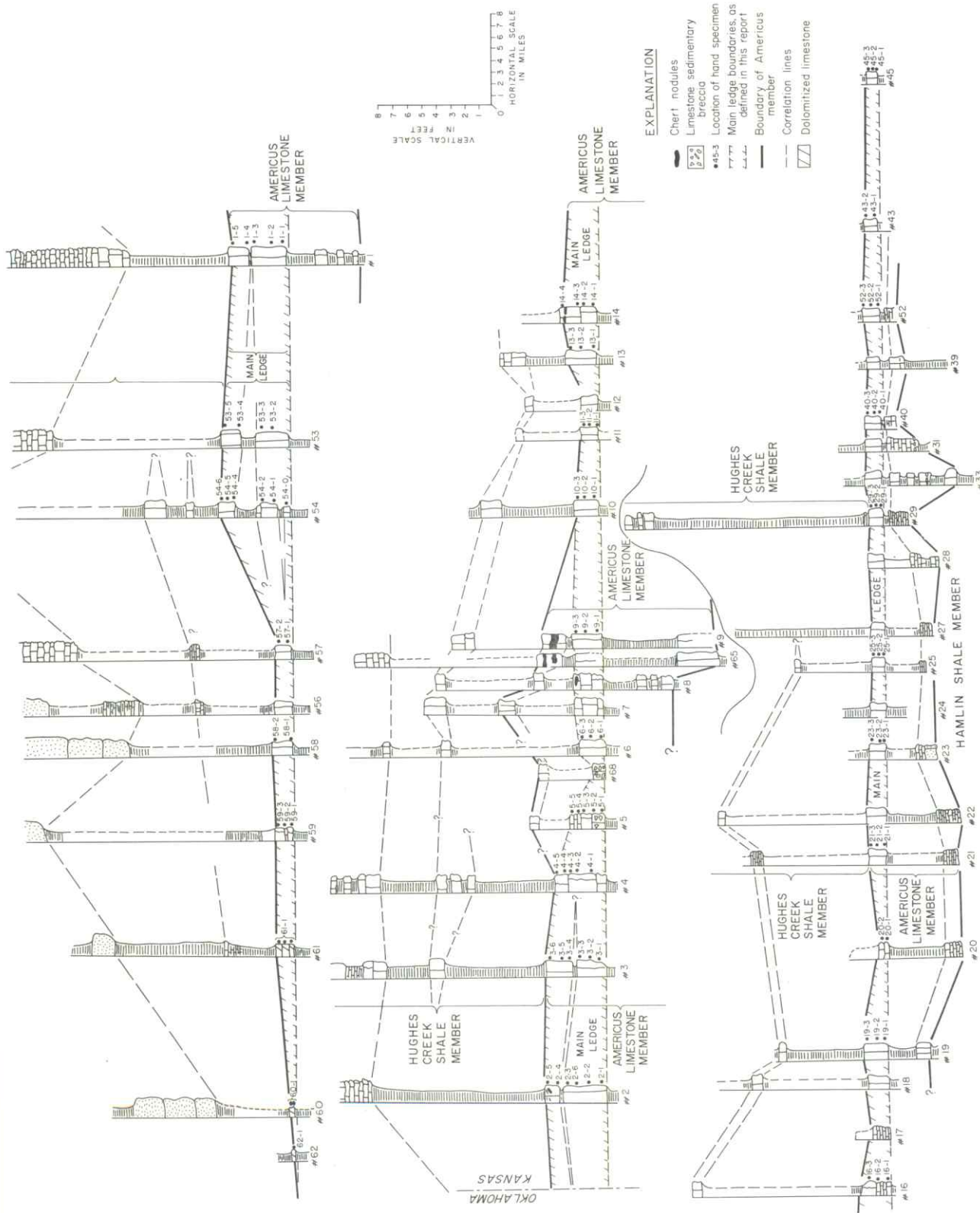


Figure 2. Stratigraphic section along outcrop belt showing main ledge and other units in Americus Limestone Member. Base of main ledge forms arbitrary horizontal datum. Some units in lower part of Hughes Creek Shale are shown for reference purposes.

The second purpose of this study was to see if factor analysis helps in making decisions and interpretations by objectively grouping and classifying geological data. Factor analysis has been widely used to interpret psychological and educational data, but relatively little used in geology. Because of the newness of factor analysis in geology, part of this report explains factor analysis and how it may be used with geological data.

Acknowledgments.--This study represents the labor of a number of persons. Daniel F. Merriam and Richard L. Schuman of the State Geological Survey assisted in the field work. Chemical analyses were made by O. Karmie Galle, Wanda N. Waugh, and Walter E. Hill, Jr., also of the State Geological Survey. Thin sections were made by Ruperto Laniz of Stanford University. Assistance in initial applications of factor analysis was given by John Imbrie of Columbia University. Several computer programs were adapted for this study by John Welsch, Steve Brophy, and Leroy Junker of the Computation Center at Stanford University. William F. Massy of Stanford University provided the computer program used for most of the factor analysis calculations. Part of the drafting for this report was done by Perfecto Mary of Stanford University, and the remainder was done under the supervision of Kenneth Badger of the State Geological Survey. The manuscript was typed by Sally Liggett and Julie Wells of the State Geological Survey. Support for Demirmen's field expenses and for part of the computer costs was provided by National Science Foundation Grant NSF-G6597. In addition the National Science Foundation's support of Stanford's Computation Center through grant NSF-GP948 helped make computing machinery available to us.

The manuscript was reviewed, partly or entirely, by Don Good, Walter E. Hill, Jr., Floyd Preston, Gary Stewart, and Ada Swineford, all of the State Geological Survey, William D. Johnson of the U.S. Geological Survey, and by John Imbrie. However, any errors and omissions are the responsibility of the authors and not the reviewers.

STRATIGRAPHY OF AMERICUS LIMESTONE

Classification

Stratigraphic classification of the Americus Limestone is summarized in Table 1. The Americus Limestone is one of many units in a sequence consisting mostly of alternating shales and limestones (Moore and others, 1951). Division of this sequence into groups, formations and members is partly a matter of convenience, and separation of the Americus Limestone from strata lying immediately above and below is arbitrary. The upper and lower boundaries of the Americus (Fig. 2) have been defined by Mudge and Yochelson (1962, Pl. 5), who also describe the paleontology and general aspects of the Americus (1962, pp. 30-33).

In much of its outcrop belt, the Americus consists of either two or three limestone units separated by shale (Fig. 2). North of locality 10, the Americus consists of two limestone units separated by shale. From locality 9 southward to locality 54, the Americus may be divided into three or more limestone units (lowermost unit not shown at all localities in Fig. 2). South of locality 57 only one limestone unit is present.

Table 1.- Stratigraphic classification of Americus Limestone.

Permian System

Lower Permian Series

Cimarronian Stage

Gearyan Stage

Chase Group

Council Grove Group

Various formations, not listed here

Foraker Limestone

Long Creek Limestone Member

Hughes Creek Shale Member

Americus Limestone Member

Admire Group

Pennsylvanian System

Since this study is directly concerned only with the main ledge of the Americus, its definition is critical. In some places the main ledge is easily defined, but in other places it is a problem to decide what is included in it. At localities 5, 6, 7, 8, 9, 29, 53, 54, and 65 (Fig. 2), it was hard to decide whether to include or exclude certain limestone beds, although lithology and continuity from locality to locality were carefully weighed in making decisions. Our correlations of units within the Americus are essentially the same as those of Mudge and Yochelson (1962, Pl. 5), and were made independently before their publication appeared.

An important question is whether the main ledge, as defined in Figure 2, is laterally continuous. No precise answer can be given, although it seems probable that some variations are due to lateral transition of limestone to shale, and vice versa. Another difficult question is whether the main ledge was deposited essentially simultaneously at different places. In detail the main ledge may differ slightly in age from place to place, but viewed broadly, it would seem to be fairly uniform in age.

General Petrology of Main Ledge

The main ledge consists mostly of poorly sorted "hash" composed of angular skeletal fragments, microcrystalline calcite, and coarse, visibly crystalline calcite (Pl. 1). In places (localities 1-6) it contains intraclasts (Pl. 2). Some of the visibly crystalline calcite formed by recrystallization of fine-grained carbonate (Harbaugh, 1961) and the rest formed by precipitation in water-filled voids. The two types of visibly crystalline calcite were lumped together for point-counting purposes.

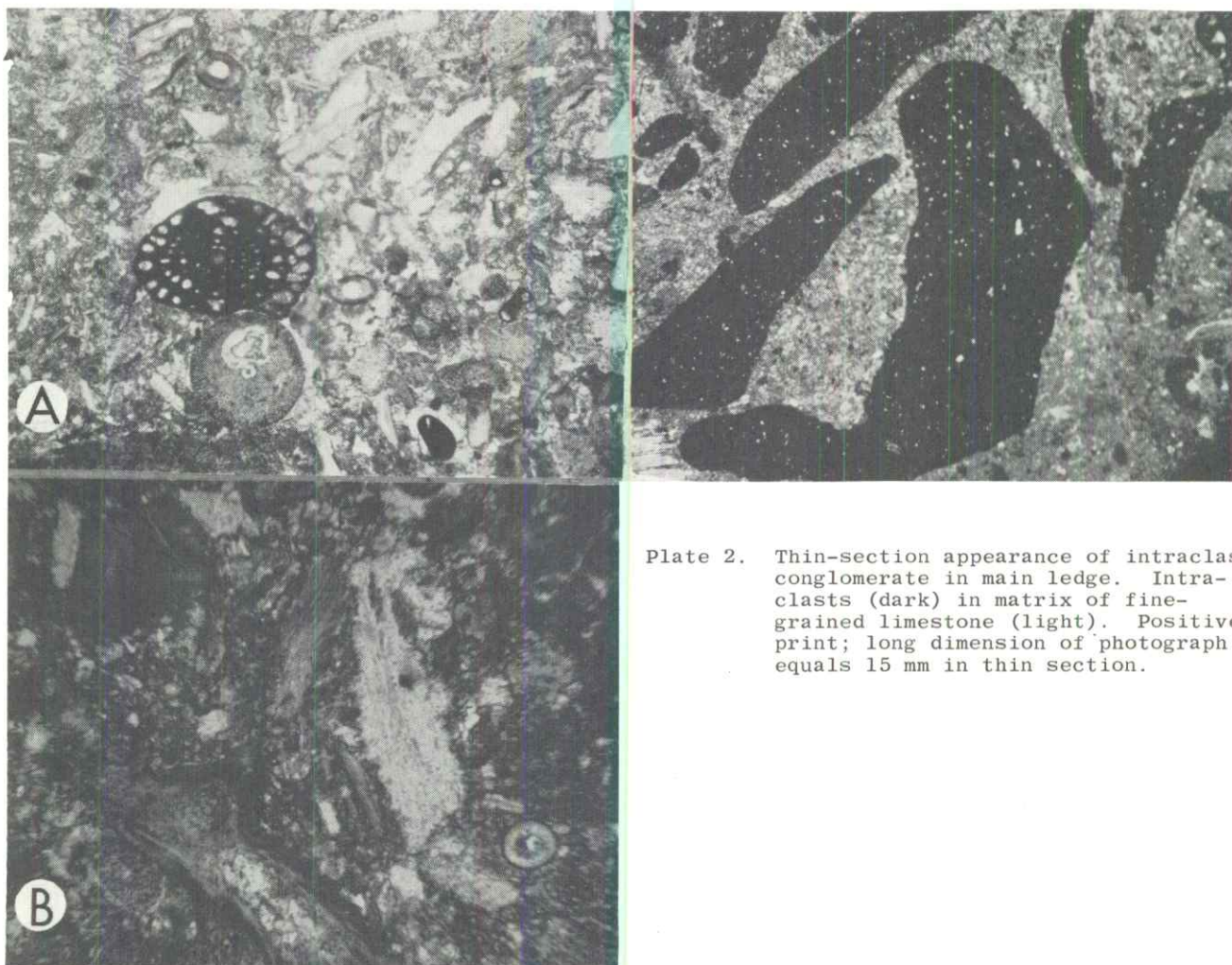


Plate 2. Thin-section appearance of intraclast conglomerate in main ledge. Intraclasts (dark) in matrix of fine-grained limestone (light). Positive print; long dimension of photograph equals 15 mm in thin section.

Plate 1. Typical thin-section appearance of Americus main ledge. Positive prints; long dimension of photograph equals 7 mm in thin section. A, Skeletal hash containing fusulinids (dark), crinoid stem plate, and fragments of fenestrate bryozoans and other unidentified debris. B, Skeletal "hash" consisting of angular, unsorted debris.

METHODS

Field Methods

Two conditions governed the location of sampling localities (Fig. 1). First, suitable exposures occur only where roads and a few streams cross the outcrop belt, severely limiting the places where samples could be collected. Second, an attempt was made to space the 29 sampling localities more or less equally along the outcrop belt. At each sampling locality, large "hand" specimens were broken off with a heavy machinist's hammer. The number of specimens collected per locality ranges from two to six, depending on the thickness of the main ledge. An attempt was made to space the hand samples at each locality more or less equally vertically. For stratigraphic correlation purposes, 15 additional localities were visited but not sampled. Section, township and range descriptions of Americus localities visited are listed in Appendix A.

Laboratory Methods

Thin-section point counting.--A 1 by 1½ inch thin section was made of each hand specimen chosen for study, the long dimension being oriented vertically. Estimates of volumetric proportions were made by point counting 94 thin sections of specimens collected at 27 of the 29 sampling localities. Point-count constituents were grouped into seven major categories: bryozoans, crinoids, fusulinids, shells, intraclasts, visibly crystalline calcite, and microcrystalline calcite. No attempt was made to subdivide these categories. For example, pelecypods, gastropods and brachiopods were all lumped together as shells. In point counting, each thin section was moved in small uniform increments on the stage of a petrographic microscope. The type of particle at the point directly beneath the cross-hairs of the microscope was identified and tallied after each movement. According to theory, the estimated volume of a particle type is proportional to the number of particles of that type encountered (Chayes, 1956, pp. 4-13). In this study, 400 points were counted per thin section and the results recalculated in percent. For example, if 56 out of 400 points encountered crinoid particles, the estimated volume of crinoid material would be 14 percent. Point-count data are tabulated in Appendix B.

Chemical analysis.--Samples for chemical analysis were prepared by blending all specimens collected at an individual sampling locality to form a single composite sample representing that locality. Approximately equal-sized hand specimens were crushed to a coarse powder with a laboratory rock crusher and then ground to a coarse powder. The powder was mixed thoroughly by stirring, quartered down by repeated passes through a sample splitter, and finely ground with a motor-driven quartz mortar and pestle so as to pass through a #100 screen.

Silica, magnesium oxide, and the R_2O_3 group were determined by wet chemical methods in the State Geological Survey laboratories using methods described by Hill, Waugh, Galle, and Runnels (1961). Strontium, titanium, manganese and vanadium were determined spectrographically, employing methods described by Runnels and Schleicher (1956). The results of wet chemical analysis of samples collected at 20 localities and spectrographic analysis of samples from 15 localities also are presented in Appendix B.

Computations

The data were statistically studied by trend surface analysis and factor analysis. Principles of these methods are outlined later in this report. All computations were made with electronic computers. The initial computing work was done on an IBM 650 computer at the Watson Computing Laboratory of Columbia University, but all later computations were made with a Burroughs 220 computer and an IBM 7090 computer at Stanford University. Computer programs used for fitting trend surfaces and for calculating correlation coefficients and distance coefficients have been described by Harbaugh (1963, 1964).

TYPES OF VARIATION IN DATA OF THIS STUDY

Variation Between Constituents and Between Samples

Two principal types of variation are present in the data of this study: (I) Variation between constituents, and (II) variation between samples. Variation between constituents may be subdivided (Table 2) into (1) variation between point-count constituents on a volume percentage basis and (2) variation in chemical constituents on a weight percentage basis. Variation between samples may be subdivided into (3) variation between localities, and (4) variation between hand specimens. Although these types of variation are interrelated, we may consider them separately. For example, point-count constituents, measured in terms of volume, are free to vary more or less independently of chemical constituents in terms of weight.

We must keep in mind the restraint placed on variation between constituents which yield a constant sum when added together. When the proportion of one constituent increases in a given sample, the other constituents, lumped together, must necessarily decline. On the other hand, variations between samples are not directly subject to this restraint because the proportion of a constituent in one sample does not, in itself, influence the proportions of constituents in other samples. In this study, we will be concerned only with variations between samples.

Variation Introduced in Point Counting

Some of the variation of point-count constituents is inherent in the process of point counting and is termed "operator error." There are two sources of variation: (1) random fluctuations due to different "routes" traversed in a thin section, and (2) errors and indecision by the person doing the point counting. The two sources are not readily separable.

To determine if significant variation was introduced in point counting, a simple experiment was performed. Demirmen, who did all the point counting, was given on three occasions,

Table 2.- Classification of principal types of variation present in data of this study. Items marked with an asterisk (miscellaneous and loss on ignition) contribute to variability and are listed for completeness, but are not considered otherwise.

I VARIATION BETWEEN CONSTITUENTS		II VARIATION BETWEEN SAMPLES	
1	2	3	4
Point-count constituents in volume percent:	Chemical constituents in weight percent	Variation between localities	Variation between hand specimens
a. Bryozoans	a. Silica	a. Average of point-count constituents at each of 27 localities	a. Point-count constituents in each of 94 hand specimens
b. Crinoids	b. R ₂ O ₃ group	b. Chemical constituents at 15 to 20 localities	
c. Fusulinids	c. Magnesia		
d. Shells	d. Manganese		
e. Intraclasts	e. Titanium		
f. Visibly crystalline calcite	f. Strontium		
g. Microcrystalline calcite	g. Vanadium		
h. Miscellaneous*	h. Calcium oxide		
	i. Loss on ignition*		

three thin sections of hand specimens from different localities. He counted 400 points in each thin section on each occasion, but the identities of the thin sections were unknown to him. The results are shown in Table 3. As might be expected, the number of points counted per constituent tended to differ on the three occasions. The question arises, are these differences significant?

Analysis of variance was employed (Moroney, 1956, pp. 371-376) to determine if variation due to operator error is significantly large compared to variation between thin sections. The results (Table 4) show that we may be 92 to 99.9 percent confident that variation is significantly greater between the three thin sections than variation due to operator error. While the results of Table 4 apply to only three thin sections, they suggest that operator error has probably not seriously influenced conclusions based on point-count data from the rest of thin sections.

Table 3.- Comparison of points counted during three trials to measure operator error.

Constituent	Thin Sec. A			Thin Sec. B.			Thin Sec. C.		
	Trial	Trial	Trial	Trial	Trial	Trial	Trial	Trial	Trial
	1	2	3	1	2	3	1	2	3
Bryozoans	40	42	46	35	30	27	60	43	55
Crinoids	54	58	50	20	22	22	50	47	43
Fusulinids	14	14	14	8	4	10	1	0	1
Shells	60	64	55	55	80	76	48	55	44
Visibly Crystalline Calcite	80	94	64	98	107	106	64	56	60
Microcrystalline Calcite	126	110	145	156	133	140	155	184	182
Miscellaneous	26	18	26	28	24	19	19	13	15

Differences of Point-count Constituents Within Localities and Between Localities

Analysis of variance also was used to determine if variation in point-count constituents in hand specimens within localities is as great as variation between localities. Six point-count constituents (bryozoans, crinoids, fusulinids, shells, visibly crystalline calcite, and microcrystalline calcite) were compared. The results (Table 5) show that for five constituents, we may be 92 to 99.5 percent confident that differences between localities are significantly greater than differences within localities. However, for microcrystalline calcite, the confidence level is very low and significant differences appear not to exist.

Variation of Constituents Between Localities

Variation in point-count constituents and chemical constituents between localities are shown graphically in Figure 3. The volume percentages for point-count constituents at each locality are averages of values of individual hand specimens at that locality, whereas weight

percentages of chemical constituents are based on an analysis of a single, composite sample at each locality. Chemical analyses are available at 20 localities, spectrographic analyses at 15 localities, and point-count data at 27 localities. Variations of each constituent are discussed below.

Bryozoans.--Bryozoans vary irregularly along the outcrop belt (Fig. 3A), forming 5 to 15 percent of the volume of the main ledge at the northern and southern ends of the outcrop belt,

Table 4.- Analysis of variance to test whether variation due to operator error is greater or less than variation between thin sections. (T. S. stands for thin sections.)

Source		Sum of Squares	Degrees of Freedom	Variance	F	Confidence Level
Bryozoans	Between T.S.	728	2	364	10.7	99%
	Within T.S.	204	6	34		
Crinoids	Between T.S.	1814	2	907	56.3	99.9+%
	Within T.S.	97	6	16		
Fusulinids	Between T.S.	267	2	133	41.4	99.9+%
	Within T.S.	19	6	3		
Shells	Between T.S.	683	2	341	4.4	92%
	Within T.S.	463	6	77		
Visibly Crystalline Calcite	Between T.S.	2873	2	1436	16.2	99.5%
	Within T.S.	531	6	89		
Microcrystalline Calcite	Between T.S.	3374	2	1687	7.1	97.5%
	Within T.S.	1417	6	236		

Table 5.- Analysis of variance to test whether significant differences in point-count constituents exist between localities as compared to within localities.

Source		Sum of Squares	Degrees of Freedom	Variance	F	Confidence Level
Bryozoans	Between loc.	141,160	26	5429	2.6	99%
	Within loc.	140,223	67	2093		
Crinoids	Between loc.	28,992	26	1115	3.1	99.5+%
	Within loc.	23,841	67	356		
Fusulinids	Between loc.	41,041	26	1578	1.8	92%
	Within loc.	58,210	67	867		
Shells	Between loc.	35,001	26	598	2.3	98%
	Within loc.	40,039	67	225		
Visibly Crystalline Calcite	Between loc.	175,087	26	6734	2.3	98%
	Within loc.	194,232	67	2899		
Micro-Crystalline Calcite	Between loc.	143,400	26	5515	1.05	40%
	Within loc.	352,175	67	5256		

and 30 to 50 percent in the central part. At locality 5, bryozoans are absent.

Crinoids.--Crinoids (Fig. 3B) do not vary as much as bryozoans, forming 5 to 15 percent, except at locality 5 where crinoids are absent, and locality 57, where crinoids rise to 23 percent.

Fusulinids.--Fusulinids (Fig. 3C) range from zero to about 19 percent, and are roughly 2 to 3 times more abundant in the southern half of the outcrop belt as the northern half. Variations in fusulinids do not generally parallel variations in bryozoans and crinoids.

Shells.--Shells (Fig. 3D) are concentrated in the northern part of the outcrop belt, but are present at all localities. Proportions of shells and fusulinids tend to vary inversely.

Intraclasts.--Intraclasts are confined to localities 1 to 6 (Fig. 3E) and consist of irregular lumps and fragments of fine-grained limestone (Plate 2). Where intraclasts are abundant (locality 5), skeletal material is low or absent.

Visibly crystalline calcite and microcrystalline calcite.--Visibly crystalline calcite in the main ledge consists of both grain-growth calcite and void-filling calcite (Harbaugh, 1961). Void-filling calcite is most common, but in places, particularly at localities 58 and 59, grain-growth calcite is abundant. The proportion of visibly crystalline calcite (Fig. 3F) generally increases toward the south and is accompanied by a decrease in microcrystalline calcite (Fig. 3G). There is a rough inverse relationship between visibly crystalline calcite and microcrystalline calcite. An increase in sorting toward the south may have been accompanied by greater winnowing of fine carbonate mud, leaving a higher proportion of voids for subsequent filling with void-filling calcite. There also appears to be a rough inverse relationship between proportions of microcrystalline calcite and bryozoans.

Calcium oxide.--Calcium oxide (CaO) varies only slightly from locality to locality (Fig. 3H), except at the extreme southern end of the outcrop belt (localities 60 and 61), where it declines due to dolomitization.

Silica.--Silica (SiO_2), ranges from 5 to 14 percent (Fig. 3I). With two exceptions, high silica values occur in the southern part of the outcrop belt. Part of the silica probably occurs as fine chalcedony. Chert nodules occur in a few places (Fig. 2), although no specimens containing chert nodules were analyzed. The R_2O_3 group also increases toward the south, suggesting that much of the rest of the silica occurs in clay and silt.

R_2O_3 group.--The R_2O_3 group, consisting of combined alumina (Al_2O_3), ferric oxide (Fe_2O_3), and titanium oxide (TiO_2), ranges from 1 to 6 percent, is lowest in the central part of the outcrop belt, increases slightly toward the north, and increases greatly toward the south (Fig. 3J). Separate determination of titanium by spectrographic analysis, reveals that titanium oxide forms 6 to 21 percent of the R_2O_3 group, and that the remainder is alumina and ferric oxide. Increase in alumina and ferric oxide toward the south probably reflects increasing proportions of clay and other terrestrially derived clastic materials.

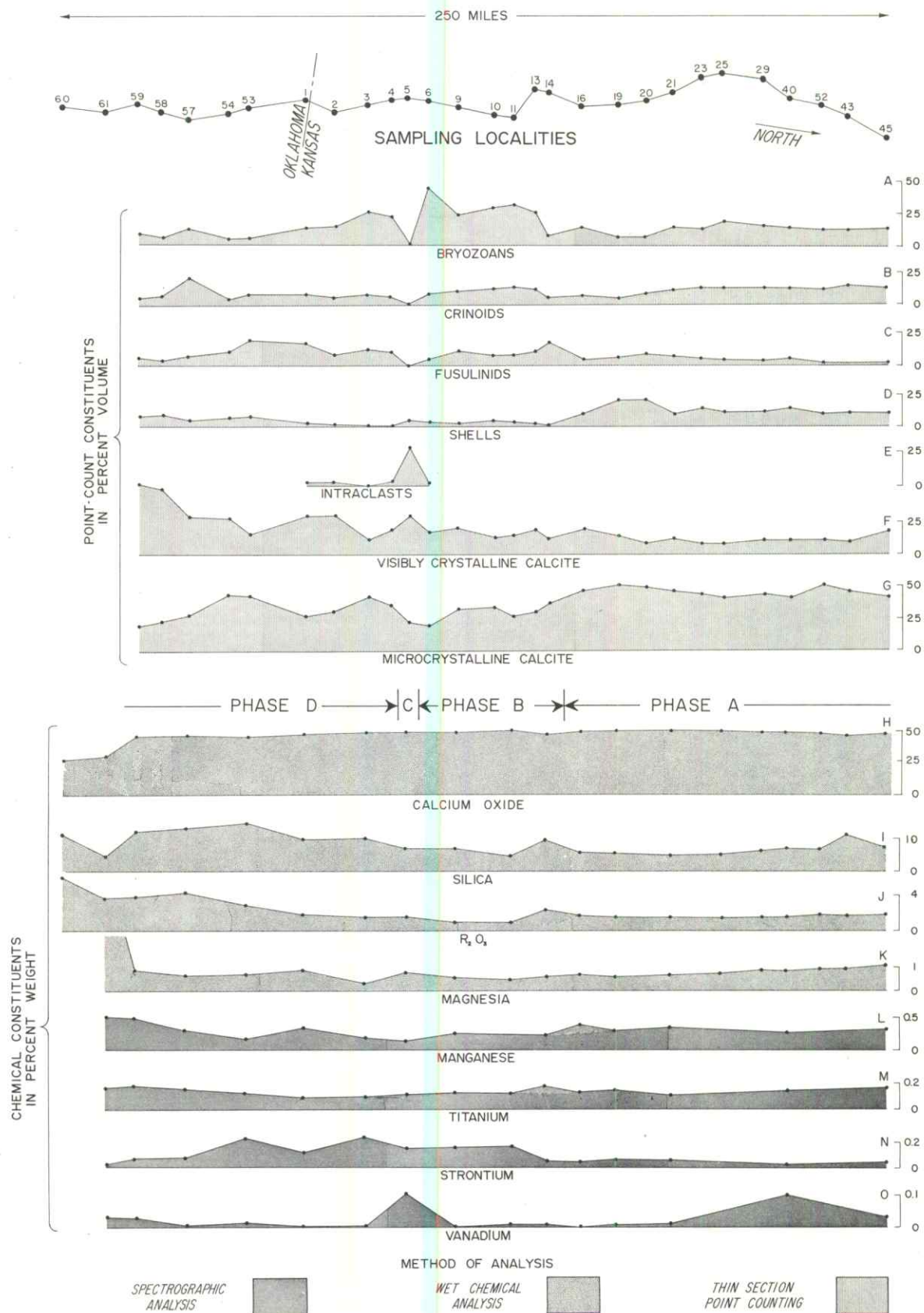


Figure 3. Variations of seven point-count constituents (A-G) and eight chemical constituents (H-O) in Americus main ledge from locality to locality.

Magnesia.--Magnesia (MgO) ranges from 0.28 to 1.16 percent (Fig. 3K), except at the southern end of the outcrop belt, where the main ledge is dolomitized and magnesia is as high as 17 percent. The association of dolomite with relatively high proportions of terrestrially derived clastics, suggests that the southern end of the outcrop belt may be close to the site of an ancient shoreline. The relationship of dolomitization to ancient shorelines has been emphasized by Fairbridge (1957, p. 154).

Manganese.--Manganese ranges from 0.16 to more than 0.5 percent (Fig. 3L). Highest values occur near the southern end of the outcrop belt. Variations from locality to locality are not great and may be of little significance.

Titanium.--Titanium ranges from about 0.10 to 0.18 percent. Variations from locality to locality are slight (Fig. 3M) and appear to be of little significance.

Strontium.--Strontium ranges from 0.03 to 0.24 percent, highest values occurring in the southern part of the outcrop belt (Fig. 3N). Variations in strontium correlate somewhat with variations in bryozoans and fusulinids, perhaps marking a cause and effect relationship between bryozoans and fusulinids, and strontium. High strontium values may indicate the former presence of aragonite that was converted subsequently to calcite, because aragonite is capable of taking up much more strontium within its crystal lattice than is calcite. In addition there is a rough inverse relationship between strontium and magnesia in the main ledge. Where strontium is high, magnesium is low, and vice versa. This also may reflect differences in the proportions of calcite and aragonite that were originally deposited, for calcite can take up more magnesium than aragonite.

Vanadium.--Vanadium ranges from about 0.003 to 0.10 percent (Fig. 3O). The two high values may represent random fluctuations of slight geological significance.

INTRODUCTION TO FACTOR ANALYSIS

We have used factor analysis to help interpret variations in the Americus Limestone. Factor analysis was initially devised by psychologists in the early 1900's. Since then there have been many advances in technique, and applications have spread beyond psychology into medicine, biology, economics, business, and lately, to geography and geology. The objective of factor analysis is to simplify by determining if relationships among a set of variables may be reduced to a smaller number of underlying fundamental variables, or "factors." Inasmuch as most geologists are not familiar with factor analysis and because some understanding of its principles is needed here, a brief description follows. For a more thorough treatment see Fruchter (1954) and Imbrie and Purdy (1962).

Correlation Coefficients

Raw data cannot be analyzed directly by factor analysis. First it is necessary to compute coefficients that express the degree of similarity between different pairs of variables.

One measure is the product-moment correlation coefficient, which may be visualized as the degree to which observations of two variables approach a straight line when plotted as points on an X-Y diagram (Fig. 4). The correlation coefficient may vary from -1 to +1. Either -1 or +1 indicates that the points form a straight line (Fig. 4A), whereas zero indicates complete dissociation or no correlation (Fig. 4C). Values between zero and +1 or -1 indicate the degree of correlation between two variables (Fig. 4B). For further discussion, see an elementary statistics book such as Moroney (1956, pp. 271-289).

Suppose we have observations on different variables and wish to correlate each variable with each other variable. This requires a correlation coefficient for each pair of variables. If there are 3 variables, 3 coefficients are needed. The number of coefficients goes up sharply as the number of variables increases. For 10 variables, 45 coefficients are needed. The number of coefficients is equal to $(n^2 - n)/2$, where n is the number of variables. A table of correlation coefficients forms a matrix (Table 6) consisting of coefficients in rows and columns in which the number of rows (or columns) is equal to the number of variables, and the matrix is symmetrical about the diagonal containing ones.

Distance Coefficients

Distance coefficients form an alternative way of expressing the degree of similarity between variables. Distance coefficients (Harbaugh, 1964) are based on the distance between two variables in a rectangular coordinate system in n -dimensional space. The closer the two points representing two variables are together, the greater the similarity, and vice versa. Generally, distance coefficients should be calculated from data in which the values are either all positive or all negative, and in which the absolute values do not exceed 1.0. Transformations of raw data are generally necessary to satisfy these conditions. A simple linear transformation is to divide each variable by the highest observed absolute value for that variable. Distance coefficients may be defined by the formula

$$d = 1.0 - \frac{\sqrt{\sum_{i=1}^n (X_{i1} - X_{i2})^2}}{n}$$

where d = distance coefficient,

X_{i1} = observations on variable 1 with absolute values not exceeding 1.0,

X_{i2} = observations on variable 2 with absolute values not exceeding 1.0,

n = number of observations, and

i = subscript values which begin at 1 and increase in increments of 1 until n is reached.

Maximum distance yields a coefficient of zero, minimum distance yields a coefficient of 1, and negative values are avoided. Distance coefficients, as defined above, may have important advantages over correlation coefficients in applications where different samples are compared with each other.

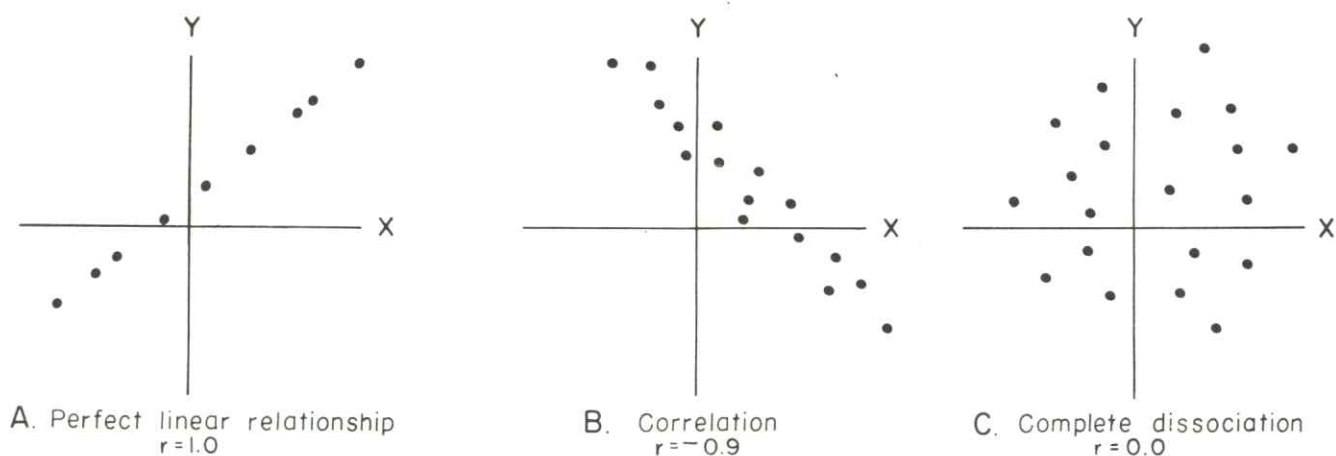


Figure 4. Degree of correlation represented by fit of line to points.

Vector Representation of Coefficients

As the number of variables becomes large, the number of coefficients between pairs of variables becomes unmanageably great. It is here that factor analysis is useful, because it may be used to simplify the relationships in a matrix of coefficients. Although factor analysis is generally accomplished by algebraic and trigonometric manipulations with electronic computers, it is convenient to think of factor analysis in graphic terms. For example, it is possible to represent graphically the correlation coefficient or distance coefficient between two variables by using two vectors separated by an angle whose cosine is equal to the coefficient between two variables (Fig. 5). If three variables are involved, we may think of three vectors emanating from a common origin, the cosine of each of the angles being equal to each coefficient represented. If 3 variables are involved, not more than three-dimensional space is needed to represent the interrelationships (Fig. 6). However, if four or more variables are involved, space of more than three dimensions (hyperspace) may be needed if the angular relationships between all pairs of vectors are correctly represented. Although we cannot visualize four or more dimensions, the algebraic and trigonometric relationships between vectors are still valid.

Table 6.- Example of 3 x 3 correlation matrix. Nine coefficients are shown but only three are needed. Each 1.0 represents correlation of variable with itself. After Fruchter (1954, p. 32).

	Variable		
	1	2	3
Variable 1	1.00	.00	.707
2	.00	1.00	-.50
3	.707	-.50	1.00

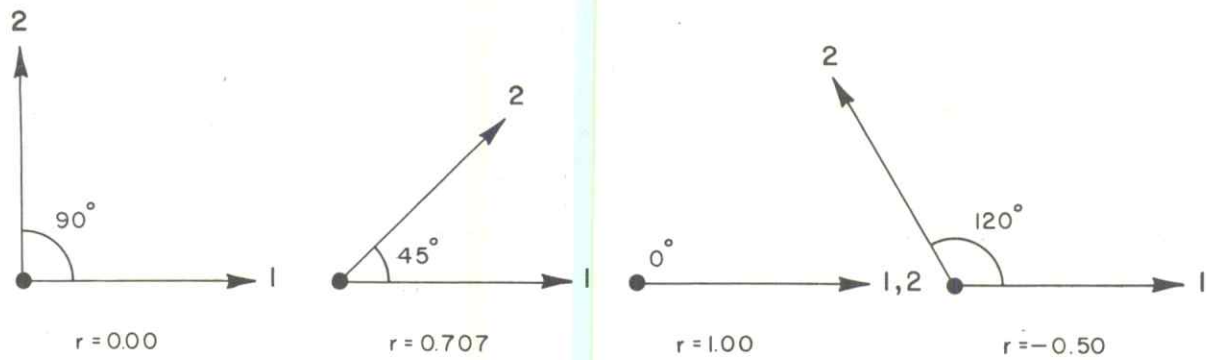


Figure 5. Representation of coefficients by vectors. Cosine of angle equals coefficient.

Table 7.- Coefficient matrix of four hypothetical variables (after Fruchter, 1954, p. 33). Matrix is symmetrical about principal diagonal containing ones.

		Variable			
		1	2	3	4
Variable	1	1.00	.80	.96	.60
	2	.80	1.00	.60	.00
	3	.96	.60	1.00	.80
	4	.60	.00	.80	1.00

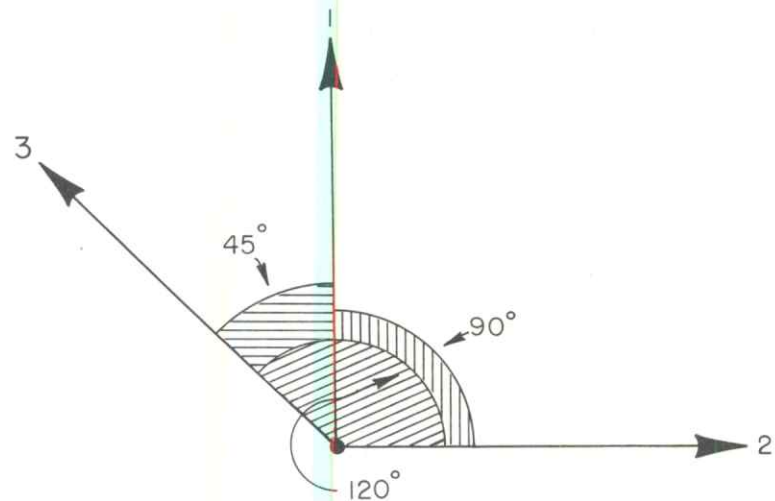


Figure 6. Vectors representing correlation coefficients between three variables (Table 6) in three-dimensional space.

Factor Axes

The objective of factor analysis is to represent the relationships between a number of variables in terms of a smaller number of underlying variables or "factors." To illustrate, consider vectors representing (Fig. 7) coefficients between four variables (Table 7). In this example, the coefficients have been chosen so that the vectors are correctly represented in two-dimensional space. It is possible to simplify the relationships among the vectors by projecting lines from the four vectors onto two arbitrarily oriented new axes (Fig. 8). These axes are termed factor axes and may be labeled Factor I and Factor II. Positive and negative directions are defined on each factor axis. If the factor axes are of unit length measured from the origin, then the value (termed "loading") of any given projection from a vector onto the axis must be between -1 and +1. Values of the projections or loadings are listed in two columns of Table 8. In this example the relationships between four variables are completely represented by the loadings on only two factors.

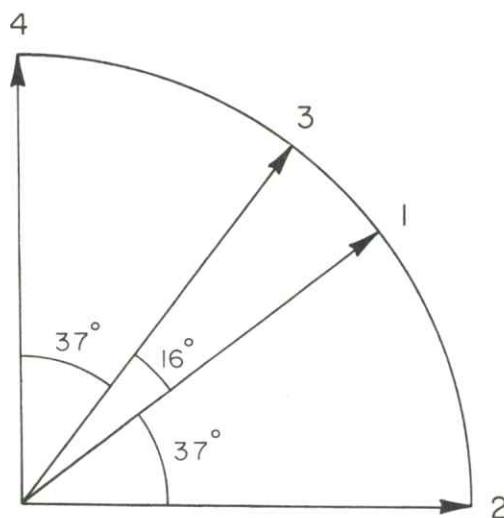


Figure 7. Vectors representing coefficients between four variables of Table 7. Coefficients have been chosen so that vectors are accommodated in two-dimensional space (after Fruchter, 1954, p. 33).

Table 8.- Hypothetical factor matrix based on coefficients of Table 7. (after Fruchter, 1954, p. 34).

		Factor		Communality, h^2
		I	II	
Variable	1	.8	.6	1.0
	2	1.0	.0	1.0
	3	.6	.8	1.0
	4	.0	1.0	1.0

In actual practice, the factor loadings generally do not completely represent the relationships in a coefficient matrix. The question then arises as to the effectiveness of the loadings in representing the relationships. The degree of representation is measured by the communality, which is the sum of the squared factor loadings for each variable. In the hypothetical example of Table 8, the communalities are 1.0, signifying complete representation. In nature, however, complete representation is virtually never attained, and if one wishes to represent relationships between a given number of variables with a smaller number of factors, the communalities will inevitably be less than one. For example, in this study, four factors represent 85 to 88 percent (mean communalities of 0.85 to 0.88 of an assumed maximum communality of 1.0) of the relationships in a 27 x 27 matrix representing 27 Americus sampling localities (Tables 10 A, B, C). By not attempting to represent all the relationships (*i.e.*, sacrificing about 12 to 15 percent), four factors, in essence, have been substituted for a larger number of variables. For 100 percent representation, it would have been necessary to extract 27 factors, which would be pointless because there would be no simplification.

The number of factors that should be extracted depends upon one's objectives. If the number of factors extracted is small (generally desirable) the degree of representation of the relationships in the correlation matrix also may be small (undesirable). Thus, one may have to sacrifice some representation to gain simplicity, and vice versa.

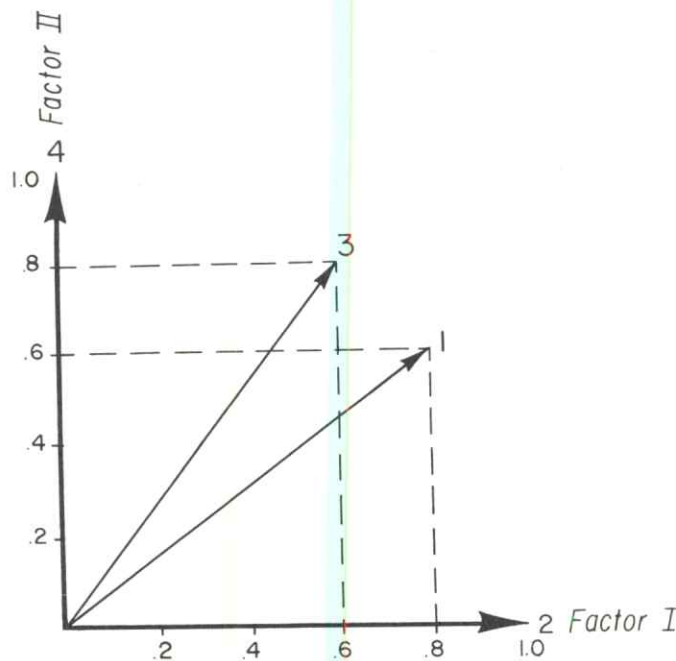


Figure 8. Vectors representing coefficients between four variables of Table 7, with factor axes I and II added. Factor I coincides with vector 2 and factor II coincides with vector 4.

Methods of Factor Analysis

Two common methods of factor analysis are the principal axis method and the centroid method. Differences between the methods are described by Fruchter (1954). Both methods have been used in this study. In the principal axis method, the factor axes are positioned so that the first factor accounts for as much of the variance as possible (*i.e.*, the sum of squares of factor loadings is maximized on this factor). Then, in turn, each successive factor axis is positioned so that it "extracts" or accounts for as much of the remaining variance as possible. Thus, after a sufficient number of factors have been extracted, much of which remains is "noise." In the methods employed here, the factor axes have been maintained at right angles (orthogonal) with respect to each other at all times.

The centroid method involves less computational labor, but with development of large electronic computers, the centroid method has become somewhat obsolete. The objectives of the centroid method are similar to those of the principal axis method, but the centroid method is less rigorous mathematically.

Rotation of Factor Axes

After factor axes have been established, it is commonly advantageous to rotate them, two axes at a time, around the origin so that the value of each loading on the two axes tends to approach either -1 or +1, or approaches zero. Ideally, the absolute values of the loadings are either high or low, permitting the most clean-cut decisions to be made. In practice, of course, intermediate values are commonly obtained, making decisions more difficult. The most common methods of rotation are the varimax and quartimax methods. Differences between the methods are explained by Fruchter (1954). Although there is merit in using both rotated and unrotated factor axes, for geological purposes, rotated factor axes generally seem to be most useful.

R-Factor and Q-Factor Analysis

Factor analysis may be used to study relationships either between tests (R-factor analysis) or between samples (Q-factor analysis). The difference is one of objective, for the same techniques and the same data may be used in both R and Q applications. Q-factor analysis was used in this study to interpret relationships between samples.

Q-FACTOR ANALYSIS OF VARIATIONS BETWEEN LOCALITIES

Calculation of Coefficients Between Pairs of Localities

Q-factor analysis was used in this study to pick out natural groupings of Americus localities. The groupings are based on variations of point-count constituents and chemical constituents on a locality-by-locality basis. Both correlation coefficients and distance coefficients were computed between all possible pairs of 27 localities, the number of coefficients of each type being $(27^2 - 27)/2 = 351$. Before computing the coefficients, however, the data were transformed by dividing each value by the highest value observed in the study for that particular constituent. This gives each constituent more or less equal weight when comparing locali-

ties. Otherwise, for example, it would be illogical to use volume percentage of bryozoans, weight percentage of silica, and proportions of the other constituents in calculating coefficients expressing the degree of similarity between pairs of localities.

To illustrate the principle in comparing localities, a list of original and transformed values of 11 constituents at localities 1 and 3 is presented in Table 9. After transforming the values, a correlation coefficient and a distance coefficient were calculated, both expressing the degree of similarity between the two localities based on the 11 constituents. The correlation coefficient can be visualized as the degree to which the points approach a straight line on an X-Y diagram (Fig. 9). The distance coefficient, on the other hand, is the complement of the mean distance within an 11-dimensional "cube," and cannot be visualized geometrically.

In this study running averages for silica, R_2O_3 group, magnesia, and strontium were used at certain localities where point-count data are available, but chemical data are lacking. This practice is not generally recommended, but it was necessary if all point-count data were to be used in comparing certain localities. Chemical constituents used in calculations (silica, R_2O_3 group, magnesia, and strontium) generally vary somewhat less from locality to locality than point-count constituents (Fig. 3). Therefore, inclusion of running averages for data at some localities probably has not seriously affected interpretation. Each running

Table 9.- Example of percentage data transformed to permit calculation of coefficients between localities. Percentages for point-count constituents are averages of specimens at each locality. Point-pairs of transformed values are plotted in Figure 9, where they are identified by numbers in left column of this table.

Number of Con- stitu- ent	Constituent observed	Highest percentage observed at any locality	Locality 1		Locality 3	
			Observed percentage	Trans- formed value	Observed percentage	Trans- formed value
1	Bryozoans	44.4	14.0	.32	25.1	.57
2	Crinoids	23.8	6.4	.27	7.7	.32
3	Fusulinids	19.0	16.5	.87	12.2	.64
4	Shells	19.3	2.6	.14	1.0	.05
5	Intraclasts	29.7	1.8	.06	0.0	.00
6	Visibly crystalline calcite	52.4	30.5	.58	10.7	.20
7	Microcrystalline calcite	50.8	25.6	.50	41.9	.82
8	Silica	13.83	9.45	.68	9.1	.68
9	R_2O_3 group	4.23	1.80	.42	1.56	.37
10	Magnesia	1.16	0.88	.76	0.28	.24
11	Strontium	0.24	0.12	.50	0.24	1.00

average is the mean of two values at the localities immediately adjacent to a locality for which no chemical analysis was available. Eighteen wet chemical analyses and 14 spectrographic analyses were used in calculating coefficients. Calcium oxide, manganese, titanium, and vanadium were omitted in calculating coefficients.

Grouping of Localities on Basis of Factor Loadings

The loadings (Tables 10A, 10B, and 10C) on each of four (rotated) factors for 27 localities are graphed on Figures 10 and 11. Loadings on centroid factors extracted from a correlation coefficient matrix are shown in Figure 10, and loadings on principal axis factors are shown in Figure 11. In the upper part of Figure 11, the factors have been extracted from a correlation coefficient matrix, and in the lower part, the factors have been extracted from a distance coefficient matrix. The graphs permit comparison of results based on the two types of coefficients and two methods of factor analysis.

In interpreting results, the first (rotated) factor is the most significant and should be the easiest to interpret. The statistical significance of the factors declines as successive factors are extracted, and the last is the least significant. The contribution of each factor to the mean communality is a quantitative measure of the importance of that factor.

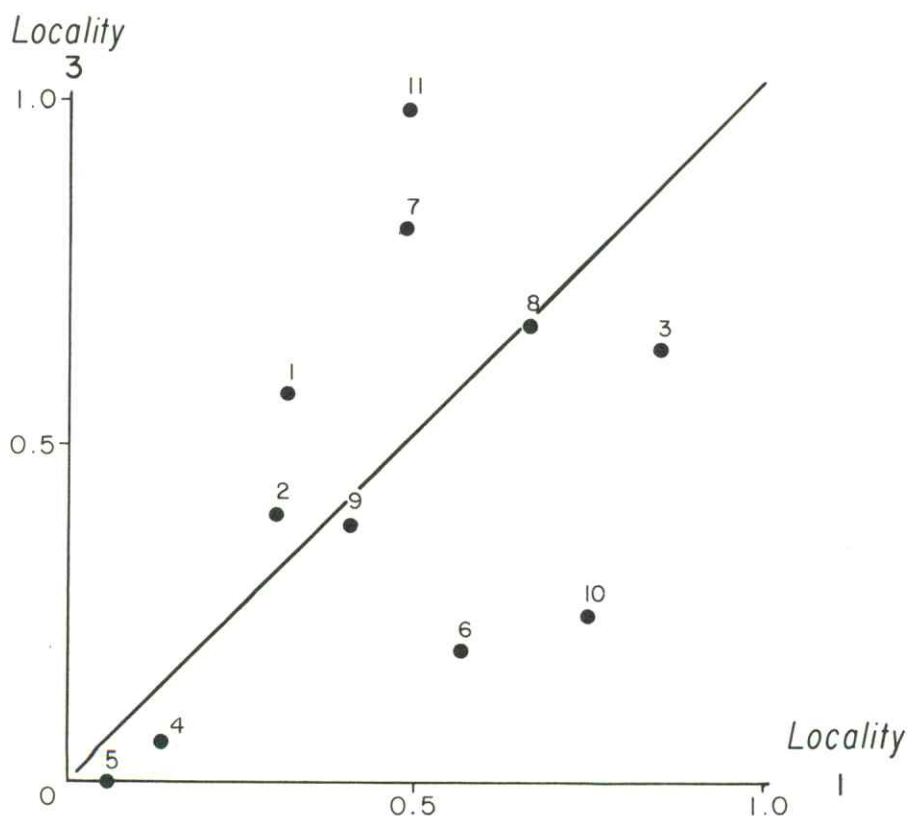


Figure 9. X-Y plot of transformed data to illustrate calculation of correlation coefficients between two localities. Numbers identify constituents listed in Table 9. Correlation coefficient is 0.47. Perfect similarity of localities would yield coefficient of 1.0 with all points on diagonal line.

It should be made clear that factors extracted from a correlation matrix, are, in themselves, only mathematical abstractions. There is no guarantee that mathematical factors correspond to actual "factors" that exist in nature. Mathematical factors may strongly reflect the influence of actual factors, but factor analysis, in itself, does not identify the actual factors. Identification is a matter of interpretation.

Table 10A.- Loadings and communalities of four rotated factors at 27 Americus localities. Factors were extracted by the centroid method from a correlation coefficient matrix. The communality at each locality is the sum of the squared factor loadings at that locality.

Local- ity Number	Factor Loadings				Communal- ity, h^2
	Factor I	Factor II	Factor III	Factor IV	
59	.17	.02	.95	.06	.93
58	.24	.10	.93	-.04	.94
57	.39	-.21	.51	.20	.50
54	.19	-.47	.64	-.48	.89
53	.11	-.60	.35	-.62	.88
1	.03	-.65	.52	-.20	.73
2	.05	-.73	.58	-.13	.89
3	.00	-.90	-.04	-.23	.86
4	.11	-.91	.21	-.16	.92
5	-.24	.39	.32	-.08	.32
6	.15	-.61	-.06	.52	.67
9	.16	-.97	-.06	-.01	.98
10	.32	-.86	-.23	.23	.95
11	.18	-.81	-.25	.40	.92
13	.39	-.71	.02	.26	.73
14	.23	-.59	.21	-.36	.58
16	.86	-.15	.27	-.14	.86
19	.78	.10	.03	-.52	.89
20	.83	.01	-.11	-.51	.96
21	.93	-.30	-.03	-.12	.96
23	.98	-.10	-.13	-.14	.99
25	.98	-.13	-.11	.12	.99
29	.98	-.09	.03	.16	.99
40	.99	-.03	.01	.00	.98
52	.97	-.05	.16	.07	.96
43	.90	-.12	.22	.14	.90
45	.90	-.06	.30	.25	.97

Average communality = .857

Table 10B.- Loadings and communalities of four rotated factors at 27 Americus localities. Factors were extracted by the principal axis method from a correlation coefficient matrix.

Locality Number	Factor Loadings				Communality h^2
	Factor I	Factor II	Factor III	Factor IV	
59	0.107	-0.272	-0.023	-0.920	0.933

58	0.189	-0.283	0.088	-0.887	0.911
57	0.012	-0.235	-0.409	-0.606	0.590
54	0.302	-0.738	0.005	-0.548	0.936
53	0.238	-0.905	-0.022	-0.204	0.917
1	-0.006	-0.745	-0.334	-0.397	0.824
2	0.175	-0.628	-0.397	-0.565	0.902
3	0.144	-0.693	-0.604	0.105	0.876
4	0.255	-0.662	-0.616	-0.215	0.929
5	-0.027	0.114	0.475	-0.514	0.503
6	0.143	0.088	-0.861	-0.171	0.800
9	0.214	-0.580	-0.742	0.020	0.933
10	0.404	-0.279	-0.853	0.087	0.976
11	0.202	-0.139	-0.951	0.069	0.970
13	0.166	-0.506	-0.783	-0.248	0.958
14	0.116	-0.905	-0.252	-0.189	0.931
16	0.790	-0.348	-0.225	-0.306	0.889
19	0.828	-0.361	0.229	-0.020	0.868
20	0.821	-0.425	0.146	0.126	0.892
21	0.834	-0.376	-0.295	-0.048	0.927
23	0.938	-0.121	-0.087	0.178	0.935
25	0.934	0.004	-0.308	0.068	0.972
29	0.951	-0.011	-0.275	-0.099	0.991
40	0.903	-0.226	-0.170	-0.129	0.912
52	0.943	-0.084	-0.182	-0.204	0.971
43	0.865	-0.070	-0.242	-0.269	0.883
45	0.729	0.218	-0.135	-0.174	0.627

Average communality = 0.880

Table 10C.-Loadings and communalities of four rotated factors at 27 Americus localities.
Factors were extracted by the principal axis method from a distance coefficient matrix.

Locality Number	Factor Loadings				Communality h^2
	Factor I	Factor II	Factor III	Factor IV	
59	0.287	0.270	-0.825	0.239	0.893
58	0.321	0.213	-0.819	0.263	0.888
57	0.266	0.431	-0.620	0.203	0.683
54	0.359	0.310	-0.559	0.566	0.859
53	0.327	0.325	-0.397	0.684	0.838
1	0.277	0.556	-0.462	0.481	0.830
2	0.366	0.606	-0.472	0.380	0.868
3	0.353	0.710	-0.227	0.418	0.855
4	0.389	0.716	-0.324	0.377	0.911
5	0.381	0.314	-0.550	0.109	0.558
6	0.408	0.736	-0.330	0.041	0.819
9	0.399	0.767	-0.260	0.315	0.915
10	0.480	0.779	-0.226	0.188	0.923
11	0.428	0.803	-0.252	0.134	0.909
13	0.373	0.756	-0.344	0.288	0.912
14	0.287	0.546	-0.329	0.609	0.859
16	0.665	0.444	-0.327	0.314	0.845
19	0.731	0.206	-0.226	0.454	0.834
20	0.734	0.248	-0.176	0.477	0.859
21	0.700	0.483	-0.235	0.322	0.882
23	0.794	0.369	-0.230	0.233	0.874
25	0.788	0.443	-0.254	0.155	0.906
29	0.816	0.401	-0.282	0.147	0.927
40	0.770	0.372	-0.274	0.289	0.889
52	0.812	0.328	-0.315	0.177	0.898
43	0.753	0.342	-0.364	0.156	0.842
45	0.685	0.308	-0.389	0.022	0.716

Average communality = 0.852

SAMPLING LOCALITIES



Figure 10. Graphs of loadings on four rotated centroid factors extracted from correlation coefficient matrix based on comparison of 11 constituents at all possible pairs of 27 localities. Largest absolute values of loadings are plotted above zero reference lines. Diagonal ruling denotes positive loadings and stippling denotes negative loadings.

SAMPLING LOCALITIES

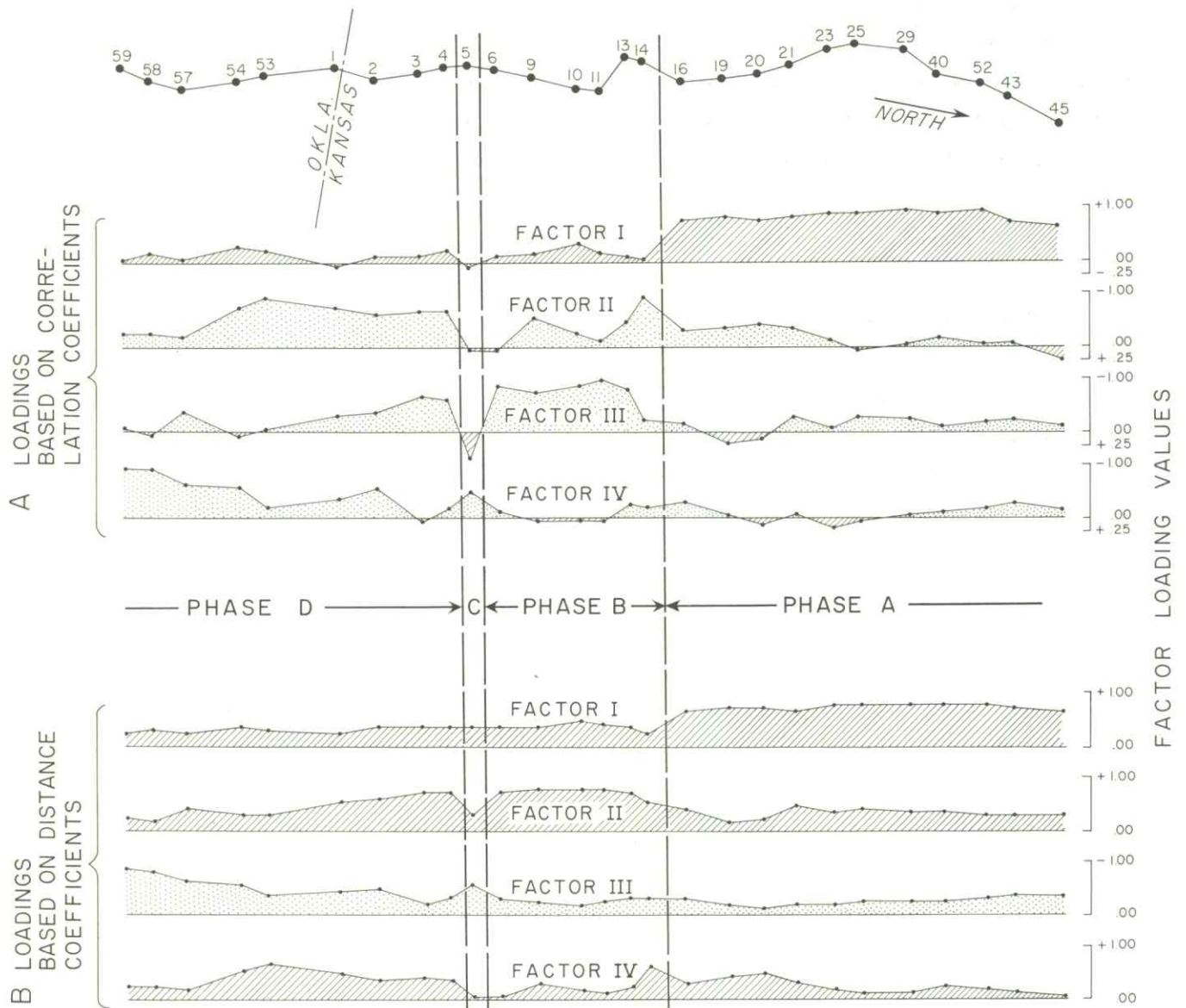


Figure 11. Graphs of loadings on four rotated principal axis factors: (A) Based on correlation coefficients; (B) based on distance coefficients. Diagonal ruling denotes positive loadings; stippling denotes negative loadings.

Comparison of the loadings of the first factor extracted (Factor I, Fig. 10, 11) shows clearly that a major change occurs between localities 14 and 16. At locality 16 and northward, loadings on factor 1 are high, whereas at 14 and southward, the loadings are relatively low. The break between 14 and 16 appears to mark the most important change along the outcrop belt and probably reflects a major change in environment of deposition. The change next in importance is adjacent to locality 5, which is distinguished sharply from adjacent localities. These changes reflect variations in the 11 constituents (Fig. 3) used in calculating coefficients, and mark the boundaries between divisions which have been arbitrarily labeled phases

A, B, C, and D. Phase A contains a high proportion of shells, low fusulinids and low strontium, phase B reflects high bryozoans and low R_2O_3 group, phase C is high in intraclasts and low in bryozoans, crinoids, fusulinids and microcrystalline calcite, and phase D is similar to Phase B but contains less bryozoans and more silica and R_2O_3 group. These divisions might have been picked out by inspecting the raw data (Fig. 3) alone, but this is difficult and subjective. On the other hand, by plotting the factor loadings, the boundaries between major groups can be picked out more readily and objectively.

Comparison of Factor Techniques

It should be noted that some factor loadings are negative, (Fig. 10, 11), signifying projections on the negative halves of factor axes. If we wish, we may reverse the algebraic signs of all the loadings on factors extracted from a given matrix without changing the geometric relationships of the factors. For convenience, factors with loadings that are predominantly negative have been plotted so that negative values lie above the zero reference line. Disregarding signs, the loadings on principal axis factors extracted from a distance coefficient matrix (lower part Fig. 11) are very similar to the loadings on centroid factors extracted from a correlation coefficient matrix (Fig. 10).

Comparison of the two sets of principal axis factors (Fig. 11) show that the loadings on factor I are very similar, but that the sequence of the next three factors differs. For example, factor IV of the distance coefficient matrix is very similar to factor II of the correlation coefficient matrix. Comparison of Figures 10 and 11 suggests that the centroid method is as effective as the principal axis method in analyzing the data of this study. Furthermore, results based on distance coefficients are similar to those based on correlation coefficients.

Before proceeding further to interpret between-variations between localities let us now consider variations between hand specimens.

VARIATIONS BETWEEN HAND SPECIMENS

Variations in bryozoans, crinoids, fusulinids and shells have been studied between hand specimens (Fig. 12), by (1) contouring percentages on vertical sections (Fig. 13 A, C, E, G), (2) fitting trend surfaces to the percentage data (Fig. 13 B, D, F, H), and (3) contouring factor loadings on vertical sections (Fig. 14). In plotting the hand specimen data on vertical sections (Fig. 12), the height of each specimen is proportional to the percentage of the distance from bottom to top of the main ledge, thus cancelling out differences in thickness of the main ledge. For example, a specimen at the top of the main ledge is 100 percent of the way from bottom to top, whereas a specimen in the middle is 50 percent of the way.

Principles of Trend Surfaces

Trend surfaces may be used to interpret geological data. A trend surface is a plane or gently curving surface fitted to data points so that the surface passes above some data points and below others.

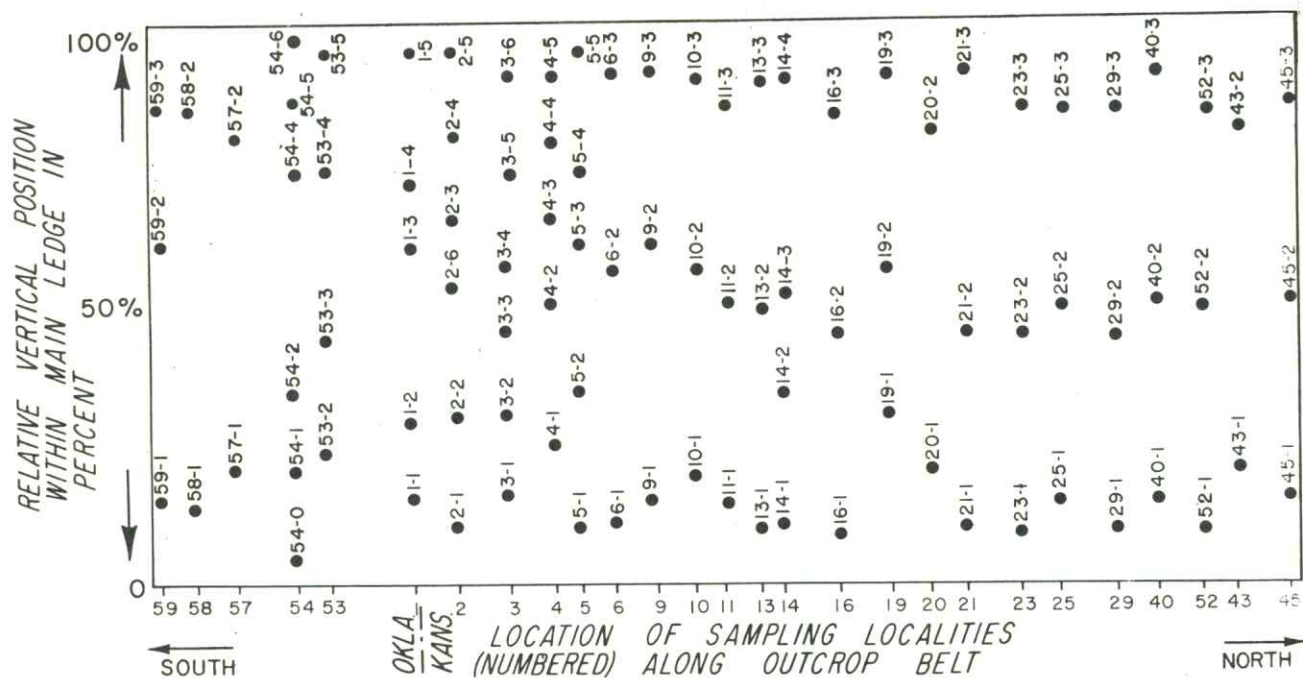


Figure 12. Location of hand specimens in main ledge with vertical position of specimens expressed as percentage of distance from bottom to top of main ledge.

Trend surfaces are commonly fitted so that the least-squares criterion is satisfied, requiring that the sum of the squared deviations from the surface is the least possible and providing a unique fit. A deviation is the difference between the trend surface value and the observed value. Both positive and negative deviations are obtained, but the squared deviations are all positive.

A trend surface may be expressed by an equation whose degree is a measure of the complexity of the surface. A plane or first-degree surface is the simplest and is represented by an equation containing variables to the first power. The equation of a second-degree surface may contain terms of the first degree, but also contains one or more terms in which the variables are squared, or consist of two first-power variables multiplied by each other. In this report only second-degree trend surfaces are shown. These surfaces are described by the general equation

$$z = A + Bx + Cy + Dx^2 + Exy + Fy^2,$$

where x and y are independent variables, z is the dependent variable, and A through F are constants containing the algebraic signs. The surfaces are parabolic and either resemble a bowl (elliptical paraboloid) or a saddle (hyperbolic paraboloid).

In this study, x and y are the horizontal and vertical coordinates that specify the locations of hand specimens in the main ledge, and z is the trend value of a constituent, such as bryozoan percentage.

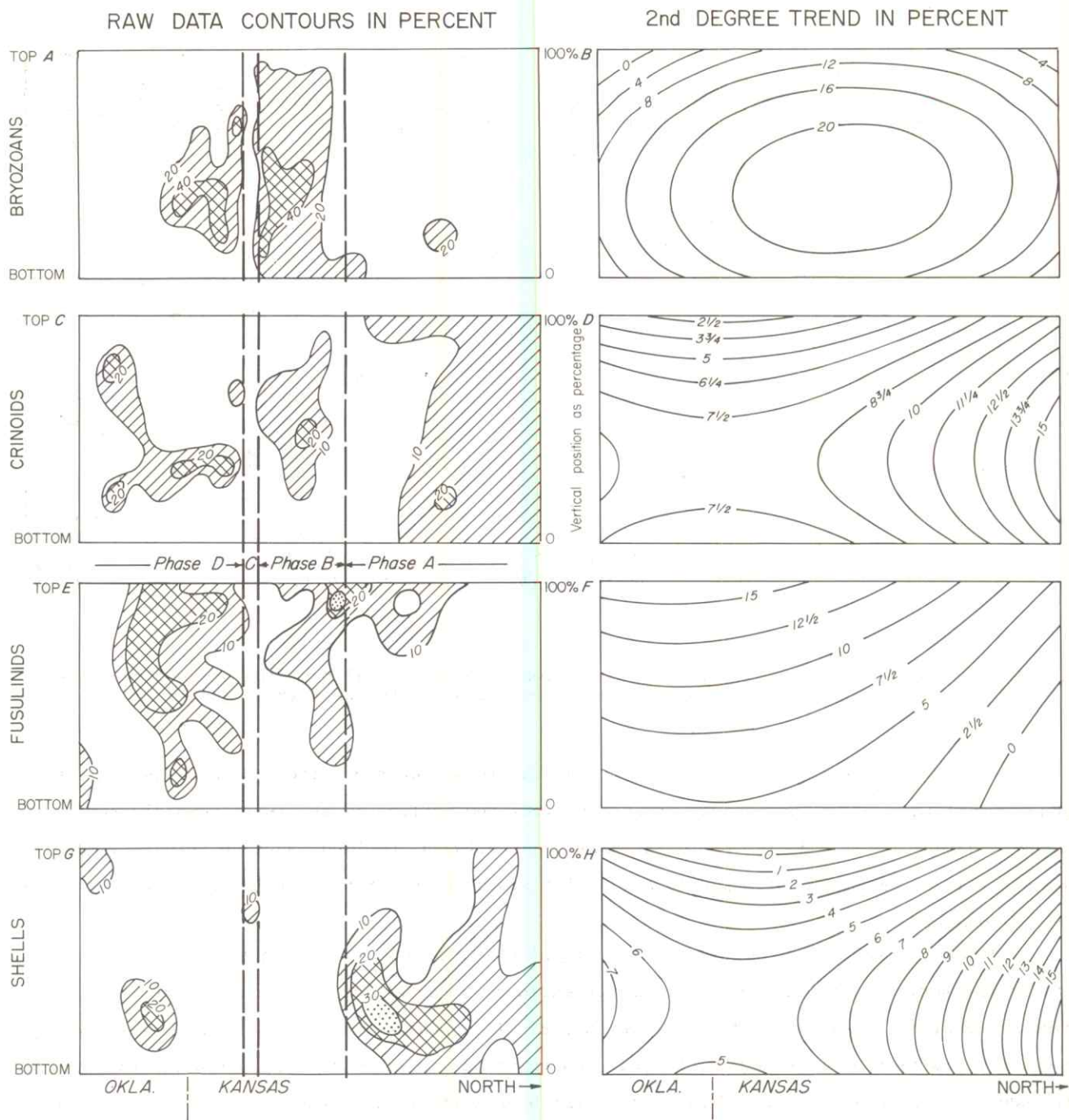


Figure 13. Contoured sections showing vertical and lateral variations of point-count constituents in hand specimens of Americus main ledge. Left column shows raw percentage values and right column contains second-degree trend surfaces. Boundaries between phases are shown by heavy dashed vertical lines.

Trend surface analysis of geologic data has been described by Krumbein (1956, 1959), Dawson and Whitten (1962), Merriam and Harbaugh (1963), and others. The computer program used for fitting trend surfaces in this study was written by Harbaugh (1963), and may be obtained in punch-card form from the Kansas Geological Survey.

Statistics of Trend Surfaces

The goodness of fit of a trend surface to the raw data is given by the percentage of total sum of squares (see Allen and Krumbein, 1962, for discussion); the higher the percentage,

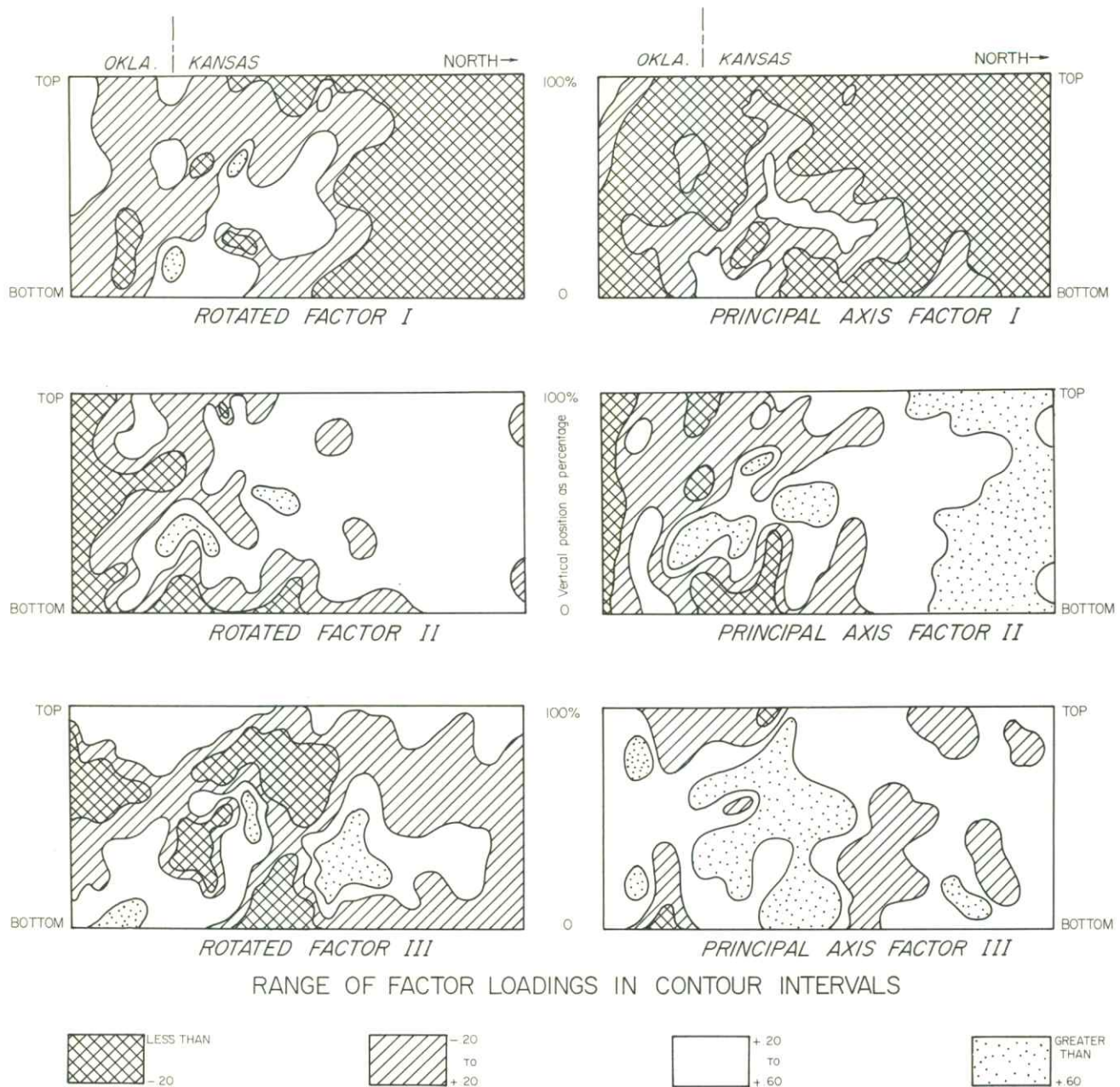


Figure 14. Contoured sections showing factor loadings. Factors are based on comparison of seven point-count constituents between pairs of 86 hand specimens. Loadings on rotated factors are shown in left column, and loadings on unrotated factors are shown in right column. Patterns distinguish range of factor loadings.

the closer the fit, and vice versa. The second-degree trend surfaces of this study (Fig. 13) fit the raw data only moderately well (Table 11). In addition to measuring the goodness of fit, we need to be confident that the fit of a trend surface is not due to chance alone. If the deviations from a trend surface are distributed more or less normally, analysis of variance may be used to determine the probability that the fit reflects a real effect and is not due to chance. The degree of confidence may be expressed as the confidence level in percent (Table 11). For example, we may be 99 percent confident that the second degree terms ($Dx^2 + Exy +$

Fy²) of the bryozoan trend surface represent a real effect, whereas only 50 percent confidence is associated with the second-degree terms of the fusulinid trend surface.

Variations Among Four Point-Count Constituents

Bryozoans.--Bryozoan percentages in hand specimens (Fig. 13A) are highest toward the center of the main ledge. The trend surface (Fig. 13B) strongly emphasizes this tendency, and suggests that bryozoans were particularly favored during the middle part of the time in which the main ledge was deposited.

Crinoids.--Comparison of crinoids (Fig. 13C) with bryozoans suggests that there is some tendency for one to exclude the other. The crinoid trend surface (Fig. 13D) is saddle shaped, whereas the bryozoan trend surface is bowl shaped.

Fusulinids.--Fusulinids (Fig. 13E) are irregularly distributed, and tend to be concentrated in the upper part of the main ledge. The second-degree trend surface (Fig. 13F) emphasizes an upward and southward increase in fusulinids, although the confidence level of the second-degree terms is low. However, the confidence level of the plane represented by the first degree terms is high (Table 11), and it also emphasizes a southward and upward increase in fusulinids.

Shells.--Shells (Fig. 13G) are highest in the lower northern parts of the main ledge, although shells also increase toward the southern end. These relationships are emphasized by the saddle-shaped trend surface (Fig. 13H). Comparison of the raw data contours suggests that shells bear a rough inverse relationship to fusulinids and bryozoans. Where shells are high, fusulinids and bryozoans tend to be low, and vice versa.

Application of Q-Factor Analysis to Variations Between Specimens

The results of Q-factor analysis applied to variations between hand specimens are shown (Fig. 14) by contoured factor loadings on vertical sections. The loadings are on three rotated and three unrotated factors which had been extracted from a correlation matrix based on 86 hand specimens and containing 3655 correlation coefficients. Although point-count analyses of 94 hand specimens were made, the largest computer program for factor analysis available to the authors when the computations were made would not accommodate more than 86 variables, requiring

Table 11.-- Percent of total sum of squares and confidence levels of trend surfaces fitted to four point-count constituents.

Constituent	Percent of total sum of squares	Confidence level of first-degree terms, in percent	Confidence level of second-degree terms, in percent
Bryozoans	14	99.9+	99
Crinoids	21	99.9+	90
Fusulinids	30	99.9+	50
Shells	25	99.9+	92

that data for eight specimens be omitted. Loadings on the first two rotated factors are most meaningful, whereas loadings on the unrotated factors are difficult to interpret. Loadings on the rotated factors based on hand specimens, except for opposite algebraic signs, are somewhat similar in geographic distribution to loadings on rotated factors based on comparisons between localities (Fig. 10, 11). Loadings on the first rotated between-specimen factor axis (Fig. 13) tend to be algebraically low toward the north, whereas the first between-locality factor loadings (Fig. 10, 11) are high toward the north. The difference in algebraic signs is not very significant, because we can reverse the algebraic signs for all loadings on a given set of factors if we wish. Little advantage appears to have been gained by factor analysis of variations between specimens. The variations are probably too erratic to permit the factor loadings to be meaningful.

DEPOSITIONAL ENVIRONMENT OF MAIN LEDGE

Phase A

Phase A represents an environment in which brachiopods and other shelled organisms flourished (Fig. 3D) and in which fine carbonate mud (microcrystalline calcite, Fig. 3G) accumulated in abundance (Fig. 15). Shelled animals were particularly common during deposition of the lower and middle part of the main ledge (Fig. 13G), and were partly succeeded by fusulinids during deposition of the upper part (Fig. 13E). The abundant carbonate mud suggests that currents and wave motion may have been relatively low, exerting little winnowing effect and perhaps signifying deeper water than in other places. In addition, mud-forming processes probably supplied more fine carbonate than was supplied in other phases. Low silica and low R_2O_3 group (Fig. 3I, 3J) suggest that phase A formed relatively far from shore, where the influx of terrestrially derived clay and other clastic materials was less than in other phases.

Phase B

Bryozoans flourished in phase B during deposition of the middle part of the main ledge. Crinoids were also favored, although less than bryozoans (Fig. 13A, 13C). Shelled animals were not favored in phase B (Fig. 13G). Judging from the scarce paleoecological data on bryozoans (Duncan, 1957) and crinoids (Laudon, 1957), the water in which phase B was probably deposited was clear, agitated and considerably less than 200 feet deep. Phases A and B have been grouped in a "northern marine shelf" (Fig. 15) which lay north of a postulated shoal area (phase C).

Phase C

Phase C is the intraclast area, where fusulinids, crinoids and bryozoans are low. The intraclasts probably formed as partly consolidated carbonate mud chips broken loose on a tidal flat or shoal (Fig. 15) that was inhospitable to fusulinids, crinoids and bryozoans. Laporte (1962) interprets a facies rich in platy, Anchicodium-like calcareous algae in the Lower Permian Cottonwood Limestone to be a shoal deposit. This deposit is termed the "Greenwood

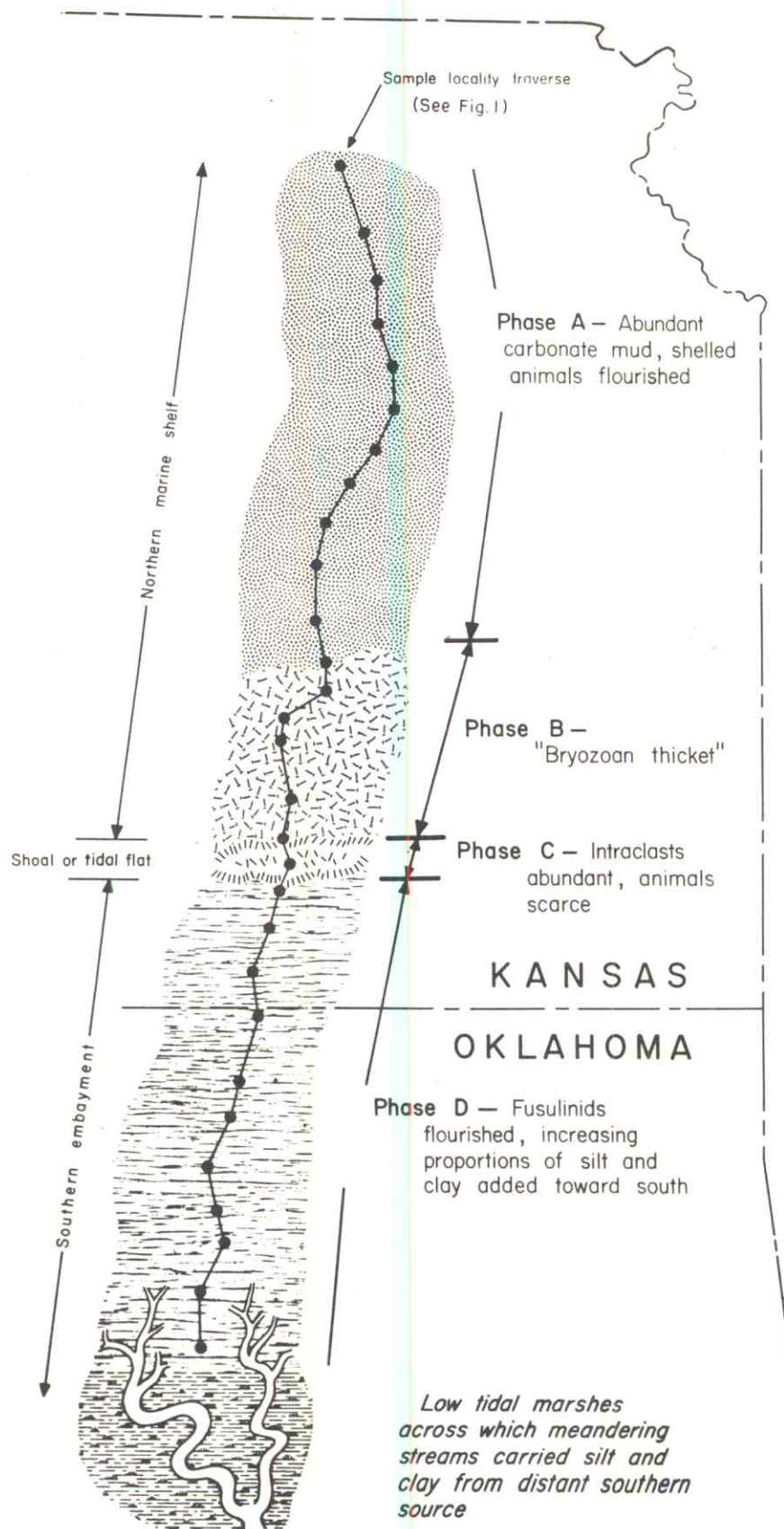


Figure 15. Map showing phases of depositional environment of main ledge.

shoal" and occurs in Greenwood County, immediately north of phase C. Perhaps shoals in both the Americus and Cottonwood were affected by controls of regional magnitude, which may have included the geographic relations of land and sea, sources of terrestrially derived sediments, and gentle tectonic warping.

Phase D

Phase D is characterized by an increase in thickness (Fig. 2) and by abundant fusulinids (Fig. 3C) which increase toward the upper part of the main ledge (Fig. 13E). Crinoids, bryozoans and shells are also abundant locally. A progressive increase in silica and R_2O_3 group toward the south, coupled with southerly transition to shales and redbeds, suggests that phase D was deposited in an embayment (Fig. 15) that lay north of a low-lying alluvial plain that was intermittently flooded by the sea. Streams and tidal channels may have meandered across this plain depositing sand and gray and red silt and clay to form alternating marine, brackish-water, and terrestrial sediments. The abundance of fusulinids in phase D suggests that fusulinids could thrive in a near-shore environment.

High strontium in phases B, C, and part of D may reflect the former presence of carbonate sediments originally containing a high proportion of aragonite, which tends to be enriched in strontium compared with calcite (Siegel, 1960, 1961). It is also possible that proportions of strontium were controlled by clay minerals carried into the depositional basin (Hirst, 1962).

SUMMARY

(1) The main ledge of the Americus Limestone was studied along its outcrop belt from Lincoln County, Oklahoma, to Pottawatomie County, Kansas, a straight-line distance of about 250 miles. The main ledge varies from about one to five feet in thickness.

(2) Point counting of thin sections shows that the main ledge consists mostly of skeletal "hash" composed of varying proportions of bryozoans, fusulinids, shells, and crinoid stem plates in a matrix of microcrystalline calcite and visibly crystalline calcite. Intraclasts occur locally.

(3) Silica, magnesia, calcium oxide, and the R_2O_3 group (iron oxide, alumina, and titanium oxide, undifferentiated) were determined by wet chemical analysis, and manganese, titanium, strontium, and vanadium were determined spectrographically.

(4) Comparison of variations in point-count and chemical constituents from locality to locality along the outcrop belt permits the major divisions of the main ledge to be picked out. Graphs on which constituents are plotted roughly suggest where major divisions or phases occur, but application of factor analysis brings out the boundaries between divisions much more sharply. Major divisions from north to south, are as follows: (a) Phase A is characterized by abundant shells and microcrystalline calcite, and is interpreted to be a deposit formed relatively far from shore, where waves and currents exerted little winnowing effect. (b) Phase B is interpreted to be a deposit formed in a bryozoan "thicket." (c) Phase C contains abundant intraclasts, but contains few bryozoans, crinoids, fusulinids, and shelled animals,

suggesting that it is a shoal or tidal flat deposit. (d) Phase D contains abundant fusulinids and increasing silica and R_2O_3 group toward the south. Phase D is interpreted to have formed in an embayment lying south of the shoal (phase C) and north of a low alluvial plain across which terrestrially derived sediments were transported.

(5) Different organisms are concentrated in different levels within the main ledge. Fusulinids are most abundant toward the top and bryozoans are abundant in the middle part.

(6) Factor analysis is a useful statistical tool in clarifying simple relations in complex data and should be useful in picking boundaries between sedimentary facies. Trend surface analysis is also useful in interpreting quantitative data.

REFERENCES

- Allen, P., and Krumbein, W.C., 1962, Secondary trend components in the top Ashdown Pebble bed; A case history: *Jour. Geol.*, v. 70, p. 507-538.
- Chayes, F., 1956, Petrographic modal analysis: John Wiley and Sons, Inc., p. 1-113.
- Dawson, K.R., and Whitten, E.H.T., 1962, The quantitative mineralogical composition and variation of the Lacorne, La Motte and Preissac granitic complex, Quebec, Canada: *Jour. Petrology*, v. 3, p. 1-37.
- Duncan, Helen, 1957, Bryozoans, in *Treatise on marine ecology and paleoecology*: Geol. Soc. Amer., Mem. 67, v. 2, p. 783-789.
- Fairbridge, R.W., 1957, The dolomite question: *Soc. Econ. Paleontologists and Mineralogists Spec. Pub.* 5, p. 125-178.
- Fruchter, B., 1954, Introduction to factor analysis: D. Van Nostrand Co., p. 1-280.
- Harbaugh, J.W., 1961, Relative ages of visibly crystalline calcite in late Paleozoic limestones: *Kansas Geol. Survey Bull.* 152, p. 91-126.
- Harbaugh, J.W., 1963, BALGOL program for trend-surface mapping using an IBM 7090 computer: *Kansas Geol. Survey Special Dist. Pub.* 3, p. 1-22.
- Harbaugh, J.W., 1964, BALGOL programs for calculation of distance coefficients and correlation coefficients using an IBM 7090 computer: *Kansas Geol. Survey Special Dist. Pub.* 9, p. 1-32.
- Hill, W.E., Jr., Waugh, W.N., Galle, O.K., and Runnels, R.T., 1961, Methods of chemical analysis for carbonate and silicate rocks: *Kansas Geol. Survey Bull.* 152, p. 1-30.
- Hirst, D.M., 1962, The geochemistry of modern sediments from the Gulf of Paria; II: The location and distribution of trace elements: *Geochem. et Cosmochim. Acta*, v. 26, p. 1147-1187.
- Imbrie, J., and Purdy, E.G., 1962, Classification of modern Bahamian carbonate sediments: *Am. Assoc. Petroleum Geologists, Mem.* 1, p. 253-272.
- Krumbein, W.C., 1956, Regional and local components in facies maps: *Am. Assoc. Petroleum Geologists Bull.*, v. 40, p. 2163-2194.
- Krumbein, W.C., 1959, Trend-surface analysis of contour-type maps with irregular control-point spacing: *Jour. Geophysical Res.*, v. 64, p. 823-834.
- Laporte, L.F., 1962, Paleoecology of the Cottonwood Limestone (Permian), Northern Mid-Continent: *Geol. Soc. America Bull.*, v. 73, p. 521-544.
- Laudon, L.R., 1957, Crinoids in *Treatise on marine ecology and paleoecology*: Geol. Soc. America, Mem. 67, v. 2, p. 961-971.
- Merriam, D.F., and Harbaugh, J.W., 1963, Computer helps map oil structures: *Oil and Gas Jour.*, v. 61, no. 47, p. 158-163.
- Moore, R.C., and others, 1951, The Kansas rock column: *Kansas Geol. Survey Bull.* 89, p. 1-132.
- Moroney, M.J., 1956, Facts from figures: Penguin Books, p. 1-472.
- Mudge, M.R., and Yochelson, E.L., 1962, Stratigraphy and paleontology of the uppermost Pennsylvanian and lowermost Permian rocks in Kansas: *U.S. Geol. Survey Prof. Paper* 323, p. 1-213.
- Runnels, R.T., and Schleicher, J.A., 1956, Chemical composition of eastern Kansas limestones: *Kansas Geol. Survey Bull.* 119, p. 1-23.
- Siegel, F.R., 1960, The effect of strontium on the aragonite-calcite ratios of Pleistocene corals: *Jour. Sed. Pet.*, v. 30, p. 297-304.
- Siegel, F.R., 1961, Variations of Sr/Ca ratios and Mg contents in recent carbonate sediments of the northern Florida Keys area: *Jour. Sed. Pet.*, v. 31, p. 336-342.
- Terriere, R.T., 1963, Petrography and environmental analysis of some Pennsylvanian limestones from central Texas: *U.S. Geol. Survey Prof. Paper* 215-E, p. 79-126.

APPENDIX A

Table 12.- Section, township and range of all localities incorporated in this report (shown graphically in Figure 2). Number of hand specimens is listed at localities where samples were collected.

Locality number	Number of hand specimens	Section	Township	Range	State
1	5	20	29 N	7 E	Oklahoma
2	6	NW NW 7	34 S	8 E	Kansas
3	6	SW 21	32 S	8 E	Kansas
4	5	NE NE 14	31 S	8 E	Kansas
5	5	NE NW 20	30 S	9 E	Kansas
6	3	NW NE 18	28 S	9 E	Kansas
7	-	SW SW 31	28 S	9 E	Kansas
8	-	SW NE 23	28 S	9 E	Kansas
9	3	SE SW 31	27 S	9 E	Kansas
10	3	SE SE 1	26 S	8 E	Kansas
11	3	SE SW 1	25 S	8 E	Kansas
12	2	SE SW 30	24 S	10 E	Kansas
13	3	NW SW 10	24 S	10 E	Kansas
14	4	NW cor 22	23 S	10 E	Kansas
16	3	SE SW 25	21 S	9 E	Kansas
17	-	SW SW 1	21 S	10 E	Kansas
18	-	SE SW 14	20 S	10 E	Kansas
19	3	SW SW 33	19 S	10 E	Kansas
20	2	NE NE 26	18 S	10 E	Kansas
21	3	NE NW 21	17 S	11 E	Kansas
22	-	NW NW 35	16 S	11 E	Kansas
23	3	SW NW 4	16 S	12 E	Kansas
24	-	NW NW 19	15 S	12 E	Kansas
25	3	NW NW 1	15 S	12 E	Kansas
27	-	NW NW 23	14 S	12 E	Kansas
28	-	NW NW 21	13 S	12 E	Kansas
29	3	SW NW 3	13 S	12 E	Kansas
31	-	SE SW 27	11 S	11 E	Kansas
33	-	SW SE 12	12 S	11 E	Kansas
39	-	NE SE 27	10 S	11 E	Kansas
40	3	NE SE 24	11 S	11 E	Kansas
43	2	NW NE 21	8 S	11 E	Kansas
52	3	NE SE 36	9 S	11 E	Kansas
53	4	NW NW 10	26 N	6 E	Oklahoma
54	6	NW SE 7	25 N	6 E	Oklahoma
56	-	NW NW 1	22 N	6 E	Oklahoma
57	2	NE NE 9	23 N	5 E	Oklahoma
58	2	NW SW 23	22 N	5 E	Oklahoma
59	3	SE 36	21 N	5 E	Oklahoma
60	1	SE SW 20	17 N	5 E	Oklahoma
61	1	SE SW 16	19 N	5 E	Oklahoma
62	0	NE NE 7	16 N	5 E	Oklahoma
65	-	NE SE 4	28 S	9 E	Kansas
68	-	NE NE 28	29 S	9 E	Kansas

APPENDIX B

Table 13.- Tabulation of basic data, Americus main ledge. Figures underlined pertain to each locality as a whole. Grand means and standard deviation values for each constituent are listed at bottom of table.

	<u>25.6</u>	<u>30.5</u>	<u>1.8</u>	<u>16.5</u>	<u>14.0</u>	<u>6.4</u>	<u>2.6</u>	<u>2.6</u>	<u>9.45</u>	<u>1.80</u>	<u>0.88</u>	<u>0.12</u>	<u>0.34</u>	<u>0.10</u>	<u>0.003</u>	<u>48.21</u>
<u>1</u>																
1-1	3.5	50.0	8.0	25.5	0.5	3.2	2.7	2.5								
1-2	13.5	0.0	0.0	7.5	47.0	20.5	8.0	2.8								
1-3	5.8	59.0	0.0	13.0	15.0	5.2	1.0	2.5								
1-4	66.2	8.2	0.0	16.2	4.5	3.0	0.2	2.8								
1-5	39.0	35.2	0.8	20.0	3.0	0.0	1.0	2.4								
<u>2</u>	<u>31.6</u>	<u>30.0</u>	<u>1.8</u>	<u>8.4</u>	<u>14.6</u>	<u>5.0</u>	<u>2.2</u>	<u>1.8</u>								
2-1	10.0	51.0	7.5	0.2	0.0	3.2	0.2	0.9								
2-2	24.5	26.0	2.0	4.0	25.5	9.5	4.0	1.6								
2-3	25.5	31.2	1.2	10.0	19.5	8.0	3.0	2.2								
2-4	45.0	22.0	0.0	17.5	9.0	5.0	1.0	2.6								
2-5	50.5	27.0	0.0	16.0	2.8	1.2	1.5	2.5								
2-6	33.0	22.5	0.0	3.0	31.0	3.0	3.5	1.1								
<u>3</u>	<u>41.9</u>	<u>10.7</u>	<u>0.0</u>	<u>12.2</u>	<u>25.1</u>	<u>7.7</u>	<u>1.0</u>	<u>2.4</u>	<u>9.07</u>	<u>1.56</u>	<u>0.28</u>	<u>0.24</u>	<u>0.16</u>	<u>0.11</u>	<u>0.004</u>	<u>49.84</u>
3-1	17.5	21.0	0.0	2.0	48.5	8.8	0.5	1.6								
3-2	1.5	0.5	0.0	15.5	54.5	22.0	4.2	3.3								
3-3	40.0	20.0	0.0	6.5	25.5	6.8	0.8	2.4								
3-4	69.5	2.0	0.0	13.0	9.2	5.0	0.2	2.4								
3-5	74.5	2.0	0.0	13.2	7.0	2.2	0.0	2.4								
3-6	23.8	19.2	0.0	23.2	6.0	1.2	0.5	2.5								
<u>4</u>	<u>35.8</u>	<u>18.0</u>	<u>0.8</u>	<u>10.0</u>	<u>21.4</u>	<u>5.7</u>	<u>1.3</u>	<u>2.2</u>								
4-1	41.5	11.0	2.5	10.8	7.0	6.5	0.2	1.9								
4-2	7.5	36.2	0.0	10.5	32.5	7.0	1.5	1.9								
4-3	21.0	1.0	0.5	6.0	53.5	10.8	2.8	3.6								
4-4	71.5	5.0	0.8	10.8	5.0	2.8	0.8	2.4								
4-5	37.5	37.0	0.2	12.0	9.0	1.5	1.0	1.1								

<u>5</u>	<u>22.4</u>	<u>30.7</u>	<u>29.7</u>	<u>0.0</u>	<u>0.1</u>	<u>0.0</u>	<u>4.2</u>	<u>3.2</u>	<u>6.39</u>	<u>1.77</u>	<u>0.80</u>	<u>0.13</u>	<u>0.13</u>	<u>0.11</u>	<u>0.10</u>	<u>50.05</u>
5-1	0.0	48.2	50.5	0.0	0.2	0.0	1.0	4.1								
5-2	0.0	57.5	40.0	0.0	0.0	0.0	2.5	3.8								
5-3	0.0	45.0	53.0	0.0	0.0	0.0	1.5	3.8								
5-4	23.5	0.8	5.0	0.0	0.2	0.0	14.5	3.8								
5-5	58.0	2.0	0.0	0.0	0.0	0.0	1.5	0.8								
<u>6</u>	<u>18.9</u>	<u>17.7</u>	<u>0.1</u>	<u>4.2</u>	<u>44.4</u>	<u>8.3</u>	<u>3.7</u>	<u>2.4</u>								
6-1	15.0	20.0	0.2	3.5	49.5	4.0	4.0	2.2								
6-2	18.5	0.0	0.0	2.0	57.5	13.5	5.0	2.5								
6-3	21.0	33.0	0.0	7.0	26.2	7.2	2.0	2.5								
<u>9</u>	<u>31.2</u>	<u>19.1</u>	<u>0.0</u>	<u>11.4</u>	<u>24.0</u>	<u>9.1</u>	<u>2.8</u>	<u>2.6</u>	<u>6.31</u>	<u>0.87</u>	<u>0.56</u>	<u>0.16</u>	<u>0.29</u>	<u>0.09</u>	<u>0.003</u>	<u>50.36</u>
9-1	30.0	23.2	0.0	7.2	24.2	6.2	5.0	2.4								
9-2	27.5	10.2	0.0	17.5	29.5	11.8	2.0	2.6								
9-3	36.0	23.8	0.0	9.8	18.0	9.2	4.1	2.8								
<u>10</u>	<u>34.5</u>	<u>13.0</u>	<u>0.0</u>	<u>6.9</u>	<u>28.5</u>	<u>11.2</u>	<u>2.8</u>	<u>2.1</u>								
10-1	17.0	24.5	0.0	5.8	35.0	9.5	2.5	1.6								
10-2	35.0	4.0	0.0	10.0	30.0	15.0	3.0	2.8								
10-3	51.2	10.5	0.0	5.0	20.5	9.0	3.0	2.0								
<u>11</u>	<u>26.8</u>	<u>16.8</u>	<u>0.0</u>	<u>7.4</u>	<u>31.1</u>	<u>13.7</u>	<u>2.1</u>	<u>2.2</u>	<u>4.88</u>	<u>1.02</u>	<u>0.48</u>	<u>0.17</u>	<u>0.26</u>	<u>0.13</u>	<u>0.012</u>	<u>52.06</u>
11-1	28.5	32.0	0.0	3.8	23.8	6.0	3.0	1.6								
11-2	5.5	5.0	0.0	10.0	47.5	27.5	2.8	2.5								
11-3	46.2	13.5	0.0	8.5	22.0	7.5	0.5	2.6								
<u>13</u>	<u>28.8</u>	<u>19.3</u>	<u>0.0</u>	<u>9.6</u>	<u>26.0</u>	<u>10.6</u>	<u>2.0</u>	<u>2.3</u>								
13-1	28.0	28.5	0.0	4.5	23.0	7.0	2.2	1.7								
13-2	13.5	22.0	0.0	14.2	34.0	11.5	2.8	2.5								
13-3	45.0	7.5	0.0	10.0	21.0	13.2	1.0	2.8								
<u>14</u>	<u>35.9</u>	<u>11.4</u>	<u>0.0</u>	<u>16.7</u>	<u>7.1</u>	<u>4.6</u>	<u>0.3</u>	<u>2.2</u>	<u>8.70</u>	<u>2.33</u>	<u>0.60</u>	<u>0.03</u>	<u>0.23</u>	<u>0.18</u>	<u>0.010</u>	<u>48.33</u>
14-1	45.0	10.0	0.0	0.0	0.2	4.8	0.0	1.9								
14-2	28.0	14.0	0.0	17.5	1.2	4.5	0.2	2.5								
14-3	36.5	11.0	0.0	8.2	20.2	4.0	0.8	2.0								
14-4	34.0	10.8	0.0	41.2	6.5	5.0	0.2	2.6								
<u>16</u>	<u>47.1</u>	<u>20.1</u>	<u>0.0</u>	<u>3.2</u>	<u>14.2</u>	<u>5.7</u>	<u>8.1</u>	<u>2.2</u>	<u>5.51</u>	<u>1.79</u>	<u>0.73</u>	<u>0.04</u>	<u>0.36</u>	<u>0.13</u>	<u>0.002</u>	<u>50.94</u>
16-1	49.2	15.8	0.0	0.5	23.0	6.8	0.8	1.6								
16-2	59.0	16.0	0.0	0.0	2.0	3.5	19.5	2.5								
16-3	33.0	28.5	0.0	9.2	17.5	6.8	3.2	2.6								

<u>19</u>	<u>49.4</u>	<u>14.7</u>	<u>0.0</u>	<u>5.5</u>	<u>5.8</u>	<u>4.2</u>	<u>19.3</u>	<u>2.6</u>	<u>4.96</u>	<u>1.73</u>	<u>0.64</u>	<u>0.06</u>	<u>0.31</u>	<u>0.13</u>	<u>0.006</u>	<u>51.32</u>
19-1	24.2	31.5	0.0	0.0	2.2	2.0	36.8	2.2								
19-2	73.8	7.5	0.0	0.0	0.0	1.2	17.5	3.1								
19-3	50.0	5.0	0.0	16.5	15.0	9.5	3.5	2.5								
<u>20</u>	<u>47.7</u>	<u>8.1</u>	<u>0.0</u>	<u>8.9</u>	<u>6.8</u>	<u>8.6</u>	<u>18.5</u>	<u>2.5</u>								
20-1	53.0	4.2	0.0	0.0	0.2	8.8	31.8	2.5								
20-2	42.5	12.0	0.0	17.8	13.2	8.5	5.2	2.5								
<u>21</u>	<u>45.8</u>	<u>12.0</u>	<u>0.0</u>	<u>6.0</u>	<u>14.2</u>	<u>10.1</u>	<u>8.4</u>	<u>2.5</u>	<u>4.17</u>	<u>1.53</u>	<u>0.73</u>	<u>0.06</u>	<u>0.38</u>	<u>0.11</u>	<u>0.009</u>	<u>51.64</u>
21-1	35.0	18.0	0.0	3.5	16.0	11.5	12.8	2.8								
21-2	51.2	13.0	0.0	5.5	13.8	7.5	6.8	2.5								
21-3	51.0	5.0	0.0	9.0	13.0	11.2	6.0	2.2								
<u>23</u>	<u>45.0</u>	<u>8.3</u>	<u>0.0</u>	<u>5.7</u>	<u>12.5</u>	<u>12.8</u>	<u>12.7</u>	<u>2.4</u>								
23-1	28.5	6.5	0.0	1.0	17.0	19.2	24.2	2.2								
23-2	49.5	10.5	0.0	3.8	14.0	11.2	9.0	2.4								
23-3	57.0	8.0	0.0	12.5	6.5	8.0	5.0	2.6								
<u>25</u>	<u>40.7</u>	<u>8.3</u>	<u>0.0</u>	<u>3.7</u>	<u>19.3</u>	<u>13.1</u>	<u>10.8</u>	<u>2.2</u>	<u>4.60</u>	<u>1.67</u>	<u>0.81</u>					<u>51.29</u>
25-1	14.2	8.8	0.0	0.8	30.0	20.0	20.8	1.7								
25-2	50.0	11.0	0.0	5.5	10.5	13.0	15.0	3.3								
25-3	58.0	5.0	0.0	5.0	17.5	6.2	4.0	1.6								
<u>29</u>	<u>44.2</u>	<u>11.2</u>	<u>0.0</u>	<u>1.8</u>	<u>16.0</u>	<u>13.0</u>	<u>10.3</u>	<u>2.0</u>	<u>5.40</u>	<u>1.46</u>	<u>0.92</u>					<u>50.66</u>
29-1	28.0	17.5	0.0	0.0	18.0	18.0	14.5	1.9								
29-2	50.0	12.0	0.0	0.5	12.5	10.8	11.0	1.7								
29-3	54.0	4.0	0.0	5.0	18.0	10.0	5.5	2.4								
<u>40</u>	<u>42.3</u>	<u>11.7</u>	<u>0.0</u>	<u>4.0</u>	<u>13.6</u>	<u>11.8</u>	<u>13.0</u>	<u>2.5</u>	<u>6.63</u>	<u>1.40</u>	<u>0.85</u>	<u>0.06</u>	<u>0.27</u>	<u>0.12</u>	<u>0.10</u>	<u>50.29</u>
40-1	49.0	10.0	0.0	0.7	12.0	10.8	15.0	2.6								
40-2	34.5	16.2	0.0	4.5	15.0	13.8	13.0	2.2								
40-3	43.2	9.0	0.0	6.8	13.8	11.0	11.0	2.5								
<u>43</u>	<u>46.6</u>	<u>10.1</u>	<u>0.0</u>	<u>0.4</u>	<u>13.0</u>	<u>14.3</u>	<u>9.7</u>	<u>2.5</u>	<u>10.15</u>	<u>1.74</u>	<u>0.99</u>					<u>48.11</u>
43-1	41.2	9.5	0.0	0.2	15.2	15.8	11.0	2.8								
43-2	51.8	10.8	0.0	0.5	10.8	13.0	8.5	2.2								
<u>45</u>	<u>41.6</u>	<u>19.5</u>	<u>0.0</u>	<u>0.6</u>	<u>14.1</u>	<u>12.5</u>	<u>9.3</u>	<u>2.7</u>	<u>6.95</u>	<u>1.82</u>	<u>1.16</u>	<u>0.04</u>	<u>0.33</u>	<u>0.14</u>	<u>0.028</u>	<u>49.54</u>
45-1	36.5	24.5	0.0	1.8	13.5	10.8	11.0	3.9								
45-2	42.0	14.0	0.0	0.0	14.5	18.0	9.5	2.4								
45-3	46.0	20.0	0.0	0.0	14.5	9.0	7.5	1.9								

<u>52</u>	<u>50.8</u>	<u>10.5</u>	<u>0.0</u>	<u>0.8</u>	<u>11.8</u>	<u>12.3</u>	<u>10.0</u>	<u>1.7</u>	<u>6.08</u>	<u>2.03</u>	<u>0.99</u>						<u>50.18</u>
52-1	50.0	12.0	0.0	0.2	15.5	15.0	5.5	2.2									
52-2	47.5	12.0	0.0	1.0	15.0	12.5	8.2	1.6									
52-3	55.0	7.5	0.0	1.0	5.0	9.5	16.2	1.4									
<u>53</u>	<u>43.1</u>	<u>16.2</u>	<u>0.0</u>	<u>19.0</u>	<u>4.9</u>	<u>6.6</u>	<u>8.4</u>	<u>2.5</u>	<u>13.83</u>	<u>3.10</u>	<u>0.66</u>	<u>0.24</u>	<u>0.16</u>	<u>0.11</u>	<u>0.018</u>		<u>44.43</u>
53-2	49.1	4.0	0.0	0.8	7.2	11.8	24.0	1.4									
53-3	28.5	27.5	0.0	21.5	7.0	10.0	5.0	3.0									
53-4	45.5	16.5	0.0	27.8	2.2	2.2	3.8	3.3									
53-5	49.0	16.8	0.0	26.2	3.2	2.5	0.8	2.5									
<u>54</u>	<u>43.9</u>	<u>28.6</u>	<u>1.1</u>	<u>10.9</u>	<u>3.2</u>	<u>4.0</u>	<u>6.2</u>	<u>2.2</u>									
54-0	68.0	15.2	6.8	0.0	1.0	2.8	2.2	1.2									
54-1	32.2	37.5	0.0	0.8	7.0	5.0	14.5	1.7									
54-2	46.5	21.5	0.0	0.5	5.0	7.5	17.5	2.0									
54-4	26.2	40.5	0.0	25.8	2.5	3.2	0.8	2.8									
54-5	45.0	27.5	0.0	21.5	1.8	2.5	1.5	3.3									
54-6	44.5	29.5	0.0	16.8	3.5	3.0	0.5	2.4									
<u>57</u>	<u>17.9</u>	<u>31.0</u>	<u>0.0</u>	<u>8.5</u>	<u>15.2</u>	<u>23.8</u>	<u>2.0</u>	<u>3.3</u>	<u>12.65</u>	<u>4.23</u>	<u>0.67</u>	<u>0.07</u>	<u>0.29</u>	<u>0.15</u>	<u>0.004</u>		<u>45.57</u>
57-1	11.8	38.0	0.0	7.5	13.5	26.5	2.5	3.6									
57-2	24.0	24.0	0.0	9.5	17.0	21.2	1.5	3.0									
<u>58</u>	<u>22.1</u>	<u>51.8</u>	<u>0.2</u>	<u>4.2</u>	<u>4.5</u>	<u>5.0</u>	<u>9.9</u>	<u>2.4</u>									
58-1	33.0	35.0	0.0	8.2	6.5	6.8	7.8	2.4									
58-2	11.2	68.8	0.5	0.2	2.5	3.2	11.5	2.4									
<u>59</u>	<u>17.6</u>	<u>52.4</u>	<u>0.0</u>	<u>5.5</u>	<u>7.5</u>	<u>4.3</u>	<u>6.5</u>	<u>5.9</u>	<u>10.73</u>	<u>3.94</u>	<u>0.95</u>	<u>0.06</u>	<u>0.5+</u>	<u>0.19</u>	<u>0.03</u>		<u>44.79</u>
59-1	21.2	35.0	0.0	12.0	10.0	5.0	7.0	2.5									
59-2	22.0	59.0	0.0	2.5	5.5	3.0	3.0	2.0									
59-3	9.5	62.0	0.0	2.0	7.0	5.0	9.5	1.4									
<u>60</u>									<u>10.59</u>	<u>6.40</u>	<u>15.97</u>						<u>27.17</u>
<u>61</u>									<u>4.72</u>	<u>3.85</u>	<u>17.82</u>	<u>0.016</u>	<u>0.5+</u>	<u>0.16</u>	<u>0.03</u>		<u>29.58</u>

Grade mean,
based on 36.13 20.10 2.65 7.86 15.03 8.16 6.13 2.37 7.588 2.302 2.374 0.0976 0.301 0.131 0.024
all data
in column:

Standard de-
viation, 18.15 15.67 9.72 8.12 13.68 5.93 7.06 6.63 0.0726 0.106 0.028 0.031
based on
all data
in column:

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