DANIEL F. MERRIAM, Editor

THE DENDROGRAPH

By

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and

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in cooperation with the
American Association of Petroleum Geologists
Tulsa, Oklahoma

COMPUTER CONTRIBUTION 48

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Editor's Remarks

COMPUTER CONTRIBUTION 48, "The dendrograph" by R.B. McCammon, is another program for graphic display. Geologists are increasingly using quantified classification techniques and will find this computer program of value in their work. For a limited time the Geological Survey will make available the program on magnetic tape for $10.00 (US) or $20.00 (US) on punched cards.

For a complete list of COMPUTER CONTRIBUTIONS, write Editor, COMPUTER CONTRIBUTIONS, Kansas Geological Survey, The University of Kansas, Lawrence, Kansas 66044.

Computer Contribution

1.	Mathematical simulation of marine sedimentation with IBM 7090/7094 computers, by J.W. Harbaugh, 1966	(out of print)
2.	A generalized two-dimensional regression procedure, by J.R. Dempsey, 1966	$0.50
3.	FORTRAN IV and MAP program for computation and plotting of trend surfaces for degrees 1 through 6, by Mont O'Leary, R.H. Lippert, and O.T. Spitz, 1966	$0.75
4.	FORTRAN II program for multivariate discriminant analysis using an IBM 1620 computer, by J.C. Davis and R.J. Sampson, 1966	$0.50
5.	FORTRAN IV program using double Fourier series for surface fitting of irregularly spaced data, by W.R. James, 1966	$0.75
6.	FORTRAN IV program for estimation of cladistic relationships using the IBM 7040, by R.L. Bartcher, 1966	$1.00
7.	Computer applications in the earth sciences: Colloquium on classification procedures, edited by D.F. Merriam, 1966	$1.00
8.	Prediction of the performance of a solution gas drive reservoir by Muskat's equation, by Apolonio Baca, 1967	$1.00
9.	FORTRAN IV program for mathematical simulation of marine sedimentation with IBM 7040 or 7094 computers, by J.W. Harbaugh and W.J. Wahlstedt, 1967	$1.00
10.	Three-dimensional response surface program in FORTRAN II for the IBM 1620 computer, by R.J. Sampson and J.C. Davis, 1967	$0.75
11.	FORTRAN IV program for vector trend analyses of directional data, by W.T. Fox, 1967	$1.00
12.	Computer applications in the earth sciences: Colloquium on trend analysis, edited by D.F. Merriam and N.C. Cocke, 1967	$1.00
13.	FORTRAN IV computer programs for Markov chain experiments in geology, by W.C. Krumbein, 1967	$1.00
14.	FORTRAN IV programs to determine surface roughness in topography for the CDC 3400 computer, by R.D. Hobson, 1967.	$1.00
15.	FORTRAN II program for progressive linear fit of surfaces on a quadratic base using an IBM 1620 computer, by A.J. Cole, C. Jordan, and D.F. Merriam, 1967.	$1.00

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ABSTRACT

The dendrograph has been developed as a two-dimensional diagram for depicting the mutual relationships among a group of objects whose pairwise similarities are given. The construction of the dendrograph is based on results of clustering using the unweighted pair-group method. The resultant hierarchical arrangement reflects both the within-group and the between-group similarity. In order to accentuate the hierarchical group structure, a rule of ordering is used to impart a pyramid shape to the dendrograph. To allow greater flexibility in the computer program for constructing dendrographs, an option is provided for the user to enter either correlation coefficients or distance functions as the measure of pairwise similarity between objects. Two examples are given to illustrate the usefulness of dendrographs in extracting meaning from multivariate data arrays.

INTRODUCTION

The search for pattern characterizes much of what geologists do at present. For the most part, the search is concentrated on the elucidation of multivariate data arrays. In many instances, this has been facilitated through the use of graphic forms of data display. One notable graphic form which has found application in a number of geological problems is the dendrogram. Although devised originally by systematic zoologists to illustrate the hierarchical relationships among organisms based on observed similarities (Mayr, Linsley, and Usinger, 1953, p. 58), the dendrogram is used increasingly by the geologist to depict the mutual relationships for (1) different samples with multiple attributes, and (2) different variables measured for a set of samples. In either situation, what is desired and usually obtained is data compression without significant loss of information. Because the dendrogram has consistently had this effect on the expression of multidimensional data, it has been used widely in geology (Purdy, 1963; Behrens, 1965; Bonham-Carter, 1965; Parks, 1966; Merriam and Sneath, 1966; Valentine and Peddicord, 1967; Rowell, 1967; McCammon, 1968a).

Recently, the unidimensionality of the dendrogram has been extended to two dimensions and the new graphic form has been termed the dendrograph (McCammon, 1968b). The dendrograph has the advantage of displaying similarities within-groups as well as between-groups of ordered objects. This tends to lessen the distortional effect inherent in any portrayal of data in fewer than the original number of dimensions.

The purpose here is to describe a computer program that produces dendrographs. A program is provided which makes it possible for anyone having access to a computer and plotter system to produce his own dendrographs. If a user does not have access to a plotter, it is possible to modify the plotting portion of the program so that the dendrograph can be produced on a printer. The user also can construct dendrographs manually using a straightedge and ruler with results computed by the program. The reason for offering the program with plotter is that with the time-sharing concept coming into vogue with computers, it is likely that most computer centers will soon have plotter capabilities.

To make the program as usable as possible, the assumption was made that the user previously would have calculated or somehow estimated the pairwise similarities of the items which he wished to arrange in hierarchical order. The reason, as numerical taxonomists have found (Sokal and Sneath, 1963, p. 166), is that the problem of deciding the best generalized measure of pairwise similarity for a given set of objects having multiple attributes is unresolved. Furthermore, if the items to be ordered represent different variables measured for a set of samples, a problem as to which measure to choose arises whenever continuous variables are to be correlated with discrete variables. Also, there is a problem if measurements for certain samples are missing. The attitude here is that the user of the program must treat problems as unique to his particular set of data, and the necessary steps are taken beforehand so that the input is a set of pairwise similarity coefficients.

One main option, however, exists in the program. In general, there are two broad classes of measures of pairwise similarity. On one hand, there are values that increase as the two objects compared more resemble one another. The notable example is the correlation coefficient defined for paired observations for two continuous variables. On the other-hand, there are values that decrease the more the two objects resemble one another. The notable example here is the Euclidean distance measure defined for two objects located as points in some coordinate space. Other measures which are analogous to the Euclidean measure but do not possess necessarily full metric properties may be used. Most of these, however, satisfy all but the triangular inequality and are called semimetrics. Both measures are accepted by the program.

During execution, the program operates on the similarity measures as if they were either metric or semimetric quantities. This requires that if correlation-type measures are entered as input, it is necessary to transform them into metric quantities. The transformation used is the arc cosine transformation. In the situation of the correlation coefficient, it transforms the correlation measure into one possessed with all properties of a metric (Blumenthal, 1953, p. 16). The transformation has a similar effect on Jaccard's coefficient. The transformation is diagrammed in Figure 1. In instances where correlation-type coefficients are used, the arc cosine transformation will have the desired effect of converting coefficients to distance-type measures. To ease the subsequent interpretation of the resultant dendrographs generated by correlation type measures, the within-group similarity measures are transformed by taking the cosine and scaling the within-group similarity axis in correlation units.

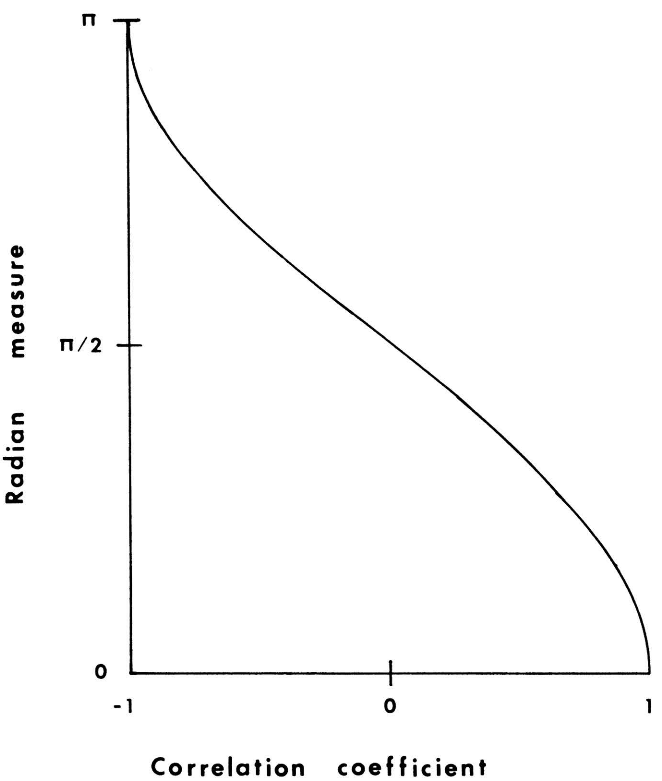

Figure 1.- Arc cosine transformation of correlation coefficient.

The method used for ordering the objects to form the dendrograph has been described previously (McCammon, 1968b). Essentially, the method is the unweighted pair-group method of clustering (UWPGM) modified to provide, in addition, intergroup similarity. The literature on cluster analysis has grown enormously over the past few years and it would be impossible here to review its critical aspects. Also it is not possible to justify the preference of UWPGM over the dozens of other algorithms that have been proposed. Perhaps because it is one of the simplest to implement and because it treats objects equally, UWPGM has gained favor among cluster analysts. The assumptions underlying the various clustering schemes now in circulation however need a closer examination. An important step in this direction has been made recently by Gower (1967). For the present, UWPGM appears as versatile as any of the other methods.

Acknowledgments.- The algorithm used to cluster objects and the computer program for constructing dendrographs on a plotter were developed originally at Gulf Research & Development Company. Special thanks go to Robert O'Hearn and Alexandra Tavlarides for assistance in development of the plotter program. Since that time the program has been rewritten for general use by the junior author. Special thanks go to Michael Goldman, David Rappaport, and Dennis Bartley of the Computer Center staff at the University of Illinois at Chicago Circle for making available the plotting subroutines for the 1627 Calcomp plotter.

PROGRAM OPERATION

Program Dimensions

The program is dimensioned currently to cluster and construct a dendrograph for 200 objects. This number may be increased simply by increasing the dimensions of the appropriate program variables provided the necessary core storage is available at execution. No change in the logic of the program is necessary. To change the dimensions of the program variables, it is necessary only to change their dimensions in the main program. This has been made possible through the use of vector arrays in the calculations. Although the object name array is a matrix, the row dimension is fixed regardless of the number of objects to be clustered.

Order of Input Cards

1. Program control and title card
2. Plotter control card

   Object name array cards
3. Variable format input data card

   Data cards

Program usage

Card 1:

Cols.	
1-4	M = number of objects to be clustered (negative M should be used if correlation measure coefficients are given; positive M should be used if distance measure coefficients are given)
5-8	KLAM = frequency with which intermediate stages of clustering are printed (if KLAM=0, only the final stage of clustering is printed)
9-80	A = problem identification which will appear as the title for the dendrograph

Card 2:

1-10	AENG = length of dendrograph in inches (between-group scale axis) (AENG $\leqslant$ 120.)
11-20	AHI = height of dendrograph in inches (within-group scale axis) (AHI $\leqslant$ 8.)
21-25	FNCR = units that tic marks along the within-group scale axis are apart (if FNCR=0., then, for +M, FNCR=1.0; for -M, FNCR=0.1)

Object name array cards

Columns 1-72 are used to read object names. Sixteen columns are reserved for each object name and there are five names to a card. The order in which object names are given must be the same as the ordering of the pairwise similarity coefficients.

Card 3:

Columns 1-72 are used to read the format for the pairwise similarity coefficients. The order in which the data are read is given as follows.

Data Cards:

The pairwise similarity coefficients are read as a lower triangular matrix with one row of the matrix read at a time. Thus,

$$\begin{matrix} c_{11} & & \\ c_{21} & c_{22} & \\ c_{31} & c_{32} & c_{33} \end{matrix}$$

is read as:

first card: c_{11}

second card: c_{21}, c_{22}

third card: c_{31}, c_{32}, c_{33}

If more than one card is needed to read any row, the data are continued on additional cards until all of the row is read.

The sequence of cards is repeated if a new problem is to be read.

Subprograms

The listed subprograms are provided along with the main program. The user need only check that the card decks are present before program execution.

IPOS	integer compare function
SEARCH	element search function
DENDRO	dendrograph plot subroutine

The following subroutines refer to plotter instructions and must be supplied by the user because plotter arrangements differ from one computer installation to the next. A more complete description follows this listing:

LIMIT	set maximum length
BGNPLT	begin plot
ENDPLT	end plot
SYMBOL	symbol plot
NUMBER	number plot
PLOT	location plot with pen control

The following are excerpts quoted from part of a user's manual prepared by the staff of the Computer Center at the University of Illinois at Chicago. The information contained in the excerpts should be sufficient for the user to prepare his own subroutines compatible with the plotter at his computer installation. The subroutines described here were designed to provide capability for plotting on a

Calcomp 1627 Plotter via the System/360 - 1800 channel adapter.

Introduction

This is a user-oriented program designed to produce an X-Y plot of digital data. These plotting subroutines produce a plot file on the disk which, upon completion, is plotted (via the S/360 - 1800 channel adaptor) on the 1627 Plotter. The plotter converts tabulated digital information into graphic form. Bar charts, flow charts, organization charts, engineering drawings, and maps are among the many graphic forms of data which can be plotted on the 1627 Plotter.

The actual recording is produced by the incremental movement of the pen on the paper surface (y-axis) and/or the paper under the pen (x-axis). The pen is mounted on a carriage that travels horizontally across the paper as viewed from the front of the plotter. The vertical plotting motion is achieved by rotation of the pin feed drum, which also acts as a platen.

The drum and the pen carriage are bi-directional; i.e., the paper moves forward or backward, and the pen moves left or right. Control is also provided to lower or raise the pen to or from the paper surface. The pen remains in the "up" or "down" position until directed to change to the opposite status.

The drum and pen-carriage movements and the pen status are controlled by digits transferred to the 1627 Plotter from the S/360 file. Each digit is decoded into a directional signal and relayed to the 1627 Plotter. Each signal to the Plotter causes a 1/100 inch incremental movement of the pen carriage and/or paper, or pen up or a pen down movement.

The maximum size of the plot in the Y-direction is limited by the length of the drum; on the plotter at this installation, this is 11 inches. The initial origin is established .54 inches above the bottom of the page and the maximum size of the plot in the X-direction is 24 inches; thus the range of values available is:

$$-0.54 < Y < 10.46 \quad \text{and} \quad -4. < X < 20.$$

All actual plotting is done in Background Partitions after the user has ended execution; because it is in background, both the S/360 and the 1800 can proceed in their normal facilities. FORTRAN is the only language required for the use of this program with the addition of the following JCL card:

```
//GO.PLOT18 DD DSN=SYS1.PLOTIN,DISP=(MOD,KEEP)
```

1. Subroutine BGNPLT

Calling Sequence:

CALL BGNPLT (I,EBCDIC)

where:

I	is the number of characters in the EBCDIC String
EBCDIC	is the Plot Identification Literal String. It is to be in quotes or H format. IT MUST BE THE PROGRAMMERS NAME.

purpose: This subroutine must be called before any plotting is to be done. It opens the plot file and gives the user exclusive use of it. The EBCDIC String will be drawn as an identifier .5 inches high along the Y axis. The pen is then moved 4 inches away from the identifier and placed .54 inches away from the edge of the paper.
ALL LIMITS ARE RESET.

example: CALL BGNPLT (9, 'NIXON R M')

Note: $I \leqslant 9$

2. Subroutine ENDPLT

Calling Sequence:
CALL BGNPLT (I,EBCDIC)

purpose: This subroutine closes and releases the user's exclusive control of the plot file. It then positions the pen for the next plot.

Note: ENDPLT must be called before another call to BGNPLT can be made.

3. Subroutine LIMIT

Calling Sequence:
CALL LIMIT (XL)

where: XL: the maximum length of the plot in inches

purpose: This subroutine allows the user to specify the maximum length of the desired plot.

example:
XL = 24.
CALL LIMIT (XL)

Note: $XL \leqslant 120.$

4. Subroutine NUMBER

Calling Sequence:
CALL NUMBER (XA,YA,HEIGHT,(ANUM / NUM),THETA,N)

where:

XA and YA: the X and Y coordinates in inches of the bottom left-hand corner of where you want the number to appear.

HEIGHT: the height in inches of the number or symbol to be plotted. For legibility it should be at least .07.

ANUM
NUM: a floating point number or a floating point variable name where the number is located.

THETA: degrees from the horizontal, which you want the number to be drawn at.

N: >0, the N places to the right of the decimal point will be drawn; =0, the integral portion will be drawn with a decimal point; <-1, no decimal point will be drawn and $\overline{N}$-1 digits will be dropped from the number.

purpose: To supply additional labeling above and beyond that supplied by AXIS.

example:
CALL NUMBER (5.0, 10.0, 14, SUM, 0.0, 0)
For example if SUM = -17.395, then
N = 3 WILL GENERATE -17.395
N = 0 WILL GENERATE -17.
N =-1WILL GENERATE -17

5. Subroutine SYMBOL

Calling Sequence:
CALL SYMBOL (XA,YA,HEIGHT,EBCDIC,THETA,NS)

where:

XA and YA: coordinates of lower left corner of first symbol to be plotted, except in the case of a negative NS, where they are the coordinates of the center of the symbol.

HEIGHT: same as in NUMBER.

EBCDIC: a Hollerith string or an address of a Hollerith string to be plotted or a number specifying a centering symbol depending on the value of NS.

THETA: same as in NUMBER

NS: If NS > 0, NS is the number of EBCDIC characters to be plotted from the EBCDIC string specified; if NS < 0, SYMBOL will expect in EBCDIC an integer specifying a centering symbol (see symbol list); if NS = -1, the pen will be raised before moving to the new location; if NS = -2, the pen will be lowered.

purpose: To provide labeling of alphanumeric information on the plotter.

example: CALL SYMBOL (1.0, 8.0, 0.25,5HLABEL, 0.0, 5)
This will plot LABEL with the left-hand corner of the L starting at (1.0, 8.0) relative to the origin.

CALL SYMBOL (1.0, 7.0, 0.14, 4, 0.0,-1)
This will draw an X centered at (1.0, 7.0) relative to the origin.

6. Subroutine PLOT

This subroutine has the entry point PLOT. Its primary purpose is the translation of data into plotter format and the writing of the necessary plot file. It also monitors the pen position and will terminate the computer run if the pen goes out of the range of these limits. The limits are:

+X = XL inches; see LIMIT for XL

-X = 4 inches

+Y = 10.46 inches

-Y = .54 inch

Entry PLOT

Calling Sequence:
CALL PLOT (X,Y,IND)

where:

X=x-coordinate in inches of new pen position relative to origin

Y=y-coordinate in inches of new pen position relative to origin

IND=0 or ±1 no change in vertical position of pen

±2 put pen down

±3 put pen up

999 dump the current buffer

If the value of IND is negative, after the pen has moved to the new position, zeros are stored as its old position. This effectively establishes the new position as a new reference point (origin) for subsequent plotting. If IND is not equal to any of the above, it is automatically set equal to 3.

purpose: To move the pen from current position to a new position. It is the basic entry for plotting.

example: CALL PLOT (10.0, 10.0, 2) This command will draw a line from where the pen currently is to the coordinates (10.0, 10.0) with the pen down.

EXAMPLES

To see how dendrographs are produced and how to interpret output from the program, let us consider two examples. The first example is based on data reported by Valentine (1966) in a study of latitude changes in the molluscan shelf fauna of western North America between Baja California, and Point Barrow, Alaska. The data on which the study was based consisted of faunal lists compiled over many years for shelled benthonic Mollusca of the classes Pelecypoda and Gastropoda recorded from depths less than 183 m. Range data were compiled for some 2077 forms. The purpose of the study was to define major marine provinces based on the fauna and to explain the observed latitude changes. Valentine employed numerical methods to analyze the data. For each degree of latitude, a faunal list was prepared. The latitude ranged from 23° N to 72° N, and for each pair, a measure of faunal resemblance was calculated. The measure used was Jaccard's coefficient defined for each pair of latitudes as the number of common species divided by the total number of different species reported for the two latitudes. The matrix formed from the set of pairwise latitude similarity coefficients constitutes the input to the dendrograph program. The input is shown in Table 1 which is the first part of the output from the program. Note that the data are entered as a lower triangular matrix. Because Jaccard's coefficient is a correlation-type measure, it is necessary to transform the coefficients by taking the arc cosine which transforms the coefficients into distance measures. The transformed input is shown next. The data now are ready for clustering. In this situation, we are not interested in the intermediate stages of clustering and thus the next output step gives the Final Results of clustering (Table 1). It is from the Final Results that we construct the dendrograph. The results contain five columns which we can interpret (starting from the left). The first column labelled NAME contains the name given to each object. In our example, the objects refer to latitudes, and they are represented by numbers ranging from 72 to 23. The numbers appear displaced to the right because there are sixteen spaces allowed for each object name and the numbers have been right adjusted. The next column labelled NP lists the objects by number in the order they appear in the dendrograph. The objects are numbered in sequence in the order they appear in the input data. For this example, the latitudes were entered starting from latitude 72N to 23N. Thus, corresponding to number 1 in column NP is latitude 72 in column NAME, corresponding to 2 is 71, 3 is 70, etc., to 50 which is latitude 23. The objects are arranged in this order in the dendrograph. The next column labelled ORDER gives the order in which the objects are joined, in this instance, latitudes, are latitudes 47 and 46. The next two are latitudes 68 and 69. The next combines latitude 67 with the previously joined latitudes 68 and 69. This is continued until all latitudes have been combined into a single group. To construct the dendrograph, the connecting lines are drawn to form groupings in the order indicated in column ORDER. The last two columns contain the distances involved in connecting the objects to form the hierarchical groupings. The first is the within-group distance and the second the between-group distance. The latter states the distance apart between two adjacent objects. The former gives the distance from the origin where two adjacent groups are connected. Because all connections are made between two objects and only one connection is possible between objects, the distances are given between those two objects for which the connecting line is drawn. In this manner, the dendrograph is constructed step-by-step in the order that the groups are formed in the clustering process. Figure 2 contains the dendrograph produced on the plotter for the data described. The latitude

Table 1.- Results of cluster analysis for Pacific Coast molluscan data taken from Valentine (1966).

LATITUDE PACIFIC COAST MOLLUSCAN RANGES

NUMBER OF GROUPS IS 50

ORIGINAL INPUT MATRIX (LOWER HALF BY ROWS)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1.0000															
2	0.8500	1.0000														
3	0.8200	0.9500	1.0000													
4	0.8200	0.9500	0.9600	1.0000												
5	0.8100	0.9500	0.9600	0.9900	1.0000											
6	0.8000	0.9300	0.9400	0.9800	0.9700	1.0000										
7	0.7500	0.8700	0.8800	0.9200	0.9100	0.9400	1.0000									
8	0.7100	0.8200	0.8300	0.8600	0.8500	0.8800	0.9200	1.0000								
9	0.6400	0.7300	0.7400	0.7700	0.7600	0.7900	0.8200	0.8900	1.0000							
10	0.6200	0.7200	0.7300	0.7500	0.7500	0.7700	0.8000	0.8700	0.9300	1.0000						
11	0.6200	0.7000	0.7100	0.7400	0.7300	0.7500	0.7900	0.8600	0.9100	0.9700	1.0000					
12	0.5700	0.6400	0.6500	0.6700	0.6700	0.6900	0.7200	0.7800	0.8400	0.8900	0.9100	1.0000				
13	0.4500	0.5000	0.5000	0.5200	0.5200	0.5300	0.5600	0.6000	0.6500	0.6900	0.7000	0.7600	1.0000			
14	0.4000	0.4600	0.4600	0.4700	0.4700	0.4800	0.5100	0.5500	0.6000	0.6400	0.6500	0.7100	0.8500	1.0000		
15	0.3500	0.3900	0.4000	0.4000	0.4000	0.4100	0.4400	0.4700	0.5100	0.5400	0.5500	0.6000	0.7200	0.8300	1.0000	
16	0.2900	0.3200	0.3300	0.3400	0.3400	0.3400	0.3600	0.3900	0.4300	0.4500	0.4500	0.5000	0.6000	0.6800	0.7600	1.0000
17	0.2400	0.2700	0.2700	0.2800	0.2800	0.2900	0.3000	0.3200	0.3500	0.3700	0.3800	0.4100	0.5000	0.5700	0.6200	0.7500
18	0.1700	0.1900	0.2000	0.2000	0.2000	0.2000	0.2200	0.2300	0.2500	0.2600	0.2700	0.2900	0.3700	0.4100	0.4600	0.5600
19	0.1300	0.1400	0.1500	0.1500	0.1500	0.1500	0.1600	0.1700	0.1800	0.1900	0.2000	0.2200	0.2800	0.3200	0.3600	0.4500
20	0.1200	0.1400	0.1400	0.1400	0.1400	0.1400	0.1500	0.1600	0.1700	0.1800	0.1800	0.2100	0.2700	0.3100	0.3500	0.4300
21	0.1200	0.1300	0.1400	0.1400	0.1400	0.1400	0.1500	0.1600	0.1700	0.1800	0.1800	0.2000	0.2600	0.3000	0.3400	0.4200
22	0.1100	0.1300	0.1300	0.1400	0.1400	0.1400	0.1500	0.1600	0.1600	0.1700	0.1700	0.2000	0.2500	0.2900	0.3200	0.4100
23	0.1100	0.1300	0.1300	0.1300	0.1300	0.1300	0.1400	0.1500	0.1600	0.1600	0.1700	0.1900	0.2500	0.2800	0.3200	0.4000
24	0.1000	0.1200	0.1200	0.1200	0.1200	0.1200	0.1300	0.1400	0.1500	0.1500	0.1600	0.1800	0.2300	0.2600	0.3000	0.3700

	17	18	19	20	21	22	23	24
17	1.0000							
18	0.6800	1.0000						
19	0.5600	0.7300	1.0000					
20	0.5100	0.6800	0.9100	1.0000				
21	0.5000	0.6600	0.8800	0.9500	1.0000			
22	0.4900	0.6500	0.8500	0.9300	0.9400	1.0000		
23	0.4800	0.6300	0.8200	0.8900	0.9100	0.9600	1.0000	
24	0.4500	0.5900	0.7700	0.8300	0.8400	0.8800	0.9100	1.0000

```
25   0.0900  0.1000  0.1000  0.1100  0.1100  0.1100  0.1100  0.1200  0.1300  0.1300  0.1400  0.1600  0.2000  0.2300  0.2600  0.3300
 0.3900  0.5100  0.6400  0.6800  0.6900  0.7200  0.7400  0.7800  1.0000

26   0.0500  0.0600  0.0600  0.0600  0.0600  0.0600  0.0700  0.0800  0.0800  0.0800  0.0900  0.1100  0.1500  0.1700  0.2100  0.2700
 0.3200  0.4200  0.5400  0.5700  0.5800  0.6100  0.6300  0.6700  0.7600  1.0000

27   0.0500  0.0600  0.0600  0.0600  0.0600  0.0600  0.0700  0.0800  0.0800  0.0800  0.0800  0.1100  0.1500  0.1700  0.2000  0.2700
 0.3100  0.4100  0.5300  0.5600  0.5700  0.6000  0.6200  0.6600  0.7500  0.9900  1.0000

28   0.0500  0.0600  0.0600  0.0600  0.0600  0.0600  0.0600  0.0700  0.0800  0.0800  0.0800  0.1000  0.1400  0.1700  0.2000  0.2600
 0.3100  0.4100  0.5200  0.5500  0.5600  0.5900  0.6100  0.6500  0.7400  0.9700  0.9700  1.0000

29   0.0500  0.0600  0.0600  0.0600  0.0600  0.0600  0.0700  0.0800  0.0800  0.0800  0.0800  0.1100  0.1500  0.1700  0.2000  0.2600
 0.3100  0.4000  0.5100  0.5500  0.5600  0.5800  0.6000  0.6400  0.7300  0.9500  0.9600  0.9800  1.0000

30   0.0500  0.0600  0.0600  0.0600  0.0600  0.0600  0.0600  0.0700  0.0700  0.0800  0.0800  0.1000  0.1400  0.1600  0.1900  0.2500
 0.3000  0.3900  0.5000  0.5300  0.5400  0.5700  0.5900  0.6200  0.7100  0.9300  0.9300  0.9500  0.9700  1.0000

31   0.0400  0.0500  0.0500  0.0500  0.0500  0.0500  0.0600  0.0700  0.0700  0.0700  0.0700  0.1000  0.1400  0.1600  0.1900  0.2500
 0.2900  0.3800  0.4800  0.5100  0.5200  0.5400  0.5600  0.6000  0.6800  0.8800  0.8900  0.9000  0.9200  0.9400  1.0000

32   0.0400  0.0500  0.0500  0.0500  0.0500  0.0500  0.0600  0.0700  0.0700  0.0700  0.0700  0.1000  0.1300  0.1600  0.1900  0.2500
 0.2800  0.3700  0.4700  0.5000  0.5100  0.5300  0.5500  0.5900  0.6700  0.8600  0.8700  0.8900  0.9000  0.9200  0.9800  1.0000

33   0.0400  0.0500  0.0500  0.0500  0.0500  0.0500  0.0600  0.0600  0.0700  0.0700  0.0700  0.0900  0.1300  0.1500  0.1800  0.2400
 0.2800  0.3600  0.4600  0.4900  0.5000  0.5200  0.5400  0.5700  0.6600  0.8500  0.8500  0.8700  0.8800  0.9100  0.9500  0.9700
 1.0000

34   0.0400  0.0500  0.0500  0.0500  0.0500  0.0500  0.0500  0.0600  0.0700  0.0700  0.0700  0.0900  0.1300  0.1500  0.1800  0.2400
 0.2800  0.3600  0.4600  0.4900  0.4900  0.5100  0.5300  0.5700  0.6500  0.8300  0.8300  0.8500  0.8600  0.8900  0.9300  0.9500
 0.9800  1.0000

35   0.0400  0.0500  0.0500  0.0500  0.0500  0.0500  0.0500  0.0600  0.0600  0.0600  0.0600  0.0900  0.1200  0.1400  0.1700  0.2200
 0.2500  0.3300  0.4200  0.4400  0.4500  0.4700  0.4800  0.5100  0.5900  0.7300  0.7400  0.7600  0.7600  0.7900  0.8200  0.8400
 0.8600  0.8800  1.0000

36   0.0300  0.0300  0.0300  0.0300  0.0300  0.0300  0.0400  0.0400  0.0400  0.0500  0.0500  0.0600  0.0900  0.1000  0.1200  0.1600
 0.1900  0.2500  0.3000  0.3100  0.3200  0.3300  0.3400  0.3600  0.4200  0.4900  0.4900  0.5000  0.5000  0.5200  0.5400  0.5500
 0.5600  0.5700  0.6300  1.0000

37   0.0200  0.0300  0.0300  0.0300  0.0300  0.0300  0.0300  0.0300  0.0400  0.0400  0.0400  0.0500  0.0800  0.0900  0.1100  0.1500
 0.1700  0.2300  0.2800  0.2900  0.3000  0.3000  0.3200  0.3400  0.3900  0.4600  0.4700  0.4800  0.4800  0.4900  0.5200  0.5300
 0.5400  0.5500  0.6100  0.8600  1.0000

38   0.0200  0.0300  0.0300  0.0300  0.0300  0.0300  0.0300  0.0300  0.0400  0.0400  0.0400  0.0500  0.0800  0.0900  0.1100  0.1400
 0.1700  0.2200  0.2700  0.2800  0.2900  0.3000  0.3100  0.3300  0.3800  0.4500  0.4500  0.4600  0.4700  0.4800  0.5100  0.5100
 0.5200  0.5400  0.5900  0.8400  0.9600  1.0000

39   0.0100  0.0200  0.0200  0.0200  0.0200  0.0200  0.0200  0.0200  0.0200  0.0200  0.0200  0.0300  0.0500  0.0600  0.0700  0.1000
 0.1100  0.1500  0.1800  0.1900  0.1900  0.2000  0.2000  0.2200  0.2600  0.2900  0.2900  0.3000  0.3000  0.3000  0.3200  0.3300
 0.3300  0.3400  0.3700  0.5400  0.5900  0.6000  1.0000

40   0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0200  0.0200  0.0200  0.0200  0.0200  0.0300  0.0400  0.0500  0.0700  0.0900
 0.1000  0.1300  0.1600  0.1600  0.1700  0.1700  0.1800  0.1900  0.2300  0.2600  0.2600  0.2600  0.2600  0.2700  0.2800  0.2900
 0.2900  0.3000  0.3300  0.4700  0.5100  0.5200  0.7400  1.0000

41   0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0200  0.0400  0.0400  0.0500  0.0700
 0.0700  0.1100  0.1300  0.1300  0.1300  0.1400  0.1400  0.1500  0.1800  0.2100  0.2100  0.2100  0.2200  0.2200  0.2400  0.2400
 0.2500  0.2500  0.2800  0.3800  0.4200  0.4300  0.5500  0.6800  1.0000

42   0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0200  0.0300  0.0400  0.0500  0.0600
```

```
0.0700  0.0900  0.1100  0.1200  0.1200  0.1200  0.1300  0.1400  0.1700  0.1900  0.1900  0.2000  0.2000  0.2000  0.2200  0.2300
0.2300  0.2300  0.2600  0.3500  0.3900  0.3900  0.4900  0.6100  0.8600  1.0000

43  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0200  0.0300  0.0400  0.0500  0.0600
0.0700  0.0900  0.1100  0.1200  0.1200  0.1200  0.1300  0.1400  0.1600  0.1900  0.1900  0.2000  0.2000  0.2000  0.2200  0.2200
0.2300  0.2300  0.2600  0.3500  0.3800  0.3900  0.4800  0.6000  0.8400  0.9800  1.0000

44  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0200  0.0300  0.0400  0.0500  0.0600
0.0600  0.0800  0.1000  0.1100  0.1100  0.1100  0.1200  0.1200  0.1500  0.1800  0.1800  0.1800  0.1900  0.1900  0.2000  0.2100
0.2100  0.2100  0.2300  0.3100  0.3400  0.3400  0.4200  0.5300  0.7400  0.8700  0.8800  1.0000

45  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0200  0.0300  0.0400  0.0400  0.0500
0.0600  0.0800  0.0900  0.1000  0.1000  0.1000  0.1100  0.1100  0.1400  0.1600  0.1600  0.1600  0.1700  0.1700  0.1800  0.1900
0.1900  0.1900  0.2100  0.2800  0.3100  0.3100  0.3900  0.4800  0.6500  0.7500  0.7600  0.8500  1.0000

46  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0200  0.0300  0.0300  0.0400  0.0400
0.0500  0.0700  0.0800  0.0900  0.0900  0.0900  0.0900  0.1000  0.1200  0.1400  0.1400  0.1400  0.1400  0.1500  0.1600  0.1600
0.1600  0.1700  0.1800  0.2400  0.2600  0.2600  0.3400  0.4100  0.5600  0.6400  0.6500  0.7200  0.8100  1.0000

47  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0200  0.0200  0.0300  0.0400  0.0400
0.0400  0.0600  0.0800  0.0800  0.0800  0.0800  0.0900  0.1000  0.1200  0.1300  0.1300  0.1400  0.1400  0.1400  0.1500  0.1600
0.1600  0.1600  0.1800  0.2100  0.2400  0.2400  0.2900  0.3600  0.4900  0.5600  0.5700  0.6300  0.7300  0.8400  1.0000

48  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0200  0.0200  0.0300  0.0300
0.0400  0.0500  0.0700  0.0700  0.0700  0.0700  0.0700  0.0800  0.1000  0.1100  0.1100  0.1200  0.1200  0.1200  0.1300  0.1300
0.1300  0.1400  0.1500  0.1800  0.2000  0.2000  0.2500  0.3100  0.4000  0.4500  0.4500  0.4900  0.5700  0.6600  0.7500  1.0000

49  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0200  0.0200  0.0300  0.0300
0.0300  0.0500  0.0600  0.0600  0.0600  0.0600  0.0700  0.0700  0.0900  0.1000  0.1000  0.1100  0.1100  0.1100  0.1200  0.1200
0.1200  0.1300  0.1300  0.1700  0.1900  0.1900  0.2400  0.2900  0.3800  0.4200  0.4300  0.4700  0.5500  0.6400  0.7300  0.8900
1.0000

50  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0100  0.0200  0.0200  0.0300  0.0300
0.0300  0.0400  0.0600  0.0600  0.0600  0.0600  0.0600  0.0700  0.0800  0.0900  0.1000  0.1000  0.1000  0.1000  0.1100  0.1200
0.1200  0.1200  0.1300  0.1600  0.1800  0.1800  0.2300  0.2800  0.3700  0.4100  0.4200  0.4600  0.5400  0.6200  0.7200  0.8700
0.9800  1.0000
```

TRANSFORMED INPUT MATRIX

1	0.0																								
2	C.5548	C.C																							
3	0.6094	0.3176	C.C																						
4	C.6C94	0.3176	0.2838	0.C																					
5	C.6266	0.3176	C.2838	0.1415	0.0																				
6	0.6435	C.3764	C.3482	C.2003	0.2456	0.0																			
7	0.7227	C.5156	C.4949	0.4027	0.4275	0.3482	0.0																		
8	0.7813	C.6C94	C.5917	0.5355	0.5548	0.4949	0.4027	0.0																	
9	0.8763	0.7525	0.7377	0.6920	0.7075	0.6600	0.6094	0.4735	0.0																
10	C.9021	C.767C	0.7525	0.7227	0.7227	0.6920	0.6435	0.5156	0.3764	0.0															
11	0.9021	C.7954	0.7813	0.7377	0.7525	0.7227	0.6600	0.5355	0.4275	0.2456	0.0														
12	C.9643	0.8763	0.8632	0.8366	0.8366	0.8093	0.7670	0.6761	0.5735	0.4735	0.4275	0.0													
13	1.104C	1.0472	1.0472	1.0239	1.0239	1.0122	0.9764	0.9273	0.8632	0.8093	0.7954	0.7075	0.0												
14	1.1593	1.C928	1.0928	1.0815	1.0815	1.0701	1.0356	0.9884	0.9273	0.8763	0.8632	0.7813	0.5548	0.0											
15	1.2132	1.17C2	1.1593	1.1593	1.1593	1.1483	1.1152	1.0815	1.0356	1.0004	0.9884	0.9273	0.7670	0.5917	0.0										
16	1.2766	1.2451	1.2345	1.2239	1.2239	1.2239	1.2025	1.1702	1.1263	1.1C40	1.1040	1.0472	0.9273	0.8230	0.7075	0.0									
17	1.3284	1.2974	1.2974	1.2870	1.2870	1.2766	1.2661	1.2451	1.2132	1.1918	1.1810	1.1483	1.0472	0.9643	0.9021	0.7227	C.C								
18	1.40CC	1.3796	1.3694	1.3694	1.3694	1.3694	1.3490	1.3387	1.3181	1.3078	1.2974	1.2766	1.1918	1.1483	1.0928	0.9764	0.8230	0.0							
19	1.4404	1.4303	1.4202	1.4202	1.4202	1.4202	1.4101	1.4000	1.3898	1.3796	1.3694	1.3490	1.2870	1.2451	1.2025	1.1040	C.9764	0.7525	C.C						
2C	1.4505	1.4303	1.4303	1.4303	1.4303	1.4303	1.4202	1.4101	1.4000	1.3898	1.3898	1.3592	1.2974	1.2556	1.2132	1.1263	1.0356	C.8230	C.4275	C.0					
21	1.45C5	1.4404	1.4303	1.4303	1.4303	1.4303	1.4202	1.4101	1.4000	1.3898	1.3898	1.3694	1.3078	1.2661	1.2239	1.1374	1.C472	C.85CC	C.4949	0.3176	C.0				
22	1.46C6	1.44C4	1.4404	1.4303	1.4303	1.4303	1.4202	1.4101	1.4101	1.4000	1.4000	1.3694	1.3181	1.2766	1.2451	1.1483	1.0587	C.8632	C.5548	0.3764	0.3482	0.0			
23	1.4606	1.44C4	1.4404	1.4404	1.4404	1.4404	1.4303	1.4202	1.4101	1.4101	1.4000	1.3796	1.3181	1.2870	1.2451	1.1593	1.0701	C.8892	C.6094	0.4735	0.4275	0.2838	0.0		
24	1.4706	1.4505	1.4505	1.4505	1.4505	1.4505	1.4404	1.4303	1.4202	1.4202	1.4101	1.3898	1.3387	1.3078	1.2661	1.1918	1.1040	0.9397	C.6920	0.5917	0.5735	0.4949	0.4275	0.0	
25	1.4807	1.47C6	1.4706	1.4606	1.4606	1.4606	1.4606	1.4505	1.4404	1.4404	1.4303	1.4101	1.3694	1.3387	1.3078	1.2345	1.1702	1.0356	C.8763	0.8230	0.8093	0.7670	0.7377	0.6761	0.0

```
26   1.5208  1.5108  1.5108  1.5108  1.5108  1.5108  1.5007  1.4907  1.4907  1.4907  1.4807  1.4606  1.4202  1.4000  1.3592  1.2974
 1.2451  1.1374  1.0004  0.9643  0.9521  0.9147  0.8892  0.8366  0.7075  0.0

27   1.5208  1.5108  1.5108  1.5108  1.5108  1.5108  1.5007  1.4907  1.4907  1.4907  1.4907  1.4606  1.4202  1.4000  1.3694  1.2974
 1.2556  1.1483  1.0122  0.9764  0.9643  0.9273  0.9021  0.8500  0.7227  0.1415  0.0

28   1.5208  1.5108  1.5108  1.5108  1.5108  1.5108  1.5108  1.5007  1.4907  1.4907  1.4907  1.4706  1.4303  1.4000  1.3694  1.3078
 1.2556  1.1483  1.0239  0.9884  0.9764  0.9397  0.9147  0.8632  0.7377  0.2456  0.2456  0.0

29   1.5208  1.5108  1.5108  1.5108  1.5108  1.5108  1.5007  1.4907  1.4907  1.4907  1.4907  1.4606  1.4202  1.4000  1.3694  1.3078
 1.2556  1.1593  1.0356  0.9884  0.9764  0.9521  0.9273  0.8763  0.7525  0.3176  0.2838  0.2003  0.0

30   1.5208  1.5108  1.5108  1.5108  1.5108  1.5108  1.5108  1.5007  1.5007  1.4907  1.4907  1.4706  1.4303  1.4101  1.3796  1.3181
 1.2661  1.1702  1.0472  1.0122  1.0004  0.9643  0.9397  0.9021  0.7813  0.3764  0.3764  0.3176  0.2456  0.0

31   1.5308  1.5208  1.5208  1.5208  1.5208  1.5208  1.5108  1.5007  1.5007  1.5007  1.5007  1.4706  1.4303  1.4101  1.3796  1.3181
 1.2766  1.1810  1.0701  1.0356  1.0239  1.0004  0.9764  0.9273  0.8230  0.4949  0.4735  0.4510  0.4027  0.3482  0.0

32   1.5308  1.5208  1.5208  1.5208  1.5208  1.5208  1.5108  1.5007  1.5007  1.5007  1.5007  1.4706  1.4404  1.4101  1.3796  1.3181
 1.2870  1.1918  1.0815  1.0472  1.0356  1.0122  0.9884  0.9397  0.8366  0.5355  0.5156  0.4735  0.4510  0.4027  0.2003  0.0

33   1.5308  1.5208  1.5208  1.5208  1.5208  1.5208  1.5108  1.5108  1.5007  1.5007  1.5007  1.4807  1.4404  1.4202  1.3898  1.3284
 1.2870  1.2025  1.0928  1.0587  1.0472  1.0239  1.0004  0.9643  0.8500  0.5548  0.5548  0.5156  0.4949  0.4275  0.3176  0.2456
 0.0

34   1.5308  1.5208  1.5208  1.5208  1.5208  1.5208  1.5208  1.5108  1.5007  1.5007  1.5007  1.4807  1.4404  1.4202  1.3898  1.3284
 1.2870  1.2025  1.0928  1.0587  1.0587  1.0356  1.0122  0.9643  0.8632  0.5917  0.5917  0.5548  0.5355  0.4735  0.3764  0.3176
 0.2003  0.0

35   1.5308  1.5208  1.5208  1.5208  1.5208  1.5208  1.5208  1.5108  1.5108  1.5108  1.5108  1.4807  1.4505  1.4303  1.4000  1.3490
 1.3181  1.2345  1.1374  1.1152  1.1040  1.0815  1.0701  1.0356  0.9397  0.7525  0.7377  0.7075  0.7075  0.6600  0.6094  0.5735
 0.5355  0.4949  0.0

36   1.5408  1.5408  1.5408  1.5408  1.5408  1.5408  1.5308  1.5308  1.5308  1.5208  1.5208  1.5108  1.4807  1.4706  1.4505  1.4101
 1.3796  1.3181  1.2661  1.2556  1.2451  1.2345  1.2239  1.2025  1.1374  1.0587  1.0587  1.0472  1.0472  1.0239  1.0004  0.9884
 0.9764  0.9643  0.8892  0.0

37   1.5508  1.5408  1.5408  1.5408  1.5408  1.5408  1.5408  1.5408  1.5308  1.5308  1.5308  1.5208  1.4907  1.4807  1.4606  1.4202
 1.4000  1.3387  1.2870  1.2766  1.2661  1.2661  1.2451  1.2239  1.1702  1.0928  1.0815  1.0701  1.0701  1.0587  1.0239  1.0122
 1.0004  0.9884  0.9147  0.5355  0.0

38   1.5508  1.5408  1.5408  1.5408  1.5408  1.5408  1.5408  1.5408  1.5408  1.5308  1.5308  1.5208  1.4907  1.4807  1.4606  1.4303
 1.4000  1.3490  1.2974  1.2870  1.2766  1.2661  1.2556  1.2345  1.1810  1.1040  1.1040  1.0928  1.0815  1.0701  1.0356  1.0356
 1.0239  1.0004  0.9397  0.5735  0.2838  0.0

39   1.5608  1.5508  1.5508  1.5508  1.5508  1.5508  1.5508  1.5508  1.5508  1.5508  1.5508  1.5408  1.5208  1.5108  1.5007  1.4706
 1.4606  1.4202  1.3898  1.3796  1.3796  1.3694  1.3694  1.3490  1.3078  1.2766  1.2766  1.2661  1.2661  1.2661  1.2451  1.2345
 1.2345  1.2239  1.1918  1.0004  0.9397  0.9273  0.0

40   1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5508  1.5508  1.5508  1.5508  1.5508  1.5408  1.5308  1.5208  1.5007  1.4807
 1.4706  1.4404  1.4101  1.4101  1.4000  1.4000  1.3898  1.3796  1.3387  1.3078  1.3078  1.3078  1.3078  1.2974  1.2870  1.2766
 1.2766  1.2661  1.2345  1.0815  1.0356  1.0239  0.7377  0.0

41   1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5508  1.5308  1.5308  1.5208  1.5007
 1.5007  1.4606  1.4404  1.4404  1.4404  1.4303  1.4303  1.4202  1.3898  1.3592  1.3592  1.3592  1.3490  1.3490  1.3284  1.3284
 1.3181  1.3181  1.2870  1.1810  1.1374  1.1263  0.9884  0.8230  0.0

42   1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5508  1.5408  1.5308  1.5208  1.5108
 1.5007  1.4807  1.4606  1.4505  1.4505  1.4505  1.4404  1.4303  1.4000  1.3796  1.3796  1.3694  1.3694  1.3694  1.3490  1.3387
 1.3387  1.3387  1.3078  1.2132  1.1702  1.1702  1.0587  0.9147  0.5355  0.0
```

```
43   1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5508  1.5408  1.5308  1.5208  1.5108
 1.5007  1.4807  1.4606  1.4505  1.4505  1.4505  1.4404  1.4303  1.4101  1.3796  1.3796  1.3694  1.3694  1.3694  1.3490  1.3490
 1.3387  1.3387  1.3078  1.2132  1.1810  1.1702  1.0701  0.9273  0.5735  0.2003  0.0

44   1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5508  1.5408  1.5308  1.5208  1.5108
 1.5108  1.4907  1.4706  1.4606  1.4606  1.4606  1.4505  1.4505  1.4202  1.3898  1.3898  1.3898  1.3796  1.3796  1.3694  1.3592
 1.3592  1.3592  1.3387  1.2556  1.2239  1.2239  1.1374  1.0122  0.7377  0.5156  0.4949  0.0

45   1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5508  1.5408  1.5308  1.5308  1.5208
 1.5108  1.4907  1.4807  1.4706  1.4706  1.4706  1.4606  1.4606  1.4303  1.4101  1.4101  1.4101  1.4000  1.4000  1.3898  1.3796
 1.3796  1.3796  1.3592  1.2870  1.2556  1.2556  1.1702  1.0701  0.8632  0.7227  0.7075  0.5548  0.0

46   1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5508  1.5408  1.5408  1.5308  1.5308
 1.5208  1.5007  1.4907  1.4807  1.4807  1.4807  1.4807  1.4706  1.4505  1.4303  1.4303  1.4303  1.4303  1.4202  1.4101  1.4101
 1.4101  1.4000  1.3898  1.3284  1.3078  1.3078  1.2239  1.1483  0.9764  0.8763  0.8632  0.7670  0.6266  0.0

47   1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5508  1.5508  1.5408  1.5308  1.5308
 1.5308  1.5108  1.4907  1.4907  1.4907  1.4907  1.4807  1.4706  1.4505  1.4404  1.4404  1.4303  1.4303  1.4303  1.4202  1.4101
 1.4101  1.4101  1.3898  1.3592  1.3284  1.3284  1.2766  1.2025  1.0587  0.9764  0.9643  0.8892  0.7525  0.5735  0.0

48   1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5508  1.5508  1.5408  1.5408
 1.5308  1.5208  1.5007  1.5007  1.5007  1.5007  1.5007  1.4907  1.4706  1.4606  1.4606  1.4505  1.4505  1.4505  1.4404  1.4404
 1.4404  1.4303  1.4202  1.3898  1.3694  1.3694  1.3181  1.2556  1.1593  1.1040  1.1040  1.0587  0.9643  0.8500  0.7227  0.0

49   1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5508  1.5508  1.5408  1.5408
 1.5408  1.5208  1.5108  1.5108  1.5108  1.5108  1.5007  1.5007  1.4807  1.4706  1.4706  1.4606  1.4606  1.4606  1.4505  1.4505
 1.4505  1.4404  1.4404  1.4000  1.3796  1.3796  1.3284  1.2766  1.1810  1.1374  1.1263  1.0815  0.9884  0.8763  0.7525  0.4735
 0.0

50   1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5608  1.5508  1.5508  1.5408  1.5408
 1.5408  1.5308  1.5108  1.5108  1.5108  1.5108  1.5108  1.5007  1.4907  1.4807  1.4706  1.4706  1.4706  1.4706  1.4606  1.4505
 1.4505  1.4505  1.4404  1.4101  1.3898  1.3898  1.3387  1.2870  1.1918  1.1483  1.1374  1.0928  1.0004  0.9021  0.7670  0.5156
 0.2003  0.0
```

FINAL RESULTS

NAME	NP	ORDER	WGR	BGR
47	26			
		1	0.1415	0.1415
46	27			
		9	0.2391	0.2731
45	28			
		8	0.2003	0.2003
44	29			
		12	0.2750	0.3290
43	30			
		26	0.4301	0.7403
48	25			
		32	0.5099	0.5907
38	35			
		21	0.3871	0.5533
39	34			
		6	0.2003	0.2003
40	33			
		13	0.2763	0.3143
41	32			
		7	0.2003	0.2003
42	31			
		38	0.6338	1.2535
34	39			
		44	0.7661	1.0336
37	36			
		29	0.4643	0.5545
35	38			
		15	0.2838	0.2838
36	37			
		47	0.9197	1.1077
56	17			
		42	0.6902	1.0164
55	18			
		35	0.5815	0.8529
49	24			
		30	0.4729	0.5559
54	19			
		27	0.4314	0.5217
50	23			
		16	0.2838	0.2838
51	22			
		20	0.3711	0.4064
52	21			
		17	0.3176	0.3176
53	20			
		48	1.1305	1.3947
24	49			
		4	0.2003	0.2003
23	50			
		23	0.3965	0.4945
25	48			

		34	0.5719	0.7474
26	47			
		40	0.6633	0.8005
27	46			
		46	0.8851	1.0465
33	40			
		43	0.7102	0.9495
28	45			
		36	0.5906	0.7121
32	41			
		31	0.5096	0.6156
29	44			
		24	0.4036	0.5053
30	43			
		5	0.2003	0.2003
31	42			
		49	1.2482	1.4719
60	13			
		33	0.5548	0.5548
59	14			
		39	0.6378	0.6793
58	15			
		45	0.7844	1.0001
57	16			
		41	0.6821	1.1818
63	10			
		10	0.2456	0.2456
62	11			
		19	0.3498	0.4019
64	9			
		25	0.4207	0.4915
61	12			
		37	0.5913	0.7420
72	1			
		28	0.4556	0.6497
65	8			
		22	0.3910	0.5315
66	7			
		18	0.3347	0.4378
71	2			
		14	0.2832	0.3323
70	3			
		11	0.2505	0.3053
67	6			
		3	0.1958	0.2230
68	5			
		2	0.1415	0.1415
69	4			

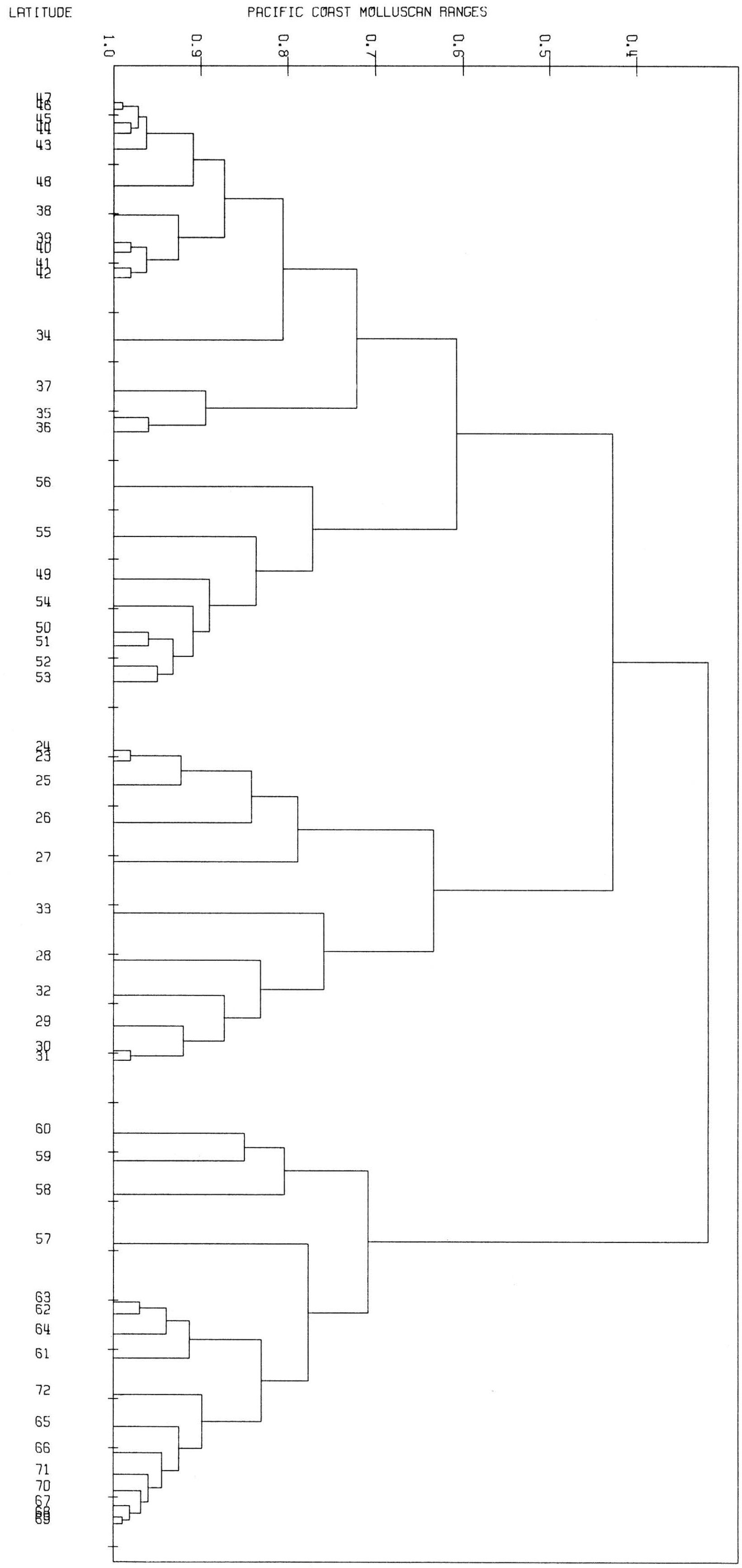

Figure 2. - Dendrograph depicting hierarchical pattern of Pacific Coast molluscan ranges.

pattern that emerges is distinct and shows clearly how a system of marine provinces could be proposed. Two points should be made. The pyramid shape of the dendrograph is not by accident. As described in the original paper (McCammon, 1968b, p. 1665), a rule given for ordering the objects as they are joined into groups results in the pyramid structure. As seen in Figure 2, it greatly enhances the interpretive quality of the dendrograph. Because the input for this problem was a set of correlation-type coefficients, the within-group distance measures were transformed by taking the cosine, and the transformed values are shown plotted in Figure 2. This strategy has been employed to preserve the sense of the original data.

As a second example, some unpublished data from a pattern recognition study are considered. The example will serve to show how distance-type similarity measures are used to produce dendrographs. Table 2 is a listing of the card input for this example. The data represent interpoint distances between gravity centers of environments based on a random sample of 500 points taken from a map of the Mississippi River deltaic plain of southeastern Louisiana prepared by Kolb and others (1958). The environments represented are natural levee, point bar, marsh, swamp, beach, lacustrine, and bay-sound. The purpose of the sampling was to obtain statistical measures of spatial order in a deltaic complex. The matrix of interpoint distances between gravity centers was constructed from the sample. It will suffice here to consider the graphic portrayal of spatial ordering of environments using the dendrograph. Table 3 lists the results of clustering the interpoint distances for the seven environments. Because distance-type measures are involved, no transformation of the coefficnets is necessary. To show more of the working of the program, an intermediate stage of clustering is listed. This was done by assigning a value of 5 to the KLAM control variable which was interpreted as a request to have the intermediate stages of clustering printed out at every fifth stage of clustering. For this example, it amounted to the listing of one intermediate stage. The form of the intermediate clustering stage is similar to the final clustering with the exception that zeroes appear where ordering has not been determined. The intermediate groups are indexed by the lowest ranking element within each group. In the example, there remains group 1 and group 4 at the fifth stage of clustering, the former having 4 elements, the latter, 3 elements. This information is contained in the two lines below the intermediate stage of clustering. The final results given next form the basis for constructing the dendrograph which is shown in Figure 3. It reveals clearly the dichotomous nature of marine versus nonmarine environments and the transitional character of the lacustrine environment. The environments can be taken as representative elements for pattern recognition in deltaic-type deposits situated elsewhere in the geologic column.

Table 2.- Card input for environment center of gravity data based on random sample of 500 points taken from map prepared by Kolb and others (1958).

```
 7    5MISSISSIPPI DELTA REGION ENVIRONMENTAL GRAVITY CENTERS

 1C.        7.        1C.

 NATURAL LEVEE        POINT BAR           SWAMP          MARSH          BEACH

    LACUSTRINE        BAY-SOUND

(7F10.1)

      C.0

      9.4       C.0

      9.8      12.5       0.0

     27.8      36.8      28.1       0.0

     55.2      64.2      54.7      27.5       0.0

     22.4      27.8      15.4      20.4      43.0       0.0

     42.8      5C.9      39.9      17.6      18.0      26.4       0.0
```

Table 3.- Results of cluster analysis of environment center of gravity data. Intermediate results given for fifth stage of clustering.

MISSISSIPPI DELTA REGION ENVIRONMENTAL GRAVITY CENTERS

NUMBER OF GROUPS IS 7

ORIGINAL INPUT MATRIX (LOWER HALF BY ROWS)

1	0.0						
2	9.4000	0.0					
3	9.8000	12.5000	0.0				
4	27.8000	36.8000	28.1000	0.0			
5	55.2000	64.2000	54.7000	27.5000	0.0		
6	22.4000	27.8000	15.4000	20.4000	43.0000	0.0	
7	42.8000	50.9000	39.9000	17.6000	18.0000	26.4000	0.0

CLUSTERING STAGE NO. 5

NP	NGR	ORD	WGR	BGR
1	1			
		1	9.4000	9.4000
2	1			
		2	10.5667	11.1500
3	1			
		3	16.2167	21.8667
6	1			
		0	0.0	0.0
4	4			
		4	17.6000	17.6000
7	4			
		5	21.0333	22.7500
5	4			

1 4

4 3

FINAL RESULTS

NAME	NP	ORDER	WGR	BGR
NATURAL LEVEE	1			
		1	9.4000	9.4000
POINT BAR	2			
		2	10.5667	11.1500
SWAMP	3			
		3	16.2167	21.8667
LACUSTRINE	6			
		6	30.9809	40.8500
BEACH	5			
		5	21.0333	22.7500
BAY-SOUND	7			
		4	17.6000	17.6000
MARSH	4			

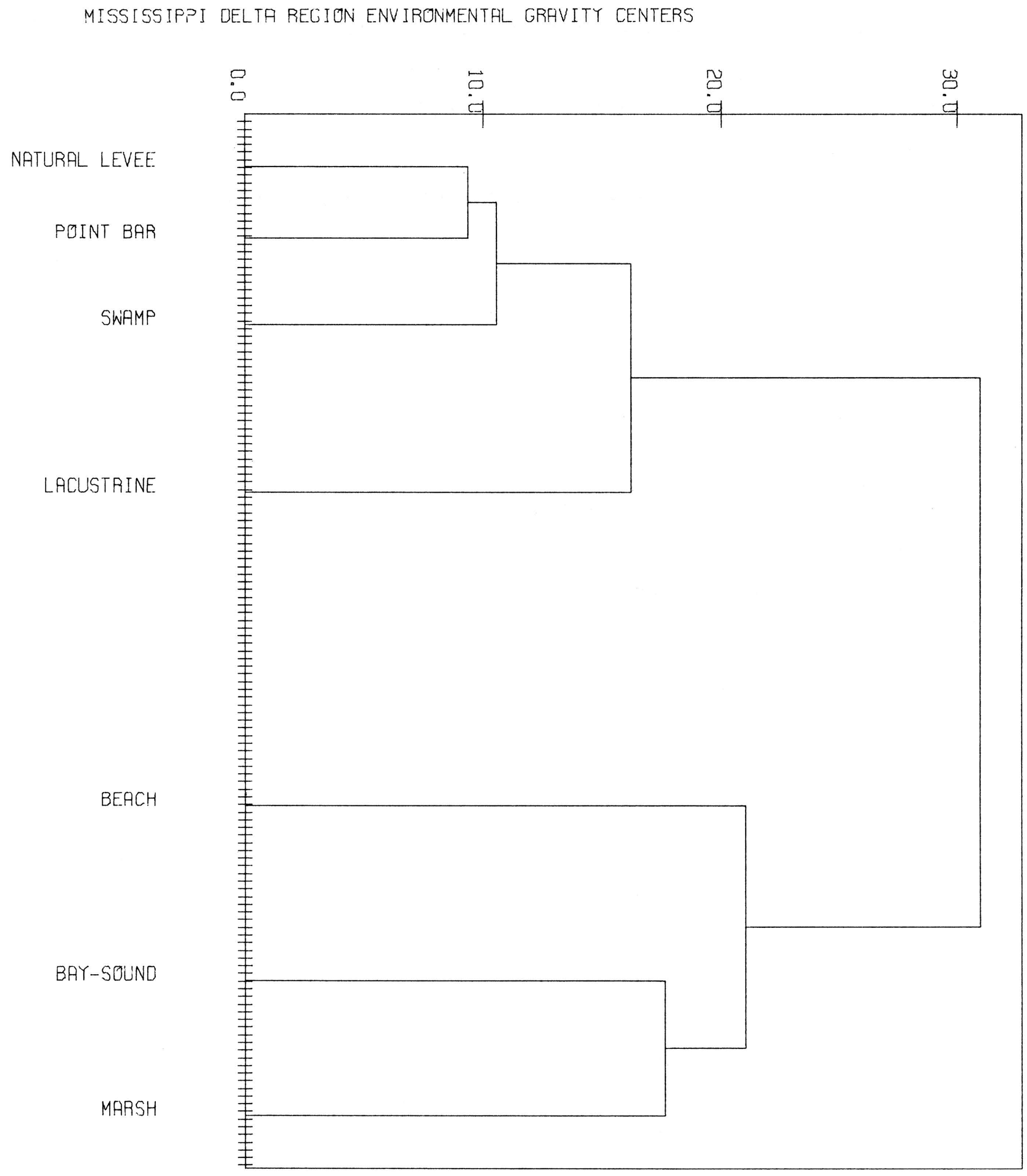

Figure 3. - Dendrograph depicting hierarchical pattern of spatial arrangement of environments in the Mississippi River deltaic plain of southeastern Louisiana.

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APPENDIX

```
C
C
C                         DENDROGRAPH COMPUTER PROGRAM
C
C
C               A COMPUTER PROGRAM TO ARRANGE A SET OF OBJECTS WHOSE
C       PAIRWISE SIMILARITY COEFFICIENTS ARE GIVEN INTO MUTUALLY EXCLUSIVE
C       HOMOGENEOUS SUBGROUPS AND TO DISPLAY THE RESULTS IN THE FORM OF A
C       DENDROGRAPH. THE METHOD USED FOR CLUSTERING IS THE UNWEIGHTED PAIR
C       GROUP METHOD. THE PRESENT PROGRAM IS DIMENSIONED TO CLUSTER UP TO
C       200 OBJECTS. THIS CAN BE EXTENDED BY INCREASING THE DIMENSIONS OF
C       THE APPROPRIATE PROGRAM VARIABLES. THE PRESENT PROGRAM USES THE
C       1627 CALCOMP PLOTTER VIA A SYSTEM/360-1800 CHANNEL ADAPTER TO DRAW
C       THE DENDROGRAPH. FEW MODIFICATIONS WOULD BE NECESSARY TO DRAW
C       DENDROGRAPHS ON A DIFFERENT PLOTTER SYSTEM.
C
C
C       ORDER OF INPUT CARDS
C         1. PROGRAM CONTROL AND TITLE CARD
C         2. PLOTTER CONTROL CARD
C           ** OBJECT NAME ARRAY CARDS **
C         3. VARIABLE FORMAT INPUT DATA CARD
C           ** DATA CARDS **
C
C
C       PROGRAM CONTROL VARIABLES
C
C            M - NUMBER OF OBJECTS TO BE CLUSTERED
C                    POSITIVE M    DISTANCE MEASURE COEFFICIENTS
C                    NEGATIVE M   CORRELATION MEASURE COEFFICIENTS
C         KLAM - FREQUENCY WITH WHICH INTERMEDIATE STAGES OF CLUSTERING
C                ARE PRINTED
C                DEFAULT OPTION - ONLY THE FINAL STAGE OF CLUSTERING IS
C                                 PRINTED
C
C
C       PLOTTING CONTROL VARIABLES
C
C         AENG - DENDROGRAPH LENGTH IN INCHES (BETWEEN-GROUP SCALE AXIS ,
C                AENG.LE.120.)
C          AHI - DENDROGRAPH HEIGHT IN INCHES (WITHIN-GROUP SCALE AXIS ,
C                AHI.LE.8.)
C         FNCR - UNITS THAT TIC MARKS ALONG THE WITHIN-GROUP SCALE AXIS
C                ARE APART
C                DEFAULT OPTIONS : +M  FNCR=1.0
C                                  -M  FNCR=0.1
C
C
C       PROGRAM VARIABLES
C
C         MATR - INPUT DATA VECTOR OF LENGTH M*(M+1)/2
C         NAME - OBJECT NAME MATRIX WITH DIMENSION 4 BY M
C         WIGR,BEGR,NP,ICRD,NGR,NN,NOM,NAR,ALIN - WORKING VECTORS OF
C                LENGTH M
```

```
C
C
C      SUBROUTINES SUPPLIED ALONG WITH MAIN PROGRAM
C
C        IPOS      INTEGER COMPARE
C        SEARCH    ELEMENT SEARCH
C        DENDRO    DENDROGRAPH PLOT
C
C
C      SUBROUTINES TO BE SUPPLIED BY USER
C
C        LIMIT     SET MAXIMUM LENGTH
C        BGNPLT    BEGIN PLOT
C        ENDPLT    END PLOT
C        SYMBOL    SYMBOL PLOT
C        NUMBER    NUMBER PLOT
C        PLOT      LOCATION PLOT WITH PEN CONTROL
C
C
C      ************
C        MAIN PROGRAM
C      ************
C
      REAL*4 MATR,NAME
      INTEGER*4 SEARCH
      DIMENSION MATR(20100),NGR(200),NN(200),NOM(200),WIGR(200),
     1BEGR(200),NP(200),IORD(200),NAME(4,200),ALIN(200),A(18),FMT(18)
  999 READ(5,10,END=100) M,KLAM,(A(I),I=1,18)
      MM=M
      MT=M
      M=IABS(M)
      READ(5,40) AENG,AHI,FNCR
      READ(5,30)((NAME(I,J),I=1,4),J=1,M)
      READ(5,20)(FMT(J),J=1,18)
      WRITE(6,11)(A(I),I=1,18),M
      DO 15 J=1,M
      JJ=(J*(J-1))/2
      WIGR(J)=0.0
      BEGR(J)=0.0
      NN(J)=J
      NP(J)=J
      NOM(J)=1
      NGR(J)=J
      IORD(J)=0
C
C         LOWER HALF OF MATRIX IS READ ROW WISE FROM LEFT TO RIGHT
C
   15 READ(5,FMT)(MATR(JJ+K),K=1,J)
      WRITE(6,91)
  102 DO 25 I=1,M
      JJ=(I*(I-1))/2
   25 WRITE(6,21) I,(MATR(JJ+J),J=1,I)
      IF(MM.GT.0) GO TO 107
      MM=(M*(M+1))/2
      DO 105 I=1,MM
      IF(MATR(I).GT.1.) MATR(I)=1.
  105 MATR(I)=ARCOS(MATR(I))
      WRITE(6,101)
      GO TO 102
  107 MH=M+1
      ML=M-1
```

```
C     ************          *****************           *****************
C             MAIN LOOP FOR CALCULATING CLUSTERS
C
      DO 95 MAIN=1,ML
      LCT=MH-MAIN
      LCM=LCT-1
      AMIN=100000.
C
C        MATRIX IS SEARCHED FCR SMALLEST WITHIN GROUP MEASURE
C
      DO 35 I=1,LCM
      K=I+1
      IMA1=NN(I)+(NN(I)*(NN(I)-1))/2
      DO 35 J=K,LCT
      IT=(NN(J)*(NN(J)-1))/2
      IMA2=NN(I)+IT
      IMA3=NN(J)+IT
      LA=NOM(I)+NOM(J)
      ANOM=(LA*(LA-1))/2
      TEMP=(MATR(IMA1)+MATR(IMA2)+MATR(IMA3))/ANOM
      IF(TEMP.GT.AMIN) GO TC 35
      ITI=I
      ITJ=J
      AMIN=TEMP
   35 CONTINUE
      LA=NCM(ITI)+NCM(ITJ)
C
C             BEGINNING ANC ENDS OF GROUPS TO BE JOINED ARE FOUND BY SEARCH
C
      JKL=NN(ITI)
      IBEG1=SEARCH(NGR,JKL)
      JKL=NN(ITJ)
      IBEG2=SEARCH(NGR,JKL)
      IEND1=IBEG1+NOM(ITI)-1
      IEND2=IBEG2+NOM(ITJ)-1
      IT=(NN(ITJ)*(NN(ITJ)-1))/2
      IMA1=NN(ITI)+(NN(ITI)*(NN(ITI)-1))/2
      IMA2=NN(ITI)+IT
      IMA3=NN(ITJ)+IT
      JKL=NOM(ITI)*NOM(ITJ)
      BEGR(IBEG2)=MATR(IMA2)/FLCAT(JKL)
      WIGR(IBEG2)=AMIN
      IORD(IBEG2)=MAIN
      TEMP=MATR(IMA1)+MATR(IMA2)+MATR(IMA3)
C
C            OUTPUT ARRAYS ARE SHIFTED INTO THE NEW ORDER
C        THE RIGHTMOST GROUP IS SHIFTED LEFT
C
      LDIF=IBEG2-IEND1-1
      KLIM=NOM(ITJ)
      IF(LDIF) 77,17,18
   18 DO 45 I=1,KLIM
      DO 45 J=1,LDIF
      KS=IBEG2+I-J
      MST=NGR(KS)
      NGR(KS)=NGR(KS-1)
      NGR(KS-1)=MST
      MST=NP(KS)
      NP(KS)=NP(KS-1)
      NP(KS-1)=MST
```

```
      MST=IORD(KS)
      IORD(KS)=IORD(KS-1)
      IORD(KS-1)=MST
      GW=WIGR(KS)
      WIGR(KS)=WIGR(KS-1)
      WIGR(KS-1)=GW
      GW=BEGR(KS)
      BEGR(KS)=BEGR(KS-1)
   45 BEGR(KS-1)=GW
   17 IF(NOM(ITI)-NOM(ITJ)) 131,141,141
  141 KNO2=NOM(ITJ)
      KS=IEND1+1
      GO TO 78
  131 KNO1=NOM(ITI)
      KNO2=NOM(ITJ)
      DO 125 I=1,KNO1
      DO 125 J=1,KNO2
      KS=IEND1+J-I
      MST=NP(KS+1)
      NP(KS+1)=NP(KS)
      NP(KS)=MST
      MST=IORD(KS+1)
      IORD(KS+1)=IORD(KS)
      IORD(KS)=MST
      GW=WIGR(KS+1)
      WIGR(KS+1)=WIGR(KS)
      WIGR(KS)=GW
      GW=BEGR(KS+1)
      BEGR(KS+1)=BEGR(KS)
  125 BEGR(KS)=GW
      KS=IBEG1+KNO2
      IORD(KS)=IORD(IBEG1)
      WIGR(KS)=WIGR(IBEG1)
      BEGR(KS)=BEGR(IBEG1)
      IORD(IBEG1)=0
      WIGR(IBEG1)=0.0
      BEGR(IBEG1)=0.0
      KNO2=NOM(ITI)
C
C             IF THE NEW RIGHT GROUP HAS MORE THAN TWO ELEMENTS
C                   ITS ORDER IS REVERSED
C
   78 IF(KNO2.LE.2) GO TO 111
      KNO1=KNO2/2
      KS=KS-1
      IEND2=IBEG1+NOM(ITI)+NOM(ITJ)
      DO 135 I=1,KNO1
      MST=NP(IEND2-I)
      NP(IEND2-I)=NP(KS+I)
      NP(KS+I)=MST
      MST=IORD(IEND2-I)
      IORD(IEND2-I)=IORD(KS+I+1)
      IORD(KS+I+1)=MST
      GW=BEGR(IEND2-I)
      BEGR(IEND2-I)=BEGR(KS+I+1)
      BEGR(KS+I+1)=GW
      GW=WIGR(IEND2-I)
      WIGR(IEND2-I)=WIGR(KS+I+1)
      WIGR(KS+I+1)=GW
  135 CONTINUE
```

```
C
C                LATEST GROUP IS PARTIALLY RELABELED IN 'NGR'
C
  111 DO 55 I=1,KLIM
   55 NGR(IEND1+I)=NN(ITI)
C
C                 ROW AND COLUMN NN(ITJ) ARE ADDED ON TO ROW AND COLUMN NN(ITI)
C
      DO 75 I=1,M
      KLA=IPOS(NN(ITI),I)
      KLE=IPOS(NN(ITJ),I)
   75 MATR(KLA)=MATR(KLA)+MATR(KLE)
      MATR(IMA1)=TEMP
      LDIF=LCT-ITJ
      IF(LDIF) 77,27,28
   28 DO 85 I=1,LDIF
      NN(ITJ+I-1)=NN(ITJ+I)
   85 NOM(ITJ+I-1)=NOM(ITJ+I)
   27 NOM(ITI)=LA
      IF(KLAM.EQ.0) GO TO 95
      KS=MAIN-(MAIN/KLAM)*KLA M
      IF(KS.NE.0) GO TO 95
      WRITE(6,31) MAIN
      WRITE(6,61) NP(1),NGR(1)
      DO 65 J=2,M
      WRITE(6,71) IORD(J),WIGR(J),BEGR(J)
   65 WRITE(6,61) NP(J),NGR(J)
      WRITE(6,41)(NN(I),I=1,LCM)
      WRITE(6,41)(NOM(J),J=1,LCM)
   95 CONTINUE
C
C          ******            END OF MAIN LOOP   *********
C
      WRITE(6,121)
      KS=NP(1)
      WRITE(6, 81)(NAME(L,KS),L=1,4),KS
      DO 205 I=2,M
      WRITE(6,181) IORD(I),WIGR(I),BEGR(I)
      KS=NP(I)
  205 WRITE(6, 81)(NAME(L,KS),L=1,4),KS
      WRITE(6,1)
      CALL DENDRO(WIGR,BEGR,NP,IORD,NAME,ALIN,M,FNCR,AENG,AHI,MT,A)
      GO TO 999
   77 WRITE(6,51) LDIF
      GO TO 999
  100 STOP
    1 FORMAT('1')
   10 FORMAT(2I4,18A4)
   11 FORMAT('1',30X,18A4,//' NUMBER OF GROUPS IS ',I3)
   20 FORMAT(18A4)
   21 FORMAT('0',I3,(1X,16F8.4))
   30 FORMAT(20A4)
   31 FORMAT('1 CLUSTERING STAGE NO. 'I3,//'   NP'5X,'NGR  ORD',4X,'WGR
     1    BGR')
   40 FORMAT(2F10.3,F5.2)
   41 FORMAT('0',33I4)
   51 FORMAT('0 LDIF = ' I4)
   61 FORMAT(' ',I3,5X,I3)
   71 FORMAT(' ',12X,I3,2F8.4)
   81 FORMAT(' ',4A4,I4)
```

```
 91 FORMAT('0',20X,'ORIGINAL INPUT MATRIX  (LOWER HALF BY ROWS)')
101 FORMAT('1 TRANSFORMED INPUT MATRIX')
121 FORMAT('1',30X,'FINAL RESULTS'//' NAME',14X,'NP',5X,'ORDER',5X,
   1'WGR     BGR')
181 FORMAT(' ',27X,I3,2X,2F8.4)
    END
```

```
C
C     ************
      INTEGER FUNCTION IPOS(IA,IB)
C     ************
C
      IF(IA.GT.IB) GO TO 1
      IPOS=IA+(IB*(IB-1))/2
      GO TO 2
    1 IPOS=IB+(IA*(IA-1))/2
    2 RETURN
      END
```

```
C
C     ************
      INTEGER FUNCTION SEARCH(NAR,NINT)
C     ************
C
      DIMENSION NAR(1)
      DO 5 J=1,200
      IF(NINT.EQ.NAR(J)) GO TO 10
    5 CONTINUE
      J=-1
   10 SEARCH=J
      RETURN
      END
```

```
C
C      ************
       SUBROUTINE DENDRO(WIGR,BEGR,NP,IORD,NAME,ALIN,M,FNCR,AENG,AHI,MT,
      1A)
C      ************
C
       REAL NAME
       INTEGER SEARCH
       DIMENSION WIGR(1),BEGR(1),NP(1),IORD(1),NAME(4,1),ALIN(1),A(1)
       MM=M-1
       IORD(1)=M+1
       IORD(M+1)=M+1
       AENG=AENG-1.
C
C      SCALE X-AXIS IN INCHES
C
       DO 15 J=3,M
    15 BEGR(J)=BEGR(J)+BEGR(J-1)
       SCAX=AENG/BEGR(M)
       UPI=BEGR(M)/AENG
       DO 25 J=1,M
    25 BEGR(J)=BEGR(J)+UPI/2.0
       TOTX=BEGR(M)+UPI/2.0
       DO 35 J=1,M
    35 BEGR(J)=BEGR(J)*SCAX
C
C      SCALE Y-AXIS IN INCHES
C
       IH=SEARCH(IORD,MM)
       TOTY=WIGR(IH)
       IF(MT.LT.0) TOTY=1.-COS(WIGR(IH))
       SCAY=AHI/TOTY
       DO 45 I=1,M
       IF(MT.LT.0) GO TO 44
       WIGR(I)=WIGR(I)*SCAY
       GO TO 45
    44 WIGR(I)=(1.-COS(WIGR(I)))*SCAY
    45 CONTINUE
       CALL BGNPLT(8,'MCCAMMON')
C
C      SET X-AXIS LIMIT
C
       XLEN=48.
       CALL LIMIT(XLEN)
C
C      SET ORIGIN
C
       XORG=0.
       YORG=2.
C
C      PRINT TITLE
C
       X=XORG-2.
       Y=.5
       CALL SYMBOL(X,Y,.125,A(1),90.,72)
C
C      DRAW IN BOUNDARY LINES
C
       CALL PLOT(XORG,YORG,3)
       X=XORG+AENG+1.
```

```
      CALL PLOT(X,YORG,2)
      Y=YORG+AHI+.4
      CALL PLOT(X,Y,2)
      CALL PLOT(XORG,Y,2)
      CALL PLOT(XORG,YORG,2)
C
C     PEN NOW LOCATED AT ORIGIN
C
C
C     PLACE TIC MARKS ALONG X-AXIS
C
      IAX=TOTX
      AX=XORG
      DO 145 I=1,IAX
      AX=AX+SCAX
      Y1=YORG+.06
      Y2=YORG-.07
      CALL PLOT(AX,Y1,3)
  145 CALL PLOT(AX,Y2,2)
C
C     LABEL AND PLACE TIC MARKS ALONG Y-AXIS
C
      IF(MT.LT.0) GO TO 36
      IF(FNCR.EQ.0.) FNCR=1.
      NTOT=TOTY/FNCR
      FNMB=0.
      FDIS=FNCR*SCAY
      GO TO 37
   36 IF(FNCR.EQ.0.) FNCR=.1
      NTOT=TOTY/FNCR
      FDIS=FNCR*SCAY
      FNMB=1.
      FNCR=-FNCR
   37 X=XORG-.425
      Y=YORG-.125
      CALL NUMBER(X,Y,.125,FNMB,0.,1)
      AI=FNMB
      AY=YORG
      IF(NTOT.EQ.0) GO TO 27
      DO 85 I=1,NTOT
      AI=AI+FNCR
      AY=AY+FDIS
      X1=XORG+.125
      X2=XORG-.125
      CALL PLOT(X1,AY,3)
      CALL PLOT(X2,AY,2)
      AYY=AY-.125
      X=XORG-.425
   85 CALL NUMBER(X,AYY,.125,AI,0.,1)
C
C     PRINT OBJECT NAMES
C
   27 DO 55 I=1,M
      AX=XORG+BEGR(I)
      IL=NP(I)
      Y=YORG-2.54
   55 CALL SYMBOL(AX,Y,.125,NAME(1,IL),90.,16)
      DO 65 I=1,M
   65 ALIN(I)=0.0
```

```
C
C     GENERATE DENDROGRAPH
C
      DO 75 I=1,MM
      IH=SEARCH(IORD,I)
      BAR=YORG+ALIN(IH)
      BAL=YORG+ALIN(IH-1)
      TOP=YORG+WIGR(IH)
      SR=XORG+BEGR(IH)
      SL=XORG+BEGR(IH-1)
      CALL PLOT(SR,BAR,3)
      CALL PLOT(SR,TOP,2)
      CALL PLOT(SL,TOP,1)
      CALL PLOT(SL,BAL,1)
      K=IORD(IH)
      J=IH
   67 J=J+1
      IF(K-IORD(J)) 68,67,67
   68 IEND=J-1
      J=IH
   77 J=J-1
      IF(K-IORD(J)) 78,77,77
   78 IBEG=J
      ALIN(IBEG)=WIGR(IH)
      ALIN(IEND)=WIGR(IH)
      CENT=(BEGR(IH)+BEGR(IH-1))/2.0
      BEGR(IBEG)=CENT
   75 BEGR(IEND)=CENT
      CALL ENDPLT
      RETURN
      END
```

CHARACTERS AVAILABLE IN SYMBOL ROUTINE (IBM 360)*

00	⊡	10	|	20	}	30	Σ	40		50	&	60	-	70	0
01	⏀	11	BS	21	{	31	÷	41	A	51	J	61	/	71	1
02	⚠	12	∧	22	μ	32	≤	42	B	52	K	62	S	72	2
03	+	13	≡	23	π	33	≥	43	C	53	L	63	T	73	3
04	×	14	→	24	Φ	34	△	44	D	54	M	64	U	74	4
05	◇	15	CR	25	θ	35	[	45	E	55	N	65	V	75	5
06	⍏	16	≠	26	ψ	36	]	46	F	56	Ø	66	W	76	6
07	⧗	17	±	27	χ	37	\	47	G	57	P	67	X	77	7
08	Z	18	_	28	ω	38	↑	48	H	58	Q	68	Y	78	8
09	Y	19	=	29	λ	39	√	49	I	59	R	69	Z	79	9
0A	⌧	1A		2A	∝	3A	†	4A	¢	5A	!	6A	∞	7A	:
0B	✳	1B	∫	2B	δ	3B	‡	4B	.	5B	$	6B	,	7B	#
0C	⧖	1C	⊃	2C	∈	3C	←	4C	<	5C	*	6C	%	7C	@
0D	|	1D	∨	2D	η	3D	×	4D	(	5D	)	6D	—	7D	'
0E	☆	1E	~	2E	UC	3E	↑	4E	+	5E	;	6E	>	7E	=
0F	_	1F	≈	2F	LC	3F	↓	4F	|	5F	¬	6F	?	7F	"

* Please note that all identifying numbers are base 16

KANSAS GEOLOGICAL SURVEY COMPUTER PROGRAM
THE UNIVERSITY OF KANSAS, LAWRENCE

PROGRAM ABSTRACT

Title (If subroutine state in title): THE DENDROGRAPH

Date: February 1970

Author, organization: Richard B. McCammon, Department of Geological Sciences, University of Illinois at Chicago, Chicago, Illinois 60680

Direct inquiries to: same as above

Name: Address:

Purpose/description: To produce a two dimensional diagram called the dendrograph in which is displayed the mutual relationships among a group of objects whose pairwise similarities are given

Mathematical method: Unweighted pair group method of clustering (UWPGM)

Restrictions, range: The program is currently dimensioned to cluster up to 200 objects. Plotting subroutines must be supplied by the user.

Computer manufacturer: IBM Model: 360/50

Programming language: FORTRAN IV

Memory required: 27 K Approximate running time: 2 - 5 minutes

Special peripheral equipment required: Plotter unit

Remarks (special compilers or operating systems, required word lengths, number of successful runs, other machine versions, additional information useful for operation or modification of program)

(continued from inside front cover)

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18. Computer applications in the earth sciences: Colloquium on time-series analysis, edited by D.F. Merriam, 1967 . . . . $1.00
19. FORTRAN II time-trend package for the IBM 1620 computer, by J.C. Davis and R.J. Sampson, 1967 . . . . $1.00
20. Computer programs for multivariate analysis in geology, edited by D.F. Merriam, 1968 . $1.00
21. FORTRAN IV program for computation and display of principal components, by W.J. Wahlstedt and J.C. Davis, 1968 . . . . $1.00
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24. Mathematical model and FORTRAN IV program for computer simulation of deltaic sedimentation, by G.F. Bonham-Carter and A.J. Sutherland, 1968 . . . . . $1.00
25. FORTRAN IV CDC 6400 computer program to analyze subsurface fold geometry, by E.H.T. Whitten, 1968 . . . . $1.00
26. FORTRAN IV computer program for simulation of transgression and regression with continuous-time Markov models, by W.C. Krumbein, 1968 . . . . $1.00
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30. Sampling a geological population, by J.C. Griffiths and C.W. Ondrick, 1968 . . . $1.00
31. Multivariate procedures and FORTRAN IV program for evaluation and improvement of classifications, by Ferruh Demirmen, 1969 . . . . $1.00
32. FORTRAN IV programs for canonical correlation and canonical trend-surface analysis, by P.J. Lee, 1969 . . . . $1.00
33. FORTRAN IV program for construction of Pi diagrams with the Univac 1108 computer, by Jeffrey Warner, 1969 . . . . $1.00
34. FORTRAN IV program for nonlinear estimation, by R.B. McCammon, 1969 . . . . $0.75
35. FORTRAN IV computer program for fitting observed count data to discrete distribution models of binomial, Poisson and negative binomial, by C. W. Ondrick and J.C. Griffiths, 1969 $0.75
36. GRAFPAC, graphic output subroutines for the GE 635 computer, by F.J. Rohlf, 1969 . . $1.00
37. An iterative approach to the fitting of trend surfaces, by A.J. Cole, 1969 . . . . $1.00
38. FORTRAN II programs for 8 methods of cluster analysis (CLUSTAN I), by David Wishart, 1969 $1.50
39. FORTRAN IV program for the generalized statistical distance and analysis of covariance matrices for the CDC 3600 computer, by R.A. Reyment, Hans-Ake Ramden, and W.J. Wahlstedt, 1969 . . . . $1.00
40. Symposium on computer applications in petroleum exploration, edited by D.F. Merriam, 1969 $1.00
41 FORTRAN IV program for sample normality tests, by D.A. Preston, 1970 . . . . $1.00
42. CORFAN-FORTRAN IV computer program for correlation, factor analysis (R- and Q-mode) and varimax rotation, by C.W. Ondrick and G.S. Srivastava, 1970 . . . $1.50
43. Minimum entropy criterion for analytic rotation, by R.B. McCammon, 1970 . . . $1.00
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48. The dendrograph, by R.B.McCammon and G. Wenninger, 1970 . . . . $1.00