DANIEL F. MERRIAM, Editor

FORTRAN IV PROGRAM FOR CANONICAL VARIATES ANALYSIS FOR THE CDC 3600 COMPUTER

By

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and

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Editor's Remarks

The "FORTRAN IV program for canonical variates analysis for CDC 3600 computer" by R.A. Reyment and H. Ramden will be made available for a limited time by the Geological Survey for $20.00 (US). An extra $10.00 charge is made if punched cards are required. This charge covers handling and postage. A complete up-to-date list of publications and programs and data decks can be obtained by writing Editor, COMPUTER CONTRIBUTIONS, Kansas Geological Survey, The University of Kansas, Lawrence, Kansas 66044.

Computer Contribution

1. Mathematical simulation of marine sedimentation with IBM 7090/7094 computers, by J.W. Harbaugh, 1966 . . . . (out of print)
2. A generalized two-dimensional regression procedure, by J.R. Dempsey, 1966 . . . . $0.50
3. FORTRAN IV and MAP program for computation and plotting of trend surfaces for degrees 1 through 6, by Mont O'Leary, R.H. Lippert, and O.T. Spitz, 1966 . . . . . $0.75
4. FORTRAN II program for multivariate discriminant analysis using an IBM 1620 computer, by J.C. Davis and R.J. Sampson, 1966 . . . . $0.50
5. FORTRAN IV program using double Fourier series for surface fitting of irregularly spaced data, by W.R. James, 1966 . . . . $0.75
6. FORTRAN IV program for estimation of cladistic relationships using the IBM 7040, by R.L. Bartcher, 1966 . . . . $1.00
7. Computer applications in the earth sciences: Colloquium on classification procedures, edited by D.F. Merriam, 1966 . . . . $1.00
8. Prediction of the performance of a solution gas drive reservoir by Muskat's equation, by Apolonio Baca, 1967 . . . . $1.00
9. FORTRAN IV program for mathematical simulation of marine sedimentation with IBM 7040 or 7094 computers, by J.W. Harbaugh and W.J. Wahlstedt, 1967 . . . . . $1.00
10. Three-dimensional response surface program in FORTRAN II for the IBM 1620 computer, by R.J. Sampson and J.C. Davis, 1967 . . . . $0.75
11. FORTRAN IV program for vector trend analyses of directional data, by W.T. Fox, 1967 . $1.00
12. Computer applications in the earth sciences: Colloquium on trend analysis, edited by D.F. Merriam and N.C. Cocke, 1967 . . . . $1.00
13. FORTRAN IV computer programs for Markov chain experiments in geology, by W.C. Krumbein, 1967 . . . . $1.00
14. FORTRAN IV programs to determine surface roughness in topography for the CDC 3400 computer, by R.D. Hobson, 1967 . . . . $1.00

(continued on inside back cover)

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FORTRAN IV PROGRAM FOR CANONICAL VARIATES ANALYSIS FOR CDC 3600 COMPUTER

by

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ABSTRACT

This FORTRAN IV program for canonical variates for a maximum of 30 variables performs several functions: (1) basic statistics for each sample are computed and the covariance matrices tested for homogeneity; (2) the generalized determinantal equation is solved and eigenvalues tested for significance; (3) the mean vectors are tested in a one-way multivariate analysis of variance for significant difference; and (4) the first two elements of the transformed mean vectors and transformed observations are plotted.

Application of the program is illustrated by two examples: observations on Upper Cretaceous echinoids from England and matrix input data from an anthropomorphic problem.

INTRODUCTION

The FORTRAN IV program accomplishes the calculations required for a canonical variate analysis including the associated multivariate analysis of variance. The theory is well known and may be consulted in many textbooks on multivariate analysis.

Canonical variate analysis, referred to as "canvar", is a logical extension of the analysis of two multivariate samples for which a comprehensive FORTRAN IV program was published recently by Reyment, Ramdén, and Wahlstedt.

Two problems are used as examples of application of the program. Data for the first example are on Upper Cretaceous echinoids from England (Nicholls, 1959). Data for the second example are from an anthropomorphic problem (Talbot and Mulhall, 1962). The examples are given in the Appendix.

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METHOD OF CANONICAL VARIATES

We may think about canvar in the following manner. Each sample has been drawn from a universe. The first universe is a swarm of points in a p-variate Cartesian space with its center at $\mu^{(1)}$, the mean vector. The swarm is dispersed about this point in an ellipsoidal shape, determined by the covariance matrix $\Sigma^{(1)}$. The second universe is represented in the same space by a swarm of points that may overlap the first swarm. The center of this ellipsoid is at $\mu^{(2)}$, and it is determined by covariance matrix $\Sigma^{(2)}$. The shape and orientation of this ellipsoid may differ from that of the first. For the theory to be strictly applicable, the two shapes and orientations should not differ significantly. More than two universes may be considered.

The calculations are based on reasoning similar to that of principal components analysis. Here, again, transformed axes are produced. The first axis is inclined in the direction of greatest variability between the k means of k samples. The second axis is perpendicular to the first and inclined in the direction of next greatest variability; similarly for the third and subsequent axes.

In general, the entire procedure may be viewed in light of a generalized multivariate analysis of variance (manova). This is based on a natural breakdown of variability into "between" and "within" components - in the present situation, the "between sums of squares and cross products" matrix and the "within sums of squares and cross products" matrix (Table 1).

Table 1.- Breakdown for manova.

Matrix	Meaning	Degrees of freedom
$B = T - W$	Among s.s.c.p. matrix	$k - 1$
$W = (n - k) S_W$	Within s.s.c.p. matrix	$n - k$
$T = (n - 1) S_T$	Total s.s.c.p. matrix	$n - 1$

S_T is the total covariance matrix and S_W is the within-groups covariance matrix,
k is the number of groups and n the total number of observations.

The canonical variates are found from the eigenvalues and eigenvectors of the determinantal equation

$$|B - zW| = 0 . \tag{1}$$

If $p > k - 1$ equation (1) will have only $k - 1$ nonzero eigenvalues. Conversely, with $p < k - 1$, there will be p eigenvalues. p is the number of dimensions.

The eigenvectors corresponding to the z_i are required for finding the canonical variates. These are found from

$$(B - zW) t = 0. \qquad (2)$$

and are the p-component vectors t. The vectors may be standardized to give the canonical axes, u,

$$\left(t' \frac{W}{n-k} t\right)^{-1/2} t = u. \qquad (3)$$

Significance of Eigenvalues

In this program only the first two canonical variates for canvar are used. It is important to test the eigenvalues for significance, because this indicates how much information is associated with the remaining roots. Bartlett's well known test, used in this program, is

$$\left((n - 1) - (p + k) / 2\right) \log_e \left[\prod_{i=f+1}^{m} (1 + z_i) \right] , \qquad (4)$$

which is approximately a chi-square variable with $(p - f)(k - f - 1)$ degrees of freedom. In this formula n is the total sample size, p the number of dimensions, k the number of groups, f the number of eigenvalues and z_i the eigenvalues. The theory behind this procedure is that the roots are tested to determine if they can be given zero values.

Multivariate Analysis of Variance

The manova is made in terms of the matrices W and T by what is known as the Wilks' Λ (lambda) criterion (see Table 1). This is the determinantal ratio

$$\Lambda = \frac{|W|}{|T|} \qquad (5)$$

Λ can range between zero and one. If it is one then there is no between-group variance-covariance. The smaller the value of the ratio, the greater is the between-group variance-covariance. An approximate test of significance is made by a chi-square test.

Homogeneity of Covariance Matrices

Canvar requires the assumption of homogeneity of covariance matrices. Available empirical evidence suggests that the method may be moderately robust (resilient) to departures from homogeneity. Nevertheless it is advisable to test sample covariance matrices in this respect. The test used in this program is from Kullback (1959, p. 318). The formula is

$$\sum_{i=1}^{k} \frac{n_i}{2} \log_e \frac{|S_w|}{|S_i|} , \qquad (6)$$

where n_i is the sample size, less one, S_w the pooled covariance matrix and S_i the covariance matrix of the i^{th} sample. This is distributed asymptotically as chi-square with $(k - 1)p(p + 1)/2$ degrees of freedom.

The foregoing remarks cover the main features of the calculations made by this program.

PROGRAM DESCRIPTION

Operating Instruction

Card 1 Column 3: Number of jobs

Card 2 Title of work in columns 1 - 72

Card 3 Columns 2-3: M = number of variables
Columns 4-5: MG = number of groups
Column 6: For matrix data put L = 0
For raw data put L = 1

MATRIX DATA

Card 4 --------Format card

Card 5 --------Columns 2-6: NI = total number of observations

Card 6 and following contain the input matrices in the following order:

(1) C (I, J) = M X MG matrix of mean vectors

(2) A (I, J) = "Among groups" sums of squares and cross products matrix

(3) B (I, J) = "within groups" sums of squares and cross products matrix

RAW DATA

The further input information needed for raw data is that of the subroutine CORREL, listed here for convenience.

Card 4 Columns 1-5: Size of sample
Column 6: For the transformation of the data to base 10 logarithms put KLØG = 1. otherwise KLØG is put = 0.
Column 7: MØUT = 1 if output from CORREL

for this sample is to be printed, otherwise MØUT = 0.

Card 5 Title for this sample in columns 1-72

Card 6 Variable format for this sample

Card 7 and following, contain the observations

List of Subroutines

Mainline Program CANVAR

The mainline program reads and prints on the formalized units LIN (read) and LUT (write). Matrix data are read in here and raw data by calling subroutine CORREL, whereafter the various sums of squares and cross products are accumulated. The generalized eigenvalues returned by subroutine EIGGEN are tested for significance by a chi-square procedure (4). The transformed canonical variate means are printed out from this section.

Subroutine CORREL

This subroutine computes means, standard deviations, sums of squares and cross products, covariances and variances and correlations from the raw data. If required, the calculations can be made on the logarithmically transformed data (base 10).

Subroutine EIGGEN

This computes the generalized eigenvalues and eigenvectors for the determinantal equation obtained from the "between groups" and "within groups" sums of squares and cross products matrices as in equations (1) and (2).

A negative eigenvalue indicates a divergency from the theoretical requirements of the matrices for canonical variates analysis and should be checked.

Subroutine GETMAT

This subroutine is a tape-rationalizing procedure. It fetches a labeled stored matrix from a place in core memory. It may be modified easily to allow recalling of a particular matrix from an external medium such as a tape or drum. GETMAT works in conjunction with PUTMAT: they must be compatible exactly.

Subroutine HDIAG

This computes the eigenvalues and eigenvectors of a square symmetric matrix. The latter part of the subroutine contains some modifications for the CDC 3600, which may be changed easily for operation with an IBM FORTRAN compiler.

Subroutine HOMMAT

This carries out an intermediate step for the test of homogeneity of covariance matrices. A byproduct of the calculations is an abridged principal component analysis for each sample. HOMMAT is called only for raw data.

Subroutine HOMO

This subroutine carries out a test for homogeneity of the covariance matrices (6) derived from the samples. It is called only where raw data are used. The result is expressed in terms of chi-square.

Subroutine MATOUT

Prints out MG X M matrices with a title and a maximum of 10 numbered columns to a page. In its present form it is adapted for the CDC 3600 which has 8 characters in each cell (NCHPW set to 8). On other computers NCHPW will have to be changed to the pertinent cell size - this involves a corresponding change in format statement 103.

Subroutine PLT 3

This subroutine plots a scatter diagram, used for the transformed means and the transformed observations, on the printer. It takes account of each sample in the latter case, plotting for each an identifying letter.

Subroutine PUTMAT

This stores an MXN matrix and labels it. The subroutine complements GETMAT.

Subroutine STAND

The subroutine standardizes the elements of the matrix of the generalized eigenvectors as an intermediate step in producing the coordinates for the plotting section of the program. The calculations of equation (3) are done here.

Subroutine TRANSF

This transforms the original observations to a canonical variate set and prepares the first two elements of each vector for plotting by PLT 3.

Subroutine WILKS

This subroutine reports on the one-way multivariate analysis of the samples; i.e. it is a test of equality of mean vectors. The test of significance is based on the Wilks lambda (5) criterion. The result is given as chi-square.

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Nicholls, D., 1959, Changes in the Chalk heart-urchin Micraster interpreted in relation to living forms: Phil. Trans. Royal Soc. London, v. 693, no. 242, p. 347-437.

Reyment, R.A., Ramdén, H., and Wahlstedt, W.J., 1969, FORTRAN IV program for the generalized statistical distance and analysis of covariance matrices for the CDC 3600 computer: Kansas Geol. Survey Computer Contr. 39, 42 p.

Talbot, P.A., and Mulhall, H., 1962, The physical anthropology of southern Nigeria: Cambridge Univ. Press, Cambridge,

APPENDIX

```
      PROGRAM CANVAR
C------------PROGRAM FOR COMPUTING CANONICAL VARIATES
C-------------------------------OUTPUT------------------------------
C----------BASIC STATISTICS FOR EACH SAMPLE (OPTIONAL)
C----------MATRIX OF MEAN VECTORS
C----------MATRIX OF CANONICAL MEAN VECTORS
C----------EIGENVALUES AND EIGENVECTORS OF THE GENERALIZED
C--------------------------DETERMINANTAL EQUATION
C----------TESTS OF SIGNIFICANCE OF THESE EIGENVALUES
C----------TEST  OF HOMOGENEITY OF COVARIANCE MATRICES
C----------WILK,S TEST OF EQUALITY OF THE MEAN VECTORS (GENERALIZED
C--------------ONE-WAY CLASSIFICATION ANALYSIS OF VARIANCE)
C----------GRAPH OF FIRST TWO TRANSFORMED MEANS
C----------GRAPH OF FIRST TWO TRANSFORMED OBSERVATIONS
C-----------------------------------INPUT-----------------------------
C----------IN THE CASE OF PROCESSED DATA
C----------MEAN VECTORS, WITHIN SUMS OF SQUARES AND CROSS PRODUCTS
C----------MATRIX AND BETWEEN SUMS OF SQUARES AND CROSS PRODUCTS MATRIX
C----------IN THE CASE OF UNPROCESSED (RAW) DATA
C---------------INFORMATION INPUTTED AS OBSERVATION VECTORS
C---- MATRIX A IS AMONG GROUPS SSP-MATRIX
C---- MATRIX B IS WITHIN GROUPS SSP-MATRIX**TOTAL=WITHIN + AMONG
C----------------------ORDER OF THE CONTROL CARDS------------------------
C------FIRST CARD*********** NUMBER OF JOBS IN COL 3
C------SECOND CARD****** TITLE OF WORK IN COLUMNS 1 THROUGH 72
C------THIRD CARD M= NUMBER OF VARIABLES IN COLS 2-3
C-----------------MG = NUMBER OF GROUPS IN COLS 4-5
C-----------------L IN COL  6 IS PUT = 0 FOR MATRIX INPUT AND = 1
C-----------------FOR RAW DATA
C----------------------THE CASE OF MATRIX DATA-------------------------
C-------CARD 4 THE VARIABLE FORMAT CARD
C-------CARD 5  COLUMNS 2-6  NI = THE TOTAL NUMBER OF OBSERVATIONS
C-------CARD 6 AND FOLLOWING CARDS CONTAIN THE INPUT MATRICES IN THE
C----------------------------FOLLOWING ORDER,,,,,,,,
C----------(1) C(I,J) = M X MG MATRIX OF MEAN VECTORS
C----------(2) A(I,J) = ,,AMONG GROUPS,, SUMS OF SQUARES AND CROSS
C------------------------PRODUCTS MATRIX
C----------(3) B(I,J) = ,,WITHIN GROUPS,, SUMS OF SQUARES AND CROSS
C------------------------PRODUCTS MATRIX
C----------------------THE CASE OF RAW DATA---------------------
C-------THE FURTHER INPUT INFORMATION NEEDED FOR RAW DATA IS THAT
C-------OF THE SUBROUTINE   CORREL    , LISTING HERE FOR CONVENIENCE,
C----------CARD 4  COLUMNS 1-5  SIZE OF THE SAMPLE
C----------COLUMN 6  FOR THE BASE 10 LOG-TRANSFORMATION OF THE DATA
C-----------------PUT KLOG = 1, OTHERWISE KLOG = 0,
C----------COLUMN 7  MOUT = 1 GIVES FULL OUTPUT FROM CORREL, IF NO
C-----------------OUTPUT REQUIRED MOUT = 0,
C----------CARD 5  TITLE FOR THIS SAMPLE IN COLUMNS 1-72,
C----------CARD 6  VARIABLE FORMAT FOR THIS SAMPLE
C----------CARD 7, AND FOLLOWING CARDS, CONTAIN THE OBSERVATIONS
C----------------------------------------------------------------------
      COMMON A(30,30),B(30,30),C(60,30),DET,DETM(30),ENI,ENUMB(30),
     1 L,M,MG,NI,NG,SS(30,30),SSD(30,30),SX(30,30),X(30,30),
     2 XD(30),XL(30),XM(30),Z(30,30)
      COMMON /DIMENS/ MDIM,MGDIM
      COMMON /MATRIX/ MATNR,MTMAT
```

```
      COMMON /PLOT/ IVPLOT(60),CHAR(25),KGROUP
      COMMON /SCRCOM/ TS(30,30),DUMMY(2700)
      COMMON /UNITS/ LIN,LUT,MTDATA
      DATA (CHAR=1HA,1HB,1HC,1HD,1HE,1HF,1HG,1HH,1HJ,1HK,1HL,1HM,1HN,1HO
     1,1HP,1HQ,1HR,1HS,1HT,1HU,1HV,1HW,1HX,1HY,1HZ)
      DIMENSION FMT(12),SUMM(30),XME(30),TITLE(12)
      MDIM=30
      MGDIM=60
C------- LIN=STANDARD INPUT UNIT (CARD-READER)
      LIN=60
C------- LUT=STANDARD OUTPUT UNIT  (PRINTER)
      LUT=61
C------- MTDATA IS A SCRATCH UNIT TO STORE SPECIAL VALUES
      MTDATA=1
      READ(LIN,7025) JOBS
      DO 7606 KLOBS=1,JOBS
      REWIND MTDATA
      MATNR=0
      WRITE(LUT,1709)
      READ(LIN,3) TITLE
      WRITE(LUT,3)TITLE
      READ(LIN,1) M,MG,L
      WRITE(LUT,8) MG,M
      MT=M
      IF(M.GE.MG) WRITE(LUT,505)
      IF (M .GT. MDIM .OR. MG .GT. MGDIM) GO TO 9900
      IF(L)100,100,200
C-------------------INPUT FOR MATRIX DATA------------------------
  100 READ(LIN,3) FMT
      READ(LIN,40) NI
      DO 5 I=1,MG
    5 READ(LIN,FMT)(C(I,J),J=1,M)
C---- MEANS READ IN AS C(I,J)
      CALL PUTMAT (C,MG,M,MGDIM,3)
      CALL MATOUT (C,MG,M,MGDIM,19,19H0    TABLE OF MEANS )
      DO 20 I=1,M
   20 READ(LIN,FMT)(A(I,J),J=1,M)
C------MATRIX A IS THE AMONG-GROUPS MATRIX
      CALL MATOUT (A,M,M,MDIM,34,34H0  AMONG GROUPS SSCP MATRIX,A(I,J) )
      DO 21 I=1,M
   21 READ(LIN,FMT)(B(I,J),J=1,M)
      CALL PUTMAT (B,M,M,MDIM,1)
      CALL MATOUT (B,M,M,MDIM,28,28H0  WITHIN GROUPS SSCP MATRIX )
      DO 23 I=1,M
      DO 23 J=1,M
   23 X(I,J) = A(I,J) + B(I,J)
      CALL PUTMAT (X,M,M,MDIM,2)
      GO TO 270
C------STARTING POINT FOR UNPROCESSED DATA
  200 N=0
      KM=0
      DO 205 I=1,M
      SUMM(I)=0.
      XME(I)=0.
      DO 205 J=1,M
      A(I,J)=0.
      B(I,J)=0.
      X(I,J)=0.
  205 TS(I,J)=0.
      KGROUP=0
```

```
      DO 2206 I=1,MG
      DO 2206 J=1,M
 2206 C(I,J)=0.
  300 KM=KM+1
      CALL CORREL
      N=N+NG
      DO 210 I=1,M
      SUMM(I)=SUMM(I)+SX(I)
      DO 210 J=1,M
      B(I,J)=B(I,J)+SSD(I,J)
  210 TS(I,J)=TS(I,J)+SS(I,J)
      DETM(KM) = DET
      ENUMB(KM) = ENI
      WRITE(LUT,7061) KM
      I=KM
      DO 211 J=1,M
  211 C(I,J)=XM(J)
      KGROUP=KGROUP+1
      IVPLOT(KGROUP)=N
      IF(KGROUP-MG)300,215,215
  215 EN = N
      DO 220 I=1,M
      XME(I)=SUMM(I)/EN
      DO 220 J=1,M
  220 X(I,J)=TS(I,J)-(SUMM(I)*SUMM(J)/EN)
      CALL MATOUT (B,M,M,MDIM,31,31H0   POOLED WITHIN MATRIX B(I,J) )
      CALL PUTMAT (B,M,M,MDIM,1)
C----- THE WITHIN MATRIX IS NOW SAVED
      DO 255 I=1,M
      DO 255 J=1,M
  255 A(I,J)=X(I,J)-B(I,J)
      CALL MATOUT (A,M,M,MDIM,24,24H0   AMONG GROUPS MATRIX    )
      CALL MATOUT (X,M,M,MDIM,16,16H0   TOTAL MATRIX )
      CALL PUTMAT (X,M,M,MDIM,2)
C-----MATRIX OF TOTAL SUMS IS NOW SAVED
      WRITE(LUT,245)N
      WRITE(LUT,250)
      WRITE(LUT,231)(XME(I),I=1,M)
      NI = N
      CALL MATOUT (C,MG,M,MGDIM,19,19H0    TABLE OF MEANS )
      CALL PUTMAT (C,MG,M,MGDIM,3)
C------MATRIX OF MEANS NOW SAVED
      WRITE(LUT,1060)
C------------------------------------------------------------------------------
  270 CALL EIGGEN
      WRITE(LUT,1058)
      K=0
      KR=0
  320 K=K+1
      PROD=1.
      DO 1300 I=K,M
 1300 PROD = PROD*(1. + XL(I))
      PROD = ALOG(PROD)
      WRITE(LUT,305) K
      WRITE(LUT,310) PROD
      CHISQ = -(FLOAT(NI-1) - .5*(FLOAT(M+MG)))*PROD
      CHISQ = ABS(CHISQ)
      WRITE(LUT,315)CHISQ
      KDF = (M-KR)*(MG-KR-1)
      WRITE(LUT,1088)KDF
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```
      KR = KR + 1
      IF(MT-K)325,325,320
  325 CONTINUE
      CALL WILKS
      IF(L.EQ.0) GO TO 7606
      CALL HOMO (DETM,M,ENUMB,NI,MG)
 7606 CONTINUE
 7027 CALL EXIT
 9900 WRITE (LUT,9901)
      GO TO 7027
C-----------------------------  FORMATS  -------------------------------------
    1 FORMAT(1X,I2,I2,I1)
    3 FORMAT(12A6)
    8 FORMAT(42H0   CANONICAL VARIATES STUDY FOR GROUPS = I2/29H0      N
     1UMBER OF VARIABLES = I2)
   40 FORMAT(1X,I5)
  231 FORMAT(1X,10F13.5)
  245 FORMAT(24H0   TOTAL SAMPLE SIZE = I5)
  250 FORMAT(21H0   TOTAL MEAN VECTOR)
  305 FORMAT(12H0 LAMBDA NO I1)
  310 FORMAT(11X,F14.7)
  315 FORMAT(15H0  CHISQUARE = F14.7)
  505 FORMAT(30H0  LESS GROUPS THAN DIMENSIONS)
 1058 FORMAT(45H1  TESTS OF SIGNIFICANCE FOR THE EIGENVALUES        )
 1060 FORMAT(27H1  OUTPUT FROM EIGENROUTINE)
 1088 FORMAT(10H0  D.F. = I3)
 1709 FORMAT(1H1)
 7025 FORMAT(1X,I2)
 7061 FORMAT(17H-  END OF SAMPLE I2)
 9901 FORMAT (25H0*** TOO BIG A PROBLEM***        )
      END


      SUBROUTINE CORREL
C------SUMMARY OF INPUT INSTRUCTIONS
C------FIRST CONTROL CARD FOR EACH GROUP DEFINED AS
C------COLUMNS 1-5 SAMPLE SIZE
C------ COLUMN 6  KLOG = 0 FOR RAW DATA AND = 1 FOR LOGARITHMS
C------MOUT = 0 FOR NO OUTPUT AND = 1 FOR ALL OUTPUT FROM SUBROUTINE
C------SECOND CARD BEARS GROUP TITLE IN COLUMNS 1 - 72
C------THIRD CARD BEARS VARIABLE INPUT FORMAT
      COMMON A(30,30),B(30,30),C(60,30),DET,DETM(30),ENI,ENUMB(30),
     1 L,M,MG,NI,NG,SS(30,30),SSD(30,30),SX(30,30),X(30,30),
     2 XD(30),XL(30),XM(30),Z(30,30)
      COMMON /DIMENS/ MDIM,MGDIM
      COMMON /SCRCOM/ DUMMY1(1800),D(30,30),RX(30,30)
      COMMON /UNITS/ LIN,LUT,MTDATA
      DIMENSION IFMT(12),XX(30),SD(30),TITLE(12)
      READ(LIN,21) NG,KLOG,MOUT
      READ(LIN,22) TITLE
      ENG=NG
      READ(LIN,22) IFMT
      DO 230 I=1,M
      SX(I)=0.
      DO 230 J=1,M
  230 SS(I,J)=0.
```

```
      CASES=ENG
  240 READ(LIN,IFMT)(XX(I),I=1,M)
      IF(KLOG) 2240,2240,2241
 2240 GO TO 2244
 2241 DO 2243 I=1,M
 2243 XX(I) = ALOG10(XX(I))
 2244 CONTINUE
      WRITE(MTDATA)(XX(I),I=1,M)
      DO 260 I=1,M
      SX(I) = SX(I) + XX(I)
      DO 260 J=I,M
      SS(I,J) = SS(I,J) + XX(I)*XX(J)
  260 SS(J,I)=SS(I,J)
C---- RAW SUMS OF SQUARES AND CROSS PRODUCTS
      CASES=CASES-1.
      IF(CASES)280,280,240
  280 DO 286 I=1,M
      DO 286 J=I,M
      SSD(I,J)=SS(I,J)-SX(I)*SX(J)/ENG
  286 SSD(J,I)=SSD(I,J)
      DO 295 I=1,M
      XM(I)=SX(I)/ENG
  295 SD(I)=SQRT(SSD(I,I)/(ENG-1.))
C     MEANS AND STANDARD DEVIATION
      IF(MOUT) 435,435,690
  690 WRITE (LUT,31) TITLE,M,NG,(XM(I),I=1,M)
      WRITE (LUT,32)(SD(I),I=1,M)
      CALL MATOUT (SS,M,M,MDIM,39,39H1RAW SUMS OF SQUARES AND CROSS PROD
     1UCTS )
      IF(MOUT.GT.0.AND.M.GT.10) WRITE(6,909)
      CALL MATOUT (SSD,M,M,MDIM,45,45H0DEVIATION SUMS OF SQUARES AND CRO
     1SS PRODUCTS)
  435 DO 441 I=1,M
      DO 441 J=I,M
      D(I,J)=SSD(I,J)/(ENG-1.)
  441 D(J,I)=D(I,J)
C---- DISPERSION MATRIX
      IF (MOUT .GT. 0) CALL MATOUT (D,M,M,MDIM,18,18H1COVARIANCE MATRIX)
  670 DO 486 I=1,M
      DO 486 J=I,M
      RX(I,J)=D(I,J)/(SD(I)*SD(J))
  486 RX(J,I)=RX(I,J)
      IF(MOUT.GT.0.AND.M.GT.10) WRITE(6,909)
C     CORRELATION MATRIX
      IF (MOUT .GT. 0)CALL MATOUT(RX,M,M,MDIM,19,19H0CORRELATION MATRIX)
      CALL HOMMAT(M,D,DET)
C----------
      ENI = ENG
  700 RETURN
C----------------------------- FORMATS -------------------------------------
   21 FORMAT(I5,2I1)
   22 FORMAT(12A6)
   31 FORMAT (26H1CORRELATION ANALYSIS FOR ,12A6/23H0NUMBER OF VARIABLES
     1 = ,I3/26H0NUMBER OF OBSERVATIONS = ,I4/13H0MEAN VECTOR /(9F14.5))
   32 FORMAT (35H0STANDARD DEVIATIONS OF THIS SAMPLE /(9F14.5))
  909 FORMAT(1H1)
      END
```

```
      SUBROUTINE EIGGEN
C----------THIS SUBROUTINE COMPUTES THE GENERALIZED EIGENVALUES
C-------AND EIGENVECTORS OF A NON-SYMMETRIC MATRIX OF THE FORM
C      B-INVERSE**A VECTOR XL CONTAINS THE CANONICAL VARIATE
C------COEFFICIENTS AS COLUMNS
      COMMON A(30,30),B(30,30),C(60,30),DET,DETM(30),ENI,ENUMB(30),
     1 L,M,MG,NI,NG,SS(30,30),SSD(30,30),SX(30,30),X(30,30),
     2 XD(30),XL(30),XM(30),Z(30,30)
      COMMON /DIMENS/ MDIM,MGDIM
      COMMON /UNITS/ LIN,LUT,MTDATA
      WRITE(LUT,1060)
      DO 1 I=1,M
      XL(I)=0.
      DO 1 J=1,M
    1 X(I,J)=0.
      CALL HDIAG(B,M,0,X,NR)
      DO 10 I=1,M
   10 XL(I)=1./SQRT(ABS(B(I,I)))
      DO 20 I=1,M
      DO 20 J=1,M
   20 B(I,J)=X(I,J)*XL(J)
      DO 35 I=1,M
      DO 35 J=1,M
      X(I,J)=0.
      DO 35 K=1,M
   35 X(I,J)=X(I,J)+B(K,I)*A(K,J)
      DO 40 I=1,M
      DO 40 J=1,M
      A(I,J)=0.
      DO 40 K=1,M
   40 A(I,J)=A(I,J)+X(I,K)*B(K,J)
      CALL HDIAG(A,M,0,X,NR)
      WRITE(LUT,1066)(A(I,I),I=1,M)
      DO 55 I=1,M
   55 XL(I)=A(I,I)
      SUMMA=0.
      DO 60 I=1,M
      SUMMA=SUMMA+XL(I)
      IF(XL(I).LT.0.) WRITE(LUT,432)
   60 WRITE(LUT,65)I,XL(I)
      DO 70 I=1,M
      DO 70 J=1,M
      A(I,J)=0.
      Z(I,J)=0.
      DO 70 K=1,M
   70 A(I,J)=A(I,J)+B(I,K)*X(K,J)
      CALL MATOUT (A,M,M,MDIM,15,15H0  EIGENVECTORS)
      DO 71 I=1,M
      DO 71 J=1,M
   71 Z(I,J)=A(I,J)
      DO 75 J=1,M
      SUM=0.
      DO 80 I=1,M
   80 SUM=SUM+(A(I,J)**2)
      DEN=SQRT(SUM)
      DO 85 I=1,M
   85 X(I,J)=A(I,J)/DEN
   75 CONTINUE
      CALL MATOUT (X,M,M,MDIM,26,26H0  NORMALIZED EIGENVECTORS)
      CALL STAND
      RETURN
```

```
C----------------------------  FORMATS  ---------------------------------------
   65 FORMAT(16H0   EIGENVALUE  I2,F14,7)
  432 FORMAT(33H0  NEGATIVE EIGENVALUE OBTAINED                )
 1060 FORMAT(49H1  SOLUTION OF GENERALIZED DETERMINANTAL EQUATION)
 1066 FORMAT(1X,12F10,4)
      END
```

```
      SUBROUTINE GETMAT (STMAT,M,N,MDIM,MATNO)
C------- GETMAT FETCHES THE STORED MATRIX NO MATNO,
C-------- THIS MATRIX IS PLACED IN THE PARAMETER STMAT,
C------- DIMENSIONED IN THE CALLING PROGRAM TO (MDIM,SOMETHING)
C------- THE ACTUAL MATRIX IS OF SIZE M*N,
C------- THIS VERSION OF GETMAT FETCHES THE MATRIX FROM A PLACE IN
C------- THE CORE MEMORY (COMMON/SAVMAT/ ) BUT THE ROUTINE CAN BE EASILY
C------- REWRITTEN TO FETCH THE MATRIX FROM AN EXTERNAL MEDIUM SUCH AS A
C------- MAGNETIC TAPE OR DRUM,
C------- NOTE THAT THE MATRICES MUST FIRST BE STORED BY ROUTINE PUTMAT
C------- WHICH MUST WORK IN THE SAME MANNER AS GETMAT,
      DIMENSION STMAT (MDIM,25)
      COMMON /MATRIX/ MATNR,MTMAT
      COMMON /SAVMAT/ IX(3),SAVE(3600)
      DATA (IX=1,901,1801)
      JX=IX(MATNO)
      DO 1 I=1,M
      DO 1 J=1,N
      STMAT(I,J)=SAVE(JX)
    1 JX=JX+1
      RETURN
      END
```

```
      SUBROUTINE HDIAG (H,N,IEGEN,U,NR)
C-------- THIS SUBROUTINE CALCULATES THE EIGENVALUES AND EIGENVECTORS
C-------- OF A SQUARE SYMMETRIC MATRIX
C--------         --------       --------       --------       --------
C---------***NOTE****NOTE****
C-------- THE LATTER PART OF THE SUBROUTINE MUST BE REWRITTEN FOR
C-------- AN IBM FORTRAN COMPILER
      DIMENSION H(30,30),U(30,30),X(30),IQ(30)
      IF(IEGEN)15,10,15
 10   DO 14 I=1,N
      DO 14 J=1,N
      IF(I-J)12,11,12
 11   U(I,J)=1,
      GO TO 14
```

```
 12   U(I,J)=0.
 14   CONTINUE
 15   NR=0
      IF(N-1)1000,1000,17
 17   NMI1=N-1
      DO 30 I=1,NMI1
      X(I)=0.
      IPL1=I+1
      DO 30 J=IPL1,N
      IF(X(I)-ABS (H(I,J)))20,20,30
 20   X(I)=ABS (H(I,J))
      IQ(I)=J
 30   CONTINUE
      RAP=7.450580596E-9
      HDTEST=1.0E38
 40   DO 70 I=1,NMI1
      IF(I-1)60,60,45
 45   IF(XMAX-X(I))60,70,70
 60   XMAX=X(I)
      IPIV=I
      JPIV=IQ(I)
 70   CONTINUE
      IF(XMAX)1000,1000,80
 80   IF(HDTEST)90,90,85
 85   IF(XMAX-HDTEST)90,90,148
 90   HDIMIN=ABS (H(1,1))
      DO 110 I=2,N
      IF(HDIMIN-ABS (H(I,I)))110,110,100
 100  HDIMIN=ABS (H(I,I))
 110  CONTINUE
      HDTEST=HDIMIN*RAP
C     RETURN IF MAX. H(I,J) LESS THAN (2**-27)ABSF(H(K,K)-MIN)
      IF(HDTEST-XMAX)148,1000,1000
 148  NR=NR+1
C     COMPUTE TANGENT, SINE AND COSINE , H(I,I),H(J,J)
 150  IF(H(IPIV,IPIV)-H(JPIV,JPIV))91,92,92
 91   A=-2.0
      GO TO 93
 92   A=2.0
 93   TANG=A*H(IPIV,JPIV)/(ABS (H(IPIV,IPIV)-H(JPIV,JPIV))+SQRT ((H(IPIV
     1,IPIV)-H(JPIV,JPIV))**2+4.0*H(IPIV,JPIV)**2))
      COSINE=1.0/SQRT (1.0+TANG**2)
      SINE=TANG*COSINE
      HII=H(IPIV,IPIV)
      H(IPIV,IPIV)=COSINE**2*(HII+TANG*(2.*H(IPIV,JPIV)+TANG*H(JPIV,JPIV
     1)))
      H(JPIV,JPIV)=COSINE**2*(H(JPIV,JPIV)-TANG*(2.*H(IPIV,JPIV)-TANG*HI
     1I))
      H(IPIV,JPIV)=0.
      IF(H(IPIV,IPIV)-H(JPIV,JPIV))152,153,153
 152  HTEMP=H(IPIV,IPIV)
      H(IPIV,IPIV)=H(JPIV,JPIV)
      H(JPIV,JPIV)=HTEMP
      IF(SINE)94,95,95
 94   HTEMP=COSINE
      GO TO 96
 95   HTEMP=-COSINE
 96   COSINE=ABS (SINE)
      SINE=HTEMP
 153  CONTINUE
```

```
      DO 350 I=1,NMI1
      IF(I-IPIV)210,350,200
200   IF(I-JPIV)210,350,210
210   IF(IQ(I)-IPIV)230,240,230
230   IF(IQ(I)-JPIV)350,240,350
240   K=IQ(I)
250   HTEMP=H(I,K)
      H(I,K)=0.
      IPL1=I+1
      X(I)=0.
      DO 320 J=IPL1,N
      IF(X(I)-ABS (H(I,J)))300,300,320
300   X(I)=ABS (H(I,J))
      IQ(I)=J
320   CONTINUE
      H(I,K)=HTEMP
350   CONTINUE
      X(IPIV)=0.
      X(JPIV)=0.
      DO 530 I=1,N
      IF(I-IPIV)370,530,420
370   HTEMP=H(I,IPIV)
      H(I,IPIV)=COSINE*HTEMP+SINE*H(I,JPIV)
      IF(X(I)-ABS (H(I,IPIV)))380,390,390
380   X(I)=ABS (H(I,IPIV))
      IQ(I)=IPIV
390   H(I,JPIV)=-SINE*HTEMP+COSINE*H(I,JPIV)
      IF(X(I)-ABS (H(I,JPIV)))400,530,530
400   X(I)=ABS (H(I,JPIV))
      IQ(I)=JPIV
      GO TO 530
420   IF(I-JPIV)430,530,480
430   HTEMP=H(IPIV,I)
      H(IPIV,I)=COSINE*HTEMP+SINE*H(I,JPIV)
      IF(X(IPIV)-ABS (H(IPIV,I)))440,450,450
440   X(IPIV)=ABS (H(IPIV,I))
      IQ(IPIV)=I
450   H(I,JPIV)=-SINE*HTEMP+COSINE*H(I,JPIV)
      IF(X(I)-ABS (H(I,JPIV)))400,530,530
480   HTEMP=H(IPIV,I)
      H(IPIV,I)=COSINE*HTEMP+SINE*H(JPIV,I)
      IF(X(IPIV)-ABS (H(IPIV,I)))490,500,500
490   X(IPIV)=ABS (H(IPIV,I))
      IQ(IPIV)=I
500   H(JPIV,I)=-SINE*HTEMP+COSINE*H(JPIV,I)
      IF(X(JPIV)-ABS (H(JPIV,I)))510,530,530
510   X(JPIV)=ABS(H(JPIV,I))
      IQ(JPIV)=I
530   CONTINUE
      IF(IEGEN)40,540,40
540   DO 550 I=1,N
      HTEMP=U(I,IPIV)
      U(I,IPIV)=COSINE*HTEMP+SINE*U(I,JPIV)
550   U(I,JPIV)=-SINE*HTEMP+COSINE*U(I,JPIV)
      GO TO 40
1000  RETURN
      END
```

```
      SUBROUTINE HOMMAT(M,D,DET)
C----------INTERMEDIATE STEP IN THE TEST FOR HOMOGENEITY OF
C----------COVARIANCE MATRICES****
C----------AS A SIDELINE IT GIVES AN ABRIDGED PRINCIPAL COMPONENTS
C----------BREAKDOWN***
      COMMON /DIMENS/ MDIM,MGDIM
      COMMON /SCRCOM/ DUMMY1(900),H(30,30),DUMMY2(1800)
      COMMON /UNITS/ LIN,LUT,MTDATA
      DIMENSION D(30,30)
      WRITE(LUT,2)
      CALL HDIAG(D,M,0,H,NR)
      DET=1.0
      DO 1 I=1,M
    1 DET = DET*D(I,I)
C----------
      WRITE(LUT,15) DET
C----------
      WRITE (LUT,3) (D(I,I),I=1,M)
      SUM = 0.0
      DO 4 I=1,M
    4 SUM = SUM + D(I,I)
      DO 5 I=1,M
      PRO = (D(I,I)/SUM)*100.
      WRITE(LUT,6) I,PRO
    5 CONTINUE
      CALL MATOUT (H,M,M,MDIM,35,35H0  ELEMENTS OF PRINCIPAL COMPONENTS)
      RETURN
C---------------------------- FORMATS -------------------------------------
    2 FORMAT(20H1 OUTPUT FROM HOMMAT  )
    3 FORMAT (33H0    PRINCIPAL COMPONENTS SUMMARY/20H0  VARIANCES FOR P
     1CA/(X,10F13.6))
    6 FORMAT(18H0  PERCENTAGE FOR I2,3H = F14.4)
   15 FORMAT(9H0  DET = E18.11)
      END
```

```
      SUBROUTINE HOMO (DETM,M,ENUMB,NI,MG)
      COMMON /DIMENS/ MDIM,MGDIM
      COMMON /SCRCOM/ B(30,30),X(30,30),DUMMY(1800)
      COMMON /UNITS/ LIN,LUT,MTDATA
      DIMENSION DETM(30),ENUMB(30)
C------THIS SUBRT COMPUTES THE TEST FOR HOMOGENEITY OF COVARIANCE
C------MATRICES...IT IS ONLY CALLED WHEN RAW DATA ARE USED
      WRITE(LUT,30)
      CALL GETMAT (B,M,M,MDIM,1)
      DO 2 I=1,M
      ENUMB(I) = ENUMB(I)-1.0
      DO 2 J=1,M
    2 B(I,J) = B(I,J)/FLOAT(NI-MG)
      CALL HDIAG(B,M,1,X,NR)
      DET2=1.0
      DO 5 I=1,M
    5 DET2 = DET2*B(I,I)
      RES = 0.0
      DO 6 I=1,MG
    6 RES = RES + ENUMB(I)*ALOG(DET2/DETM(I))
```

```
      WRITE(LUT,9) RES
      KDF = (MG-1)*M*(M+1)/2
      WRITE(LUT,8) KDF
      RETURN
C---------------------------  FORMATS  ------------------------------------
    8 FORMAT(8H0   DF = I3)
    9 FORMAT(9H0 CHSQ = F14,6)
   23 FORMAT(12F10,6)
   30 FORMAT(41H1     HOMOGENEITY OF COVARIANCE MATRICES         )
      END




      SUBROUTINE MATOUT (STMAT,MG,M,NDIM,NCHAR,TITLE)
C------- MATOUT PRINTS OUT THE MG*M MATRIX STMAT,
C------- STMAT IS SUPPOSED TO BE DIMENSIONED TO (NDIM,SOMETHING)
C------- BEFORE WRITING THE MATRIX A HEAD TITLE IS WRITTEN
C------- THE TITLE LENGTH IS NCHAR CHARACTERS WHICH ARE WRITTEN IN
C------- COLUMNS 1 TO NCHAR,
C------- IN CD 3600 EACH CELL CONTAINS 8 CHARACTER SO NCHPW IS SET TO 8
C------- IF NCHPW IS CHANGED , DO NOT FORGET TO CHANGE FORMAT 103 TOO,
      DIMENSION STMAT (NDIM,25),TITLE(1)
      COMMON /UNITS/ LIN,LUT,MTDATA
      DATA(NCHPW=8)
      IW=(NCHAR+NCHPW-1)/NCHPW
      WRITE (LUT,103) (TITLE(J),J=1,IW)
      MEND = 0
    1 MBEG = MEND + 1
      MEND = MBEG + 9
      IF(MEND,GT,M)MEND = M
      WRITE(LUT,101)(J,J=MBEG,MEND)
      DO 2 I=1,MG
    2 WRITE(LUT,102) I,(STMAT(I,J),J=MBEG,MEND)
      IF(MEND,EQ,M) RETURN
      GO TO 1
C---------------------------  FORMATS  ------------------------------------
  101 FORMAT(1H0,10I12)
  102 FORMAT(2X,I2,2X,10F12,5)
  103 FORMAT (17A8)
      END




      SUBROUTINE PLT3 (X1,Y1,N1,IVPLOT,CHAR,MG)
C------- PLT3 PLOTS THE N1 COORDINATES IN X1,Y1 ON THE PRINTER,
C------- IVPLOT IS AN ARRAY THAT CONTAINS INDEX LIMITS FOR DIFFERENT
C-------  GROUPS OF DATA, EACH GROUP ARE WRITTEN WITH A SPECIAL SYMBOL,
C------- MG IS THE NUMBER OF GROUPS, IF MG IS NEGATIVE EACH X1,Y1 PAIR
C-------  IS PLOTTED WITH A NEW SYMBOL,
C------- CHAR CONTAINS THE SYMBOLS THAT ARE TO BE USED, ONE CHARACTER IN
C-------  EACH CELL IN A1-FORMAT,
```

```
      COMMON /UNITS/ LIN,LUT,MTDATA
      DIMENSION X1(10),Y1(10),IVPLOT(10),CHAR(10),ALINE(101)
      DATA(BLANK=1H ),(DASH=1H-),(UP=1HI),(DOT=1H,),(AST=1H*)
      XMAX = X1(1)
      XMIN = XMAX
      YMAX = Y1(1)
      YMIN = YMAX
      DO 1 I=2,N1
      IF(X1(I),GT,XMAX) XMAX = X1(I)
      IF(X1(I),LT,XMIN) XMIN = X1(I)
      IF(Y1(I),GT,YMAX) YMAX = Y1(I)
      IF(Y1(I),LT,YMIN) YMIN = Y1(I)
    1 CONTINUE
      KEY = 1
      ZMAX = XMAX
      ZMIN = XMIN
    4 RANGE = ZMAX - ZMIN
      SCALE = 1,E-9
    5 SCALE = 10,*SCALE
      IF(SCALE,LT,RANGE) GO TO 5
      ZMIN = ZMIN/SCALE
      ZMAX = ZMAX/SCALE
      MIN = 20,0*(ZMIN - 0,025)
      MAX = 20,0*(ZMAX + 0,025)
      BOTTOM = 0,05*FLOAT(MIN)
      TOP = 0,05*FLOAT(MAX)
      RANGE = 0,1*(TOP-BOTTOM)
    6 IF(BOTTOM,LE,ZMIN) GO TO 7
      BOTTOM = BOTTOM - RANGE
      GO TO 6
    7 IF(TOP,GE,ZMAX) GO TO 8
      TOP = TOP + RANGE
      GO TO 7
    8 YINC = 0,01*(TOP -BOTTOM)
      IF(KEY,EQ,2) GO TO 11
      KEY = 2
      ZMAX = YMAX
      ZMIN = YMIN
      XSCALE = SCALE
      XTOP = TOP
      XINC = YINC*SCALE
      XBOT = BOTTOM*SCALE
      GO TO 4
C
   11 YLOW = TOP + YINC
      YINC = 2,0*YINC
C
      KEY = 5
      WRITE(LUT,1000) XSCALE,SCALE
   15 KKEY = 10
      IF(KEY.NE.5) GO TO 19
      DO 18 I=1,101
      IF(KKEY.NE.10) GO TO 16
      ALINE(I) = DOT
      GO TO 17
   16 ALINE(I) = DASH
   17 KKEY = KKEY - 1
      IF(KKEY.EQ.0) KKEY = 10
   18 CONTINUE
      GO TO 23
```

```
   19 DO 22 I=1,101
      IF(KKEY.NE.10) GO TO 20
      ALINE(I) = UP
      GO TO 21
   20 ALINE(I) = BLANK
   21 KKEY = KKEY - 1
      IF(KKEY.EQ.0) KKEY = 10
   22 CONTINUE
   23 CONTINUE
C     PUT POINTS ON ALINE
      YHIGH = YLOW
      YLOW = YHIGH -YINC
      YHS = SCALE*YHIGH
      YLS = SCALE*YLOW
      DO 24 I=1,N1
      IF(Y1(I).GT.YHS.OR.Y1(I).LE.YLS) GO TO 24
      INDEX = (X1(I) - XBOT)/XINC
      INDEX = INDEX + 1
      IF(INDEX.GT.101) INDEX = 101
      MGA=IABS(MG)
      IF (MG .GT. 0) GO TO 102
      IF (MGA .EQ. 0) MGA=1
      L=I-1
      L=L-L/MGA*MGA+1
      GO TO 100
  102 DO 10 L=1,MG
   10 IF (I .LE. IVPLOT(L)) GO TO 103
      L=MG
  103 IF (L .LE. 25) GO TO 100
      L=L-1
      L=L-L/25*25+1
  100 IF (ALINE (INDEX) .EQ. UP .OR. ALINE(INDEX) .EQ. DASH .OR.
     1 ALINE (INDEX) .EQ. DOT .OR. ALINE(INDEX) .EQ. BLANK) GO TO 101
      IF (ALINE (INDEX) .EQ. CHAR(L) ) GO TO 24
      ALINE (INDEX) =AST
      GO TO 24
  101 ALINE(INDEX)=CHAR(L)
   24 CONTINUE
C     WRITE ALINE OUT
      IF(KEY.NE.5) GO TO 28
      WRITE(LUT,1001) TOP,ALINE
      GO TO 29
   28 WRITE(LUT,1002) ALINE
   29 CONTINUE
C
      KEY = KEY - 1
      IF(KEY.EQ.0) KEY = 5
      TOP = TOP - YINC
      IF(TOP.GE.BOTTOM) GO TO 15
      IF(KEY.NE.4) GO TO 15
C
      XINC = 10.*XINC/XSCALE
      ALINE(1) = XBOT/XSCALE
      DO 30 I=2,11
   30 ALINE(I) = ALINE(I-1) +XINC
      WRITE(LUT,1003) (ALINE(I),I=1,11)
      RETURN
C------------------------------ FORMATS ------------------------------
 1000 FORMAT(22H1SCALE FACTOR ON X IS ,  E9.2,4X,21HSCALE FACTOR ON Y IS
     1 ,E9.2/1H )
 1001 FORMAT(F10.3,1X,101A1)
```

```
 1002 FORMAT(11X,101A1)
 1003 FORMAT(5X,11F10.3)
      END


      SUBROUTINE PUTMAT (STMAT,M,N,MDIM,MATNO)
C------- PUTMAT STORES MATRIX STMAT AND NOTES IT AS MATRIX NO. MATNO.
C------- STMAT IS SUPPOSED TO BE DIMENSIONED TO (MDIM,SOMETHING).
C------- THE ACTUAL SIZE OF STMAT IS M*N ELEMENTS.
C------- PUTMAT CORRESPONDS TO SUBROUTINE GETMAT AND THIS VERSION
C------- STORES THE MATRICES IN MEMORY AT COMMON/SAVMAT/. IT IS EASY
C------- TO REWRITE PUTMAT AND GETMAT TO STORE THE MATRICES ON AN
C------- EXTERNAL MEDIUM LIKE MAGNETIC TAPE OR DRUM.
      DIMENSION STMAT (MDIM,25)
      COMMON /MATRIX/ MATNR,MTMAT
      COMMON /SAVMAT/ IX(3),SAVE(3600)
      DATA (IX=1,901,1801)
      JX=IX(MATNO)
      DO 1 I=1,M
      DO 1 J=1,N
      SAVE(JX)=STMAT(I,J)
    1 JX=JX+1
      RETURN
      END


      SUBROUTINE STAND
C----------STANDARDIZES THE COORDINATES THROUGH THE MATRIX OF
C----------EIGENVECTORS
      COMMON A(30,30),B(30,30),C(60,30),DET,DETM(30),ENI,ENUMB(30),
     1 L,M,MG,NI,NG,SS(30,30),SSD(30,30),SX(30,30),X(30,30),
     2 XD(30),XL(30),XM(30),Z(30,30)
      COMMON /DIMENS/ MDIM,MGDIM
      COMMON /PLOT/ IVPLOT(60),CHAR(25),KGROUP
      COMMON /UNITS/ LIN,LUT,MTDATA
      DIMENSION F(30)
      DO 998 I=1,M
      SX(I)=0.0
  998 XM(I)=0.0
      CALL GETMAT (B,M,M,MDIM,1)
      DO 210 I=1,M
      DO 210 J=1,M
  210 B(I,J) = B(I,J)/FLOAT(NI-MG)
      K=1
  555 CONTINUE
  570 DO 556 I=1,M
  556 XD(I) =  Z(I,K)
      DO 560 I=1,M
      DO 560 J=1,M
  560 XM(I) = XM(I) + B(I,J)*XD(J)
      DO 565 I=1,M
  565 DEN = DEN + XM(I)*XD(I)
      SX(K) = DEN
      K = K+1
      IF(K-M)570,570,571
  571 DO 572 J=1,M
      DO 572 I=1,M
  572 Z (I,J) =  Z(I,J)/ SQRT(SX(J))
      CALL MATOUT  (Z,M,M,MDIM,32,32H0  ADJUSTED CANONICAL VARIATES   )
```

```
      CALL GETMAT (C,MG,M,MGDIM,3)
      DO 51 I1=1,MG
      DO 50 I2=1,M
      F(I2)=0
      DO 50 I3=1,M
   50 F(I2)=F(I2)+C(I1,I3)*Z(I3,I2)
      DO 51 I2=1,M
   51 C(I1,I2)=F(I2)
      DO 275 K=1,MG
      WRITE(LUT,280) K
      WRITE(LUT,231)(C(K,J),J=1,M)
  275 CONTINUE
      WRITE(LUT,350)
      CALL PLT3 (C,C(1,2),MG,MG,CHAR,-25)
      IF(L.EQ.0) GO TO 666
      CALL TRANSF ( Z,M,NI)
      WRITE(LUT,600)
  666 RETURN
C----------------------------------  FORMATS  ----------------------------------------
  231 FORMAT(1X,10F13.5)
  280 FORMAT(30H0   CANONICAL VARIATE MEAN NO,I2)
  350 FORMAT(35H1   PLOT OF CANONICAL MEANS FOLLOWS)
  600 FORMAT(50H1  PLOT OF TRANSFORMED OBSERVATIONS FOR 1 AND 2         )
      END


      SUBROUTINE TRANSF (A,M,N)
      COMMON /DIMENS/ MDIM,MGDIM
      COMMON /PLOT/ IVPLOT(60),CHAR(25),KGROUP
      COMMON /SCRCOM/ B(1300,2),DUMMY(1000)
      COMMON /UNITS/ LIN,LUT,MTDATA
      DIMENSION A(30,30),X(30),Y(30)
C------THE EIGENVECTORS ARE PLACED IN MATRIX A
C------THE TOTAL DATA IS IN MATRIX B WITH A MAXIMUM OF 1300 OBSERVATIONS
      REWIND MTDATA
      DO 20 KOUNT=1,N
      DO 14 I=1,M
   14 Y(I) = 0.0
      READ(MTDATA)(X(I),I=1,M)
      DO 1 J=1,M
      DO 1 I=1,M
    1 Y(J) = Y(J) + X(I)*A(I,J)
      B(KOUNT,1)=Y(1)
      B(KOUNT,2)=Y(2)
   20 CONTINUE
      WRITE(LUT,60)
      DO 5 I=1,N
      DO 4 J=1,KGROUP
    4 IF (I .LE. IVPLOT(J)) GO TO 40
      J=KGROUP
   40 IF (J .LE.25) GO TO 5
      J=J-J/25*25+1
    5 WRITE(LUT,10)I,B(I,1),B(I,2),CHAR(J)
      CALL PLT3 (B,B(1,2),N,IVPLOT,CHAR,KGROUP)
      RETURN
C----------------------------------  FORMATS  ----------------------------------------
   10 FORMAT(15X,I3,6X,F15.6,10X,F15.6,5X,A1)
   60 FORMAT(39H0                  TRANSFORMATION MATRIX                  )
      END
```

```
      SUBROUTINE WILKS
      COMMON A(30,30),B(30,30),C(60,30),DET,DETM(30),ENI,ENUMB(30),
     1 L,M,MG,NI,NG,SS(30,30),SSD(30,30),SX(30,30),X(30,30),
     2 XD(30),XL(30),XM(30),Z(30,30)
      COMMON /DIMENS/ MDIM,MGDIM
      COMMON /UNITS/ LIN,LUT,MTDATA
      WRITE(LUT,150)
      KDF = M*(MG-1)
      CALL GETMAT (B,M,M,MDIM,1)
      CALL GETMAT (X,M,M,MDIM,2)
      CALL HDIAG (B,M,0,SS,NR)
      CALL HDIAG (X,M,0,Z,NR)
      DO 5 I=1,M
      XL(I) = B(I,I)
      IF(B(I,I).LE.0.)GO TO 90
      XD(I) = X(I,I)
      IF(X(I,I).LE.0.) GO TO 95
    5 CONTINUE
      PROD = 1.
      PRODD = 1.
      DO 10 I=1,M
      PROD = PROD*XL(I)
   10 PRODD = PRODD*XD(I)
      WLAMDA=PROD/PRODD
      CHISQ = -(FLOAT(NI-M) - (FLOAT(M) + FLOAT(MG))*0.5)*ALOG(WLAMDA)
      WRITE (LUT,15) PROD,PRODD,WLAMDA,CHISQ,KDF
      RETURN
   90 MX=1
      GO TO 99
   95 MX=2
   99 WRITE (LUT,100) MX
      RETURN
C--------------------------- FORMATS -------------------------------------
   15 FORMAT (10H0  DETB = E18.9,9H  DETX = E18.9/18H0  WILKS LAMBDA =
     1F10.5/15H0  CHISQUARE = F12.3,7H  DF = I2 )
  100 FORMAT (8H0 MATRIX,I2,22H NOT POSITIVE DEFINITE)
  150 FORMAT(28H1  TEST OF EQUALITY OF MEANS )
      END
```

EXAMPLE 1

QUADRIVARIATE ECHINOID DATA (NICHOLLS)

CANONICAL VARIATES STUDY FOR GROUPS = 8

NUMBER OF VARIABLES = 4

OUTPUT FROM HOMMAT

DET = 4,54568677885-012

PRINCIPAL COMPONENTS SUMMARY

VARIANCES FOR PCA

0,045371	0,002353	0,000366	0,000116

PERCENTAGE FOR 1 = 94,1186

PERCENTAGE FOR 2 = 4,8811

PERCENTAGE FOR 3 = 0,7590

PERCENTAGE FOR 4 = 0,2414

ELEMENTS OF PRINCIPAL COMPONENTS

	1	2	3	4
1	0,52090	0,13920	0,52571	0,65796
2	0,55743	0,04433	0,36659	-0,74359
3	0,57685	0,30725	-0,75266	0,07969
4	0,29187	-0,94035	-0,15082	0,08838

END OF SAMPLE 1

OUTPUT FROM HOMMAT

DET = 1,34896993442-012

PRINCIPAL COMPONENTS SUMMARY

VARIANCES FOR PCA

0,035464	0,000894	0,000371	0,000115

PERCENTAGE FOR 1 = 96,2548

PERCENTAGE FOR 2 = 2,4278

PERCENTAGE FOR 3 = 1,0060

PERCENTAGE FOR 4 = 0,3114

ELEMENTS OF PRINCIPAL COMPONENTS

	1	2	3	4
1	0,50114	0,07877	-0,38589	-0,77055
2	0,56658	0,15406	-0,50152	0,63540
3	0,52735	0,41505	0,74124	0,01419
4	0,38697	-0,89319	0,22390	0,04824

END OF SAMPLE 2

OUTPUT FROM HOMMAT

DET = 7,39457759797-013

PRINCIPAL COMPONENTS SUMMARY

VARIANCES FOR PCA

0,022201	0,002171	0.000390	0,000039

PERCENTAGE FOR 1 = 89,5154

PERCENTAGE FOR 2 = 8,7526

PERCENTAGE FOR 3 = 1,5735

PERCENTAGE FOR 4 = 0,1585

ELEMENTS OF PRINCIPAL COMPONENTS

	1	2	3	4
1	0,56771	0,17986	0,30586	-0,74284
2	0,56231	0,24092	0,42891	0,66468
3	0,43416	-0,88563	-0,15471	0,05362
4	0,41595	0,35343	-0,83579	0,05933

END OF SAMPLE 3

OUTPUT FROM HOMMAT

DET = 1,33938128830-012

PRINCIPAL COMPONENTS SUMMARY

VARIANCES FOR PCA

0,027917	0,001110	0,000626	0,000069

PERCENTAGE FOR 1 = 93,9266

PERCENTAGE FOR 2 = 3,7360

PERCENTAGE FOR 3 = 2,1050

PERCENTAGE FOR 4 = 0,2323

ELEMENTS OF PRINCIPAL COMPONENTS

	1	2	3	4
1	0,51980	-0,25362	0,34898	-0,73736
2	0,59327	-0,16546	0,41327	0,67072
3	0,42883	-0,36696	-0,82461	0,03825
4	0,44039	0,87957	-0,16567	-0,07048

END OF SAMPLE 4

OUTPUT FROM HOMMAT

DET = 1,60798379677-013

PRINCIPAL COMPONENTS SUMMARY

VARIANCES FOR PCA

0,006785	0,000687	0,000554	0,000062

PERCENTAGE FOR 1 = 83,8794

PERCENTAGE FOR 2 = 8,4979

PERCENTAGE FOR 3 = 6,8542

PERCENTAGE FOR 4 = 0,7685

ELEMENTS OF PRINCIPAL COMPONENTS

	1	2	3	4
1	0,51044	-0,15727	0,17411	-0,82729
2	0,57211	0,15091	0,65984	0,46317
3	0,41142	-0,75521	-0,40338	0,31252
4	0,49282	0,61818	-0,60958	0,05827

END OF SAMPLE 5

OUTPUT FROM HOMMAT

DET = 3,34760786348-013

PRINCIPAL COMPONENTS SUMMARY

VARIANCES FOR PCA

0,008196	0,000689	0,000536	0,000111

PERCENTAGE FOR 1 = 85,9886

PERCENTAGE FOR 2 = 7,2275

PERCENTAGE FOR 3 = 5,6234

PERCENTAGE FOR 4 = 1,1605

ELEMENTS OF PRINCIPAL COMPONENTS

	1	2	3	4
1	0,51402	-0,33865	-0,29002	-0,73279
2	0,57669	-0,13594	-0,47080	0,65368
3	0,58591	0,69140	0,41630	-0,07329
4	0,24477	-0,62354	0,72176	0,17421

END OF SAMPLE 6

OUTPUT FROM HOMMAT

DET = 4,01412451675-011

PRINCIPAL COMPONENTS SUMMARY

VARIANCES FOR PCA

0,034727	0,003068	0,001453	0,000259

PERCENTAGE FOR 1 = 87,8998

PERCENTAGE FOR 2 = 7,7661

PERCENTAGE FOR 3 = 3,6779

PERCENTAGE FOR 4 = 0,6563

ELEMENTS OF PRINCIPAL COMPONENTS

	1	2	3	4
1	0,59134	-0,19871	0,48542	0,61254
2	0,59903	-0,10161	0,19896	-0,76893
3	0,37683	0,88734	-0,23995	0,11422
4	0,38663	-0,40350	-0,81683	0,14317

END OF SAMPLE 7

OUTPUT FROM HOMMAT

DET = 2,06863500754-013

PRINCIPAL COMPONENTS SUMMARY

VARIANCES FOR PCA

0,006691 0,001002 0,000451 0,000068

PERCENTAGE FOR 1 = 81,4720

PERCENTAGE FOR 2 = 12,2044

PERCENTAGE FOR 3 = 5,4906

PERCENTAGE FOR 4 = 0,8330

ELEMENTS OF PRINCIPAL COMPONENTS

	1	2	3	4
1	0,56067	-0,18058	0,40714	-0,69805
2	0,54897	-0,11616	0,41770	0,71461
3	0,53624	-0,24119	-0,80859	0,02148
4	0,31101	0,94643	-0,07711	-0,04001

END OF SAMPLE 8

POOLED WITHIN MATRIX B(I,J)

	1	2	3	4
1	1,40501	1,46620	1,20740	0,89464
2	1,46620	1,58664	1,29529	0,96367
3	1,20740	1,29529	1,35395	0,75357
4	0,89464	0,96367	0,75357	0,83131

AMONG GROUPS MATRIX

	1	2	3	4
1	0,23438	0,19917	0,21390	0,18895
2	0,19917	0,22179	0,23438	0,29738

3	0,21390	0,23438	0,33328	0,43531
4	0,18895	0,29738	0,43531	0,88268

TOTAL MATRIX

	1	2	3	4
1	1,63938	1,66538	1,42130	1,08359
2	1,66538	1,80842	1,52967	1,26105
3	1,42130	1,52967	1,68722	1,18888
4	1,08359	1,26105	1,18888	1,71399

TOTAL SAMPLE SIZE = 201

TOTAL MEAN VECTOR

1,66640 1,67721 1,49623 2,29956

TABLE OF MEANS

	1	2	3	4
1	1,70277	1,68090	1,50159	2,24003
2	1,63293	1,63278	1,44638	2,21360
3	1,65323	1,68497	1,50200	2,27712
4	1,61002	1,63860	1,45376	2,31044
5	1,74403	1,76557	1,58049	2,42014
6	1,66015	1,66958	1,51709	2,35598
7	1,66323	1,67612	1,45012	2,26499
8	1,68300	1,70143	1,54282	2,38864

SOLUTION OF GENERALIZED DETERMINANTAL EQUATION

3,2467 0,8462 0,1223 0,1043

EIGENVALUE 1 3,2466742

EIGENVALUE 2 0,8461712

EIGENVALUE 3 0,1223404

EIGENVALUE 4 0,1042624

EIGENVECTORS

	1	2	3	4
1	-1,83993	-3,94400	-0,89652	0,64296
2	0,12973	4,34435	-0,44351	-1,19180
3	0,64160	-0,54657	1,58175	-0,49037
4	1,83829	-0,56072	-0,50173	0,43730

NORMALIZED EIGENVECTORS

	1	2	3	4
1	-0,68603	-0,66626	-0,46271	0,42717
2	0,04837	0,73389	-0,22890	-0,79181
3	0,23922	-0,09233	0,81636	-0,32579
4	0,68542	-0,09472	-0.25895	0,29054

ADJUSTED CANONICAL VARIATES

	1	2	3	4
1	-25,56107	-38,74362	-7,19085	4,46615
2	1,80230	42,67649	-3,55734	-8,27848
3	8,91333	-5,36925	12,68688	-3,40620
4	25,53836	-5,50818	-4,02429	3,03758

CANONICAL VARIATE MEAN NO, 1
30,09561 -14,63739 -8,18792 -4.62095

CANONICAL VARIATE MEAN NO, 2
30,62720 -13,54327 -8,10852 -4,42669

CANONICAL VARIATE MEAN NO, 3
32,32019 -12,75075 -7,99029 -4,76461

CANONICAL VARIATE MEAN NO, 4
33,76226 -12,98009 -8,26069 -4.30819

CANONICAL VARIATE MEAN NO, 5
34,49681 -14,03812 -8,50962 -4,85927

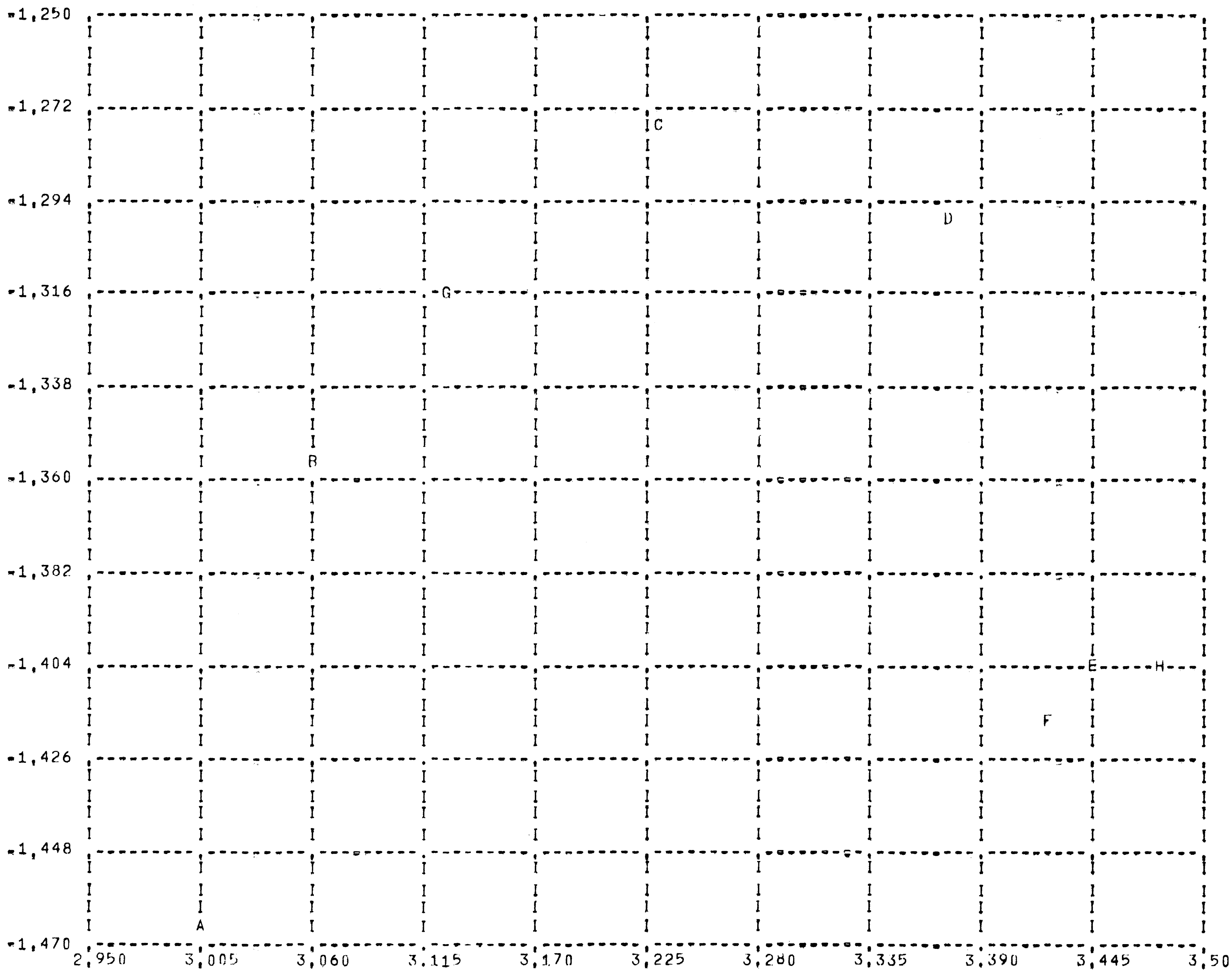
CANONICAL VARIATE MEAN NO, 6
34,26398 -14,19114 -8,11114 -4,41813

CANONICAL VARIATE MEAN NO, 7
31,27645 -13,17068 -8.64007 -4,50681

CANONICAL VARIATE MEAN NO, 8
34,80082 -14,03512 -8,19370 -4,56824

PLOT OF CANONICAL MEANS FOLLOWS

SCALE FACTOR ON X IS 1,00+001 SCALE FACTOR ON Y IS 1,00+001



TRANSFORMATION MATRIX

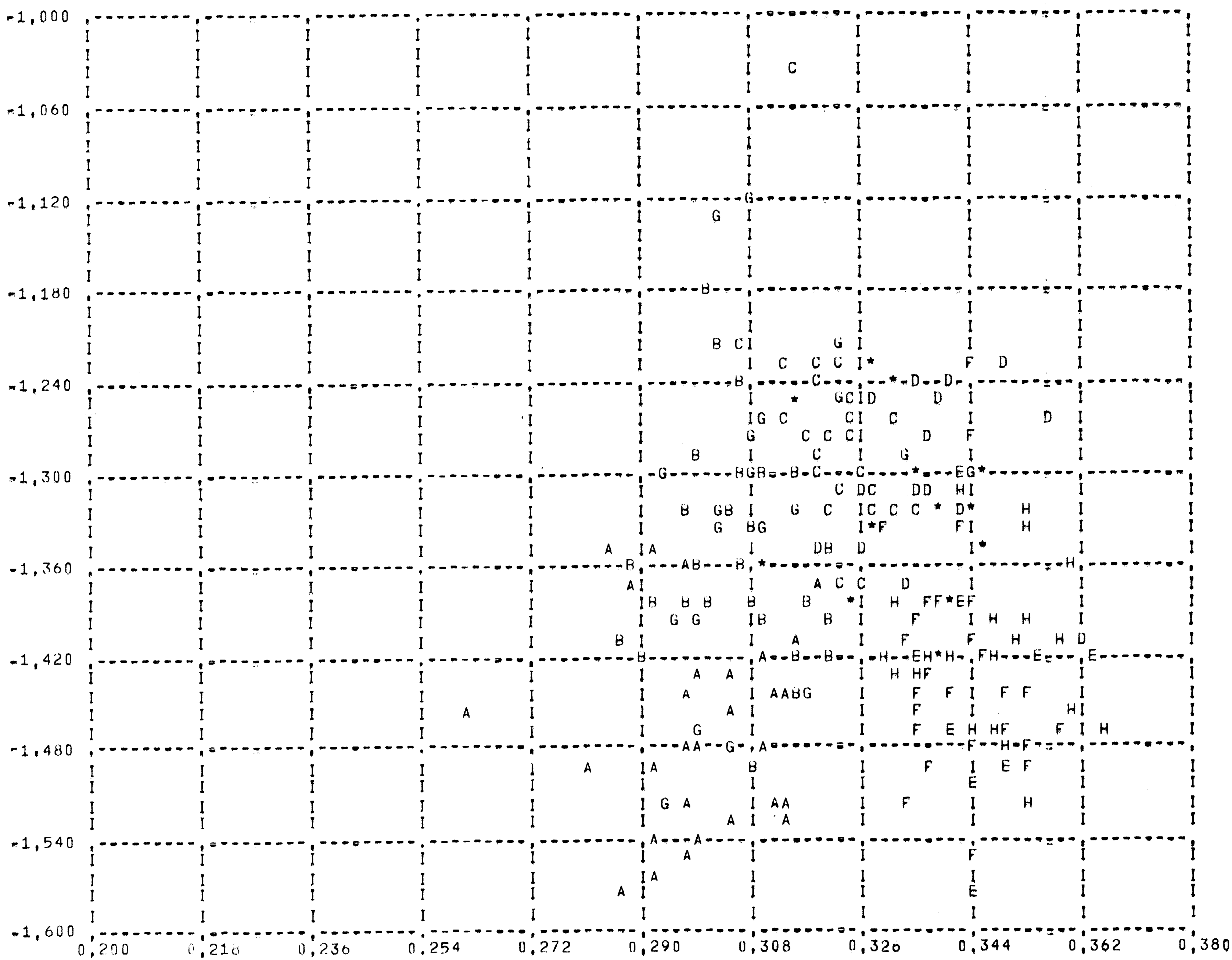
1	29.747655	-15.198586	A
2	28.147211	-14.966214	A
3	29.740079	-15.470204	A
4	31.172192	-15.173944	A
5	31.188338	-14.382835	A
6	28.609175	-13.425965	A
7	28.830465	-13.779984	A
8	29.900590	-14.744183	A
9	29.938222	-15.377861	A
10	29.163266	-13.473598	A
11	29.796929	-14.479261	A
12	29.319540	-15.425130	A
13	30.500155	-15.316794	A
14	31.107638	-14.784194	A
15	31.599472	-14.135922	A
16	30.587652	-14.318804	A
17	29.282345	-15.665207	A
18	28.816639	-15.720507	A
19	31.935864	-13.720647	A
20	29.327143	-14.959846	A
21	32.435298	-13.888002	A
22	29.721732	-13.602300	A
23	31.369217	-15.203215	A
24	31.468267	-14.382751	A
25	31.468604	-15.221663	A
26	26.266894	-14.524114	A
27	30.477050	-14.574115	A
28	29.841067	-14.767878	A
29	31.014576	-14.154495	A
30	30.075033	-14.283375	A
31	31.716317	-13.834124	B
32	30.282452	-12.120579	B
33	30.140409	-13.888503	B
34	32.095790	-13.487536	B
35	31.047092	-13.588590	B
36	30.938922	-13.789282	B
37	30.977559	-13.830032	B
38	28.977394	-13.563321	B
39	29.084825	-14.217397	B
40	30.664382	-13.020435	B
41	29.959527	-13.582877	B
42	31.667163	-14.178664	B
43	30.775694	-13.585984	B
44	30.237693	-11.762058	B
45	29.775906	-13.292205	B
46	29.329229	-13.793265	B
47	31.139372	-13.027462	B
48	31.566038	-13.054644	B
49	29.879859	-13.299308	B
50	31.620415	-14.480632	B
51	31.097909	-13.910358	B
52	32.209308	-13.972338	B
53	29.851327	-13.883648	B
54	30.528022	-13.259512	B
55	30.805377	-13.375431	B
56	32.092908	-14.196037	B
57	29.923500	-12.875052	B
58	28.707615	-14.117635	B
59	30.789600	-12.400341	B

60	30.934530	-14.910924	B
61	33.265183	-12.419050	C
62	31.735379	-12.742501	C
63	33.529079	-13.228380	C
64	32.291508	-13.723304	C
65	30.712166	-12.133902	C
66	32.561901	-13.803571	C
67	31.882782	-12.971678	C
68	32.428413	-12.523443	C
69	31.900544	-12.881834	C
70	32.691245	-13.719721	C
71	32.636205	-12.990411	C
72	31.545432	-10.401313	C
73	32.281513	-13.145960	C
74	32.918595	-12.331101	C
75	31.922620	-12.363180	C
76	31.406621	-12.241170	C
77	31.681391	-12.498604	C
78	32.030168	-12.317615	C
79	33.190555	-12.596503	C
80	31.438128	-12.589402	C
81	32.782646	-13.226760	C
82	32.236401	-12.757098	C
83	32.239817	-13.221011	C
84	32.453463	-12.772601	C
85	32.519180	-12.629722	C
86	32.941080	-13.070498	C
87	33.258079	-13.207082	C
88	32.004419	-12.345527	C
89	32.333757	-12.277878	C
90	32.787509	-13.391809	C
91	33.628448	-12.378364	D
92	34.073894	-12.424991	D
93	32.923090	-13.314857	D
94	33.326712	-13.771505	D
95	34.402188	-13.228189	D
96	33.873997	-12.522374	D
97	33.520686	-13.137731	D
98	31.990580	-13.506135	D
99	35.819039	-12.630479	D
100	32.803933	-12.237486	D
101	36.216618	-14.064223	D
102	34.980590	-12.306902	D
103	33.820454	-13.119815	D
104	34.305859	-13.182271	D
105	33.735222	-12.726171	D
106	32.766744	-13.477201	D
107	32.850489	-12.492700	D
108	32.682083	-13.120139	D
109	33.562210	-14.191314	E
110	34.573092	-15.797107	E
111	34.607815	-13.034756	E
112	33.950635	-13.287529	E
113	34.974522	-14.905500	E
114	34.160873	-14.710522	E
115	34.152512	-13.809421	E
116	34.373958	-13.849031	E
117	36.417225	-14.156958	E
118	34.353880	-13.006611	E
119	34.702880	-13.530820	E

120	33,500943	-12,971825	E
121	34,490147	-15,037935	E
122	35,625819	-14,140832	E
123	34,005613	-14,141612	E
124	33,773728	-14,926366	F
125	33,440834	-15,150679	F
126	34,465970	-12,255561	F
127	33,543202	-14,649669	F
128	34,511375	-12,804301	F
129	33,944321	-13,877170	F
130	33,437281	-14,081180	F
131	33,631987	-14,506346	F
132	34,991262	-14,660788	F
133	35,069963	-14,478308	F
134	35,334066	-14,492346	F
135	35,939232	-14,656191	F
136	34,018612	-13,254745	F
137	35,457867	-14,903879	F
138	34,432543	-15,531560	F
139	33,557210	-14,451342	F
140	34,459929	-14,053586	F
141	33,776168	-13,817037	F
142	34,412990	-13,886276	F
143	33,016611	-13,343563	F
144	34,648611	-14,242088	F
145	34,191409	-13,804864	F
146	33,582692	-13,916941	F
147	35,454466	-14,755747	F
148	34,217309	-14,395478	F
149	34,434007	-14,746860	F
150	33,725127	-14,296791	F
151	34,313931	-13,356085	F
152	33,872850	-14,247417	F
153	31,089674	-13,635039	G
154	30,982432	-13,362428	G
155	31,555945	-12,533700	G
156	30,437844	-13,392473	G
157	32,342279	-12,109552	G
158	29,616477	-13,902008	G
159	30,353206	-13,205714	G
160	30,342267	-11,352945	G
161	32,355097	-12,477648	G
162	31,878302	-14,458763	G
163	30,059850	-14,658031	G
164	34,443532	-13,027629	G
165	33,195043	-12,359846	G
166	33,590131	-12,967900	G
167	33,468532	-12,900049	G
168	29,380529	-15,157748	G
169	31,073554	-12,641524	G
170	30,887547	-12,774402	G
171	29,527179	-13,035652	G
172	31,692406	-13,215807	G
173	30,904957	-12,943750	G
174	29,943429	-13,964055	G
175	30,595513	-14,825906	G
176	30,919171	-11,193681	G
177	36,560350	-14,631573	H
178	34,947361	-14,775228	H
179	34,884643	-14,240732	H

180	34,777970	-13,912151	H
181	34,756451	-13,052109	H
182	35,889784	-14,061180	H
183	34,470189	-14,720764	H
184	33,654184	-14,306528	H
185	35,357543	-14,016218	H
186	35,268841	-14,067559	H
187	34,920892	-14,643276	H
188	33,692138	-14,205129	H
189	36,185203	-14,510549	H
190	34,508868	-13,279092	H
191	34,273849	-13,164411	H
192	35,380790	-13,946156	H
193	34,194971	-14,150588	H
194	33,276994	-14,310835	H
195	35,425272	-13,370719	H
196	36,085542	-13,557957	H
197	34,640724	-13,539802	H
198	32,975939	-14,208979	H
199	35,308788	-13,202792	H
200	33,228273	-13,875850	H
201	35,354956	-15,127905	H

SCALE FACTOR ON X IS 1,00+002 SCALE FACTOR ON Y IS 1,00+001



TESTS OF SIGNIFICANCE FOR THE EIGENVALUES

LAMBDA NO 1

2,2738438

CHISQUARE = 441,1256940

D,F, = 28

LAMBDA NO 2

0,8277077

CHISQUARE = 160,5752860

D,F, = 18

LAMBDA NO 3

0,2145938

CHISQUARE = 41,6311954

D,F, = 10

LAMBDA NO 4

0,0991776

CHISQUARE = 19,2404566

D,F, = 4

TEST OF EQUALITY OF MEANS

DETB = 5,654815569-003 DETX = 5,494602164-002

WILKS LAMBDA = 0,10292

CHISQUARE = 434,304 DF = 28

HOMOGENEITY OF COVARIANCE MATRICES

CHSQ = 237,905759

DF = 70

ANTHROPOMETRIC DATA ON BIAFRANS

CANONICAL VARIATES STUDY FOR GROUPS = 17

TABLE OF MEANS

	1	2	3	4	5	6	7	8
1	191,40000	152,20000	110,90000	120.70000	142,60000	109.30000	47,80000	42,30000
2	191,20000	149,70000	108,20000	114.20000	140,30000	109.10000	44,60000	42,30000
3	190,00000	145,90000	108,80000	113.60000	138,20000	105.90000	45,70000	42,10000
4	191,10000	146,40000	108,90000	113.80000	138,60000	105.60000	45,00000	42,30000
5	192,60000	147,60000	108,40000	115.30000	140,30000	108.60000	45,90000	42,50000
6	191,10000	146,00000	109,60000	113.40000	140,10000	107.60000	44,60000	41,70000
7	188,70000	144,20000	106,00000	114.10000	138,20000	106.30000	43,60000	41,90000
8	190,10000	144,80000	107,80000	112.90000	138,20000	104.60000	46,70000	42,40000
9	189,50000	144,30000	108,60000	114.00000	138,30000	106.50000	44,90000	41,40000
10	191,60000	145,80000	108,50000	113.30000	138,00000	106.00000	44,80000	41,80000
11	193,00000	146,20000	108,00000	114.90000	139,70000	107.90000	44,20000	42,20000
12	192,90000	146,40000	109,10000	115.40000	139,80000	107,20000	46,40000	43,40000
13	193,80000	146,90000	108,90000	118.30000	140,20000	107.00000	45,30000	41,80000
14	192,30000	145,80000	108,70000	114.50000	139,90000	107.10000	45,60000	41,50000
15	190,60000	142,10000	107,00000	114.50000	137,20000	107.30000	45,00000	42,50000
16	192,70000	142,50000	108,20000	113.80000	136,60000	104.40000	45,00000	42,00000
17	191,10000	146,90000	107,00000	115.10000	137,20000	105.00000	45,30000	42,80000

WITHIN GROUPS SSCP MATRIX

	1	2	3	4	5	6	7	8
1	38,41200	8,92200	9,42500	12.66200	10,94700	6.63600	3,79000	4,04800
2	8,92200	18,01500	8,00300	4.18800	8,52700	5.59500	1,10500	1,51800
3	9,42500	8,00300	18,25000	5,91000	10,48700	6.36000	0,72100	2,59300
4	12,66200	4,18800	5,91000	37.45900	8,44600	7.37800	11,98600	1,97800
5	10,94700	8,52700	10,48700	8.44600	22,70800	11,51100	2,20100	4,09200
6	6,63600	5,59500	6,36000	7.37800	11,51100	25,99000	2,20800	3,51300
7	3,79000	1,10500	0,72100	11.98600	2,20100	2.20800	10,18100	0,76300
8	4,04800	1,51800	2,59300	1.97800	4,09200	3.51300	0,76300	9,42900

AMONG GROUPS SSCP MATRIX,A(I,J)

	1	2	3	4	5	6	7	8
1	1,27800	0,09400	0,25500	0.60200	0,41600	0.38900	-0,03600	0,06600
2	0,09400	4,73800	1,00100	2.81100	2,46600	2.00300	0,94800	0,26200
3	0,25500	1,00100	1,02200	0.81200	1,17000	0.86600	0,44100	-0,13500
4	0,60200	2,81100	0,81200	3.24500	1,78300	1.43000	0,89600	0,20500
5	0,41600	2,46600	1,17000	1.78300	2,39200	2.05600	0,61800	-0,04100
6	0,38900	2,00300	0,86600	1.43000	2,05600	2.20600	0,25200	-0,04500
7	-0,03600	0,94800	0,44100	0,89600	0,61800	0.25200	0,77500	0,16300
8	0,06600	0,26200	-0,13500	0.20500	-0,04100	-0.04500	0,16300	0,24300

SOLUTION OF GENERALIZED DETERMINANTAL EQUATION

EIGENVALUE	1	0,3616871	EIGENVALUE	5	0,0307268
EIGENVALUE	2	0,1144466	EIGENVALUE	6	0,0240432
EIGENVALUE	3	0,1066501	EIGENVALUE	7	0,0118406
EIGENVALUE	4	0,0443698	EIGENVALUE	8	0,0055013

EIGENVECTORS

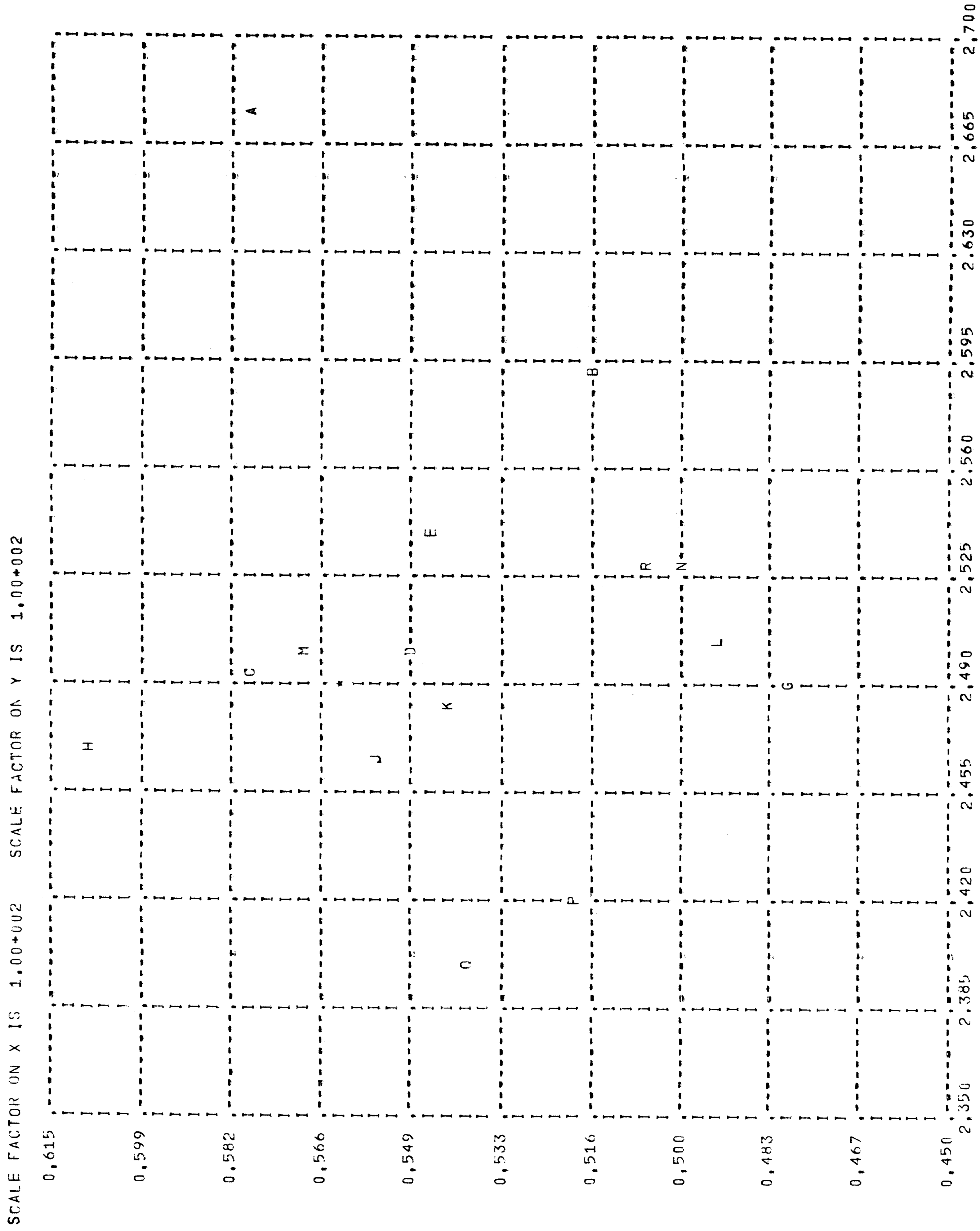
	1	2	3	4	5	6	7	8
1	-0,06951	-0,05059	0,01269	0.02326	-0,09285	0.12702	-0,01285	-0,04397
2	0,23169	-0,03459	-0,08599	-0.06386	0,04205	0.08591	0,00985	-0,02599
3	-0,07297	0,14504	0,03807	0.09954	0,12650	0.07135	0,14407	0,07258
4	0,06100	-0,12664	0,01930	0.14666	0,03810	-0.07367	0,00153	0,01754
5	0,03156	0,06212	0,11217	-0.02819	-0,06827	-0.04511	-0,20358	0,11209
6	0,01311	-0,01919	0,10945	-0.06644	-0,05032	-0.04751	0,14375	-0,09387
7	-0,00391	0,32448	-0,13058	-0.01135	-0,13010	-0.00741	-0,00848	-0,14282
8	0,00411	-0,04948	-0,18250	-0.02632	-0,13407	-0.06632	0,12917	0,20774

NORMALIZED EIGENVECTORS

	1	2	3	4	5	6	7	8
1	-0,26507	-0,12943	0,04372	0.11341	-0,35108	0.61414	-0,04065	-0,14411
2	0,88359	-0,08849	-0,29616	-0.31134	0,15900	0.41536	0,03115	-0,08516
3	-0,27827	0,37107	0,13113	0.48533	0,47835	0.34496	0,45585	0,23787
4	0,23263	-0,32398	0,06646	0.71505	0,14406	-0.35620	0,00484	0,05747
5	0,12035	0,15891	0,38634	-0.13743	-0,25814	-0.21809	-0,64415	0,36732
6	0,05001	-0,04909	0,37698	-0.32396	-0,19027	-0.22970	0,45482	-0,30764
7	-0,01490	0,83011	-0,44975	-0.05535	-0,49196	-0.03581	-0,02682	-0,46804
8	0,01566	-0,12659	-0,62858	-0.12833	-0,50698	-0.32067	0,40871	0,68079

ADJUSTED CANONICAL VARIATES

	1	2	3	4	5	6	7	8
1	-0,68498	-0,48494	0,11852	0.21180	-0,82555	1.10406	-0,10928	-0,36630
2	2,28331	-0,33155	-0,80282	-0.58147	0,37389	0.74671	0,08373	-0,21648
3	-0,71908	1,39026	0,35545	0.90642	1,12482	0.62014	1,22534	0,60462
4	0,60113	-1,21385	0,18015	1.33543	0,33876	-0.64035	0,01300	0,14608
5	0,31100	0,59538	1,04725	-0.25666	-0,60702	-0.39206	-1,73149	0,93368
6	0,12924	-0,18394	1,02189	-0.60503	-0,44741	-0.41294	1,22259	-0,78197
7	-0,03851	3,11013	-1,21914	-0.10338	-1,15684	-0.06437	-0,07210	-1,18968
8	0,04048	-0,47429	-1,70390	-0.23966	-1,19215	-0.57647	1,09863	1,73047


SCALE FACTOR ON X IS 1.00+002 SCALE FACTOR ON Y IS 1.00+002
0.615
0.599
0.582
0.566
0.549
0.533
0.516
0.500
0.483
0.467
0.450
2.350
2.385
2.420
2.455
2.490
2.525
2.560
2.595
2.630
2.665
2.700
A
B
C
D
E
G
H
J
K
L
M
N
O
P
R

THE CANONICAL MEANS

CANONICAL VARIATE MEAN NO. 1	267,57119	57,78730	92,34007	95,93965	-176,66167	187,94617	59,03158	45.63381
CANONICAL VARIATE MEAN NO. 2	259,41614	51,56448	93,48096	87,26545	-177,48271	189,53678	59,41980	45.48217
CANONICAL VARIATE MEAN NO. 3	249,65229	57,82306	90,02506	91,37274	-175,76884	188,31992	59,38497	45.90629
CANONICAL VARIATE MEAN NO. 4	250,20947	55,04149	90,45056	91,77599	-175,84700	189,73851	58,64259	47.27174
CANONICAL VARIATE MEAN NO. 5	254,07307	54,56491	93,16544	90,55350	-180,34468	188,94152	58,86504	44.89606
CANONICAL VARIATE MEAN NO. 6	249,26843	56,19855	96,07308	90,69900	-175,97200	189,08773	58,67923	46.99731
CANONICAL VARIATE MEAN NO. 7	249,09961	48,00747	93,64032	90,23865	-175,82252	183,64398	56,38094	46,97003
CANONICAL VARIATE MEAN NO. 8	247,17607	60,80567	87,37969	90,80356	-178,55749	187,73656	56,71562	45,74692
CANONICAL VARIATE MEAN NO. 9	246,83687	55,62549	94,13717	92,41186	-174,61292	186.36106	58,91481	45,73790
CANONICAL VARIATE MEAN NO.10	248,33662	54,23308	91,63544	91,25298	-176,09073	190.28580	59,03410	45.40329
CANONICAL VARIATE MEAN NO.11	250.42588	49,39113	95.36260	91,38068	-178,78211	189,15250	58,18482	46.24282
CANONICAL VARIATE MEAN NO.12	250.36510	56.75711	90.33386	92,79081	-180,94124	188,96988	59,69765	47,07429
CANONICAL VARIATE MEAN NO.13	252,85354	49.96932	94,77227	96,89769	-177,71322	189,27479	56,81818	46.00885
CANONICAL VARIATE MEAN NO.14	249,12479	56,27434	94,65538	92,02105	-178,25046	189,33656	56,88602	44,88605
CANONICAL VARIATE MEAN NO.15	242,31318	51,97734	93,22443	92,66584	-179,09118	184,08073	60,74032	45.04880
CANONICAL VARIATE MEAN NO.16	239,92273	53,75768	90,71276	95,05934	-177,30487	189,61134	58,94971	45.65854
CANONICAL VARIATE MEAN NO.17	252,99460	50,62094	86,31101	92,07061	-177,18168	188,59024	58,59142	45.87498

TESTS OF SIGNIFICANCE FOR THE EIGENVALUES

LAMBDA NO 1

0,6331137

CHISQUARE = 1079,1423739

D,F, = 128

LAMBDA NO 2

0,3243893

CHISQUARE = 552,9216029

D,F, = 105

LAMBDA NO 3

0.2160313

CHISQUARE = 368,2254338

D,F, = 84

LAMBDA NO 4

0,1146938

CHISQUARE = 195,4955648

D,F, = 65

LAMBDA NO 5

0,0712802

CHISQUARE = 121,4970785

D,F, = 48

LAMBDA NO 6

0,0410160

CHISQUARE = 69,9116959

D,F, = 33

LAMBDA NO 7

0,0172572

CHISQUARE = 29,4149599

D,F, = 20

LAMBDA NO 8

0,0054862

CHISQUARE = 9,3512320

D,F, = 9

TEST OF EQUALITY OF MEANS

DETE = 4,250459650+009 DETX = 8,005596602+009

WILKS LAMBDA = 0,53094

CHISQUARE = 1074,711 DF = 28

KANSAS GEOLOGICAL SURVEY COMPUTER PROGRAM
THE UNIVERSITY OF KANSAS, LAWRENCE

PROGRAM ABSTRACT

Title (If subroutine state in title): FORTRAN IV program for canonical variates analysis for the CDC 3600 computer

Date: 12 August 1969

Author, organization: Richard A. Reyment and Hans-Åke Ramdén (University of Uppsala) Paleontological Institute

Direct inquiries to: Author, or

Name: D.F. Merriam

Address: Kansas Geological Survey
University of Kansas, Lawrence

Purpose/description: For computing canonical variates for homogeneous covariance matrices, multivariate analysis of variance and tests for homogeneity of covariance matrices and significance of generalized eigenvalues.

Mathematical method: Canonical variate method of Bartlett and multivariate analysis of variance of Wilks.

Restrictions, range: The program accepts matrices up to 30x30

Computer manufacturer: CDC

Model: 3600

Programming language: FORTRAN IV

Memory required: 32 K

Approximate running time:

Special peripheral equipment required:

Remarks (special compilers or operating systems, required word lengths, number of successful runs, other machine versions, additional information useful for operation or modification of program)

The two examples given took about 3 minutes to run, exclusive of compiling time.

(continued from inside front cover)

15. FORTRAN II program for progressive linear fit of surfaces on a quadratic base using an IBM 1620 computer, by A.J. Cole, C. Jordan, and D.F. Merriam, 1967 . . . $1.00
16. FORTRAN IV program for the GE 625 to compute the power spectrum of geological surfaces, by J.E. Esler and F.W. Preston, 1967 . . . . $0.75
17. FORTRAN IV program for Q-mode cluster analysis of nonquantitative data using IBM 7090/7094 computers, by G.F. Bonham-Carter, 1967 . . . . $1.00
18. Computer applications in the earth sciences: Colloquium on time-series analysis, edited by D.F. Merriam, 1967 . . . . $1.00
19. FORTRAN II time-trend package for the IBM 1620 computer, by J.C. Davis and R.J. Sampson, 1967 . . . . $1.00
20. Computer programs for multivariate analysis in geology, edited by D.F. Merriam, 1968 . $1.00
21. FORTRAN IV program for computation and display of principal components, by W.J. Wahlstedt and J.C. Davis, 1968 . . . . $1.00
22. Computer applications in the earth sciences: Colloquium on simulation, edited by D.F. Merriam and N.C. Cocke, 1968 . . . . $1.00
23. Computer programs for automatic contouring, by D.B. McIntyre, D.D. Pollard, and R. Smith, 1968 . . . . $1.50
24. Mathematical model and FORTRAN IV program for computer simulation of deltaic sedimentation, by G.F. Bonham-Carter and A.J. Sutherland, 1968. . . . . $1.00
25. FORTRAN IV CDC 6400 computer program to analyze subsurface fold geometry, by E.H.T. Whitten, 1968 . . . . $1.00
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27. Stepwise regression and nonpolynomial models in trend analysis, by A.T. Miesch and J.J. Connor, 1968 . . . . $1.00
28. KWIKR8 a FORTRAN IV program for multiple regression and geologic trend analysis, by J.E. Esler, P.F. Smith, and J.C. Davis, 1968 . . . . $1.00
29. FORTRAN IV program for harmonic trend analysis using double Fourier series and regularly gridded data for the GE 625 computer, by J.W. Harbaugh and M.J. Sackin, 1968 . . $1.00
30. Sampling a geological population, by J.C. Griffiths and C.W. Ondrick, 1968 . . . $1.00
31. Multivariate procedures and FORTRAN IV program for evaluation and improvement of classifications, by Ferruh Demirmen, 1969 . . . . $1.00
32. FORTRAN IV programs for canonical correlation and canonical trend-surface analysis, by P.J. Lee, 1969 . . . . $1.00
33. FORTRAN IV program for construction of Pi diagrams with the Univac 1108 computer, by Jeffrey Warner, 1969 . . . . $1.00
34. FORTRAN IV program for nonlinear estimation, by R.B. McCammon, 1969 . . . . $0.75
35. FORTRAN IV computer program for fitting observed count data to discrete distribution models of binomial, Poisson and negative binomial, by C.W. Ondrick and J.C. Griffiths, 1969. $0.75
36. GRAFPAC, graphic output subroutines for the GE 635 computer, by F.J. Rohlf, 1969 . . $1.00
37. An iterative approach to the fitting of trend surfaces, by A.J. Cole, 1969 . . . . $1.00
38. FORTRAN II programs for 8 methods of cluster analysis (CLUSTAN I), by Davis Wishart, 1969 $1.50
39. FORTRAN IV program for the generalized statistical distance and analysis of covariance matrices for the CDC 3600 computer, by R.A. Reyment, Hans-Ake Ramden, and W.J. Wahlstedt, 1969. . . . . $1.00
40. Symposium on computer applications in petroleum exploration, edited by D.F. Merriam, 1969 $1.00
41. FORTRAN IV program for sample normality tests, by D.A. Preston, 1970 . . . . $1.00
42. CORFAN-FORTRAN IV computer program for correlation, factor analysis (R-and Q-mode) and varimax rotation, by C.W. Ondrick and G.S. Srivastava, 1970 . . . $1.50
43. Minimum entropy criterion for analytic rotation, by R.B. McCammon, 1970 . . . $1.00
44. FORTRAN IV CDC 6400 computer program for constructing isometric diagrams, by W.B. Wray, Jr., 1970 . . . . $1.00
45. An APL language computer program for use in electron microprobe analysis, by D.G.W. Smith and M.C. Tomlinson, 1970. . . . . $1.00
46. FORTRAN IV program for Q-mode cluster analysis on distance function with printed dendrogram, by J.M. Parks, 1970 . . . . $1.00
47. FORTRAN IV program for canonical variates analysis for the CDC 3600 computer, by R.A. Reyment and H. Ramden, 1970 . . . . $1.00