DANIEL F. MERRIAM, Editor

AN ITERATIVE APPROACH TO THE FITTING OF TREND SURFACES

By

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Editor's Remarks

This program, "An iterative approach to the fitting of trend surfaces", by A.J. Cole is a modification and improvement of a similar program published in 1967 as COMPUTER CONTRIBUTION 15. Contouring routines are especially valuable to earth scientists, who represent much of their data in map form. Graphics, therefore, is an important aspect in presenting results of research in addition to being the subject of research itself. Other publications in this series of interest to those working with visual display are COMPUTER CONTRIBUTIONS 23 and 36.

For a limited time the Geological Survey will make available on magnetic tape the program as described in this COMPUTER CONTRIBUTION for $20.00 (U.S.). An extra charge of $10.00 is made if punched cards are required.

For an up-to-date list of available COMPUTER CONTRIBUTIONS write the Editor, Kansas Geological Survey, The University of Kansas, Lawrence, Kansas 66044, U.S.A.

(continued on inside back cover)

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A.J. Cole

INTRODUCTION

The method described in this paper is a further development and considerable improvement on that described in a previous Computer Contribution (Cole, Jordan, and Merriam, 1967). The basic problem is the same as described in that publication, namely the problem of fitting a surface to a scattered data set of xyz-coordinates and plotting the corresponding contour lines using a digital plotter. The contouring technique finally is the same as described in the above publication and the reader is referred to that paper for details. Other approaches to the general problem are listed in the references at the end of this paper. The method employed in this paper has been described by Cole (1968).

Acknowledgments.- Thanks are due to W.A. Read and J.M. Dean of the Institute of Geological Sciences (Edinburgh) who supplied many data sets, scrutinized the results and reviewed the manuscript. Machine time was made available through the Computing Laboratory of the University of St. Andrews (Scotland).

DESCRIPTION OF METHOD. PART 1 (JCMAP1)

The general technique will be described first, followed by a discussion of options which are included in the main program. The first step is to choose some trend-surface fit for the data. Polynomial surfaces of degree 1, 2 and 3 are fitted by least squares and, unless an option is exercised, that with the best statistical fit is chosen for the trend-surface fit. It is easy to modify the program to use another surface function if a better one is known for a particular data set. Having chosen this function, it is evaluated at the grid points of a rectangular grid G. In the standard form of the program this is a 33 by 33 grid but in practice there is little point in having more grid points than about 16 times the number of data points. An option permits the user's specification of the number of grid rows and columns. The grid point values are used as starting values for the iterative process and the trend function is not used again.

Each of the original data points is treated independently. For each data point the nearest grid point is taken as the center C of a 3 x 3 square of grid points, but using points twice the basic cell dimensions of the grid away from the center (Fig. 1). The coarser grid is chosen so as not to localize the effect of the given data point too much. Choice of the total number of grid points of G is now seen to be that which makes an average of only one data point in the larger nine point square if the data points are reasonably evenly distributed over the map area.

The nine grid points with weight 1 and the current data point P_i with weight 4 then are fitted by least squares to a second-degree (quadratic) polynomial. New values using this polynomial next are calculated at the grid points forming the 5 x 5 square with center C, but with the distance between adjacent points equal to the basic cell dimensions of G. During the first iteration nothing more is done with the data point P_i. During later iterations, to complete the fitting process, the value of the quadratic at the xy-coordinates of P_i is checked. If it differs by more than a user chosen percentage of the standard deviation of the original data values, the fit is repeated with progressively higher weightings for P_i. This process is repeated for each data point.

At the completion of this stage the surface represented by values at points of G may be thought of as a smooth basic surface with quadratic bumps on it. The surface is not suitable for contouring because there will be discontinuities at the edge of the bumps. These are smoothed by a process described by Cole and Davie (1969). The particular smoothing formula used is a generalization in two dimensions of a method des-

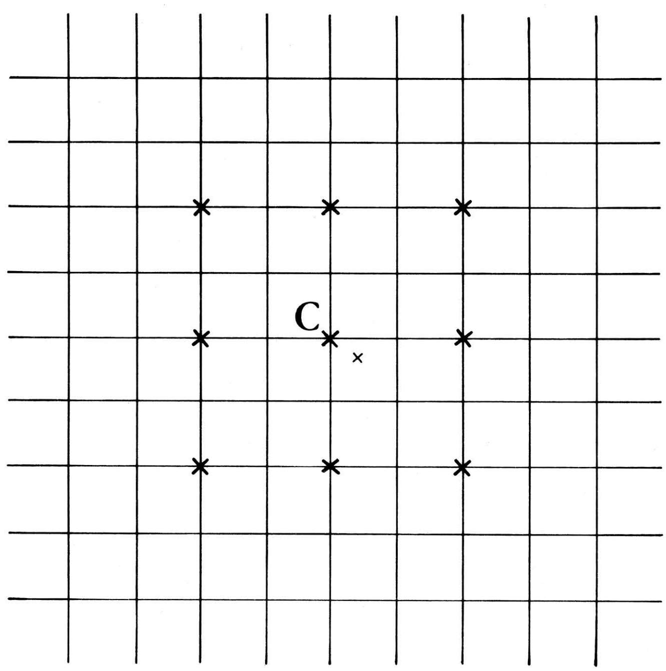

Figure 1.- Data point and nine grid points used for local quadratic fit.

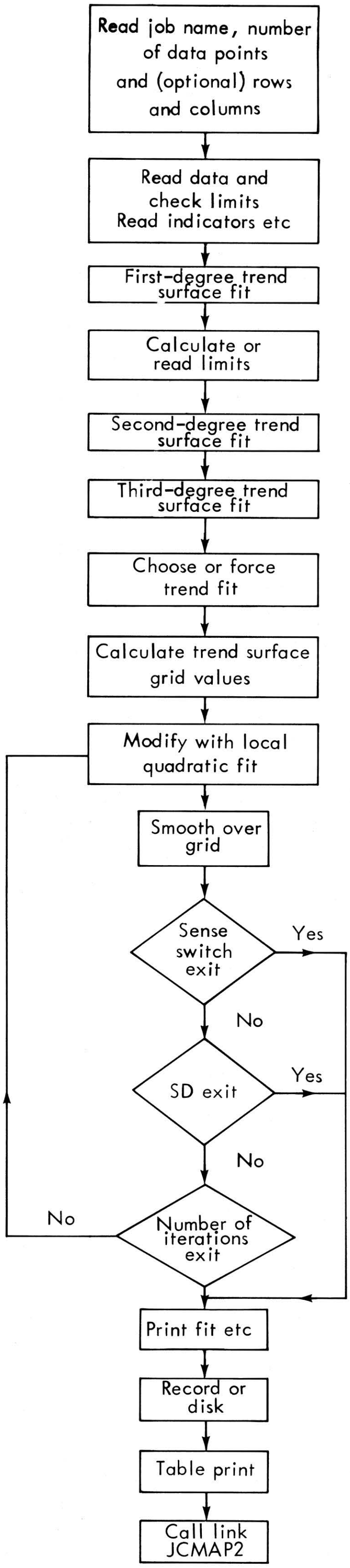

Figure 2a.-Flow diagram for JCMAP1

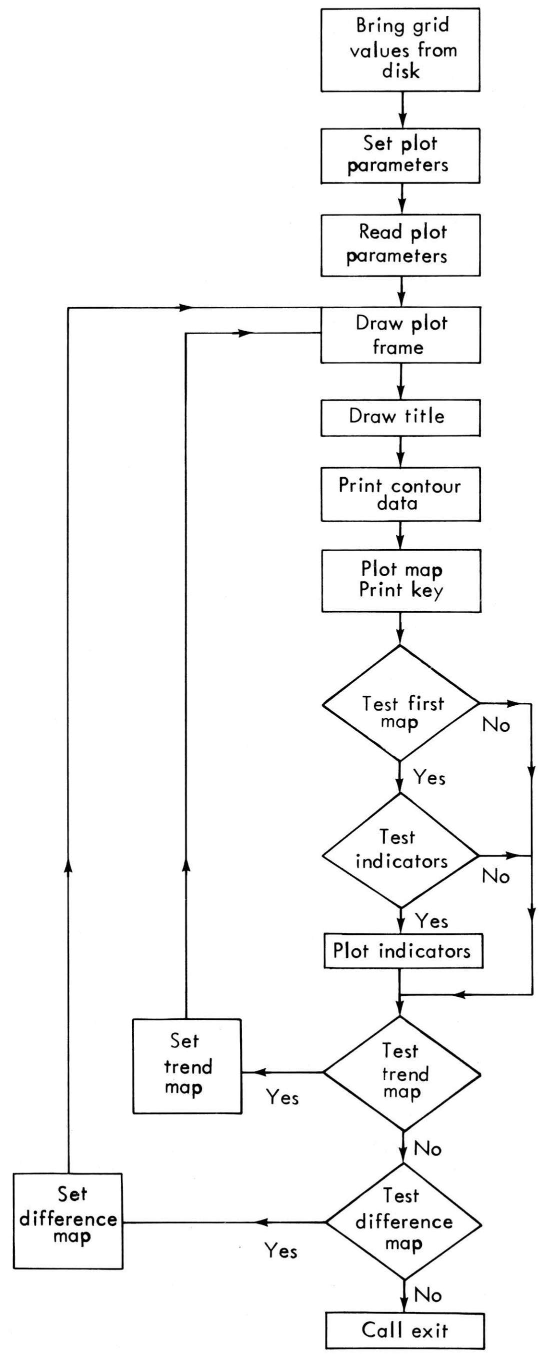

Figure 2b.-Flow diagram for JCMAP2

cribed by Lanczos (1957) and Hildebrand (1956). It is a quadratic smoothing using a 3 x 3 basic square of grid points of G based on the point at which the correction is to be made as center and gives the correction

$$\frac{-1}{9}\ \delta^4{}_{xxyy} f_0$$

where $\delta^4{}_{xxyy} f_0$ denotes the central difference of order four with differences taken twice in the x-direction and twice in the y-direction. The order in which this differencing is accomplished is of no significance. The smoothing, being quadratic, does not affect the value at the center of a bump, but does smooth the intersection of the local quadratic with the trend surface, even where the trend surface is itself a quadratic polynomial.

The error at the original data points is computed also and may be used to determine whether or not the iteration has proceeded far enough. The criterion used is a comparison of the standard deviation of the errors with the standard deviation of the z-values of the original data. The iterative process may be terminated also after a given number of complete steps. It is advisable to use this condition as a safety exit, because if some data points are particularly bad the iterative process may not converge. In practice if the original trend fit is 50 percent or more, approximately three iterations should produce significant improvements. For poor initial fits up to ten or twelve iterations may be required.

If another iteration is necessary the quadratic bumps and smoothing sections of the program are repeated. In addition to the options mentioned the following also are available. In the basic program minimum and maximum x, y values are set from the original data. If desired, the extreme values may be specified by the user. This enables plots of specified areas to be produced and more rational scale factors to be calculated. Intermediate printouts of errors may be suppressed by setting INDPR to a nonzero value. The trend-fit base can be forced to be linear, quadratic or cubic by setting IBASE equal to 1, 2 or 3 respectively. If IBASE is set equal to zero then the program chooses the best fit base.

A flow diagram for JCMAP1 is given in Figure 2a.

DESCRIPTION OF METHOD. PART 2 (JCMAP2)

This part of the program sets up the data and parameters for plotting. Three maps are offered, namely the original trend-surface fit, the final modified surface and the difference map between the two. The contour intervals to be plotted for the first two maps are read in as data and printed in a key to the contours. It is assumed that the final modified map always is required but that the trend fit and difference maps are optional. If the difference map is required the contour intervals may be specified by the user. Otherwise they are calculated by the program. Because the range of values in the difference map is not known in advance, it is usually better to let the program calculate the intervals. In situations in which this does not give a satisfactory result the program may be run again, but this time specifying the levels. An option permits the user to specify whether or not data points are to be marked with crosses on the final modified map. Points where errors exceed a user's specified proportion of the standard deviation of the original data may be marked with a small triangle if desired. The apex of this triangle will point up if the error is positive; down if it is negative. This facility assists in identifying a poor fit area.

A flow diagram for JCMAP2 is given in Figure 2b.

ADVANTAGES OF METHOD

There are six main advantages to this approach.

(1). The initial trend-surface fit is not critical to the method. It is, therefore, chosen so that sparse data areas are not given unjustified detail. This is always a problem when using an overall method of fit. Also the initial data are used at every iteration and there is, consequently, no round off error to accumulate.

(2). Data points, after the initial trend fit, are not interdependent. Bad data points do not, therefore, affect the entire map surface as in pure trend-surface methods.

(3). Particularly bad points are easily detected on the final map and also on the difference map.

(4). The statistical fit is usually good, hence the difference map is a good approximation to the residual map for the chosen trend fit.

(5). An objective difference map is produced and, therefore, does not incorporate any preconceived ideas of the user.

(6). The method is fast, being practically linear per iteration with respect to the number of data points used. This is, of course, of great importance when numerous data points are available.

PROGRAM DESCRIPTION

LEAD CARDS

Card 1	Column 1	Blank
	Columns 2-80	Contains job name
Card 2	Column 1	Blank
	Columns 2-4	Number of sets of xyz-values that are read. Value must be between 1 and 240 and must be right justified.
	Columns 5-7	Optional specification of number of grid rows to be

	used.
Columns 8-10	Optional specification of number of grid columns to be used.

DATA CARDS

This is the first of a set of N cards containing xyz-values where N is the value specified in columns 2-4 of Card 2.

Columns 1-11	Blank
Columns 12-26	Contain x-coordinate. If no decimal point is punched, it is assumed to be between columns 20 and 21.
Columns 27-41	Contain y-coordinate. If no decimal point is punched, it is assumed to be between columns 35 and 36.
Columns 42-56	Contain z-coordinate. If no decimal point is punched, it is assumed to be between columns 50 and 51.

TRAILER CARDS

Card 1	Columns 1-4	Numeric height of plot, that is, actual length of y-axis. If no decimal point is punched it is assumed to be after column 4.
	Columns 5-8	Percentage of SD at which iteration is to stop.
	Columns 9-10	Maximum number of iterations allowed.
	Columns 11-14	Percentage of SD at which error indicators are to be plotted.
	Column 15	Read plot limits (next card) if $\neq 0$.
	Column 16	Print errors at each iteration if 0.
	Column 17	Base indicator. If 0 then best base is chosen. If 1, 2 or 3 then linear quadratic or cubic base is forced.

Card 2 (Optional dependent on column 15 of card 1 above).

	Columns 1-15	Minimum x-plot limit. If no decimal point is punched it is assumed to be between columns 9 and 10.
	Columns 16-30	Maximum x-plot limit. If no decimal point is punched it is assumed to be between columns 25 and 26.
	Columns 31-45	Minimum y-plot limit. If no decimal point is punched it is assumed to be between columns 39 and 40.
	Columns 46-60	Maximum y-plot limit. If no decimal point is punched it is assumed to be between columns 54 and 55.
Card 3	Columns 1-15	Minimum contour value required. If no decimal point is punched it is assumed to be between columns 9 and 10.
	Columns 16-30	Contour interval. If no decimal point is punched it is assumed to be between columns 24 and 25.
	Columns 31-35	Number of contours required. Must be a right justified integer.
	Column 36	Plot error indicators if $\neq 0$.
	Column 37	Plot point markers if $\neq 0$.
	Column 38	Plot basic fit map if 1.

	Column 39	Plot error map if 1.
	Column 40	Read contour start value, interval and number of values for difference map if ≠ 0.
Card 4	Column 1	Blank
	Column 2-80	Title card. Usually, but no necessarily the same as Leader Card 1.
Card 5 (Optional dependent on column 40 of card 3 above).		
	Columns 1-35	The same as card 3 above but with reference to difference map.

Error Messages

In addition to the usual output, programmer error messages may be printed during the run. Possible errors are listed below.

1. Number of data points outside allowable range (1,240). This can occur in JCMAP2.
2. Determinant is singular (JC101). Only occurs if determinant of any of the least-squares equations vanishes.
3. JCCONT errors. These errors only occur if the contour has 1, 3 or 4 intersections with the grid cell at bottom left-hand vertex (n, m) where n,m are rows and columns counted from the bottom left-hand corner. In situations 1 and 3, no contours are drawn. In situation 4, an arbitrary join of the points in pairs is made.

Program Listing

Table 1 is a listing of JCMAP1 and JCMAP2. The subroutines JCCONT, JC101, JCKU11, PLOT CHAR and POINT are exactly the same as those listed in Computer Contribution 15 (Cole, Jordan and Merriam, 1967). The programs are written in FORTRAN IID and require an IBM 1620 model II with 60k core storage, line printer, plotter and disk storage package. With minor modifications, the program is adaptable to other machines.

SAMPLE PROBLEM

The data used here (Table 2) are the same as that of McIntyre (1967). Figures 3, 4 and 5 show the final modified fit, third-degree trend fit and difference map. The corresponding printer output is shown in Table 2. For comparative purposes Figures 6 and 7 show the final modified fits starting with second- and first-degree trend fits respectively. The quadratic had a moderately good statistical fit whereas the linear fit was low. Even with such a poor starting surface the final result is good.

Table 1. -Listing of program, JCMAP1 and JCMAP2.

```
*LISTPRINTER
*LDISKJCMAP1
      DIMENSION X(240),Y(240),WW(10,11),C(11),W(110),CL(3),CQ(6),CC(10),
     1S(10),ZERR(240),W1(33),W2(33),W3(33),Z(240),P(33,33),DUMMY(110)
      COMMON N,XMAXI,XMINI,YMAXI,YMINI,YL,ERRPER,X,Y,ZERR,NC,NR,XNC,XNR
      EQUIVALENCE(WW,DUMMY),(W1(1),DUMMY(1)),(W2(1),DUMMY(34)),(W3(1),
     1DUMMY(67))
      DEFINE DISK (10,300)
      READ 20
   20 FORMAT (80H THIS FORMAT WILL BE FILLED BY JOB NAME
     1                                )
      READ 95,N,NR,NC
      IF(NR)99,90,99
   90 NR=33
      NC=33
   99 XNR=NR-1
      XNC=NC-1
   95 FORMAT (1X,3I3)
C-----------------------------------
C DATA PARAMETERS CHECKED WITH JCKU11
C-----------------------------------
  599 NERR=0
```

```
      CALL JCKU11 (1,240,N,NKR)
      KAW=1
      IF(NKR)710,710,700
  700 PRINT 705,KAW
  705 FORMAT (1X,14H PROGRAM ERROR, I3)
      NERR=1
      GO TO (710,235),KAW
  710 IF(NERR) 100,100,706
  706 TYPE 707
  707 FORMAT (12HINVALID DATA)
      CALL EXIT
C-----------------------------------------------------------
C READ IN X,Y,Z- COORDINATES AND CALCULATE GLOBAL FITS
C-----------------------------------------------------------
  100 READ 105,X(1),Y(1),Z(1)
  105 FORMAT (11X,3F15.6)
      DO 106 I=1,10
      DO 106 J=1,11
  106 WW(I,J)=0.0
      XMAX=X(1)
      XMIN=X(1)
      YMAX=Y(1)
      YMIN=Y(1)
      ZMAX=Z(1)
      ZMIN=Z(1)
      T5=0.0
      T6=0.0
      T=0.0
      DO 110 I=1,N
      IF(I-1)171,170,171
  171 READ 105,X(I),Y(I),Z(I)
      IF(XMAX-X(I))135,140,140
  135 XMAX=X(I)
  140 IF(XMIN-X(I))150,150,145
  145 XMIN=X(I)
  150 IF(YMAX-Y(I))155,160,160
  155 YMAX=Y(I)
  160 IF(YMIN-Y(I))161,161,165
  165 YMIN=Y(I)
  161 IF(ZMAX-Z(I))166,167,167
  166 ZMAX=Z(I)
  167 IF(ZMIN-Z(I))170,170,169
  169 ZMIN=Z(I)
  170 C(11)=Z(I)
      C(10)=1.0
      C(9)=Y(I)
      C(8)=X(I)
      C(7)=Y(I)*Y(I)
      C(6)=X(I)*Y(I)
      C(5)=X(I)*X(I)
      C(4)=C(7)*Y(I)
      C(3)=C(7)*X(I)
      C(2)=C(5)*Y(I)
      C(1)=C(5)*X(I)
      T5=T5+Z(I)
      DO 1171 K=1,10
      DO 1171 L=K,11
 1171 WW(K,L)=WW(K,L)+C(L)*C(K)
      DO 1172 K=2,10
      KK=K-1
      DO 1172 L=1,KK
```

```
1172 WW(K,L)=WW(L,K)
 110 T=T+Z(I)*Z(I)
     READ 215,YL,PER,NIT,ERRPER,INDXY,INDPR,IBASE
 215 FORMAT (2F4.0,I2,F4.0,3I1)
     T6=T
     J=1
     DO 1173 K=8,10
     DO 1173 L=8,11
     W(J)=WW(K,L)
1173 J=J+1
     G=W(12)*W(12)
     CALL JC101(W,3,S)
     CL(1)=S(1)
     CL(2)=S(2)
     CL(3)=S(3)
     WK=N
     T=T-G/WK
     T3=T
     T1=0.0
     T2=0.0
     T4=0.0
     DO 205 I=1,N
     G=X(I)*S(1)+Y(I)*S(2)+S(3)
     T1=T1+G
     E=Z(I)-G
     ZERR(I)=E
     T4=T4+E
 205 T2=T2+G*G
     E=T4/WK
     T1=T1+T4
     T2=T2+2.0*T1*E+T4*E
     T=((T2-T1*T1/WK)/T)*100.0
     FL=T
     PRINT 20
     PRINT 210,T
 210 FORMAT (24HOPLANE PERCENTAGE FIT IS,F8.3,//)
     PRINT 200,S(3),S(1),S(2)
 201 FORMAT (1HO,/)
 200 FORMAT (30HOGLOBAL PLANE FIT COEFFICIENTS, 3F10.4)
     IF(INDPR)216,217,216
 217 DO 218 I=1,N
 218 ZERR(I)=ZERR(I)+E
     PRINT80
  80 FORMAT(12HOZ RESIDUALS)
     PRINT 306,(ZERR(I),I=1,N)
 216 I=YL+1.
     CALL JCKU11(1,29,I,NKR)
     KAW=2
     IF(NKR)250,250,700
 250 G=M1-1
     T1=0.0
     T2=0.0
     T4=0.0
     J=1
     DO 247 K=5,10
     DO 247 L=5,11
     W(J)=WW(K,L)
 247 J=J+1
     CALL JC101(W,6,S)
     DO 2500I=1,N
     ZQ=ACPOLY(2,X(I),Y(I),S)
```

```
      E=Z(I)-ZQ
      T1=T1+ZQ
      ZERR(I)=E
      T2=T2+ZQ*ZQ
 2500 T4=T4+E
      E=T4/WK
      T1=T1+T4
      T2=T2+2.0*T1*E+T4*E
      IF(INDPR)2503,2504,2503
 2504 DO 1501 I=1,N
 1501 ZERR(I)=ZERR(I)+E
 2503 DO 2502 I=1,6
 2502 CQ(I)=S(I)
      T=((T2-T1*T1/WK)/T3)*100.0
      ZSD=SQRT(WK*T6-T5*T5)/WK
      ZMEAN=T5/WK
      FQ=T
      PRINT 2501,T,(S(I),I=1,6)
 2501 FORMAT(29HOQUADRATIC PERCENTAGE FIT IS , F8.3,//,34H GLOBAL QUADRA
     1TIC FIT COEFFICIENTS,6F10.4)
      PRINT 80
      PRINT 306,(ZERR(I),I=1,N)
      T1=0.0
      T2=0.0
      T4=0.0
      J=1
      DO 2506 K=1,10
      DO 2506 L=1,11
      W(J)=WW(K,L)
 2506 J=J+1
      CALL JC101(W,10,S)
      DO 1502 J=1,10
 1502 CC(J)=S(J)
      DO 1500 I=1,N
      ZCUB=ACPOLY(3,X(I),Y(I),S)
      ZERR(I)=Z(I)-ZCUB
      T1=T1+ZCUB
      T2=T2+ZCUB*ZCUB
 1500 T4=T4+ZERR(I)
      E=T4/WK
      T1=T1+T4
      T2=T2+2.0*T1*E+T4*E
      ERR=ZSD*PER/100.0
      T=((T2-T1*T1/WK)/T3)*100.0
       FC=T
      PRINT 1505,T,(CC(I),I=1,10)
 1505 FORMAT(24HOCUBIC PERCENTAGE FIT IS, F8.3,//,30HOGLOBAL CUBIC FIT C
     1OEFFICIENTS,//,10F10.4,//)
      IF(INDPR) 1510,1515,1510
 1515 PRINT80
      PRINT306,(ZERR(I),I=1,N)
      PRINT 1506,ZMEAN,ZSD
 1506 FORMAT (19HOMEAN VALUE OF Z IS,F10.3,5X,27H STANDARD DEVIATION OF
     1Z IS,F8.3,//)
 1510 III=1
      I=IBASE+1
C-----------------------------------------------------------------------
C  CHOOSE GLOBAL FIT SURFACE
C-----------------------------------------------------------------------
      GO TO(1511,418,424,423),I
```

```
 1511 IF (FL-FQ)421,421,422
  421 IF(FQ-FC)423,423,424
  423 NFIT=3
      DO 417 J=1,10
  417 S(J)=CC(J)
      GO TO 419
  424 NFIT =2
      DO 416 J=1,6
  416 S(J)=CQ(J)
      GO TO 419
  422 IF(FL-FC)423,423,418
  418 NFIT=1
      DO 414 J=1,3
  414 S(J)=CL(J)
  419 PRINT 301,NFIT
  301 FORMAT(38HOCHOSEN GLOBAL FIT SURFACE IS OF ORDER,I4,//)
      PRINT 1507
 1507 FORMAT (1H1)
  235 IF(INDXY)220,251,220
  251 XMINI=XMIN
      XMAXI=XMAX
      YMINI=YMIN
      YMAXI=YMAX
      GO TO 420
  220 READ 225,XMINI,XMAXI,YMINI,YMAXI
  225 FORMAT (4F15.4)
  420 XQ=XMINI
      DX=(XMAXI-XMINI)/XNC
      DY=(YMAXI-YMINI)/XNR
      DX2=2.0*DX
      DY2=2.0*DY
      NC1=NC-1
      NR1=NR-1
C-----------------------------------------------------------------------
C  CHOOSE RESTRICTED X,Y,Z SET IF REQUIRED
C-----------------------------------------------------------------------
      IF(INDXY)1440,1430,1440
 1440 I=1
 1300 IF(X(I)-XMINI)1425,1305,1305
 1305 IF(X(I)-XMAXI)1306,1306,1425
 1306 IF(Y(I)-YMINI)1425,1307,1307
 1307 IF(Y(I)-YMAXI)1308,1308,1425
 1308 I=I+1
      IF(N-I)1430,1300,1300
 1425 N=N-1
      IF(N-I)1430,1435,1435
 1435 X(I)=X(N+1)
      Y(I)=Y(N+1)
      Z(I)=Z(N+1)
      GO TO 1300
C-----------------------------------------------------------------------
C  CALCULATE GLOBAL FIT GRID VALUES
C-----------------------------------------------------------------------
 1430 DO 300 I=1,NC
      YQ=YMINI
      DO 305 J=1,NR
      P(J,I)=ACPOLY(NFIT,XQ,YQ,S)
  305 YQ=YQ+DY
  300 XQ=XQ+DX
      IDC=150
```

```
      DO 302 I=1,NC
  302 RECORD(IDC) (P(I,J),J=1,NR)
      ITER=0
  307 ZERMAX=0.0
      SUM=0.0
      SUMSQ=0.0
  306 FORMAT(/,(8F12.4))
      NK=N
C-----------------------------------------------------------------------
C  MODIFY GRID VALUES BY QUADRATIC BUMPS
C-----------------------------------------------------------------------
      DO 310I=1,N
      XNWT=4.0
      L=(X(I)-XMINI)/DX-0.5
      M=(Y(I)-YMINI)/DY-0.5
      IF(L-1)315,320,320
  315 L=1
      GO TO 325
  320 IF(L-NC+4)325,325,330
  330 L=NC-4
  325 IF(M-1)335,340,340
  335 M=1
      GO TO 345
  340 IF(M-NR+4)345,345,350
  350 M=NR-4
  345 DO 355 J=1,42
  355 W(J)=0.0
      MM=M+4
      LL=L+4
      XQ=L-1
      XQ=XQ*DX+XMINI
      DO 360 J=L,LL,2
      YQ=M-1
      YQ=YQ*DY+YMINI
      DO 365 K=M,MM,2
      C(1)=XQ*XQ
      C(2)=XQ*YQ
      C(3)=YQ*YQ
      C(4)=XQ
      C(5)=YQ
      C(6)=1.0
      C(7)=P(K,J)
      JJ=1
      DO 370 KK=1,6
      DO 370 KL=1,7
      W(JJ)=W(JJ)+C(KL)*C(KK)
  370 JJ=JJ+1
  365 YQ=YQ+DY2
  360 XQ=XQ+DX2
      C(1)=X(I)*X(I)
      C(2)=X(I)*Y(I)
      C(3)=Y(I)*Y(I)
      C(4)=X(I)
      C(5)=Y(I)
      C(6)=1.0
      C(7)=Z(I)
      JJ=1
      DO 380 KK=1,6
      DO 380 KL=1,7
      W(JJ)=W(JJ)+C(KL)*C(KK)*XNWT
```

```
  380 JJ=JJ+1
      CALL JC101 (W,6,S)
      XQ=L-1
      XQ=XQ*DX+XMINI
      DO 385 J=L,LL
      YQ=M-1
      YQ=YQ*DY+YMINI
      DO 390 K=M,MM
      P(K,J)=ACPOLY(2,XQ,YQ,S)
  390 YQ=YQ+DY
  385 XQ=XQ+DX
      ZERR(I)=Z(I)-ACPOLY(2,X(I),Y(I),S)
      WG=ZERR(I)
      SUM=SUM+WG
      SUMSQ=SUMSQ+WG*WG
      IF(ABS(WG)-ABS(ZERMAX))316,316,311
  311 ZERMAX=WG
C-----------------------------------------------------------------------
C  TEST FOR END OF ITERATION
C-----------------------------------------------------------------------
  316 IF(ITER)310,310,312
  312 IF(ABS(WG-ERR))313,310,310
  313 IF(XNWT-12.0)314,310,310
  314 XNWT=XNWT+4.0
      GO TO 345
  310 CONTINUE
      SD=SQRT(WK*SUMSQ-SUM*SUM)/WK
      IF(INDPR)402,401,402
  401 PRINT80
      PRINT 306,(ZERR(I),I=1,N)
      SE=0.0
      IF (SENSE SWITCH 4)1416,402
  402 DO 400 I=1,NC
      W1(I)=P(1,I)
      W2(I)=P(2,I)
  400 W3(I)=P(3,I)
C------------------------------------------------------------
C     SMOOTH
C------------------------------------------------------------
      T=1.0/9.0
      DO 405 I=2,NR1
      DO 410 J=2,NC1
      TT=T*(W3(J+1)+W3(J-1)+W1(J+1)+W1(J-1)-2.0*(W3(J)+W2(J+1)+W2(J-1)+W
     11(J))+4.0*W2(J))
      IF(ABS(SE)-ABS(TT))411,410,410
  411 SE=TT
  410 P(I,J)=P(I,J)-TT
      L=I+2
      DO 415 K=1,NC
      W1(K)=W2(K)
      W2(K)=W3(K)
  415 W3(K)=P(L,K)
  405 CONTINUE
 1416 ITER=ITER+1
      PRINT 403,SD,SE
  403 FORMAT(/,31HOSTANDARD DEVIATION OF ERROR IS,F10.4,//,32H MAXIMUM S
     1MOOTHING CORRECTION IS,F12.4,//)
C-------------------------------------------------------
C     FORCED EXIT
C-------------------------------------------------------
      IF(SENSE SWITCH1)425,426
```

```
  426 IF(SD-ERR)425,425,428
  428 IF(ITER-NIT)307,425,425
  425 IDC=1
      PRINT 427,ITER,ZERMAX
  427 FORMAT(24HONUMBER OF ITERATIONS IS,I5, 5X, 30H Z ERROR OF MAXIMUM
     1MODULUS IS, F8.3,///)
      DO 430 I=1,NC
  430 RECORD(IDC)(P(I,J),J=1,NR)
      T1=0.0
      T2=0.0
      T3=0.0
      G=0.0
      DO 500 I=1,N
      WG=Z(I)-ZERR(I)
      T1=T1+WG
      T3=T3+Z(I)
      G=G+Z(I)*Z(I)
  500 T2=T2+WG*WG
      T3=G-T3*T3/WK
      T1=(T2-T1*T1/WK)/T3*100.0
      PRINT 505,T1
  505 FORMAT(26HOOVERALL PERCENTAGE FIT IS, F8.3)
      PRINT 510
C-----------------------------------------------------------------------
C  PRINT TABLE
C-----------------------------------------------------------------------
 510  FORMAT(1H1,2X,1HI,10X,1HX,14X,1HY,11X,1HZ, 9X,5HZCALC,10X,4HZERR,
     17X,7HPERCENT,7X,6H SDPER,/)
      DO 515 I=1,N
      ZCALC=Z(I)+ZERR(I)
      IF(Z(I))520,525,520
  520 ZPER=ZCALC/Z(I)*100.0
      GO TO 530
  525 ZPER=999.0
 530  SDPER=ABS(ZERR(I)/ZSD*100.0)
      IF(SDPER-ERRPER)535,540,540
  535 PRINT 545,I,X(I),Y(I),Z(I),ZCALC,ZERR(I),ZPER,SDPER
  545 FORMAT(I6,7F13.4)
      GO TO 514
  540 PRINT 550,I,X(I),Y(I),Z(I),ZCALC,ZERR(I),ZPER,SDPER
  550 FORMAT(I6,7F13.4,4X,1H*)
  514 IF(ZERR(I))513,512,512
  513 ZERR(I)=-SDPER
      GO TO 515
  512 ZERR(I)=SDPER
  515 CONTINUE
      PRINT 1507
      CALL LINK(JCMAP2)
      END
```

```
*LDISKJCMAP2
*LIST PRINTER
      DIMENSION B(50,33),X(240),Y(240),ZERR(240)
      COMMON N,XMAXI,XMINI,YMAXI,YMINI,YL,ERRPER,X,Y,ZERR,NC,NR,XNC,XNR
      DEFINE DISK(10,300)
      ICOUNT=1
      IDT=1
      DO 5 I=1,NC
    5 FETCH(IDT) (B(I,J),J=1,NR)
C-----------------------------------------------------------------------
C     COMPUTE MAP PARAMETERS
C-----------------------------------------------------------------------
      XQ=XMAXI-XMINI
      YQ=YMAXI-YMINI
      XL=XQ*YL/YQ
      T=YMINI-0.5*(YMAXI-YMINI)/YL
      CALL PLOT(201,0.0,1.0,1.0,1.0,0.0,1.0,1.0,1.0)
      CALL PLOT(99)
      CALL PLOT(98,0.0,-8.0)
      CALL PLOT(98,0.0,-7.25)
      CALL PLOT(8)
      CALL PLOT(201,XMINI,XMAXI,XL,XQ,YMINI,YMAXI,YL,YQ)
      READ10,CMIN,CINC,NUMC,IERIND,IPOINT,IBASIC,JERR,IDIF
   10 FORMAT(2F15.6,I5,5I1)
      READ25
   11 CALL PLOT (90,XMINI,YMINI)
      CALL PLOT (90,XMAXI,YMINI)
      CALL PLOT (90,XMAXI,YMAXI)
      CALL PLOT (90,XMINI,YMAXI)
      CALL PLOT(90,XMINI,YMINI)
      CALL PLOT(99)
      CALL PLOT(90,XMINI,T)
      GO TO (12,300,400),ICOUNT
   12 CALL CHAR(0,0.14,0)
   25 FORMAT(80H THIS FORMAT WILL BE FILLED BY JOB NAME
     1                                         )
      PRINT 26
   26 FORMAT(1H1)
      PRINT 25
      PRINT 15,XMINI,XMAXI,YMINI,YMAXI,CMIN,CINC,NUMC
   15 FORMAT(24H0 CONTOUR MAP PARAMETERS,//,19H MINIMUM X-VALUE IS,F15.6
     1,6X,18HMAXIMUM X-VALUE IS,F15.6,//,19H MINIMUM Y-VALUE IS,F15.6,5X
     2,19H MAXIMUM Y-VALUE IS,F15.6,//,25H MINIMUM CONTOUR VALUE IS,F15.
     36,5X,20H CONTOUR INTERNAL IS,F15.6,5X,28H NUMBER OF CONTOUR LEVELS
     4 IS,I6,//)
      W=(XMAXI-XMINI)/XL
      WW=(YMAXI-YMINI)/YL
      PRINT 30,W,WW,ERRPER
   30 FORMAT(19H0 X-SCALE FACTOR IS,F12.3,5X,19H  Y-SCALE FACTOR IS,
     1F12.3,//,29H0 ERROR INDICATORS PLOTTED AT,F4.0,2X,9H  PERCENT,
     1///,22H KEY TO CONTOUR LEVELS,/)
      W=CMIN
      DO 35 I=1,NUMC
      PRINT 40,I,W
   40 FORMAT(I4,1H=,F12.4)
   35 W=W+CINC
      XD=(XMAXI-XMINI)/XNC
      YD=(YMAXI-YMINI)/XNR
      GO TO 36
  300 CALL CHAR(0,0.14,0)
```

```
 301 FORMAT(24H GLOBAL FIT SURFACE PLOT)
     GO TO 36
 400 CALL CHAR(0,0.14,0)
 401 FORMAT(15H DIFFERENCE MAP)
 36  CALL JCCONT(B,CMIN,CINC,NUMC,NC,NR,XMINI,XD,YMINI,YD,XL,YL)
     GO TO (37,76,76),ICOUNT
 37  DO 50 I=1,N
     CALL PLOT(99)
     IF(IERIND)61,55,61
 61  IF(ABS(ZERR(I))-ERRPER)55,60,60
 60  CALL PLOT(90,X(I),Y(I))
     IF(ZERR(I))65,70,70
 65  CALL POINT (8,0.1,1,1)
     GO TO 50
 70  CALL POINT (7,0.1,1,1)
     GO TO 50
  55 IF(IPOINT)50,50,75
 75  CALL PLOT(90,X(I),Y(I))
     CALL POINT(1,0.08,1,1)
  50 CONTINUE
C-----------------------------------------------------------------------
C   MOVE FOR NEXT PLOT
C-----------------------------------------------------------------------
  76 S=3.0*(XMAXI-XMINI)/XL+XMAXI
     CALL PLOT(99)
     CALL PLOT(98,S,YMINI)
     CALL PLOT(99)
     CALL PLOT(8)
     CALL PLOT(201,XMINI,XMAXI,XL,XQ,YMINI,YMAXI,YL,YQ)
     ICOUNT=ICOUNT+1
     IF(IBASIC)200,205,200
 200 IDT=150
     IBASIC=0
     DO 210 I=1,NC
 210 FETCH(IDT)(B(I,J),J=1,NR)
     GO TO 11
 205 IF(JERR)220,100,220
 220 JERR=0
     IDT=1
     DO 221 I=1,NC
     FETCH(IDT)(B(I,J),J=1,NR)
 221 CONTINUE
     R=0.0
     IF (IDIF) 222,223,222
 222 READ 270, CMIN,CINC,NUMC
 270 FORMAT (2F15.6,I5)
     IDT=150
     DO 500 I=1,NC
     FETCH(IDT) (ZERR(J),J=1,NR)
     DO 500 J=1,NR
 500 B(I,J)=B(I,J)-ZERR(J)
     GO TO 234
 223 IDT=150
     DO 226I=1,NC
     FETCH(IDT)(ZERR(J),J=1,NR)
     DO 225 J=1,NR
     B(I,J)=B(I,J)-ZERR(J)
     IF(R-ABS(B(I,J)))230,225,225
 230 R=ABS(B(I,J))
 225 CONTINUE
```

```
  226 CONTINUE
      RC=1.0
      RI=0.2
      RM=5.0
      RM1=2.0
      IF(R-RC)231,232,233
  231 RM=0.5
      RM1=0.2
  265 RC=RC*RM
      RI=RI*RM
      IF(R-RC)260,232,241
  260 RT=RM
      RM=RM1
      RM1=RT
      GO TO 265
  233 RC=RC*RM
      RI=RI*RM
      IF(R-RC)232,232,243
  243 RT=RM
      RM=RM1
      RM1=RT
      GO TO 233
  241 RC=RC/RM
      RI=RI/RM
 232  CMIN=-RC+RI/2.0
      CINC=RI
      NUMC=10
  234 PRINT 250
  250 FORMAT(//, 37HOKEY TO DIFFERENCE MAP CONTOUR LEVELS ,//)
      W=CMIN
      DO 255 I=1,NUMC
      PRINT 40,I,W
  255 W=W+CINC
      GO TO 11
  100 CALL EXIT
      END
```

Table 2. –Sample problem data and results.

KANSAS DATA
100

COMPUTER CONTRIBUTION 23   3 ITERATIONS

0.057	0.051	2.70
0.041	0.274	4.00
0.085	0.339	4.70
0.061	0.343	4.70
0.034	0.385	4.90
0.028	0.497	5.40
0.012	0.923	2.80
0.169	0.059	2.90
0.125	0.178	3.50
0.182	0.270	4.40
0.116	0.375	5.30
0.159	0.492	6.70
0.150	0.553	6.80
0.132	0.691	5.80
0.171	0.745	5.00
0.132	0.830	4.10
0.136	0.891	3.80
0.177	0.893	3.30
0.165	0.949	2.80
0.235	0.962	2.20
0.278	0.905	2.70
0.218	0.640	6.30
0.241	0.633	6.40
0.209	0.351	5.80
0.293	0.189	4.90
0.274	0.169	4.50
0.232	0.089	3.60
0.350	0.030	3.60
0.353	0.144	4.80
0.339	0.229	5.50
0.339	0.431	7.90
0.394	0.571	6.80
0.325	0.602	6.70
0.320	0.649	5.80
0.365	0.718	4.70
0.385	0.723	4.50
0.301	0.745	4.50
0.378	0.755	4.30
0.364	0.805	3.50
0.331	0.848	3.30
0.373	0.899	3.30
0.352	0.998	3.30
0.417	0.965	4.10
0.419	0.933	4.00
0.458	0.794	4.20
0.403	0.691	4.80
0.407	0.390	7.70
0.488	0.388	8.00
0.446	0.276	6.50
0.442	0.275	6.40
0.423	0.157	5.20

0.518	0.033	3.90
0.593	0.264	6.40
0.501	0.473	7.00
0.552	0.477	6.80
0.523	0.552	6.00
0.536	0.678	5.10
0.553	0.786	4.90
0.512	0.971	5.00
0.651	0.993	4.70
0.674	0.937	4.60
0.660	0.926	4.70
0.666	0.758	5.70
0.690	0.731	5.90
0.641	0.726	5.80
0.605	0.433	6.50
0.614	0.282	6.50
0.623	0.160	5.40
0.696	0.111	4.90
0.627	0.081	4.60
0.694	0.036	4.10
0.719	0.084	4.60
0.736	0.215	5.60
0.784	0.287	5.10
0.723	0.311	5.00
0.716	0.421	5.10
0.794	0.489	4.80
0.746	0.852	4.20
0.757	0.902	3.90
0.763	0.976	4.10
0.830	0.990	3.40
0.806	0.949	3.00
0.842	0.921	2.70
0.853	0.835	3.50
0.897	0.771	4.30
0.876	0.663	5.20
0.865	0.578	6.10
0.806	0.512	4.40
0.826	0.461	4.50
0.833	0.052	4.20
0.977	0.213	3.40
0.959	0.406	3.30
0.914	0.457	4.00
0.906	0.531	5.00
0.961	0.536	4.60
0.936	0.541	4.90
0.955	0.612	4.90
0.925	0.642	5.60
0.964	0.764	4.80
0.951	0.831	3.40

06.50005030025000

2.0   1.0   000061111

COMPUTER CONTRIBUTION 23

KANSAS DATA   3 ITERATIONS

KANSAS DATA COMPUTER CONTRIBUTION 23 3 ITERATIONS

PLANE PERCENTAGE FIT IS 5.888

GLOBAL PLANE FIT COEFFICIENTS 5.3408 .0363 -1.0274

Z RESIDUALS

-2.5905	-1.0608	-.2956	-.2907	-.0465	.5687	-1.5929	-2.3864
-1.6625	-.6701	.3401	1.8588	2.0218	1.1642	.4183	-.3929
-.6303	-1.1298	-1.5718	-2.1610	-1.7211	1.6087	1.7007	.8121
-.2573	-.6772	-1.6578	-1.7228	-.4057	.3820	2.9896	2.0314
1.9658	1.1142	.0835	-.1120	-.0863	-.2789	-1.0270	-1.1816
-1.1307	-1.0283	-.2645	-.3975	-.3417	.1544	2.7450	3.0400
1.4264	1.3255	.0050	-1.4258	1.3087	2.1268	1.9291	1.2072
.4362	.3465	.6381	.3556	.1973	.2865	1.1136	1.2850
1.1817	1.5819	1.4265	.2008	-.3521	-.6804	-1.2291	-.6807
.4532	.0254	-.0476	.1656	-.0673	-.2926	-.5416	-.2658
-.9539	-1.3951	-1.7252	-1.0139	-.2813	.5084	1.3215	-.4441
-.3972	-1.1177	-1.7575	-1.6586	-.9045	.1717	-.2251	.0809
.1531	.8850	.2090	-1.1216				

QUADRATIC PERCENTAGE FIT IS 64.116

GLOBAL QUADRATIC FIT COEFFICIENTS -7.9798 .8627 -11.4845 7.4888 10.6228 2.0412

Z RESIDUALS

-.2565	-.3929	-.2263	-.0788	.2145	.7005	.6396	-.7742
-.8988	-.8132	.0913	1.1558	1.3810	.9738	.2618	.2095
.4357	-.2809	-.1327	-.9463	-1.2310	.7905	.7633	.1651
-.2956	-.5012	-.6213	-.4021	-.2258	-.0602	1.6659	.5316
.6660	-.0566	-.9442	-1.1587	-.8056	-1.1517	-1.5717	-1.3374
-.9817	.1454	.4200	-.0293	-1.1052	-1.0602	1.3996	1.6484
.5428	.4500	-.0231	-.2320	.5849	.5501	.3752	-.3886
-.8992	-.4716	1.3007	1.3835	.7209	.6758	.3216	.4271
.1767	.1763	.6372	.2987	.4078	.1714	.3158	.4361
.4800	-.2045	-.6712	-.9090	-.9399	-.2493	-.0644	.9199
.6708	-.2912	-.7004	-.6002	.0019	.2263	.8170	-1.2775
-1.0675	.8989	-.2620	-1.3398	-1.0361	-.0935	-.1072	.0177
.2807	.8606	.9073	-.1247				

CUBIC PERCENTAGE FIT IS 71.915

GLOBAL CUBIC FIT COEFFICIENTS

-2.8725 10.0782 12.1519 6.4911 -8.4122 -21.9681 -27.8441 12.8367 23.2577 -.2440

Z RESIDUALS

1.1860	-.4789	-.5322	-.4109	-.2581	.0307	.1590	.1470
-.6051	-.9456	-.2691	.7768	1.0126	.7015	.1781	.1251
.4357	-.1608	-.0013	-.6673	-.9121	.6669	.6747	-.1147
-.3417	-.4549	-.1164	.0217	-.2569	-.2486	1.4449	.5455
.6587	.0057	-.7218	-.9116	-.6203	-.8685	-1.2446	-.9997

-.6033	.4730	.7445	.3251	-.7551	-.8445	1.1698	1.4410
.2504	.1591	-.2044	-.3069	.2714	.4744	.3347	-.3196
-.6434	-.1524	1.4790	1.1091	.5997	.6304	.5680	.6696
.4468	.0950	.3475	-.0585	.0568	-.1493	.0088	.0973
.2095	-.3196	-.8378	-.9161	-.7656	-.2725	-.2883	.3127
-.3140	-.9073	-1.2826	-.7910	-.0225	.4651	1.1126	-1.0675
-.8790	.6740	.0416	-.9143	-.6922	.2559	.3317	.4118
.6338	1.1452	.8354	-.5030				

MEAN VALUE OF Z IS 4.795 STANDARD DEVIATION OF Z IS 1.234

CHOSEN GLOBAL FIT SURFACE IS OF ORDER 3

Z RESIDUALS

.4497	-.1692	-.1580	-.0119	-.0631	.0155	.0507	.0407
-.1918	-.2999	-.0437	.2370	.2824	.2194	.0110	.0209
.1297	-.1157	.0107	-.2146	-.2434	.2246	.1193	.0190
-.1219	-.0812	-.0634	.0064	-.0638	-.0650	.4580	.1727
.1578	-.1046	-.2344	-.1307	-.1261	-.0807	-.3457	-.1456
-.0615	.1644	.3143	.0064	-.2073	-.1484	.2913	.4084
.0815	-.0015	-.0565	-.1048	.0916	.1216	.0911	-.1329
-.2119	-.0218	.5101	.3782	.0551	.0334	.1787	.1199
.0040	.0049	.0675	-.0257	.0202	-.0540	.0016	.0242
.0667	-.1044	-.2658	-.2870	-.2482	-.0536	-.0785	.1323
-.1248	-.2481	-.2858	-.1988	.0071	.1554	.3253	-.2103
-.1227	.2518	.0086	-.3618	-.1437	.0617	.0787	.0279
.1459	.2140	.2974	-.2094				

STANDARD DEVIATION OF ERROR IS .1848

MAXIMUM SMOOTHING CORRECTION IS -.1924

Z RESIDUALS

.1776	-.0676	-.0548	.0102	-.0204	.0170	.0265	.0132
-.1000	-.1292	-.0168	.0458	.1255	.1123	-.0042	-.0145
.1101	-.0678	.0308	-.0867	-.0839	.0606	.0454	.0705
-.0149	-.0458	-.0414	.0090	-.0213	-.0191	.1864	.0743
.0935	-.0898	.0251	-.0319	-.0133	.0312	-.1386	-.0087
-.0730	.0666	.1619	-.0135	-.0771	-.0862	.0679	.1584
.0365	-.0144	-.0230	-.0487	.0208	.0506	.0647	-.1019
-.0878	.0038	.1981	.1415	-.0196	-.0049	.0294	.0451
-.0363	-.0145	.0430	-.0154	.0025	-.0356	-.0060	.0129
.0636	.0064	-.1429	-.1304	.0359	-.0220	.0393	.1591
.0501	-.1080	-.1033	-.0003	-.0866	-.0437	.1573	-.1278
-.0259	.1169	.0055	-.1383	-.0528	-.0332	.0262	.0028
-.0397	.1354	.1794	-.1071				

STANDARD DEVIATION OF ERROR IS .0804

MAXIMUM SMOOTHING CORRECTION IS -.1640

Z RESIDUALS

.0774	-.0287	-.0358	.0125	-.0062	.0028	.0202	.0065
-.0592	-.0604	-.0075	.0223	.0646	.0612	-.0189	-.0297
.0954	-.0633	.0349	-.0724	-.0359	.0133	.0295	.0715
.0156	-.0339	-.0311	.0094	-.0130	-.0093	.0939	.0460
.0713	-.0840	.0325	-.0252	.0004	.0434	-.0890	.0264
-.0648	.0386	.1213	-.0123	-.0247	-.0621	.0091	.0856
.0289	-.0158	-.0114	-.0268	.0028	.0269	.0510	-.0934
-.0447	.0019	.0946	.0792	-.0156	.0055	.0205	.0383
-.0372	-.0277	.0456	-.0153	.0004	-.0243	-.0042	.0088
.0531	.0142	-.0999	-.0647	.0684	-.0247	.0504	.1222
.0528	-.0831	-.0510	.0584	-.1096	-.0713	.1305	-.1215
.0148	.0688	.0038	-.0602	-.0217	-.0323	.0209	-.0003
-.0813	.1238	.1352	-.0709				

STANDARD DEVIATION OF ERROR IS .0566

MAXIMUM SMOOTHING CORRECTION IS -.1241

NUMBER OF ITERATIONS IS 3   Z ERROR OF MAXIMUM MODULUS IS .135

OVERALL PERCENTAGE FIT IS 97.687

I	X	Y	Z	ZCALC	ZERR	PERCENT	SDPER
1	.0570	.0510	2.7000	2.7774	.0774	102.8685	6.2715
2	.0410	.2740	4.0000	3.9712	-.0287	99.2824	2.3240
3	.0850	.3390	4.7000	4.6641	-.0358	99.2374	2.9019
4	.0610	.3430	4.7000	4.7125	.0125	100.2676	1.0187
5	.0340	.3850	4.9000	4.8937	-.0062	99.8733	.5025
6	.0280	.4970	5.4000	5.4028	.0028	100.0525	.2299
7	.0120	.9230	2.8000	2.8202	.0202	100.7229	1.6391
8	.1690	.0590	2.9000	2.9065	.0065	100.2253	.5292
9	.1250	.1780	3.5000	3.4407	-.0592	98.3063	4.8000
10	.1820	.2700	4.4000	4.3395	-.0604	98.6250	4.8989
11	.1160	.3750	5.3000	5.2924	-.0075	99.8567	.6145
12	.1590	.4920	6.7000	6.7223	.0223	100.3329	1.8061
13	.1500	.5530	6.8000	6.8646	.0646	100.9510	5.2370
14	.1320	.6910	5.8000	5.8612	.0612	101.0564	4.9617
15	.1710	.7450	5.0000	4.9810	-.0189	99.6219	1.5306
16	.1320	.8300	4.1000	4.0702	-.0297	99.2738	2.4109
17	.1360	.8910	3.8000	3.8954	.0954	102.5119	7.7294
18	.1770	.8930	3.3000	3.2366	-.0633	98.0803	5.1296
19	.1650	.9490	2.8000	2.8349	.0349	101.2498	2.8339
20	.2350	.9620	2.2000	2.1275	-.0724	96.7090	5.8627
21	.2780	.9050	2.7000	2.6640	-.0359	98.6682	2.9115
22	.2180	.6400	6.3000	6.3133	.0133	100.2113	1.0783
23	.2410	.6330	6.4000	6.4295	.0295	100.4623	2.3959
24	.2090	.3510	5.8000	5.8715	.0715	101.2334	5.7928
25	.2930	.1890	4.9000	4.9156	.0156	100.3198	1.2691
26	.2740	.1690	4.5000	4.4660	-.0339	99.2456	2.7488
27	.2320	.0890	3.6000	3.5688	-.0311	99.1343	2.5233
28	.3500	.0300	3.6000	3.6094	.0094	100.2614	.7622
29	.3530	.1440	4.8000	4.7869	-.0130	99.7284	1.0553
30	.3390	.2290	5.5000	5.4906	-.0093	99.8294	.7593
31	.3390	.4310	7.9000	7.9939	.0939	101.1887	7.6044
32	.3940	.5710	6.8000	6.8460	.0460	100.6766	3.7259
33	.3250	.6020	6.7000	6.7713	.0713	101.0646	5.7761
34	.3200	.6490	5.8000	5.7159	-.0840	98.5511	6.8047
35	.3650	.7180	4.7000	4.7325	.0325	100.6928	2.6368
36	.3850	.7230	4.5000	4.4747	-.0252	99.4386	2.0456
37	.3010	.7450	4.5000	4.5004	.0004	100.0090	.0329
38	.3780	.7550	4.3000	4.3434	.0434	101.0103	3.5179
39	.3640	.8050	3.5000	3.4109	-.0890	97.4560	7.2100
40	.3310	.8480	3.3000	3.3264	.0264	100.8014	2.1417
41	.3730	.8990	3.3000	3.2351	-.0648	98.0363	5.2473
42	.3520	.9980	3.3000	3.3386	.0386	101.1703	3.1272
43	.4170	.9650	4.1000	4.2213	.1213	102.9591	9.8243
44	.4190	.9330	4.0000	3.9876	-.0123	99.6915	.9990
45	.4580	.7940	4.2000	4.1752	-.0247	99.4117	2.0007
46	.4030	.6910	4.8000	4.7378	-.0621	98.7055	5.0311
47	.4070	.3900	7.7000	7.7091	.0091	100.1184	.7383
48	.4880	.3880	8.0000	8.0856	.0856	101.0701	6.9323
49	.4460	.2760	6.5000	6.5289	.0289	100.4461	2.3480
50	.4420	.2750	6.4000	6.3841	-.0158	99.7527	1.2815
51	.4230	.1570	5.2000	5.1885	-.0114	99.7792	.9296
52	.5180	.0330	3.9000	3.8731	-.0268	99.3106	2.1768
53	.5930	.2640	6.4000	6.4028	.0028	100.0439	.2278
54	.5010	.4730	7.0000	7.0269	.0269	100.3853	2.1844
55	.5520	.4770	6.8000	6.8510	.0510	100.7504	4.1321
56	.5230	.5520	6.0000	5.9065	-.0934	98.4419	7.5699
57	.5360	.6780	5.1000	5.0552	-.0447	99.1224	3.6240
58	.5530	.7860	4.9000	4.9019	.0019	100.0403	.1599
59	.5120	.9710	5.0000	5.0946	.0946	101.8926	7.6627
60	.6510	.9930	4.7000	4.7792	.0792	101.6866	6.4190
61	.6740	.9370	4.6000	4.5843	-.0156	99.6592	1.2692
62	.6600	.9260	4.7000	4.7055	.0055	100.1189	.4525
63	.6660	.7580	5.7000	5.7205	.0205	100.3608	1.6653
64	.6900	.7310	5.9000	5.9383	.0383	100.6499	3.1053
65	.6410	.7260	5.8000	5.7627	-.0372	99.3577	3.0161
66	.6050	.4330	6.5000	6.4722	-.0277	99.5724	2.2505
67	.6140	.2820	6.5000	6.5456	.0456	100.7022	3.6962
68	.6230	.1600	5.4000	5.3846	-.0153	99.7149	1.2465
69	.6960	.1110	4.9000	4.9004	.0004	100.0092	.0368
70	.6270	.0810	4.6000	4.5756	-.0243	99.4702	1.9734
71	.6940	.0360	4.1000	4.0957	-.0042	99.8967	.3428
72	.7190	.0840	4.6000	4.6088	.0088	100.1913	.7125
73	.7360	.2150	5.6000	5.6531	.0531	100.9493	4.3049
74	.7840	.2870	5.1000	5.1142	.0142	100.2800	1.1564
75	.7230	.3110	5.0000	4.9000	-.0999	98.0000	8.0972
76	.7160	.4210	5.1000	5.0352	-.0647	98.7311	5.2400
77	.7940	.4890	4.8000	4.8684	.0684	101.4268	5.5457
78	.7460	.8520	4.2000	4.1752	-.0247	99.4106	2.0044
79	.7570	.9020	3.9000	3.9504	.0504	101.2947	4.0887
80	.7630	.9760	4.1000	4.2222	.1222	102.9808	9.8962
81	.8300	.9900	3.4000	3.4528	.0528	101.5541	4.2787
82	.8060	.9490	3.0000	2.9168	-.0831	97.2273	6.7355
83	.8420	.9210	2.7000	2.6489	-.0510	98.1100	4.1321
84	.8530	.8350	3.5000	3.5584	.0584	101.6685	4.7289
85	.8970	.7710	4.3000	4.1903	-.1096	97.4506	8.8766
86	.8760	.6630	5.2000	5.1286	-.0713	98.6285	5.7747
87	.8650	.5780	6.1000	6.2305	.1305	102.1404	10.5729
88	.8060	.5120	4.4000	4.2784	-.1215	97.2377	9.8417
89	.8260	.4610	4.5000	4.5148	.0148	100.3302	1.2034
90	.8330	.0520	4.2000	4.2688	.0688	101.6393	5.5754
91	.9770	.2130	3.4000	3.4038	.0038	100.1134	.3123
92	.9590	.4060	3.3000	3.2397	-.0602	98.1747	4.8773
93	.9140	.4570	4.0000	3.9782	-.0217	99.4566	1.7600
94	.9060	.5310	5.0000	4.9676	-.0323	99.3535	2.6175
95	.9610	.5360	4.6000	4.6209	.0209	100.4546	1.6934
96	.9360	.5410	4.9000	4.8996	-.0003	99.9920	.0315
97	.9550	.6120	4.9000	4.8186	-.0813	98.3396	6.5877
98	.9250	.6420	5.6000	5.7238	.1238	102.2114	10.0282
99	.9640	.7640	4.8000	4.9352	.1352	102.8169	10.9488
100	.9510	.8310	3.4000	3.3290	-.0709	97.9123	5.7476

KANSAS DATA COMPUTER CONTRIBUTION 23 3 ITERATIONS

CONTOUR MAP PARAMETERS

MINIMUM X-VALUE IS .012000 MAXIMUM X-VALUE IS .977000

MINIMUM Y-VALUE IS .030000 MAXIMUM Y-VALUE IS .998000

MINIMUM CONTOUR VALUE IS 2.000000 CONTOUR INTERNAL IS 1.000000

NUMBER OF CONTOUR LEVELS IS 6

X-SCALE FACTOR IS .148 Y-SCALE FACTOR IS .148

ERROR INDICATORS PLOTTED AT 25. PERCENT

KEY TO CONTOUR LEVELS

1=	2.0000
2=	3.0000
3=	4.0000
4=	5.0000
5=	6.0000
6=	7.0000

KEY TO DIFFERENCE MAP CONTOUR LEVELS

1=	-4.5000
2=	-3.5000
3=	-2.5000
4=	-1.5000
5=	-.5000
6=	.5000
7=	1.5000
8=	2.5000
9=	3.5000
10=	4.5000

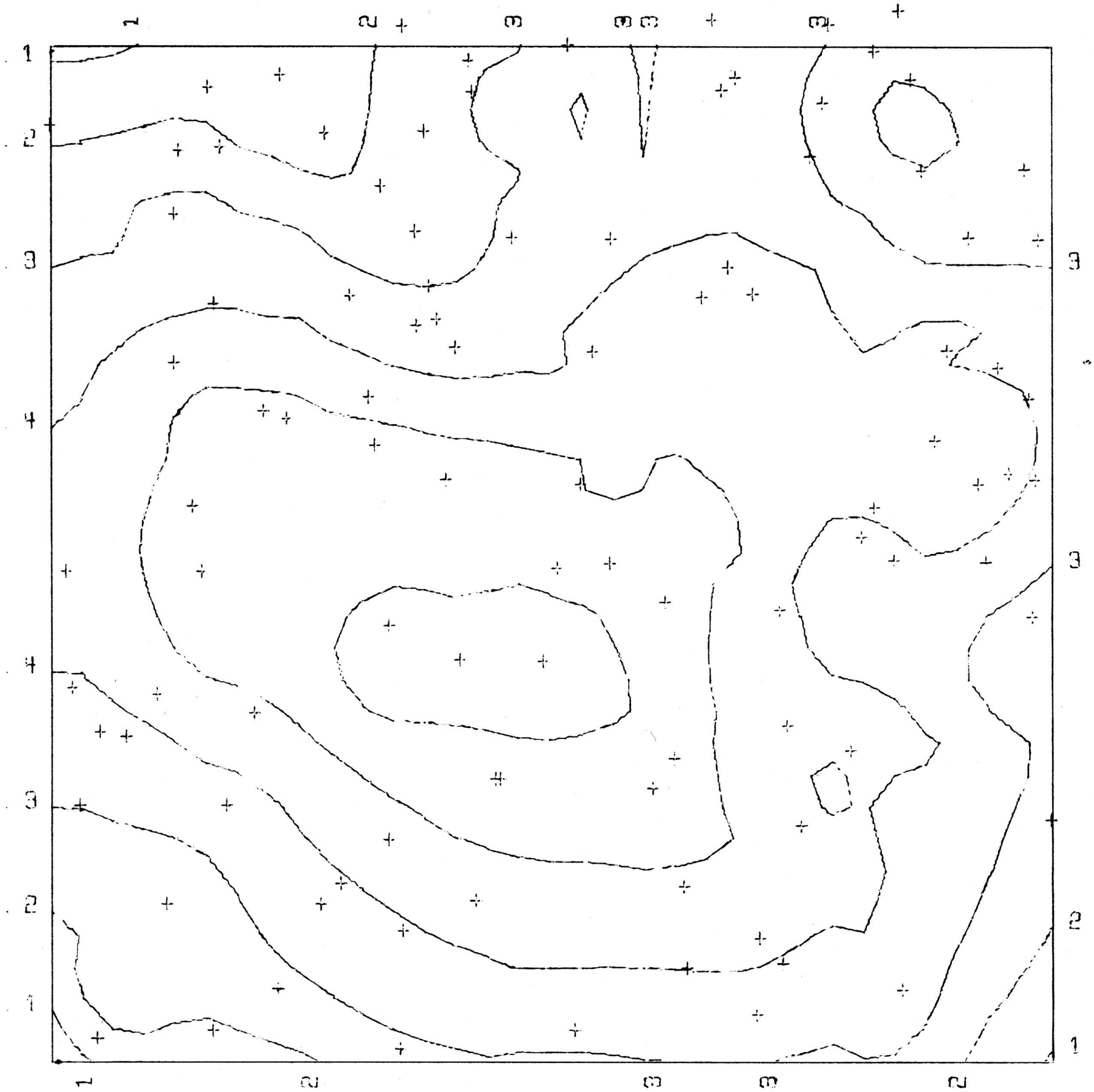

Figure 3. –Kansas data from Computer Contribution 23, cubic base, 3 iterations.

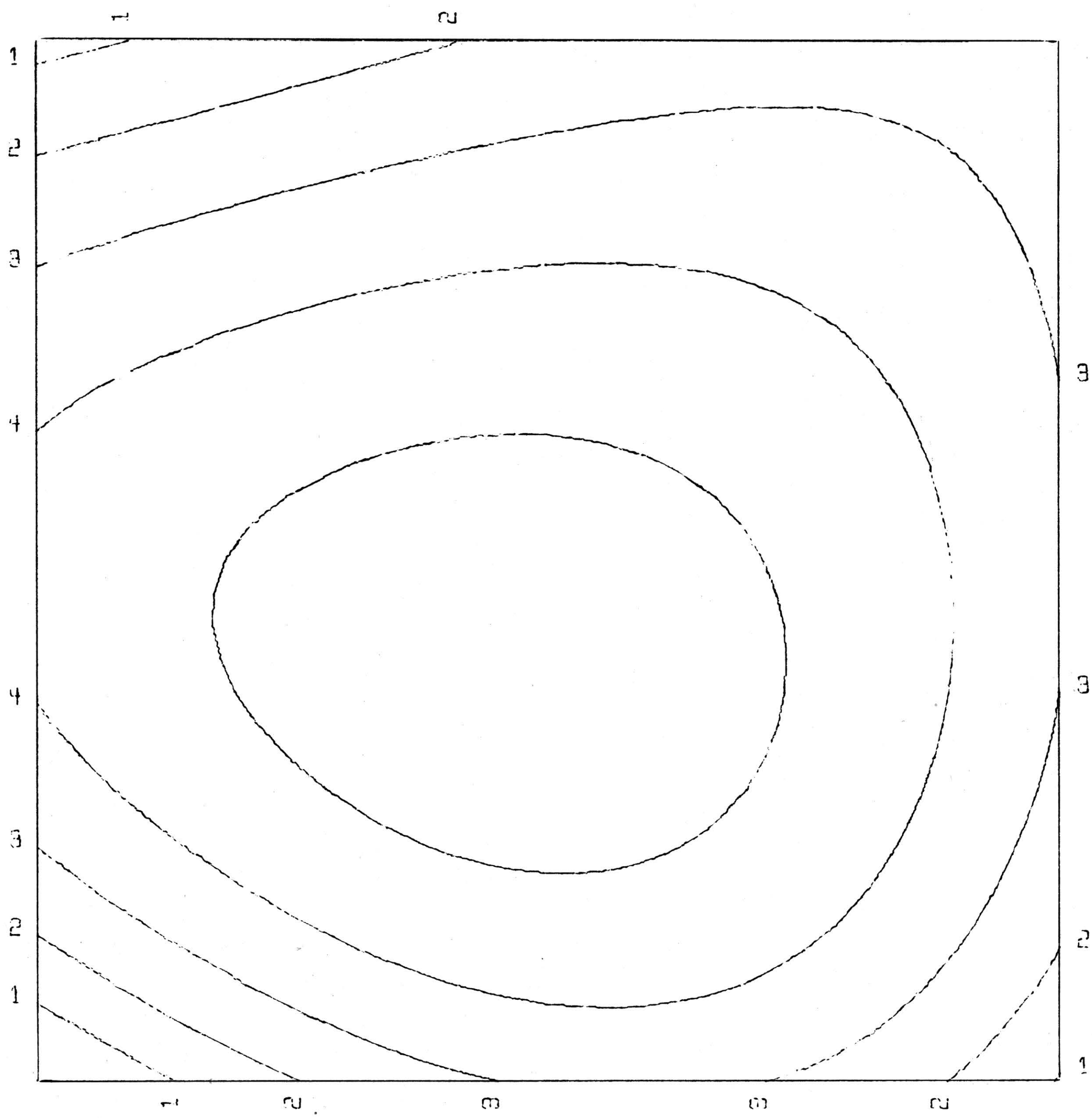

Figure 4. –Third–degree trend surface.

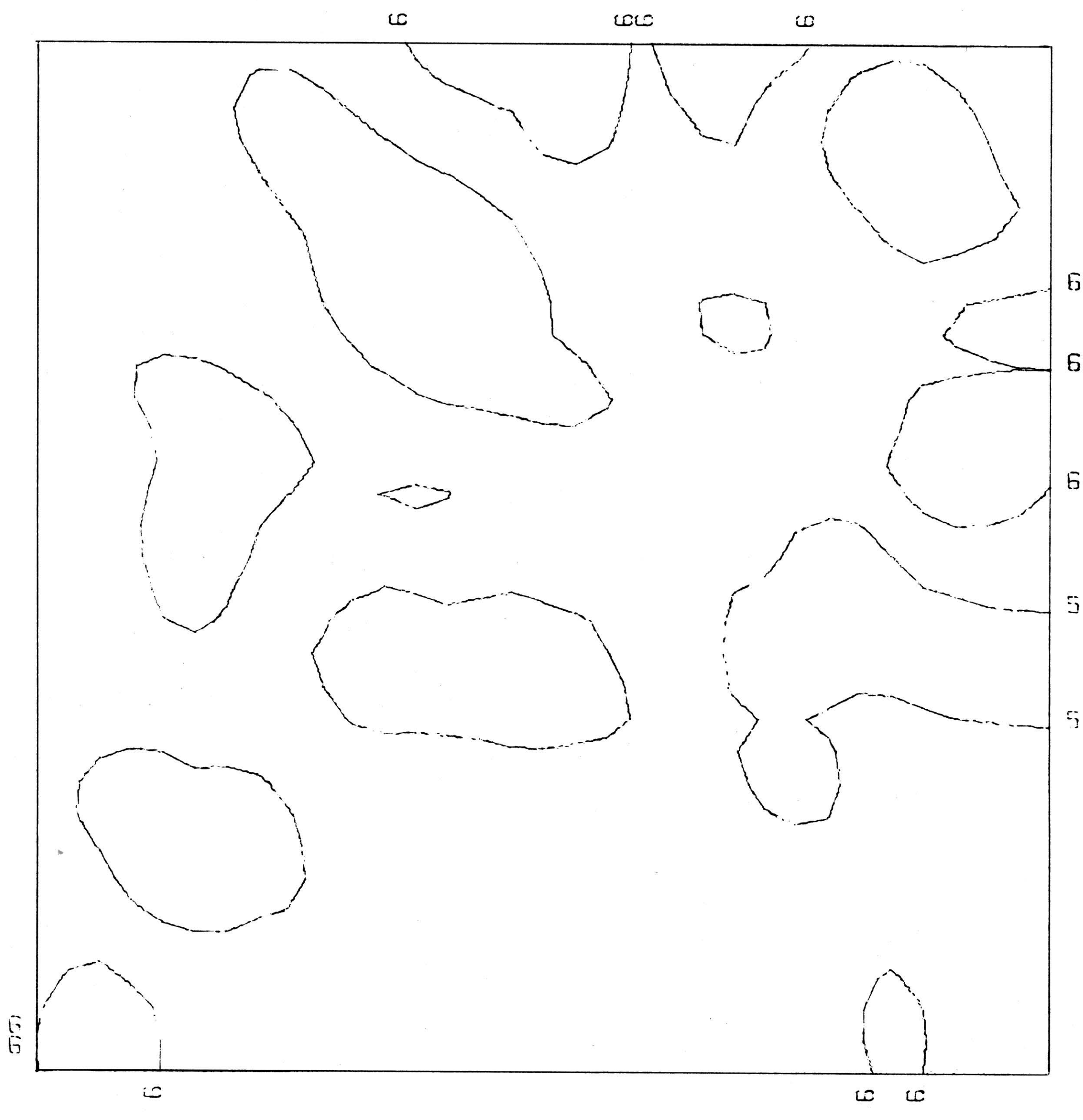

Figure 5. -Difference map.

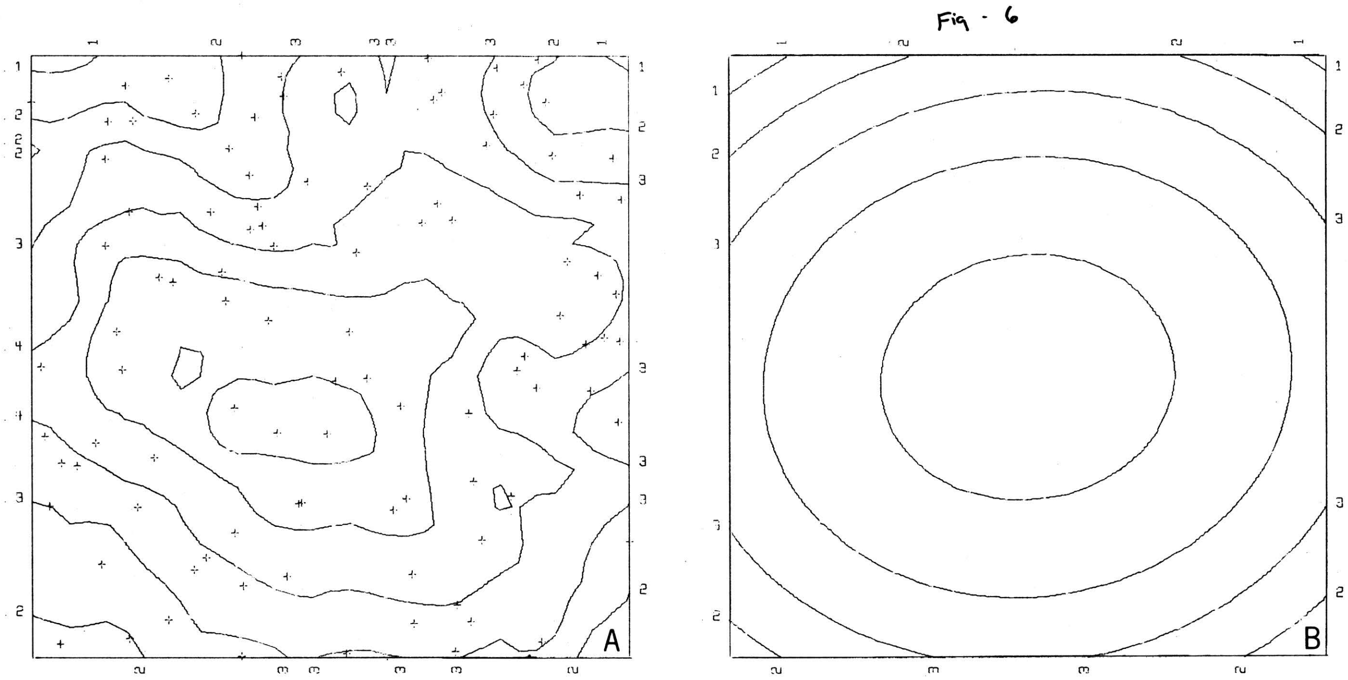

Figure 6. –A, Kansas data from Computer Contribution 23, quadratic base, 3 iterations; B, second-degree trend surface.

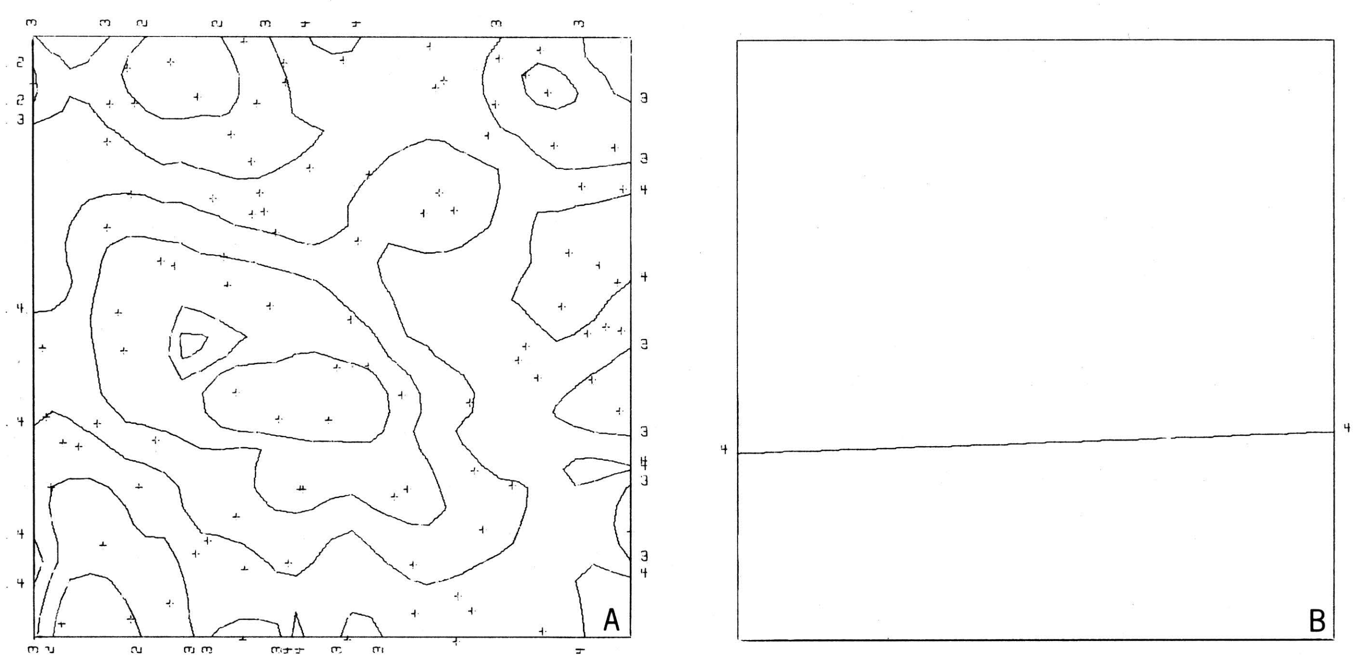

Figure 7. –A, Kansas data from Computer Contribution 23, linear base, 12 iterations; B, first-degree trend surface.

SELECTED REFERENCES

Allen, P., and Krumbein, W.C., 1962, Secondary trend components in the Top Ashdown Pebble Bed: a case history: Jour. Geology, v. 70, no. 5, p. 507-538.

Batcha, J.P., and Reese, J.R., 1964, Surface determination and automatic contouring for mineral exploration, extraction and processing: Colorado Sch. Mines Quart., v. 59, no. 4, p. 1-14.

Cole, A.J., Jordan, C., and Merriam, D.F., 1967, FORTRAN II program for progressive linear fit of surfaces on a quadratic base using an IBM 1620 computer: Kansas Geol. Survey Computer Contr. 15, 54 p.

Cole, A.J., 1968, Algorithm for the production of contour maps from scattered data: Nature, v. 220, no. 5162, p. 92-94.

Cole, A.J., and Davie, A.J., 1969, Local smoothing by polynomials in N dimensions: Computer Jour., v. 12, no. 1, p. 72-76.

Duff, P.M.D., and Walton, E.K., 1964, Trend surface analysis of sedimentary features of the Modiolaris Zone, East Pennine Coalfield, England, in Deltaic and shallow marine deposits: Developments in Sedimentology, v. 1, p. 114-122.

Good, D.I., 1964, FORTRAN II trend-surface program for the IBM 1620: Kansas Geol. Survey Sp. Dist. Publ. 14,54 p.

Harbaugh, J.W., 1963, BALGOL program for trend-surface mapping using an IBM 7090 computer: Kansas Geol. Survey Sp. Dist. Publ. 3, 17 p.

Harbaugh, J.W., 1964, Application of four-variable trend hypersurfaces in oil exploration (abs.): Stanford Univ. Publ., Geol. Sciences, v. 9, no. 2, p. 693.

Hildebrand, F.B., 1956, Introduction to numerical analysis: McGraw-Hill Book Co., Inc., New York, 511 p.

IBM, 1965, Numerical surface techniques and contour map plotting: IBM, Data Processing Application, White Plains, New York, 36 p.

Krumbein, W.C., 1959, Trend surface analysis of contour-type maps with irregular control-point spacing: Jour. Geophysical Res., v. 64, no. 7, p. 823-834.

Krumbein, W.C., 1966, Classification of map surfaces based on the structure of polynomial and Fourier coefficient matrices, in Computer applications in the earth sciences: Colloquium on classification procedures: Kansas Geol. Survey Computer Contr. 7, p. 12-18.

Lanczos, C., 1956, Applied analysis: Prentice Hall, Inc., Englewood Cliffs, New Jersey, 608 p.

Lippert, R.H., 1964, Investigation of structure and structural development by trend surface analysis: Unpub. master's thesis, Univ. Kansas, 42 p.

McIntyre, D.B., 1967, Trend-surface analysis of noisy data: Kansas Geol. Survey Computer Contr. 12., p. 45-46.

McIntyre, D.B., Pollard, D.D., and Smith, R., 1968, Computer programs for automatic contouring: Kansas Geol. Survey Computer Contr. 23, 76 p.

Merriam, D.F., 1964, Use of trend-surface residuals in interpreting geologic structures: Stanford Univ. Publ., Geol. Sciences, v. 9, no. 2, p. 686-692.

Merriam, D.F., 1965, Geology and the computer: New Scientist, v. 26, no. 444, p. 513-516.

Merriam, D.F., 1966, Geologic use of the computer: Wyoming Geol. Assoc., 20th Ann. Field Conf., p. 109-112.

Merriam, D.F. 1967, Computer aids exploration geologists: Oil and Gas Jour., v. 65, no. 4, p. 118-121.

Merriam, D.F., and Harbaugh, J.W.,1963, Computer helps map oil structures: Oil and Gas Jour., v. 61, no. 47, p. 158-159, 161-163.

Merriam, D.F., and Harbaugh, J.W.,1964, Trend-surface analysis of regional and residual components of geologic structure in Kansas: Kansas Geol. Survey Sp. Dist. Publ. 11, 27 p.

Merriam, D.F., and Lippert, R.H., 1964, Pattern recognition studies of geologic structure using trend-surface analysis: Colorado Sch. Mines Quart., v. 59, no. 4, p. 237-245.

Merriam, D.F., and Lippert, R.H., 1966, Geologic model studies using trend-surface analysis: Jour. Geology, v. 74, no. 3, p. 344-357.

Merriam, D.F., and Sneath, P.H.A., 1966, Quantitative comparison of contour maps: Jour. Geophysical Res., v. 71, no. 4, p. 1105-1115.

Ojakangas, D.R., and Basham, W.L., 1964, Simplified computer contouring of exploration data: Stanford Univ. Publ., Geol. Sciences, v. 9, no. 2, p. 757-770.

Peikert, E.W., 1965, Model for three-dimensional mineralogical variation in granitic plutons based on the Glen Alpine Stock, Sierra Nevada, California: Geol. Soc. America Bull., v. 76, no. 3, p. 331-348.

Read, W.A., and Merriam, D.F., 1966, Trend-surface analysis of stratigraphic thickness data from some Namurian rocks east of Stirling, Scotland: Scottish Jour. Geology, v. 2, pt. 1, p. 96-100.

Slack, H.A., and others, 1963, Now - map making made accurate, objective: Oil and Gas Jour., v. 61, no. 31, p. 158-160, 162, 164, 166, 168, 170.

Smith, J.W., and Harbaugh, J.W., 1966, Stratigraphic and geographic variation of shale-oil specific gravity from Colorado's Green River Formation: U.S. Bur. Mines Rept. Invest. 6883, 11 p.

Tobler, W.R., 1965, Automation in the preparation of Thematic maps: Cartographic Jour., v. 2, no. 1, p. 1-7.

Whitten, E.H.T., 1959, Composition trends in a granite: model variation and ghost stratigraphy in part of the Donegal Granite, Eire: Jour. Geophysical Res., v. 64, no. 7, p. 835-848.

Whitten, E.H.T., 1961, Systematic quantitative areal variation in five granitic massifs from India, Canada, and Great Britain: Jour. Geology, v. 69, no. 6, p. 619-646.

Whitten, E.H.T., 1963a, Application of quantitative methods in the geochemical study of granitic massifs: Royal Soc. Canada Sp. Publ. 6, p. 75-123.

Whitten, E.H.T., 1963b, A surface-fitting program suitable for testing geological models which involve areally-distributed data: Office of Naval Research, Geography Branch, Tech. Rept. 2, ONR Task No. 389-135, 56 p.

KANSAS GEOLOGICAL SURVEY COMPUTER PROGRAM
THE UNIVERSITY OF KANSAS, LAWRENCE

PROGRAM ABSTRACT

Title (If subroutine state in title):

An iterative approach to the fitting of trend surfaces

Date: 6 June 1969

Author, organization: A.J. Cole

Computing Laboratory, University of St. Andrews, St. Andrews, Fife, Scotland

Direct inquiries to: As above

Name: Address:

Purpose/description: Trend-surface analysis includes a mapping technique which plots initial polynomial overall fit, iterative final fit and difference map

Mathematical method: Iterative quadratic bump and smoothing technique on polynomial least-square base

Restrictions, range: Not more than 240 points

Computer manufacturer: IBM Model: 1620

Programming language: FORTRAN II - D

Memory required: 60 K Approximate running time: 30 min., iteration, 100 points

Special peripheral equipment required: Graph plotter, disks

Remarks (special compilers or operating systems, required word lengths, number of successful runs, other machine versions, additional information useful for operation or modification of program)

Program is being converted to run on IBM System 360/Model 44. Details from A.J. Cole.

(continued from inside front cover)

7. Computer applications in the earth sciences: Colloquium on classification procedures, edited by D.F. Merriam, 1966 . . . . $1.00
8. Prediction of the performance of a solution gas drive reservoir by Muskat's Equation, by Apolonio Baca, 1967 . . . . $1.00
9. FORTRAN IV program for mathematical simulation of marine sedimentation with IBM 7040 or 7094 computers, by J.W. Harbaugh and W.J. Wahlstedt, 1967 . . . . . $1.00
10. Three-dimensional response surface program in FORTRAN II for the IBM 1620 computer, by R.J. Sampson and J.C. Davis, 1967 . . . . $0.75
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24. Mathematical model and FORTRAN IV program for computer simulation of deltaic sedimentation, by G.F. Bonham-Carter and A.J. Sutherland, 1968 . . . . . $1.00
25. FORTRAN IV CDC 6400 computer program for analysis of subsurface fold geometry, by E.H.T. Whitten, 1968 . . . . $1.00
26. FORTRAN IV computer program for simulation of transgression and regression with continuous-time Markov models, by W.C. Krumbein, 1968 . . . . . . $1.00
27. Stepwise regression and nonpolynomial models in trend analysis, by A.T. Miesch and J.J. Connor, 1968 . . . . $1.00
28. KWIKR8 a FORTRAN IV program for multiple regression and geologic trend analysis, by J.E. Esler, P.F. Smith, and J.C. Davis, 1968 . . . . $1.00
29. FORTRAN IV program for harmonic trend analysis using double Fourier series and regularly gridded data for the GE 625 computer, by J.W. Harbaugh and M.J. Sackin, 1968 . . $1.00
30. Sampling a geological population (workshop on experiment in sampling), by J.C. Griffiths and C.W. Ondrick, 1968 . . . . $1.00
31. Multivariate procedures and FORTRAN IV program for evaluation and improvement of classifications, by Ferruh Demirmen, 1969 . . . . $1.00
32. FORTRAN IV programs for canonical correlation and canonical trend-surface analysis, by P.J. Lee, 1969 . . . . $1.00
33. FORTRAN IV program for construction of Pi diagrams with the Univac 1108 computer, by Jeffrey Warner, 1969 . . . . $1.00
34. FORTRAN IV program for nonlinear estimation, by R.B. McCammon, 1969 . . . . . $0.75
35. FORTRAN IV computer program for fitting observed count data to discrete distribution models of binomial, Poisson and negative binomial, by C.W. Ondrick and J.C. Griffiths, 1969 . . . . $0.75
36. GRAFPAC, graphic output subroutines for the GE 635 computer, by F.J. Rohlf, 1969 . . $1.00
37. An iterative approach to the fitting of trend surfaces, by A.J. Cole $1.00