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FORTRAN IV PROGRAMS FOR CANONICAL CORRELATION AND CANONICAL TREND-SURFACE ANALYSIS

By

P. J. LEE

McMaster University

in cooperation with the
American Association of Petroleum Geologists
Tulsa, Oklahoma

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Editor's Remarks

For a limited time the Geological Survey will make available on magnetic tape the canonical correlation and canonical trend-surface programs as described here for $20.00 (US). An extra charge of $10.00 is made if punched cards are required.

*Active Member, °Associate Member, ⁺Junior Member, American Association of Petroleum Geologists.

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By

P.J. Lee*

INTRODUCTION

In the statistical technique of simple linear regression a dependent variable, Y, is related to an independent variable, X, by an equation of the type

$$Y = bX + \varepsilon,$$

where ε is a random variable. The "strength of the relationship" of Y and X may be expressed by the correlation coefficient, r.

In multiple linear regression, the dependent variable is related to several independent variables by an equation of this type

$$Y = \Sigma b_i X_i + \varepsilon.$$

The multiple correlation coefficient serves as a measure of the "strength of the relationship". More precisely, the square of the multiple correlation coefficient gives the proportion of the total sums of squares of Y which may be attributed to variation of the X_i.

Multiple correlation techniques have been applied frequently in geology. A typical application might relate a bulk property of a rock (e.g. permeability) to a series of mineralogical or textural properties (e.g. mean grain size, sorting, skewness, etc.).

It is not unusual, however, for geologists to make two (or more) <u>sets</u> of measurements on a single specimen (e.g. size and compositional parameters, trace element and major elements, or chemical and modal analyses) or at a single locality (e.g. formation thickness, sand percentage, etc., and size or compositional parameters). It may be of interest to attempt to relate one <u>set</u> of variables to another <u>set</u> of variables, to discover equations of the type

$$U = \Sigma a_i X_i \text{ , and}$$

$$V = \Sigma b_i Y_i \text{ ,}$$

where the coefficients a_i and b_i are chosen to give the largest possible correlation between U and V. This technique is called <u>canonical correlation</u>.

As an extension of this technique, the X_i might be chosen to be geographical coordinates (and their polynomials) of a set of spatially distributed variables, Y_i. In this instance, the technique would be similar to that of trend analysis, except that the trend which is determined is not the trend of any particular variable, Y, but is the common trend of a set of variables, Y_i. This technique may be called <u>canonical trend-surface analysis</u>.

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CANONICAL CORRELATION

Suppose a number of rock specimens are collected from a formation. On each specimen, the size parameters (mean sorting coefficient and skewness) and petrographic composition (percentage of quartz, feldspar, rock fragments) are determined. It may be uncertain whether or not a relationship is present between the grain-size parameters and petrographic variables. A simple correlation coefficient indicates a possible relationship between one grain-size parameter and one petrographic parameter. Nine correlation coefficients thus are required to describe all possible relationships. A further study of the relationship between the two sets of variables is, however, possible.

Canonical correlation analysis was introduced by Hotelling in 1936 (the paper was read in 1935) as a technique to understand the relationship between two sets of variables. The relationship is summarized and expressed by a simple index. For example, the grain-size and petrographic parameters may be weighted and combined into two linear combinations

$$U = a_1 \text{ (median)} + a_2 \text{ (So)} + a_3 \text{ (Sk)} \text{ , and}$$

$$V = b_1 \text{ (quartz)} + b_2 \text{ (feldspar)} + b_3 \text{ (rock fragments)} \text{ ,}$$

where the a_i and b_i are weights to be determined

*Present address: Fisheries Research Board of Canada, Freshwater Institute, Winnipeg 19, Manitoba, Canada.

such that U and V have maximum simple correlation. The coefficient of correlation of U and V is called the canonical correlation coefficient, and measures the strength of the link between the two sets of variables. The nature of the link is indicated by the relative sizes of the weighting coefficients, a_i and b_i. For standardized variables the larger coefficients indicate those variables which are correlated most closely between the two sets. In this situation, canonical correlation is used in a correlation model, where each of the two sets, U and V, is defined as subject to random error fluctuations, i.e. they are random variables or variates.

Another possible application of canonical correlation is as a tool to predict a combination of two or more dependent variates from a set of independent variates. Here, canonical correlation is used as a regression model of the type

$$U = c + kV,$$

where c and k are determined in the same way as in linear regression analysis. The canonical correlation coefficient then may be considered as the simple correlation coefficient between U and V.

Computational Procedures

The computations for canonical correlation are carried out in five major steps.

STEP 1 Transformation of variates if desired.

STEP 2 Calculation of correlation matrix

$$\left[\begin{array}{c|c} R_{11} & R_{12} \\ \hline R_{21} & R_{22} \end{array}\right], \qquad (1)$$

where R_{11} is correlation matrix of set Z_1 (contains p_1 variates), and R_{22} is correlation matrix of set Z_2 (contains p_2 variates). $R_{12} = R'_{21}$ is the correlation matrix between the two sets. It is assumed that $p_1 \geq p_2$.

STEP 3 Computation of the eigenvalues and eigenvectors of equation (2) by using a Jacobilike method (Eberlein, 1962).

$$\left| R_{22}^{-1} \; R_{21} \; R_{11}^{-1} \; R_{12} - \Lambda^2 I \right| = 0 \qquad (2)$$

The column matrix A_i which is the eigenvector of equation (2) contains the canonical coefficients of p_2 variates of i-th canonical root. $A_i'A_i = 1$.

STEP 4 The matrix, B_i, which contains the canonical coefficients of p_1 variates of i-th canonical root is calculated as

$$B_i = R_{11}^{-1} R_{12} A_i / \lambda i \quad (\lambda i \neq 0), \qquad (3)$$

where $B_i'B_i = 1$. Finally, the canonical variates U and V of i-th canonical root are given by the equations

$$U_i = A_i' Z_1, \text{ and } V_i = B_i' Z_2. \qquad (4)$$

The numerical approach of this program is different from the canonical correlation program given by Cooley and Lohnes (1962) which was adopted in the Scientific Subroutine Package for IBM System/360.

STEP 5 Statistical test of canonical root. Bartlett's statistic (Bartlett, 1938)

$$\chi^2 = -\left[N - \tfrac{1}{2}(p_1 + p_2 + 1)\right] \log_e L, \qquad (5)$$

where $L = \prod^{p_2} (1 - \lambda_i^2)$, follows approximately a chi-square distribution with $(p_1 - r + 1)(p_2 - r + 1)$ degrees of freedom. The assumption is made that Z_1 and Z_2 follow a multivariate normal distribution with zero means.

Detailed discussion of canonical correlation is given by Anderson (1958, p. 288-306), Wilks (1963, p. 587-590), and Lee (1968).

Example - Recent Sediment From Buttonwood Sound

Recent sediment specimens were collected by Lynts (1966) from 19 stations located in Buttonwood Sound, Florida Bay. Three samples of size 19 were taken on August 14th, 17th, and 20th, 1962, respectively. A sample of size 16 was taken on February 9th, 1963. For each location, the environmental parameters, depth of water, temperature, salinity, pH, and Eh, of the sediment-water interface were measured. The dominant element of macroflora in the Sound was grass, in particular, turtle grass. The distribution of turtle grass was mapped as very dense, dense-moderate, and patchy. The abundance of the turtle grass was coded by the writer as follows: very dense— 1, dense-moderate— 0, and patchy— -1.

The problem here is to determine to what extent the environmental factors, depth, temperature, salinity, pH, and abundance of turtle grass, are related to size parameters of sediments. Four samples were grouped together and considered as a single sample of size 73, because the turtle grass map is not specific to any one of the four sampling dates.

From examination of the simple correlation coefficients of all possible pairs of the seven variates (Table 1), it may be concluded that the turtle grass is related positively to silt and related negatively to sand. The simple correlation coefficients between the sediments and depth, temperature, salinity, and pH

Table 1.- Simple correlation coefficients of sediments from Buttonwood Sound

Turtle grass	1.000					
Depth	-0.061	1.000				
Temperature	0.091	-0.059	1.000			
Salinity	-0.085	0.086	-0.976	1.000		
pH	-0.194	0.242	0.122	-0.070	1.000	
Sand	-0.665	0.073	-0.192	0.194	0.173	1.000
Silt	0.458	0.084	0.015	0.001	-0.139	-0.602

are small.

A canonical analysis was performed on this set of data. The first canonical root is equal to 0.683 which is significant at the 1 percent level. The second canonical correlation (0.242) is not significant at the 5 percent level. This would suggest that in studying the relation between these two sets, attention may be restricted to the first pair of canonical variates which is listed in equation (6) and (7)

$$U_1 = -0.98(\text{turtle grass}) - 0.00(\text{depth}) - 0.07(\text{temperature}) + 0.13(\text{salinity}) + 0.10(\text{pH}), \text{ and} \quad (6)$$

$$V_1 = 1.00(\text{sand}) - 0.09(\text{silt}). \quad (7)$$

It is interpreted from the large coefficients that the distribution of sand is correlated closely with the abundance of turtle grass. Sands are more abundant in regions of less grass in Buttonwood Sound. Lynts (1966) arrived at the same conclusion by comparing the turtle grass map with sediment-size distribution map.

Principal component analysis also was performed on the correlation matrix of the same set of data. The rotated and normalized eigenvectors of the last principal component are practically zero. The eigenvectors of the sixth and fifth eigenvalues indicate the same relationship on sand, silt and turtle grass as that of the first eigenvalues (Table 2). These two principal components imply that there is slight interaction between the variates of sand and silt.

The coefficients of the first component are loaded highly with turtle grass and sand, with coefficients of opposite sign. This indicates that the first canonical variate is similar to the first component obtained by an ordinary principal component analysis. This component is interpreted as a "sediment stabilizer" factor.

It is interesting to examine the physical meaning of the second canonical correlation even though it is not statistically significant. The corresponding canonical variates are

Table 2.- Rotated and normalized seven principal components for sediment from Buttonwood Sound

Variate \ Component	1	2	3	4	5	6	7
Turtle grass	0.93	0.04	-0.03	0.09	-0.21	0.27	-0.00
Temperature	0.04	0.99	-0.02	-0.07	-0.01	0.06	-0.11
Salinity	-0.03	-0.99	0.04	0.02	-0.01	-0.06	-0.10
Depth	-0.02	-0.05	0.99	-0.12	-0.05	-0.03	-0.00
Sand	-0.37	-0.13	0.09	-0.08	0.34	-0.85	0.00
pH	-0.08	0.07	0.13	-0.98	0.06	-0.06	-0.00
Silt	0.20	-0.01	0.06	0.06	-0.94	0.24	-0.00
Eigenvalues	2.362	1.902	1.208	0.705	0.518	0.283	0.022
Cumulative percent	33.74	60.91	78.17	88.24	95.64	99.69	100.00

$$U_2 = 0.55(\text{sand}) + 0.84(\text{silt}), \text{ and} \quad (8)$$

$$V_2 = 0.05(\text{turtle grass}) + 0.26(\text{depth}) + 0.56(\text{temperature}) + 0.78(\text{salinity}) - 0.12(\text{pH}) \quad (9)$$

The second canonical correlation suggests that the amount of sand and silt could be predicted by certain combinations of environmental factors. If all variates are standardized (the origin of each variate is placed at its mean, and standard deviation is used as the unit of measurement), then the larger positive V_2 values implies higher percentages of both sand and silt, whereas larger negative V_2 values imply lower percentages of sand and silt. If the percentage of sand is high and the percentage of silt is low, or vice versa, then a small V_2 value will be given by equation (9). The variance of equation (8) explained by equation (9) is equal to 6 percent. However, this interpretation is merely an illustrative example to show multivariate prediction.

Limitations To Canonical Correlation

(1) The results of canonical correlation analysis may be difficult to interpret. Linear combinations of sets of variates need not have any intrinsic meaning. If the result shows a low canonical correlation, no doubt correlations do not exist between the two sets, no matter what types of linear combinations are used. Thus, the canonical correlation is the maximum correlation between linear combinations of the two sets of variates. The coefficients of the linear combination are not stable as variates are removed from (or new variates added to) the analysis. The technique is best regarded as an exploratory tool which will

give some idea of the complicated structure of a multivariate relation.

(2) For small sample sizes the shape of a sample cluster of the sets of variates may not be well defined. In this situation there may be a large sampling error in calculating simple correlation coefficients between any two variates in one set. Consequently, the direction of the canonical variates U, V will not necessarily pass through the center of the sample cluster of one set of variates and the signs and magnitudes of the coefficients (i.e. the direction cosines of the canonical variates) may not correspond to expectations.

A better result is produced by a more clearly defined sample cluster, that is in general by a larger sample size.

(3) If there is a relationship between the variates which changes at different levels of another variate (Fig. 1), i.e. if there is strong interaction between any two variates, the results of canonical correlation will be ambiguous.

(4) If the variates within one set are highly intercorrelated (i.e. the correlation matrix R_{11} or R_{22} of equation 2 is singular or nearly singular), and if the nature of the problem studied defined its canonical roots poorly (i.e. if the limiting eigenvalues of equation 2 are poorly defined), then the results of canonical correlation will be meaingless.

(5) Variates are assumed to be distributed normally for the purposes of making a statistical test on the canonical correlation coefficients (though not for the basic computational analysis).

Comparison With Other Statistical Techniques

(1) Multiple Correlation and Regression.- If in a special situation of canonical correlation, the number of variates of one set is equal to one, then the canonical correlation will be the same as the multiple correlation. In a regression model, canonical correlation analysis permits the estimation of two or more dependent variates, whereas only one dependent variate may be estimated in conventional multiple regression. Natural variation is multivariate and consequently there are advantages to a model which analyses the variation of several dependent variates simultaneously.

(2) Principal Component Analysis.- Both canonical correlation analysis and principal component analysis result from orthogonal rotation. In canonical correlation, the purpose of rotation is to inquire how two linear combinations should be constructed so that the simple correlation coefficient between them is as large as possible; whereas in principal component analysis, the purpose of rotation is to seek the direction of the maximum variance or variation of a sample cluster.

In order to demonstrate geometrically the differences between principal component and canonical correlation, consider an example of two variates. Suppose a set of bivariate observations is made on X_1 and X_2, and each observation is plotted on a two-dimensional space (Fig.2A) forming a swarm of points. In principal component analysis an orthogonal rotation is carried out such that the first principal component, C_1, passes through the cluster and accounts for or "explains" the maximum variance (or variation) of the sample; whereas the second principal component, C_2, which is perpendicular to C_1, "explains" the maximum variance in another direction (Fig.2B).

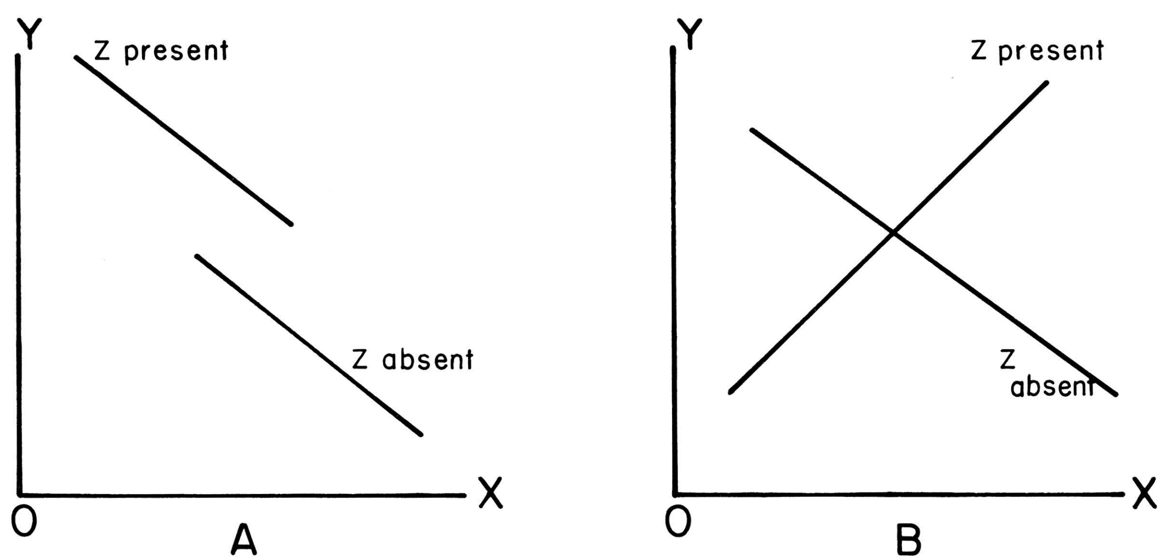

Figure 1.- A, No interactions between variates X and Y, simple correlation between them is high. B, Strong interaction occurs between variates X and Y, simple correlation between them is equal to zero.

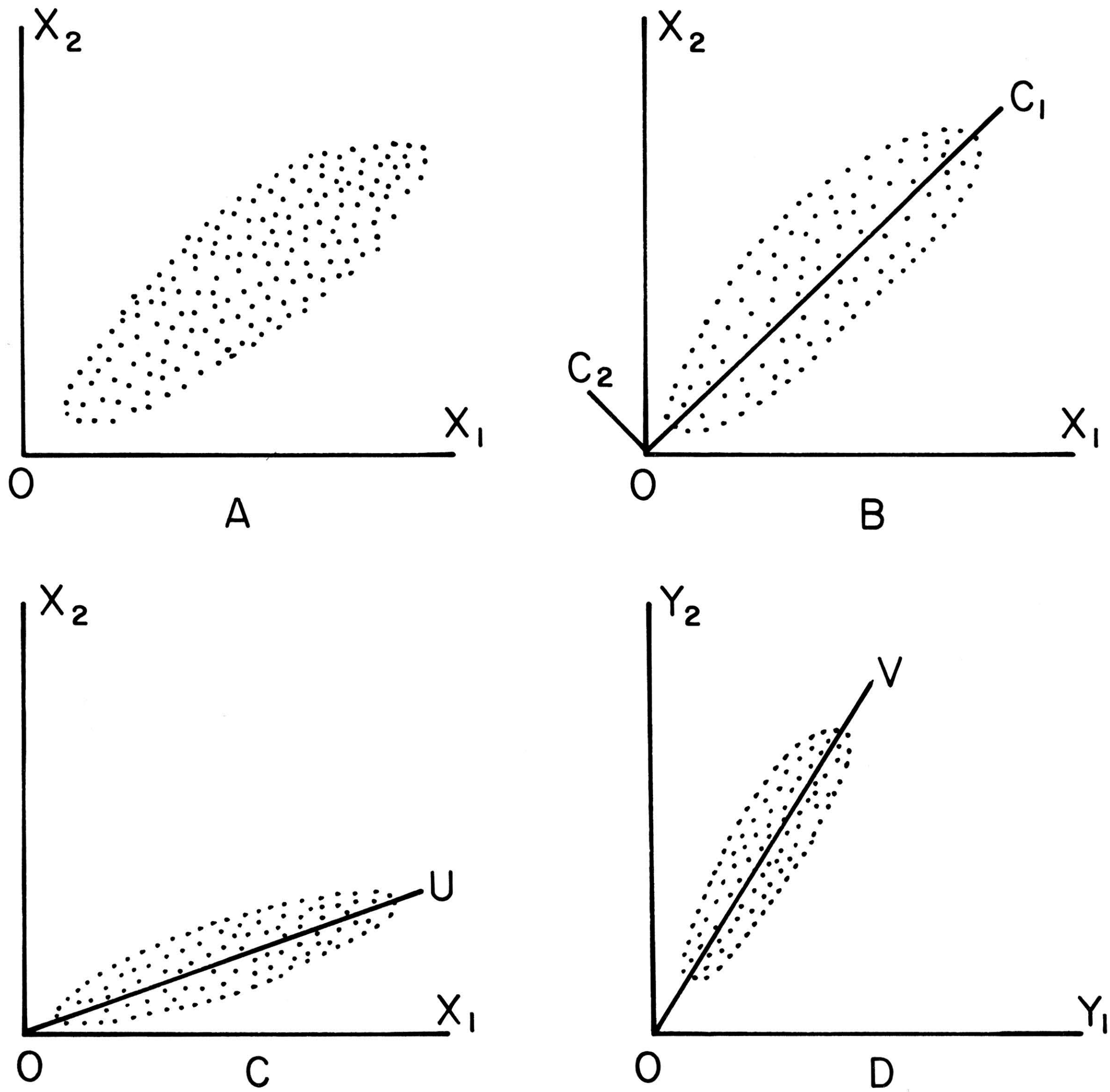

Figure 2.- A, Well-defined sample cluster of bivariate observations. B, Principal component analysis on bivariate observation. C, Canonical variate U passing through a sample cluster in $X_1 - X_2$ coordinates. D, Canonical variate V passing through a sample cluster in $Y_1 - Y_2$ coordinates.

The principal components C_1 and C_2 are linear combinations of variates X_1 and X_2.

In canonical correlation, assume that one set consists of two variates, X_1 and X_2, and the second set consists of variates Y_1 and Y_2. A set of observations is made on these four variates. For the convenience of graphic representation, the observations are plotted on coordinates, X_1-X_2, and Y_1-Y_2, respectively, forming a swarm of points in each diagram (Figs.2C and 2D), but actually the observations are considered properly as one set of points in a four-dimensional space. Then there will be a canonical variate passing through each swarm of points, respectively, in each of the two-dimensional spaces. We can imagine that the origins of these two systems join together and that an orthogonal rotation is carried out such that the angle between U and V is the least possible. After determination of U and V in terms of the variates of each set, the U and V may be treated in a further two-dimensional space as $U = c + k V$, where c and k are constants.

Principal component analysis tends to express interactions by extra principal components correlated with the interacting components. But the way in which slight interactions can be detected from the results of canonical correlation analysis has not yet been discovered.

For solving a problem, the sample size for a canonical correlation analysis usually should be larger than that used for principal component analysis.

Principal component solution reveals more extensive information about internal relations of a set of measures than canonical correlation and is likely to be a powerful tool for displaying these internal relations. On the other hand, where the purpose is to investigate the relations between two sets of variates, or to predict many dependent variates, the technique of canonical correlation is more appropriate.

Other Applications

(1) Matching factor patterns.- Suppose a separate factor or principal component analysis has been conducted for a rock body at different geographic localities or different stratigraphic sections. It may be that the factor patterns obtained at the various localities seem different but that canonical transformations exist such that some of the factors or principal components in one set would be found to correspond closely to factors or principal components in the other set. The technique has been generalized for m factor pattern by Horst (1961a, 1961b) to m canonical transformations which will yield new patterns having maximum similarity. A geologic example is given by Lee (1968).

(2) Q-technique canonical correlation.- Canonical correlation is used in this situation for dealing with sample space. Suppose there are two sets of samples collected from different groups (such as localities, or different formations), and the problem is to determine to what extent the two sets of samples are similar, based on observable variates. The canonical root provides an index of the maximum degree of similarity between the two groups of samples. The technique is applicable to samples of small size. A detailed discussion is given by Lee (1968).

Instructions For Using Program

Input to program

1. Title card

   The project title and the investigator's name will be punched on one card.

2. Control card (all digits are right justified)

Cols. 1-5	Total number of observations.
Cols. 6-7	Number of variables of left-hand set, maximum of 30.
Cols. 8-9	Number of variables of right-hand set, maximum of 30. The number of variables of left-hand set must always be greater than or equal to that of right-hand set.
Cols. 10-11	Total number of variables subject to transformations, blank or zero for no transformation.
Col. 12	1 for correlation matrix input 2 for raw data matrix input

3. Transformation card(s) (optional)

   Assume the 3-digit transformation code to be of the general form IIJ. II defines the subscript of the variables. J defines the type of transformation.

   J=1 $\sin^{-1}\sqrt{x}$,

   J=2 $\log_{10} x$,

   J=3 $\log_{10}(x + 1.0)$

   J=4 $\log_e x$

   J=5 $\log_e (x + 1.0)$

   J=6 $\sqrt{x}$

   Twenty-six variables are punched on one card, if there are more than twenty-six variables to be transformed, then two or more cards will be used.

4. Variable name card(s)

   The output of the program is designed for self-explanation, so the variable names will be input together with data. Twelve characters are used for the names of each variable. They should be punched in a continuous sequence and also in the same order as in the data matrix or correlation matrix. Six names are punched on one card.

5. Data format card

   This is used for the input data card.

6. Data cards

   Each observation has its own separate data card(s) two or more observations sharing one data card is not allowed. The data of left-hand set must be followed by the data of right-hand set. The correlation matrix must be stored row by row, each row going up to the diagonal elements.

7. Job-done card

   The investigator may run more than one project at a time. Each project must contain all instruction cards mentioned above. A card with a word, DONE, punched on the first four columns is placed at the end of the deck.

Output from program

1. Project title and the investigator's name,
2. Correlation matrix,
3. Mean, variance, and standard deviation of each variable, if raw data is input,
4. Canonical roots, chi-square values, and significant level,
5. Canonical variates both of left-and right-hand sets.

CANONICAL TREND SURFACES

Trend-surface analysis is now a familiar technique in geology and geography. The technique attempts to determine the "trend" of an areally distributed variate by constructing a surface from a polynomial expansion of the X, Y coordinates of the sample localities. One of the limitations of the technique is that it can be applied only to one variate at a time. Commonly more than one variate has been measured at each locality, so that it seemed worth while to seek a multivariate trend surface which would summarize the areal variation of a set of variates simultaneously.

A multivariate trend surface was constructed by using the principle of canonical correlation analysis. This trend surface which is called canonical trend surface is a parsimonious summarization of areal variations of a set of geologic variates from a single population. By use of this type of trend it is possible to reveal the underlying pattern of geographic variations common to a set of variates. Details of the theory and computational procedures of canonical trend is given by Lee and Middleton (1967) and Lee (1968, in press).

The geographic coordinates, X and Y, and their various powers and cross-products constitute one set of p_1 variates, and the p_2 geologic variates, Z_i, constitute the second set. The function of canonical trend analysis is to find one linear combination for the (X,Y) coordinates, and one for Z_i such that these two linear combinations have a maximum correlation.

Example - Permian System in Western Kansas And Eastern Colorado

The stratigraphic data of upper Permian rocks in western Kansas and eastern Colorado were used as an example to illustrate the validity of the canonical trend in a practical situation. The hypothesis is that the area studied shows the characteristics of a sedimentary basin. In other words, the question is asked whether or not it is possible to evaluate a trend which is common to the thickness of sand, shale, carbonate, and evaporate. Thus, the thickness of sand, shale, carbonate, and evaporite were treated by canonical trend-surface analysis. The raw data were published by Krumbein (1962), and has been analyzed by using factor-vector method (Krumbein and Imbrie, 1963).

The data were subjected to canonical trend analysis. After three iterations, the highest canonical root that could be reached was 0.9464 which indicates that a cubic polynomial is the most predictable response surface. The canonical variate for the four sediments was

$$U = 0.516(\text{sand}) + 0.408(\text{shale}) - 0.104(\text{carbonate}) + 0.746(\text{evaporite}). \quad (10)$$

The canonical variate for the polynomial was

$$V = -0.407\,X - 0.914\,Y + 0.002\,X^2 - 0.001\,XY - 0.001\,Y^2 \quad (11)$$

Equation (10) mainly indicates the variations in thickness of evaporite, shale, and sand. Equation (11) and Figure 3 show that evaporite, shale, and sand thicken toward the southwest corner of Kansas. The variance of the linear function (10) explained by the linear function (11) is equal to 91 percent (the square of 0.9464). The canonical trend establishes the hypothesis suggested before carrying out the analysis.

The essential approach to the interpretation of a canonical trend surface is to visualize the geologic implications of the variations of all or most of the variates being displayed realistically in a single map.

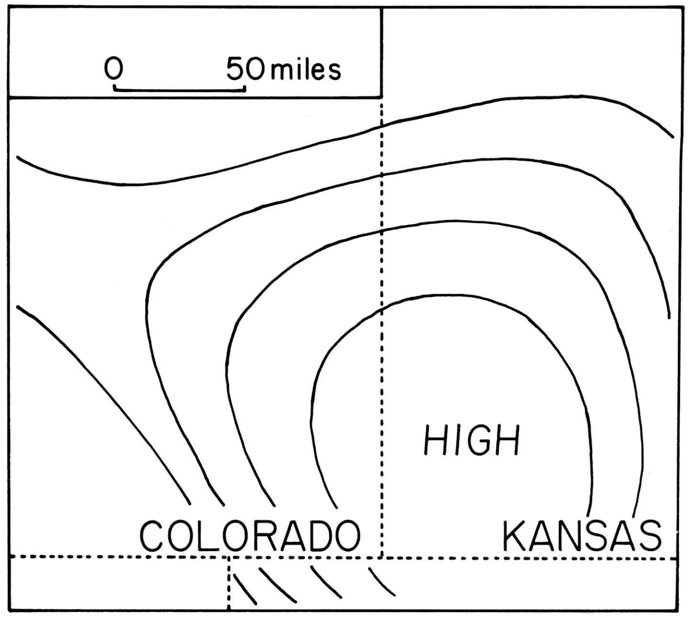

Figure 3.- Canonical trend surface for thickness of sand, shale, and evaporite of Permian unit in western Kansas and eastern Colorado.

Instructions For Using Program

Input to program

1. Title card
   The project title and the investigator's name are punched on one card.
2. Control card (all digits are right justified)
   - Cols. 1-3 Total number of observations or sample size, maximum of 300.
   - Cols. 4-5 Number of dependent variables, maximum of 15.
   - Cols. 6-7 Total number of variables subjected to transformations.
   - Col. 8 Order of canonical polynomial, maximum of 6. If a number 6, was punched on column 8 the program would stop computing the higher order polynomial (≤ 6) if the difference of two successive canonical correlation was less than 0.05.
   - Cols. 9-16 Scale of trend map. In general we

use an arbitrary system for sample location. In order to have an actual scale for the map printed by the printer, the distance in meters which is equal to 1 unit in the arbitrary coordinate system is punched on columns 9-16. The input format is F8.0.

3. Transformation card

This instruction card enables the investigator to transform the dependent variables by using various transformations (see instructions for using canonical correlation program).

4. Variable name card

Twelve characters are used for the names of each variable. They should be punched in the same order as in the data matrix. Six names are punched on one card.

5. Data format card

This format is used for the input data cards. No more than 80 characters are allowed.

6. Data cards

The X-, Y-coordinates must be in the first two places and are followed by the data. The origin of the coordinate may be placed anywhere, but the convention for the sign of the reference coordinate is the same as the Cartesian coordinate system. The maximum length of the X-axis (horizontal axis) must be greater than or equal to that of the Y-axis (vertical axis). Each sample has its own separate data card(s). Two or more samples sharing one data card are not allowed.

<u>Output from program</u>

1. Project title and investigator's name,
2. Mean, and standard deviation of each geologic variable,
3. Covariance matrix for the standardized geologic variable,
4. Record of successive evaluation of the degree of the trend surface,
5. Each canonical correlation coefficient has one set of the following outputs
   - (I) Trend-surface equation and linear function of geologic variables,
   - (II) Observed, calculated, and residual values with X-, Y-coordinates,
   - (III) Contoured trend-surface map with explanation, and
   - (IV) Residual map.

CONCLUDING REMARKS

If there are two sets of variates relating to one set of samples, various statistical techniques may be adapted according to the nature of the problem discussed. We may, for example, factorize the two sets together, or we may factorize each set independently, and try to correlate factors, or we may perform a canonical correlation analysis between the two sets of variates in order to yield a maximum correlation.

There are no standard rules for discovering relationships among a set of variates. The easy availability of computer programs for statistical analysis will certainly enable a geologist to look at his own data in different ways.

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APPENDIX I. Complete FORTRAN IV source listing of canonical correlation program.

```
C       CANONICAL CORRELATION ANALYSIS PROGRAM
C       P J LEE AUGUST 1966
C       DEPARTMENT OF GEOLOGY,MCMASTER UNIVERSITY,HAMILTON,CANADA
C
C       SUBROUTINES REQUIRED: EBERVC, MATRIX, MATINV, AND ROSIE
C
C
        DIMENSION Q(1830),COR(30,30),A2(30,30),B(30,30),SUMSQ(60),
       1B1(30,30),TITLE(20),NTYPE(60),FMT(20),SUM(60),D(60),CHI(37),
       2NTRA(60)
        COMMON VEC(30,30),VARIAT(180),NUMBER(30),B2(30,30)
        DATA CHI/  6.6349,  9.2103, 11.3449, 13.2767, 15.0863, 16.8119, 18
       1.4753, 20.0902, 21.6660, 23.2093, 24.7250, 26.2170, 27.6883, 29.14
       213, 30.5779, 31.9999, 33.4087, 34.8053, 36.1908, 37.5662, 38.9321,
       3 40.2894, 41.6384, 42.9798, 44.3141, 45.6417, 46.9630, 48.2782, 49
       4.5879, 50.8922, 63.6907, 76.1539, 88.3794,100.4250,112.3290,124.11
       560,135.8070/
        DATA HALT/4HDONE/
        DO 28 I=1,30
     28 NUMBER(I)=I
     90 READ(5,2) TITLE
      2 FORMAT(20A4)
        IF(TITLE(1).EQ.HALT) STOP
        READ(5,1) N,NL,NR,MTRA,I1
      1 FORMAT(I5,3I2,I1)
        WRITE(6,3) TITLE
      3 FORMAT(1H1,30X,20A4)
        WRITE(6,4)
      4 FORMAT(///)
        IF(MTRA.EQ.0) GOTO 17
        READ(5,103) (NTRA(I),NTYPE(I),I=1,MTRA)
    103 FORMAT(26(I2,I1))
     17 M=NL+NR
        NV=3*M
        S=N
        M1=(M+1)*M/2
        READ(5,7) (VARIAT(I),I=1,NV)
      7 FORMAT(18A4)
        READ(5,2) FMT
        IF(I1.EQ.1) GOTO 9
        DO 10 J=1,M
        SUM(J)=0.0
     10 SUMSQ(J)=0.0
        DO 11 J=1,M1
     11 Q(J)=0.0
        DO 20 NSAMPL=1,N
        READ(5,FMT) (D(J),J=1,M)
        IF(MTRA.EQ.0) GOTO 13
        DO 104 ITRA=1,MTRA
        JTRA=NTRA(ITRA)
        KTYPE=NTYPE(ITRA)
        IF(KTYPE.EQ.1.OR.KTYPE.EQ.2.OR.KTYPE.EQ.3.OR.KTYPE.EQ.4.OR.KTYPE.
       1EQ.5.OR.KTYPE.EQ.6) GOTO 18
        WRITE(6,16)
     16 FORMAT(1H0,37H WRONG CODE NUMBER FOR TRANSFORMATION)
        STOP
     18 GOTO(14,15,105,106,107,108), KTYPE
     14 D(JTRA)=ARSIN(SQRT(D(JTRA)))
```

```
      GOTO 104
   15 D(JTRA)=ALOG10(D(JTRA))
      GOTO 104
  105 D(JTRA)=ALOG10(D((JTRA)+1.0))
      GOTO 104
  106 D(JTRA)=ALOG(D(JTRA))
      GOTO 104
  107 D(JTRA)=ALOG(D(JTRA)+1.0)
      GOTO 104
  108 D(JTRA)=SQRT(D(JTRA))
  104 CONTINUE
   13 DO 19 J=1,M
      SUM(J)=SUM(J)+D(J)
   19 SUMSQ(J)=SUMSQ(J)+D(J)*D(J)
      M2=M-1
      DO 20 K=1,M2
      DO 20 K1=K,M
      L=K1*(K1-1)/2+K
   88 IF(K1-K) 21,22,21
   22 Q(L)=1.000
      GOTO 20
   21 Q(L)=Q(L)+D(K)*D(K1)
   20 CONTINUE
      DO 23 I=1,M2
      DO 23 J=I,M
      K=J*(J-1)/2+I
      IF(I-J) 24,25,24
   25 Q(K)=1.000
      GOTO 23
   24 Q(K)=(S*Q(K)-SUM(I)*SUM(J))/SQRT((S*SUMSQ(I)-SUM(I)*SUM(I))*
     1(S*SUMSQ(J)-SUM(J)*SUM(J)))
   23 CONTINUE
      Q(K+1)=1.000
      WRITE(6,99)
   99 FORMAT(1H1,30X,5H MEAN,11X,9H VARIANCE,7X,19H STANDARD DEVIATION/)
      IV1=0
      DO 101 I=1,M
      AMEAN=SUM(I)/S
      VAR=(S*SUMSQ(I)-SUM(I)**2)/(S*(S-1.0))
      DEV=SQRT(VAR)
      IV=IV1+1
      IV1=IV+2
  101 WRITE(6,100) ((VARIAT(J),J=IV,IV1),AMEAN,VAR,DEV)
  100 FORMAT(1H ,10X,3A4,5X,F8.3,9X,E12.4,9X,F8.3)
      GOTO 26
    9 READ(5,FMT) (Q(I),I=1,M1)
   26 DO 27 L=1,M,10
      WRITE(6,30) N
   30 FORMAT(1H1,10X,50H CORRELATION MATRIX    THE SAMPLE SIZE IS EQUAL
     1TO,I5///)
   96 J2=(L-1)*3+1
      J3=J2+2
      DO 27 J=L,M
      K=0
      DO 29 I=L,J
      LIMIT=L+9
      IF(I.GT.LIMIT) GOTO 86
      KK=J*(J-1)/2+I
      K=K+1
   29 D(K)=Q(KK)
```

```
86 WRITE(6,84) ((VARIAT(J1),J1=J2,J3),(D(K1),K1=1,K))
84 FORMAT(1H0,5X,3A4,10F8.3)
   J2=J3+1
   J3=J2+2
27 CONTINUE
   DO 32 I=1,NL
   DO 32 J=I,NL
   L=J*(J-1)/2+I
   A2(I,J)=Q(L)
32 A2(J,I)=A2(I,J)
   CALL MATINV(A2,NL,0,IERR)
   IF(IERR.EQ.0) GOTO 34
   WRITE(6,85)
85 FORMAT(1H0,85H CORRELATION MATRIX ON THE LEFT HAND SET CANNOT BE
  1INVERTED TRANSFORMATION IS NEEDED)
   GOTO 90
34 DO 50 J=1,NR
   DO 50 I=1,NL
   K=(J+NL-1)*(J+NL)/2+I
50 COR(I,J)=Q(K)
   CALL MATRIX(NR,NL,NL,A2,COR,B)
   DO 52 K=1,NR
   DO 52 I=1,NR
   B1(K,I)=0.0
   DO 52 J=1,NL
52 B1(K,I)=B1(K,I)+COR(J,K)*B(J,I)
   DO 59 I=1,NR
   DO 59 J=I,NR
   L=(J+NL-1)*(J+NL)/2+I+NL
   A2(I,J)=Q(L)
59 A2(J,I)=A2(I,J)
   CALL MATINV(A2,NR,0,IERR)
   IF(IERR.EQ.0) GOTO 55
   WRITE(6,56)
56 FORMAT(1H1,85H CORRELATION MATRIX ON THE RIGHT HAND SET CANNONT BE
  1INVERTED TRANSFORMATION IS NEEDED)
   GOTO 90
55 CALL MATRIX(NR,NR,NR,A2,B1,B2)
   DO 60 I=1,NR
   DO 60 J=1,NR
   IF(I.EQ.J) GOTO 61
   VEC(I,J)=0.0
   GOTO 60
61 VEC(I,J)=1.000
60 CONTINUE
   CALL EBERVC(NR,1,200,0.01,0.001,1000.0,30,1)
   M3=NR-1
   DO 62 I=1,M3
   IJ=I+1
   DO 62 J=IJ,NR
   IF(B2(I,I).GE.B2(J,J)) GOTO 62
   TEMP=B2(I,I)
   B2(I,I)=B2(J,J)
   B2(J,J)=TEMP
   DO 63 K=1,NR
   D(K)=VEC(K,I)
   VEC(K,I)=VEC(K,J)
63 VEC(K,J)=D(K)
62 CONTINUE
   WRITE(6,65)
```

```
65 FORMAT(1H1,10X,47H STATISTICAL TEST OF SUCCESSIVE CANONICAL ROOTS)
   WRITE(6,64)
64 FORMAT(1H0,16H CANONICAL ROOTS,8X,17H CHI SQUARE VALUE,5X,18H DEGR
  1EE OF FREEDOM,5X,18H SIGNIFICANT LEVEL)
   WL=1.000
   NROOT=0
   DO 66 I=1,NR
   JJ=NR-I+1
   IF(B2(JJ,JJ).LT.0.000) B2(JJ,JJ)=0.0
   WL=WL*(1.00-B2(JJ,JJ))
   L=(NR-JJ+1)*(NL-JJ+1)
   CHISQ=-1.00*(S-0.5*(FLOAT(M)+1.000))*ALOG(WL)
   IF(B2(JJ,JJ).NE.0.000) GOTO 67
   GOTO 68
67 NROOT=NROOT+1
   B2(JJ,JJ)=SQRT(B2(JJ,JJ))
68 IF(L.GE.30) GOTO 69
   IF(CHISQ.GE.CHI(L)) GOTO 70
71 F=-0.01
   GOTO 66
70 F=0.01
   GOTO 66
69 IF(L.GT.100) GOTO 51
   L=(L-25)/10+30
   IF(CHISQ.GE.CHI(L)) GOTO 70
   GOTO 71
51 CHI1=0.5*(2.3263+SQRT(2.0*FLOAT(L)-1.000))**2
   IF(CHISQ.GE.CHI1) GOTO 70
   GOTO 71
66 WRITE(6,72) B2(JJ,JJ),CHISQ,L,F
72 FORMAT(1H0,6X,F8.3,15X,F8.3,18X,I4,12X,F6.2)
   WRITE(6,12)
12 FORMAT(1H0,56H 0.01 SIGNIFICANT AT 1 PERCENT LEVEL,-0.01 INSIGNIFI
  1CANT)
   WRITE(6,73)
73 FORMAT(1H1,10X,41H CANONICAL VARIATE ON THE RIGHT HAND SET //)
   NLL=NL*3
   CALL ROSIE(NROOT,NR,NLL)
   CALL MATRIX(NROOT,NL,NR,B,VEC,COR)
   DO 77 I=1,NROOT
   DO 77 J=1,NL
77 COR(J,I)=COR(J,I)/B2(I,I)
   DO 78 I=1,NROOT
   D(I)=0.0
   DO 78 J=1,NL
78 D(I)=D(I)+COR(J,I)*COR(J,I)
   DO 79 I=1,NROOT
   DO 79 J=1,NL
79 VEC(J,I)=COR(J,I)/SQRT(D(I))
   WRITE(6,89)
89 FORMAT(///)
   WRITE(6,80)
80 FORMAT(1H0,10X,40H CANONICAL VARIATE ON THE LEFT HAND SET //)
   CALL ROSIE(NROOT,NL,0)
   GOTO 90
83 STOP
   END
```

```
      SUBROUTINE MATRIX(M,K,L,P,X,C)
      DIMENSION P(30,30),X(30,30),C(30,30)
      DO 201 I=1,M
      DO 201 J=1,K
      C(J,I)=0.0
      DO 201 K1=1,L
  201 C(J,I)=C(J,I)+P(J,K1)*X(K1,I)
      RETURN
      END
```

```
      SUBROUTINE EBERVC(N,IN,NBMAX,EPS,EPS1,EF,NN,IND)
      COMMON AV(30,30),VARIAT(180),NUMBER(30),A(30,30)
      DO 16 II=1,IN
      EPS=EPS/EF
      EPS1=EPS1/EF
      NB=0
   18 DR=0.0
      DI=0.0
      DO 17 I=2,N
      IJ=I-1
      DO 17 J=1,IJ
      C=A(I,J)+A(J,I)
      D=A(I,I)-A(J,J)
      IF(EPS.LE.ABS(C)) GOTO 23
   21 CC=1.0
      SS=0.0
      GOTO 22
   23 CC=D/C
      SIG=SIGN(1.0,CC)
      COT=CC+SIG*SQRT(1.0+CC*CC)
      SS=SIG/SQRT(1.0+COT*COT)
      CC=SS*COT
      DR=DR+1.0
   22 E=A(I,J)-A(J,I)
      IF(EPS.GT.ABS(E)) GOTO 31
      CO=CC*CC-SS*SS
      SI=2.0*SS*CC
      H=0.0
      G=0.0
      HJ=0.0
      DO 40 K=1,N
      IF(K.EQ.I) GOTO 40
      IF(K.EQ.J) GOTO 40
      H=H+A(I,K)*A(J,K)-A(K,I)*A(K,J)
      S1=A(I,K)*A(I,K)+A(K,J)*A(K,J)
      S2=A(J,K)*A(J,K)+A(K,I)*A(K,I)
      G=G+S1+S2
      HJ=HJ+S1-S2
   40 CONTINUE
      D=D*CO+C*SI
      H=2.0*H*CO-HJ*SI
      F=(2.0*E*D-H)/(4.0*(E*E+D*D)+2.0*G)
      IF(EPS1.GT.ABS(F)) GOTO 31
      CH=1.0/SQRT(1.0-F*F)
      SH=F*CH
      DI=DI+1.0
```

```
      GOTO 36
   31 CH=1.0
      SH=0.0
   36 C1=CH*CC-SH*SS
      C2=CH*CC+SH*SS
      S1=CH*SS+SH*CC
      S2=SH*CC-CH*SS
      IF((ABS(S1)+ABS(S2)).EQ.0.0) GOTO 17
      DO 52 L=1,N
      A1=A(L,I)
      A2=A(L,J)
      A(L,I)=C2*A1-S2*A2
      A(L,J)=C1*A2-S1*A1
      IF(IND.LT.0) GOTO 52
      A1=AV(L,I)
      A2=AV(L,J)
      AV(L,I)=C2*A1-S2*A2
      AV(L,J)=C1*A2-S1*A1
   52 CONTINUE
      DO 53 L=1,N
      A1=A(I,L)
      A2=A(J,L)
      A(I,L)=C1*A1+S1*A2
      A(J,L)=C2*A2+S2*A1
      IF(IND.GT.0) GOTO 53
      A1=AV(I,L)
      A2=AV(J,L)
      AV(I,L)=C1*A1+S1*A2
      AV(J,L)=C2*A2+S2*A1
   53 CONTINUE
   17 CONTINUE
      IF((DR+DI).LT.0.5) GOTO 16
      NB=NB+1
      IF(NB.NE.NBMAX) GOTO 18
   16 CONTINUE
      EPS=EPS*EF**IN
      EPS1=EPS1*EF**IN
      IF(IND.LE.0) GOTO 70
      DO 80 I=1,N
      SUM=0.0
      DO 81 J=1,N
   81 SUM=SUM+AV(J,I)**2
      SUM=SQRT(SUM)
      DO 82 J=1,N
   82 AV(J,I)=AV(J,I)/SUM
   80 CONTINUE
      RETURN
   70 DO 90 I=1,N
      SUM=0.0
      DO 91 J=1,N
   91 SUM=SUM+AV(I,J)**2
      SUM=SQRT(SUM)
      DO 92 J=1,N
   92 AV(I,J)=AV(I,J)/SUM
   90 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE MATINV(A,N,M,IERR)
C     ADAPTED FROM MULTIVARIATE PROCEDURES FOR THE BEHAVIOURAL
C     SCIENCES BY COOLEY AND LOHNES, P.198.
C
C
      DIMENSION IPIVOT(30),A(30,30),INDEX(30,2),PIVOT(30)
      EQUIVALENCE (IROW,JROW),(ICOLUM,JCOLUM),(AMAX,T,SWAP)
      IERR=0
      DETERM=1.0
      DO 20 J=1,N
   20 IPIVOT(J)=0
      DO 550 I=1,N
      AMAX=0.0
      DO 105 J=1,N
      IF(IPIVOT(J)-1) 60,105,60
   60 DO 100 K=1,N
      IF(IPIVOT(K)-1) 80,100,740
   80 IF(ABS(AMAX)-ABS(A(J,K))) 85,100,100
   85 IROW=J
      ICOLUM=K
      AMAX=A(J,K)
  100 CONTINUE
  105 CONTINUE
      IPIVOT(ICOLUM)=IPIVOT(ICOLUM)+1
      IF(IROW-ICOLUM) 140,260,140
  140 DETERM=-DETERM
      DO 200 L=1,N
      SWAP=A(IROW,L)
      A(IROW,L)=A(ICOLUM,L)
  200 A(ICOLUM,L)=SWAP
  260 INDEX(I,1)=IROW
      INDEX(I,2)=ICOLUM
      PIVOT(I)=A(ICOLUM,ICOLUM)
      DETERM=DETERM*PIVOT(I)
      IF(PIVOT(I).GT.-0.0005.AND.PIVOT(I).LT.0.0005) GOTO 760
       A(ICOLUM,ICOLUM)=1.000
      DO 350 L=1,N
  350 A(ICOLUM,L)=A(ICOLUM,L)/PIVOT(I)
      DO 550 L1=1,N
      IF(L1-ICOLUM) 400,550,400
  400 T=A(L1,ICOLUM)
      A(L1,ICOLUM)=0.0
      DO 450 L=1,N
  450 A(L1,L)=A(L1,L)-A(ICOLUM,L)*T
  550 CONTINUE
      DO 710 I=1,N
      L=N+1-I
      IF(INDEX(L,1)-INDEX(L,2)) 630,710,630
  630 JROW=INDEX(L,1)
      JCOLUM=INDEX(L,2)
      DO 705 K=1,N
      SWAP=A(K,JROW)
      A(K,JROW)=A(K,JCOLUM)
      A(K,JCOLUM)=SWAP
  705 CONTINUE
  710 CONTINUE
  740 RETURN
  760 IERR=1
      RETURN
      END
```

```
      SUBROUTINE ROSIE(NROOT,NOV,IVAR)
      COMMON VEC(30,30),VARIAT(180),NUMBER(30),B2(30,30)
      IV1=IVAR
      DO 1 L=1,NROOT,10
      LIMIT=L+9
      IPRINT=NROOT
      IF(NROOT.GT.LIMIT) IPRINT=LIMIT
      WRITE(6,2) (NUMBER(J),J=1,IPRINT)
    2 FORMAT(1HO,2X,15H CANONICAL ROOT,1X,10I8//)
      DO 1 K=1,NOV
      IV=IV1+1
      IV1=IV+2
    1 WRITE(6,3) ((VARIAT(J),J=IV,IV1),(VEC(K,J),J=L,IPRIN1))
    3 FORMAT(1H ,5X,3A4,10F8.2)
      RETURN
      END
```

APPENDIX II. Input for sample problem of canonical correlation.

```
CANONICAL CORRELATION ANALYSIS ON BUTTONWOOD SOUND, FLORIDA BAY
   73 5 2  2
TURTLE GRASSDEPTH           TEMPERATURE SALINITY    PH          SAND
SILT
(F2.0,F3.1,3F5.1,2F5.0)
 0 12  281  253   79   24   38   38
-1 20  282  265   77   66    5   29
-1 23  297  265   77   68    6   26
 0  9  295  257   69   37   20   43
 1 21  296  261   73   25   21   54
 0 24  300  259   76   30   44   26
 0 26  310  258   79   38   11   51
 0 24  301  255   80   23   35   42
 0 24  300  253   80   67   11   22
 0 24  324  257   80   24   62   14
 0 23  304  261   81   29   41   30
 1 18  320  258   76   12   37   51
 0 20  301  263   73   23   51   26
 0 26  299  261   73   25   54   21
 0 24  300  259   78   20   45   35
 0 24  310  263   77   31   31   38
 0 18  311  265   79   58    8   34
 1 24  298  262   81   11   71   18
 0 23  301  261   79   46   27   27
 0 12  321  262   78   25   45   30
-1 20  311  265   79   74   13   13
-1 23  321  267   82   52   19   29
 0  9  319  264   79   33   40   27
 1 21  316  258   73   18   45   37
 0 24  320  262   82   42   29   29
 0 26  314  264   78   23   32   45
 0 24  317  263   79   24   63   13
 0 24  310  261   76   43   25   32
 0 24  308  267   78   37   44   19
 0 23  309  263   81   32   23   45
 1 18  311  256   79   12   29   59
 0 20  311  258   79   20    7   73
 0 26  313  253   80   24   38   38
 0 24  314  262   79   12   25   63
 0 20  314  254   82   38   10   52
 0 18  316  289   80   34   40   26
 1 24  316  259   82    6   53   41
 0 23  318  256   89   44   28   28
 0 12  303  271   76   15   25   60
-1 20  301  266   80   88    6    6
-1 23  300  269   77   46   36   18
 0  9  304  264   77   21   59   20
 1 21  307  261   76   13   52   35
 0 24  304  261   76   36   37   27
 0 26  307  268   80   36   40   24
 0 24  310  270   78   25   38   37
 0 24  310  270   78   70   24    6
 0 24  307  268   77   26   37   37
 0 23  310  265   80   18   47   35
 1 18  314  261   73    9   55   36
 0 20  309  264   81   38   31   31
 0 26  308  259   79   14   43   43
 0 24  305  264   79    2   35   63
 0 20  312  265   81   41   15   44
 0 18  304  266   82   34   28   38
 1 24  309  266   82    6   31   63
 0 23  313  259   79   32   54   14
 0 12  200  386   78   25   32   43
-1 23  195  400   79   74   11   15
-1 24  200  408   78   70   12   18
 0 15  195  393   79   44   29   27
 0 27  202  400   77   35   40   25
 0 27  203  393   80   34   44   22
 0 24  205  393   80   18   35   47
 0 26  205  393   80   41   38   21
 0 26  207  408   77   40   39   21
 0 26  210  393   78   29   47   24
 1 15  209  386   70   18   42   40
 0 24  209  386   78   26   54   20
 0 18  211  393   79   45   35   20
 0 15  212  393   80   41   47   12
 1 26  213  379   78   41   40   19
 0 26  220  386   76   52   28   20
DONE
```

APPENDIX III. Output for sample problem of canonical correlation.

CANONICAL CORRELATION ANALYSIS ON BUTTONWOOD SOUND, FLORIDA BAY

	MEAN	VARIANCE	STANDARD DEVIATION
TURTLE GRASS	0.041	0.2622E 00	0.512
DEPTH	2.141	0.2083E 00	0.456
TEMPERATURE	28.551	0.1867E 02	4.321
SALINITY	29.104	0.3005E 02	5.481
PH	7.836	0.8711E-01	0.295
SAND	33.603	0.3280E 03	18.111
SILT	34.137	0.2290E 03	15.131

CORRELATION MATRIX   THE SAMPLE SIZE IS EQUAL TO 73

TURTLE GRASS	1.000						
DEPTH	-0.061	1.000					
TEMPERATURE	0.091	-0.059	1.000				
SALINITY	-0.085	0.087	-0.976	1.000			
PH	-0.194	0.242	0.122	-0.070	1.000		
SAND	-0.701	0.070	-0.208	0.202	0.133	1.000	
SILT	0.476	0.057	-0.017	0.047	-0.124	-0.631	1.000

STATISTICAL TEST OF SUCCESSIVE CANONICAL ROOTS

CANONICAL ROOTS	CHI SQUARE VALUE	DEGREE OF FREEDOM	SIGNIFICANT LEVEL
0.294	6.242	4	-0.01
0.716	55.820	10	0.01

0.01 SIGNIFICANT AT 1 PERCENT LEVEL,-0.01 INSIGNIFICANT

CANONICAL VARIATE ON THE RIGHT HAND SET

CANONICAL ROOT	1	2
SAND	1.00	0.53
SILT	-0.00	0.85

CANONICAL VARIATE ON THE LEFT HAND SET

CANONICAL ROOT	1	2
TURTLE GRASS	-0.98	0.05
DEPTH	0.02	0.12
TEMPERATURE	-0.20	0.59
SALINITY	0.01	0.79
PH	0.02	-0.09

APPENDIX IV. Complete FORTRAN IV source listing of canonical trend-surface program.

```
C       CANONICAL TREND SURFACE PROGRAM
C       P J LEE JULY 1966
C       DEPARTMENT OF GEOLOGY, MCMASTER UNIVERSITY, HAMILTON, CANADA
C
C       SUBROUTINES REQUIRED: EBERVC, MATINV
C
C
        REAL IPOLY
        DOUBLE PRECISION Q(903),R11(27,27),R22(15,15),R12(27,15),
       1PRO(27,15),PRO1(15,15),SUMSQ(42),SUM(42),A(9,9)
        DIMENSION SIG(300),E(42),DOT(10),P(42),D(42),NTRA(15),NTYPE(15),
       1POLY(50,50),B(300),C(300),FMT(20),DM(300,17),TITLE(20),VARIAT(45),
       2IPOLY(50,50),RESIDU(300),TERM(27),MAPING(300,2)
        COMMON VEC(15,15),PRO2(15,15)
        EQUIVALENCE (POLY,IPOLY)
        DATA TERM/4H   X,4H   Y,4H  X2,4H  XY,4H  Y2,4H  X3,4H X2Y,4H XY2,
       14H  Y3,4H  X4,4H X3Y,4HX2Y2,4H XY3,4H  Y4,4H  X5,4H X4Y,4HX3Y2,
       24HX2Y3,4H XY4,4H  Y5,4H  X6,4H X5Y,4HX4Y2,4HX3Y3,4HX2Y4,4H XY5,
       34H  Y6/
        DATA DOT/2H 0,2H 1,2H 2,2H 3,2H 4,2H 5,2H 6,2H 7,2H 8,2H 9/
        DATA SAMPLE,BLANK/2H *,2H  /
        READ(5,2) TITLE
      2 FORMAT(20A4)
        READ(5,1) N,M,MTRA,NORDER,FEET
      1 FORMAT(I3,2I2,I1,F8.0)
        S=N
        WRITE(6,3) TITLE
      3 FORMAT(1H1,20X,20A4///)
        IF(MTRA.EQ.0) GOTO 13
        READ(5,201) (NTRA(I),NTYPE(I),I=1,MTRA)
    201 FORMAT(26(I2,I1))
     13 MU=M*3
        READ(5,6) (VARIAT(I),I=1,MU)
      6 FORMAT(18A4)
        READ(5,2) FMT
        XMAX=-100000.0
        XMIN=100000.0
        YMAX=-100000.0
        YMIN=100000.0
        M90=M+27
        IM90=(27+M+1)*(27+M)/2
        DO 20 I=1,M90
        SUM(I)=0.0
     20 SUMSQ(I)=0.0
        DO 21 I=1,IM90,1
     21 Q(I)=0.0
        DO 131 ISAMPL=1,N
        READ(5,FMT) X,Y,(D(J),J=1,M)
        IF(MTRA.EQ.0) GOTO 11
        DO 15 ITRA=1,MTRA
        JTRA=NTRA(ITRA)
        KTYPE=NTYPE(ITRA)
        IF(KTYPE.EQ.1.OR.KTYPE.EQ.2.OR.KTYPE.EQ.3.OR.KTYPE.EQ.4.OR.
       1KTYPE.EQ.5.OR.KTYPE.EQ.6) GOTO 202
     19 WRITE(6,12)
     12 FORMAT(1H0,30H WRONG CODE FOR TRANSFORMATION)
        STOP
    202 GOTO(9,10,301,302,303,304),KTYPE
```

```
  9 D(JTRA)=ARSIN(SQRT(D(JTRA)))
    GOTO 15
 10 D(JTRA)=ALOG10(D(JTRA))
    GOTO 15
301 D(JTRA)=ALOG10(D(JTRA)+1.0)
    GOTO 15
302 D(JTRA)=ALOG(D(JTRA))
    GOTO 15
303 D(JTRA)=ALOG(D(JTRA)+1.0)
304 D(JTRA)=SQRT(D(JTRA))
 15 CONTINUE
 11 DO 16 J=1,M
 16 DM(ISAMPL,J)=D(J)
    DM(ISAMPL,M+1)=X
    DM(ISAMPL,M+2)=Y
    XMAX=AMAX1(X,XMAX)
    YMAX=AMAX1(Y,YMAX)
    XMIN=AMIN1(X,XMIN)
    YMIN=AMIN1(Y,YMIN)
    DO 130 I=1,M
    SUM(I)=SUM(I)+D(I)
130 SUMSQ(I)=SUMSQ(I)+D(I)*D(I)
131 CONTINUE
    DO 126 I=1,M
    SUMSQ(I)=DSQRT((S*SUMSQ(I)-(SUM(I))**2)/(S*(S-1.0)))
126 SUM(I)=SUM(I)/S
    WRITE(6,96)
 96 FORMAT(1H1,30X,5H MEAN,5X,19H STANDARD DEVIATION/)
    IV1=0
    DO 93 I=1,M
    IV=IV1+1
    IV1=IV+2
 93 WRITE(6,95) ((VARIAT(J),J=IV,IV1),SUM(I),SUMSQ(I))
 95 FORMAT(1H0,10X,3A4,5X,F10.3,7X,F12.3)
    WRITE(6,106)
106 FORMAT(///)
    DO 127 J=1,N
    DO 127 I=1,M
127 DM(J,I)=(DM(J,I)-SUM(I))/SUMSQ(I)
    DO 129 I=1,M90
129 SUM(I)=0.0
    DO 8 IS=1,N
    DO 128 J=1,M
128 D(J)=DM(IS,J)
    X=DM(IS,M+1)
    Y=DM(IS,M+2)
    L=M
    A(1,1)=1.0
    DO 17 I=1,6
    DO 18 J=1,I
    A(J,I+1)=A(J,I)*X
    L=L+1
 18 D(L)=A(J,I+1)
    A(I+1,I+1)=A(I,I)*Y
    L=L+1
 17 D(L)=A(I+1,I+1)
    DO 8 I=1,M90
 22 SUM(I)=SUM(I)+D(I)
    DO 8 J=I,M90
    L=(J-1)*J/2+I
```

```
      Q(L)=Q(L)+D(J)*D(I)
    8 CONTINUE
      DO 25 I=1,M90
      DO 25 J=I,M90
      K=(J-1)*J/2+I
   25 Q(K)=(S*Q(K)-SUM(I)*SUM(J))/(S*(S-1.0))
      J3=0
      WRITE(6,102) N
  102 FORMAT(1H0,10X,19H COVARIANCE  MATRIX,10X,21H THE SAMPLE SIZE IS
     1I5//)
      DO 103 J=1,M
      K=0
      DO 104 I=1,J
      KK=J*(J-1)/2+I
      K=K+1
  104 E(K)=Q(KK)
      J2=J3+1
      J3=J2+2
      WRITE(6,105) ((VARIAT(J1),J1=J2,J3),(E(K1),K1=1,K))
  105 FORMAT(1H0,5X,3A4,15F7.3)
  103 CONTINUE
      ROOT=0.000
      ITERM=0
      WRITE(6,113)
  113 FORMAT(1H1,10X,53H RECORD OF SUCCESSIVE EVALUATION TREND SURFACE D
     1EGREE///)
  125 DO 27 NPOWER=1,NORDER
      IPOWER=NPOWER
      ITERM=IPOWER+ITERM+1
      IF(M.GT.ITERM) GOTO 37
      NL=ITERM
      NR=M
      DO 32 I=1,NR
      DO 32 J=1,NL
      K=(J+M-1)*(J+M)/2+I
   32 R12(J,I)=Q(K)
   33 DO 35 I=1,NR
      DO 35 J=I,NR
      L=J*(J-1)/2+I
      R22(I,J)=Q(L)
   35 R22(J,I)=R22(I,J)
      CALL MATINV(R22,NR,15,IERR)
      IF(IERR.EQ.0) GOTO 29
   39 WRITE(6,34)
   34 FORMAT(1H0,45H GEOLOGICAL VARIATE MATRIX CANNOT BE INVERTED)
      STOP
   29 DO 31 I=1,NL
      DO 31 J=I,NL
      K=(J+M-1)*(J+M)/2+I+M
      R11(I,J)=Q(K)
   31 R11(J,I)=R11(I,J)
      CALL MATINV(R11,NL,27,IERR)
      IF(IERR.EQ.0) GOTO 36
   43 WRITE(6,30)
   30 FORMAT(1H0,42H X-Y COORDINATES MATRIX CANNOT BE INVERTED)
      STOP
   37 NL=M
      NR=ITERM
   42 DO 44 I=1,NR
      DO 44 J=I,NR
```

```
      K=(J+M-1)*(J+M)/2+I+M
      R22(I,J)=Q(K)
   44 R22(J,I)=R22(I,J)
      CALL MATINV(R22,NR,15,IERR)
      IF(IERR.EQ.1) GOTO 43
   56 DO 45 I=1,NL
      DO 45 J=1,NR
      K=(J+M-1)*(J+M)/2+I
   45 R12(I,J)=Q(K)
      DO 40 I=1,NL
      DO 40 J=I,NL
      K=J*(J-1)/2+I
      R11(I,J)=Q(K)
   40 R11(J,I)=R11(I,J)
      CALL MATINV(R11,NL,27,IERR)
      IF(IERR.EQ.1) GOTO 39
   36 DO 46 I=1,NL
      DO 46 J=1,NR
      PRO(I,J)=0.0
      DO 46 K=1,NL
   46 PRO(I,J)=PRO(I,J)+R11(I,K)*R12(K,J)
      DO 47 I=1,NR
      DO 47 J=1,NR
      PRO1(I,J)=0.0
      DO 47 K=1,NL
   47 PRO1(I,J)=PRO1(I,J)+R12(K,I)*PRO(K,J)
      DO 48 I=1,NR
      DO 48 J=1,NR
      PRO2(I,J)=0.0
      DO 48 K=1,NR
   48 PRO2(I,J)=PRO2(I,J)+R22(I,K)*PRO1(K,J)
      DO 49 I=1,NR
      DO 49 J=1,NR
      VEC(I,J)=0.0
      IF(I.EQ.J) VEC(I,J)=1.000
   49 CONTINUE
      CALL EBERVC(NR,1,200,0.01,0.001,1000.0,15,1)
      TEMP=-100.0
      DO 50 J=1,NR
      IF(TEMP.GT.PRO2(J,J)) GOTO 50
      TEMP=PRO2(J,J)
      JJ=J
   50 CONTINUE
      CANON=SQRT(PRO2(JJ,JJ))
      DIF=CANON-ROOT
      ROOT=CANON
      WRITE(6,52) IPOWER,ROOT
   52 FORMAT(1H ,10X,24H THE DEGREE IS EQUAL TO I2,5X,23H THE CANONICAL
     1ROOT IS F10.4)
      IF(DIF.LT.0.05.OR.ROOT.GE.0.95000) GOTO 51
   27 CONTINUE
   51 WRITE(6,53) IPOWER,ROOT
   53 FORMAT(1H0,10X,57H THE DEGREE OF THE MOST PREDICTABLE SURFACE IS
     1EQUAL TO I2,3X,48H THE CORRESPONDING CANONICAL ROOT IS EQUAL TO
     1F7.4//)
      NR1=NR-1
      DO 171 I=1,NR1
      JP=I+1
      DO 171 J=JP,NR
      IF(PRO2(I,I).GE.PRO2(J,J)) GOTO 171
```

```
      TEMP=PRO2(I,I)
      PRO2(I,I)=PRO2(J,J)
      PRO2(J,J)=TEMP
      DO 174 K=1,NR
      TEMP=VEC(K,J)
      VEC(K,J)=VEC(K,I)
  174 VEC(K,I)=TEMP
  171 CONTINUE
      TOTALX=XMAX-XMIN
      TOTALY=YMAX-YMIN
      INDEX=TOTALY/TOTALX*50.0
      DELTA=TOTALX/50.0
      TX=50.0/TOTALX
      TY=FLOAT(INDEX)/TOTALY
      DO 172 JJ=1,NR
      INDEX1=1
      PRO2(JJ,JJ)=SQRT(PRO2(JJ,JJ))
      WRITE(6,173) PRO2(JJ,JJ)
  173 FORMAT(1H1,10X,37H TREND SURFACE FOR THE CANONICAL ROOT,2X,F8.4//)
      WRITE(6,114)
  114 FORMAT(1H0,10X,34H THE EQUATION OF THE TREND SURFACE///)
      DO 54 I=1,NL
      E(I)=0.0
      DO 54 J=1,NR
   54 E(I)=E(I)+PRO(I,J)*VEC(J,JJ)
      RR=0.0
      DO 55 I=1,NL
      E(I)=E(I)/PRO2(JJ,JJ)
   55 RR=RR+E(I)*E(I)
      DO 57 I=1,NL
   57 E(I)=E(I)/SQRT(RR)
      J3=0
      IF(M.GT.ITERM) GOTO 59
      DO 60 J=1,M
      J2=J3+1
      J3=J2+2
   60 WRITE(6,58) (VARIAT(J1),J1=J2,J3),VEC(J,JJ)
   58 FORMAT(1H ,5X,3A4,3X,F6.3)
      GOTO 112
   59 DO 132 J=1,M
      J2=J3+1
      J3=J2+2
  132 WRITE(6,58) (VARIAT(J1),J1=J2,J3),E(J)
  112 DO 64 I=1,ITERM,10
      IPRINT=I+9
      LIMIT=ITERM
      IF(ITERM.GT.IPRINT) LIMIT=IPRINT
      WRITE(6,115)
  115 FORMAT(//)
      WRITE(6,62) (TERM(J),J=I,LIMIT)
   62 FORMAT(1H0,10(6X,A4))
      IF(M.GT.ITERM) GOTO 65
   61 WRITE(6,63) (E(IK),IK=I,LIMIT)
   63 FORMAT(1H0,10F10.4//)
      GOTO 64
   65 WRITE(6,63) (VEC(J,JJ),J=I,LIMIT)
   64 CONTINUE
      WRITE(6,97)
   97 FORMAT(1H0,40H    NOTE X4Y2=X**4*Y**2,X3=X**3,AND SO ON)
      BSUM=0.0
```

```
      CSUM=0.0
      BSS=0.0
      CSS=0.0
      BCR=0.0
      DO 72 NSAMPL=1,N
      L=0
      A(1,1)=1.0
      DO 73 I=1,IPOWER
      DO 74 J=1,I
      A(J,I+1)=A(J,I)*DM(NSAMPL,M+1)
      L=L+1
   74 P(L)=A(J,I+1)
      A(I+1,I+1)=A(I,I)*DM(NSAMPL,M+2)
      L=L+1
   73 P(L)=A(I+1,I+1)
  108 B(NSAMPL)=0.0
      C(NSAMPL)=0.0
      IF(M.GT.ITERM) GOTO 75
      DO 76 J=1,ITERM
   76 B(NSAMPL)=B(NSAMPL)+E(J)*P(J)
      DO 77 I=1,M
   77 C(NSAMPL)=C(NSAMPL)+VEC(I,JJ)*DM(NSAMPL,I)
      GOTO 81
   75 DO 78 J=1,ITERM
   78 B(NSAMPL)=B(NSAMPL)+VEC(J,JJ)*P(J)
      DO 79 I=1,M
   79 C(NSAMPL)=C(NSAMPL)+E(I)*DM(NSAMPL,I)
   81 BSUM=BSUM+B(NSAMPL)
      BSS=BSS+B(NSAMPL)**2
      CSUM=CSUM+C(NSAMPL)
      CSS=CSS+C(NSAMPL)**2
   72 BCR=BCR+B(NSAMPL)*C(NSAMPL)
      DENON=S*BSS-BSUM**2
      ALPHA=(BSS*CSUM-BSUM*BCR)/DENON
      BETA=(S*BCR-BSUM*CSUM)/DENON
      DO 133 I=1,N
      B(I)=ALPHA+BETA*B(I)
  133 RESIDU(I)=C(I)-B(I)
      S2=0.0
      DO 116 I=1,N
  116 S2=S2+RESIDU(I)
      S2=-1.0*S2/S
      DO 117 I=1,N
  117 RESIDU(I)=RESIDU(I)+S2
      WRITE(6,118) S2
  118 FORMAT(1H0,10X,55H THE CONSTANT OF THE TREND SURFACE EQUATION IS E
     1QUAL TO,F10.4)
      WRITE(6,80)
   80 FORMAT(1H1,13H X-COORDINATE,5X,13H Y-COORDINATE,5X,15H OBSERVED VA
     1LUE,5X,17H CALCULATED VALUE,5X,9H RESIDUAL///)
      WRITE(6,82) ((DM(I,M+1),DM(I,M+2),C(I),B(I),RESIDU(I)),I=1,N)
   82 FORMAT(1H ,F10.4,8X,F10.4,8X,F10.4,12X,F10.4,8X,F10.4)
      DO 66 LENGTH=1,INDEX
      Y=YMAX-DELTA*(FLOAT(LENGTH)-1.0)
      DO 66 LWIDTH=1,50
      X=XMIN+DELTA*(FLOAT(LWIDTH)-1.0)
      L=0
      A(1,1)=1.0
      DO 67 I=1,IPOWER
      DO 68 J=1,I
```

```
      A(J,I+1)=A(J,I)*X
      L=L+1
   68 P(L)=A(J,I+1)
      A(I+1,I+1)=A(I,I)*Y
      L=L+1
   67 P(L)=A(I+1,I+1)
  107 POLY(LENGTH,LWIDTH)=S2
      IF(M.GT.ITERM) GOTO 69
      DO 70 J=1,ITERM
   70 POLY(LENGTH,LWIDTH)=POLY(LENGTH,LWIDTH)+E(J)*P(J)
      GOTO 66
   69 DO 71 J=1,ITERM
   71 POLY(LENGTH,LWIDTH)=POLY(LENGTH,LWIDTH)+VEC(J,JJ)*P(J)
   66 POLY(LENGTH,LWIDTH)=ALPHA+BETA*POLY(LENGTH,LWIDTH)
      PMAX=-100000.0
      PMIN=100000.0
      DO 83 I=1,N
      PMAX=AMAX1(PMAX,B(I))
      PMIN=AMIN1(PMIN,B(I))
   83 CONTINUE
      DIFF=PMAX-PMIN
   86 GRAD=DIFF/10.0
      DO 88 LENGTH=1,INDEX
      DO 88 LWIDTH=1,50
      DO 121 I=1,10
      IGRAD=I
      DISC=GRAD*(FLOAT(I)-1.0)+PMIN
      IF(DISC.GE.POLY(LENGTH,LWIDTH)) GOTO 123
  121 CONTINUE
      IGRAD=10
  123 IPOLY(LENGTH,LWIDTH)=DOT(IGRAD)
   88 CONTINUE
      DO 134 I=1,N
      DTEMPX=(DM(I,M+1)-XMIN)*TX
      DTEMPY=(YMAX-DM(I,M+2))*TY
      MAPING(I,1)=DTEMPX
      MAPING(I,2)=DTEMPY
      IF(MAPING(I,1).EQ.0) MAPING(I,1)=1
      IF(MAPING(I,2).EQ.0) MAPING(I,2)=1
  134 CONTINUE
      DO 137 I=1,INDEX
      DO 137 J=1,50
      DO 138 K=1,N
      IF(MAPING(K,1).EQ.J.AND.MAPING(K,2).EQ.I).GOTO 139
  138 CONTINUE
      GOTO 137
  139 IPOLY(I,J)=SAMPLE
  137 CONTINUE
  150 SCALE=TOTALX*4.0/100.0*FEET
      MSCALE=SCALE
      WRITE(6,90) MSCALE
   90 FORMAT(1H1,35X,5H ----,I8,5H FEET,5X,18H * SAMPLE LOCALITY///)
  153 WRITE(6,91)
   91 FORMAT(1H ,13X,105H **********************************************
     1**********************************************************)
      WRITE(6,92) ((IPOLY(I,J),J=1,50),I=1,INDEX)
   92 FORMAT(1H ,13X,2H *,101X,2H */14X,2H *,50A2,1X,2H *)
      WRITE(6,124)
  124 FORMAT(1H ,13X,2H *,101X,2H *)
      WRITE(6,91)
```

```
      WRITE(6,122)
  122 FORMAT(1H0,10X,7H LEGEND/)
      DO 109 I=1,10
      DISC=GRAD*(FLOAT(I)-1.0)+PMIN
  109 WRITE(6,110) (DOT(I),DISC)
  110 FORMAT(1H0,11X,A2,2H =,F8.3)
      IF(INDEX1.EQ.2) GOTO 172
      PMAX=-100000.0
      PMIN=100000.0
      DO 151 I=1,N
      PMAX=AMAX1(PMAX,RESIDU(I))
      PMIN=AMIN1(PMIN,RESIDU(I))
  151 CONTINUE
      GRAD=(PMAX-PMIN)/10.0
      DO 161 I=1,N
      DO 159 J=1,10
      JDOT=J
      DISC=GRAD*(FLOAT(J)-1.0)+PMIN
      IF(DISC.GE.RESIDU(I)) GOTO 160
  159 CONTINUE
      JDOT=9
  160 SIG(I)=DOT(JDOT)
  161 CONTINUE
      DO 152 I=1,INDEX
      DO 152 J=1,50
      DO 157 K=1,N
      IF(MAPING(K,1).EQ.J.AND.MAPING(K,2).EQ.I) GOTO 156
      IPOLY(I,J)=BLANK
  157 CONTINUE
      GOTO 152
  156 IPOLY(I,J)=SIG(K)
  152 CONTINUE
      INDEX1=2
      WRITE(6,158)
  158 FORMAT(1H1,10X,17H THE RESIDUAL MAP//)
      GOTO 153
  172 CONTINUE
  100 STOP
      END
```

```
      SUBROUTINE EBERVC(N,IN,NBMAX,EPS,EPS1,EF,NN,IND)
      COMMON AV(15,15),A(15,15)
      DO 16 II=1,IN
      EPS=EPS/EF
      EPS1=EPS1/EF
      NB=0
   18 DR=0.0
      DI=0.0
      DO 17 I=2,N
      IJ=I-1
      DO 17 J=1,IJ
      C=A(I,J)+A(J,I)
      D=A(I,I)-A(J,J)
      IF(EPS.LE.ABS(C)) GOTO 23
   21 CC=1.0
      SS=0.0
      GOTO 22
   23 CC=D/C
      SIG=SIGN(1.0,CC)
      COT=CC+SIG*SQRT(1.0+CC*CC)
      SS=SIG/SQRT(1.0+COT*COT)
      CC=SS*COT
      DR=DR+1.0
   22 E=A(I,J)-A(J,I)
      IF(EPS.GT.ABS(E)) GOTO 31
      CO=CC*CC-SS*SS
      SI=2.0*SS*CC
      H=0.0
      G=0.0
      HJ=0.0
      DO 40 K=1,N
      IF(K.EQ.I) GOTO 40
      IF(K.EQ.J) GOTO 40
      H=H+A(I,K)*A(J,K)-A(K,I)*A(K,J)
      S1=A(I,K)*A(I,K)+A(K,J)*A(K,J)
      S2=A(J,K)*A(J,K)+A(K,I)*A(K,I)
      G=G+S1+S2
      HJ=HJ+S1-S2
   40 CONTINUE
      D=D*CO+C*SI
      H=2.0*H*CO-HJ*SI
      F=(2.0*E*D-H)/(4.0*(E*E+D*D)+2.0*G)
      IF(EPS1.GT.ABS(F)) GOTO 31
      CH=1.0/SQRT(1.0-F*F)
      SH=F*CH
      DI=DI+1.0
      GOTO 36
   31 CH=1.0
      SH=0.0
   36 C1=CH*CC-SH*SS
      C2=CH*CC+SH*SS
      S1=CH*SS+SH*CC
      S2=SH*CC-CH*SS
      IF((ABS(S1)+ABS(S2)).EQ.0.0) GOTO 17
      DO 52 L=1,N
      A1=A(L,I)
      A2=A(L,J)
      A(L,I)=C2*A1-S2*A2
      A(L,J)=C1*A2-S1*A1
      IF(IND.LT.0) GOTO 52
```

```
      A1=AV(L,I)
      A2=AV(L,J)
      AV(L,I)=C2*A1-S2*A2
      AV(L,J)=C1*A2-S1*A1
   52 CONTINUE
      DO 53 L=1,N
      A1=A(I,L)
      A2=A(J,L)
      A(I,L)=C1*A1+S1*A2
      A(J,L)=C2*A2+S2*A1
      IF(IND.GT.0) GOTO 53
      A1=AV(I,L)
      A2=AV(J,L)
      AV(I,L)=C1*A1+S1*A2
      AV(J,L)=C2*A2+S2*A1
   53 CONTINUE
   17 CONTINUE
      IF((DR+DI).LT.0.5) GOTO 16
      NB=NB+1
      IF(NB.NE.NBMAX) GOTO 18
   16 CONTINUE
      EPS=EPS*EF**IN
      EPS1=EPS1*EF**IN
      IF(IND.LE.0) GOTO 70
      DO 80 I=1,N
      SUM=0.0
      DO 81 J=1,N
   81 SUM=SUM+AV(J,I)**2
      SUM=SQRT(SUM)
      DO 82 J=1,N
   82 AV(J,I)=AV(J,I)/SUM
   80 CONTINUE
      RETURN
   70 DO 90 I=1,N
      SUM=0.0
      DO 91 J=1,N
   91 SUM=SUM+AV(I,J)**2
      SUM=SQRT(SUM)
      DO 92 J=1,N
   92 AV(I,J)=AV(I,J)/SUM
   90 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE MATINV(A,N,NN,IERR)
C     ADAPTED FROM MULTIVARIATE PROCEDURES FOR THE BEHAVIOURAL
C     SCIENCES BY COOLEY AND LOHNES, P.198.
C
C
      DOUBLE PRECISION A(NN,NN),PIVOT(27),AMAX
      DIMENSION IPIVOT(27),INDEX(27,2)
      EQUIVALENCE (IROW,JROW),(ICOLUM,JCOLUM),(AMAX,T,SWAP)
      IERR=0
      DETERM=1.0
      DO 20 J=1,N
   20 IPIVOT(J)=0
      DO 550 I=1,N
      AMAX=0.0
      DO 105 J=1,N
      IF(IPIVOT(J)-1) 60,105,60
   60 DO 100 K=1,N
      IF(IPIVOT(K)-1) 80,100,740
   80 IF(DABS(AMAX)-DABS(A(J,K))) 85,100,100
   85 IROW=J
      ICOLUM=K
      AMAX=A(J,K)
  100 CONTINUE
  105 CONTINUE
      IPIVOT(ICOLUM)=IPIVOT(ICOLUM)+1
      IF(IROW-ICOLUM) 140,260,140
  140 DETERM=-DETERM
      DO 200 L=1,N
      SWAP=A(IROW,L)
      A(IROW,L)=A(ICOLUM,L)
  200 A(ICOLUM,L)=SWAP
  260 INDEX(I,1)=IROW
      INDEX(I,2)=ICOLUM
      PIVOT(I)=A(ICOLUM,ICOLUM)
      DETERM=DETERM*PIVOT(I)
      IF(PIVOT(I).GT.-0.0005.AND.PIVOT(I).LT.0.0005) GOTO 760
       A(ICOLUM,ICOLUM)=1.000
      DO 350 L=1,N
  350 A(ICOLUM,L)=A(ICOLUM,L)/PIVOT(I)
      DO 550 L1=1,N
      IF(L1-ICOLUM) 400,550,400
  400 T=A(L1,ICOLUM)
      A(L1,ICOLUM)=0.0
      DO 450 L=1,N
  450 A(L1,L)=A(L1,L)-A(ICOLUM,L)*T
  550 CONTINUE
      DO 710 I=1,N
      L=N+1-I
      IF(INDEX(L,1)-INDEX(L,2)) 630,710,630
  630 JROW=INDEX(L,1)
      JCOLUM=INDEX(L,2)
      DO 705 K=1,N
      SWAP=A(K,JROW)
      A(K,JROW)=A(K,JCOLUM)
      A(K,JCOLUM)=SWAP
  705 CONTINUE
  710 CONTINUE
  740 RETURN
  760 IERR=1
      RETURN
      END
```

APPENDIX V. Input for sample problem of canonical trend surface.

```
CANONICAL TREND ANALYSIS ON PERMIAN SYSTEM FROM KANSAS AND COLORADO    P J LEE
 30 4  3  211200
SAND         SHALE         CARBONATE    EVAPORITE
(2F5.1,5X,4F5.0)
  185- 260  608  365  148   20   75
  235- 285  640  224  304   14   98
  260- 230  464  104  242   18  100
  450- 220  532  157  238    0  137
  550- 230  562  120  316    0  126
  555- 140  530   30  461    0   39
   20- 295  447  293  116   12   26
  115- 330  844  451  311   42   40
  230- 340  906  337  432   60   77
  310- 355  845  266  350   24  205
  290- 380  915  295  355   43  222
  360- 400 1139  179  643   20  297
  370- 365 1118  180  568    0  370
  385- 420 1224  207  758   11  248
  480- 345 1162  130  659   13  360
  510- 330 1003  224  542   21  216
  555- 310  721  229  400   12   80
  620- 300  775  223  477   28   47
  115- 430  614  255  272   28   59
  225- 495  702  237  341   39   85
  260- 500  933  275  435   41  182
  310- 485 1001  348  450   17  186
  425- 440 1204  277  610   10  307
  410- 510 1144  310  520   12  302
  380- 550 1048  362  510   12  164
  430- 530 1114  246  528   32  308
  420- 550 1023  295  501   18  209
  570- 460  955  267  502   24  162
  575- 510 1005  271  637    8   89
  340- 580 1126  270  558   68  230
```

APPENDIX VI. Output for sample problem of canonical trend surface.

CANONICAL TREND ANALYSIS ON PERMIAN SYSTEM FROM KANSAS AND COLORADO P J LEE

	MEAN	STANDARD DEVIATION
SAND	247.567	87.018
SHALE	439.467	155.699
CARBONATE	21.567	16.850
EVAPORITE	168.200	101.715

COVARIANCE MATRIX THE SAMPLE SIZE IS 30

SAND	1.000			
SHALE	-0.138	1.000		
CARBONATE	0.462	-0.047	1.000	
EVAPORITE	-0.117	0.663	-0.097	1.000

RECORD OF SUCCESSIVE EVALUATION TREND SURFACE DEGREE

THE DEGREE IS EQUAL TO 1 THE CANONICAL ROOT IS 0.7804
THE DEGREE IS EQUAL TO 2 THE CANONICAL ROOT IS 0.8810
THE DEGREE IS EQUAL TO 3 THE CANONICAL ROOT IS 0.9464

THE DEGREE OF THE MOST PREDICTABLE SURFACE IS EQUAL TO 3
THE CORRESPONDING CANONICAL ROOT IS EQUAL TO 0.9464

TREND SURFACE FOR THE CANONICAL ROOT 0.9464

THE EQUATION OF THE TREND SURFACE

SAND	0.516
SHALE	0.408
CARBONATE	-0.104
EVAPORITE	0.746

X	Y	X2	XY	Y2	X3	X2Y	XY2	Y3
-0.4031	-0.9149	0.0146	-0.0048	-0.0145	-0.0001	0.0003	0.0002	0.0000

NOTE X4Y2=X**4*Y**2,X3=X**3,AND SO ON

THE CONSTANT OF THE TREND SURFACE EQUATION IS EQUAL TO 0.0000

X-COORDINATE	Y-COORDINATE	OBSERVED VALUE	CALCULATED VALUE	RESIDUAL
18.5000	-26.0000	-0.7406	-1.2937	0.5532
23.5000	-28.5000	-0.9628	-0.6816	-0.2812
26.0000	-23.0000	-1.8469	-1.5947	-0.2521
45.0000	-22.0000	-1.1607	-1.0923	-0.0684
55.0000	-23.0000	-1.2566	-1.0149	-0.2416
55.5000	-14.0000	-2.0489	-2.2093	0.1604
2.0000	-29.5000	-1.5615	-1.1965	-0.3650
11.5000	-33.0000	-0.1959	-0.8965	0.7006
31.0000	-35.5000	0.1300	0.5782	-0.4481
23.0000	-34.0000	-0.3948	-0.0700	-0.3248
36.0000	-40.0000	1.0806	1.1523	-0.0716
29.0000	-38.0000	0.3227	0.6376	-0.3148
37.0000	-36.5000	1.5488	0.9287	0.6201
38.5000	-42.0000	1.2438	1.3327	-0.0888
48.0000	-34.5000	1.3371	0.6505	0.6866
51.0000	-33.0000	0.4828	0.3324	0.1504
55.5000	-31.0000	-0.8015	-0.2234	-0.5780
62.0000	-30.0000	-0.9761	-1.1118	0.1357
11.5000	-43.0000	-1.2350	-1.4280	0.1930
22.5000	-49.5000	-1.0382	-0.5006	-0.5375
26.0000	-50.0000	0.1325	0.0103	0.1223
31.0000	-46.5000	0.7820	0.7993	-0.0172
42.5000	-44.0000	1.7104	1.4494	0.2611
41.0000	-51.0000	1.6215	1.3960	0.2255
38.0000	-55.0000	0.8916	0.9637	-0.0721
43.0000	-53.0000	1.1837	1.3404	-0.1567
42.0000	-55.0000	0.7637	1.1873	-0.4235
57.0000	-46.0000	0.2185	0.2740	-0.0554
57.5000	-51.0000	0.1589	0.1423	0.0166
34.0000	-58.0000	0.6106	0.1394	0.4713

---- 506379 FEET * SAMPLE LOCALITY

```
*****************************************************************************************************
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* 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 *
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* 2 3 3 3 3 3 4 4 4 4 5 5 5 5 6 6 6 7 7 7 7 * 8 8 8 8 8 9 9 9 9 9 9 9 9 9 9 9 9 8 8 8 8 7 7 7 6 6 5 5 *
* 2 2 2 3 3 3 4 4 4 4 5 5 5 6 6 6 6 7 7 7 7 8 8 8 8 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 8 8 8 7 7 6 6 5 5 *
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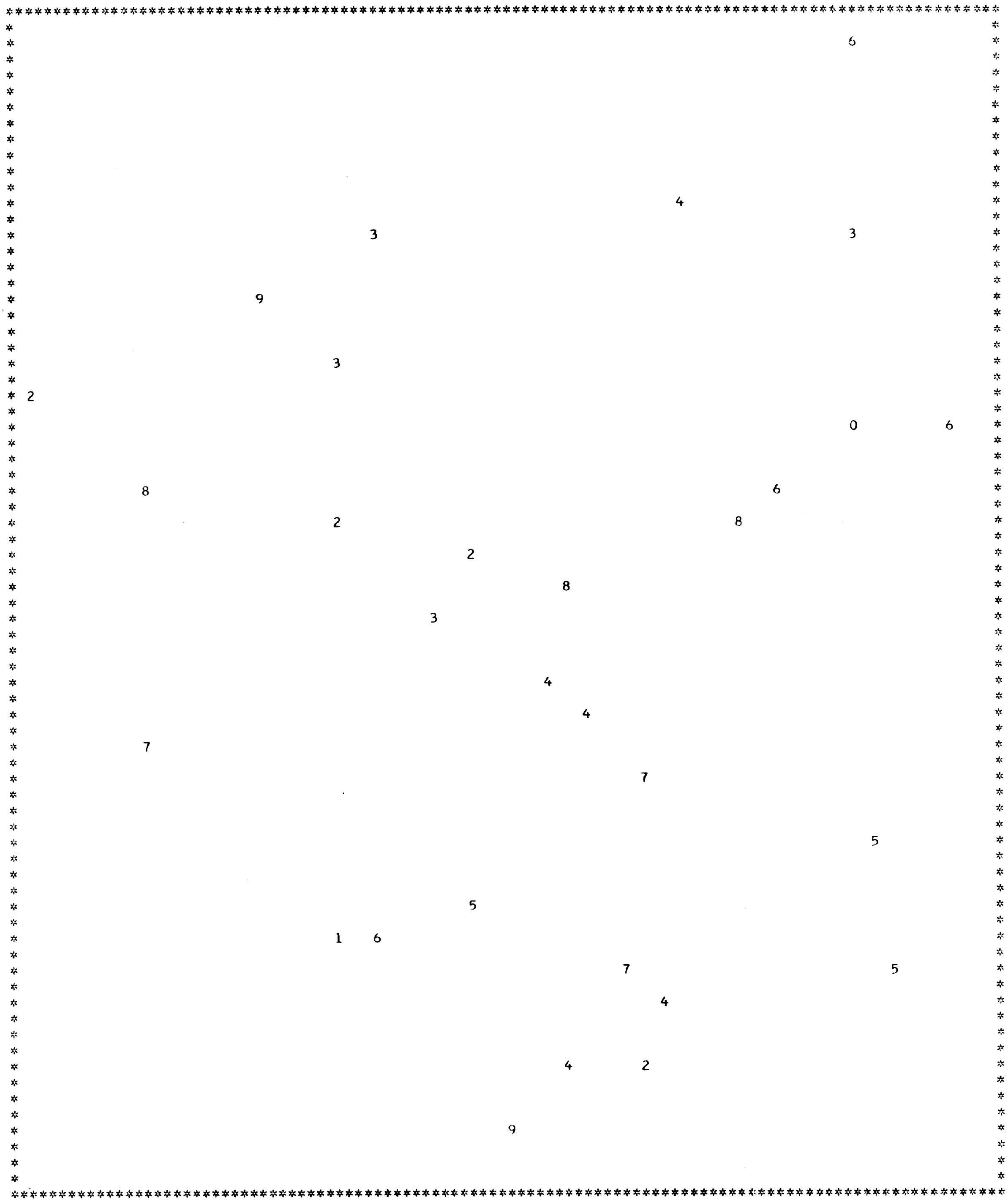
* 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 2 3 3 4 5 5 6 6 7 7 8 8 9 9 9 9 9 9 9 9 9 9 9 9 9 9 8 8 7 7 6 5 4 *
*****************************************************************************************************
```

LEGEND

0 = -2.209	5 = -0.380
1 = -1.843	6 = -0.014
2 = -1.478	7 = 0.352
3 = -1.112	8 = 0.718
4 = -0.746	9 = 1.083

THE RESIDUAL MAP



LEGEND

0 =	-0.578	5 =	0.061
1 =	-0.450	6 =	0.189
2 =	-0.322	7 =	0.317
3 =	-0.194	8 =	0.445
4 =	-0.067	9 =	0.573

TREND SURFACE FOR THE CANONICAL ROOT 0.8671

THE EQUATION OF THE TREND SURFACE

SAND	0.757
SHALE	-0.113
CARBONATE	0.616
EVAPORITE	-0.184

X	Y	X2	XY	Y2	X3	X2Y	XY2	Y3
0.1807	-0.9832	-0.0060	0.0049	-0.0230	0.0001	0.0001	0.0001	-0.0002

NOTE X4Y2=X**4*Y**2,X3=X**3,AND SO ON

THE CONSTANT OF THE TREND SURFACE EQUATION IS EQUAL TO 0.0001

X-COORDINATE	Y-COORDINATE	OBSERVED VALUE	CALCULATED VALUE	RESIDUAL
18.5000	-26.0000	1.3449	0.9334	0.4116
23.5000	-28.5000	-0.2565	0.8108	-1.0671
26.0000	-23.0000	-1.1130	-0.5538	-0.5590
45.0000	-22.0000	-1.3743	-2.2745	0.9003
55.0000	-23.0000	-1.7330	-1.1140	-0.6190
55.5000	-14.0000	-2.4639	-2.5063	0.0424
2.0000	-29.5000	0.5376	0.8094	-0.2717
11.5000	-33.0000	2.8431	1.9043	0.9388
31.0000	-35.5000	0.2478	0.2509	-0.0031
23.0000	-34.0000	2.3547	1.2650	1.0898
36.0000	-40.0000	-1.0349	-0.3683	-0.6665
29.0000	-38.0000	1.1608	0.5450	0.6159
37.0000	-36.5000	-1.8355	-0.5313	-1.3042
38.5000	-42.0000	-1.1153	-0.6325	-0.4827
46.0000	-34.5000	-1.8430	-1.3810	-0.4620
51.0000	-33.0000	-0.3868	-1.3241	0.9374
55.5000	-31.0000	-0.3232	-0.8622	0.5390
62.0000	-30.0000	0.2137	0.7124	-0.4986
11.5000	-43.0000	0.6193	1.2072	-0.5878
22.5000	-49.5000	0.7679	0.9118	-0.1438
26.0000	-50.0000	0.9279	0.7713	0.1562
31.0000	-48.5000	0.6670	0.3568	0.3103
42.5000	-44.0000	-0.5421	-0.9449	0.4029
41.0000	-51.0000	-0.1074	-0.2303	0.1230
38.0000	-55.0000	0.6022	0.6296	-0.0273
43.0000	-53.0000	0.0507	0.0125	0.0383
42.0000	-55.0000	0.1637	0.4818	-0.3179
57.0000	-46.0000	0.2239	-0.5634	0.7875
57.5000	-51.0000	-0.2924	0.2331	-0.5254
34.0000	-58.0000	1.6959	1.4533	0.2427

---- 506879 FEET * SAMPLE LOCALITY

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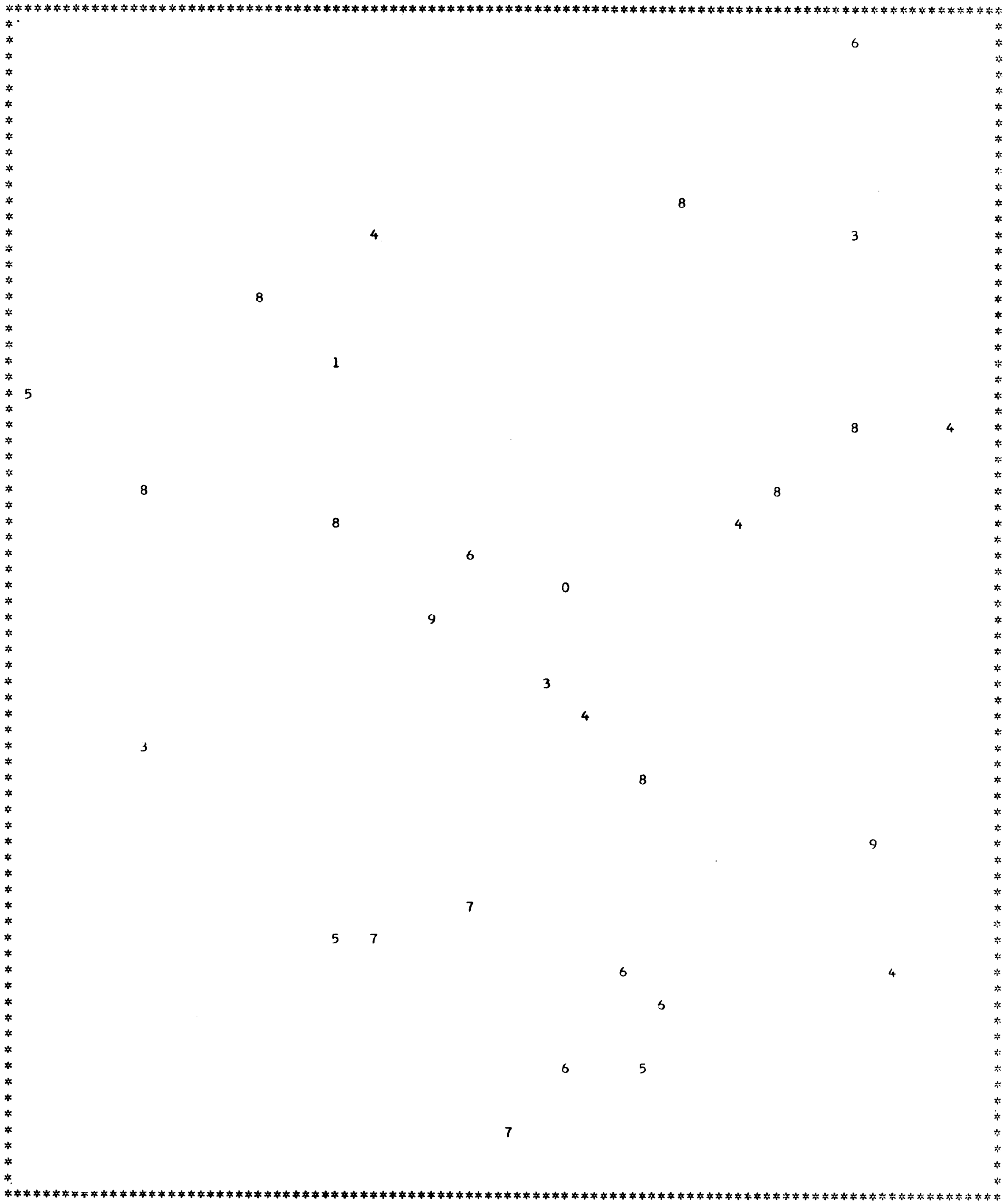
* 0 1 2 3 4 5 5 6 6 7 7 7 8 8 8 8 8 8 8 8 8 8 8 8 8 7 7 7 * 7 7 6 * 6 6 6 6 6 6 6 6 6 6 6 7 7 7 8 8 9 *
* 0 1 2 3 3 4 5 5 6 7 7 7 8 8 8 8 8 8 8 8 8 8 8 8 8 8 8 7 7 7 7 7 7 7 7 6 6 6 6 6 6 7 7 7 7 8 8 8 9 9 *
* 0 0 1 2 3 4 5 5 6 6 7 7 8 8 8 8 8 8 9 9 9 8 8 8 8 * 8 8 8 8 8 7 7 7 7 7 7 7 7 7 7 7 8 8 8 8 9 9 9 9 *
* 0 0 1 2 3 3 4 5 6 6 7 7 8 8 8 8 9 9 9 9 9 9 9 9 9 9 9 8 8 8 8 8 8 8 8 8 8 8 8 8 8 8 8 9 9 9 9 9 9 9 *
```

LEGEND

0 =	-2.506	5 =	-0.301
1 =	-2.065	6 =	0.140
2 =	-1.624	7 =	0.581
3 =	-1.183	8 =	1.022
4 =	-0.742	9 =	1.463

THE RESIDUAL MAP



LEGEND

0 =	-1.304	5 =	-0.107
1 =	-1.065	6 =	0.132
2 =	-0.825	7 =	0.372
3 =	-0.586	8 =	0.611
4 =	-0.347	9 =	0.850

TREND SURFACE FOR THE CANONICAL ROOT 0.7282

THE EQUATION OF THE TREND SURFACE

SAND	0.087
SHALE	0.737
CARBONATE	-0.093
EVAPORITE	-0.663

X	Y	X2	XY	Y2	X3	X2Y	XY2	Y3
0.5857	-0.8101	-0.0120	0.0220	-0.0058	0.0002	0.0001	0.0004	0.0001

NOTE X4Y2=X**4*Y**2,X3=X**3,AND SO ON

THE CONSTANT OF THE TREND SURFACE EQUATION IS EQUAL TO 0.0000

X-COORDINATE	Y-COORDINATE	OBSERVED VALUE	CALCULATED VALUE	RESIDUAL
18.5000	-26.0000	-0.6466	-0.3713	-0.2753
23.5000	-28.5000	-0.1656	-0.3185	0.1529
26.0000	-23.0000	-0.6146	-0.6986	0.0841
45.0000	-22.0000	-0.7220	-0.6460	-0.0760
55.0000	-23.0000	-0.3179	0.1317	-0.4496
55.5000	-14.0000	0.8461	0.6746	0.1715
2.0000	-29.5000	-0.5064	-0.4545	-0.0520
11.5000	-33.0000	0.3179	0.0321	0.2858
31.0000	-35.5000	-0.6587	-0.2108	-0.4479
23.0000	-34.0000	0.4360	-0.0525	0.4885
36.0000	-40.0000	0.0642	-0.1698	0.2341
29.0000	-38.0000	-0.8221	-0.0818	-0.7402
37.0000	-36.5000	-0.6552	-0.2978	-0.3575
38.5000	-42.0000	1.0063	-0.1361	1.1424
48.0000	-34.5000	-0.2810	-0.2781	-0.0030
51.0000	-33.0000	0.1535	-0.1789	0.3324
55.5000	-31.0000	0.4225	0.1336	0.2890
62.0000	-30.0000	0.9078	0.9758	-0.0680
11.5000	-43.0000	-0.1094	0.0236	-0.1330
22.5000	-49.5000	-0.0310	-0.0656	0.0346
26.0000	-50.0000	-0.1913	-0.0558	-0.1354
31.0000	-48.5000	0.0596	-0.0132	0.0729
42.5000	-44.0000	-0.0038	-0.0632	0.0594
41.0000	-51.0000	-0.3755	0.0794	-0.4549
38.0000	-55.0000	0.5290	0.0069	0.5221
43.0000	-53.0000	-0.5515	0.1703	-0.7219
42.0000	-55.0000	0.0926	0.1523	-0.0596
57.0000	-46.0000	0.3426	0.7942	-0.4516
57.5000	-51.0000	1.5506	1.1355	0.4151
34.0000	-58.0000	-0.0762	-0.2171	0.1409

---- 506879 FEET * SAMPLE LOCALITY

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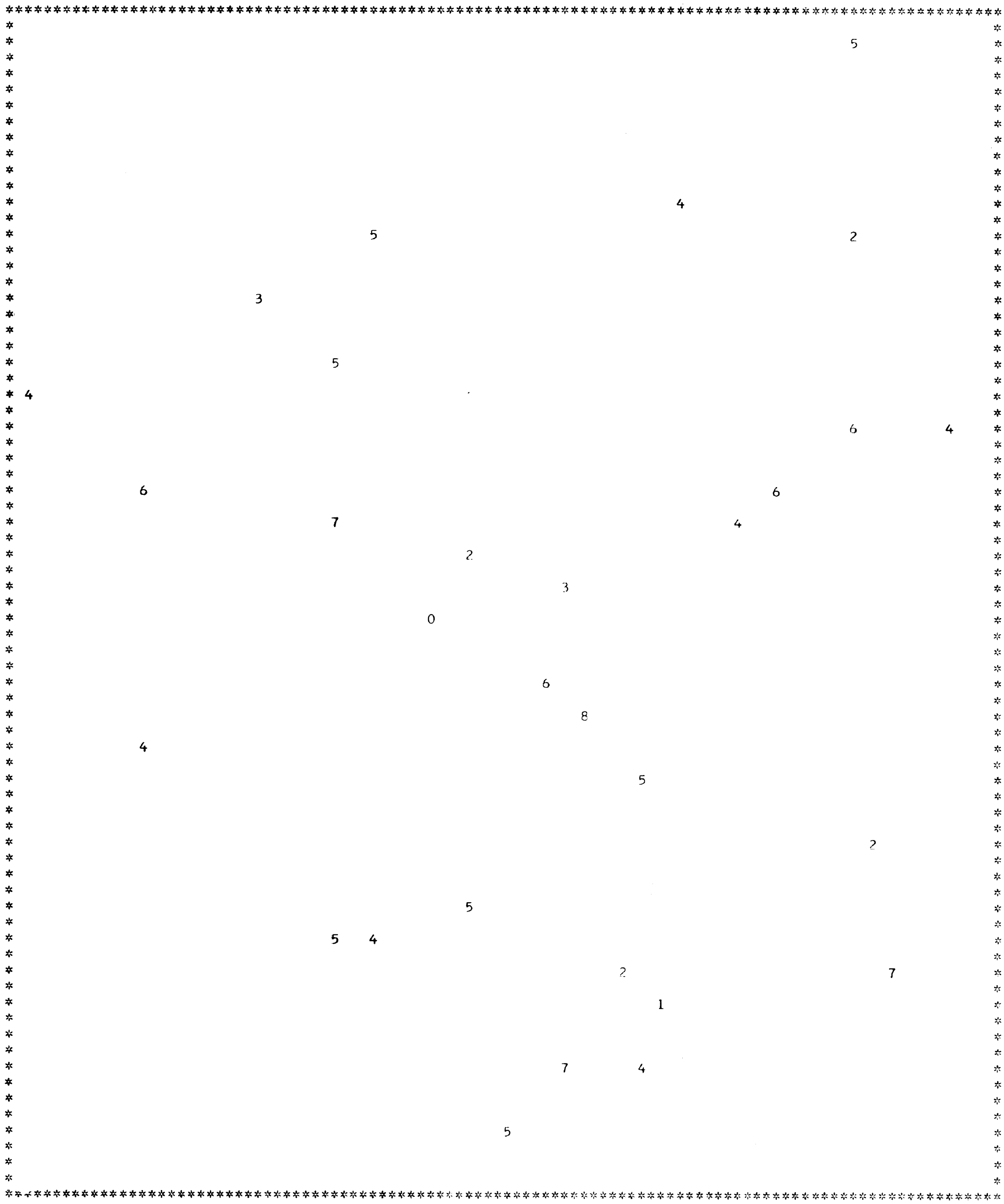
LEGEND

0 =	-0.699	5 =	0.218
1 =	-0.515	6 =	0.402
2 =	-0.332	7 =	0.585
3 =	-0.148	8 =	0.769
4 =	0.035	9 =	0.952

THE RESIDUAL MAP



LEGEND

0	=	-0.740	5	=	0.201
1	=	-0.552	6	=	0.389
2	=	-0.364	7	=	0.578
3	=	-0.175	8	=	0.766
4	=	0.013	9	=	0.954

TREND SURFACE FOR THE CANONICAL ROOT 0.5201

THE EQUATION OF THE TREND SURFACE

SAND	-0.630
SHALE	0.139
CARBONATE	0.762
EVAPORITE	0.057

X	Y	X2	XY	Y2	X3	X2Y	XY2	Y3
0.6705	-0.7415	-0.0124	0.0074	-0.0164	0.0001	-0.0001	-0.0001	-0.0002

NOTE X4Y2=X**4*Y**2,X3=X**3,AND SO ON

THE CONSTANT OF THE TREND SURFACE EQUATION IS EQUAL TO 0.0000

X-COORDINATE	Y-COORDINATE	OBSERVED VALUE	CALCULATED VALUE	RESIDUAL
18.5000	-26.0000	-1.2328	-0.2893	-0.9434
23.5000	-28.5000	-0.3318	-0.0378	-0.2940
26.0000	-23.0000	0.6634	-0.0629	0.7263
45.0000	-22.0000	-0.5172	0.2222	-0.7394
55.0000	-23.0000	-0.1860	0.3255	-0.5115
55.5000	-14.0000	0.5458	0.0780	0.4678
2.0000	-29.5000	-1.1296	-1.4474	0.3178
11.5000	-33.0000	-0.7343	-0.3789	-0.3554
31.0000	-35.5000	-0.0825	0.0575	-0.1400
23.0000	-34.0000	1.0335	0.0287	1.0048
36.0000	-40.0000	0.6789	-0.0443	0.7233
29.0000	-38.0000	0.5811	0.0690	0.5121
37.0000	-36.5000	-0.2589	-0.0080	-0.2509
38.5000	-42.0000	0.1443	-0.1176	0.2619
48.0000	-34.5000	0.7665	-0.0287	0.7952
51.0000	-33.0000	0.2631	0.0354	0.2277
55.5000	-31.0000	-0.3830	0.1606	-0.5435
62.0000	-30.0000	0.4344	0.3629	0.0715
11.5000	-43.0000	0.0267	0.2790	-0.2523
22.5000	-49.5000	0.7307	0.6357	0.0950
26.0000	-50.0000	0.6843	0.5205	0.1638
31.0000	-48.5000	-0.9140	0.2142	-1.1282
42.5000	-44.0000	-0.5064	-0.2343	-0.2722
41.0000	-51.0000	-0.7378	-0.1446	-0.5932
38.0000	-55.0000	-1.2004	0.1929	-1.3932
43.0000	-53.0000	0.6405	-0.1678	0.8083
42.0000	-55.0000	-0.4269	-0.0294	-0.3975
57.0000	-46.0000	0.0217	-0.4124	0.4341
57.5000	-51.0000	-0.6515	-0.4988	-0.1527
34.0000	-58.0000	2.0783	0.7207	1.3576

---- 506879 FEET   * SAMPLE LOCALITY

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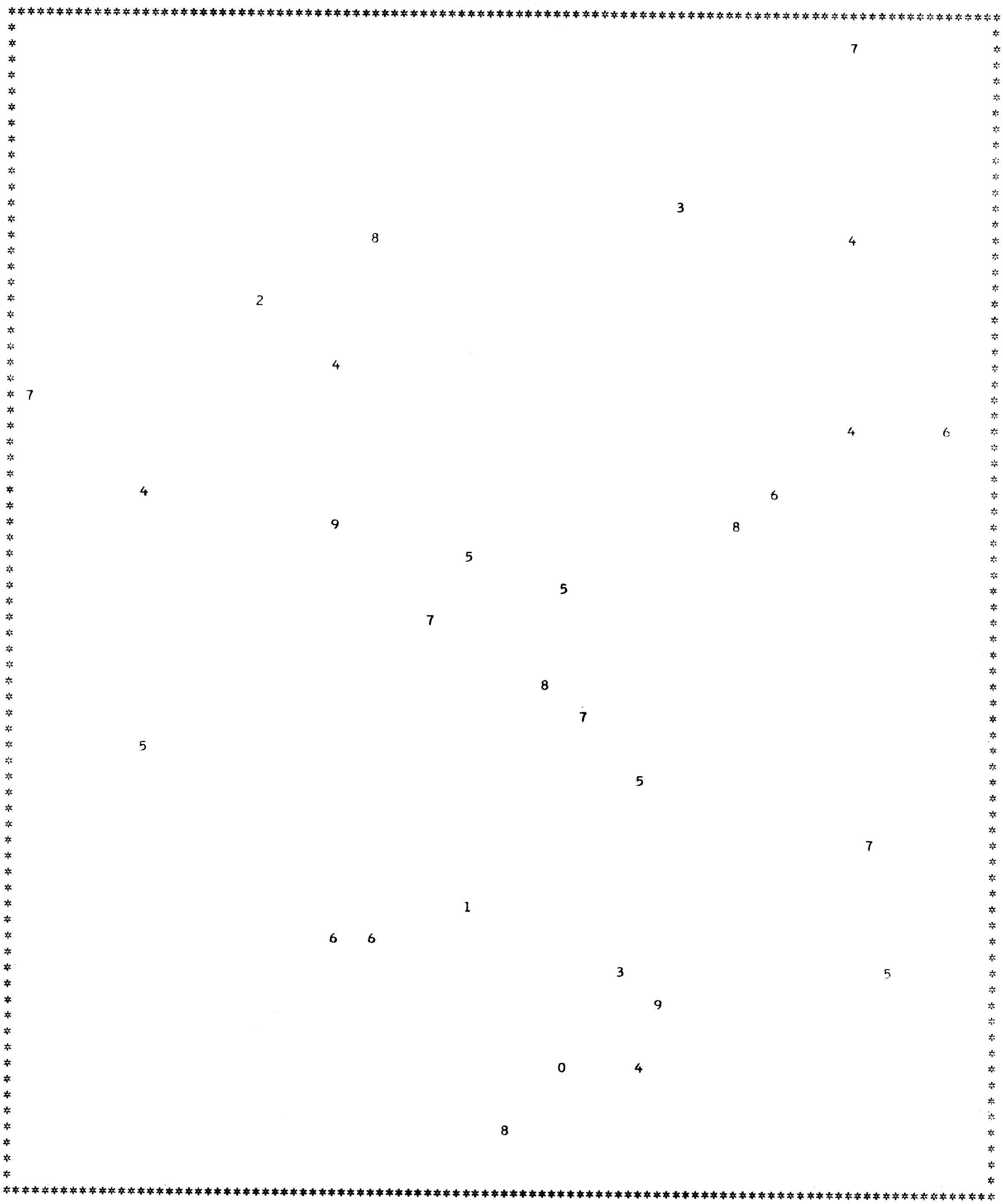
LEGEND

0 = -1.447	5 = -0.363
1 = -1.231	6 = -0.147
2 = -1.014	7 = 0.070
3 = -0.797	8 = 0.287
4 = -0.580	9 = 0.504

THE RESIDUAL MAP



LEGEND

0 = -1.393	5 = -0.018
1 = -1.118	6 = 0.257
2 = -0.843	7 = 0.532
3 = -0.568	8 = 0.807
4 = -0.293	9 = 1.082

KANSAS GEOLOGICAL SURVEY COMPUTER PROGRAM
THE UNIVERSITY OF KANSAS, LAWRENCE

PROGRAM ABSTRACT

Title (If subroutine state in title): Canonical correlation program

Date: August 1966

Author, organization: P.J. Lee, Freshwater Institute, Winnipeg 19, Canada

Direct inquiries to: Dr. D.F. Merriam

Name:

Address: Kansas Geological Survey
Lawrence, Kansas 66044

Purpose/description: Computes canonical correlation coefficient and canonical variates between two sets of variables. Bartlett's statistic is used to test the statistical significance of the canonical roots.

Mathematical method:

Restrictions, range: Accepts up to 99999 samples having up to 30 variables in each set.

Computer manufacturer: IBM

Model: System/ 360 Model 65

Programming language: FORTRAN IV (G)

Memory required: 65 K bytes K

Approximate running time: Less than 2 minutes for the given example

Special peripheral equipment required: None

Remarks (special compilers or operating systems, required word lengths, number of successful runs, other machine versions, additional information useful for operation or modification of program)

KANSAS GEOLOGICAL SURVEY COMPUTER PROGRAM
THE UNIVERSITY OF KANSAS, LAWRENCE

PROGRAM ABSTRACT

Title (If subroutine state in title):

Canonical trend-surface program

Date: July 1966

Author, organization: P.J. Lee, Freshwater Institute, Winnipeg 19, Canada

Direct inquiries to: Dr. D.F. Merriam

Name:

Address: Kansas Geological Survey
Lawrence, Kansas 66044

Purpose/description: Computes canonical trend surface for a set of variables over an area with irregular sampling grid. Trend map and residual map are printed by printer.

Mathematical method:

Restrictions, range: Accepts up to 300 samples having up to 15 geologic variables, polynomial order up to 6.

Computer manufacturer: IBM

Model: System/360 Model 65

Programming language: FORTRAN IV(G)

Memory required: 101 K bytes K

Approximate running time: About 2 minutes for the given example

Special peripheral equipment required: None

Remarks (special compilers or operating systems, required word lengths, number of successful runs, other machine versions, additional information useful for operation or modification of program)

COMPUTER CONTRIBUTIONS

1. Mathematical simulation of marine sedimentation with IBM 7090/7094 computers, by J.W.Harbaugh, 1966.
2. A generalized two-dimensional regression procedure, by J.R.Dempsey, 1966.
3. FORTRAN IV and MAP program for computation and plotting of trend surfaces for degrees 1 through 6, by Mont O'Leary, R.H.Lippert, and O.T.Spitz, 1966.
4. FORTRAN II program for multivariate discriminant analysis using an IBM 1620 computer, by J.C.Davis and R.J.Sampson, 1966.
5. FORTRAN IV program using double Fourier series for surface fitting of irregularly spaced data, by W.R. James, 1966.
6. FORTRAN IV program for estimation of cladistic relationships using the IBM 7040, by R.L.Bartcher, 1966.
7. Computer applications in the earth sciences: Colloquium on classification procedures, edited by D.F. Merriam, 1966.
8. Prediction of the performance of a solution gas drive reservoir by Muskat's Equation, by Apolonio Baca, 1967.
9. FORTRAN IV program for mathematical simulation of marine sedimentation with IBM 7040 or 7094 computers, by J.W.Harbaugh and W.J.Wahlstedt, 1967.
10. Three-dimensional response surface program in FORTRAN II for the IBM 1620 computer, by R.J.Sampson and J.C.Davis, 1967.
11. FORTRAN IV program for vector trend analyses of directional data, by W.T.Fox, 1967.
12. Computer applications in the earth sciences: Colloquium on trend analysis, edited by D.F.Merriam and N.C.Cocke, 1967.
13. FORTRAN IV computer programs for Markov chain experiments in geology, by W.C.Krumbein, 1967.
14. FORTRAN IV programs to determine surface roughness in topography for the CDC 3400 computer, by R.D. Hobson, 1967.
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