DANIEL F. MERRIAM, Editor

FORTRAN IV COMPUTER PROGRAM FOR SIMULATION OF TRANSGRESSION AND REGRESSION WITH CONTINUOUS-TIME MARKOV MODELS

By

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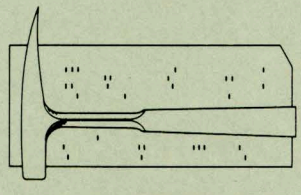

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Editor's Remarks

We are very pleased and proud to announce that the American Association of Petroleum Geologists is now helping to support our effort in computer applications in the earth sciences. We look forward to long and continued cooperation between our two organizations. Partial sponsorship of publication of the COMPUTER CONTRIBUTIONS by the largest geological organization in the world indicates the growing importance of computer applications in petroleum exploration and exploitation.

More than 50,000 computers are presently in use and organizations in the mineral sciences are major consumers of these services. Earth scientists have been using programs adapted from other disciplines, especially in areas of correlation, classification, and trend analysis. The Kansas Geological Survey, however, has been publishing computer programs in a regularly numbered series for about three years. Joint sponsorship of the series now insures better distribution and availability to earth scientists everywhere.

The high editorial standards of the series will be maintained, and active, associate, and junior members of the Association who are on the editorial board are indicated. In publishing this series, we are working closely with the Editor and Managing Editor of the Association to improve our effort. Through the years we have found the series extremely useful and valuable to earth scientists and in particular to those engaged in the petroleum industry.

With cooperation of the American Association of Petroleum Geologists, we now will be able to make the series available to more individuals and organizations interested in this important field. For those interested in back issues of the COMPUTER CONTRIBUTIONS, an up-to-date list can be obtained by writing the Editor, Computer Contributions, Kansas Geological Survey, The University of Kansas, Lawrence, Kansas 66044.

It is most appropriate that the first publication in this joint effort be one by William C. Krumbein, a pioneer in computer utilization. The program, "FORTRAN IV Computer Program for Simulation of Transgression and Regression with Continuous-time Markov Models" will be of interest to stratigraphers, sedimentologists, and structural geologists to help unravel some of their problems in sedimentary basin analysis. As stated in the text, this paper is "...concerned with a stochastic simulation model in which the pattern of lithologic succession is examined in terms of the length of time that the system remains in a given state, once it has entered that state. The model is also based on transgressive-regressive motion of a strandline, with the resulting lithologic units developing as responses to the movement of sedimentary environments laterally and through time."

For a limited time, the Kansas Geological Survey will make available a magnetic tape of the program BOREHOLE as described in this publication for $15.00. An extra charge of $10.00 is made for a punched deck of cards.

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*Active Member, °Associate Member, +Junior Member, American Association of Petroleum Geologists.

FORTRAN IV COMPUTER PROGRAM FOR SIMULATION OF TRANSGRESSION AND REGRESSION WITH CONTINUOUS-TIME MARKOV MODELS[1/]

by

W. C. Krumbein

ABSTRACT

Continuous-time Markov models based on lateral movement of shallow marine and nonmarine environments during transgression and regression through continuous time can be adapted to a variety of lateral-shift processes, such as progressive facies changes across a sedimentary basin, onlap-offlap relations among stratigraphic rock bodies, etc. In this report a clastic-wedge model is used to generate equally spaced borehole or outcrop sections that show the type of deposits formed as the environments shift back and forth.

The states of the transgressive-regressive system can be defined in terms of the position of the strandline through time, with the accumulating sediments representing responses to particular environments as they successively occupy a given monitoring station. Simulations are performed with a transition-rate matrix instead of the conventional transition-probability matrix, although the latter can also be adapted to lateral-shift processes. These two kinds of matrices can be transformed from one to another, and a computer program for this purpose is listed in this report. The main simulation program is illustrated with an experimental example, and the body of the text discusses some of the implications involved in structuring Markov models as continuous-time, discrete-state mechanisms.

INTRODUCTION

Simulation models for stratigraphic analysis can be structured in a variety of ways to produce sections ranging from completely deterministic sequences to completely random successions of beds. The states of the system can be defined as the lithologic components that make up the stratigraphic body, or they can be defined in terms of the underlying geological process that controls the types and successions of rock types present. Simulation models also can be classified according to their emphasis on the vertical arrangement of beds in a single section, or on lateral relations among adjacent vertical sections, where one or more marker beds are present. In addition to these different ways of defining states and focussing on vertical or lateral relations, simulation models can be structured so that changes of state are controlled by transition probabilities or transition rates.

This paper is concerned with a stochastic simulation model in which the pattern of lithologic succession is examined in terms of the length of time that the system remains in a given state, once it has entered that state. The model is also based on transgressive-regressive motion of a strandline, with the resulting lithologic units developing as responses to the movement of sedimentary environments laterally and through time. A preliminary report on the model used here was published as part of the Kansas Geological Survey's fourth Computer Colloquium (in Merriam and Cocke, 1968, p. 11-21). Some excerpts from the preliminary report are included later for completeness, in part with respect to the development of a stochastic simulation model from a deterministic counterpart.

There is a fairly close relationship between transition probabilities and transition rates, and some implications involved in selecting one or the other for simulation studies are developed. Two computer programs are listed in the Appendix, one for transforming transition probabilities into transition rates, and the other for simulating a transgressive-regressive process.

Computer programs listed in the Appendix are illustrated with a conceptual transgressive-regressive clastic-wedge model, in which transition rates are arbitrarily (though in part realistically) selected to develop sedimentary cycles of varying lateral extent, and with varying duration in time. This example is in the category of "geological experimentation" as advocated by Harbaugh (1966; see also Harbaugh and Merriam, 1968, Chapter 8). In these studies the experimental parameters may be varied to give insight into geological processes that are only partly understood, or into processes for which critical quantitative data may not yet be available.

The topic of using actual stratigraphic data for setting up transition-rate matrices, both for the lateral-shift model and for simulation of single stratigraphic sections, is developed later. Other topics, such as ways to define states, the use of transition probabilities instead of rates in lateral-shift models, and the application of rate matrices to simulation of single stratigraphic sections, also are discussed in the last

1/ This work was supported by the Geography Branch, Office of Naval Research, under Contract Nonr-1228(36), ONR Task No. 389-150.

part of this paper.

Acknowledgments.- I am much indebted to the Geography Branch of the Office of Naval Research for continuing financial support in conducting these studies, and for opportunities extended by Dr. John W. Tukey of Princeton University for me to participate in his Statistical Techniques Research Group during the spring of 1966. Paul Tukey, at that time an undergraduate at Princeton, prepared the basic BOREHOLE program, and devised the "clock mechanism" that allocates the appropriate number of output lines to each state on the basis of the random interval in continuous time that the state is occupied. Dr. L. L. Sloss of Northwestern University gave me the benefit of his wide stratigraphic experience as it relates to time and thickness in stratigraphic columns. Wolfgang Scherer, a graduate student at Northwestern, contributed much time and effort in taking care of computer program details as this manuscript was being written.

RELATIONS BETWEEN TRANSITION PROBABILITIES AND TRANSITION RATES

In a conventional first-order, discrete-state, discrete-time Markov chain, transitions occur at fixed discrete-time intervals $\Delta t = (t_r - t_{r-1})$. Thus a transition occurs at every tick of a conceptual "Markovian clock." The transition may be from a given state to itself, or from one state to another. The probability that the system will be in state j at time t_r, given that it was in state i at time t_{r-1}, is expressed as the transition probability p_{ij}. The diagonal probabilities, p_{ii}, control the within-state transitions, and the off-diagonal probabilities p_{ij} (where $j \neq i$), control the between-state probabilities.

The transition probabilities of the discrete-time model are not explicit functions of continuous time inasmuch as they are based on observations of state at discrete clock ticks. In a continuous-time Markov model, on the other hand, these probabilities may be expressed as $p_{ij}(t)$, which is the probability that the system will be in state j at time t, given that it was in state i at time zero. Coleman (1964, p. 129) points out that $p_{ij}(t)$ sums up all the paths by which the system may have come to state j from state i, including all the times between zero and t at which jumps were made. Thus, interest in a continuous-time model shifts from the probability of a state change, p_{ij} (where $j \neq i$), to the rate of transfer, q_{ij}, from state i to state j. It has been shown (Doob, 1953, p. 239) that the q_{ij}'s are the derivatives of the $p_{ij}(t)$'s with respect to t as t approaches zero.

The transition rate matrix, with elements q_{ij} (where $j \neq i$), can be set up directly if rates of sedimentation are known. However, it is commonly more convenient in practice (for initial investigations at least) to transform the discrete-time $[p_{ij}]$ matrix to its corresponding continuous-time $[q_{ij}]$ matrix. This may be done in several ways, and I am indebted to Dr. John Hartigan of the Statistics Department at Princeton University for the following transformation equations.

The probability p_{ii} that a system with k states is still in state i at time t is:

$$p_{ii} = e^{-\sum_{j \neq i}^{k} q_{ij} \Delta t} \qquad (1)$$

where

$\sum_{j \neq i}^{k} q_{ij}$ is the rate of leaving state i for some other state j.

The probabilities p_{ij} (where $i \neq j$) that the system is not still in state i at time t are:

$$p_{ij} = \frac{q_{ij}}{\sum_{j \neq i}^{k} q_{ij}} \left[1 - e^{-\sum_{j \neq i}^{k} q_{ij} \Delta t} \right] \qquad i \neq j. \qquad (2)$$

We note from equation (2) that the expression in parentheses on the right is $(1 - p_{ii})$ by substitution from equation (1), and this quantity is identically $\sum_{j \neq i}^{k} p_{ij}$, inasmuch as $p_{ii} + \sum_{j \neq i}^{k} p_{ij} = 1$. By substituting this last relation in equation (2), we have:

$$p_{ij} = \frac{q_{ij}}{\sum_{j \neq i}^{k} q_{ij}} \sum_{j \neq i}^{k} p_{ij} \qquad i \neq j, \qquad (3)$$

as well as its equivalent in terms of q_{ij}:

$$q_{ij} = \frac{p_{ij}}{\sum_{j \neq i}^{k}} \sum_{j \neq i}^{k} q_{ij} \qquad i \neq j. \qquad (4)$$

The quantity $\sum_{j \neq i}^{k} q_{ij}$ is the sum of the off-diagonal elements in the $[q_{ij}]$ matrix. We shall simplify the notation by calling this M_i, the rate at which the system leaves a given state i for some

other state j. Similarly, the quantity $\sum_{j \neq i}^{k} p_{ij}$ is the sum of the off-diagonal elements in the $[p_{ij}]$ matrix. We may call this quantity $p_i = (1 - p_{ii})$, the probability that the system leaves state i for another state j. With these notational changes, equations (1), (3), and (4) become simply:

$$p_{ii} = e^{-M_i \Delta t} \qquad (1a)$$

$$p_{ij} = \frac{q_{ij}}{M_i} p_i \qquad i \neq j \qquad (3a)$$

$$q_{ij} = \frac{p_{ij}}{p_i} M_i \qquad i \neq j. \qquad (4a)$$

In transforming a $[p_{ij}]$ matrix to its equivalent $[q_{ij}]$ matrix, the first step is to solve equation (1a) for M_i:

$$M_i = \frac{-\log_e p_{ii}}{\Delta t} . \qquad (5)$$

Application of these equations for transforming a $[p_{ij}]$ matrix to its equivalent $[q_{ij}]$ matrix is illustrated by the following example, where the states A, B, C, and D are sandstone, shale, siltstone, and lignite, respectively, in a vertical stratigraphic section. Table 1, top, is the 4 x 4 transition probability matrix based on observations of the state of the system at vertical intervals of five feet. The first step is to compute M_i (i = 1, 2, 3, 4) for each row. The problem here is to choose Δt. In the observations, vertical distance s was used instead of time t, so that in the transition-rate model the deposition is continuous, with state changes occurring at random thickness intervals rather than at random time intervals.

It would be desirable to have information on the relation between time and thickness in these models. How much time is required for deposition of five feet of sediment? The answer may be different for different lithologic components and for different sedimentary environments. Moreover, if compaction effects are included, perhaps the observations of state need to be spaced at different intervals for each lithologic component in the system, in order to equalize the time increments. As a practical compromise, with an arbitrary assumption that five feet of sediment require a fixed but unspecified unit of time, the Δt in equation (5) can be considered as 1.0. Thus, the implication in the corresponding rate matrix is that for each $\Delta s = \Delta t = 1.0$, five feet of sediment will accumulate. Although such a compromise may be "reasonable" at present, it indicates that considerably more thought needs to be given to ways of structuring stratigraphic observations in order to apply transition-rate matrices rigorously. Similar problems arise in developing matrices for lateral strandline movement, as will be seen.

During the remainder of this paper the word "time" will be used in discussing the transition-rate matrix with the understanding that the word "thickness" may be substituted for stratigraphic purposes where appropriate. Where the distinction is necessary, it will be brought out in the context.

For state A in Table 1, $M_1 = (-\log_e 0.62)/1.0 = 0.478$; for state B, $M_2 = 0.462$; for state C, $M_3 = 1.109$; and for state D, $M_4 = 1.967$. These are the total "rates" for leaving the designated states. They have dimensions (1/T), and may be interpreted as the reciprocal of the time (thickness of deposition) required to leave the state. Thus, the expectation of the system remaining in state A, after entering it, is 1/0.478 = 2.09 time units, equivalent in this example to 2.09 x 5 = 10.45 feet.

Once the M_i's are computed, the off-diagonal elements for each row are obtained by equation (4a). From the first row of Table 1, top, we have $q_{12} = (0.14/0.38)(0.478) = 0.176$; q_{13} also is 0.176; and $q_{14} = (0.10/0.38)(0.478) = 0.126$. Similar computations for the other three rows yield the $[q_{ij}]$ matrix of Table 1, bottom. The diagonals have been left blank, with the total rates of transfer, M_i's, entered in the row margins.

The elements $q_{ij} / \sum_{j \neq i}^{k} q_{ij}$ in equation (2) can be interpreted as conditional probabilities that a transition occurs out of state i to state j (Karlin, 1966, p. 228). The probabilistic element is thus preserved in the $[q_{ij}]$ matrix, in that the probabilities of moving from state A to state B, C, or D are proportional to their relative probabilities in the untransformed $[p_{ij}]$ matrix. Moreover, the probability that state A reverts directly to itself is zero in the $[q_{ij}]$ matrix, and it can only be re-entered by way of states B, C, or D.

The transformation of the p_{ij}'s (where $j \neq i$) to their corresponding q_{ij}'s has in effect eliminated the diagonal elements, inasmuch as interest has been shifted from transitions within a given state (i.e., to itself), to the time required for the system to leave the given state. Inasmuch as the probability for a given state to revert directly to itself is zero in the $[q_{ij}]$ matrix, a zero can be entered in each diagonal element of Table 1, bottom. This is appropriate for the present model, although for some types of Markov models with continuous time the negative of M_i is entered (Karlin, 1966, p. 225). For pre-

sent purposes zeros are placed in the main diagonal when the transition matrix is prepared as computer input.

Transformation of the $[q_{ij}]$ matrix to its corresponding $[p_{ij}]$ matrix can also be accomplished with equations (1a) to (4a). The first step is to compute p_{ii} for each row of the matrix in equation (1a). For the top row in the $[q_{ij}]$ matrix of Table 1 (where $\Delta t = 1.0$), this is $p_{ii} = e^{-0.478} = 0.619$, or 0.62 rounded off. The sum of the off-diagonal probabilities in the top row is $(1.00 - p_{11}) = 0.38$. From equation (3a), where $(1 - p_{ii}) = p_{i}$, the individual p_{ij}'s ($i \neq j$) are respectively the rate ratios (0.176/0.478), (0.176/0.478), (0.126/0.478), each multiplied by $p_{i} = 0.38$. This yields the values $p_{12} = 0.14$, $p_{13} = 0.14$, and $p_{14} = 0.10$, in agreement with the original $[p_{ij}]$ matrix at the top of Table 1.

Computations shown in the preceding example are used in computer program PEQUMAT, listed in the Appendix. Details for using the program and for choosing Δt also are included there.

Table 1.- Transformation of $[p_{ij}]$ to $[q_{ij}]$.

Transition Probability Matrix, with Δs = 5.0 feet

	A	B	C	D	$\sum_{j \neq i} p_{ij}$
A	0.62	0.14	0.14	0.10	0.38
B	0.06	0.63	0.20	0.11	0.37
C	0.19	0.31	0.33	0.17	0.67
D	0.21	0.48	0.17	0.14	0.86

Transition Rate Matrix from above, with Δt = 1.0

	A	B	C	D	M_i
A		.176	.176	.126	.478
B	.075		.250	.137	.462
C	.315	.512		.282	1.109
D	.480	1.098	.389		1.967

A-Sandstone, B-Shale, C-Siltstone, D-Lignite

The distribution of time intervals during which the system is in any given state i is the exponential distribution with parameter M_i (Karlin, 1966, p. 228). Thus, a typical realization starting in any state i involves taking an observation from the exponential distribution which determines the "waiting time" in state i (that is, the time required to leave state i after having once entered it). At the end of that time the system shifts to state j, with probability q_{ij}/M_i, where $j \neq i$. A second observation is then drawn from the exponential distribution having as parameter the total rate applicable to the new state. This in turn determines the random length of time that the system is in the second state, and so on through a succession of realizations.

In computer simulation it is convenient to use uniform (i.e., rectangularly distributed) random numbers, and to convert them to their exponential equivalents. This is done by choosing a random number U in the range 0.0000 to 1.000, and setting it equal to the integral from 0 to t_c of the exponential frequency distribution with parameter M_i for the state involved, and solving for t_c:

$$\int_0^{t_c} M_i e^{-M_i t} dt = U. \qquad (6)$$

Here t_c is the time required (the waiting time) for the system to leave state i for some state j, and the number U is a random area under the frequency curve, the total area of which is 1.000. This expression can be simplified to the following, where t_c is found simply by taking the natural log of the reciprocal of the random number, and dividing it by the rate M_i:

$$t_c = \frac{\log_e(1/U)}{M_i} \qquad (7)$$

An example from the bottom matrix of Table 1 illustrates the procedure. Suppose the system starts in state A. Draw a random number U. Say it is 0.6623. $\text{Log}_e(1/0.6623) = 0.4121$. This is divided by the marginal rate for state A, 0.478, to give 0.862 time units, or 5 x 0.862 = 4.31 feet of sandstone. The state to which the system shifts next is controlled by the conditional probability q_{ij}/M_i (where $j \neq i$), which for a shift to state B is (0.176/0.478) = 0.368, to state C is the same, and to state D is (0.126/0.478) = 0.264. A uniform random number in the range 0.001 to 1.000 is now selected. If it is in the range 0.001 to 0.368, the system shifts to state B; if it is in the range 0.369 to 0.736, the shift is to C; and if in the range 0.737 to 1.000 the

shift is to D. Thus the simulation procedure requires drawing two random numbers for each realization, the first to determine the random time interval during which a given state is occupied, and the second to determine which state succeeds the one just vacated.

Simulation program BOREHOLE in the Appendix uses this procedure, as described later. The material is included here to complete the mathematical portion of this paper.

EXPERIMENTAL LATERAL-SHIFT MODEL

We start with a completely deterministic model, and introduce stochastic elements into it. Some portions of the preliminary report (Krumbein, 1968) are included here with modifications that make the model somewhat more realistic. The model involves lateral migration of three environments that produce a band of littoral sand in the shore environment, with marine mud (shale) on its seaward side, and undifferentiated nonmarine deposits (lagoonal silts, lignitic shales, alluvial sands, etc.) on its landward side. Observation stations are placed at convenient lateral positions to monitor the forward and backward motion of the three environments.

In this model the width of the shore sand belt is assumed to remain fixed during the cycles. This is not extreme for a clastic wedge, inasmuch as differences in thickness between transgressive and regressive sands express themselves in the angle at which the strandline sand cuts the time lines. This angle is controlled by the rate of shoreline movement; small angles are associated with high transition rates and thin sand.

Figure 1, top, modified from the preliminary report (Krumbein, 1968) shows one way in which the states may be defined for a simple model in which the rate of shoreline movement is constant. The landward and seaward limits of the band of littoral sand are shown as successive segments at equally spaced time intervals, so placed that the position of the sand is unambiguously stated with respect to the three monitoring stations. These positions are boundaries between zones labeled A, B, C, and D from left to right. The state of the transgressive-regressive system is now defined as the position of the littoral sand within and between these zones. Thus, the first state (at the bottom of the diagram) is designated as AA because both sand limits lie in zone A. The second state is designated as AB because the sand limits straddle monitoring position I and hence the sand lies in both zones A and B. This procedure is followed through the cycle, with the regression states indicated as CD', CC', etc. This means that once state AA' is reached, the system moves to state AA for the next cycle.

For each state in the system there is a definite lithology associated with each monitoring position. In state AA all three stations receive nonmarine deposits. In state AB position I receives littoral sand, and nonmarine beds continue at stations II and III. In state BB marine shale occurs at position I, but the littoral sand has not yet reached position II, so that this station and station III still receive nonmarine deposits. Thus, in a simulation experiment three stratigraphic sections or boreholes can be generated as the cycle proceeds, and each station receives an unambiguous deposit for each state of the system.

The manner in which the states of the system are related to the monitoring stations can be expressed in various ways. In Figure 1, two intermediate states (BB and CC) represent conditions during which the strandline sand lies between monitoring stations. An alternative to this is also shown in Figure 1, bottom, in which the shoreline sand is always present at one of the monitoring stations, except at the extremes of the cycles. The designation of states, as shown at the left, is now reduced from 14 to 10, for the same number of stations as in Figure 1, top. In this arrangement the states are still unambiguously defined with respect to the deposits formed at each monitoring station. These alternative ways of setting up the model control the size of the transition matrix required to implement the model.

We may use Figure 1, top, as a basis for the initial model. In its deterministic form the model states that the rates of transgression and regression are equal, but field and subsurface observation suggests that in general transgression is much more rapid than regression. Thus the rates of leaving successive states in transgression can be chosen as some multiple of the rates in regression, say 4 to 1. The deterministic path of a cycle can be shown diagrammatically as in Figure 2, top. Here the arrows representing transgression rates are all 4.0, and the regression rates are all 1.0.

A stochastic element that permits reversals at any stage of the cycle can now be introduced in either or both of two ways. The first, illustrated in the preliminary report, assumed that although the total rate of moving from one state to the next is 4.0, the probability of a reversal to the preceding state is 0.25, whereas the probability of continuing in the deterministic path is 0.75. This distributes the total rate of moving out of a state into two portions, one of which is three times as large as the other. This is shown in the center of Figure 2. It is perhaps more realistic to indicate reversal by crossing from forward progress of the cycle to its counterpart in the backward movement of the strandline. This will automatically produce cycles having variable lateral extent. In this scheme the forward rate in transgression is 3.0, and the arrow with rate 1.0 crosses the diagram to its corresponding regressive state in the bottom diagram of Figure 2. Similarly, during regression the reversal changes over to its comparable state with rate 1/4, whereas the rate for continuing regression is 3/4. This also implies a probability of reversal of 0.25.

The diagrams in Figure 2 contain deterministic

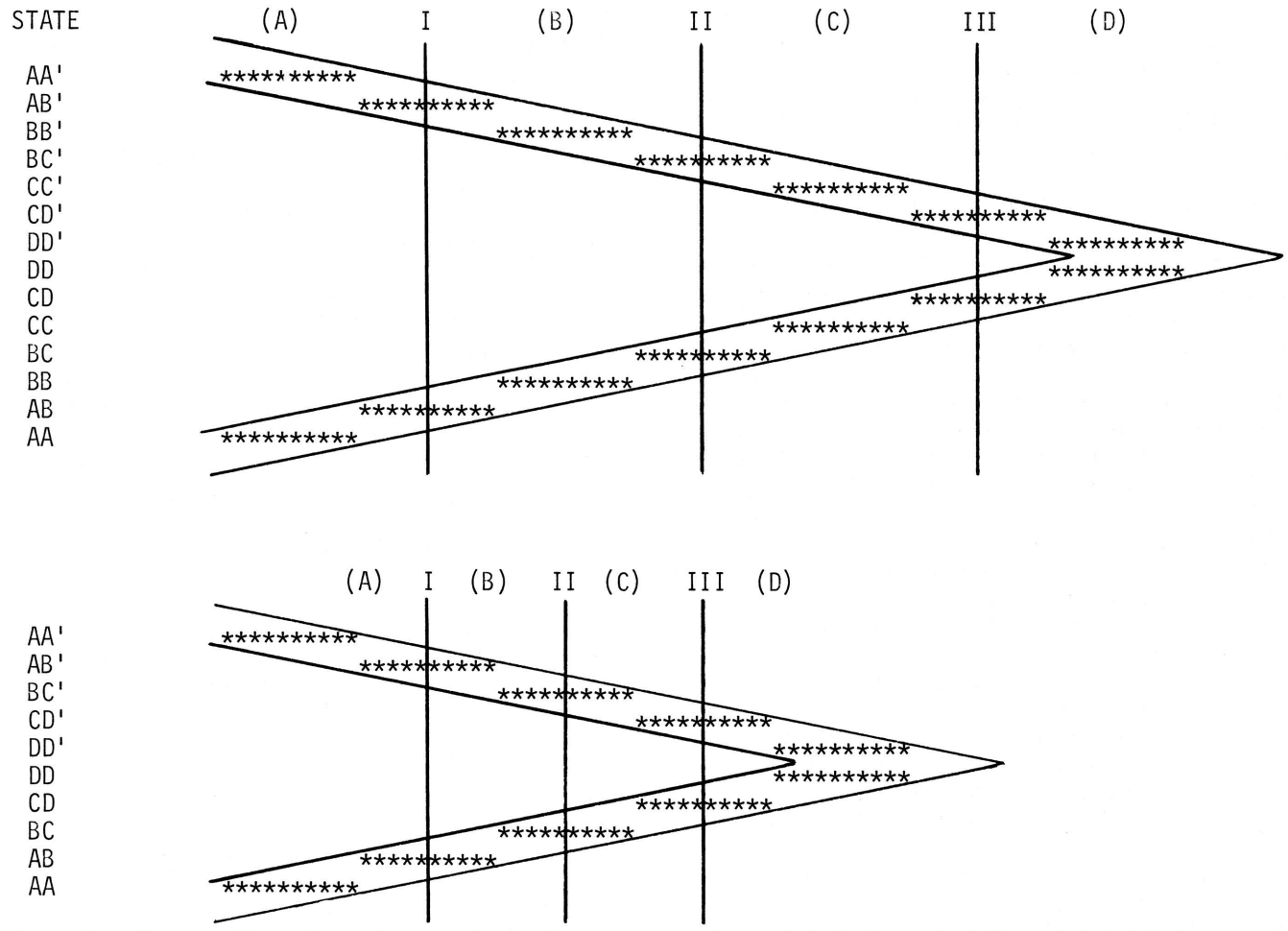


Figure 1.- Two versions of continuous-time, discrete-state deterministic lateral-shift models, showing successive positions of strandline sand in relation to three monitoring stations. See text for discussion.

elements at the extremes of the cycles. Thus, in the bottom diagram the path from state DD to DD' is fixed with a rate of 4.0 and the passage from DD' to CD' is also fixed with a transition rate of 1.0. In similar manner, at the start of the cycle the path from AA to AB is fixed at a rate of 4.0, and the last stage of regression from AA' to AA is fixed with a rate of 1.0. These are introduced to assure direct reversal when the extremes of the cycles are reached. There is, of course, much flexibility in the way the paths may be structured.

The transition rate matrices for the two lower diagrams in Figure 2 are given in Tables 2 and 3. For the simple reversal to the preceding state in the cycle, the individual q_{ij}'s move diagonally downward through the matrix on both sides of the main diagonal, as in Table 2. In the reversal pattern that switches from transgression to regression or vice versa, the individual q_{ij}'s form a cross pattern through the transition rate matrix in Table 3.

One advantage of a change from state CD to CD' rather than back to state CC, for example, is that successive simulations vary in their lateral extent from one cycle to the next, as mentioned. An even more interesting variant can be introduced by weighting the rates so that the occurrence of maximum lateral extent of transgression is less probable than cyclical movements of intermediate or short lateral extent. Thus, in the classical Mesa Verde-Mancos case there are relatively few sands that extend to the eastern end of the Book Cliffs; the bulk of the transgressive-regressive phases is clustered relatively closer to the Price area near the western edge of the Book Cliffs.

IMPLEMENTING THE SIMULATION MODEL

Conceptual clastic-wedge models that yield "reasonable" simulations of real-world cycles require a considerable amount of preliminary experimentation. The transition rates may be chosen

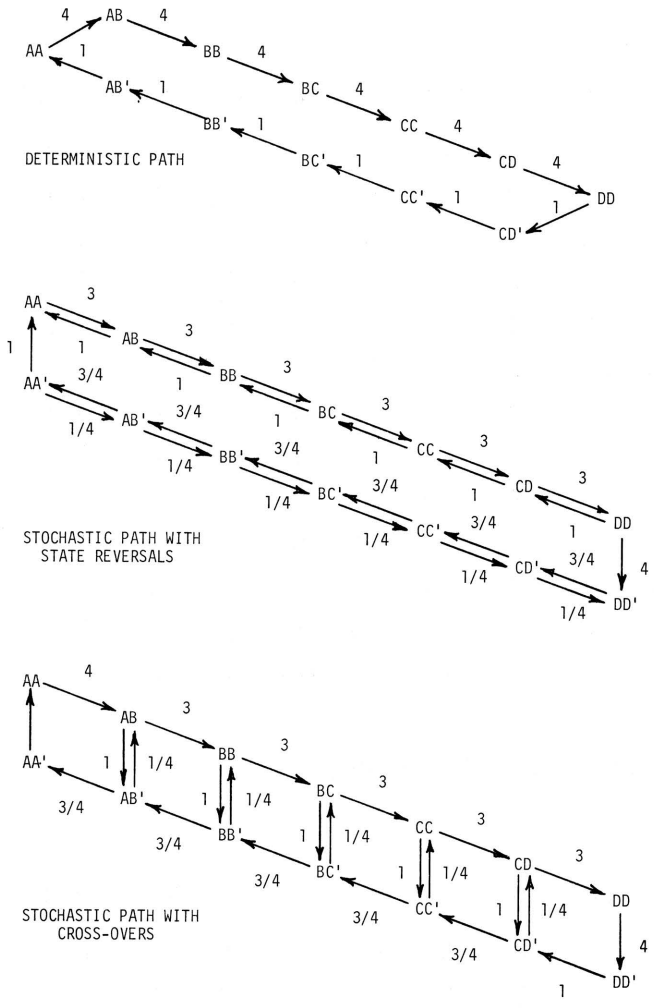


Figure 2.- Diagrammatic transgressive-regressive models. Top, completely deterministic scheme with transgression rates four times as great as regression rate. Center, simple pattern of state reversals superimposed on deterministic model. This adds a stochastic component by permitting reversals to the previous state with probability 1/3 that of progressive movement through the cycle. Bottom, alternative way of superimposing stochastic elements. In this diagram progression through the cycle permits a switch from transgression to regression and vice versa, thus generating subcycles of varying lateral extent within the major cycle.

artibrarily, or based on observed stratigraphic sections. We shall use the $[q_{ij}]$ matrix from Table 3 for the simulation experiment. As stated, the relative total rates in this matrix, and the conditional probabilities, based on the off-diagonal q_{ij}'s, have been arbitrarily chosen to establish different rates of transgression and regression, as well as different probabilities for continued progress or reversal.

The exponential time distribution of equation (6) may need to be truncated to avoid extreme durations of the system in any one state. If the range of the random numbers that control t_c in equation (7) is from 0.0000 to 1.0000, the time in any given state can range from zero to infinity. In practice the cost of machine time suggests setting limits that constrain the thicknesses of the transgressive and regressive sands. This can be illustrated by the example in Table 4. There the truncated limits to the random numbers are 0.0100 and 0.9900. The reciprocals of these are 100.0000 and 1.0101, and their natural logs are 4.6052 and 0.0099, respectively. These extremes are divided by the fastest and slowest rates (4.00 and 1.00) to obtain the minimum and maximum time intervals in any given state, which from Table 4 are 4.6052 and 0.0025 units. Hence, if one line of output is assigned to the shortest interval, it will require (4.6052/0.0025) = 184 lines (about three pages of output), if the random number happens to be 0.9900 for any state with rate 1.00.

The simulation program has a built-in procedure for specifying the time interval to be associated with each unit of sediment thickness (i.e., one line of output), with the condition that every state must be represented by at least one line of output. The interval of time to be associated with one line of output is controlled by the relation:

$$1 \text{ thickness unit} = C_1 \frac{\log_e(1/U_{max})}{M_{max}} \text{ time units} \qquad (8)$$

where U_{max} is the upper limit of the range of random numbers (0.9900 in the previous example), and M_{max} is the maximum marginal rate for any state (4.00 here). The constant C_1 is a conversion factor which can be specified on the control card.

Although the time interval required to deposit unit thickness of a given type of sediment in a given type of environment is in general not quantitatively known, the relation in equation (8) can be used as a convenient control for maximum thicknesses in experimental simulations. This control is the same for each lithologic type, which implies the assumption that the depositional site is subject to a uniform rate of subsidence, with deposition controlled by some prevailing base level. Under this assumption the stratigraphic sections developed as output could be thought of as "equal time sections." If these in turn are "corrected" for relative compaction of the different lithologies, the "time section" can be converted to its corresponding "thickness section."

The manner in which the program allocates an appropriate number of output lines for the time interval associated with each state deserves comment here. In the particular simulation example given in the Appendix, the exponential time distribution was arbitrarily truncated at values of 0.3000 and 0.8000. By the method illustrated in Table 4, the limiting time intervals in any given state are 0.0558 and

		Succeeding State, j														
		AA	AB	BB	BC	CC	CD	DD	DD'	CD'	CC'	BC'	BB'	AB'	AA'	MARGIN
Given State, i	AA	X	4.0	0	0	0	0	0	0	0	0	0	0	0	0	4.0
	AB	1.0	X	3.0	0	0	0	0	0	0	0	0	0	0	0	4.0
	BB	0	1.0	X	3.0	0	0	0	0	0	0	0	0	0	0	4.0
	BC	0	0	1.0	X	3.0	0	0	0	0	0	0	0	0	0	4.0
	CC	0	0	0	1.0	X	3.0	0	0	0	0	0	0	0	0	4.0
	CD	0	0	0	0	1.0	X	3.0	0	0	0	0	0	0	0	4.0
	DD	0	0	0	0	0	0	X	4.0	0	0	0	0	0	0	4.0
	DD'	0	0	0	0	0	0	0	X	1.0	0	0	0	0	0	1.0
	CD'	0	0	0	0	0	0	0	1/4	X	3/4	0	0	0	0	1.0
	CC'	0	0	0	0	0	0	0	0	1/4	X	3/4	0	0	0	1.0
	BC'	0	0	0	0	0	0	0	0	0	1/4	X	3/4	0	0	1.0
	BB'	0	0	0	0	0	0	0	0	0	0	1/4	X	3/4	0	1.0
	AB'	0	0	0	0	0	0	0	0	0	0	0	1/4	X	3/4	1.0
	AA'	1.0	0	0	0	0	0	0	0	0	0	0	0	0	X	1.0

Table 2.- Transition-rate matrix for transgressive-regressive model with reversals, I.

		Succeeding State, j														
		AA	AB	BB	BC	CC	CD	DD	DD'	CD'	CC'	BC'	BB'	AB'	AA'	MARGIN
Given State, i	AA	X	4.0	0	0	0	0	0	0	0	0	0	0	0	0	4.00
	AB	0	X	3.0	0	0	0	0	0	0	0	0	0	1.0	0	4.00
	BB	0	0	X	3.0	0	0	0	0	0	0	0	1.0	0	0	4.00
	BC	0	0	0	X	3.0	0	0	0	0	0	1.0	0	0	0	4.00
	CC	0	0	0	0	X	3.0	0	0	0	1.0	0	0	0	0	4.00
	CD	0	0	0	0	0	X	3.0	0	1.0	0	0	0	0	0	4.00
	DD	0	0	0	0	0	0	X	4.0	0	0	0	0	0	0	4.00
	DD'	0	0	0	0	0	0	0	X	1.0	0	0	0	0	0	1.00
	CD'	0	0	0	0	0	1/4	0	0	X	3/4	0	0	0	0	1.00
	CC'	0	0	0	0	1/4	0	0	0	0	X	3/4	0	0	0	1.00
	BC'	0	0	0	1/4	0	0	0	0	0	0	X	3/4	0	0	1.00
	BB'	0	0	1/4	0	0	0	0	0	0	0	0	X	3/4	0	1.00
	AB'	0	1/4	0	0	0	0	0	0	0	0	0	0	X	3/4	1.00
	AA'	1.0	0	0	0	0	0	0	0	0	0	0	0	0	X	1.00

Table 3.- Transition-rate matrix for transgressive-regressive model with reversals, II.

1.2030 (arbitrary) units. If one line of output is allocated to the shortest time interval, there are (1.2030/0.0558) = 21.56 output lines for the longest interval. This is not particularly an extreme ratio, and from equation (8) C_1 comes out as 1.00. If it were decided to limit the thickest sand to ten output lines, C_1 would be found by substituting (1.2030/10) = 0.1203 in the left of equation (8), as representing the thickness accumulated during one time unit. This is equated to C_1 times the

Extreme Random Number, U	$\text{Log}_e(1/U)$	Waiting time for lowest rate (1.00)	Waiting time for highest rate (4.00)
0.0100	4.6052	4.6052	1.1513
0.9900	0.0099	0.0099	0.0025

Table 4.- Maximum and minimum "waiting times" for changes of state.

minimum time interval, 0.0558 units. This gives $0.1203 = 0.0558\ C_1$, which yields the value 2.16 for the constant.

When C_1 is greater than 1.0, there will be transitions that occur at intervals smaller than the time interval allotted to each line of output. The program is so arranged that it assigns a minimum of one line of printout for each state occupied, in order to provide a complete output record of all state transitions.

The decision to choose C_1 at some value greater than 1.0 is normally based on the amount of printout anticipated from a given simulation. These vary rather widely, depending in part on the interplay of the forward and backward motions of the strand as displayed in the bottom diagram of Figure 2. A comparatively short simulation from program BOREHOLE is shown in Figure 3. This is the graphic output of an experiment described in the Appendix in connection with Tables A-7 and A-8. The figure is introduced here to provide a text example that shows some features of such simulations. At the left are the times on the program's internal continuous "clock mechanism" that notes when a given state was entered, and the length of time (as represented by the corresponding number of printout lines) that the system remains in that state. The state symbols are also listed; in this case the symbol $\neq$ indicates regression states, and the arrow at the leading edge of the belt of strandline sand indicates the direction in which the strand is moving. The dashed lines represent marine shale and the slashes are undifferentiated nonmarine deposits.

The simulation represented in Figure 3 is based on the model shown in the bottom diagram of Figure 2, using the $[q_{ij}]$ matrix of Table 3, with $C_1 = 1.00$. The complete cycle, from AA through DD and back to AA$\neq$ required an interval of 10.16 time units, with one line of output representing 0.06 time unit. The simulation shows a short subcycle from AA to BB and back, terminating at time 1.74. Then follows a larger subcycle which proceeds to state DD at time 3.35, followed by regression with minor switches, such as at time 4.41, when there is a momentary change to transgressive motion, followed again by regression at time 4.53. Although the strandline sand "piles up" in some states, this means that duration in that state is longer than for states with only one or a few lines of printout. The distance between the three boreholes (edged by blank spaces) is considered to be equal, but the actual spacing in the output is controlled by the width of the printout sheet. It provides an interesting study to cut out the logs from a simulation, repaste them at a greater distance from each other, and draw "correlation lines" on the strandline sand. This illustrates graphically the difference between a lateral-shift model and a series of independent simulations of vertical stratigraphic columns, such as are obtained from conventional $[p_{ij}]$ matrices based on transitions along a single vertical section.

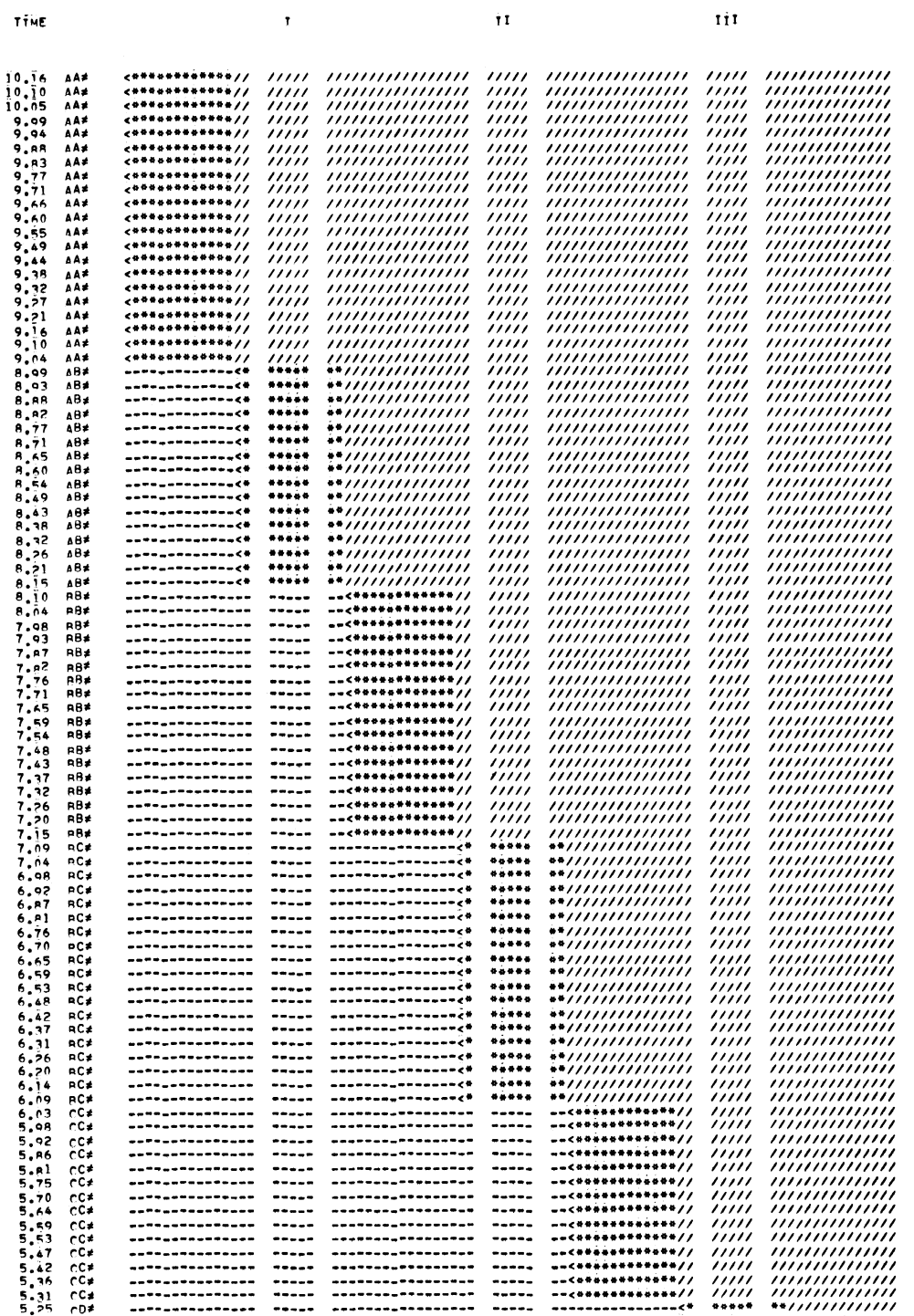
CONCLUDING REMARKS

The reader interested primarily in the program and its operation may refer directly to the Appendix at this point. This section of the text is designed to bring up several points that are involved in designing simulation models in a process-response framework, the difficulties involved in structuring real-world data for either lateral or vertical continuous-time models, choices between transition probabilities or transition rates, and developing relations between thickness and time.

Process and Response in Simulation Models.- If the states of a stratigraphic system are defined as its lithologic components, the model is essentially a response model. That is, the transition probabilities are expressed in terms of the rock succession itself, rather than directly from considerations of the inferred process that produced the stratigraphic sequence. The transition probability matrix indirectly reflects the process by the relative magnitude of its probabilities, which act as the "driving forces" on the simulations. Each simulation of a stratigraphic section from a transition probability matrix is an independent event, however, in that successive simulations may display widely different arrangements of individual lithologies and thicknesses. This is unlike a lateral succession of outcrops or boreholes in the real world, where marker beds occur that permit direct stratigraphic correlation.

Despite these limitations, the transition probability matrix in simple response models is a useful descriptor of the stratigraphic succession on which it is based. This aspect of stratigraphic analysis deserves detailed study for comparisons among different types of cyclic deposits, for estimation of the degree of "variety" (statistical entropy) inherent in the section, and as a basis for identifying the underlying process elements.

One advantage of stratigraphic models that define the system in terms of underlying processes is that lateral continuity can be readily built into the model. Thus, if the model is set up in terms of sedimentary environments that temporally succeed each other at a given geographic point as the system shifts to and fro laterally, the types and thicknesses of deposits formed become responses to a specified process model. Where such models can be set up con-


TIME
I
II
III

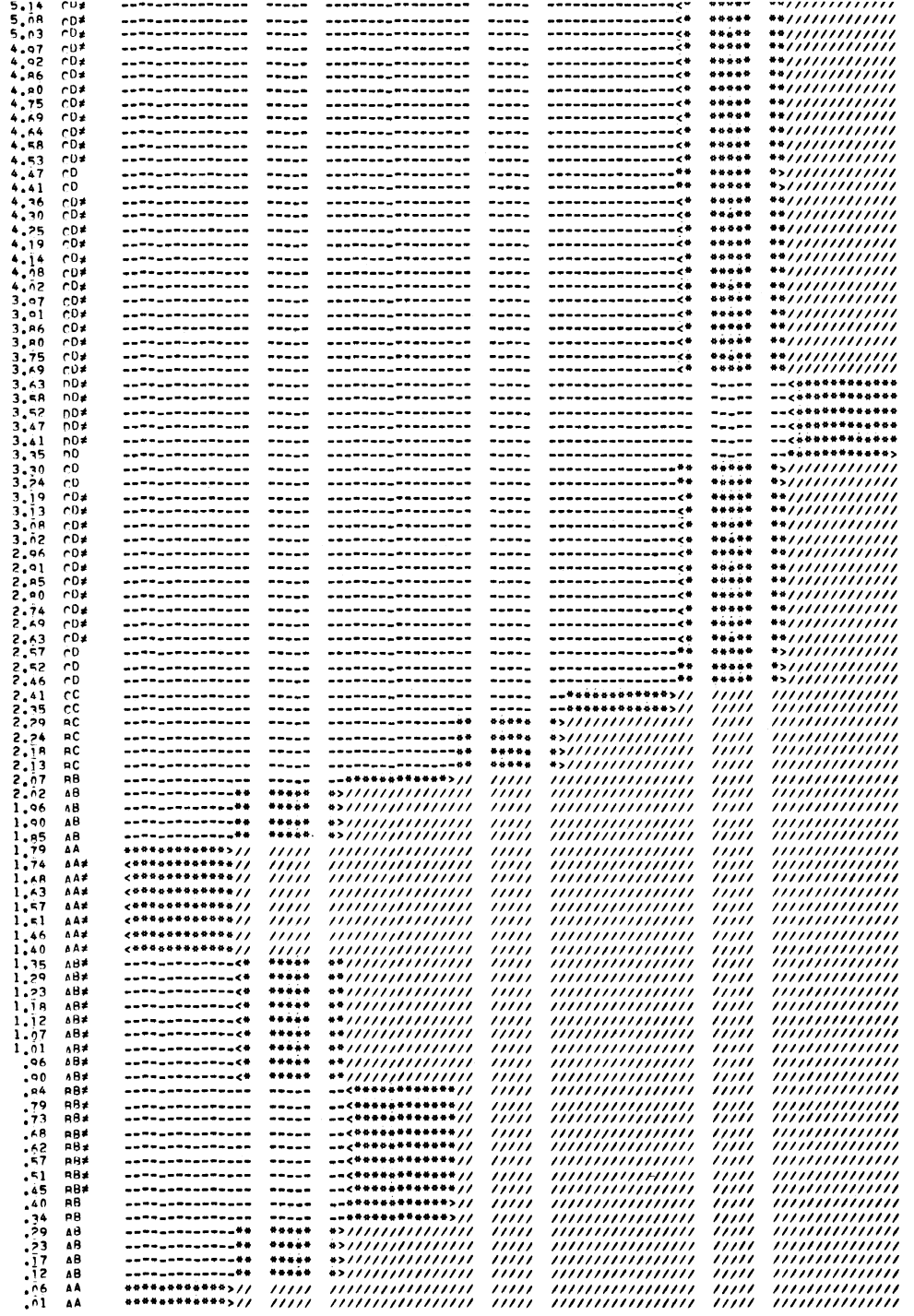


Figure 3.- One complete transgressive-regressive cycle from program BOREHOLE, using the transition-rate matrix of Table 3. The partial subcycle from time 0.01 to 1.74 is clearly seen, with the major cycle extending through state DD at time 3.35, and regressing to state AA at time 8.99. This is shorter than the average simulation, but was selected to show the entire sequence.

ceptually and implemented with realistic transition probabilities or rates of state change, simulations can be used more effectively to test the adequacy of the process model for generating particular types of rock successions.

Process models can be developed deterministically from differential equations, as in the evaporite model of Briggs and Pollack (1967). On this sort of model can be superimposed such stochastic elements as may be required to introduce random fluctuations in the ensuing responses. In fact, a complete spectrum can be discerned from purely deterministic simulation models, to stochastic models with a deterministic core, leading to completely stochastic models in which complex feedback mechanisms within the system itself (Oertel and Walton, 1967) give rise to rhythmic patterns of sedimentation without direct control by any one external factor such as tectonic oscillations, etc. This last avenue has considerable potential for exploring complex geological systems.

The lateral-shift model emphasized here uses transition rates rather than transition probabilities, to permit setting up the model in continuous time. This provides a more realistic approach to the process without a corresponding increase in mathematical complexity.

Transition rates can also be used in conventional simulation studies of single stratigraphic sections, in which the states are the lithologic components of the stratigraphic unit. In this case the sediments accumulate continuously through time, with the several lithologies occurring as discrete states. As in conventional Markov chain experiments, vertical distance (thickness) may be used instead of time, which gives rise to continuous accumulation through the vertical dimension. It is possible to include erosional episodes in such models if desired.

Use of observed data for structuring rate matrices.- Simulation experiments based on a conceptual model may lead to interesting and instructive results. However, the adequacy of such experimental models for describing real-world transgressive-regressive deposits must in the long run be tested with actual stratigraphic data. Similarly, the relation of the model to the mechanisms that produce cyclical deposits in nature needs to be evaluated with real-world data examined in the light of underlying theory.

The use of measured outcrop or borehole sections for setting up transition matrices for lateral-shift models poses a number of problems. Inasmuch as the characteristic marker beds in transgressive-regressive sequences transect time lines at an angle, the first problem is the identification of one or more time lines within a stratigraphically correlated unit arranged as a cross section normal to the strandline. The stratigraphic section is then redrawn with the time lines arranged horizontally. Conceptually, this changes the thickness axis to a time axis.

In cyclical deposits a principle developed by Sears, Hunt, and Hendricks (1941) can be used to identify time lines. The principle asserts in effect that if the stratigraphic position of the deepest water portion of a given cycle can be determined, this stratigraphic position may be accepted as a time-equivalent horizon in a succession of outcrops or boreholes.

A second procedure that may lead to identification of time-equivalent horizons is the format concept introduced by Forgotson (1957). In this approach stratigraphic correlation is conducted by carrying one or more characteristic mechanical log pips (i.e., a marker defined unit) through a cross section even in the presence of lateral facies changes. In this way it is sometimes possible to infer which beds are time-transgressive, and are thus useful as marker beds for defining the states of a transgressive-regressive system.

Broad regional studies (Weimer, 1960) show that four Upper Cretaceous transgressive-regressive cycles can be recognized over a large area from New Mexico to Montana, with numerous smaller cyclical episodes within the major cycles. The time-transgressive nature of the sands is well shown in Weimer's diagrams, but a practical difficulty arises when more limited segments of the cross sections are assembled for detailed study. The angle at which the time-transgressive marker beds cut the time lines is so small that it cannot be detected in practice, and the limited segments do not include the whole lateral sequence of movement. Even for the definitely cyclical events seen in the Mesa Verde-Mancos (Upper Cretaceous) sections along the Book Cliffs in Utah, there are difficulties, within any practical range of vertical exaggeration, to develop a transition matrix with enough tally entries to give a reasonable picture of the actual cycles.

For major episodes of transgression and regression on a continental scale, it is even more difficult to detect any but the major cyclical events in a segment of limited lateral extent. In this situation it may also be questioned whether broad environments follow each other systematically in a lateral sense across the craton.

Another situation that deserves mention is the extraordinarily widespread extent of thin cyclothem members in parts of the North American Midcontinent Pennsylvanian. These suggest the almost instantaneous occupation of large areas by a given sedimentary environment, with relatively abrupt changes to other equally widespread environments through time. The structuring of such data into a model that involves lateral movement of successive environments obviously also presents numerous difficulties.

The decision to use a conceptual model with "reasonable" transition rates in this paper was based largely on a lack of sufficient data on time-transgressive sands that could be clearly related to time lines in relatively limited areas.

A second problem, mentioned earlier, concerns relations between time and thickness in sedimentary deposits. If observational data are structured into a diagram with time as the vertical axis, and lateral distance as the horizontal axis, it is not difficult to compile a transition probability matrix. This can be transformed to its corresponding transition rate matrix providing an appropriate Δt is known for equation (5). It will be recalled that in the example given for conversion of the p_{ij}'s to q_{ij}'s, Δt was set to 1.0 undefined time unit that represents 5 feet of sediment accumulation. The effects of compaction after sedimentation are ignored here, however, with the result that the q_{ij}'s are really transformations involving an interval of Δs instead of Δt.

What these remarks sum to is that in some aspects of geological simulation, conventional ways of structuring stratigraphic data lag behind the requirements of the models. Although this lends some support to the use of frankly experimental models, it emphasizes the need for development of field and subsurface procedures that furnish the kinds of observational data needed to test the newer kinds of models being introduced into geology. There is also need for structuring the conceptual models on underlying physical processes of sedimentation. One contribution of experimental models, especially comprehensive ones like that of Harbaugh (1966) is that they point out the need for critical information of types that are rare or absent in the geological literature, as well as the need for better understanding of the theory of sedimentary processes. These considerations also involve the interplay among purely deterministic models, deterministic models with an overlay of stochastic elements, and purely stochastic models without a core of determinism. The model given in the Appendix has such a core in that the system follows a prescribed path, with departures controlled by the probabilistic elements in the model.

Transition probabilities vs. transition rates.- Although the experimental transgressive-regressive model used in this report is based directly on arbitrarily selected transition rates, lateral-shift models also can be based on transition probabilities. If the transition-rate matrix of Table 3 is converted to its corresponding transition-probability matrix with $\Delta t = 1.0$, the model becomes a discrete-time, discrete-state model. Program BOREHOLE, developed for transition rates, cannot be used directly for simulations with the $[p_{ij}]$ matrix, but it is comparatively easy to convert program MARCHAIN (Krumbein, 1967) into a lateral-shift model by simply changing the input matrix and the auxiliary control cards. It is to be emphasized that in the continuous-time model the thickness of the sediments is controlled by a random time element, whereas in the discrete-time model the thicknesses are controlled by the magnitude of the diagonal elements in the $[p_{ij}]$ matrix.

This is brought out by Table 5, which lists the $[p_{ij}]$ matrix corresponding to Table 3, with Δt chosen as 1.0. It will be noted that for all transgressive states, where $M_i = 4.0$, the p_{ij} elements are 0.018, whereas for the regressive states they are all 0.368. The off-diagonal nonzero probabilities are in the same relative proportion as are the off-diagonal q_{ij}'s in Table 3. That is, if the total rate is high, the diagonal probabilities are low, and vice versa. As a result, simulations with the lateral-shift model tend to have thin transgressive sandstones in comparison with the regressive sandstones. In Table 3 the relative rates are 4 to 1, whereas in Table 5, the relative probabilities are 0.368 to 0.018, or about 20.4 to 1. Nevertheless, the relative probabilities for continued progressive movement through the cycles as opposed to a cross over to the corresponding reverse path are still 3 to 1.

Just as the lateral-shift model can be implemented with either kind of matrix, so the simulation of single stratigraphic sections can be performed with either kind of matrix. For example, the $[q_{ij}]$ matrix of Table 1 represents a single stratigraphic section, and program BOREHOLE can readily be adapted to generate a single rock column instead of a system based on lateral migration of a strandline.

This interplay between simulations based on transition rates and transition probabilities is illustrated in Figures 4 and 5. Figure 4 is part of a lateral-shift simulation obtained with a slightly modified version of program MARCHAIN. It uses the $[p_{ij}]$ matrix of Table 5, which corresponds to the $[q_{ij}]$ matrix of Table 3, as mentioned. In this simulation, the thickness in any given state is controlled by the diagonal probabilities, and the changes of state are controlled by the off-diagonal probabilities. The form of input that was used in MARCHAIN for Figure 4 is shown in Table A-10 of the Appendix.

Figure 4 may be compared with Figure 3, inasmuch as they have the same basic model activated by the same driving force as represented by transforming the matrix of Table 3 to its counterpart in Table 5. It may be seen that the individual stages of the cycle appear to "move faster" in Figure 4; that is, one sees fewer thick accumulations of strandline sand in the boreholes generated by the discrete-time model. It seems likely that the two outputs can be made equivalent inasmuch as the information content of both matrices is the same. Ultimately, however, when sedimentation rates are better known, it may be that simulations based on rate matrices as in Figure 3 will be regarded as time simulations, whereas those based on transition probabilities as in Figure 4 may be expressed as the corresponding thickness simulations.

Just as lateral-shift models may be treated in these two ways, so the generation of single strati-

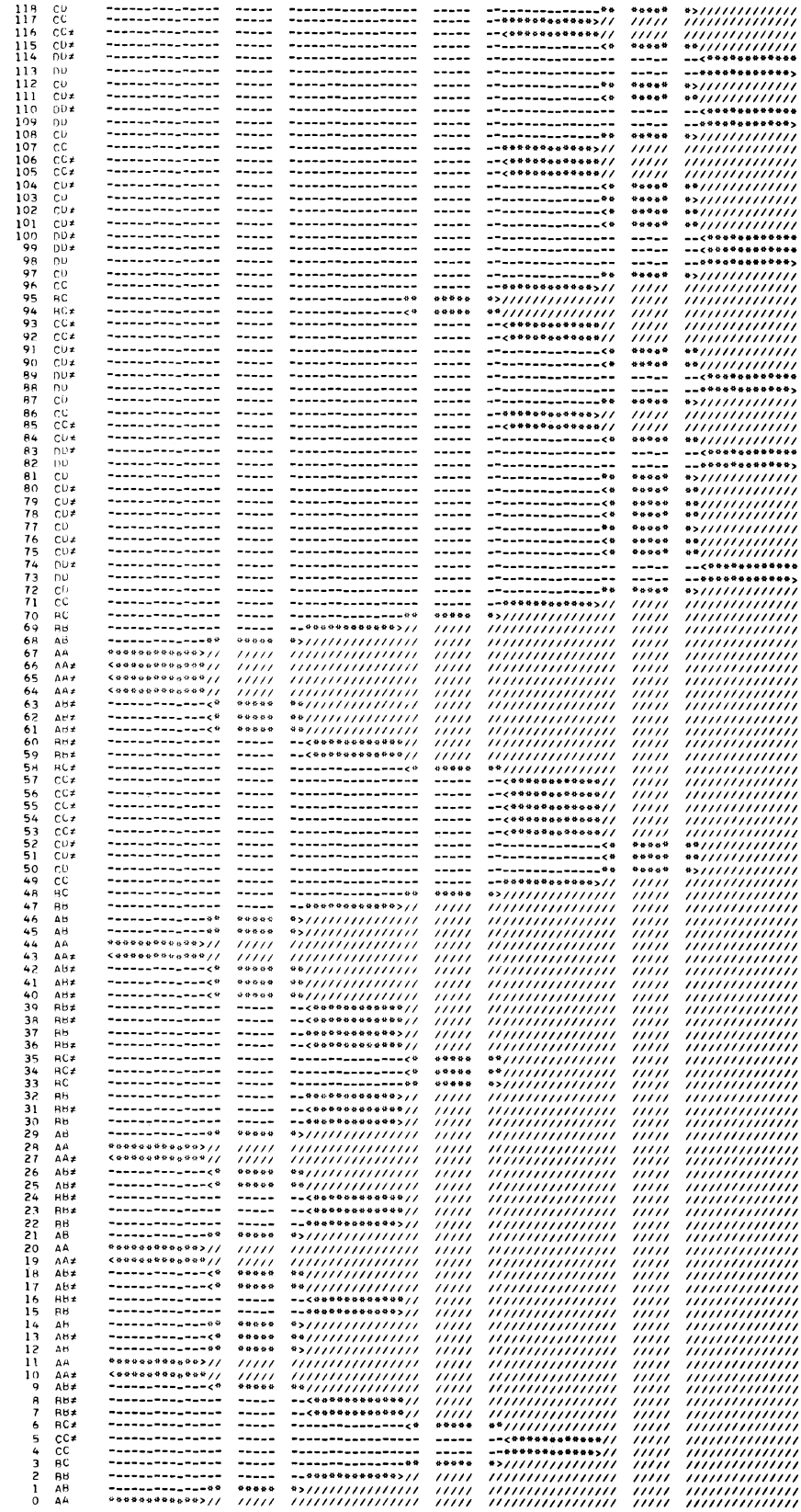


Figure 4.- Part of a transgressive-regressive cycle using the transition probabilities of Table 5. This is a discrete-time in contrast to Figure 3, which is a continuous-time simulation.

		Succeeding State, j													
		AA	AB	BB	BC	CC	CD	DD	DD'	CD'	CC'	BC'	BB'	AB'	AA'	SUM
Given State, i	AA	.018	.982	0	0	0	0	0	0	0	0	0	0	0	0	1.000
	AB	0	.018	.736	0	0	0	0	0	0	0	0	0	.246	0	1.000
	BB	0	0	.018	.736	0	0	0	0	0	0	0	.246	0	0	1.000
	BC	0	0	0	.018	.736	0	0	0	0	0	.246	0	0	0	1.000
	CC	0	0	0	0	.018	.736	0	0	0	.246	0	0	0	0	1.000
	CD	0	0	0	0	0	.018	.736	0	.246	0	0	0	0	0	1.000
	DD	0	0	0	0	0	0	.018	.982	0	0	0	0	0	0	1.000
	DD'	0	0	0	0	0	0	0	.368	.632	0	0	0	0	0	1.000
	CD'	0	0	0	0	0	.158	0	0	.368	.474	0	0	0	0	1.000
	CC'	0	0	0	0	.158	0	0	0	0	.368	.474	0	0	0	1.000
	BC'	0	0	0	.158	0	0	0	0	0	0	.368	.474	0	0	1.000
	BB'	0	0	.158	0	0	0	0	0	0	0	0	.368	.474	0	1.000
	AB'	0	.158	0	0	0	0	0	0	0	0	0	0	.368	.474	1.000
	AA'	.632	0	0	0	0	0	0	0	0	0	0	0	0	.368	1.000

Table 5.- Transition-probability matrix transformed from transition-rate matrix of Table 3.

graphic columns can be handled in continuous or discrete time. Figure 5 shows simulations based on the two matrices of Table 1, which represent a portion of the Oficina Formation in Venezuela. I am indebted to Wolfgang Scherer for use of his $[p_{ij}]$ matrix (Scherer, 1968), which was used with program MARCHAIN to develop the vertical section in the right-hand side of Figure 5. The left-hand column was obtained by using program BOREHOLE. The input matrices and state symbols used in these simulations are given in Appendix Tables A-11, A-12, and A-13.

No attempt was made to find "matching" sequences in the simulations; Figure 5 shows the starting portions of the two simulations, with the time scale and states along the left margin of the first column, and the succession of transitions and states along the left margin of the right-hand column. The continuous-time simulation was started in state A, and the discrete-time simulation began in state D, where the state definitions are those at the bottom of Table 1. As with Figures 3 and 4, Figure 5 is introduced to show the variety introduced if one has a choice of two ways for studying the same phenomenon.

The program adaptations in BOREHOLE and MARCHAIN were made during the final stages of preparation of this manuscript, so that Figures 4 and 5 are inserted mainly for the ideas they may give readers. I have not yet fully sorted the implications of using continuous versus discrete time in stratigraphic simulation models, or the possible advantages of one model over the other in specific cases. It is possible, by adjustments in Δt and in the constant C_1 mentioned earlier, that the two versions of the model could be brought into closer apparent agreement. In my opinion such comparisons are more meaningful if examined in their relation to real-world data. For the present, the intention here is to emphasize that lateral-shift models can be treated on the basis of transition probabilities as well as transition rates.

What emerges from these remarks is that simulation studies based either on discrete time or continuous time, and involving either a lateral-shift process or the generation of a single stratigraphic column, can all be performed with two basic programs. It is my belief that continuous-time models will be in the long run more meaningful in stratigraphic analysis than discrete-time models, though final judgment can come only from comparison of these two models with real-world situations. One advantage of the continuous-time model appears to be the greater ease with which erosional episodes can be introduced into the process model. This can be done by having an additional random control on "clock reversals," that is, random time intervals during which the depositional clock runs backward and wipes out part of the previously deposited sediments. This random control could be made operative at the end of each cycle, or anywhere within a cycle. In the first instance, the net effect would be to insert disconformities between the basal sand of the new cycle and the uneroded remnants of previous cycles.

It is apparent that many more questions can be raised than can be presently answered in the domain of geological simulation. The remarks, opinions, and speculations in this concluding part of the paper suggest (1) how wide open the field of geological simulation is for continued study and research, (2) the need for analytical (as opposed to descriptive)

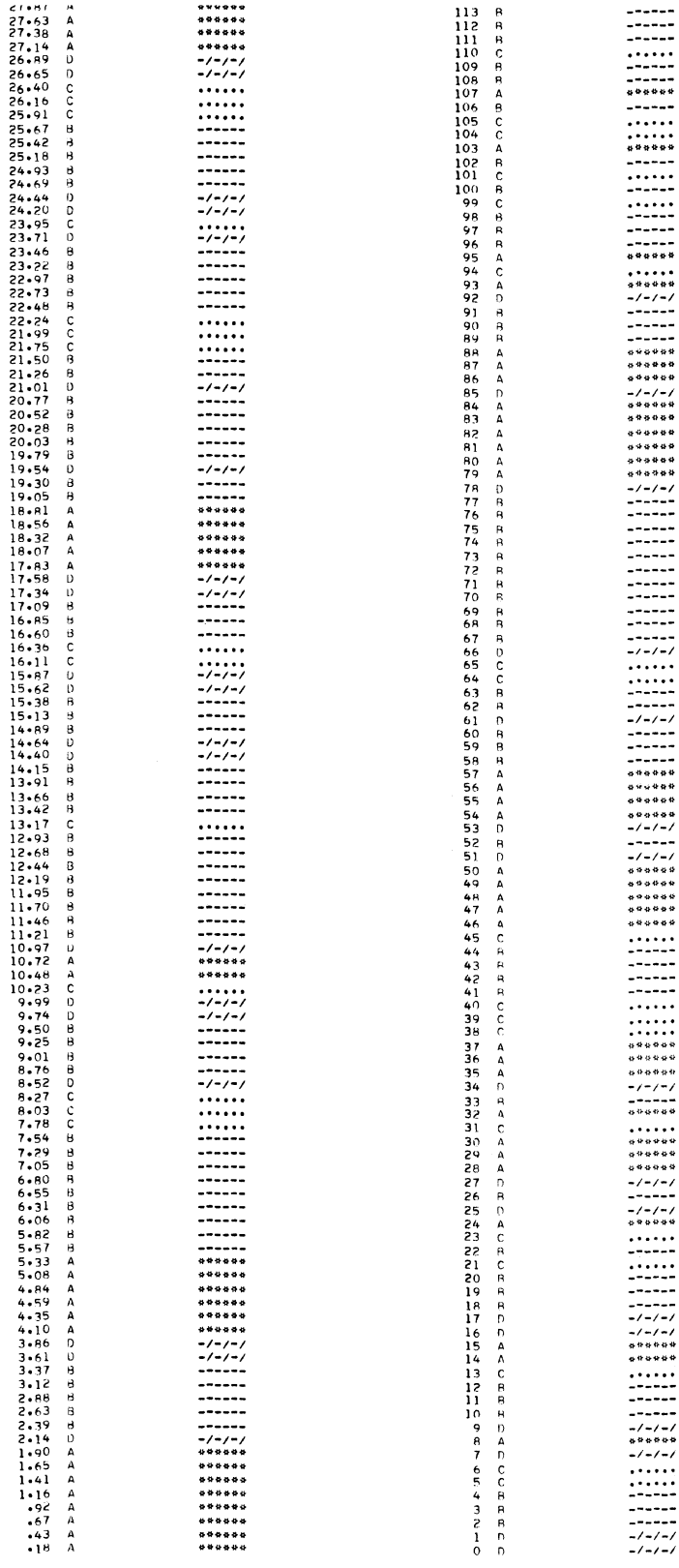


Figure 5.- Single-section stratigraphic simulations. Left column generated by program BOREHOLE with $[q_{ij}]$ matrix of Table 1. Right column generated by program MARCHAIN with $[p_{ij}]$ matrix of Table 1. The state symbols are ****** = sandstone; ..... = siltstone; ------ = shale; and -/-/-/ = lignite. Original $[p_{ij}]$ data from Scherer (1968).

simulation models that include the underlying physical theory of the process being simulated, and (3) the need for developing new ways of examining and recording natural data for testing the adequacy of the many simulation models proposed in the geological literature. This adequacy refers not only to agreement of the model with theory and observed earth phenomena, but concerns its potential usefulness as a predicting device in the search for oil, gas, and ore.

Further implications of Markov Models.- Several additional points that deserve comment with respect to Markov models in stratigraphy include the implications of the exponential distribution of waiting times for state changes in the continuous-time model. When time is converted to thickness by equation (8), the thickness distribution will also be exponential. This suggests in effect that the thickness distribution of beds is a one-parameter Gamma distribution. Most sedimentological studies strongly suggest that bed thickness distributions are lognormal. An interesting question is raised by this point, in that the conventional discrete-state, discrete-time Markov chain that corresponds to its equivalent $[q_{ij}]$ matrix has a geometric distribution of discrete states (bed thicknesses) controlled by the magnitude of the diagonal transition probabilities, p_{ii}.

Potter and Blakely (1968) use an embedded Markov chain in which the transitions from one lithology to another are recorded, with a separate record kept of bed thicknesses. In their model the p_{ii}'s are identically zero, so that the matrix gives the sequence of states through which the system passes without regard for the length of time required for the state transitions. Thus there may be no necessary relation of the embedded model to any particular kind of waiting-time distribution, inasmuch as $p_{ii} = 0$, rather than $p_{ii} = e^{-M_i \Delta t}$ as in the equal-intercept Markov chain, where the diagonals reflect transitions from a given state to itself. These considerations raise interesting and challenging questions regarding the kinds of "practical" bed-thickness (and hence waiting-time) distributions that can be used with Markov models based either on discrete or continuous time.

The arbitrary practice of limiting the exponential waiting-time distribution to fixed limits (as in using random numbers from 0.3 to 0.8, instead of from 0.0 to 1.0) also bears on this problem. In the absence of pertinent field data an arbitrary but "reasonable" choice was made. However, problems raised by such truncation are not unique to the exponential distribution. Even with the lognormal distribution there is a finite probability that a given bed may be a mile or more thick. Field data, however, on actual bed thicknesses permit the setting of reasonable limits on the maximum thickness allowable in simulations. It would be interesting to examine the point at which nature truncates the lognormal probability distribution of bed thicknesses; similarly, factual data are needed on the maximum waiting time that nature permits in the exponential waiting-time distribution.

Another element that bears on the question of an optimum Markov model for stratigraphic simulation is the influence of the constant C_1 in equation (8). As stated, this is a conversion factor between time and thickness. Inasmuch as the quantity $\log_e(1/U_{max})/M_{max}$ has the dimension time from equation (7), and the thickness had dimension length, it follows that C_1 has the dimensions (L/T), and hence is a rate, assumed constant for all lithologies as emphasized earlier in the text. How good is this assumption for clastic wedges? Even the choice of Δt in equation (5), or more conveniently Δs for thickness, comes into the picture. If Δs is taken as very small, there will be virtually no state transitions in a simulation of reasonable length; if Δs is taken as very large, details may be lost. Until relations between time and accumulated thickness are better known, the choice here is also arbitrary.

Finally, there is the question of the adequacy of first-order Markov models as simulation devices in stratigraphy. Schwarzacher (1967) has convincingly shown that stratigraphic Markov models should be at least of second order, with a two-step memory that need not necessarily involve the two immediately preceding steps. In the Kansas Pennsylvanian he found that the state of the system at time t_r was controlled by events at time t_{r-31} and t_{r-1}. I have not had an opportunity in this study for comparing second-order chains with their equivalent continuous-time equivalents.

All of these questions illustrate an interesting contribution of models to conventional stratigraphic procedures, suggested to me by Paul Potter of Indiana University. It is "That one sees here the feedback contribution of models - a reappraisal of the field that would perhaps not otherwise come about."

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APPENDIX

INTRODUCTION

The two programs listed here are written in a modified version of FORTRAN IV for the CDC 6400 computer. Normally, there is relatively little difficulty in making these programs compatible with standard FORTRAN IV. Fortunately, the two programs are fairly short.

Most programs in the Geology Program Library at Northwestern University have an underlying similarity in their organization, and in the types of data formats used. These similarities involve the use of project numbers, four title cards, and at least one major control card. Programs also are arranged so that several data decks can be placed one behind the other, rather than using the program deck separately with each set of data.

The following comments apply to both programs in this Appendix:

Title Cards. - Provision is made for four title cards, so that the specific problem can be completely documented. These are of the following form:

Card 1. Enter the digit 1 in column 1, and leave column 2 blank. The initial digit is a carriage control, so that the title is repeated as new segments of output are printed. Columns 3 to 70 inclusive may be used for any appropriate title.

Cards 2, 3, and 4. Enter a zero in column 1, leave column 2 blank, and proceed as with card 1 in columns 3 to 70. If only two cards are required for the title, for example, include two blank cards that have a zero entered in column 1, to fulfill the requirement for four cards.

Control Cards. - These furnish the computer with specific details regarding a particular problem, such as size of matrix to be read in, format of the data cards, etc. The control card for each program is described in detail later.

Input Matrices. - Program PEQUMAT is designed to accept input matrices up to 20 x 20, but these dimensions can readily be enlarged. Program BOREHOLE accepts matrices up to 41 x 41. Input formats for reading the matrices differ in these programs, as is described more fully for each program. The special input format for BOREHOLE was designed to facilitate making changes in the rate matrices for experimental purposes without requiring that the entire matrix be repunched.

Computing Center Lead Cards. - The sequence of lead cards for programs differs at different installations. Inasmuch as every computing center has somewhat different lead cards, details are not included here.

Input Assemblies for Data Decks. - Lead cards are followed by assemblies for the data decks. These are as follows:

The input assembly for program PEQUMAT consists of:

4 TITLE CARDS, format as described for title cards above,
CONTROL CARD, as described later,
INPUT MATRIX, read in by rows,
END-OF-RECORD CARD,
END-OF-INFORMATION CARD.

The input assembly for program BOREHOLE consists of:

4 TITLE CARDS, as described above
CONTROL CARD, as described later,
INPUT MATRIX, read in as described later,
BLANK CARD,
STATE-NAME CARDS, as described below,
LITHOLOGIC STATE SYMBOL CARDS, as described below,
END-OF-RECORD CARD,
END-OF-INFORMATION CARD.

If more than one problem is to be run consecutively, the additional input data decks are each followed by an end-of-record card, with the end-of-information card inserted behind the last problem.

PROGRAM PEQUMAT

The original version of this program was written for the CDC 3400 in 1966, and modified for the CDC 6400 in 1968. I am indebted to Mrs. Betty Benson for preparing the program on the basis of text equations (1a) to (4a), and to Wolfgang Scherer for later modifications. The program consists of two interlocked parts, one of which converts a $[p_{ij}]$ matrix to its corresponding $[q_{ij}]$ matrix, and the other for the reverse operation. Thus, the initial input can be a transition probability matrix or a transition rate matrix.

The program first lists the input matrix and records the value of Δt used, then makes the appropriate transformation, which is printed out in unrounded and rounded form. The unrounded form is kept in storage, and the program automatically takes this transformed matrix and re-transforms it to the original input matrix. The sequence thus is a loop, depending upon which matrix is used as input:

$$[p_{ij}] \longrightarrow [q_{ij}] \longrightarrow [p_{ij}] \qquad \text{PQ loop}$$

$$[q_{ij}] \longrightarrow [p_{ij}] \longrightarrow [q_{ij}] \qquad \text{QP loop}$$

The example of input and output given in Table A-2 is the transformation of the rate matrix in text Table 3 to the transition probability matrix of Table 5 via the QP loop. This uses equation (1a) directly to obtain the diagonal probabilities p_{ii} for each state (i.e., for each row of the rate matrix). This diagonal probability is used to obtain $(1 - p_{ii}) = P_i$, the sum of the off-diagonal probabilities in

```
1  KRUMBEIN PROJECT 01 0254  BOREHOLE RESEARCH
0  P(I,J) MATRIX FROM HYPOTHETICAL T-R MATRIX
0
0
0102540141  1.00(10X,14F4.2)
01025400010.004.000.000.000.000.000.000.000.000.000.000.000.000.00
01025400020.000.003.000.000.000.000.000.000.000.000.001.000.00
01025400030.000.000.003.000.000.000.000.000.000.001.000.000.00
01025400040.000.000.000.003.000.000.000.000.001.000.000.000.00
01025400050.000.000.000.000.003.000.000.000.001.000.000.000.00
01025400060.000.000.000.000.000.003.000.001.000.000.000.000.00
01025400070.000.000.000.000.000.000.004.000.000.000.000.000.00
01025400080.000.000.000.000.000.000.000.001.000.000.000.000.00
01025400090.000.000.000.000.000.250.000.000.000.750.000.000.00
01025400100.000.000.000.000.250.000.000.000.000.000.750.000.00
01025400110.000.000.000.250.000.000.000.000.000.000.000.750.00
01025400120.000.000.250.000.000.000.000.000.000.000.000.000.75
01025400130.000.250.000.000.000.000.000.000.000.000.000.000.75
01025400141.000.000.000.000.000.000.000.000.000.000.000.000.00
```

Table A-1.- P(I,J) matrix from hypothetical T-R matrix.

each row. Equation (3a) is then used to obtain the individual p_{ij}'s where $j \neq i$, by multiplying P_i by the ratio of the corresponding off-diagonal q_{ij}'s to the total rate for the state, M_i.

The PQ loop uses text equations (5) and (4a) as described in detail in the illustrative example of text Table 1.

The control card for PEQUMAT has the following form:

Columns

01-06 NPROJ, the problem project number in format (A6). In the example this is 010254

07-09 N, the size of the input matrix in format (I3). For the example given, this is 014

10 INDIC, a code to indicate the kind of input matrix. If the input is a $[p_{ij}]$ matrix, enter 0. If it is a $[q_{ij}]$ matrix, enter 1.

11-16 DELT, the desired value of Δt in format (F6.2). See the text discussion of equation (5). For many purposes this will be selected as 001.00.

17-80 FMT, the format in fields of (8A8), used in reading the input matrix. This matrix is read in by rows for PEQUMAT, and the input example of Table A-1 has the input format (10X, 14F4.2).

Input for matrix cards in this program is as follows:

Columns

01-06 Project number in (A6) format, example 010352

07-10 Row number of matrix in (I4), example 0001

11-70 Input fields of any convenient size. If probabilities or rates are entered to two or three decimal places, fields can be kept uniform, say from 3 to 6 digits each, with format F3.2, F6.3, or the like.

71-80 Blank.

Table A-1 is a direct listing of the complete set of input cards for the example given here. This deck carries project number 010254 and the row number for the input matrix. These are not read in, and hence the 10X. The input deck in the Table also illustrates the use of only two title cards with information; therefore, it includes two blank title cards with a zero punched in column 1. The titles are followed directly by the control card, which carries the code INDIC = 1, to indicate that the input is a $[q_{ij}]$ matrix. The 14 matrix cards follow the control card, one card per row.

The complete output for PEQUMAT in the example is shown in Table A-2. The title cards are listed first. These are followed by a printout of the input deck for subsequent comparison with the recheck on the matrix transformation. The calculated $[p_{ij}]$ matrix is printed out in unrounded and rounded form, the latter for visual check with the original input matrix.

The double printout of the transformed matrices was originally designed as part of some larger loops that permitted the use of several Δt's on the same matrix, to see how this choice influenced the transformed matrices. Most users will probably wish to simplify the program by deleting the unrounded printout, to obtain output that shows the input matrix, the rounded transformed matrix, and the rounded recheck matrix. Table A-3 lists program PEQUMAT.

PROGRAM BOREHOLE

This program is written in a modified version of FORTRAN IV for the CDC 6400 computer. The original version, written by Paul Tukey at Princeton in 1966, used the IBM 7094; subsequently some details of input and output were modified by Mrs. Betty Benson. The version listed here was finalized in May, 1968, with some alterations by Wolfgang Scherer.

In its structure the program is straightforward, in that it first reads and prints four title cards, and then reads a control card that specifies the size of the $[q_{ij}]$ matrix, the upper and lower limits of the random numbers used in the simulation, the value of the constant C_1 in equation (8) of the text, etc. It also reads the input matrix, the state names, and the stratigraphic symbol cards, one for each state in the system. These initial cards and the input matrix based on text Table 3, are described next.

The control card for BOREHOLE has the following form:

Columns

01-05 DIM, the dimensions of the $[q_{ij}]$ input matrix. This has format (I5), and in the example is 00014

06-10 NTRIAL, in format (I5), the number of

Table A-2.- Output for program PEQUMAT.

KRUMBEIN PROJECT 01 0254 BOREHOLE RESEARCH

P(I,J) MATRIX FROM HYPOTHETICAL T-R MATRIX

DELTA T = 1.00

Q(I,J) MATRIX

	1	2	3	4	5	6	7	8	9	+	-	*	/	(	MG
1	0.00000	4.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	4.00000
2	0.00000	0.00000	3.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000	0.00000	4.00000
3	0.00000	0.00000	0.00000	3.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000	0.00000	0.00000	4.00000
4	0.00000	0.00000	0.00000	0.00000	3.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	4.00000
5	0.00000	0.00000	0.00000	0.00000	0.00000	3.00000	0.00000	0.00000	0.00000	1.00000	0.00000	0.00000	0.00000	0.00000	4.00000
6	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	3.00000	0.00000	0.00000	1.00000	0.00000	0.00000	0.00000	0.00000	4.00000
7	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	4.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	4.00000
8	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000
9	0.00000	0.00000	0.00000	0.00000	0.00000	.25000	0.00000	0.00000	0.00000	.75000	0.00000	0.00000	0.00000	0.00000	1.00000
10	0.00000	0.00000	0.00000	0.00000	.25000	0.00000	0.00000	0.00000	0.00000	0.00000	.75000	0.00000	0.00000	0.00000	1.00000
11	0.00000	0.00000	0.00000	.25000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	.75000	0.00000	0.00000	1.00000
12	0.00000	0.00000	.25000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	.75000	0.00000	1.00000
13	0.00000	.25000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	.75000	1.00000
14	1.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000

P(I,J) MATRIX

	1	2	3	4	5	6	7	8	9	+	-	*	/	(	MG
1	.01832	.98168	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000
2	0.00000	.01832	.73626	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	.24542	0.00000	1.00000
3	0.00000	0.00000	.01832	.73626	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	.24542	0.00000	0.00000	1.00000
4	0.00000	0.00000	0.00000	.01832	.73626	0.00000	0.00000	0.00000	0.00000	0.00000	.24542	0.00000	0.00000	0.00000	1.00000
5	0.00000	0.00000	0.00000	0.00000	.01832	.73626	0.00000	0.00000	0.00000	.24542	0.00000	0.00000	0.00000	0.00000	1.00000
6	0.00000	0.00000	0.00000	0.00000	0.00000	.01832	.73626	0.00000	.24542	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000
7	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	.01832	.98168	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000
8	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	.36788	.63212	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000
9	0.00000	0.00000	0.00000	0.00000	0.00000	.15803	0.00000	0.00000	.36788	.47409	0.00000	0.00000	0.00000	0.00000	1.00000
10	0.00000	0.00000	0.00000	0.00000	.15803	0.00000	0.00000	0.00000	0.00000	.36788	.47409	0.00000	0.00000	0.00000	1.00000
11	0.00000	0.00000	0.00000	.15803	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	.36788	.47409	0.00000	0.00000	1.00000
12	0.00000	0.00000	.15803	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	.36788	.47409	0.00000	1.00000
13	0.00000	.15803	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	.36788	.47409	1.00000
14	.63212	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	.36788	1.00000

P(I,J) MATRIX (ROUNDED)

	1	2	3	4	5	6	7	8	9	+	-	*	/	(	MG
1	.01800	.98200	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000
2	0.00000	.01800	.73600	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	.24500	0.00000	1.00000
3	0.00000	0.00000	.01800	.73600	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	.24500	0.00000	0.00000	1.00000
4	0.00000	0.00000	0.00000	.01800	.73600	0.00000	0.00000	0.00000	0.00000	0.00000	.24500	0.00000	0.00000	0.00000	1.00000
5	0.00000	0.00000	0.00000	0.00000	.01800	.73600	0.00000	0.00000	0.00000	.24500	0.00000	0.00000	0.00000	0.00000	1.00000
6	0.00000	0.00000	0.00000	0.00000	0.00000	.01800	.73600	0.00000	.24500	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000
7	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	.01800	.98200	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000
8	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	.36800	.63200	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000
9	0.00000	0.00000	0.00000	0.00000	0.00000	.15800	0.00000	0.00000	.36800	.47400	0.00000	0.00000	0.00000	0.00000	1.00000
10	0.00000	0.00000	0.00000	0.00000	.15800	0.00000	0.00000	0.00000	0.00000	.36800	.47400	0.00000	0.00000	0.00000	1.00000
11	0.00000	0.00000	0.00000	.15800	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	.36800	.47400	0.00000	0.00000	1.00000
12	0.00000	0.00000	.15800	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	.36800	.47400	0.00000	1.00000
13	0.00000	.15800	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	.36800	.47400	1.00000
14	.63200	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	.36800	1.00000

Q(I,J) MATRIX

	1	2	3	4	5	6	7	8	9	+	-	*	/	(	MG
1	0.00000	4.01738	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	4.01738
2	0.00000	0.00000	3.01099	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	1.00230	0.00000	4.01738
3	0.00000	0.00000	0.00000	3.01099	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	1.00230	0.00000	0.00000	4.01738
4	0.00000	0.00000	0.00000	0.00000	3.01099	0.00000	0.00000	0.00000	0.00000	0.00000	1.00230	0.00000	0.00000	0.00000	4.01738
5	0.00000	0.00000	0.00000	0.00000	0.00000	3.01099	0.00000	0.00000	0.00000	1.00230	0.00000	0.00000	0.00000	0.00000	4.01738
6	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	3.01099	0.00000	1.00230	0.00000	0.00000	0.00000	0.00000	0.00000	4.01738
7	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	4.01738	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	4.01738
8	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	.99967	0.00000	0.00000	0.00000	0.00000	0.00000	.99967
9	0.00000	0.00000	0.00000	0.00000	0.00000	.24992	0.00000	0.00000	0.00000	.74975	0.00000	0.00000	0.00000	0.00000	.99967
10	0.00000	0.00000	0.00000	0.00000	.24992	0.00000	0.00000	0.00000	0.00000	0.00000	.74975	0.00000	0.00000	0.00000	.99967
11	0.00000	0.00000	0.00000	.24992	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	.74975	0.00000	0.00000	.99967
12	0.00000	0.00000	.24992	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	.74975	0.00000	.99967
13	0.00000	.24992	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	.74975	.99967
14	.99967	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	.99967

Q(I,J) MATRIX (ROUNDED)

	1	2	3	4	5	6	7	8	9	+	-	*	/	(	MG
1	0.00000	4.01700	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	4.01700
2	0.00000	0.00000	3.01100	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	1.00200	0.00000	4.01700
3	0.00000	0.00000	0.00000	3.01100	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	1.00200	0.00000	0.00000	4.01700
4	0.00000	0.00000	0.00000	0.00000	3.01100	0.00000	0.00000	0.00000	0.00000	0.00000	1.00200	0.00000	0.00000	0.00000	4.01700
5	0.00000	0.00000	0.00000	0.00000	0.00000	3.01100	0.00000	0.00000	0.00000	1.00200	0.00000	0.00000	0.00000	0.00000	4.01700
6	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	3.01100	0.00000	1.00200	0.00000	0.00000	0.00000	0.00000	0.00000	4.01700
7	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	4.01700	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	4.01700
8	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000
9	0.00000	0.00000	0.00000	0.00000	0.00000	.25000	0.00000	0.00000	0.00000	.75000	0.00000	0.00000	0.00000	0.00000	1.00000
10	0.00000	0.00000	0.00000	0.00000	.25000	0.00000	0.00000	0.00000	0.00000	0.00000	.75000	0.00000	0.00000	0.00000	1.00000
11	0.00000	0.00000	0.00000	.25000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	.75000	0.00000	0.00000	1.00000
12	0.00000	0.00000	.25000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	.75000	0.00000	1.00000
13	0.00000	.25000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	.75000	1.00000
14	1.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000

```
      PROGRAM PEQUMAT (INPUT,TAPE60=INPUT,OUTPUT)
C
C  FOR WILLIAM C. KRUMBEIN, GEOLOGY .......AUGUST 1966, BETTY BENSON
C  REVISED FOR CDC 6400, BETTY BENSON, APRIL 1968
C  ADDITIONAL MODIFICATIONS, W. SCHERER, MAY 1968
C
      COMMON /AA/ P(20,20),Q(20,20),PMARG(20),QMARG(20),LABELP,LABELQ
      COMMON /BB/ TITL(36),FMT(8),N,DELT
      LABELP=6HP(I,J)
      LABELQ=6HQ(I,J)
C
    3 READ 100,TITL
      IF (EOF,60) 4,5
    4 STOP
    5 READ 101,NPROJ,N,INDIC,DELT,FMT
  100 FORMAT (9A8)
  101 FORMAT (A6,I3,I1,F6.2,8A8)
      IF (INDIC) 10,10,20
C
   10 CALL IN (P,PMARG,LABELP)
      CALL PTOQ
      CALL QTOP
      GO TO 3
C
   20 CALL IN (Q,QMARG,LABELQ)
      CALL QTOP
      CALL PTOQ
      GO TO 3
      END
      SUBROUTINE PTOQ
      COMMON /AA/ P(20,20),Q(20,20),PMARG(20),QMARG(20),LABELP,LABELQ
      COMMON /BB/ TITL(36),FMT(8),N,DELT
C
      DO 10 I=1,N
      QMARG(I)= - ALOG(P(I,I))/DELT
      SUMOTRO = PMARG(I)-P(I,I)
      P(I,I)=0.0
      DO 10 J=1,N
   10 Q(I,J) = (P(I,J)/SUMOTRO) * QMARG(I)
      CALL OUT (Q,QMARG,LABELQ)
      CALL ROUND (Q,QMARG,LABELQ)
      RETURN
C
C      ************************************************************
C
      ENTRY QTOP
      DO 20 I=1,N
      P(I,I) = EXP((-QMARG(I))*DELT)
      SUMOTRO = 1.0-P(I,I)
      DO 20 J=1,N
      IF (I-J) 21,20,21
   21 P(I,J) = SUMOTRO * (Q(I,J)/ QMARG(I))
   20 CONTINUE
      DO 12 I=1,N
      PMARG(I)=0.0
      DO 12 J=1,N
   12 PMARG(I)=PMARG(I)+P(I,J)
      CALL OUT (P,PMARG,LABELP)
      CALL ROUND (P,PMARG,LABELP)
```

```
      RETURN
      END
      SUBROUTINE IN (MATRIX,MARG,LABEL)
      COMMON /BB/ TITL(36),FMT(8),N,DELT
      DIMENSION MATRIX(20,20),MARG(20),JLABEL(20)
      REAL MATRIX,MARG
      DATA (JMARG=2RMG)
      DO 3 J=1,20
      K=J+33B
    3 ENCODE(10,106,JLABEL(J)) K
  106 FORMAT (8XR2)
C
      DO 5 I=1,N
    5 READ(60,FMT) (MATRIX(I,J),J=1,N)
      DO 6 I=1,N
      MARG(I)=0.0
      DO 6 J=1,N
    6 MARG(I)=MARG(I) + MATRIX(I,J)
C
C     ****************************************************************
C
      ENTRY OUT
      PRINT 100,TITL
      PRINT 116, DELT
      PRINT 102,LABEL
      GO TO 4
C
C     ****************************************************************
C
      ENTRY ROUND
      DO 8 I=1,N
      MJ=1000. * MARG(I) +.5
      MARG(I)=MJ / 1000.
      DO 8 J=1,N
      MIJ=1000. * MATRIX(I,J) +.5
    8 MATRIX (I,J) = MIJ / 1000.
      PRINT 105,LABEL
C
    4 PRINT 103,(JLABEL(L),L=1,N),JMARG
      DO 7 I=1,N
    7 PRINT 104,I,(MATRIX(I,J),J=1,N),MARG(I)
      RETURN
C
  100 FORMAT (9A8)
  102 FORMAT(/30XA7,*MATRIX* / 30X,13(1H-)/)
  103 FORMAT (4X,16(XR2,5X))
  104 FORMAT (I4,16F8.5)
  105 FORMAT (///30XA7,*MATRIX (ROUNDED)* / 30X,23(1H-)/)
  116 FORMAT (30X,*DELTA T = *F6.2//)
      END
```

simulations (trials) desired. Each simulation starts in state AA, passes through state DD, and continues until it terminates in state AA. This assures that the output will include all states, with a varying number of subcycles, as shown in text Figure 3, further described below. If only a single trial is desired, enter 00001 in this field.

11-14 HILIM, in format (F4.2). This is the upper limit of the uniform random numbers to be used for setting the random times in states. This limit was chosen arbitrarily for this example as 0.80.

15-18 LOLIM, in format (F4.2). This is the lower limit for the random numbers used for setting random times in states. This number was chosen as 0.30 for the present example.

19-22 C_1, in format (F4.2) is the constant referred to in text equation (8). The value

1.00 was chosen for the example, so that one line of printout is associated with the minimum time in any state, 0.06 units. See the text discussion following equation (8).

23-27 PRINT, with format (L5). This control sets the starting point for RANSET(T), which is a standard computing center pseudorandom number routine used in the program. If the letter T is entered in this field, right justified, the computer will use its clock time to set the series in operation. Table A-5 shows how the initial program output records this time.

28-37 NPROJ, the problem project number, in a field of (A10). In the example this is 0000010254

38-47 UNIT, in a field of (F10.0). This is an unused option in which a selected value of the total right-hand side of equation (8) may be entered. This is now taken care of by the C_1 entry in columns 19-22; hence this last field is normally left blank.

48-80 Blank.

The four title cards (three of them blank except for the initial zero) and the control card are shown in the top five lines of Table A-4. Table A-5 shows how the program displays the controls and the starting time of the random numbers for waiting times in each state. Note that the entry T in the (L5) field of PRINT uses the computer clock as the starting point for the random number sequence. The material in Table A-5 constitutes the first portion of BOREHOLE output.

The control card is followed by the input matrix. The procedure for reading this was designed to provide maximum flexibility in modifying experimental matrices without repunching the entire matrix. Inasmuch as the $[q_{ij}]$ matrix of text Table 3 contains mainly zero elements, the machine first zeros a matrix of size DIM (here 14 x 14), and the nonzero entries are then read in one at a time. Thus, reference to text Table 3 shows that the first line has 4.0 in its second column; the second line has 3.0 in its third column, and 1.00 in its thirteenth column; and so on. The matrix input cards are prepared for each nonzero entry as follows:

Columns

01-05 Row of matrix in format (I5), example 00001 for first row

06-10 Column of matrix in format (I5), example 00002 for second column

11-15 q_{ij} in format (F5.0), example 003.0

16-80 Blank.

It would not be difficult to change this to direct readin of the complete matrix by rows, as in PEQUMAT, but for experimental studies, where only one or two q_{ij} elements are changed, the present scheme permits convenient insertion of rate changes. The full set of these input cards in the example is shown in Table A-4, starting on line six.

Once the $[q_{ij}]$ matrix is read in, the program reads the symbols for all states in the system. In accordance with the bottom diagram in text Figure 2, the system starts with AA, followed by transgressive states through DD, succeeded by regression states DD' to AA'. These state symbols have the format (12A6). Table A-4 includes these state symbols at the end of the matrix input. Two additional titles are read which head the main output sheet and identify the monitoring positions. These are shown in Table A-4 directly after the state label cards.

An important set of input cards shows the graphical representation of each transgressive and regressive state. These are listed as the last items in Table A-4, but their form can be seen more clearly in the program output of Table A-7. Therefore, discussion at this point will move to another aspect of program BOREHOLE that is relatively unique. This is the "clock mechanism" in the program, and it deserves special attention. As mentioned, the simulation starts in state AA, and as the system shifts from this to other states, waiting time in each state is cumulated until the total time for the simulation is recorded. The relation given in equation (8) is then used to determine the time allocated to one line of printed simulation output. The specified condition is that every state which is occupied during the simulation must have at least one line of printed output.

This condition is imposed because progress through the $[q_{ij}]$ matrix is such that no states can be skipped during any given part of a cycle. Thus, if the system starts in state AA and proceeds to state CD before a reversal occurs, the output must show all intermediate states AB, BB, BC, and CC. The time allotted to one line of output is controlled by the upper and lower limits of the random numbers, the magnitude of the largest rate M_{max}, and the constant C_1. If C_1 is chosen as 1.0, the minimum time in any given state controls the time interval associated with one line of printout. See the text discussion on Table 4 for details. If C_1 is greater than 1.0, some of the time intervals associated with a given state may be less than that allotted to a single line. In this case one printout line is assigned to that state, and the time deficit is absorbed by adjacent states.

Table A-6 shows the timing procedure for a simulation in which the shortest waiting time in any state is 0.06, but in which C_1 was chosen as 2.16. The clock timing for successive lines of printout in this case is 0.12 unit. The first column of Table A-6 lists a set of states and the waiting time in each. These waiting times are cumulated to the end of each

Table A-4.- Input for example problem.

```
1 PROJECT 01 0254     14X14 Q(I,J) MATRIX FOR T-R MODEL      3 BOREHOLES
0
0
0
   14     20.800.30 1.0     T     010254
    1      2  4.0
    2      3  3.0
    2     13  1.0
    3      4  3.0
    3     12  1.0
    4      5  3.0
    4     11  1.0
    5      6  3.0
    5     10  1.0
    6      7  3.0
    6      9  1.0
    7      8  4.0
    8      9  1.0
    9      6 0.25
    9     10 0.75
   10      5 0.25
   10     11 0.75
   11      4 0.25
   11     12 0.75
   12      3 0.25
   12     13 0.75
   13      2 0.25
   13     14 0.75
   14      1  1.0

 AA     AB     BB     BC     CC     CD     DD    -DD    -CD    -CC    -BC    -BB
-AB    -AA
    BOREHOLE RECORDS                                              BLOCKED CROSS-SECTIO
N OF SANDSTONE
                            I                          II                          III

AA      **************//  /////  ////////////////////  /////  ////////////////////  ////
/  ////////////////
AB      -------------**  *****  **////////////////////  /////  ////////////////////  ////
/  ////////////////
BB      ---------------  -----  --**************//////  /////  ////////////////////  ////
/  ////////////////
BC      ---------------  -----  ----------------******  *****  **////////////////////  ////
/  ////////////////
CC      ---------------  -----  ----------------------  -----  --**************//////  ////
/  ////////////////
CD      ---------------  -----  ----------------------  -----  ----------------******  ****
*  **//////////////
DD      ---------------  -----  ----------------------  -----  ----------------------  ----
-  --**************
DD-     ---------------  -----  ----------------------  -----  ----------------------  ----
-  --0*************
CD-     ---------------  -----  ----------------------  -----  ----------------0*  ****
*  **//////////////
CC-     ---------------  -----  ----------------------  -----  --0*************//  ////
/  ////////////////
BC-     ---------------  -----  ----------------0*  *****  **////////////////////  ////
/  ////////////////
BB-     ---------------  -----  --0*************//  /////  ////////////////////  ////
/  ////////////////
AB-     -------------0*  *****  **////////////////////  /////  ////////////////////  ////
/  ////////////////
AA-     0*************//  /////  ////////////////////  /////  ////////////////////  ////
/  ///////////////1
```

Table A-5.- Initial output of BOREHOLE, listing program controls.

```
CONTROLS
--------

  DIM=   14
HILIM=  .80
LOLIM=  .30
   CI= 1.00
TRIAL=    2
PRINT=    T
STARTING TIME FOR RANDOM NUMBERS =*347024.0000
```

state, as shown in the third column of the Table. The last two columns show the individual times at which output lines occur. At the beginning of the simulation, the clock selects an initial printing time of less than 0.12, to pick up state AA if its waiting time is less than 0.12 units. In the example given, the first line was printed at time 0.01, the next line at 0.13, and so on, at intervals of 0.12.

As Table A-6 shows, two states, CC and DD, have waiting times shorter than 0.12, but they are given one line each. In this example, the cumulative time limits for each state happen to include the clock times at which the lines are printed, and they are thus automatically picked up.

Output from program BOREHOLE.- Table A-5 shows the initial output of the program, and Tables A-7 through A-9 show the rest of the output for the example displayed. Table A-7 lists the $[q_{ij}]$ input matrix, based on the line entries in the input deck of Table A-4. This is followed by the state symbols for the system, which are also listed in Table A-4. Table A-7 also shows the cumulated probabilities for state changes. It is to be emphasized that this cumulated matrix is obtained directly from the $[q_{ij}]$ matrix, without transforming it to its corresponding $[p_{ij}]$ matrix. That is, the cumulated matrix of state probabilities shows that in state AA, for example, the transition to state AB has probability 1.0 or certainty. This is obtained by taking the q_{12} value of the top matrix and dividing it by the marginal rate: 1.00/1.00 = 1.00. The second line of the cumulated state probabilities matrix is obtained from the q_{ij} values in the second line of the top matrix. The probability of moving from state AB to BB is 3.00/4.00 = 0.75, and the probability of moving to state -AB is 1.00/4.00 = 0.25, in accordance with the text discussion following equation (5). Thus the cumulated values jump from zero to 0.75 for state BB, and rise to 1.00 for state -AB. These probabilities are used with uniform random numbers in the range 0.00 to 1.00 for determining which state succeeds a given state. See the text discussion in the paragraph following equation (7) for the procedure in selecting the state to which the system moves after its waiting time in a given state has elapsed.

Table A-6.- Segment of BOREHOLE printout, showing time allocation to printout lines.

State	Time in State	Cumulated Time to End of State	Passage of Time on Clock	Printout State
			.	.
CD'	0.475	6.022	5.92	CD'
			5.80	CD'
			5.68	CD'
			5.56	CD'
DD'	0.345	5.547	5.44	DD'
			5.32	DD'
DD	0.099	5.202	5.20	DD
CD	0.260	5.103	5.08	CD
			4.95	CD
CC	0.073	4.843	4.83	CC
CC'	0.902	4.770	4.71	CC'
			4.59	CC'
			.	.
			.	.

The actual sequence of steps in a simulation is shown by the summary output material in Table A-8. The first column of this table shows that this "trial" (simulation) required passage through 24 states to complete the whole cycle. The second column indicates the state names, and shows that in this example the system progressed from state AA to state BB, then switched over to regression state -BB, and regressed to state -AA, after which it resumed transgression. State DD was reached on the fifteenth step, with only one switch to regressive state -CD, and from event 16 through 24 regression was continuous except for one switchover to state CD at 18. Reference to the bottom diagram of text Figure 2 makes clear the path followed in this simulation.

The third column of the summary at the top of Table A-8 lists the actual random number in the range 0.30 to 0.80 that was associated with each state. The next three columns show the reciprocal, its natural log, and the transition time (waiting time) for each state. Column 7 has these times cumulated to a total time interval of 10.160 units for the simulation. The last two columns of the table show the second (uniform) random number used to select the succeeding state. Thus, the first number is 0.0331, which falls in the cumulated value of 1.00 listed in the first line of the state probability matrix of Table A-7. The second number is 0.3348, which falls within the 0.75 value in line 2 of the state probability matrix, indicating forward progress from state AB to BB.

The compilation at the bottom of Table A-8 summarizes the total time in each state and its percentage value. Although some states are occupied more than once, it is evident that in this simulation the percentage of time spent in transgressive states is considerably smaller than that in regressive states. This is of course expected from the much higher rates for transgression than for regression.

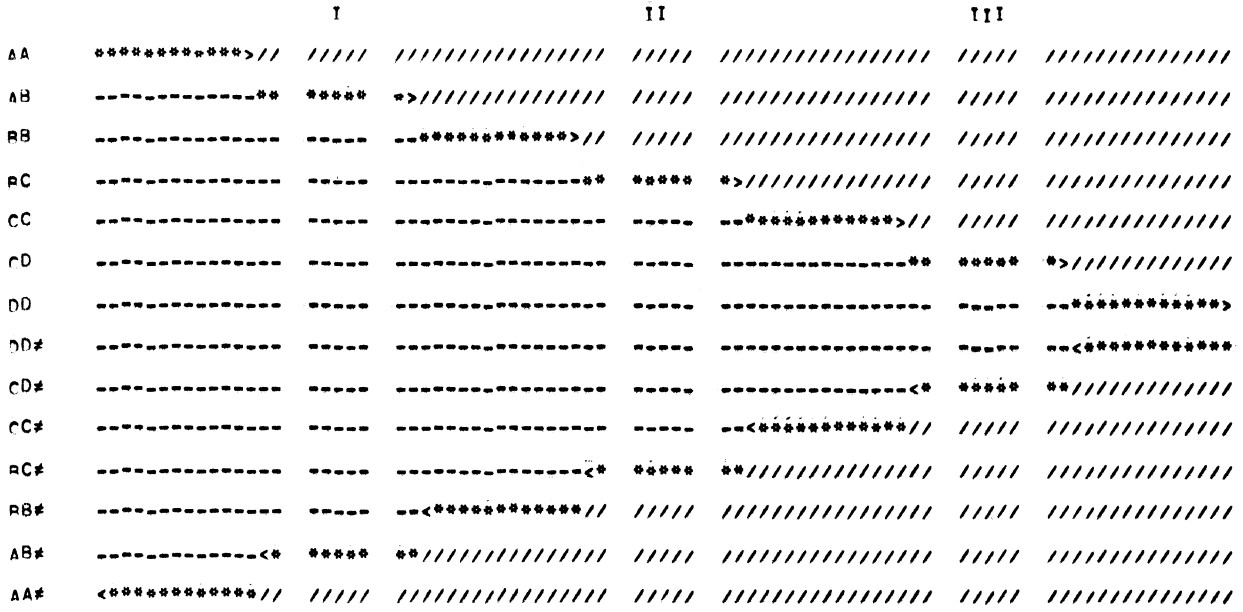
The graphic display of the simulation summarized in Table A-8 is shown in text Figure 3. In preparing the graphic display, the machine simply picks up the appropriate state symbol from the array in the center of Table A-7. These symbol lines

Table A-7. - List of $[q_{ij}]$ input matrix, based on line entries in input deck of Table A-4.

PROJECT 01 0254 14X14 Q(I,J) MATRIX FOR T-R MODEL 3 BOREHOLES
AUGUST 2, 1966 -DD STATE ADDED

MATRIX OF TRANSITION RATES, Q(I,J)

	AA	AB	BB	BC	CC	CD	DD	-DD	-CD	-CC	-BC	-BB	-AB	-AA	MARGIN
AA	0.00	4.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	4.00
AB	0.00	0.00	3.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	4.00
BB	0.00	0.00	0.00	3.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	4.00
BC	0.00	0.00	0.00	0.00	3.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	4.00
CC	0.00	0.00	0.00	0.00	0.00	3.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	4.00
CD	0.00	0.00	0.00	0.00	0.00	0.00	3.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	4.00
DD	0.00	0.00	0.00	0.00	0.00	0.00	0.00	4.00	0.00	0.00	0.00	0.00	0.00	0.00	4.00
-DD	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00
-CD	0.00	0.00	0.00	0.00	0.00	.25	0.00	0.00	0.00	.75	0.00	0.00	0.00	0.00	1.00
-CC	0.00	0.00	0.00	0.00	.25	0.00	0.00	0.00	0.00	0.00	.75	0.00	0.00	0.00	1.00
-BC	0.00	0.00	0.00	.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	.75	0.00	0.00	1.00
-BB	0.00	0.00	.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	.75	0.00	1.00
-AB	0.00	.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	.75	1.00
-AA	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00



MATRIX OF STATE PROBABILITIES FROM Q(I,J) MATRIX

	AA	AB	BB	BC	CC	CD	DD	-DD	-CD	-CC	-BC	-BB	-AB	-AA	MARGIN
AA	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	4.00
AB	0.00	0.00	.75	.75	.75	.75	.75	.75	.75	.75	.75	.75	1.00	1.00	4.00
BB	0.00	0.00	0.00	.75	.75	.75	.75	.75	.75	.75	.75	1.00	1.00	1.00	4.00
BC	0.00	0.00	0.00	0.00	.75	.75	.75	.75	.75	.75	1.00	1.00	1.00	1.00	4.00
CC	0.00	0.00	0.00	0.00	0.00	.75	.75	.75	.75	1.00	1.00	1.00	1.00	1.00	4.00
CD	0.00	0.00	0.00	0.00	0.00	0.00	.75	.75	1.00	1.00	1.00	1.00	1.00	1.00	4.00
DD	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	4.00
-DD	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
-CD	0.00	0.00	0.00	0.00	0.00	.25	.25	.25	.25	1.00	1.00	1.00	1.00	1.00	1.00
-CC	0.00	0.00	0.00	0.00	.25	.25	.25	.25	.25	.25	1.00	1.00	1.00	1.00	1.00
-BC	0.00	0.00	0.00	.25	.25	.25	.25	.25	.25	.25	.25	1.00	1.00	1.00	1.00
-BB	0.00	0.00	.25	.25	.25	.25	.25	.25	.25	.25	.25	.25	1.00	1.00	1.00
-AB	0.00	.25	.25	.25	.25	.25	.25	.25	.25	.25	.25	.25	.25	1.00	1.00
-AA	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table A-8. - Sequence of steps in a simulation.

PROJECT 0102 TRIAL 2

	STATE	RANDA	1.0/RANDA	(LOG E)	TRANSITION TIME	TIME CUM.	RANDB	NEXT STATE
1.	AA	.7184	1.3920	.33073	.083	.083	.0331	AB
2.	AB	.4198	2.3820	.86794	.217	.300	.3348	BB
3.	BB	.6160	1.6234	.48451	.121	.421	.9980	-BB
4.	-BB	.6228	1.6056	.47348	.473	.894	.3681	-AB
5.	-AB	.6329	1.5801	.45747	.457	1.352	.9075	-AA
6.	-AA	.6794	1.4720	.38660	.387	1.738	.1264	AA
7.	AA	.7237	1.3817	.32331	.081	1.819	.5112	AB
8.	AB	.4171	2.3972	.87432	.219	2.038	.6379	BB
9.	BB	.7907	1.2648	.23488	.059	2.096	.1024	BC
10.	BC	.3824	2.6150	.96125	.240	2.337	.1604	CC
11.	CC	.6605	1.5141	.41482	.104	2.440	.2977	CD
12.	CD	.5446	1.8361	.60763	.152	2.592	.8772	-CD
13.	-CD	.5487	1.8225	.60020	.600	3.193	.1235	CD
14.	CD	.5670	1.7636	.56737	.142	3.334	.3712	DD
15.	DD	.7594	1.3168	.27524	.069	3.403	.7314	-DD
16.	-DD	.7786	1.2843	.25023	.250	3.653	.1933	-CD
17.	-CD	.4831	2.0698	.72743	.727	4.381	.1967	CD
18.	CD	.5787	1.7280	.54696	.137	4.518	.8675	-CD
19.	-CD	.4590	2.1787	.77872	.779	5.296	.5309	-CC
20.	-CC	.4536	2.2044	.79045	.790	6.087	.5815	-BC
21.	-BC	.3555	2.8133	1.03496	1.034	7.121	.6971	-BB
22.	-BB	.3643	2.7453	1.00988	1.010	8.131	.7989	-AB
23.	-AB	.4073	2.4551	.89818	.898	9.029	.7242	-AA
24.	-AA	.3227	3.0993	1.13118	1.131	10.160	.8794	AA

STATE	TOTAL TIME IN STATE	0/0 TIME IN STATE
AA	.164	1.6
AB	.436	4.3
BB	.180	1.8
BC	.240	2.4
CC	.104	1.0
CD	.430	4.2
DD	.069	.7
-DD	.250	2.5
-CD	2.106	20.7
-CC	.790	7.8
-BC	1.034	10.2
-BB	1.483	14.6
-AB	1.356	13.3
-AA	1.518	14.9

are so arranged that they show the three boreholes edged with blank spaces, so that as simulation proceeds the corresponding log in each well is built up.

Some remarks are needed here to explain the seeming change in state symbols as given in the text, in the input deck of Table A-4, and in output Table A-8 and various text figures. During preparation of these experimental runs the CDC system at Northwestern University had just changed from 3400 to 6400, and occasional system inconsistencies appeared in the output. These should not interfere with the program, except that in reading this report, it is necessary to accept the complete equivalency of such regressive state symbols as AA', -AA, AA≠, etc. These are partially due to different symbols associated with IBM keypunches 026 and 29.

Table A-9 lists program BOREHOLE, and completes the formal body of this Appendix. However, some remarks are appropriate on the simulation experiments in text Figures 4 and 5, which were obtained by making a few changes in BOREHOLE, and by using MARCHAIN directly as described in Krumbein (1967). Tables A-10 to A-13 are included here to show the types of input used in these special simulations. The tables are taken directly from the initial outputs, which list the input material and hence show the form of the lithologic symbol cards, matrices, etc.

Table A-10 is part of the initial output of MARCHAIN from which text Figure 4 was made. The table lists the lithologic symbol cards used as input, which have the same form as the input symbols for BOREHOLE in the center of Table A-7. The $[p_{ij}]$ matrix of Table 5 is listed and the matrix just below this is the cumulated matrix as used with the uniform random numbers drawn for MARCHAIN. The lower part of Table A-10 shows the last part of the actual simulation, of which the lower part was used in text Figure 4.

In order to adapt BOREHOLE to the simulation of a single stratigraphic column, some internal changes need to be made. As stated earlier, a simulation run (a "trial") is specified internally as passage from AA through DD back to AA'. These instructions, which are covered on the second page of Table A-9, can be changed to a request for a simulation containing say 25 or 50 state changes. With this modification, the $[q_{ij}]$ matrix of text Table 1, with lithologic symbols similar to those used in MARCHAIN can be used as input.

Tables A-11 and A-12 show the controls, the $[q_{ij}]$ matrix input, the lithologic symbol cards, and the matrix of state probabilities for changes of state. The symbol cards for generating the single stratigraphic column are the characteristic cards for MARCHAIN as shown in Table A-13. The simulation instructions called for 50 states, which came to a total time interval of 46.77 units. Table A-12 shows the last part of the simulation, of which the early portion, extending to about time 25.00, was used on the left side of Figure 5. The controls at the top of Table A-11 indicate the random number limits as well as the chosen value of the constant C_1.

Table A-13 is included here to complete the input story. This table shows the initial characteristic output of program MARCHAIN, which lists the input of lithologic symbol cards, the $[p_{ij}]$ matrix from text Table 1, its cumulated version, and the end of the 500 transitions called for in the simulation. Roughly the first 100 of these transitions were used in preparing the right-hand column of text Figure 5.

This adaptability of programs BOREHOLE and MARCHAIN to lateral-shift models and to generation of single stratigraphic sections, means that the experimental stratigrapher now has programs available for comparative experiments based either on transition probabilities or transition rates. One is reminded of the widened horizons opened for comparative map studies several years ago when the conventional polynomial model was augmented by the double Fourier series.

Table A-9. - Program listing for BOREHOLE.

```
      PROGRAM BORHOLE(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C
C     ORIGINAL PROGRAM BY PAUL TUKEY, PRINCETON, 1966
C     MODIFIED BY BETTY BENSON, JULY, 1966
C     CONVERTED TO CDC 6400 BY BETTY BENSON, APRIL 1968.
C     CONTROL CARD CHANGES BY WOLFGANG SCHERER, MAY, 1968
C
      DIMENSION TMTX(41,41),MARG(41)
      COMMON /AA/ PRINT ,UNIT
      COMMON /BB/ KSTATE(501),TIM (500),K,LBL(41),KTRIAL,EMAX
      COMMON /CC/ TITL(36)
      EQUIVALENCE (KSTATE(501),TIME0)
C
      INTEGER DIM
      REAL MARG,LOLIM
      LOGICAL PRINT,VV
C
C                                                    INITIALIZE
      TIME0=0.0
      T=FLOAT(TIME(0))  $  CALL RANSET(T)
C
      READ 99,TITL
      READ 100,DIM,NTRIAL,HILIM,LOLIM,C1,PRINT,NPROJ,UNIT
      PRINT 99,TITL
      WRITE(6,303) DIM,HILIM,LOLIM,C1,NTRIAL,PRINT
      PRINT 1002,T
   99 FORMAT (9A8)
  100 FORMAT (2I5,3F4.2,L5,A10,F10.0)
  303 FORMAT(//* CONTROLS*/1X,8(1H-)//*   DIM=*,I5/* HILIM=*,F5.2/
     .* LOLIM=*,F5.2/*     C1=*,F5.2/*NTRIAL=*,I5/* PRINT=*,L5)
      DO 90 I=1,DIM
      DO 90 J=1,DIM
   90 TMTX(I,J)=0
      MID = (DIM+1)/2
C
      CALL RDMTX (TMTX,LBL,DIM,MARG)
      CALL RDBCD(DIM)
      EMAX=MARG(1)
      DO 7 I=1,DIM
      IF(MARG(I).GT.EMAX) EMAX=MARG(I)
    7 CONTINUE
      IF (UNIT .GT.0) GO TO 5
      UNIT = C1   * ALOG  (1.0/HILIM) / EMAX
C
C                                    CONVERT TMTX TO RATIOS (CUMULATIVE)...PRINT
    5 DO 2 I=1,DIM
      IF(MARG(I).EQ.0.0) GO TO 2
      DO 3 J=1,DIM
    3 TMTX(J,I)=TMTX(J,I)/MARG(I)
      DO 6 J=2,DIM
    6 TMTX(J,I)=TMTX(J,I)+TMTX(J-1,I)
    2 CONTINUE
      WRITE(6,304) TITL
  304 FORMAT (4(9A8/) / 30X,
     .*MATRIX OF STATE PROBABILITIES FROM Q(I,J) MATRIX* /  30X,
     .*------------*                ---                 -----   *)
      CALL KRIS  (TMTX,LBL,DIM,MARG)
C                                    INITIALIZE FOR THIS TRIAL
```

```
      KTRIAL=0
    4 KTRIAL = KTRIAL +1
      IF (PRINT) WRITE(6,300) NPROJ,KTRIAL
      TCUM=0.0
      VV=.FALSE.
      ISTATE=1
C                                   FIND TIME FOR THIS STATE
      DO 20 K=1,500
      IF (ISTATE.EQ.MID) VV=.TRUE.
      IF (MARG(ISTATE) .EQ.0) 12,13
   12 TIM (K) = RLOG=RINV=RANDA=0.0
      GO TO 14
   13 RANDA = RANF(-1)
      IF (RANDA.LT.LOLIM .OR.  RANDA.GT.HILIM) GO TO 13
      RINV=1.0/RANDA
      RLOG =       ALOG  (RINV)
      TIM (K) = RLOG/MARG(ISTATE)
      TCUM=TCUM+TIM (K)
C                                        FIND NEXT STATE
   14 KSTATE(K) = IOLD=ISTATE
       RANDB=RANF(-1)
      DO 15 J=1,DIM
      ISTATE=J
      IF (RANDB .LT. TMTX(J,IOLD)) GO TO 16
   15 CONTINUE
   16 IF (PRINT) WRITE(6,301) K,LBL(IOLD),      RANDA,RINV,RLOG,TIM (K),
     .  TCUM,RANDB,LBL(ISTATE)
      IF (IOLD.EQ.DIM .AND. VV) GO TO 10
   20 CONTINUE
      K=500
C                                              THIS CYCLE IS FINISHED
   10 CALL CUMUL (TCUM,DIM)
      CALL OUTPUT (DIM)
      IF (KTRIAL .LT. NTRIAL) GO TO 4
      STOP
C
 1000 FORMAT (120X4HSEC=F10.4)
 1001 FORMAT (110X4HRANDI4,XF10.3)
 1002 FORMAT (1X,*STARTING TIME FOR RANDOM NUMBERS =*,F12.4)
  300 FORMAT (*1PROJECT*,A8,3X5HTRIAL,I3/// 10X5HSTATE,7X5HRANDA,
     .  3X9H1.0/RANDA,3X9H  (LOG E),3X15HTRANSITION TIME,3X9HTIME CUM.,
     .  7X5HRANDB,3X10HNEXT STATE/)
  301 FORMAT (I4,1H.,5XA6,2F11.4,F12.5,4X,2F13.3,F13.4,5XA6)
      END
      SUBROUTINE RDMTX(TMTX,LBL,DIM,MARG)
      DIMENSION TMTX(41,41),LBL(41),MARG(41)
      COMMON /CC/ TITL(36)
      INTEGER DIM
      REAL MARG
C
C     NOTE   DATA FOR MATRIX MUST TERMINATE WITH A BLANK CARD
      WRITE(6,301) TITL
 4    READ(5,100) J,I,ENTRY
      IF(J.EQ.0) GO TO 3
      TMTX(I,J)=ENTRY
      GO TO 4
 3    CONTINUE
      READ(5,200)(LBL(I),I=1,DIM)
 100  FORMAT(2I5,F10.4)
```

```
 200  FORMAT(12A6)
  301 FORMAT (4(9A8/) / 30X,
     .  *MATRIX OF TRANSITION RATES, Q(I,J)* / 30X,
     .  *----------              --- -*      * )
      DO 2 I=1,DIM
      MARG(I)=0.0
      DO 2 J=1,DIM
    2 MARG(I) = MARG(I) + TMTX(J,I)
C
      ENTRY KRIS
      IF(DIM.GT.19) RETURN
      LBLMARG = 6HMARGIN
      WRITE(6,300) (LBL(I),I=1,DIM),LBLMARG
      DO 1 I=1,DIM
    1 WRITE(6,400) LBL(I),(TMTX(J,I),J=1,DIM),MARG(I)
      RETURN
  300 FORMAT (1H0,9X,20A6)
  400 FORMAT (1H0,A6,3X,20(F4.2,2X))
      END
      SUBROUTINE RDBCD(DIM)
      DIMENSION TITLE1(16),TITLE2(16),LINE(41,16)
      COMMON /BB/ KSTATE(501),TIM (500),K,LBL(41),KTRIAL,EMAX
      COMMON /AA/ PRINT ,UNIT
      IN TEGER DIM,TITLE1,TITLE2
      LOGICAL PRINT
C
      READ(5,102) (TITLE1(J), J=1,16)
      READ(5,102) (TITLE2(J), J=1,16)
      DO 20  I=1,DIM
   20 READ (5,102) (LINE(I,J), J=1,16)
  102 FORMAT (10A8/6A8)
C
      WRITE(6,200) TITLE1,TITLE2
      DO 22  I=1,DIM
   22 WRITE (6,103) (LINE(I,J),J=1,16)
  103 FORMAT(1H0,9X,15A8,A4)
      RETURN
  100 FORMAT (10A8)
  101 FORMAT (6A8)
  200 FORMAT (*1 INPUT FOR GRAPHS ...* // (1H0,9X,15A8,A6))
  201 FORMAT (1H0,9X,15A8,A6)
C
C     ******************************************************************
C
      ENTRY OUTPUT
      DO 2 I=2,K
 2    TIM (I)=TIM (I)+TIM (I-1)
      IF (PRINT) WRITE(6,400) TITLE1,TITLE2
      ENOUGH = TIM (K) - 1000.*UNIT
      CLOC =TIM (K)
C
      DO 3 II=1,K
      I=K+1-II
      ISTATE=KSTATE(I)
    4 IF (PRINT) WRITE(6,500) CLOC ,(LINE(ISTATE,J),J=1,16)
      CLOC =CLOC -UNIT
      IF(CLOC .LT.ENOUGH) GO TO 6
      IF(CLOC .GT.TIM (I-1)) GO TO 4
 3    CONTINUE
```

```
      WRITE(6,501)
      RETURN
 6    LEFT=CLOC /UNIT
      IF(PRINT)WRITE(6,600) LEFT
      RETURN
C
  400 FORMAT (1H1/10X,15A8,A6///4X4HTIME,2X,15A8,A6///*(*)
  500 FORMAT (X,F7.2,2X,15A8,A6)
  501 FORMAT (*)*)
 600  FORMAT(////*PRINTING TERMINATED FOR EXCESSIVE OUTPUT.*,I4,* LINES
     1REMAIN TO BE PRINTED.*/*)*)
      END
      SUBROUTINE CUMUL (TCUM,DIM)
      DIMENSION TOT(41),PCT(41),CUMTOT(41),CUMAVE(41),FREQ(500)
      COMMON /AA/ PRINT,UNIT
      COMMON /BB/ KSTATE(501),TIM (500),K,LBL(41),KTRIAL,EMAX
      INTEGER DIM,FREQ
      LOGICAL PRINT
C
      DATA (CUMTOT=41(0.0))
      DATA (ACUMT=0.0),(NSTATES=0),(FREQ=500(0))
      DATA (KHI=0)
C                                   GET TOTALS AND PERCENTS THIS CYCLE...PRINT
      DO 10 I=1,DIM
   10 TOT(I)=0.0
      DO 1 I=1,K
      ISTATE=KSTATE(I)
    1 TOT(ISTATE) = TOT(ISTATE)+ TIM (I)
      DO 2 I=1,DIM
    2 PCT(I) = 100. * TOT(I)/TCUM
      IF (PRINT) WRITE(6,100) (LBL(I),TOT(I),PCT(I),I=1,DIM)
C               ACCUMULATE INFORMATION FOR THIS CYCLE TO OVERALL TOTALS
      FREQ(K)=FREQ(K)+1
      NSTATES = NSTATES +K
      KHI=MAX0(KHI,K)
      DO 5 J=1,DIM
    5 CUMTOT(J) = CUMTOT(J) + TOT(J)
      ACUMT=ACUMT+TCUM
      RETURN
  100 FORMAT(///* STATE*,4X,*TOTAL TIME IN STATE*,4X,*0/0 TIME IN STATE*
     1 //(XA5,6XF9.3,13XF9.1))
      END
```

Table A-10.- Part of initial output of MARCHAIN from which text Figure 4 was made.

```
PROJECT 01 0254     14X14 P(I,J) MATRIX FOR T-R MODEL      3 BOREHOLES

 TEST RUN FOR PROGRAM GEOMARK


                        LITHOLOGIC SYMBOLS  - INPUT

A    AA    **********>//  /////  ////////////////  /////  ////////////////  /////  ////////////////
B    AB    ----------**  *****  *>////////////////  /////  ////////////////  /////  ////////////////
C    BB    ----------  -----  --**********>////  /////  ////////////////  /////  ////////////////
D    BC    ----------  -----  ----------**  *****  *>////////////////  /////  ////////////////
E    CC    ----------  -----  ----------  -----  -**********>//  /////  ////////////////
F    CD    ----------  -----  ----------  -----  ------------**  *****  *>////////////////
G    DD    ----------  -----  ----------  -----  -----------  -----  --**********>
H    DD≠   ----------  -----  ----------  -----  -----------  -----  --<**********
I    CD≠   ----------  -----  ----------  -----  -----------<*  *****  **////////////////
J    CC≠   ----------  -----  ----------  -----  --<**********//  /////  ////////////////
K    BC≠   ----------  -----  ----------<*  *****  **////////////////  /////  ////////////////
L    BB≠   ----------  -----  --<**********//  /////  ////////////////  /////  ////////////////
M    AB≠   ----------<*  *****  **////////////////  /////  ////////////////  /////  ////////////////
N    AA≠   <**********//  /////  ////////////////  /////  ////////////////  /////  ////////////////

          TRANSITION PROBABILITY MATRIX INPUT
          -----------------------------------
            A      B      C      D      E      F      G      H      I      J      K      L      M      N
     A    .018   .982  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000
     B   0.000   .018   .736  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000   .246  0.000
     C   0.000  0.000   .018   .736  0.000  0.000  0.000  0.000  0.000  0.000   .246  0.000  0.000  0.000
     D   0.000  0.000  0.000   .018   .736  0.000  0.000  0.000  0.000  0.000   .246  0.000  0.000  0.000
     E   0.000  0.000  0.000  0.000   .018   .736  0.000  0.000  0.000  0.000   .246  0.000  0.000  0.000
     F   0.000  0.000  0.000  0.000  0.000   .018   .736  0.000   .246  0.000  0.000  0.000  0.000  0.000
     G   0.000  0.000  0.000  0.000  0.000  0.000   .018   .982  0.000  0.000  0.000  0.000  0.000  0.000
     H   0.000  0.000  0.000  0.000  0.000  0.000  0.000   .368   .632  0.000  0.000  0.000  0.000  0.000
     I   0.000  0.000  0.000  0.000  0.000   .158  0.000  0.000   .368   .474  0.000  0.000  0.000  0.000
     J   0.000  0.000  0.000  0.000   .158  0.000  0.000  0.000  0.000   .368   .474  0.000  0.000  0.000
     K   0.000  0.000  0.000   .158  0.000  0.000  0.000  0.000  0.000  0.000   .368   .474  0.000  0.000
     L   0.000  0.000   .158  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000   .368   .474  0.000
     M   0.000   .158  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000   .368   .474
     N    .632  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000   .368

          CUMULATED PROBABILITY MATRIX
          ----------------------------
     A    .018 1.000 1.000 1.000 1.000 1.000 1.000 1.000 1.000 1.000 1.000 1.000 1.000 1.000
     B   0.000   .018   .754   .754   .754   .754   .754   .754   .754   .754   .754   .754 1.000 1.000
     C   0.000  0.000   .018   .754   .754   .754   .754   .754   .754   .754   .754 1.000 1.000 1.000
     D   0.000  0.000  0.000   .018   .754   .754   .754   .754   .754   .754 1.000 1.000 1.000 1.000
     E   0.000  0.000  0.000  0.000   .018   .754   .754   .754   .754 1.000 1.000 1.000 1.000 1.000
     F   0.000  0.000  0.000  0.000  0.000   .018   .754   .754 1.000 1.000 1.000 1.000 1.000 1.000
     G   0.000  0.000  0.000  0.000  0.000  0.000   .018 1.000 1.000 1.000 1.000 1.000 1.000 1.000
     H   0.000  0.000  0.000  0.000  0.000  0.000  0.000   .368 1.000 1.000 1.000 1.000 1.000 1.000
     I   0.000  0.000  0.000  0.000  0.000   .158   .158   .158   .526 1.000 1.000 1.000 1.000 1.000
     J   0.000  0.000  0.000  0.000   .158   .158   .158   .158   .526 1.000 1.000 1.000 1.000 1.000
     K   0.000  0.000  0.000   .158   .158   .158   .158   .158   .158   .526 1.000 1.000 1.000 1.000
     L   0.000  0.000   .158   .158   .158   .158   .158   .158   .158   .158   .526 1.000 1.000 1.000
     M   0.000   .158   .158   .158   .158   .158   .158   .158   .158   .158   .158   .526 1.000 1.000
     N    .632   .632   .632   .632   .632   .632   .632   .632   .632   .632   .632   .632   .632 1.000

                              CLOCK=*55590.979

          TRANSITIONS=   500

                              START= A

                                        I                       II                      III
500  CD    ----------  -----  ----------  -----  ----------**  *****  *>////////////////
499  CD    ----------  -----  ----------  -----  ----------**  *****  *>////////////////
498  CD≠   ----------  -----  ----------  -----  ----------<*  *****  **////////////////
497  DD≠   ----------  -----  ----------  -----  -----------  -----  --<**********
496  DD≠   ----------  -----  ----------  -----  -----------  -----  --<**********
495  DD≠   ----------  -----  ----------  -----  -----------  -----  --<**********
494  DD    ----------  -----  ----------  -----  -----------  -----  --**********>
493  CD    ----------  -----  ----------  -----  ----------**  *****  *>////////////////
492  CD≠   ----------  -----  ----------  -----  ----------<*  *****  **////////////////
491  DD≠   ----------  -----  ----------  -----  -----------  -----  --<**********
490  DD    ----------  -----  ----------  -----  -----------  -----  --**********>
489  CD    ----------  -----  ----------  -----  ----------**  *****  *>////////////////
488  CD≠   ----------  -----  ----------  -----  ----------<*  *****  **////////////////
487  CD≠   ----------  -----  ----------  -----  ----------<*  *****  **////////////////
486  DD≠   ----------  -----  ----------  -----  -----------  -----  --<**********
485  DD    ----------  -----  ----------  -----  -----------  -----  --**********>
484  CD    ----------  -----  ----------  -----  ----------**  *****  *>////////////////
483  CC    ----------  -----  ----------  -----  -**********>//  /////  ////////////////
482  BC    ----------  -----  ----------**  *****  *>////////////////  /////  ////////////////
481  BB    ----------  -----  --**********>//  /////  ////////////////  /////  ////////////////
480  AB    ----------**  *****  *>////////////////  /////  ////////////////  /////  ////////////////
479  AA    **********>//  /////  ////////////////  /////  ////////////////  /////  ////////////////
```

Table A-11.- Controls, $[q_{ij}]$ matrix input, lithologic symbol cards and matrix of state probabilities for changes of state.

KRUMBEIN PROJECT NO. 01 0254 (SCHERER WELL NO. 3)

4 X 4 Q(I,J) MATRIX USED IN PROGRAM BOREHOLE

CONTROLS

```
  DIM=    4
HILIM=  .80
LOLIM=  .30
   C1= 2.16
TRIAL=    2
PRINT=    T
STARTING TIME FOR RANDOM NUMBERS =*239376.0000
```

MATRIX OF TRANSITION RATES, Q(I,J)

	A	B	C	D	MARGIN
A	0 00	.18	.18	.13	.48
B	.08	0 00	.25	.14	.46
C	.31	.51	0 00	.28	1.10
D	.48	1.10	.39	0 00	1.97

INPUT FOR GRAPHS ...

A	******
B	------
C	.....
D	-/-/-/

MATRIX OF STATE PROBABILITIES FROM Q(I,J) MATRIX

	A	B	C	D	MARGIN
A	0 00	.37	.74	1.00	.48
B	.16	.16	.70	1.00	.46
C	.28	.74	.74	1.00	1.10
D	.24	.80	1.00	1.00	1.97

Table A-12. Last part of simulation, of which the early portion was used for left side of Figure 5.

46.77	B	------
46.52	B	------
46.28	B	------
46.03	B	------
45.79	D	-/-/-/
45.54	B	------
45.30	B	------
45.05	B	------
44.81	B	------
44.56	B	------
44.32	C	.....
44.07	C	.....
43.83	A	******
43.58	A	******
43.34	A	******
43.09	A	******
42.85	A	******
42.60	A	******
42.36	C	.....
42.11	C	.....
41.87	B	------
41.62	B	------
41.38	B	------
41.13	B	------
40.89	C	.....
40.64	C	.....
40.40	C	.....
40.15	C	.....
39.91	B	------
39.66	B	------
39.42	B	------
39.17	B	------
38.93	B	------
38.68	B	------
38.44	B	------
38.19	B	------
37.95	C	.....
37.70	C	.....
37.46	C	.....
37.21	C	.....
36.97	B	------
36.72	B	------
36.48	B	------
36.23	B	------
35.99	B	------
35.74	D	-/-/-/
35.50	D	-/-/-/
35.25	C	.....
35.01	C	.....
34.76	C	.....
34.52	C	.....
34.27	A	******
34.03	A	******
33.78	D	-/-/-/
33.54	D	-/-/-/
33.29	D	-/-/-/
33.05	A	******
32.80	A	******
32.56	A	******
32.31	A	******
32.07	A	******
31.82	A	******
31.58	A	******
31.33	A	******
31.09	B	------
30.84	B	------
30.60	B	------
30.35	C	.....
30.11	C	.....
29.86	C	.....
29.62	C	.....
29.37	B	------
29.13	B	------
28.88	B	------
28.64	C	.....
28.39	C	.....
28.15	C	.....

Table A-13.- Initial characteristic output of program MARCHAIN.

KRUMBEIN PROJECT NO. 01 0254 (SCHERER WELL NO. 3)

4 X 4 P(I,J) MATRIX USED IN PROGRAM GEOMARK

LITHOLOGIC SYMBOLS - INPUT

A	A	******
B	B	------
C	C	••••••
D	D	-/-/-/

TRANSITION PROBABILITY MATRIX INPUT

	A	B	C	D
A	.620	.140	.140	.100
B	.060	.630	.200	.110
C	.190	.310	.330	.170
D	.210	.480	.170	.140

CUMULATED PROBABILITY MATRIX

A	.620	.760	.900	1.000
B	.060	.690	.890	1.000
C	.190	.500	.830	1.000
D	.210	.690	.860	1.000

CLOCK=*55590.979

TRANSITIONS= 500

START= D

500	A	******
499	A	******
498	A	******
497	A	******
496	A	******
495	A	******
494	A	******
493	A	******
492	C	••••••
491	C	••••••
490	A	******
489	A	******
488	B	------
487	D	-/-/-/
486	D	-/-/-/
485	B	------
484	B	------

KANSAS GEOLOGICAL SURVEY COMPUTER PROGRAM
THE UNIVERSITY OF KANSAS, LAWRENCE

PROGRAM ABSTRACT

Title (If subroutine state in title):

FORTRAN IV computer program for simulation of transgression and regression with continuous-time Markov models

Date: Final versions, May 1968

Author, organization: W.C. Krumbein, Department of Geology
Northwestern University, Evanston, Illinois

Direct inquiries to: Author, or

Name: D.F. Merriam Address: Kansas Geological Survey
University of Kansas, Lawrence

Purpose/description: PEQUMAT transforms a transition probability matrix to its corresponding transition rate matrix and vice versa. BOREHOLE simulates lateral-shift phenomena, such as transgression and regression with a continuous-time discrete-state Markov model.

Mathematical method: PEQUMAT is based on mathematical relations between the elements of a $[p_{ij}]$ matrix and its corresponding $[q_{ij}]$ matrix for a given Δt, as described in the text. BOREHOLE uses random numbers for waiting times and state changes.

Restrictions, range:

Computer manufacturer: Control Data Corporation Model: 6400

Programming language: Modified FORTRAN IV

Memory required: K Approximate running time:

Special peripheral equipment required:

Remarks (special compilers or operating systems, required word lengths, number of successful runs, other machine versions, additional information useful for operation or modification of program)

COMPUTER CONTRIBUTIONS

Kansas Geological Survey
University of Kansas
Lawrence, Kansas

Computer Contribution

1. Mathematical simulation of marine sedimentation with IBM 7090/7094 computers, by J.W. Harbaugh, 1966 . . . . . $1.00
2. A generalized two-dimensional regression procedure, by J.R. Dempsey, 1966 . . . . . $0.50
3. FORTRAN IV and MAP program for computation and plotting of trend surfaces for degrees 1 through 6, by Mont O'Leary, R.H. Lippert, and O.T. Spitz, 1966 . . . . . $0.75
4. FORTRAN II program for multivariate discriminant analysis using an IBM 1620 computer, by J.C. Davis and R.J. Sampson, 1966 . . . . . $0.50
5. FORTRAN IV program using double Fourier series for surface fitting of irregularly spaced data, by W.R. James, 1966 . . . . . $0.75
6. FORTRAN IV program for estimation of cladistic relationships using the IBM 7040, by R.L. Bartcher, 1966 . . . . . $1.00
7. Computer applications in the earth sciences: Colloquium on classification procedures, edited by D.F. Merriam, 1966 . . . . . $1.00
8. Prediction of the performance of a solution gas drive reservoir by Muskat's Equation, by Apolonio Baca, 1967 . . . . . $1.00
9. FORTRAN IV program for mathematical simulation of marine sedimentation with IBM 7040 or 7094 computers, by J.W. Harbaugh and W.J. Wahlstedt, 1967 . . . . . $1.00
10. Three-dimensional response surface program in FORTRAN II for the IBM 1620 computer, by R.J. Sampson and J.C. Davis, 1967 . . . . . $0.75
11. FORTRAN IV program for vector trend analyses of directional data, by W.T. Fox, 1967 . . . $1.00
12. Computer applications in the earth sciences: Colloquium on trend analysis, edited by D.F. Merriam and N.C. Cocke, 1967 . . . . . $1.00
13. FORTRAN IV computer programs for Markov chain experiments in geology, by W.C. Krumbein, 1967 . . . . . $1.00
14. FORTRAN IV programs to determine surface roughness in topography for the CDC 3400 computer, by R.D. Hobson, 1967 . . . . . $1.00
15. FORTRAN II program for progressive linear fit of surfaces on a quadratic base using an IBM 1620 computer, by A.J. Cole, C. Jordan, and D.F. Merriam, 1967 . . . . . $1.00
16. FORTRAN IV program for the GE 625 to compute the power spectrum of geological surfaces, by J.E. Esler and F.W. Preston, 1967 . . . . . $0.75
17. FORTRAN IV program for Q-mode cluster analysis of nonquantitative data using IBM 7090/7094 computers, by G.F. Bonham-Carter, 1967 . . . . . $1.00
18. Computer applications in the earth sciences: Colloquium on time-series analysis, D.F. Merriam, editor, 1967 . . . . . $1.00
19. FORTRAN II time-trend package for the IBM 1620 computer, by J.C. Davis and R.J. Sampson, 1967 . . . . . $1.00
20. Computer programs for multivariate analysis in geology, D.F. Merriam, editor, 1968 . . . $1.00
21. FORTRAN IV program for computation and display of principal components, by W.J. Wahlstedt and J.C. Davis, 1968 . . . . . $1.00
22. Computer applications in the earth sciences: Colloquium on simulation, D.F. Merriam and N.C. Cocke, editors, 1968 . . . . . $1.00
23. Computer programs for automatic contouring, by D.B. McIntyre, D.D. Pollard, and R. Smith, 1968 . . . . . $1.50
24. Mathematical model and FORTRAN IV program for computer simulation of deltaic sedimentation, by G.F. Bonham-Carter and A.J. Sutherland, 1968 . . . . . $1.00
25. FORTRAN IV CDC 6400 computer program for analysis of subsurface fold geometry, by E.H.T. Whitten, 1968 . . . . . $1.00
26. FORTRAN IV computer program for simulation of transgression and regression with continuous-time Markov models, by W.C. Krumbein, 1968 . . . . . $1.00