DANIEL F. MERRIAM, Editor

FORTRAN IV PROGRAM FOR THE GE 625 TO COMPUTE THE POWER SPECTRUM OF GEOLOGICAL SURFACES

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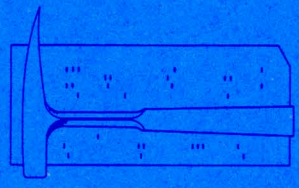

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Editor's Remarks

A recent world public opinion inquiry was made among earth scientists for information in regard to the forthcoming International Geological Congress in Praha. It was emphasized by Jiří Krupička of Czechoslovakia, that we are accumulating knowledge at an incredibly fast pace. Because of the nature of the acquired data, much of which is difficult to compartmentalize, more questions in geology are being raised than answered. More than ever a synthesis of available material is needed. It would seem that many old convictions now are being replaced by new and more up-to-date ideas. In other words, we are attempting better interpretations based on more complete information.

New information, however, is being generated at an ever increasing rate and it is almost impossible to keep up. As a result, there is needless duplication of effort. This situation, according to Krupička, demands a new approach. He states, "The new methods and techniques born by the modern scientific and technical revolution are being vehemently employed for the acquisition of new special data. Should they not, especially the most recent methods of the computer technique, be even preferentially used for a better evaluation of the existing data, for a more convenient storage, a much faster distribution, and above all, for an objective pretreatment in such a way that the highest possible quantity of information could be treated simultaneously by one mind?" Of course, these new methods are being utilized by earth scientists world-wide, as shown by the distribution of the COMPUTER CONTRIBUTION Series.

COMPUTER CONTRIBUTIONS are one of many publications making the latest results available to scientists. It is an important outlet because it is the only series featuring computer techniques and applications. As aptly stated by W.C. Krumbein and F.A. Graybill (An introduction to statistical models in geology, McGraw-Hill Book Co., 1965), "...The high-speed computer has opened new doors in preliminary data evaluation and in the selection of appropriate models for analyzing sets of geological observations...."

With the readily available third-generation computers, effort is now being directed towards formation and maintenance of geologic information files. Various techniques are being developed and tested to utilize this vast amount of information. The program reported here, "FORTRAN IV program for the GE 625 to compute the power spectrum of geologic surfaces," by J.E. Esler and F.W. Preston, is just such a technique. This program is designed to test for periodicity in geologic data. Geologists and other earth scientists should find an immediate use for the program in analyzing data in a variety of problems. It will be useful especially to those who are concerned with describing and analyzing topographic or structural surfaces.

For a limited time, the Kansas Geological Survey will make available this program on magnetic tape for $10.00.

An up-to-date listing of available COMPUTER CONTRIBUTIONS and computer programs available can be obtained by writing Editor, COMPUTER CONTRIBUTION Series, Kansas Geological Survey, the University of Kansas, Lawrence, Kansas 66044. Your comments and suggestions in regard to this series will be most appreciated.

FORTRAN IV PROGRAM FOR THE GE 625 TO COMPUTE THE POWER SPECTRUM OF GEOLOGICAL SURFACES

By

JAMES E. ESLER and FLOYD W. PRESTON

NUMERICAL CHARACTERIZATION OF SURFACES

Geologists and geographers have always been interested in the description and characterization of surfaces. Methods used to date mainly have been written descriptions and mathematical approximations. In the first category are discussions of landforms by early geologists and geographers. These were poetic and pictorial, but not useful in an operational sense. Descriptions by one investigator could not, in general, be used by another investigator to reconstruct a three-dimensional likeness of the surface.

To overcome the inadequacies of verbal descriptions, earth scientists have turned to numerical approximations of surfaces. These possess a degree of objectivity not present in verbal accounts, and are consistent and reproducible by other investigators. Statistics of numerically defined surfaces can be computed, and the surfaces can be compared by numerical techniques.

Least squares is the most widely used surface-fitting criterion, although there are others such as minimax and absolute deviation criteria. Mathematical models fitted to observed surfaces by least squares include polynomial and Fourier series approximations. Excellent reviews of these forms are given by Krumbein (1966a, 1966b) and James (1966).

All presently used mathematical representations of surfaces built around the least-squares criterion are based on the general linear regression model. A geologic surface is considered to be composed of a regional component (trend) and a residual component. The residual component in turn is composed of a local variation confounded with random error. Components are separated by the regression as a means of estimating variances in the surface due to regional effects and local effects, random fluctuation or error being neglected. In some geologic studies this assumption seems reasonable, as in trend-surface mapping of well-log data where random measurement errors are extremely small compared to other factors influencing the surface. If the residuals do not contain systematic variations, the trend is assumed to contain all geologically significant variation (regional and local) and represents an appropriate generalization of the surface.

Utility of the general linear regression model for description of geologic surfaces depends upon the ability of the investigator to determine the point where regional and residual components are most effectively separated.

Well-conceived methods have been presented to determine the order of an equation necessary to describe major elements of a surface. Despite impressive efforts in trend analysis there is growing uneasiness as to the appropriate interpretation of "trend," particularly when surfaces of high order are generated. The basic problem is distinguishing between "trend" and "random" components. In the phraseology of electronics engineering, these elements would be called "signal" and "noise." In this regard, an admonition from the electronics engineer may be cited: if one wishes to extract signal from noise, one must know, a priori, the inherent nature of either the signal or the noise. This a priori insight does not seem to be available for most geological and geographical surfaces.

Another model, which assumes the surface to have little or no "trend" above a first- or possibly a second-degree polynomial, may be considered. The observed surface essentially is noise superimposed upon this weak trend. The noise or variability is a fundamental characteristic of the surface and is considered both necessary and sufficient for description of the surface. Surfaces that are similar are assumed to have similar variabilities; surfaces with different variabilities are assumed to be different. This supposes that judgments of similarity based on variability agree with judgments of similarity made by conventional visual inspection. The measure of variability used is the power spectrum.

In this model, the surface is considered to be oscillatory or undulating in two perpendicular directions such as east-west and north-south. Within a given frequency band for these oscillations, all frequencies are considered present. The power spectrum measures the relative contribution of each frequency to the variability or oscillatory nature of the surface.

USE OF POWER SPECTRUM IN DESCRIBING GEOLOGICAL SURFACES

Geologists are accustomed to thinking of a map as a scaled replica of the original region. More properly, maps should be considered as generalizations created from information that is available concerning the original surface. If the data are surface elevations plotted as a function of position in an east-west, north-south coordinate system, the result is a topographic map. Two sets of data at the same location but representing elevations of top and bottom of one particular geological unit can be used to prepare an isopach map. The actual surface or thickness of the unit is only approximately represented by the diagram. The map is neither the surface itself nor a true scaled representation, but is only a generalization.

A less conventional method of map generalization has been developed by Krumbein (1966b), who introduced the concept of "coefficient space." If the method of least squares is used to fit a polynomial or Fourier series in two independent directions to surface data, the resulting trend surface is a generalization of the original surface. As Krumbein points out, however, the same "information" concerning the trend surface also is contained in the set of polynomial or Fourier coefficients. These coefficients may be displayed as a two-dimensional array. If an investigator is searching for regularity or order in the surface, this may be more readily found in the array of coefficients than in the surface itself. The power-spectrum method of surface description, as discussed by Preston (1966) and by Rayner (1967), can be considered as surface representation in a "coefficient space" sense. However, the power-spectrum technique utilizes a "frequency space." As in "coefficient space" techniques, points on the power-spectrum map do not correspond to points in the sample space.

THE POWER SPECTRUM

The concept of the power spectrum has been developed by electrical engineers and used to analyze signals that are largely random in nature.

The method has been successfully used to detect signals contaminated by random noise (Blackman and Tukey, 1959; Diamantides, 1961), to analyze sea waves (Pierson, 1960), and to characterize and classify porous media (Preston, Green, and Aldenderfer, 1966). Most work has been concerned with one-dimensional random data (Lee, 1960; Blackman and Tukey, 1959). A few workers have extended their investigations to two dimensions, especially W. J. Pierson, who studied the configuration of a wind-generated oceanic surface. The program described here follows his method of development. If a geological profile or cross section can be expressed as an unknown function, f(x), then various approximations can be used to represent f(x). One common method of approximating the function f(x) is through the use of Fourier series, here given in its one-dimensional trigonometric form:

$$f(x) \cong \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{2\pi nx}{L} + b_n \sin \frac{2\pi nx}{L}\right) \quad (1)$$

where

a_n , b_n = Coefficients of the zeroth through the nth harmonic
L = fundamental period or sample length
x = distance along measurement axis

Using Euler's Identity, equation (1) can be represented in exponential form:

$$f(x) \cong \sum_{n=-\infty}^{\infty} F(n)\, e^{i \frac{2\pi nx}{L}} \quad (2)$$

where

$$F(n) = 1/2 \sqrt{a_n - ib_n} \quad (3)$$

The magnitude of the function F(n) is the absolute value $|F(n)|$ of the amplitude of the nth harmonic in the function f(x). The set of values of $|F(n)|$ for all n is called the amplitude spectrum. The square of the amplitude spectrum, $S = |F(n)|^2$, is the power spectrum of the function f(x).

In analysis of stationary random functions, another method of deriving the power spectrum has been developed. It begins by defining the autocorrelation function Q as:

$$Q(\tau) = \frac{1}{T_1} \int_{-\frac{1}{2}T_1}^{+\frac{1}{2}T_1} f(x)\, f(x+\tau)\, dx \quad (4)$$

where

τ = value of lag

T_1 = period of the fundamental angular frequency, w_1

The relationship between T_1 and w_1 is:

$$T_1 = 2\pi / w_1 \quad (5)$$

Lee (1960) has shown that Q is an <u>even</u> function, that is, f(t) = f(-t). Q is also the Fourier transform of the power spectrum. Since the autocorrelation function is even, the sine term in the transform is zero. Therefore, the power spectrum S is given by:

$$S = \frac{1}{T_1}\int_{-\frac{1}{2}T_1}^{+\frac{1}{2}T_1} Q(T)\cos nw_1 T\, dT \qquad (6)$$

The finite, discrete approximation for the autocorrelation function, where $X_1, X_2, X_3, \ldots X_n$ are given values of the signal or function equally spaced in time or space, can be approximated by:

$$Q_p \cong \frac{1}{n-p}\sum_{k=1}^{n-p} X_k\, X_{k+p} \qquad (7)$$

where

$$p = 0, 1, 2, \ldots \tau$$

and τ is the maximum value of lag.

In order to weight values of Q properly to allow for extension into the negative p region, the following definitions are used:

$$Q_o^* = Q_o$$

$$Q_p^* = 2Q_p \qquad p = 1, 2, \ldots \tau-1$$

$$Q_\tau^* = Q_\tau$$

A finite approximation of the Fourier Transform will then give a rough approximation of the power spectrum:

$$S_h \cong \frac{1}{2\tau}\sum_{p=o}^{p=\tau} Q_p^* \cos\frac{\pi p h}{\tau} \qquad (8)$$

$$h = 0, 1, 2, \ldots \tau$$

Experience has shown that this rough estimate of the power spectrum should be "smoothed" by a moving average. Two sets of weights commonly used for the smoothing function are:

$$SS_h = .25\, S_{h-1} + .50\, S_h + .25\, S_{h+1} \qquad (9a)$$

$$SS_h = .23\, S_{h-1} + .54\, S_h + .23\, S_{h+1} \qquad (9b)$$

where the assumption is made that

$$S_{-1} = S_1$$

$$S_{\tau+1} = S_{\tau-1}$$

Applications discussed in this paper use the weighting function given in Equation 9a.

Each of the values of SS_h can be considered as an estimate of the contribution to the total sample variance made by frequencies in the range between $2\pi(h-1/2)/2\tau\Delta t$ and $2\pi(h+1/2)/2\tau\Delta t$, where Δt represents the time or space interval between successive measurements.

Equations for the two-dimensional case are similar to those described above and can be developed by analogy. The autocorrelation function is given by the formula:

$$Q_{(p,q)} = \frac{1}{(n-p)(m-|q|)}\sum_{k=|q|}^{m}\sum_{i=1}^{n-p} X_{i,k}\, X_{i+p,\,k+q}$$

for

$$p = 0, 1, 2, \ldots \tau$$

$$q = -\tau, -(\tau-1), \ldots -1 \qquad (10)$$

and

$$Q_{(p,q)} = \frac{1}{(n-p)(m-|q|)}\sum_{k=1}^{m-|q|}\sum_{i=1}^{n-p} X_{i,k}\, X_{i+p,\,k+q}$$

for

$$p = 0, 1, 2, \ldots \tau$$

$$q = 0, 1, 2, \ldots \tau$$

where m and n represent the number of data points along the two coordinates.

In order to weight values of Q properly, the following adjustments are made:

$$Q^*(p,q) = 2Q(p,q) \qquad p = 1 \text{ to } (\tau-1),\ q = -(\tau-1) \text{ to } (\tau-1)$$

$$Q^*(0,q) = Q(0,q) \qquad q = -(\tau-1) \text{ to } (\tau-1)$$

$$Q^*(\tau,q) = Q(\tau,q) \qquad q = -(\tau-1) \text{ to } (\tau-1)$$

$$Q^*(p,\tau) = Q(p,\tau) \qquad p = 1 \text{ to } (\tau-1)$$

$$Q^*(p,-\tau) = Q(p,-\tau) \qquad p = 1 \text{ to } (\tau-1)$$

$$Q^*(0,\tau) = 1/2\,(0,\tau)$$

$$Q^*(0,-\tau) = 1/2\,(0,-\tau)$$

$$Q^*(\tau,\tau) = 1/2\,Q(\tau,\tau)$$

$$Q^*(\tau,-\tau) = 1/2\,Q(\tau,-\tau)$$

These formulas assume that the maximum lag used in each of the two dimensions is the same. This assumption greatly simplifies calculation of the Fourier transform. For most problems, this seems a reasonable assumption and does not greatly decrease the generality.

Unsmoothed estimates of the power spectrum are then given by:

$$S(r,s) \cong \frac{1}{2\tau^2} \sum_{q=-\tau}^{\tau} \sum_{p=0}^{\tau} Q^*(p,q) \cos \frac{\pi}{\tau}(rp + sq) \qquad (11)$$

$$r = 0, 1, 2, \ldots \tau$$

$$s = -\tau, -(\tau-1), \ldots -1, 0, 1, \ldots \tau$$

The following conditions are used so that the smoothing process may be continued to the edge of the data set:

$$S(-1,b) = S(1,b) \quad b = -\tau \text{ to } \tau$$

$$S(\tau+1,b) = S(\tau-1,b) \quad b = -\tau \text{ to } \tau$$

$$S(a,\tau+1) = S(a,\tau-1) \quad a = 0 \text{ to } \tau$$

$$S(a,-\tau-1) = S(a,-\tau+1) \quad a = 0 \text{ to } \tau$$

$$S(-1,\tau+1) = S(1,\tau-1)$$

$$S(\tau+1,\tau+1) = S(\tau-1,\tau-1)$$

$$S(\tau+1,-\tau-1) = S(\tau-1,-\tau+1)$$

$$S(-1,-\tau-1) = S(1,-\tau+1)$$

The smoothing function used is an extension of the one-dimensional case (Equation 9a), a product of two linear smoothing functions which gives a square array of nine values. This function is a two-dimensional moving average with .25, .50, and .25 weightings for points in each direction.

	0.25	0.50	0.25
0.25	0.0625	0.125	0.0625
0.50	0.125	0.250	0.125
0.25	0.0625	0.125	0.0625

The smoothed spectrum is given by:

$$SS(r,s) = 0.0625 [S(r-1,s-1) + S(r-1,s+1) + S(r+1,s-1) + S(r+1,s+1)] + 0.125 [S(r-1,s) + S(r+1,s) + S(r,s-1) + S(r,s+1)] + 0.250 [S(r,s)] \qquad (13)$$

Final values of the smoothed power spectrum, SS(r,s), represent contributions to total variance made by frequencies between $2\pi(r-1/2)/2\tau\Delta t$ and $2\pi(r+1/2)/2\tau\Delta t$ in the r direction, and between $2\pi(s-1/2)/2\tau\Delta t$ and $2\pi(s+1/2)/2\tau\Delta t$ in the s direction.

Removal of Linear Trend

Data often will contain a linear trend which distorts the power spectrum. This trend can easily be removed using standard least-squares techniques, by fitting a plane of the form $X'(j,k) = Aj+Bk+C$ to the data points and using residuals for calculating the power spectrum. Pierson (1960) has shown that for equally spaced data the constants A, B, and C can be found from the equation:

$$\begin{bmatrix} \frac{mn(n+1)(2n+2)}{6} & \frac{n(n+1)m(m+1)}{4} & \frac{mn(n+1)}{2} \\ \frac{n(n+1)m(m+1)}{4} & \frac{mn(m+1)(2m+2)}{6} & \frac{mn(m+1)}{2} \\ \frac{mn(n+1)}{2} & \frac{mn(m+1)}{2} & \frac{mn}{1} \end{bmatrix} \cdot \begin{bmatrix} A \\ B \\ C \end{bmatrix}$$

$$= \begin{bmatrix} \sum_{j=1}^{n} \sum_{k=1}^{m} jX_{j,k} \\ \sum_{j=1,k=1}^{n,m} kX_{j,k} \\ \sum_{j=1}^{n} \sum_{k=1}^{m} X_{j,k} \end{bmatrix} \qquad (14)$$

Solving for A, B, and C and subtracting $X'(j,k)$ from the data matrix will then yield a new data matrix better adapted to power spectrum analysis.

Choice of Parameters

Accuracy and usefulness of the power spectrum depends upon the choice of certain parameters, such as the sampling interval Δt and the maximum value of lag τ. Proper choices for these parameters will minimize aliasing, or failure to recognize higher

harmonics, and will maximize degrees of freedom and resolution. Bendat (1966) suggested the sampling interval Δt should be given by:

$$\Delta t = \frac{1}{2f_c} \tag{15}$$

where f_c is the lowest frequency of interest in the record along either of the two axes. This gives two points per cycle at the cutoff frequency f_c. Wherever possible, more points should be used for improved results. An accurate autocorrelation function, Q, can be formed by taking

$$\Delta t = \frac{1}{4f_c} \tag{16}$$

If the power spectrum is of prime concern, a spacing of

$$\Delta t = \frac{2}{5f_c} \tag{17}$$

is sufficient. Values of Δt as close as possible to $1/2f_c$ are, of course, most economical, as fewer points are needed.

The maximum value of lag, τ, should be limited by the smaller of the two dimensions of the data matrix. Several limits have been suggested for τ in the one dimensional case. For instance, Granger and Hatanaka (1964) recommended $\tau \leq m/3$ and Blackman (1965) recommends $\tau \leq m/10$, as does Crowson (1963) who performed a very thorough error analysis. Because a two dimensional problem involves a larger number of data points a value of $\tau \leq m/4$ should provide adequate results.

Another important aspect to consider in the choice of τ is the equivalent resolution bandwidth, β_c, desired for power spectrum calculations. Bendat has determined β_c to be given by:

$$\beta_c = \frac{1}{\tau \Delta t} \tag{18}$$

For a given Δt, β_c will decrease as τ increases.

The degrees of freedom of each spectral estimate in the one-dimensional case have a Chi Square distribution with

$$\text{d.f.} = 2\left(\frac{n}{\tau} - \frac{1}{4}\right). \tag{19}$$

Pierson (1960) has expanded this formula to two dimensions to give:

$$\text{d.f.} = 1.58\left(\frac{n}{\tau} - \frac{1}{2}\right)\left(\frac{m}{\tau} - \frac{1}{2}\right) \tag{20}$$

Equation (20) may underestimate the true degrees of freedom. Values of 1/4 instead of 1/2 therefore may weight equation (20) more properly.

PROGRAM DESCRIPTION

The program to compute two-dimensional power spectra consists of a mainline routine, PSMAIN, and two subroutines, PS and PLOT. PSMAIN reads raw data and processes it according to instructions on the control cards. PS levels the data if necessary, and computes autocorrelation functions and power spectra. It also produces any punched or printed output requested. Printed output is plotted by subroutine PLOT, which can be called by PS or PSMAIN, according to options on the control cards.

Two forms of printed output for the autocorrelation function and rough and smoothed power function are provided by subroutine PLOT. The first option is a numerical listing of the functions as a matrix of three digit integers with a scaling factor giving their order of magnitude. This plotting system does not depict symmetry associated with evenness of either the autocorrelation function or the power spectrum. Alternately, a contour plot may be produced, in which each symbol represents tenths of the maximum value. A scaling factor indicates approximate values of contours. This plotting system shows symmetry of the functions about their origins.

OPERATIONAL INSTRUCTIONS FOR PSMAIN

CARD 1

Col. 1	NMBR	- Number of data sets to be run

CARD 2

Col. 1	KALL	- 1 = calculate power spectrum 2 = plot previously punched autocorrelation function or power spectrum
Col. 2	IPLOTA	- Plot autocorrelation function*
Col. 3	IPLOTR	- Plot rough power spectrum*
Col. 4	IPLOTS	- Plot smoothed power spectrum*
Col. 5	IPNCHA	- Punch autocorrelation function#
Col. 6	IPNCHS	- Punch smoothed power spectrum#

CARD 3

Col. 1-78 IDENT - Any alphanumeric title

CARD 4

Col. 1-3 LAG - Maximum value of lag

Col. 4 IBID - If KALL = 1, then
- 0 = begin with auto-correlation
- 1 = begin with power spectrum
- 2 = begin with removed linear trend
- 3 = begin with removed linear trend and punch leveled data

If KALL = 2, then
- 1 = plot autocorrelation function
- 2 = plot smoothed power spectrum
- 3 = rough plot spectrum

Col. 5-7 N - Number of data points along horizontal axis

Col. 8-10 M - Number of data points along vertical axis

Col. 11-70 IFMT - Format to read data. Left parenthesis must be in column 11. Right parenthesis follows format statement

Col. 71 IFIRST - 1 = vary N first
2 = vary M first

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*For plotting options, 0 represents omit, +1 is numerical plot, +2 is contour plot, and +3 is both.
#For punching options, 0 represents omit and +1 is punch.

DATA CARDS

There are N x M data points which are arranged on the cards in the manner specified by the variable FORMAT statement on card 4.
Note: Card No. 1 appears only once. Cards No. 2, 3, and 4 are placed before each data set to be processed. Cards No. 3 and 4 are included in any punched output for identification purposes and need not be changed when used for input.

SYMBOLIC DICTIONARY

Subroutine PS

The following calling sequence is used for subroutine PS:

PS (D, M, N, LAG, IBEGN, IPLOTA, IPLOTR, IPLOTS, IDENT, IPNCHA, IPNCHS, Q)

Variable	Type	S/A	I/O	Comments
D	R	A	I	The data matrix, D(N,M)
M	I	S	I	Second (horizontal) dimension of D
N	I	S	I	First (vertical) dimension of D
LAG	I	S	I	Maximum value of lag, τ
IBEGN	I	S	I	Switch to control starting point of calculation (0 to begin with autocorrelation, +1 with power spectrum, +2 or +3 with removed linear trend. +3 will also punch leveled data).
IPLOTA	I	S	I	Switch for plotting autocorrelation function (0 is off, +1 is numerical plot, +2 is contour plot, +3 is both).
IPLOTR	I	S	I	Switch for plotting rough power spectrum. (Same as for IPLOTA).
IPLOTS	I	S	I	Switch for plotting smoothed power spectrum (Same as for IPLOTA).
IDENT	I	S	I	78 character identifying title
IPNCHA	I	S	I	Switch to punch autocorrelation function (0 is off, +1 is on)

IPNCHS	I	S	I	Switch to punch power spectrum (0 is off, +1 is on)
Q	R	A	I/O	Input for autocorrelation function, output for power spectrum.

Subroutine PLOT

The following calling sequence is used for subroutine PLOT:

PLOT (Q, LAG, METH, ID, IDENT)

Variable	Type	S/A	I/O	Comments
Q	R	A	I	Autocorrelation function or power spectrum to be plotted.
LAG	I	S	I	Maximum value of lag
METH	I	S	I	Switch to determine plotting method (+1 is numerical list, +2 is contour map, +3 is both).
ID	I	S	I	Identifies Q as autocorrelation function (with +1), smoothed power spectrum (with +2), or rough spectrum (with +3)
IDENT	I	A	I	78 character identifying title

S = single variable A = subscripted variable R = real I = integer

Note: Symbols in contour map may be changed by changing card No. 9 in the plotting routine.

INTERPRETATION OF SAMPLE OUTPUT

Sea Surface Example

The two-dimensional power spectrum is primarily a method for study of the frequency components or "wave forms" present in an undulating surface. Such a surface could be topography in an area of rolling hills, a geologic datum in a folded area, or as in this example, the undulating surface of the sea. Stereophotographs of a region of the sea surface permit measurement of water elevation at every intersection on an imaginary grid. Sample data are from Data Set 2A of the Stereo Wave Observation Project (SWOP) described by Pierson (1960). Data are elevations in feet above an arbitrary datum close to mean sea level. Elevations were read at all intersections on a 50 x 70 grid covering an area approximately 1800 by 2700 feet. By the method of least squares, a plane may be passed through the data and the deviation of each data point from this plane determined. Deviations correspond to elevations of points on the surface of the sea under the condition that there is no planar trend in the elevations. These leveled data appear as Table 1 (p. 15).

Following removal of the trend, the autocorrelation function is computed. Results are given in Figure 1. Numbers in the body of Figure 1 represent amplitude of the autocorrelation function at various lags in two perpendicular directions. One direction was chosen to correspond to direction of the prevailing wind, the other is at right angles. Rows and columns in the illustration are indexed according to "lags" of the autocorrelation function. The data may also be contoured as shown in Figure 2. Integers represent values of the autocorrelation function as fractional parts of the maximum value. For example, the digit 1 represents 0.1 of the maximum value; dashes represent negative values.

The power spectrum of the SWOP data is shown in Figure 3 in raw or unsmoothed form, in Figure 4 as a smoothed spectrum, and in Figure 5 as a scaled contour map. The scales for these figures correspond to those of Figure 2. Each "lag" corresponds to a given frequency (or period) contribution to the overall undulating character of the surface. To convert these lag values to period, the equation shown below is used. Here period has a distance rather than time connotation

because the surface is undulating in space.

$$\lambda = \frac{2\tau_m \Delta L}{(\tau \pm 1/2)} \qquad (21)$$

where

λ = wave length or period in feet represented by lag τ; frequency is the reciprocal $1/\lambda$.
ΔL = grid interval in feet.
τ_m = maximum number of lags, not to exceed 1/10 to 1/3 of total sampling interval.
Here, $\tau_m = 20$ for each direction.
Note that one direction has a lag of -20 to +20.
τ = a particular lag.

The term $\pm$ 1/2 indicates that the value of the power spectrum at each lag does not correspond to a precise period or frequency, but rather to all frequencies in a band one lag in width.

In the example given for this data, ΔL is 30 feet and the maximum lag is 20. A lag of +3 in the x direction (column heading +3) and +5 in the y direction (row heading +5) shows a power spectrum value of 13. This corresponds to contributions made by oscillations with periods of

$$\lambda_x = \frac{(2)\ (20)\ (30)}{(3 \pm 1/2)} = 343 \text{ ft. to } 480 \text{ ft.}$$

$$\lambda_y = \frac{(2)\ (20)\ (30)}{(5 \pm 1/2)} = 218 \text{ ft. to } 267 \text{ ft.}$$

The presence of a range of values for a period (or frequency) arises from the fact that data were obtained from a sampled grid rather than from a continuous function.

Figure 4 can be regarded as a map, but in a frequency (λ_x, λ_y) rather than a distance (x,y) domain. In a sense, it represents a transformation of the original data so that frequency information is displayed rather than elevation information. Thus, there is no correspondence between a point in the original x, y coordinate space and the frequency space of Figure 4.

The spectral characterization of a dendritic drainage pattern may be considered as a geological example. The original area is a region taken from the Lone Star 7-1/2 minute Quadrangle, Douglas County, Kansas (Preston and Harbaugh, 1965). The topographic map of this area is shown as Figure 6. Figure 7 shows data from a 25 x 50 grid on this map printed and hand contoured. For this example, only the scaled power spectrum map is shown (Fig. 8). It is possible that a similar pattern is characteristic of all dendritic drainages. If this technique is to be used to characterize and compare other areas, some map orientation convention must be developed, such as aligning the primary grid parallel to the main or average stream direction.

In the Lone Star Quadrangle data, significant values in the power spectrum are concentrated vertically. This reflects greater variation of the data along a vertical axis, as is evident in the topographic map. If the grid axes are rotated, the power spectrum also rotates. If an adjacent area in this physiographic provenance were analyzed, the spectrum would be nearly identical. It is this similarity in the spectra of similar landforms that could provide a basis for landform classification.

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Listing of FORTRAN IV statements in power-spectrum program.

```
C                                                                       010
C       PSMAIN IS THE MAINLINE ROUTINE FOR PS AND PLOT SUBROUTINES. IT  020
C    WILL PROCESS UP TO NINE DATA DECKS ACCORDING TO OPTIONS ON THE     030
C    CONTROL CARDS.                                                    040
C         PROGRAMMED BY J. E. ESLER, KANSAS UNIVERSITY                 050
C          AUGUST 1966.                                                060
C                                                                       070
      DIMENSION D(100,100),Q(25,50),IDENT(13),IFMT(10)                  080
      READ(5,8) NMBR                                                    090
      DO 100 J=1,NMBR                                                   100
      READ(5,8)KALL,IPLOTA,IPLOTR,IPLOTS,IPNCHA,IPNCHS,IDENT,LAG,IBID,  110
     1N,M,IFMT,IFIRST                                                   120
    8 FORMAT(6I1/13A6/I3,I1,2I3,10A6,I1)                                130
      LP1=LAG+1                                                         140
      LLP1=LAG+LP1                                                      150
      IF(KALL.EQ.2) GO TO 10                                            160
      IF(IBID.NE.1) GO TO (30,40),IFIRST                                170
      READ(5,IFMT)((Q(K,L),K=1,LP1),L=1,LLP1)                           180
      GO TO 20                                                          190
   30 READ(5,IFMT)((D(K,L),K=1,N),L=1,M)                                200
      GO TO 20                                                          210
   40 READ(5,IFMT)((D(K,L),L=1,M),K=1,N)                                220
   20 CALL PS(D,M,N,LAG,IBID,IPLOTA,IPLOTR,IPLOTS,IDENT,IPNCHA,IPNCHS,Q) 230
      GO TO 100                                                         240
   10 IF(IBID.EQ.0) WRITE(6,6)J                                         250
      READ(5,IFMT)((Q(K,L),K=1,LP1),L=1,LLP1)                           260
    6 FORMAT(1H1,35H ERROR IN CONTROL CARD FOR DATA SET,I3,48H. RAW OR L 270
     1EVELED DATA CANNOT BE PLOTTED BY PLOT./1X,35H WILL ATTEMPT TO SKIP 280
     2 TO NEXT DECK.)                                                   290
   50 KK=IPLOTA                                                         300
      IF(IBID.EQ.2)KK=IPLOTS                                            310
      CALL PLOT(Q,LAG,KK,IBID,IDENT)                                    320
  100 CONTINUE                                                          330
      CALL EXIT                                                         340
      END                                                               350


C                                                                  PS   010
C       PS IS A SUBROUTINE TO COMPUTE THE TWO DIMENSIONAL POWER SPECT- PS 020
C    RUM OF A TWO DIMENSIONAL ARRAY OF RANDOM DATA.  OPTIONS IN THE ARGU-PS 030
C    MENT LIST DETERMINE MATERIAL TO BE PLOTTED OR PUNCHED ON CARDS.    PS 040
C    ALL PLOTTING IS DONE BY THE SUBROUTINE PLOT, WHICH IS CALLED INTER- PS 050
C    NALLY.  PLOT MUST BE INCLUDED WHEN THE PROGRAM IS RUN.             PS 060
C         PROGRAMMED BY J. E. ESLER, KANSAS UNIVERSITY                  PS 070
C          AUGUST 1966.                                                 PS 080
C                                                                       PS 090
      SUBROUTINE PS(D,M,N,LAG,IBEGN,IPLOTA,IPLOTR,IPLOTS,IDENT,IPNCHA,  PS 100
     1IPNCHS,Q)                                                         PS 110
      DIMENSION D(100,100),Q(25,50),SR(27,52),IDENT(13)                 PS 120
      IF(LAG.GT.0) GO TO 11                                             PS 130
      WRITE(6,12)LAG                                                    PS 140
   12 FORMAT(1X,43HILLEGAL VALUE OF LAG ENTERED INTO PS. LAG =,I3)      PS 150
      RETURN                                                            PS 160
   11 IF(IBEGN.LE.1) GO TO 21                                           PS 170
```

```
C          OPTIONAL DATA LEVELING                                          PS   180
      MN=M*N                                                               PS   190
      A11=FLOAT(MN*(N+1)*(2*N+2))/6.                                       PS   200
      A12=FLOAT(MN*(N+1)*(M+1))*0.25                                       PS   210
      A13=FLOAT(MN*(N+1))*0.5                                              PS   220
      A22=FLOAT(MN*(M+1)*(2*M+2))/6.                                       PS   230
      A32=FLOAT(MN*(M+1))*0.5                                              PS   240
      A33=MN                                                               PS   250
      DO 10 J=1,N                                                          PS   260
      DO 10 K=1,M                                                          PS   270
      A21=A21+FLOAT(J)*D(J,K)                                              PS   280
      A23=A23+FLOAT(K)*D(J,K)                                              PS   290
   10 A31=A31+D(J,K)                                                       PS   300
      T1=A12*A13-A11*A32                                                   PS   310
      T2=A13**2-A11*A33                                                    PS   320
      T3=A21*A13-A11*A31                                                   PS   330
      T4=A12**2-A11*A22                                                    PS   340
      C3=((A21*A12-A23*A11)*T1-T3*T4)/(T1**2-T2*T4)                        PS   350
      C2=(T3-C3*T2)/T1                                                     PS   360
      C1=(A31-A33*C3-A32*C2)/A13                                           PS   370
      DO 20 J=1,N                                                          PS   380
      DO 20 K=1,M                                                          PS   390
   20 D(J,K)=D(J,K)-C1*FLOAT(J)-C2*FLOAT(K)-C3                             PS   400
C          OPTIONAL PUNCH OF LEVELED DATA                                  PS   410
      IF(IBEGN.NE.3) GO TO 21                                              PS   420
      WRITE(7,82) IDENT                                                    PS   430
      WRITE(7,22)LAG,M,N                                                   PS   440
      WRITE(7,81)((D(J,K),J=1,N),K=1,M)                                    PS   450
   22 FORMAT(I3,1H0,2I3,8H(5E16.9),52X,1H1)                                PS   460
   21 LP1=LAG+1                                                            PS   470
      LLP1=LP1+LAG                                                         PS   480
      IF(IBEGN.EQ.1) GO TO 71                                              PS   490
      KK=-1                                                                PS   500
      IQU=LP1                                                              PS   510
      IQL=1                                                                PS   520
C          COVARIANCE MATRIX                                               PS   530
   30 DO 40 IQ=IQL,IQU                                                     PS   540
      IT=LP1-IQ                                                            PS   550
      IF(IT.LT.0) IT=-IT                                                   PS   560
      KU=M-IT                                                              PS   570
      KL=1                                                                 PS   580
      IF(IQ.GT.LP1) GO TO 50                                               PS   590
      KU=M                                                                 PS   600
      KL=IT+1                                                              PS   610
   50 DO 40 IP=1,LP1                                                       PS   620
      NMP=N-IP+1                                                           PS   630
      SUM=0.                                                               PS   640
      DO 60 K=KL,KU                                                        PS   650
      KPQ=K+IT*KK                                                          PS   660
      DO 60 J=1,NMP                                                        PS   670
      JPP=J+IP-1                                                           PS   680
   60 SUM = SUM+D(J,K)*D(JPP,KPQ)                                          PS   690
   40 Q(IP,IQ)=SUM/FLOAT(NMP*(M-IT))                                       PS   700
      LP2=LAG+2                                                            PS   710
      IF(IQU.NE.LP1)GO TO 70                                               PS   720
      IQU=LLP1                                                             PS   730
      IQL=LP2                                                              PS   740
      KK=1                                                                 PS   750
      GO TO 30                                                             PS   760
   70 LPL=LAG+LAG                                                          PS   770
```

```
C        OPTIONAL PLOT OF COVARIANCE MATRIX                              PS    780
      IF(IPLOTA.NE.0) CALL PLOT(Q,LAG,IPLOTA,1,IDENT)                    PS    790
C        OPTIONAL PUNCH OF COVARIANCE MATRIX                             PS    800
      IF(IPNCHA.NE.1) GO TO 71                                           PS    810
      WRITE(7,82)IDENT                                                   PS    820
      WRITE(7,84)LAG                                                     PS    830
      WRITE(7,81)((Q(IP,IQ),IP=1,LP1),IQ=1,LLP1)                         PS    840
   71 DO 80 IP=2,LAG                                                     PS    850
      DO 80 IQ=2,LPL                                                     PS    860
   80 Q(IP,IQ)=Q(IP,IQ)*2.                                               PS    870
      Q(1,LLP1)=.5*Q(1,LLP1)                                             PS    880
      Q(1,1)=.5*Q(1,1)                                                   PS    890
      Q(LP1,LLP1)=.5*Q(LP1,LLP1)                                         PS    900
      Q(LP1,1)=.5*Q(LP1,1)                                               PS    910
   81 FORMAT(5E16.9)                                                     PS    920
   82 FORMAT(13A6)                                                       PS    930
   83 RLAG=LAG                                                           PS    940
   84 FORMAT(I3,1H1,6X,8H(5E16.9))                                       PS    950
      PI=3.1415926                                                       PS    960
      RC=COS(PI*RLAG)                                                    PS    970
      RS=SIN(PI*RLAG)                                                    PS    980
      CRP=1.                                                             PS    990
      SRP=0.                                                             PS   1000
      CT=RC                                                              PS   1010
      ST=RS                                                              PS   1020
      CI=COS(PI/RLAG)                                                    PS   1030
      SI=SIN(PI/RLAG)                                                    PS   1040
C        ROUGH POWER SPECTRUM BY FOURIER TRANSFORM                       PS   1050
      DO 90 IR=1,LP1                                                     PS   1060
      CSQ=-1.                                                            PS   1070
      SSQ=0.                                                             PS   1080
      DO 100 IS=1,LLP1                                                   PS   1090
      CIS=CT                                                             PS   1100
      SIS=ST                                                             PS   1110
      SUM=0.                                                             PS   1120
      DO 110 IQ=1,LLP1                                                   PS   1130
      CIQ=CT                                                             PS   1140
      SIQ=ST                                                             PS   1150
      DO 120 IP=1,LP1                                                    PS   1160
      SUM=SUM+Q(IP,IQ)*CT                                                PS   1170
      TEMP=CT                                                            PS   1180
      CT=CT*CRP-ST*SRP                                                   PS   1190
  120 ST=ST*CRP+TEMP*SRP                                                 PS   1200
      CT=CIQ*CSQ-SIQ*SSQ                                                 PS   1210
  110 ST=SIQ*CSQ+CIQ*SSQ                                                 PS   1220
      SR(IR+1,IS+1)=SUM/FLOAT(2*LAG**2)                                  PS   1230
      CT=-CIS                                                            PS   1240
      ST=-SIS                                                            PS   1250
      TEMP=CSQ                                                           PS   1260
      CSQ=CSQ*CI-SSQ*SI                                                  PS   1270
  100 SSQ=SSQ*CI+TEMP*SI                                                 PS   1280
      CT=RC                                                              PS   1290
      ST=RS                                                              PS   1300
      TEMP=CRP                                                           PS   1310
      CRP=CRP*CI-SRP*SI                                                  PS   1320
   90 SRP=SRP*CI+TEMP*SI                                                 PS   1330
      LLP3=LLP1+2                                                        PS   1340
      DO 130 IR=2,LP2                                                    PS   1350
      SR(IR,1)=SR(IR,3)                                                  PS   1360
  130 SR(IR,LLP1+2)=SR(IR,LLP1)                                          PS   1370
```

```
      DO 140 IS=1,LLP3                                                  PS  1380
      SR(1,IS)=SR(3,IS)                                                 PS  1390
  140 SR(LP1+2,IS)=SR(LP1,IS)                                           PS  1400
      IF(IPLOTR.EQ.0) GO TO 142                                         PS  1410
      DO 141 IR=1,LP1                                                   PS  1420
      DO 141 IS=1,LLP1                                                  PS  1430
  141 Q(IR,IS)=SR(IR+1,IS+1)                                            PS  1440
      CALL PLOT(Q,LAG,IPLOTR,3,IDENT)                                   PS  1450
C        SMOOTHING OF POWER SPECTRUM                                    PS  1460
  142 DO 150 IR=1,LP1                                                   PS  1470
      DO 150 IS=1,LLP1                                                  PS  1480
  150 Q(IR,IS)=0.0625*(SR(IR+2,IS+2)+SR(IR+2,IS)+SR(IR,IS)+SR(IR,IS+2)) PS  1490
     1+0.125*(SR(IR+1,IS+2)+SR(IR+1,IS)+SR(IR+2,IS+1)+SR(IR,IS+1))+0.25*PS  1500
     2SR(IR+1,IS+1)                                                     PS  1510
C        OPTIONAL PUNCH OF SMOOTH SPECTRUM                              PS  1520
      IF(IPNCHS.NE.1)GO TO 151                                          PS  1530
      WRITE(7,82)IDENT                                                  PS  1540
      WRITE(7,152)LAG                                                   PS  1550
  152 FORMAT(I3,1H2,6X,8H(5E16.9))                                      PS  1560
      WRITE(7,81)((Q(IP,IQ),IP=1,LP1),IQ=1,LLP1)                        PS  1570
C        OPTIONAL PLOT OF SMOOTH SPECTRUM                               PS  1580
  151 IF(IPLOTS.NE.0)CALL PLOT(Q,LAG,IPLOTS,2,IDENT)                    PS  1590
      RETURN                                                            PS  1600
      END                                                               PS  1610


C                                                                       PLOT 010
C        PLOT IS THE OUTPUT SUBROUTINE FOR THE POWER SPECTRUM SUBROUTINEPLOT 020
C   PS. IT IS CALLED INTERNALLY FROM PS ACCORDING TO OPTIONS IN THE PS  PLOT 030
C   ARGUMENT LIST. PLOT CAN ALSO BE USED SEPARATELY TO PLOT DATA PUNCHEDPLOT 040
C   OUT BY PS, BY CALLING IT FROM THE MAINLINE ROUTINE, MAIN.           PLOT 050
C        PROGRAMMED BY J. E. ESLER, KANSAS UNIVERSITY                   PLOT 060
C         AUGUST 1966.                                                  PLOT 070
C                                                                       PLOT 080
      SUBROUTINE PLOT(Q,LAG,METH,ID,IDENT)                              PLOT 090
      DIMENSION IP(12),Q(25,50),IS(97),IHOLD(97,24),IDENT(13)           PLOT 100
      DATA IP/1H ,1H1,1H2,1H3,1H4,1H5,1H6,1H7,1H8,1H9,1H*,1H-/          PLOT 110
      RMAX=Q(1,1)                                                       PLOT 120
      M=LAG+1                                                           PLOT 130
      N=M+LAG                                                           PLOT 140
      F=.49999999                                                       PLOT 150
      DO 10 J=1,M                                                       PLOT 160
      DO 10 K=1,N                                                       PLOT 170
   10 IF(Q(J,K).GT.RMAX) RMAX=Q(J,K)                                    PLOT 180
      IF(METH.EQ.2) GO TO 100                                           PLOT 190
C        NUMERICAL PLOT                                                 PLOT 200
      IF(ID.EQ.1) WRITE(6,1)IDENT                                       PLOT 210
      IF(ID.EQ.2) WRITE(6,2)IDENT                                       PLOT 220
      IF(ID.EQ.3)WRITE(6,9) IDENT                                       PLOT 230
    1 FORMAT(1H1,29H AUTOCORRELATION FUNCTION FOR,1X,13A6)              PLOT 240
    2 FORMAT(1H1,19H POWER SPECTRUM FOR,1X,13A6)                        PLOT 250
    9 FORMAT(1H1,25H ROUGH POWER SPECTRUM FOR,1X,13A6)                  PLOT 260
      WRITE(6,5)                                                        PLOT 270
    5 FORMAT(1X,52H NUMERICAL LISTING, SYMMETRY ABOUT ORIGIN NOT SHOWN.)PLOT 280
      WRITE(6,6)LAG                                                     PLOT 290
    6 FORMAT(1X,24H MAXIMUM VALUE OF LAG IS,I3)                         PLOT 300
      PWR=10000.                                                        PLOT 310
   20 R=RMAX*PWR+F                                                      PLOT 320
      IF(R.LT.1000.) GO TO 30                                           PLOT 330
      PWR=PWR*.1                                                        PLOT 340
```

```
      GO TO 20                                                          PLOT 350
   30 WRITE(6,4)PWR                                                     PLOT 360
      DO 40 K=1,N                                                       PLOT 370
      KI=N-K+1                                                          PLOT 380
      DO 50 J=1,M                                                       PLOT 390
      IS(J)=Q(J,KI)*PWR+F                                               PLOT 400
   50 IF(Q(J,KI).LT.0.) IS(J)=Q(J,KI)*PWR-F                             PLOT 410
   40 WRITE(6,3)(IS(J),J=1,M)                                           PLOT 420
    3 FORMAT(1X,25I5)                                                   PLOT 430
      IF(METH.EQ.1) RETURN                                              PLOT 440
C        CONTOUR MAP PLOT                                               PLOT 450
  100 S=RMAX*.1                                                         PLOT 460
      IF(ID.EQ.1) WRITE(6,1)IDENT                                       PLOT 470
      IF(ID.EQ.2) WRITE(6,2)IDENT                                       PLOT 480
      IF(ID.EQ.3) WRITE(6,9) IDENT                                      PLOT 490
      WRITE(6,7)                                                        PLOT 500
    7 FORMAT(1X,35H CONTOUR MAP PLOT SHOWING SYMMETRY.)                 PLOT 510
      WRITE(6,6)LAG                                                     PLOT 520
      SC=1./S                                                           PLOT 530
      WRITE(6,4) S                                                      PLOT 540
    4 FORMAT(1X,18H SCALING FACTOR IS,F12.5///)                         PLOT 550
      L2=47+N                                                           PLOT 560
      NDX2=M                                                            PLOT 570
      L=49-N                                                            PLOT 580
      F=1.4999999                                                       PLOT 590
      DO 110 K=1,M                                                      PLOT 600
      NMK=N-K+1                                                         PLOT 610
      NDX1=M                                                            PLOT 620
      L1=L                                                              PLOT 630
      NDX3=48                                                           PLOT 640
      DO 120 J=1,M                                                      PLOT 650
      NN=Q(NDX1,K)*SC+F                                                 PLOT 660
      IF(Q(NDX1,K).LT.0.) NN=12                                         PLOT 670
      IS(L1)=IP(NN)                                                     PLOT 680
      IF(NDX1.EQ.1) GO TO 140                                           PLOT 690
      HOLD=(Q(NDX1,K)+Q(NDX1-1,K))/2.                                   PLOT 700
      NN=HOLD*SC+F                                                      PLOT 710
      IF(HOLD.LT.0.) NN=12                                              PLOT 720
      IS(L1+1)=IP(NN)                                                   PLOT 730
  140 NN=Q(J,NMK)*SC+F                                                  PLOT 740
      IF(Q(J,NMK).LT.0.) NN=12                                          PLOT 750
      IS(NDX3)=IP(NN)                                                   PLOT 760
      IF(J.EQ.M) GO TO 160                                              PLOT 770
      HOLD=(Q(J,NMK)+Q(J+1,NMK))/2.                                     PLOT 780
      NN=HOLD*SC+F                                                      PLOT 790
      IF(HOLD.LT.0.) NN=12                                              PLOT 800
      IS(NDX3+1)=IP(NN)                                                 PLOT 810
  160 NDX1=NDX1-1                                                       PLOT 820
      NDX3=NDX3+2                                                       PLOT 830
  120 L1=L1+2                                                           PLOT 840
      IS48=IS(48)                                                       PLOT 850
      IS47=IS(47)                                                       PLOT 860
      HOLD=(Q(2,K)+Q(1,NMK))/2.                                         PLOT 870
      NN=HOLD*SC+F                                                      PLOT 880
      IF(HOLD.LT.0.) NN=12                                              PLOT 890
      IS(47)=IP(NN)                                                     PLOT 900
      NN=Q(1,NMK)*SC+F                                                  PLOT 910
      IF(Q(1,NMK).LT.0.) NN=12                                          PLOT 920
      IS(48)=IP(NN)                                                     PLOT 930
      WRITE(6,8)(IS(LL),LL=L,L2)                                        PLOT 940
    8 FORMAT(2X,130A1)                                                  PLOT 950
```

```
      IS(47)=IS47                                                    PLOT 960
      IS(48)=IS48                                                    PLOT 970
      HOLD=(Q(1,K)+Q(2,NMK))/2.                                      PLOT 980
      NN=HOLD*SC+F                                                   PLOT 990
      IF(HOLD.LT.0.) NN=12                                           PLOT1000
      IS(49)=IP(NN)                                                  PLOT1010
      NMJ=L2+1                                                       PLOT1020
      DO 170 J=L,L2                                                  PLOT1030
      NMJ=NMJ-1                                                      PLOT1040
  170 IHOLD(NMJ,NDX2)=IS(J)                                          PLOT1050
  110 NDX2=NDX2-1                                                    PLOT1060
      DO 200 J=2,M                                                   PLOT1070
  200 WRITE(6,8)(IHOLD(LL,J),LL=L,L2)                                PLOT1080
      RETURN                                                         PLOT1090
      END                                                            PLOT1100
```

Table 1.-Sample data.

```
1
133310
SWOP DATA SET 2A
 202 50 70(16F5.2)                                                                1
-1.23-0.93-3.22-1.61-3.30-3.80-1.29 0.82 1.93 0.13 2.04 0.75 2.26 0.96 1.27-0.92
-1.11 3.29 3.10-0.29-2.58-2.18-2.57-0.06-2.35 1.85-0.74-0.63-1.22-2.42 0.29-1.40
-1.29 2.41 0.92-2.37 0.43 4.64-0.55 1.56-2.03-1.83-4.52-4.01-2.21-0.70 1.71 0.22
 2.52 2.43-3.03-1.52-2.32 0.89 0.90-0.59-1.09 1.62 0.73 0.04 1.54 0.95 2.16 0.17
 0.27-1.52-0.51 2.29 2.70-0.19-2.88-0.78-0.67 0.54-1.75 0.15 1.66-0.33-2.52-2.82
-0.81 0.60-1.89-0.49 0.42-0.47 1.44-1.16-1.95 1.86-1.83 0.37-1.42-5.01-3.50-0.90
-0.49-1.48 1.83 2.03-4.33-1.52-0.42 4.49 1.20-1.19-1.29 2.62-0.17-2.16-2.36 1.95
 1.36 0.87-0.63-1.42-0.11 2.90 1.60 9.21 0.62-2.07 0.13 1.84-0.95 1.96 1.16 0.47
-3.02-1.51-0.21 0.40-1.29-0.78 1.52 2.53 0.84-1.25 1.95 3.26 0.37 4.27-1.72-5.61
-4.60-2.30-2.59-3.68-0.27 0.43-1.33-3.32-1.51 2.89 1.90-0.99 0.12 2.32-0.27-3.96
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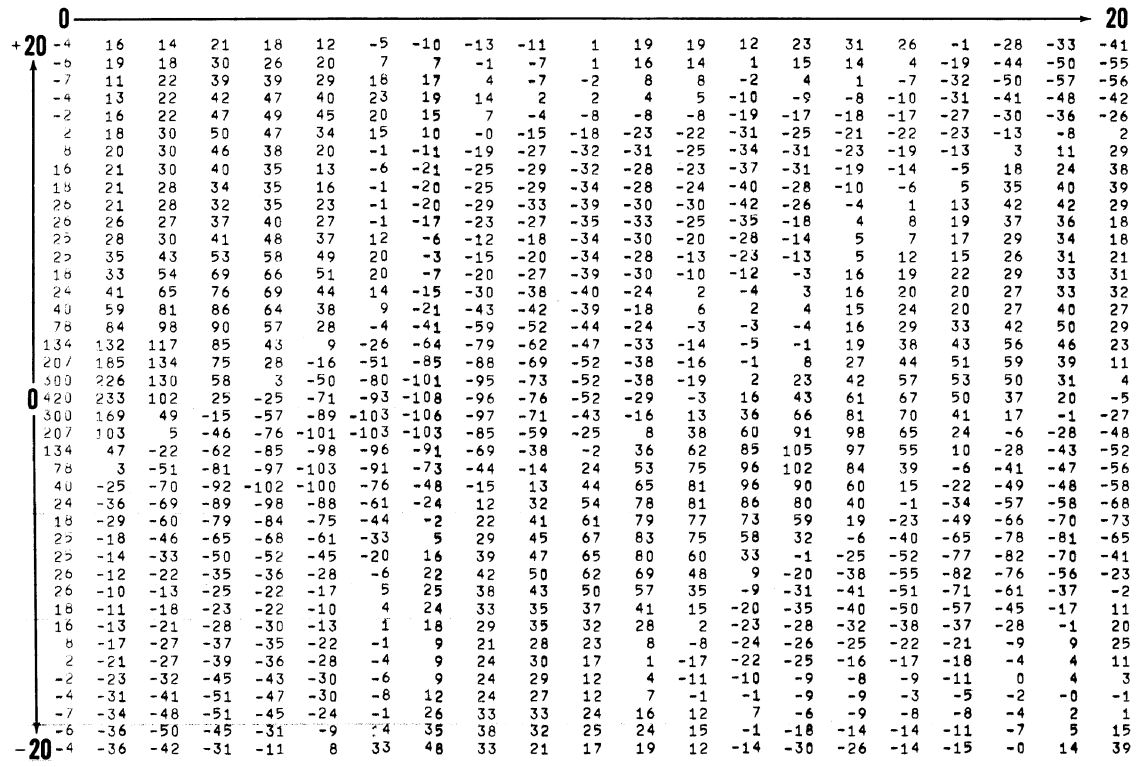

Figure 1. -Autocorrelation function for SWOP data set 2A numerical listing, symmetry about origin not shown. Maximum value of lag is 20; scaling factor is 100.00000.

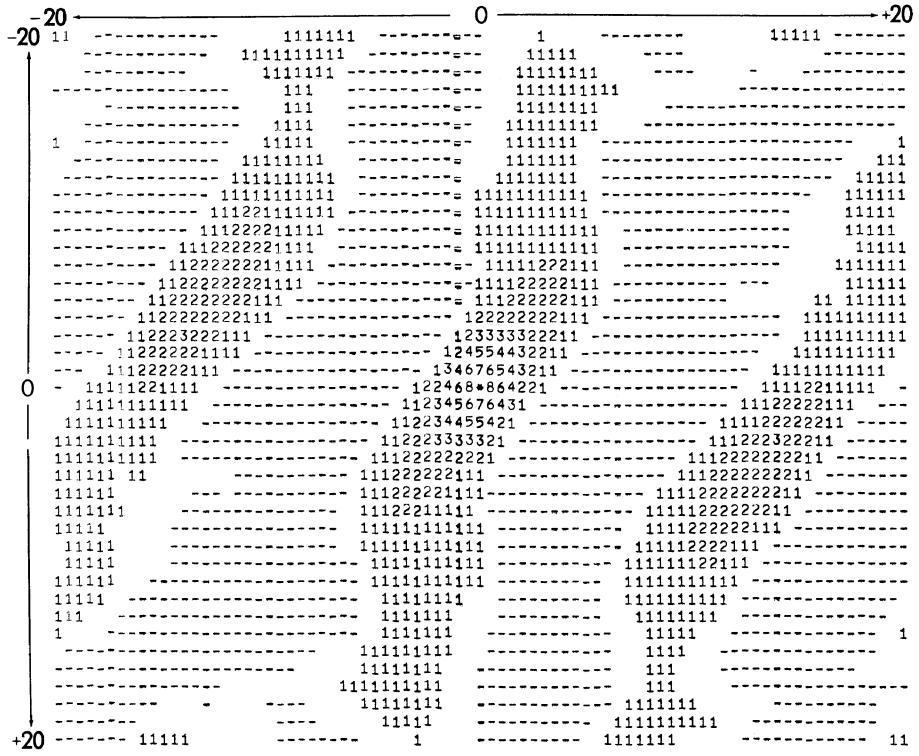

Figure 2. -Autocorrelation function for SWOP data set 2A contour map plot showing symmetry. Maximum value of lag is 20; scaling factor is 0.42040.

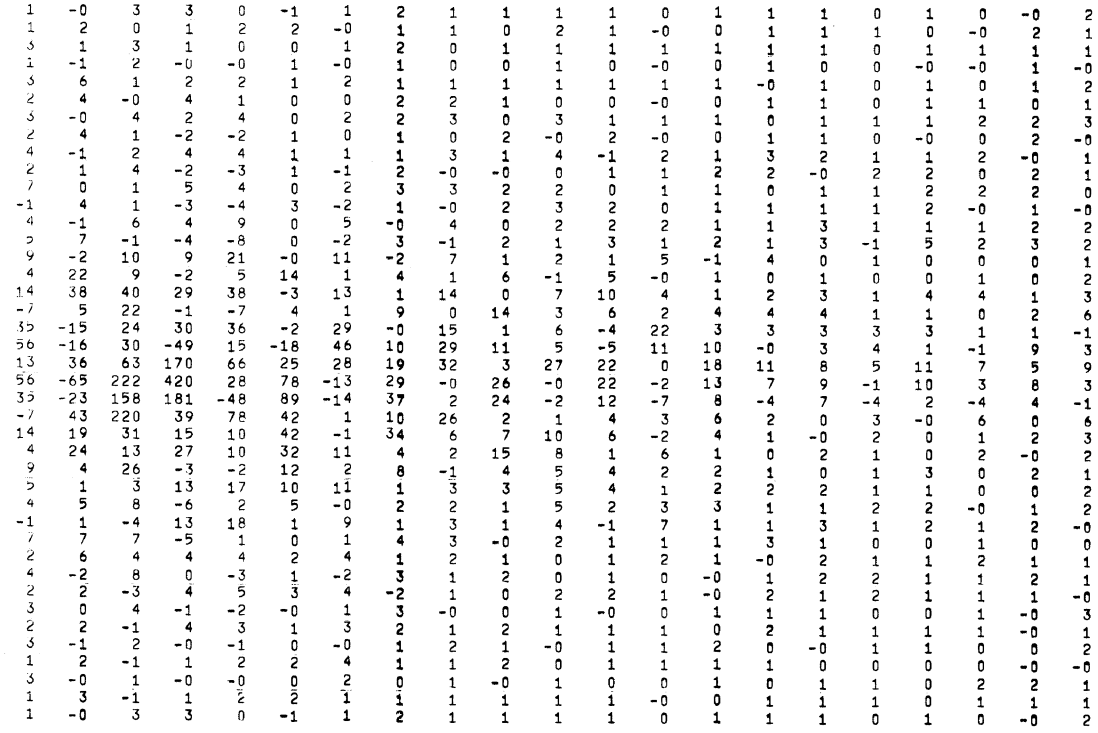

Figure 3. -Rough power spectrum for SWOP data set 2A numerical listing, symmetry about origin not shown. Maximum value of lag is 20; scaling factor is 999.99999.

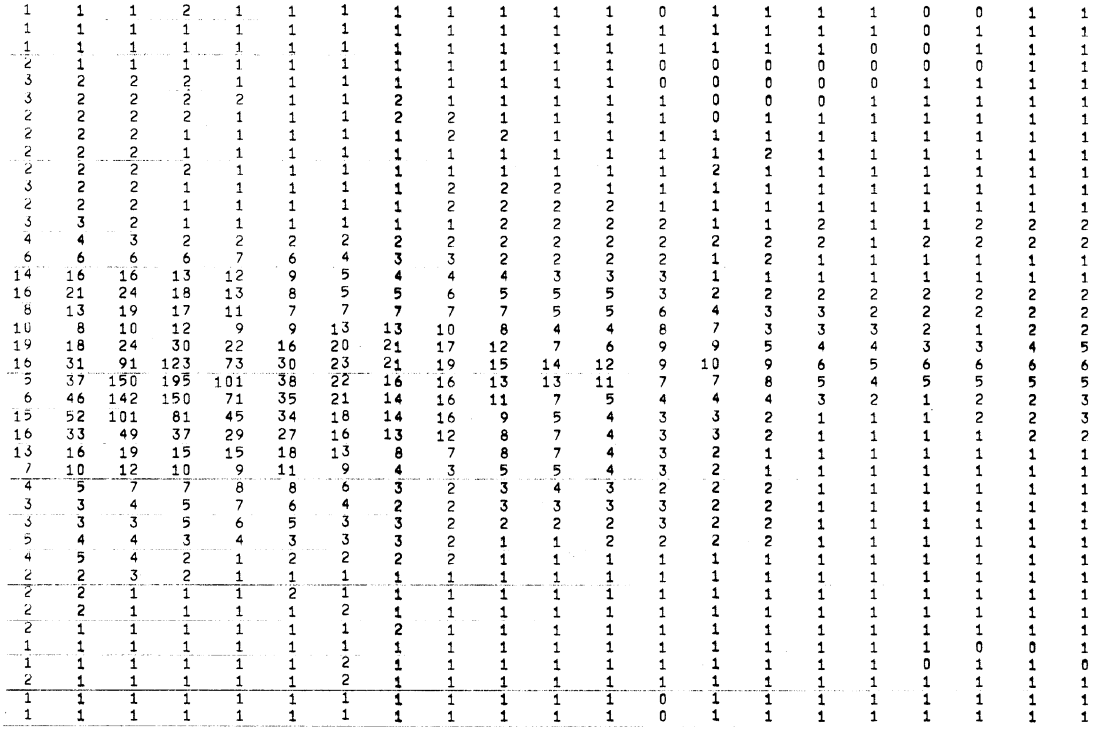

Figure 4. -Power spectrum for SWOP data set 2A numerical listing, symmetry about origin not shown. Maximum value of lag is 20; scaling factor is 999.99999.

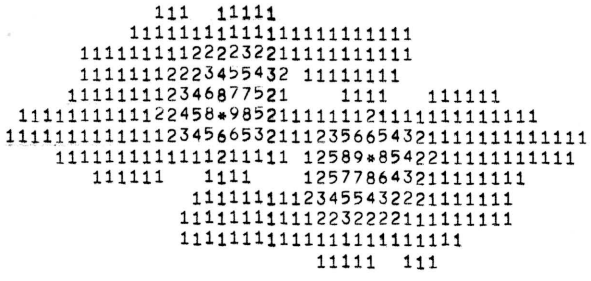

Figure 5. -Power spectrum for SWOP data set 2A contour map plot showing symmetry. Maximum value of lag is 20; scaling factor is 0.01949.

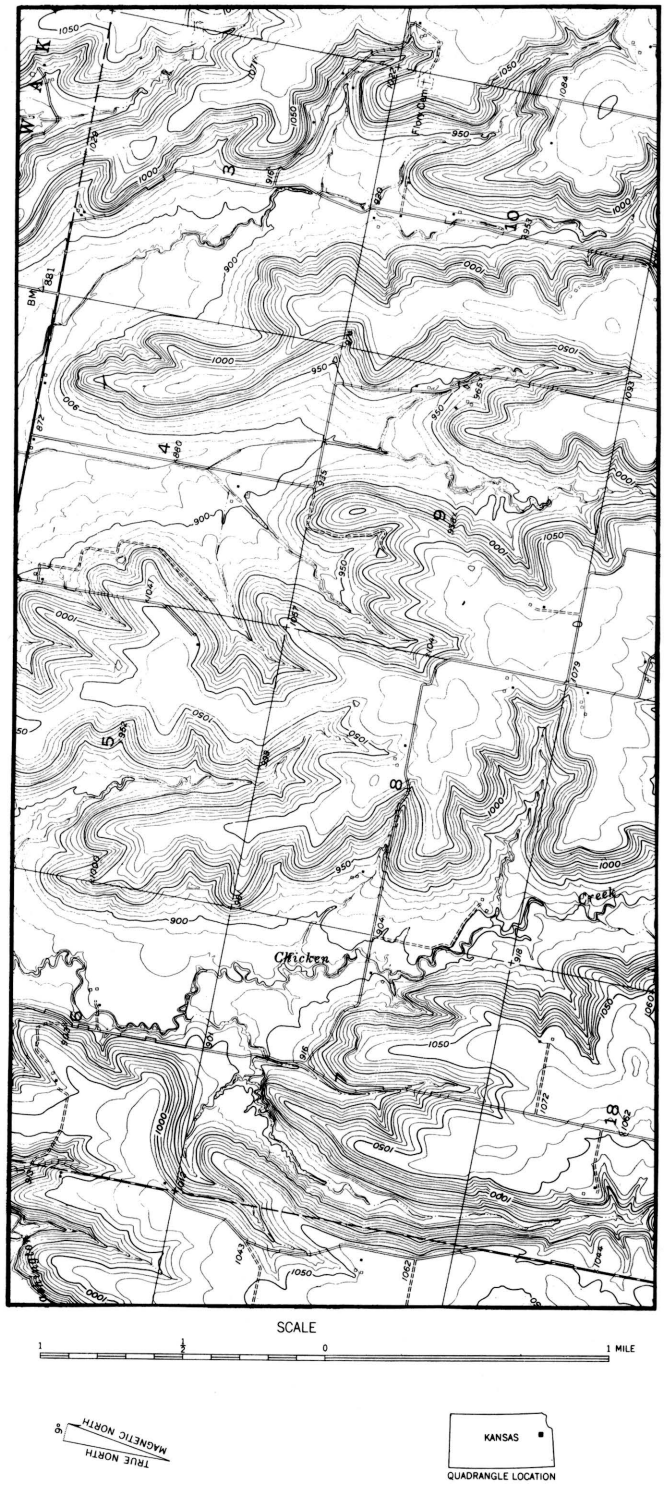

Figure 6. -Topographic map of northern part of Lone Star Quadrangle, Kansas; area is about ten miles southwest of Lawrence (from Preston and Harbaugh, 1965).

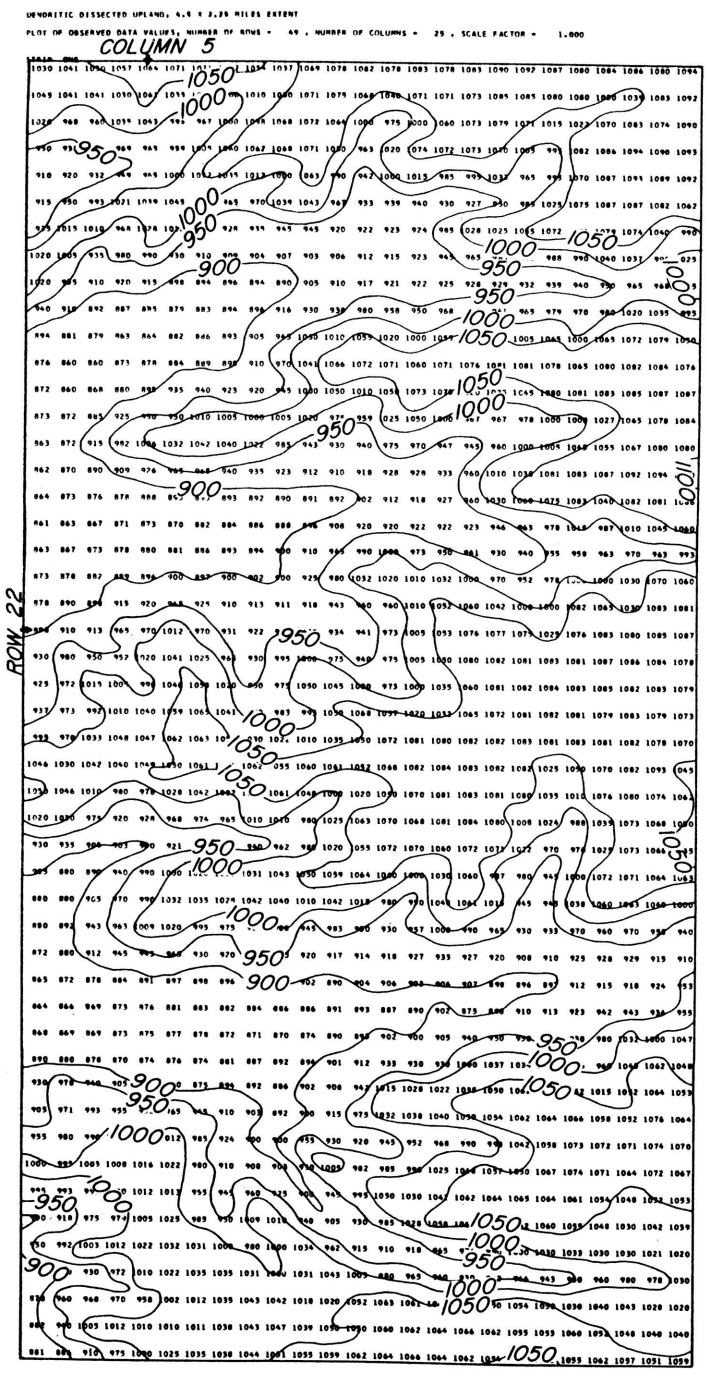

Figure 7. -Data values taken from topographic map (Fig. 6) and recontoured manually for comparison purposes. Data array contains 49 rows and 25 columns. Grid points are spaced about 1/10 mile apart (from Preston and Harbaugh, 1965).

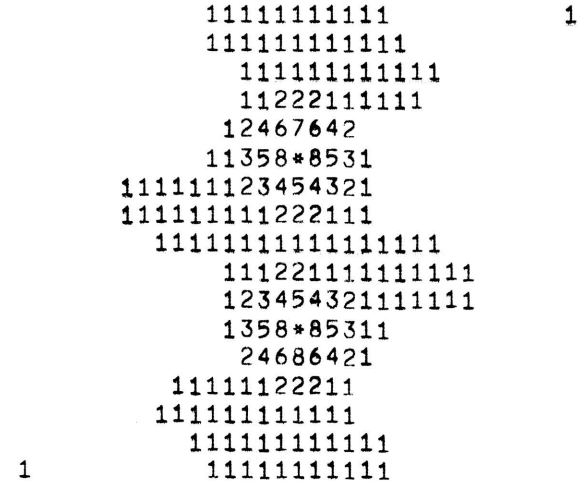

Figure 8. -Power spectrum for C 6 BALGOL data dendritic topo data - Lone Star Quadrangle. Maximum value of lag is 8; scaling factor is 94165.27488.

KANSAS GEOLOGICAL SURVEY COMPUTER PROGRAM
THE UNIVERSITY OF KANSAS, LAWRENCE

PROGRAM ABSTRACT

Title (If subroutine state in title):

PSMAIN (mainline program title)

Subroutines PS and PLOT

Computer: GE 625 Date: October 9, 1967

Programming language: FORTRAN IV

Author, organization: James E. Esler and Floyd W. Preston

University of Kansas, Lawrence, Kansas, 66044

Direct inquiries to: Authors or

Name: D. F. Merriam Address: Kansas Geological Survey

University of Kansas, Lawrence

Purpose/description: Two-dimensional power spectrum can be used for numerical description of land forms and possibly their classification.

Mathematical method: Fourier transform and autocorrelation function.

Restrictions, range: Data must be gridded, maximum is 100 by 100 points.

Storage requirements:

Equipment specifications: Memory 20K 40K 60K K 32

Automatic divide: Yes No Indirect addressing Yes No

Other special features required

Additional remarks (include at author's discretion: fixed/float, relocatability; optional: running time, approximate number of times run successfully, programming hours) Running time increases rapidly as the size of data array is increased. Maximum array will run in about 20 minutes. Program has also been run on the IBM 7040.

COMPUTER CONTRIBUTIONS

Kansas Geological Survey
University of Kansas
Lawrence, Kansas

Computer Contribution

1. Mathematical simulation of marine sedimentation with IBM 7090/7094 computers, by J.W. Harbaugh, 1966 . . . . . $1.00
2. A generalized two-dimensional regression procedure, by J.R. Dempsey, 1966 . . . . . $0.50
3. FORTRAN IV and MAP program for computation and plotting of trend surfaces for degrees 1 through 6, by Mont O'Leary, R.H. Lippert, and O.T. Spitz, 1966 . . . . . $0.75
4. FORTRAN II program for multivariate discriminant analysis using an IBM 1620 computer, by J.C. Davis and R.J. Sampson, 1966. . . . . $0.50
5. FORTRAN IV program using double Fourier series for surface fitting of irregularly spaced data, by W.R. James, 1966 . . . . . $0.75
6. FORTRAN IV program for estimation of cladistic relationships using the IBM 7040, by R.L. Bartcher, 1966 . . . . . $1.00
7. Computer applications in the earth sciences: Colloquium on classification procedures, edited by D.F. Merriam, 1966 . . . . . $1.00
8. Prediction of the performance of a solution gas drive reservoir by Muskat's Equation, by Apolonio Baca, 1967 . . . . . $1.00
9. FORTRAN IV program for mathematical simulation of marine sedimentation with IBM 7040 or 7094 computers, by J.W. Harbaugh and W.J. Wahlstedt, 1967 . . . . . $1.00
10. Three-dimensional response surface program in FORTRAN II for the IBM 1620 computer, by R.J. Sampson and J.C. Davis, 1967. . . . . $0.75
11. FORTRAN IV program for vector trend analyses of directional data, by W.T. Fox, 1967 . . $1.00
12. Computer applications in the earth sciences: Colloquium on trend analysis, edited by D.F. Merriam and N.C. Cocke, 1967 . . . . . $1.00
13. FORTRAN IV computer programs for Markov chain experiments in geology, by W.C. Krumbein, 1967. . . . . $1.00
14. FORTRAN IV programs to determine surface roughness in topography for the CDC 3400 computer, by R.D. Hobson, 1967. . . . . $1.00
15. FORTRAN II program for progressive linear fit of surfaces on a quadratic base using an IBM 1620 computer, by A.J. Cole, C. Jordan, and D.F. Merriam, 1967 . . . . . $1.00
16. FORTRAN IV program for the GE 625 to compute the power spectrum of geological surfaces, by J.E. Esler and F.W. Preston, 1967 . . . . . $0.75