DANIEL F. MERRIAM, Editor

FORTRAN IV COMPUTER PROGRAMS FOR MARKOV CHAIN EXPERIMENTS IN GEOLOGY

By

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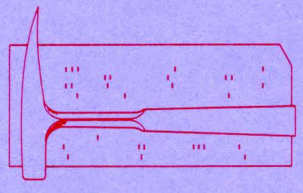

COMPUTER CONTRIBUTION 13
State Geological Survey
The University of Kansas, Lawrence
1967

Editor's Remarks

Geological modeling is new and exciting - perhaps one of the most exciting and promising facets of geology. It is obvious from the reference list that this work is new because most citations are papers less than three years old. It is exciting because the geologist can now delve into some fundamental problems and for the first time can bring the field into the laboratory so to speak. This series of computer programs by Dr. W.C. Krumbein represents a report on use of Markov chains for experiments in geology.

Although examples cited are mainly stratigraphic, Markov process models are being used in hydrological, meteorological, and volcanological studies. Undoubtedly other geological applications will be found now that computer programs are available. As Dr. Krumbein points out, Markov chains are "the simplest of stochastic process models and they afford a point of entry into a large class of probabilistic mechanisms that can be useful in geology." Here then is a beginning.

For those who would like to experiment with these computer programs, the Kansas Survey will make card decks available for a limited time for $15.00. A complete list of available programs and data decks can be obtained by writing Editor, Computer Contributions. Programs are available in FORTRAN II, FORTRAN IV, and ALGOL, that will run on the Burroughs B5500, CDC 3400, GE 625, and IBM 1620, 7040, 7090 and 7094 computers.

The Kansas Survey, in addition to distributing information through the regular Computer Contribution series, makes available reprints of interest. A list is included on the inside back cover, and they are available upon request. To help disseminate information, the Survey is also co-sponsoring a series of colloquia on timely subjects. Two have already been held - one on classification procedures (Computer Contribution 7) and the other on trend analysis (Computer Contribution 12). A complete list of computer publications is available by writing the Editor.

Because of wide interest in quantitative methods (i.e. computer applications) in the earth sciences, preliminary plans are being made for the formation of an international interest group. Although only in the formative stage, this group probably will be within the framework of an existing organization. Interest and need, however, are obvious because the Kansas Survey presently is distributing information worldwide in response to requests. Programs are being adapted for use in Australia, Bolivia, Brazil, Canada, Czechoslovakia, Great Britain, India, Israel, Japan, Libya, Mexico, New Zealand, Poland, South Africa, and Sweden.

Computer use in geology now is truly international. It has and is affecting change in the earth sciences! Part of the change can be seen in this publication. The use of Markov chains in geological modeling, undreamed of just a few short years ago, is well demonstrated here, and it is hoped new vistas will be opened and insight gained from their application.

FORTRAN IV COMPUTER PROGRAMS FOR MARKOV CHAIN EXPERIMENTS IN GEOLOGY [1]

By

W. C. KRUMBEIN

ABSTRACT

Probabilistic models provide a mechanism for computer simulation of a wide variety of geological processes. This paper emphasizes first-order Markov chains because of their intuitive appeal and rapidly growing applications. Examples are based on stratigraphic analysis, but other uses of the model are discussed briefly.

First-order Markov chains are among the simplest stochastic process models, and they afford a point of entry into a large class of probabilistic mechanisms that can be useful in geology. This paper is designed to stimulate experimentation with these models, in part by providing relatively simple computer programs that cover four aspects of such analysis. These programs, listed in the Appendix, include one for testing the presence of a Markov property, a second for raising the transition probability matrix to successive powers in order to estimate the equilibrium states (i.e., the fixed probability vector) of a system, and a third for using the transition matrix in simulation studies. A fourth program for simulating independent-events processes (non-Markovian) can be used with the fixed probability vector of a Markov chain to obtain some interesting comparisons among different ways of structuring a sequence of observations or events.

INTRODUCTION

The expanding literature on stochastic models in geology demonstrates growing interest in application of probabilistic mechanisms to studies in stratigraphy, sedimentation, paleontology, geomorphology, petrology, and other aspects of earth science. These mechanisms may be incorporated into conceptual geological models in various ways, and the present tendency in large part is to invoke Markov processes, implemented by transition probability matrices or expressed as random walks.

Interest in these models, especially among computer-oriented graduate students, is such that it seems desirable to encourage experimental modeling of a variety of geological processes, both to increase familiarity with the Markov process, and as an introduction to other classes of stochastic process models.

Four computer programs in current use at Northwestern University are used to illustrate experiments that have been performed. Although it is necessary in discussing these experiments to stress methodology, the intent here is also to raise questions regarding geological implications of structuring observed data as Markov chains. An important objective of this paper is to emphasize the point that if a process has Markov properties, this mainly means that preceding events have some influence on succeeding events. Such knowledge is important in explaining statistical "driving forces" that control simulation output, and with further analysis this knowledge can lead to better understanding of the real-world geological process that gives rise to the observed phenomenon.

Acknowledgments.-I am indebted to numerous statisticians and geologists for aid in understanding Markov processes and for various stratigraphic, sedimentary, and other examples. W.R. James kindly permitted adaptation of his program for estimating the fixed probability vector, which is listed as STOCHEX in the Appendix. T. A. Jones was helpful in developing the Markov test which is listed as TESTMARK in the Appendix. Mrs. Betty Benson of the Vogelback Computing Center at Northwestern University wrote the final versions of all programs, and adapted MARCHAIN (for Markov transitions) to NO MEM for independent-events trials with fixed probability vectors.

STOCHASTIC PROCESSES

Markov chains belong to the large class of stochastic process models. A stochastic process may be defined (Bartlett, 1960, p. 1) as "some possible actual, e.g. physical, process in the real world, that has some random or stochastic element involved in its structure." It is informative to consider the

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[1] This work was supported by Contract Nonr-1228 (36), ONR Task No. 388-078, Office of Naval Research, Geography Branch.

position occupied by Markov models within the conceptual spectrum that includes classical deterministic models at one extreme, and purely random (independent-events) models at the other. If a given geological process operating through time or space (e.g. the development of drainage basins by stream processes or the rounding of pebbles by shore agents along a beach) is thought of as a system comprising a particular set of states, then in a classical deterministic model the state of the system in time or space can be exactly predicted from knowledge of the functional relation specified by the underlying differential equations. At the other extreme, in the purely random model, the state of the system at any instant or point in time or space is wholly independent of its state at any other instant or point, and depends only on underlying fixed probabilities, as in the simplest case of tossing a coin.

Within this spectrum the first-order Markov model is characterized by the condition that the state of the system at time t_r, point X_r, or event E_r is dependent upon the state at time t_{r-1}, point X_{r-1}, or event E_{r-1}. Thus in the conceptual range from complete dependence to complete independence, the simplest Markov model occupies an intermediate position of partial dependence. There is, however, an element of probability in Markov models that precludes exact prediction of future events, and in this respect the Markov model has some resemblance to the completely random model.

The degree of dependency of a given state upon previous states is commonly expressed in terms of the "memory" involved in the process. In the classical path-dependent deterministic model the state at instant t_r depends upon all previous states; hence the process is conceived as having a long memory. On this basis a first-order Markov process has a short memory and a purely random process has no memory. The terms dependency, predictability, and memory clearly represent gradations rather than mutually exclusive categories. Thus, some models normally classified as deterministic are path-independent, and permit exact prediction of future states although no specified path is involved in moving from one state to the other. Similarly, a stochastic process model may have a deterministic core, with a specified kind of randomness superimposed upon it. Some models, however, can be developed either in a deterministic or probabilistic framework, as for example thermodynamic models (Ishida, 1966). Coleman (1964, p. 526) points out that in many situations a deterministic approach yields a simpler mathematical model than the stochastic approach. In such models coefficients of the deterministic form may simply be mean values of the probability distributions in corresponding stochastic models.

It would appear from these statements that the choice between deterministic and stochastic process models is partly one of convenience. Where the underlying physical process can be expressed as a differential equation, and where emphasis is placed on mean values rather than probability distributions, the deterministic approach has the advantage of greater mathematical simplicity. Where the underlying process is complex and is subject to influences that cannot be exactly evaluated, however, the probabilistic approach may be more flexible, in that changes of state can be rigorously examined in terms of their relative probabilities of occurrence. An example of this situation is seen in cyclical sedimentary sequences, where an underlying pattern of rock succession can be discerned, but in which the actual sequence of rock types can only be predicted in terms of relative probabilities.

Markov process models, as well as other kinds of stochastic process models, are being applied in a variety of earth-science fields. The examples in this paper chiefly concern stratigraphic sections, but other applications are cited as an introduction to the rapidly expanding literature. Leopold and Langbein (1962) described development of long stream profiles as a random walk. Their model can be expressed as a first-order Markov chain with an absorbing state, the latter introduced to terminate a given simulation. In the same paper and also discussed in Leopold, Wolman, and Miller (1964, p. 217; 416) are examples of random walks in development of drainage basins and stream junctures. Scheidegger (1966) described stream orders and branching processes in a Markov framework, and Pattison (1965) and Loucks and Lynn (1966) give excellent introductions to setting-up Markov process models in hydrology. Pattison, for example, used first-order and sixth-order Markov chains in the study of hourly rainfall rates; Loucks and Lynn are concerned with daily stream flow, and their paper gives additional references in hydrology and meteorology. Heller and Shinozuka (1966) afford a good example of contrast between purely random and Markov models in fatigue studies of material specimens. Wickman (1966) suggests that some volcanoes display Markov properties in their eruption patterns. Agterberg (1966) applies multivariate Markov models to two basaltic rock series to extract trend factors from major oxide data.

Harbaugh (1966) presents a combinatorial probabilistic model with Markov elements for simulating geological processes. This is the most comprehensive simulation model introduced into geology, and it permits direct "geological experimentation" on a computer by simulating processes of sediment dispersal, development of sedimentary facies, faunal migration, basin filling during tectonic subsidence, etc. The paper is accompanied by a complete listing of the simulation program (see Harbaugh and Wahlstedt, 1967, for a FORTRAN

listing). Harbaugh's computer output is supplied as maps and stratigraphic cross sections.

TRANSITION PROBABILITY MATRICES

Simulation of stratigraphic sections affords a point of entry into the most simple Markov process model, the regular Markov chain. Vistelius (see Vistelius, 1949; Vistelius and Feigel'son, 1965; and Vistelius and Faas, 1965) appears to be the first to have used a Markov chain for describing and analyzing sedimentary cycles. Griffiths (1966) reviews Vistelius' earlier work and provides an excellent discussion of transition probability matrices, the generation of tree diagrams from them, and use of such matrices to determine an "equilibrium state" representing proportions of each rock type in the sedimentary cycles.

Carr and others (1966) furnish an additional example. They measured thickness and number of transitions among sandstone, shale, and limestone in the Chesteran Series (Upper Mississippian) of Indiana, to obtain a matrix of transitions from one lithology to another, as well as thickness frequency distributions of each of the three rock types. Then, by selecting an initial state (sandstone, say), a thickness is taken randomly from the sandstone thickness distribution. Next, the succeeding rock type is selected in accordance with the probabilities in the transition matrix. Say it is shale. Next, a thickness is taken at random from the shale thickness distribution, and the succession is generated step by step in this fashion.

Alternatively, the transition matrix may be set up in such a way that it takes thickness into account. Figure 1 illustrates several procedures on a fictitious stratigraphic section composed of sandstone, shale, and limestone, designated as A, B, and C, respectively. If any rock types show variations (such as a change in shale color or of sandstone texture), these may be designated as "multistory lithologies" as the term is used by Carr and others (1966). In Figure 1, for example, the second shale from the bottom, and the topmost sandstone, are in this category.

In setting up the probability matrix for transitions from one lithology to its successor, observations of the changes displayed are made along the right-hand side of the section in Figure 1. Thus, starting at the bottom, the first transition is from sandstone (A) to shale (B). This is indicated by a tally mark in the upper right matrix in accordance with the following rule: the rows of the matrix represent the given state and the columns represent the state to which the transition proceeds. Transition AB is accordingly indicated by a tally mark in column B of row A. The next transition is from B to C, which gives a tally mark in column C of row B. If the transition occurs in a multistory rock, as in the change from B to B (about 30 feet from the bottom of the section) this means that state B shows a transition to itself, and the tally mark is placed in column B of row B.

The uppermost matrix on the right of Figure 1 shows the tally marks for rock transitions. In this procedure thickness of each rock type also is recorded, to build up thickness-frequency distributions. When a sufficient sample is taken, tally marks in each row are counted, and the total is used to compute transition probabilities for that row. Once the transition matrix and the thickness distributions are obtained, simulation experiments are performed as described by Carr and others (1966, p. 1163), as mentioned earlier.

The left-hand side of the stratigraphic section in Figure 1 shows alternative ways of constructing transition matrices, using fixed vertical intervals for observation. An important consideration here is the magnitude of the interval. If a 10-foot interval is used, transitions shown in the second matrix on the right of Figure 1 are obtained. In only one instance (the top sand) are two successive 10-foot marks in the same kind of lithology, so that only a single tally occurs in the main diagonal. Moreover, some lithologies are missed entirely, as for example the thin limestone near the top.

If the measuring interval is reduced to 5 feet, the number of transitions from a given state into itself increases, as shown by the increased number of entries in the main diagonal. Finally, for an interval of 1 foot (which is carried out only for the bottom 20 feet as an example), the number of transitions from a given state into itself becomes more pronounced. With an infinitesmally small vertical interval, the matrix tends to have probability 1.0 in the diagonals and 0.0 elsewhere, for any sample of finite length. Thus, in the limit, the most likely transition is from a given state to itself - i.e., for the system to remain in its present state.

It is evident that judgment is required regarding the appropriate interval to use. If it is too large, some lithologies are entirely missed; if too small, the probability of leaving any one state in a given sample of observations becomes impractically small. Experiments with a variety of stratigraphic sections suggests an interval of from 2 to 10 feet.

Table 1 shows a transition probability matrix on the left, and a tally matrix on the right, based on 309 transitions measured at fixed vertical intervals of about 8 feet from sections in the Chesteran Series (Upper Mississippian) of southern Illinois. In performing a simulation experiment with the matrix on the left, one starts in a random state or arbitrarily with sandstone, say, and draws random numbers to select the succeeding transitions. Inasmuch as the probability for state A returning to itself is 0.74, the chances are good that state A will be succeeded by itself one or more times before

the system moves to states B or C. The simulation experiment adds 8 feet of rock for each transition, so that if the first several events are A, A, A, B, C, B, B, A, ..., the simulated section will have 24 feet of sand, followed by 8 feet of shale, followed by 8 feet of limestone, followed by 16 feet of shale, and so on. The thicknesses of individual rock occurrences are thus controlled by transitions from a given state back into itself. The off-diagonal elements control the changes from one lithology to another, as well as the relative proportions of each rock type in the simulation.

Inspection of the tally numbers in the right-hand matrix of Table 1 shows that the total number of transitions is greatest for shale, and the matrix on the left shows that the largest probability for transitions from a given state into itself occurs with sand. The implication is that the total thickness of shale in the section is greater than the total thickness of sand, but that the individual sandstone occurrences are, on the average, thicker than the shale beds.

These remarks indicate an interesting difference between alternative ways of structuring

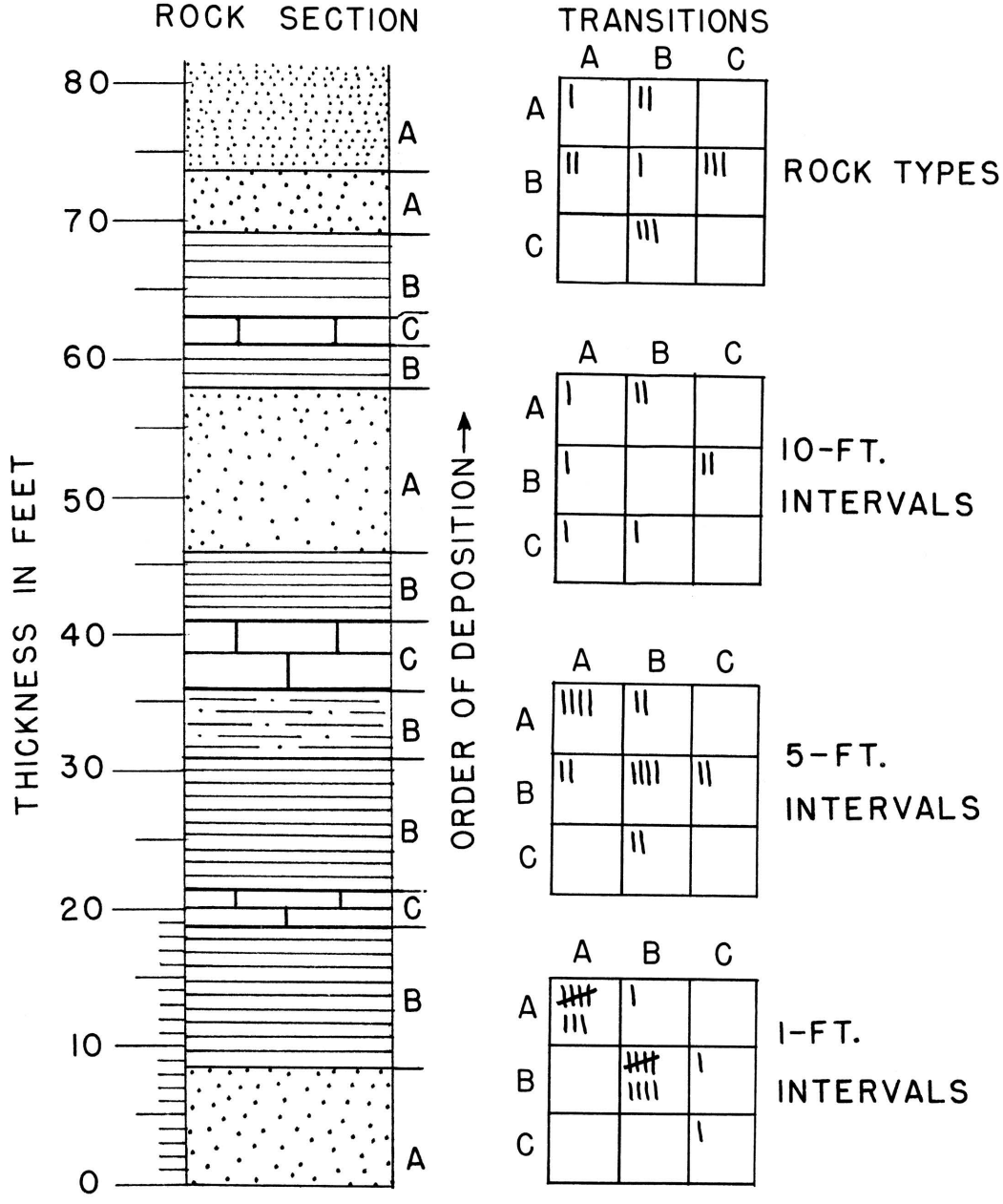


Figure 1.- Fictitious rock section, showing ways of structuring transition matrices.

Table 1.-Transition probability matrix and tally matrix for 309 Chesteran transitions. Vertical interval = 8 Feet.

Transition Probability Matrix*

	A	B	C
Sand (A)	0.74	0.23	0.03
Shale (B)	0.10	0.61	0.29
Lime (C)	0.05	0.38	0.57

Corresponding Tally Matrix

	A	B	C	Totals
Sand (A)	59	18	2	79
Shale (B)	14	86	41	141
Lime (C)	4	34	51	89
Totals	77	138	94	309

*The probabilities in each row are forced to add to 1.00. Thus, 0.74 is rounded slightly downward, and 0.05 is rounded upward.

stratigraphic data for transition probability matrices. If the matrix is based on fixed vertical intervals, simulation output includes not only the succession of lithologies, but also the thickness of each occurrence. If the matrix is based on transitions from one rock type to another, the matrix controls the lithologic succession (and occurrence of multistory events), but thickness represents independent events based on random drawings from the corresponding thickness frequency distributions. Interesting and informative simulation experiments can be performed by structuring the same stratigraphic unit in these two ways.

Figure 2 shows part of a computer simulation with MARCHAIN, involving 500 transitions from the left matrix of Table 1. The simulated section, 4000 feet thick, required 6 seconds of machine time. The total run yielded summary values shown in Table 2. Relative frequencies of the various possible transitions, as realized in this particular simulation experiment, are shown in matrix form at the bottom of the table. Deviations between these entries and the corresponding input transition probabilities of Table 1 are not extreme for a sample of 500.

Table 3 summarizes data on which the transition matrix was constructed, and affords additional checks on simulation output. Observational data consisted in noting lithologic composition of two Chesteran sections at fixed vertical intervals of 8 feet, which yielded 311 equally spaced observations on the state of the system. Sandstone (state A) occurred 79 times, arranged as 20 individual groups separated by shale or limestone. Inasmuch as each observation represents an 8-foot interval, total sandstone thickness was estimated as 79 x 8 = 632 feet, and the average sandstone thickness was computed as 632/20 = 31.6 feet. On the basis of similar estimates for the other rock types, remaining columns of Table 3 were prepared. Comparison of Tables 2 and 3 shows satisfactory agreement on the whole, although the percentage of limestone in the sample of 500 transitions is somewhat high, mainly at the expense of sandstone, which is low.

These results, supplemented by similar analyses of other sections, indicate that a single matrix can be used in Markov simulation to include the thickness factor, although other questions arise in structuring the transition matrix in this manner. The most important relates to the vertical interval selected for measurement, inasmuch as this influences the extent to which thin beds may be missed. On the other hand, there is no need for multistory lithologies in order to have entries in the matrix diagonals. In fact, multistory lithologies can be considered as separate states, such as state C_1 representing thinbedded limestone, and state C_2 thickbedded limestone.

TREE DIAGRAMS AND FIXED PROBABILITY VECTORS

Griffiths (1966) presents an excellent discussion on construction, from transition probability matrices, of diagrams that show probabilities associated with each state through a succession of cycles. Such tree diagrams are illuminating for studying the probable sequences of beds after any given number of events or transitions. An introduction to tree diagrams and their construction is given in Kemeny, Snell, and Thompson (1957, Chap. IV, Sections 7, 13). Figure 3, in the Appendix, shows a portion of such a tree.

Griffiths (1966) and Carr and others (1966) also illustrate the interesting fact that if the

transition probability matrix contains only positive elements (i.e., if it is a regular Markov chain), probabilities associated with each successive branching of the tree diagram can be had directly by raising the transition probability matrix to the corresponding power. Moreover, for regular chains, as successively higher powers are computed, the original probability transition matrix approaches a matrix in which each row becomes a fixed probability vector, that does not change during succeeding higher powers. As an example, the transition probability matrix of Table 1 (left) yields the following matrix at its 43rd power:

	A	B	C
A	0.2335	0.4480	0.3184
B	0.2335	0.4480	0.3184
C	0.2335	0.4480	0.3184

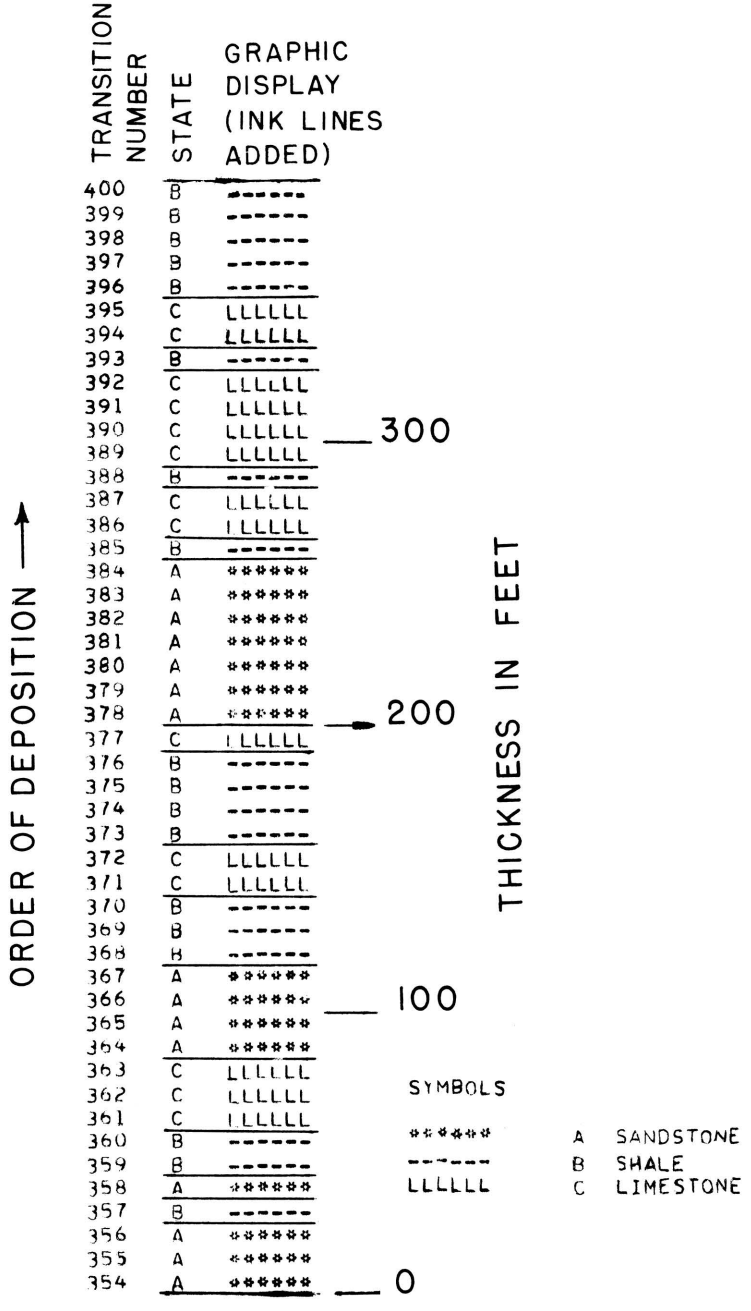


Figure 2.-Part of simulation run from transition probability matrix of Table 1.

Table 2.-Results of simulation experiment with matrix of Table 1.

Rock Type	Number of Occurrences	Thickness (Feet)	% of Total Thickness	Average Bed Thickness (Feet)	% of Total Beds
Sandstone	28	944	23.5	33.7	16.5
Shale	75	1736	43.4	23.1	44.1
Limestone	67	1328	33.1	19.8	39.4
Totals	170	4008	100.0		100.0

Relative frequencies of transitions realized in a 500-transition simulation experiment

	A	B	C
A	0.77	0.18	0.05
B	0.08	0.65	0.27
C	0.07	0.32	0.61

When rounded to two decimal places (0.23, 0.45, 0.32), stability is reached at the 16th power. Computer program STOCHEX in the Appendix was used to obtain this equilibrium matrix.

In general, the fixed probability vector represents the equilibrium state of the system, but its interpretation depends upon the structure of the transition probability matrix. For the matrix of Table 1, where relative rock thickness is expressed by diagonal entries, the stratigraphic unit in its equilibrium state has approximately 23 percent sand, 45 percent shale, and 32 percent limestone. These proportions do not differ greatly from percentage thickness of each rock type in column 4 of Tables 2 and 3. Experiments with matrices based on transitions from one lithology to another yielded fixed probability vectors related to percentage of occurrences of each rock type rather than percentage thickness.

The fixed probability vector can be used as an independent-events model for generating simulated stratigraphic sections without memory implications involved in the Markov model. As stated earlier, each event in a purely random model is independent of preceding events, which means that any tendency for sandstone to be succeeded by shale (which may have a high probability in the observed section) now depends only on relative probabilities of sand, shale, and limestone in the fixed probability vector. Zeller (1964) used random numbers to generate stratigraphic sections in this manner, with thickness dependent upon drawing the same rock type one or more times in succession. Computer program NO MEM can be used for such simulations. An alternative would be to use the thickness frequency distribution of each rock type to assign a thickness to each individual state, with repetitions shown as multistory events. In this latter model both rock succession and thickness represent independent-events processes.

TESTS FOR THE MARKOV PROPERTY

A question of basic importance has been deferred to this section, after some properties of transition probability matrices were discussed. It is possible to generate transition matrices from any succession of events. These need not proceed at fixed intervals of time or space, but can be completely sporadic. One could note successions of thundershowers during a given time span, using amount of rainfall per shower to define a convenient number of states in the system. Here each shower is a discrete event, with no necessarily fixed time interval between successive showers. Similarly, a transition probability matrix could be generated by noting the sequence of 1's, 2's, ..., 6's in successive rolls of a die, regardless of the time

Table 3.-Observed data: State of section examined at vertical intervals of 8 feet.

Rock Type	Number of Occurrences	Thickness (Feet)	% of Total Thickness	Average Bed Thickness (Feet)	% of Total Beds
Sandstone	20	632	25.4	31.6	18.0
Shale	52	1112	44.7	21.4	46.8
Limestone	39	744	29.9	19.1	35.2
Totals	111	2488	100.0		100.0

interval between rolls. Wave runup on a beach can be considered as a succession of discrete events, with the several states defined by distance of runup. The time interval between successive runups is related to the period of the wave involved, so that commonly events do not occur at fixed time intervals. A succession of glacial varves, with states defined by varve thickness, is an example of a discrete series in which each event is associated with a fixed time interval of one year.

The fact that transition probability matrices can be prepared from any succession of events does not necessarily mean that the original physical process is a Markov process. By structuring the sequence of events into a transition probability matrix, however, the model has become Markovian. Thus, a transition probability matrix prepared from a sample of N successive rolls of a six-sided die does not alter the fact that rolling dice is an independent-events process. Nevertheless, the matrix can be used for simulation experiments (which now follow probabilities in the matrix, and hence states are no longer completely independent); and the transition matrix itself can be raised to its N-th power to reach an equilibrium state. The resulting fixed probability vector for a six-sided die will generally not have probability 0.1667 for each state, but will have probabilities related to the particular sample that was drawn. Such a "die-rolling" experiment yielded the following fixed probabilities at P^5: 0.194, 0.149, 0.174, 0.161, 0.151, and 0.173. These are near the theoretical values, but they indicate that it is the finite observed sample (in this case 583 "rolls") and not the theoretical population probabilities that controls the equilibrium proportions.

It is evident that an important substantive question in setting up Markov models is whether the process under study actually has the Markov property. Markov processes range from relatively simple first-order regular Markov chains to complex models with memories that extend backward for two or more steps. Moreover, as mentioned, models may have discrete states and discrete time (or space); or they may operate in continuous time (or space) with either discrete or continuous states. Statistical criteria are available for testing some alternative ways of structuring a set of successive events, and here we consider only a test of the null hypothesis for an independent-events process as against the alternative of a simple first-order Markov chain. Computer program TESTMARK is designed for this simplest situation as an introduction to a relatively complex subject.

Anderson and Goodman (1957) and Billingsley (1961) discuss the application of the Chi-square distribution to this problem, and we shall describe the procedure used by Anderson and Goodman, on which TESTMARK is based. Conditions of the test are that the transition probability matrix is stationary, and that events occur at equally spaced intervals of time. An example is afforded by arrays in Table 1, based on equal vertical distances for observations of state. Substitution of space for time is valid for an ergodic matrix, and probabilities remain constant during simulation runs. The test is conducted by using both the probability matrix and the tally matrix in Table 1. The sample statistic is computed as follows:

$$-2 \log_e \lambda = 2 \sum_{ij}^{m} n_{ij} \log_e (\hat{p}_{ij} / \hat{p}_j)$$

where the quantity $(-2 \log_e \lambda)$ for $\underline{m}$ states is asymptotically distributed as Chi-square with $(m - 1)^2$ d.f., n_{ij} is the number of tally marks in the ij-th cell, $\hat{p}_{ij}$ is the transition probability in the same cell, and $\hat{p}_j$ is the marginal probability for the column. The null hypothesis tested by this statistic is that events at successive points in space or time are statistically independent, as against the alternative hypothesis that the observations are from

a first-order Markov chain.

In using the data in Table 1 for this test, the $\hat{p}_j$ values are first computed by adding tally numbers for each column in the right-hand matrix, and converting these to proportions. Thus, the sum of the first column is 77, and 77/309 = 0.25, which is $\hat{p}_1$. For the upper left element (ij = 11), the computations are as follows: the number n_{11} of tallies on the right is 59. The probability $\hat{p}_{11}$ for this cell is 0.74, and $\hat{p}_1$ (for the first column) is 0.25. Hence, we compute 59 $\log_e$ (.74/.25) = 64.22. These computations are performed for each matrix element, and summed algebraically. The sum is doubled and compared with tabled values of Chi-square distribution for the α chosen and appropriate degrees of freedom.

Data in Table 1 give the value $-2 \log_e \lambda$ = 159.6, which is larger than the tabled value (9.49) for α = 0.05 and 4 d.f. Thus, the hypothesis of an independent-trials process is rejected. The die-rolling experiment mentioned earlier, when subjected to the same test, gave the result $-2 \log_e \lambda$ = 24.8, which is less than the tabled value (37.65) for α = 0.05 and 25 d.f. Hence, for the die-rolling results the hypothesis of an independent-trials process was not rejected.

A different approach to examining sets of data for the Markov property is based on autocorrelation techniques. Grant (1957, p. 325) for example, in discussing criteria to distinguish residuals from the main trend in map data, used the following relation to determine whether a set of residuals is autocorrelated:

$$\varepsilon_i = \rho \varepsilon_{i-1} + u_i \quad |\rho| < 1$$

where ε_i is the i-th residual and ε_{i-1} is the next preceding residual. The u_i values represent a random variable distributed normally with mean zero, and independent of ε_{i-1}, ε_{i-2}, ..., and of u_{i-1}, u_{i-2}, ... The null hypothesis is that ρ, the population correlation coefficient, is zero in the model, so that if the hypothesis is accepted, residuals are not autocorrelated. For present purposes, if the null hypothesis is rejected, (i.e., if $\rho \neq 0$), presumably there is a Markov property in the equally spaced sequence of numbers.

The autocorrelation procedure is applied directly to the sequence of observed values, whereas the Chi-square test, as noted, is based on the transition probability matrix supplemented by the tally matrix. In the Chi-square test the data are first structured as if they had a first-order Markov property, so that if the null hypothesis of an independent-trials process is rejected, the transition probability matrix can be used directly in MARCHAIN for simulation experiments.

It is to be emphasized that this Chi-square test applies rigorously only to regular first-order Markov chains, although Anderson and Goodman (1957) give tests for higher order chains. Computer program TESTMARK in the Appendix is based on this first-order test only, and it will not accept transitional probability matrices with zero entries along the diagonal or in off-diagonal elements. The program is offered in its present form for convenience only; interested readers will likely wish to generalize the tests.

PROBLEMS IN STRATIGRAPHIC SIMULATION

Although it is generally accepted that cyclical sedimentary deposits have Markov properties, the simple first-order Markov chain has some limitations in simulating real-world cyclical sequences. One is that the transition probability matrix, with a one-step memory, includes no mechanism for the occurrence of stratigraphic marker beds or groups of characteristic beds at particular positions in the simulated sections. Such beds not only occur prominently in natural cyclical sequences, but provide an important basis for stratigraphic correlation.

A set of simulations, each based on the same first-order Markov transition probability matrix, compare favorably with the natural section and with each other in their percentages of lithologic types, bed thickness, and other gross attributes, but the occurrence of any particular beds in a given interval is wholly dependent upon the transition probabilities. The chance of having several prominent limestones of approximately equal thickness in equivalent parts of two or more simulations is small.

The development of a model which simulates stratigraphic sections that have well-defined marker beds, such that successive simulations can be "stratigraphically correlated" requires a more complex memory system than is present in the simple Markov chain. Several avenues are open, such as developing a Markov model with nonstationary transition probabilities, or using a combination of the simple Markov model with stationary transition probabilities, and a "feedback matrix" that introduces fairly regularly spaced marker beds. Thus, for a cyclical sequence in which several prominent limestones occur at roughly 200-foot intervals, the complete model would require a memory extending backward in time or space such that the probability of obtaining a moderately thick limestone increases rapidly as each 200-foot segment of section is simulated, with the probability decreasing to its stationary value again as soon as the marker bed is deposited. Some fairly ingenious (if empirical)

models can be developed to generate such sections, and they provide stimulating exercises for students.

The problem of introducing stratigraphically correlatable beds into a simulation is perhaps most effectively handled (within the framework of a first-order Markov chain) by expressing the transitions as lateral shifts of some specified attribute, such as the successive positions occupied by a strandline sand during transgressive-regressive cycles. In such a model several "monitoring positions" can be set up which follow the lateral shifts and produce two or more stratigraphic sections that are directly correlatable. For this adaptation it is more effective to use transition rates (q_{ij}) obtained from the transition probabilities (p_{ij}) by transformation. The transition rates convert the Markov process from one with discrete states and discrete time (or space) into a Markov process with discrete states but with continuous time (or space). Models of this sort were experimentally developed in the spring of 1966 at Princeton University, where I had the privilege of working with Professor J.W. Tukey and his Statistical Techniques Research Group. A computer program written by Paul Tukey implemented the experiments. This modification of the simple chain used here will be presented in a later paper.

CONCLUDING REMARKS

There are many phenomena in geology where the state of a system at time t_{r-1} exerts a dominant influence on the state at time t_r, although earlier events, at time t_{r-2}, t_{r-3}, ..., may also exert some influence. The Markov chain can be extended to such longer memory events, as in the cited reference to Pattison (1965), who used events as far back as t_{r-6} to develop a sixth-order Markov chain.

A main reason for confining this paper to first-order Markov chains is that the principles underlying the discrete-time, discrete-state Markov process are adequately brought out by the simplest model. Kemeny and Snell (1960), or an equivalent reference book, is a useful adjunct here.

An important byproduct of experimentation with Markov processes is that the student or researcher begins to see his observations in a new framework of a set of states that succeed each other in some patterned way. This insight commonly leads to fuller understanding of the underlying geological process. This process may have strong deterministic elements in its structure, as well as probabilistic elements that contribute fluctuations of varying magnitude to observational data. Although these fluctuations are random in the commonly accepted sense of the word, it is probably erroneous to view them as haphazard fluctuations without rhyme or reason.

A random variable is a mathematical entity that arises from probabilistic mechanisms, just as conventional nonstochastic (systematic) variables are associated with deterministic mechanisms. An important difference between these classes of variables is that in the deterministic case outcome of an experiment is exactly predictable, whereas in the probabilistic case outcome of an experiment is controlled by an underlying set of probabilities. Hence, the outcome cannot be predicted except in terms of relative likelihoods associated with a set of possible outcomes.

Examination of probabilistic elements in a geological phenomenon greatly extends and strengthens usefulness of statistical models in geology. Bartlett (1960, p. 1) points out that stochastic processes represent the dynamic part of statistical theory - the statistics of change - in contrast to the more static statistical problems treated by conventional statistical methods. Where probabilistic elements can be expressed as frequency distributions whose mean values are coefficients or exponents in deterministic expressions, the probabilistic model may be more meaningful than the classical path-dependent deterministic model. Briggs and Pollack (1967), however, suggest that when "...sufficient knowledge exists concerning the processes and the geology, there is no need to invoke random, probabilistic processes."

Certainly an understanding of the real-world process is essential in developing probabilistic or deterministic models. It is appropriate here to quote Coleman (1965, p. 528), who writes in the context of mathematical sociology, that "... there is an ever-present danger with probabilistic models that we will use them to say little or nothing - but to say it elegantly - about the behavior at hand." This danger is obviously present in geology as well. Perhaps experimentation with simple stochastic process models, checked and tested by substantive analysis of real-world phenomena, can lead to balanced judgment regarding the optimum approach to specific geological problems.

REFERENCES

Agterberg, F.P., 1966, The use of multivariate Markov schemes in geology: Jour. Geology, v. 74, p. 764-785.

Anderson, T.W., and Goodman, L.A., 1957, Statistical inference about Markov chains: Ann. Math. Stat., v. 28, p. 89-110.

Bartlett, M.S., 1960, An introduction to stochastic processes with special reference to methods and applications: The Univ. Press, Cambridge, 312 p.

Billingsley, P., 1961, Statistical methods in Markov chains: Ann. Math. Stat., v. 32, p. 12-40.

Briggs, L.I., and Pollack, H.N., 1967, Digital model of evaporite sedimentation: Science, v. 155, no. 3761, p. 453-456.

Carr, D.D., and others, 1966, Stratigraphic sections, bedding sequences, and random processes: Science, v. 154, no. 3753, p. 1162-1164.

Coleman, J.S., 1964, Introduction to mathematical sociology: Free Press, Glencoe, Illinois, 554 p.

Grant, F., 1957, A problem in the analysis of geophysical data: Geophysics, v. 22, p. 309-344.

Griffiths, J.C., 1966, Future trends in geomathematics: Pennsylvania State Univ., Mineral Industries, v. 35, p. 1-8.

Harbaugh, J.W., 1966, Mathematical simulation of marine sedimentation with IBM 7090/7094 computers: Kansas Geol. Survey Computer Contr. 1, 52 p.

Harbaugh, J.W., and Wahlstedt, W.J., 1967, FORTRAN IV program for mathematical simulation of marine sedimentation with IBM 7040 or 7094 computers: Kansas Geol. Survey Computer Contr. 9, 40 p.

Heller, R.A., and Shinozuka, M., 1966, Development of randomized load sequences with transition probabilities based on a Markov process: Technometrics, v. 8, p. 107-114.

Ishida, K., 1966, Stochastic approach to nonequilibrium thermodynamics of first-order chemical reactions: Jour. Phys. Chem., v. 70, p. 3806-3811.

Kemeny, J.G., Snell, J.L., and Thompson, G.L., 1957, Introduction to finite mathematics: Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 372 p.

Kemeny, J.G., and Snell, J.L., 1960, Finite Markov chains: D.Van Nostrand Co., Inc., Princeton, New Jersey, 210 p.

Leopold, L.B., and Langbein, W.B., 1962, The concept of entropy in landscape evolution: U.S. Geol. Survey Prof. Paper 500-A, p. A1-A20.

Leopold, L.B., Wolman, M.G., and Miller, J.P., 1964, Fluvial processes in geomorphology: W.H. Freeman and Co., San Francisco, 522 p.

Loucks, D.P., and Lynn, W.R., 1966, Probabilistic models for predicting stream quality: Water Res. Res., v. 2, p. 593-605.

Pattison, A., 1965, Synthesis of hourly rainfall data: Water Res. Res., v. 1, p. 489-498.

Scheidegger, A.E., 1966, Stochastic branching processes and the law of stream orders: Water Res. Res., v. 2, p. 199-203.

Vistelius, A.B., 1949, On the question of the mechanism of the formation of strata: Doklady Akademii, Nauk SSSR, v. 65, p. 191-194.

Vistelius, A.B., and Feigel'son, T.S., 1965, On the theory of bed formation: Doklady Akademii, Nauk SSSR, v. 164, p. 158-160.

Vistelius, A.B., and Faas, A.V., 1965, On the character of the alternation of strata in certain sedimentary rock masses: Doklady Akademii, Nauk SSSR, v. 164, p. 629-632.

Wickman, F.E., 1966, Repose period patterns of volcanoes; V. General discussion and a tentative stochastic model: Arkiv for Mineralogi och Geologi, v. 4, p. 351-367.

Zeller, E.J., 1964, Cycles and psychology, in Symposium on Cyclic Sedimentation: Kansas Geol. Survey Bull. 169, v. 2, p. 631-636.

Additional References Not Cited.

Adelman, I.G., 1958, A stochastic analysis of the size distribution of firms: Jour. Am. Stat. Assoc., v. 53, p. 893-904. (Example of discrete states with unequal class intervals).

Agterberg, F.P., 1966, Markov schemes for multivariate well data: Min. Ind. Experiment Sta., Pennsylvania State Univ., Spec. Publ. 2-65, p. Y1-Y18. (Theory and application of first-order Markov process to study of chemical elements in a reef).

Amorocho, J., and Hart, W.E., 1964, Critique of current methods in hydrologic systems investigation: Trans. Am. Geophys. Union, v. 45, p. 307-321. (First-order and higher order Markov chains [p. 318]).

Clark, W.A.V., 1965, Markov chain analysis in geography: an application to the movement of rental housing areas: Ann. Assoc. Am. Geog., v. 55, p. 351-359. (Study of rentals in several cities for three 10-year intervals).

Graf, D.L., Blyth, C.R., and Stemmler, R.S., 1967, One-dimensional disorder in carbonates: Illinois Geol. Survey Circ. 408, 61 p. (First-order Markov model applied to crystallographic defects in carbonate crystals).

Potter, P.E., and Blakely, R.F., 1966, Synthetic generation of stratigraphic sections and bedding sequences: (abs.), General Meetings Program, Geol. Soc. America, p. 148. (Markov model of fluviatile sandstone body, including permeability and mean grain size).

Rogers, A., 1966, A Markovian policy model of interregional migration: Regional Sci. Assoc. Papers, v. 17, p. 205-224. (Interregional migration under controlled and uncontrolled political conditions).

Scheidegger, A.E., and Langbein, W.B., 1966, Probability concepts in geomorphology: U.S. Geol. Survey Prof. Paper 500-C, p. C1-C14. (Markov processes continuous in time and space for slope development).

Vistelius, A.B., 1966, Genesis of the Mt. Belaya granodiorite, Kamchatka (an experiment in stochastic modeling): Doklady Akademii Nauk SSSR, v. 167, p. 1115-1118. (Application of a Markov chain in the study of the sequence of mineral grains in a thin section).

APPENDIX

The four programs listed here are written in a modified version of FORTRAN IV, which is used with the CDC 3400 computer. Normally there is little difficulty in making these programs compatible with standard FORTRAN IV. All programs are fairly short, and are kept as separate programs partly for greater convenience in making such modifications as the user may wish. For readers who prefer an all-inclusive program, the four short programs can be combined readily, especially inasmuch as two of them, MARCHAIN and NO MEM, are similar.

The four programs are listed in the order of their normal use. That is, after the transition probability matrix has been compiled, it (and sometimes the tally matrix on which it is based) is used with TESTMARK. If the Markov property is present, the transition probability matrix is normally used with MARCHAIN for a simulation run (say 500 transitions). The transition probability matrix is then used with STOCHEX to obtain the fixed probability vector, and if desired, this vector can be used as input to NO MEM.

If TESTMARK does not reject the hypothesis of an independent-events process, the matrix being tested can be used with STOCHEX to find the fixed probability vector for simulation with NO MEM.

Although the preceding remarks are based on the assumption that observed data are used to generate the transition probability matrix, there is no reason why completely arbitrary matrices cannot be used as input. Harbaugh (1966), for example, suggests that geological experimentation can be conducted with probabilities that are "reasonable" for the processes to be simulated. By using a range of values it is possible to compare the simulation output derived from various assumptions regarding the geological phenomenon under study.

Virtually all programs in the geology program library at Northwestern University have an underlying similarity in their organization, and in the kinds of data formats used. These similarities involve the use of project numbers, four title cards, and at least one major control card. Programs also are arranged so that several data decks can be placed one behind the other, rather than using the program separately for each set of data.

The following comments apply to all four programs that follow in this Appendix:

A. Title Cards.-Provision is made for 4 title cards, so that the specific problem can be completely documented. These are of the following form:

Card 1. Enter the digit 1 in column 1, and leave column 2 blank. The initial digit is a carriage control, so that titles are repeated as new segments of output are printed. Cols. 3 to 70 inclusive may be used for any appropriate title.

Cards 2, 3, 4. Enter a zero in column 1, leave column 2 blank, and proceed as with the first card.

Examples of titles for TESTMARK are given in Tables A-1 and A-2.

B. Control Cards.-These furnish the computer with specific details regarding a particular problem, such as size of matrix or vector to be read in, number of simulations required, a choice of output format where this applies, and format of the data cards. The control card for each program is described in detail with each program listed.

C. Input Matrices and Vectors.-The programs listed here are designed to accept input matrices up to 20x20 in size, and (for NO MEM) a row vector up to 1x20 in size. Input is read in by rows, and input cards contain the following items:

Cols. 1-6 Project number (example 010275)

Cols. 7-10 Row number of matrix (example 0001)

Cols. 11-70 Input fields of any convenient size. Where probabilities are entered to two or three decimal places, fields can be kept uniform, say from 3 to 6 digits each. The tally matrix of Table 1, which has whole numbers, had its first card punched as follows:

010275 1 59 18 02

in which the project number is a field of A6, the matrix row number is A4, and the n_{ij} are in 3F3.0. All entries are right-justified. The transition probability matrices are usually punched to 2 or 3 decimal places. IN NO INSTANCE IS IT NECESSARY TO INCLUDE MARGINAL TOTALS IN THE INPUT MATRICES OR VECTORS.

D. Computing Center Lead Cards.-The sequence of cards used with FORTRAN 3400 source decks at the Vogelback Computing Center are as follows:

JOB CARD, with Computing Center Account Number
SOURCE DECK FOR PROGRAM
SCOPE CARD
LOAD CARD
RUN CARD

These cards are not included in the program listings, inasmuch as various computing centers follow different procedures.

Program TESTMARK

The underlying equation on which this program is based was given in the section on tests

for the Markov property. Input for the program has four title cards, one control card, and either one or two matrices. The first matrix is the "tally matrix" discussed in connection with Table 1 of the text, whose elements are n_{ij}, number of transitions in the original observational data. The second matrix is the transition probability matrix derived from the tally matrix, as illustrated in Table 1 of the text.

Two choices are available to the program user. In the first choice both matrices are read in (WITHOUT MARGINAL VALUES), and in the second choice the tally matrix alone is used, and the program computes the transition probability matrix. In either choice, the program computes marginal totals of the tally matrix, and computes $\hat{p}_j$ for each column. It then extracts the n_{ij} and $\hat{p}_{ij}$ from successive ij positions in the two matrices (i.e., the input tally matrix and the computed or input transition probability matrix). The ratio $\hat{p}_{ij}/\hat{p}_j$ is computed and transformed to its natural logarithm. This is then multiplied by its corresponding n_{ij}, and the products are arranged into a square matrix whose subscripts have the same values as the input matrix. This is called the N LOG P matrix in the printed output, and its elements are

$$[n_{ij} \log_e(\hat{p}_{ij}/\hat{p}_j)].$$

The N LOG P matrix is summed and multiplied by 2 to give the statistic for testing with Chi-square for $(m - 1)^2$ d.f., and the critical α chosen. The N LOG P matrix is convenient for locating the particular elements that contribute most strongly to the final statistic. For example, in Table A-1 the only positive elements are along the diagonals, and for this example these control the large positive value obtained. For the 3 x 3 matrix used, m = 3, and hence d.f. = 4, for which the critical value of Chi-square at $\alpha = 0.05$ is 9.49, as described in the text.

In operating TESTMARK, the first 4 cards are TITLE CARDS, already described. The CONTROL CARD is next, with the following form:

Cols. 1-6 Project number, A6. This is for record only and is not read in. Columns may be left blank if desired.

Cols. 7-9 Size of input matrix or matrices, I3, (i.e., 004 means four rows and four columns).

Col. 10 Choice code, I1. Choice 1 means that both the tally matrix and transition probability matrix are read in. Choice 2 means that only the tally matrix is read in.

Cols. 11-45 Format statement for tally matrix, 7A5. If the matrix is 4 x 4, say, with whole numbers not exceeding 999, the format would be (10X, 4F3.0). Note that the first 10 columns of the input matrix are not read.

Cols. 46-70 Format statement for transition probability matrix if choice 1 is used. This normally differs from the tally matrix because probabilities are entered to 2 or 3 decimal places. Thus, the format statement corresponding to the preceding example, when 3 decimal places are used, is (10X, 4F3.3).

Tables A-1 and A-2 are examples of the two choices of output for the matrices in Table 1 of the text. The printout is rounded to 2 decimal places, which means that occasionally the computed $\hat{p}_{ij}$ matrix may not always add to 1.00 for each row. Refer to the footnote to Table 1 in the text and note that in Table A-2 the computed $\hat{p}_{ij}$ in the top and bottom rows do not add to 1.00. This particular example is used to show that the effect of these differences on the final output, 2(SUM OF N LOG P MATRIX) differs only in the first decimal place.

Program TESTMARK is listed in Table A-3. Some remarks about this program, and some changes that are recommended as student exercises, are appropriate here. In the first place, the program will not accept tally matrices with zero entries for any n_{ij}, including the diagonals. The assumption of the test is that each element is positive. Occurrence of a few zeroes, however, especially in large matrices, can be handled by testing each element against zero. If zero is present, skip this element and proceed to the next. Statistical limitations imposed by zero elements can be met approximately by omitting one d.f. for each zero. Thus if two zeroes occur in a 6 x 6 matrix, the degrees of freedom would be $(m - 1)^2 - 2 = (6 - 1)^2 - 2 = 23$. These remarks are tentative only, and if zeroes occur in the diagonals, the test may fail by not rejecting the null hypothesis of an independent-trials process even if a Markov property is present.

In its present form TESTMARK does not have a Chi-square library built into it, nor does it print out the $(m - 1)^2$ d.f. The latter can readily be taken care of by simply picking the matrix size from the control card, subtracting 1 from it, and squaring the remainder. Similarly, where the $\hat{p}_{ij}$ matrix is computed, rounding difficulties are usually avoided if the printout is carried to 3 decimal places.

TABLE A-1

KRUMBEIN PROJECT 01 0275

CHESTER DATA, 8-FOOT INTERVALS

3 STATES, SAND,SHALE,LIME

INPUT IS 2 MATRICES

TALLY MATRIX INPUT

	A	B	C	MG
A	59	18	2	79
B	14	86	41	141
C	4	34	51	89
MG	77	138	94	309

TRANSITION PROBABILITY MATRIX (INPUT)

A	0.74	0.23	0.03
B	0.10	0.61	0.29
C	0.05	0.38	0.57

P(J) ROW VECTOR

0.25 0.45 0.30

KRUMBEIN PROJECT 01 0275

CHESTER DATA, 8-FOOT INTERVALS

3 STATES, SAND,SHALE,LIME

INPUT IS 2 MATRICES

N LOG P MATRIX

A	64.217	-11.945	-4.633
B	-12.783	26.814	-1.961
C	-6.425	-5.491	32.024

2(SUM OF N LOG P MATRIX)= 159.638

TABLE A-2

KRUMBEIN PROJECT 01 0275

CHESTER DATA, 8-FOOT INTERVALS

3 STATES, SAND,SHALE,LIME

INPUT IS TALLY MATRIX

TALLY MATRIX INPUT

	A	B	C	MG
A	59	18	2	79
B	14	86	41	141
C	4	34	51	89
MG	77	138	94	309

TRANSITION PROBABILITY MATRIX COMPUTED

A	0.75	0.23	0.03
B	0.10	0.61	0.29
C	0.04	0.38	0.57

P(J) ROW VECTOR

0.25 0.45 0.30

KRUMBEIN PROJECT 01 0275

CHESTER DATA, 8-FOOT INTERVALS

3 STATES, SAND,SHALE,LIME

INPUT IS TALLY MATRIX

N LOG P MATRIX

A	64.760	-12.114	-4.973
B	-12.882	26.804	-1.851
C	-6.851	-5.310	32.295

2(SUM OF N LOG P MATRIX)= 159.756

TIME IN MINUTES 000.48 ASSEMBLY, 000.06 EXECUTE

TABLE A-3

```
      PROGRAM TESTMARK
C          FOR W.C.KRUMBEIN, GEOLOGY....JAN 1967....BETTY BENSON
C
      DIMENSION T(20,20),P(20,20),LAMBDA(20,20)
      DIMENSION S(20),HTOT(20),VTOT(20)
      DIMENSION TITL(40),FMT1(7),FMT2(7),LET(20),MING(3)
C
      REAL LAMBDA
      DATA (IBL=1H ),(MARG=2RMG)
      DATA (MING=24H(XR2,X00F6.0,F12.0)       )
      DATA (LET=2R A,2R B,2R C,2R D,2R E,2R F,2R G,2R H,2R I,2R J
     .          ,2R K,2R L,2R M,2R N,2R O,2R P,2R Q,2R R,2R S,2R T)
C
C
C                                                READ TITLES, CONTROL CARD
    2 READ 100,TITL
  100 FORMAT (10A8)
      IF (EOF,60) 50,3
   50 STOP
    3 PRINT 100,TITL
      READ 101,N,KRIS,FMT1,FMT2
  101 FORMAT (6XI3,I1,7A5,7A5)
      MING(1)=MING(1) .AND. 7777777777777700B
      NTEN=(N/10)*64  $  NUNIT=MOD(N,10)
      MING(1)=MING(1).OR.NTEN.OR.NUNIT
C
C                                  READ TALLY MATRIX, COMPUTE ROW-COLUMN SUMS
      DO 4 I=1,N
    4 READ FMT1,(T(I,J),J=1,N)
      TOT=0.0
      DC 5 I=1,N
    5 HTOT(I)=VTOT(I)=0.0
      DO 7 I=1,N
      DO 6 J=1,N
      HTOT(I) = HTOT(I) + T(I,J)
    6 VTOT(J) = VTOT(J) + T(I,J)
    7 TOT = TOT + HTOT(I)
C
C                                       PRINT TALLY MATRIX AND MARGINAL SUMS
      PRINT 130,(LET(J),J=1,N),IBL,MARG
      DO 15 I=1,N
   15 PRINT MING,LET(I),(T(I,J),J=1,N),HTOT(I)
      PRINT 132
      PRINT MING,MARG,(VTOT(J),J=1,N),TOT
C
C                                         READ OR COMPUTE PROBABILITY MATRIX
      GO TO (10,8) KRIS
    8 DO 9 I=1,N
    9 READ FMT2,(P(I,J),J=1,N)
      MESSG=8H (INPUT)
      GO TO 12
   10 DO 11 I=1,N
      DO 11 J=1,N
   11 P(I,J) = T(I,J) / HTOT(I)
      MESSG = 8HCOMPUTED
   12 PRINT 121,MESSG
```

TABLE A-3, CONT.

```
      DO 16 I=1,N
   16 PRINT 107,LET(I),(P(I,J),J=1,N)
C                                                    COMPUTE P(J)S
      DO 13 J=1,N
   13 S(J) = VTOT(J) / TOT
      PRINT 108,(S(J),J=1,N)
C
C                                    COMPUTE, PRINT N LOG P AS MATRIX
      SUM=0.0
      PRINT 100,TITL
      PRINT 109
      DO 26 I=1,N
      DO 25 J=1,N
      LAMBDA(I,J) = ALOG(P(I,J)/S(J)) * T(I,J)
   25 SUM=SUM+LAMBDA(I,J)
   26 PRINT 104,LET(I),(LAMBDA(I,J),J=1,N)
C                                          PRINT OUT FINAL ANSWER
      SUM=2*SUM
      PRINT 110,SUM
      GO TO 2
C
  104 FORMAT (3XR2,2X,16F8.3 / 7X,4F8.3)
  107 FORMAT (XR2,X,20F6.2)
  108 FORMAT (// 5X*P(J) ROW VECTOR*/ 5X,15(1H-)/ 4X,20F6.2)
  109 FORMAT (//7X,*N LOG P MATRIX* / 7X,14(1H-)/)
  110 FORMAT (//7X,*2(SUM OF N LOG P MATRIX)=*F9.3)
  121 FORMAT (// 5X*TRANSITION PROBABILITY MATRIX*,A15/ 5X,29(1H-)/)
  130 FORMAT (//5X*TALLY MATRIX INPUT* / 5X,18(1H-) //4X,22(4XR2))
  132 FORMAT (X)
      END
                    000089    CARDS
```

Program MARCHAIN

This program accepts as input a transition probability matrix of dimensions up to 20 x 20. The matrix is read into the program by rows, and number of transitions to be included in the simulation is specified on a control card. At the end of the simulation run the computer tallies the transitions and prints out a transition probability matrix generated by the simulation, for direct comparison with the input matrix. It also lists percentage occurrences of each state in the simulation.

The program is illustrated with the same sample used in TESTMARK in order to carry the same problem throughout the text and examples. The main input to MARCHAIN is the transition probability matrix, and the user has two choices of output, to be illustrated later. The input consists of four TITLE CARDS, described earlier, and a CONTROL CARD as follows:

Cols. 1-6 Project number, A6. This is for record only and is not read in.

Cols. 7-10 Size of input matrix, I4. For a 3 x 3 matrix this is simply 0003.

Cols. 11-14 Number of transitions in simulation run, I4. Normally 500 or 1000 are called for, i.e., 0500 or 1000.

Cols. 15-18 Starting state. In the present version the user specifies this by using the code A = 1, B = 2,... , S = 19, T = 20. If code 0 is used, the initial state is selected at random from a uniform distribution. Carr and others (1966) use a random start based on the fixed probability vector.

Cols. 19-22 Type of output desired. Two forms of output are available. These are coded as 1 = POSITION OUTPUT and 2 = LITHO OUTPUT. (See further remarks below).

Cols. 23-70 Format statement. This is variable. The first 10 digits are not read in (they include the project number and the row number of the input matrix), so that the format statement for a data card with four fields of 6 digits with 2 decimal places is (10X, 4F6.2).

The control card is followed by the transition probability matrix, which is read in by rows. For this example, the format statement is (10X, 3F3.2). This input matrix is printed out directly after the titles, as shown in Table A-4.

An essential part of the program is a pseudo-random number generator, called RANFSET(T), similar to equivalent random number subroutines in all computing centers. The series is commonly started by the time on the Computing Center clock, which is printed out for reference.

As Table A-4 shows, probabilities in the matrix are first cumulated from left to right, for use of the random numbers. Thus, in the example shown, the starting position is designated as state A, and if the random number falls in the range 0.000 to 0.740 inclusive, the transition is AA. If the random number lies in the range 0.741 to 0.970, the transition is AB, with the range 0.971 to 1.000 leading to the transition AC. If AB is drawn, the drawing shifts to the second row of the matrix, and the next transition now is determined by the cumulative probabilities in the second row of the matrix.

The two forms of output are illustrated in Tables A-4 and A-5. If choice 1 is used, the simulation proceeds from left to right, with the first state (A) occupying the bottom line of the graphic output, and each succeeding state (up to 20 maximum) occupying succeedingly higher positions. This is called "position output." The output scheme is easily visualized, and each row of output, defined by the enclosing dashed lines, contains 100 transitions (the example in Table A-4 is cut short to fit the page, and has about 70 transitions).

The second choice of output is called "litho output" and is arranged as a stratigraphic column, with the first transition at the bottom, so that the order of deposition is upward along the column. This is choice 2 on the control card, and it requires a series of symbol cards to represent the rocks involved, one for each type. These are arranged in the order A, B, C, etc. of the input matrix. As many as 20 lithologies can be accommodated. The format of these cards is as follows:

Cols. 1-16 The name of the rock,

Cols. 17-22 Symbol selected.

This gives a standard width of 6 spaces for the column. The symbol cards are inserted just behind the control card and ahead of the input matrix. They are printed directly after the titles in the output of Table A-5. If choice 1 for position output is used, these cards are not accepted by the program, inasmuch as the position routine is built into the program.

The last part of output for MARCHAIN consists of a count of each kind of transition, shown in the bottom parts of Tables A-4 and A-5. These counts are converted to probabilities for comparison with the input matrix. Finally, the number and percentage of counts in each state is indicated. It should be mentioned that the initial state is not included in the counts, so that in calling for 500 transitions, there are actually 501 occurrences of the states. The output matrices and percentages are based on the 500 transitions in the simulation. Note that each simulation is a separate experiment, and that the output counts for each state tend to vary from one simulation to another.

Table A-6 lists the MARCHAIN program.

TABLE A-4

KRUMBEIN PROJECT 01 0275

CHESTER DATA, 8-FOOT INTERVALS

3 STATES, SAND,SHALE,LIME

P(I,J) MATRIX BASED ON 309 TRANSITIONS

TRANSITION PROBABILITY MATRIX INPUT

	A	B	C
A	0.740	0.230	0.030
B	0.100	0.610	0.290
C	0.050	0.380	0.570

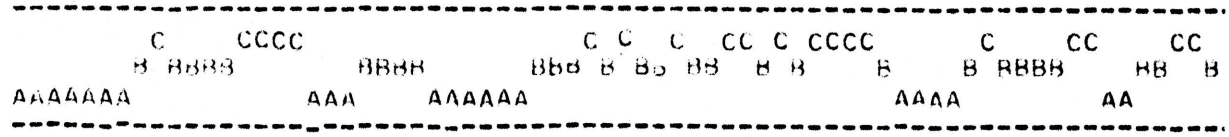
CUMULATED PROBABILITY MATRIX

A	0.740	0.970	1.000
B	0.100	0.710	1.000
C	0.050	0.430	1.000

CLOCK= 940.971

TRANSITIONS= 500

START= A



TRANSITION MATRIX COUNTS FROM OUTPUT

88	34	0
21	132	59
12	57	87

PROBABILITY MATRIX FROM OUTPUT

	A	B	C
A	0.727	0.281	0.000
B	0.094	0.592	0.309
C	0.077	0.365	0.558

	COUNTS	PERCENT
A	121	24.20
B	223	44.60
C	156	31.20

TABLE A-5

KRUMBEIN PROJECT 01 0275

CHESTER DATA, 8-FOOT INTERVALS

3 STATES, SAND,SHALE,LIME

P(I,J) MATRIX BASED ON 309 TRANSITIONS

		SYMBOLS
A	SANDSTONE	******
B	SHALE	------
C	LIMESTONE	LLLLLL

TRANSITION PROBABILITY MATRIX INPUT

	A	B	C
A	0.740	0.230	0.030
B	0.100	0.610	0.290
C	0.050	0.380	0.570

CUMULATED PROBABILITY MATRIX

A	0.740	0.970	1.000
B	0.100	0.710	1.000
C	0.050	0.430	1.000

CLOCK= 940.990

TRANSITIONS= 500

START= A

500	C	LLLLLL
499	C	LLLLLL
498	B	------
497	B	------
496	B	------
495	A	******
494	A	******
493	A	******
492	A	******
491	A	******
490	A	******
489	B	------
488	B	------
487	B	------
486	A	******
485	A	******

TABLE A-5, CONT.

```
30   C   LLLLLL
29   C   LLLLLL
28   B   ------
27   B   ------
26   C   LLLLLL
25   C   LLLLLL
24   B   ------
23   B   ------
22   B   ------
21   B   ------
20   B   ------
19   C   LLLLLL
18   C   LLLLLL
17   C   LLLLLL
16   C   LLLLLL
15   B   ------
14   C   LLLLLL
13   B   ------
12   C   LLLLLL
11   B   ------
10   A   ******
 9   A   ******
 8   A   ******
 7   A   ******
 6   A   ******
 5   A   ******
 4   A   ******
 3   A   ******
 2   A   ******
 1   A   ******
```

TRANSITION MATRIX COUNTS FROM OUTPUT

	97	29	0
	20	145	62
	8	53	86

PROBABILITY MATRIX FROM OUTPUT

	A	B	C
A	0.776	0.232	0.000
B	0.088	0.639	0.273
C	0.054	0.358	0.581

	COUNTS	PERCENT
A	125	25.00
B	227	45.40
C	148	29.60

TIME IN MINUTES 000.59 ASSEMBLY, 000.12 EXECUTE

TABLE A-6

```
      PROGRAM MARCHAIN
C                                                    DECEMBER 1966
C                FOR W.C. KRUMBEIN.......... BETTY BENSON
      DIMENSION X(20,20),TRANS(20,20),TITL(36),INGA(20,100),PILLAR(2000)
      DIMENSION LSYM(20), KRISTIE(100),LNAME(20,2)
      DIMENSION LET(20),KOUNT(20),PC(20),FMT(6)
C
      INTEGER PILLAR
      DATA (LET=1HA,1HB,1HC,1HD,1HE,1HF,1HG,1HH,1HI,1HJ,1HK,1HL,1HM,1HN,
     .  1HO,1HP,1HQ,1HR,1HS,1HT)
C          LTYPE=1 FOR 'POSITION' OUTPUT
C          LTYPE=2 FOR LITHO
C
C                                        READ CONTROLS
    2 READ 100,TITL
  101 FORMAT (6X,4I4,6A8)
      IF (EOF,60) 3,4
    3 STOP
    4 CONTINUE
      PRINT 100,TITL
      READ 101,N,NDRAW,ISTART,LTYPE,FMT
  100 FORMAT (9A8)
      IF (LTYPE.EQ.1) GO TO 1
      READ 119,((LNAME(I,J),J=1,2),LSYM(I),I=1,N)
      PRINT 120,(LET(I),(LNAME(I,J),J=1,2),LSYM(I),I=1,N)
    1 CONTINUE
C                                          READ,PRINT MATRIX
      DO 5 I=1,N
    5 READ FMT,(X(I,J),J=1,N)
      PRINT 102
      PRINT 114,(LET(J),J=1,N)
      DO 6 I=1,N
    6 PRINT 103,LET(I),(X(I,J),J=1,N)
C
C                    INITIALIZE...CONVERT TO CUM. MATRIX, PRINT
      DO 8 I=1,N
      KOUNT(I)=0
      DO 8 J=1,N
    8 TRANS(I,J)=0
      DO 7 I=1,N
      DO 7 J=2,N
    7 X(I,J) = X(I,J) + X(I,J-1)
      PRINT 104
      DO 9 I=1,N
    9 PRINT 103,LET(I),(X(I,J),J=1,N)
C
C                              SET (AND PRINT) STARTING STATE
      T = TIMEF(0)   $  CALL RANFSET(T)  $  PRINT 111,T
      PRINT 116,NDRAW
      IF (ISTART.GT.0) GO TO 23
      ISTART= T*1000.
      ISTART = MOD(ISTART,N)+1
   23 I=ISTART  $  PRINT 105,LET(I)
C                                          MAKE SPECIFIED NO. OF DRAWS
      DO 50 L=1,NDRAW
      IF (LTYPE.EQ.2) GO TO 14
```

TABLE A-6, CONT.

```
C                                    INITIALIZE EVERY 100 DRAWS FOR TYPE 1
      IF (MOD(L,100) .EQ.1) 13,14
   13 LL=0
      DO 10 M=1,N
      DO 10 J=1,100
   10 INGA(M,J) = 1H
   14 LL=LL+1
      RANDA = RANF(-1)
      DO 11 J=1,N
   11 IF (RANDA .LE. X(I,J)) GO TO 12
   12 TRANS(I,J) = TRANS(I,J) +1.0
      KOUNT(J) = KOUNT(J) +1
      I=J
      GO TO (15,17) LTYPE
   17 PILLAR(L)=J
      GO TO 50
   15 INGA(J,LL) = LET(J)
C                                          PRINT OUT EVERY 100 DRAWS ON TYPE 1
   25 IF (LL.LT.100) GO TO 50
      PRINT 113 $  PRINT 115
      DO 40 M=1,N
      MREV=N+1-M
   40 PRINT 106,(INGA(MREV,K),K=1,100)
   49 PRINT 115
   50 CONTINUE
C                                             PRINT OUT SINGLE LITHO-TYPE PILLAR
      IF (LTYPE .EQ.1) GO TO 30
      DO 28 L=1,NDRAW
      LREV=NDRAW +1 -L
      J=PILLAR(LREV)
   28 PRINT 118,LREV,LET(J),LSYM(J)
C
C                                  PRINT OUT END MATRIX, KOUNTS
   30 PRINT 107
      DO 18 I=1,N
   18 PRINT 108,(TRANS(I,J),J=1,N)
      PRINT 112
      PRINT 114,(LET(J),J=1,N)
      DO 21 I=1,N
      DO 20 J=1,N
   20 TRANS(I,J) = TRANS(I,J)/KOUNT(I)
   21 PRINT 103,LET(I),(TRANS(I,J),J=1,N)
C
      PRINT 109
      DRAW=NDRAW
      DO 19 I=1,N
      PC(I) = 100. * KOUNT(I) / DRAW
   19 PRINT 110,LET(I),KOUNT(I),PC(I)
      GO TO 2
C
  102 FORMAT (//10X,35HTRANSITION PROBABILITY MATRIX INPUT/10X,35(1H-))
  103 FORMAT (4XA4,20F6.3)
  104 FORMAT (//10X28HCUMULATED PROBABILITY MATRIX/10X,28(1H-))
  105 FORMAT (//1H(,29X,*START= *A1)
  106 FORMAT (10X,100A1)
  107 FORMAT (//10X,36HTRANSITION MATRIX COUNTS FROM OUTPUT/10X,37(1H-)/
     .  1H))
  108 FORMAT (8X,20(F5.0,X))
  109 FORMAT (//10X6HCOUNTS,4X7HPERCENT/10X6H------,4X7H-------)
```

TABLE A-6,CONT.

```
110 FORMAT (3XA1,I10,F12.2)
111 FORMAT (//30X*CLOCK=*F10.3)
112 FORMAT (/10X*PROBABILITY MATRIX FROM OUTPUT* / 10X30(1H-))
113 FORMAT (1H-)
114 FORMAT (8X,20(3XA3))
115 FORMAT (10X,100(1H-))
116 FORMAT (//10X*TRANSITIONS=*,I6)
117 FORMAT (X)
118 FORMAT (50XI5,3XA1,3XA6)
119 FORMAT (2A8,A6)
120 FORMAT (1H0,39X*SYMBOLS*//(20XA1,2X,2A8,XA6))
    END
```

Program STOCHEX

This program accepts as input a transition probability matrix $\underline{P}$ of dimensions up to 20x20. The matrix is read in by rows, and the program raises the matrix by successive powers up to $\underline{P}^{50}$. At this point it checks the columns to see whether the equilibrium state has been reached. If so, the program stops. If not, it uses the 10th power of the matrix (which has been stored without rounding), and raises this matrix to successive tenth powers of the original matrix, (i.e., $\underline{P}^{10} \times P^{10} = \underline{P}^{20}$) and so on, until $\underline{P}^{500}$ is obtained. The test is applied again, and if stabilization has not occurred, it uses the stored $\underline{P}^{100}$ and raises this to the 50th power, yielding $\underline{P}^{5000}$. At this point the program stops whether stability has been reached or not.

The program input for STOCHEX includes 4 title cards, the control card described below, and the transition probability matrix. The control card is fairly simple and has the following fields:

CONTROL CARD.

Cols. 1-3	Size of matrix, I3. Thus, 003 represents a 3 x 3 matrix.
Cols. 4-9	Test criterion, F6.4. This is usually entered as 1.0000, and if the test is satisfied by the time $\underline{P}^{50}$ is reached, the program stops. If one wishes to force the program through all the loops, a criterion value of 0.9999 can be used.
Cols. 10-57	Format statement, 6A8. This is variable, but follows the general form for Marchain. Note that this control card does not carry the project number.
Cols. 58-80	Blank.

Table A-7 shows two pages of output for the transition probability matrix of Table 1 in the text. The first page of Table A-7 has the titles and the input matrix, with the first four powers of $\underline{P}$. The second page shows the last several powers in the first loop, with stability having been reached at the 43rd power. The end of the output, shown at the bottom of Table A-7 shows that the criterion has been satisfied. The HI/LO column ratios are obtained by taking the largest and smallest values in each column of the matrix, and when these satisfy the criterion to 5 decimal places, the program terminates. Had one of the HI/LO values been, say, 1.00010, the next loop would have been entered. Program STOCHEX is listed in Table A-8.

Successive powers of the transition probability matrix, as they are printed out by STOCHEX, have several interesting features, as Griffiths (1966) points out. We may illustrate one of these by preparing a simple tree diagram showing how the probability of being in a particular state varies with successive events, depending upon the starting state. The diagram in Figure 3 starts in state A, and the first set of branches to the right show the probabilities of remaining in state A as against moving to state B or to state C. These values are taken from the top row of the original matrix in Table A-7.

Each of the three states has stationary probabilities associated with itself and with the other two states, so that the second set of branches on the right has probabilities taken from the corresponding row of the original matrix. If the probabilities along each branch are multiplied, however, the values listed at the branch ends are obtained. These sum to 1.0000, and the probability of being in A, B, or C (given that the start was in state A) add to exactly the values in the top row of the power 2 matrix in Table A-7.

If the starting state had been B, the probabilities of the three lithologies after two steps would be the values in the second row of the power 2 matrix. Griffiths (1966) develops this theme more fully, and shows that a study of the successive branches is illuminating for evaluating the probability that the system is in some particular state after n events in its approach to the fixed probability vector, which is the "equilibrium state" of the system.

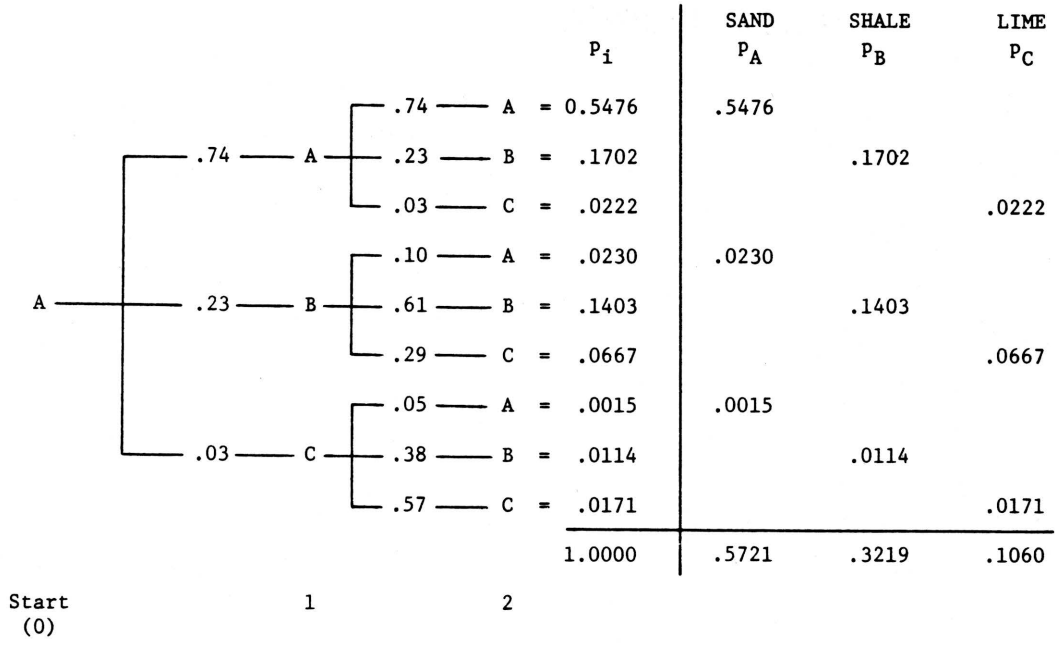


Figure 3.-Diagram showing probabilities of being in particular states after two steps, starting with state A (Sandstone).

TABLE A-7

KRUMBEIN PROJECT 01 0275

CHESTER DATA, 5-FOOT INTERVALS

3 STATES, SAND,SHALE,LIME

P(I,J) MATRIX BASED ON 309 TRANSITIONS

MAXIMUM POWER 50

STARTING MATRIX (ORIGINAL MATRIX)

	A 1	A 2	A 3
A 1	0.7400	0.2300	0.0300
A 2	0.1000	0.6100	0.2900
A 3	0.0500	0.3800	0.5700

POWER 2

	A 1	A 2	A 3
A 1	0.5721	0.3219	0.1060
A 2	0.1495	0.5053	0.3452
A 3	0.1035	0.4599	0.4366

POWER 3

	A 1	A 2	A 3
A 1	0.4608	0.3682	0.1709
A 2	0.1784	0.4738	0.3478
A 3	0.1444	0.4703	0.3853

POWER 4

	A 1	A 2	A 3
A 1	0.3864	0.3950	0.2180
A 2	0.1968	0.4622	0.3410
A 3	0.1732	0.4665	0.3603

TABLE A-7, CONT.

```
                POWER   48

          A 1         A 2         A 3
A 1   0.2335      0.4480      0.3184
A 2   0.2335      0.4480      0.3184
A 3   0.2335      0.4480      0.3184

                POWER   49

          A 1         A 2         A 3
A 1   0.2335      0.4480      0.3184
A 2   0.2335      0.4480      0.3184
A 3   0.2335      0.4480      0.3184

                POWER   50

          A 1         A 2         A 3
A 1   0.2335      0.4480      0.3184
A 2   0.2335      0.4480      0.3184
A 3   0.2335      0.4480      0.3184

KRUMBEIN PROJECT 01 0275

CHESTER DATA, 8-FOOT INTERVALS

3 STATES,  SAND,SHALE,LIME

P(I,J) MATRIX BASED ON 309 TRANSITIONS

                    STOP CRITERION=    1.0000

                    HI/LO COLUMN RATIOS
                    (FORCED TO 5 DECIMAL PLACES FOR TEST)

                     1   1.00000
                     2   1.00000
                     3   1.00000

TIME IN MINUTES 000.49 ASSEMBLY, 000.08 EXECUTE
```

TABLE A-8

```
      PROGRAM STOCHEX
C
C   ADAPTED FROM W.R.JAMES PROGRAM STOCHFIX
C   FOR W.C.KRUMBEIN, DEC.1966.......BETTY BENSON
C
C      *** INPUT 4 TITL, 1 MASTER CARD, NO NINES CARD
C         STOCHFIX EXTENDED, TO GO THRU 3 LOOPS, STARTING WITH ORIGINAL
C          MATRIX, 10TH-POWER MATRIX, AND 100TH-POWER MATRIX
C
      DIMENSION TITL(36),FMT(6),P(20,20),R(20,20),S(20,20),SAVE(20,20)
      DIMENSION          MPOW(3),AMARG(20),VOYR(20),LABEL(3)
C
      DATA (MPOW=50,500,5000) , (IA=1HA)
      DATA (LABEL=8HORIGINAL,8HPOWER 10,8HPOW. 100)
C
                                                READ CONTROLS, ORIG MATRIX
    1 READ 1000, TITL
      IF(EOF,60)2,3
```

TABLE A-8, CONT.

```
    2 STOP
    3 READ 1001,N,CRIT,FMT
      DO 5 I=1,N
    5 READ FMT,(P(I,J),J=1,N)
      LOOP=1
C
C                                                    INITIALIZE THIS LOOP
   14 PRINT 1000,TITL
      PRINT 1002,MPOW(LOOP)
      PRINT 1003,LABEL(LOOP),(IA,J,J=1,N)
      DO 6 I=1,N
    6 PRINT 1004,IA,I,(P(I,J),J=1,N)
      DO 15 I=1,N  $  DO 15 J=1,N
   15 S(I,J)=P(I,J)
      LARK=LINC=10**(LOOP-1)
C                                          RAISE TO SPECIFIED POWER
      DO 30 L=2,50
      DO 29 K=1,N
      DO 28 I=1,N
      A=0.0
      DO 27 J=1,N
   27 A=A+P(I,J)*S(J,K)
   28 R(I,K)=A
   29 CONTINUE
      LARK=LARK+LINC
      PRINT 1005,LARK,(IA,J,J=1,N)
      DO 20 I=1,N
   20 PRINT 1004,IA,I,(R(I,J),J=1,N)
      DO 16 I=1,N  $  DO 16 J=1,N
   16 S(I,J)=R(I,J)
      IF (L.NE.10) GO TO 30
      DO 17 I=1,N  $  DO 17 J=1,N
   17 SAVE(I,J)=S(I,J)
   30 CONTINUE
C                                        COMPUTE,PRINT HILO RATIOS
      DO 33 J=1,N
      AHI=ALO=S(1,J)
      DO 32 I=2,N
      IF (S(I,J).GT.AHI)  AHI=S(I,J)
      IF (S(I,J).LT.ALO)  ALO=S(I,J)
   32 CONTINUE
      VOYR(J)=AHI/ALO
      IVOY = (1.E5 * VOYR(J))  +.5
   33 VOYR(J) = IVOY / 1.E5
      PRINT 1000,TITL
      PRINT 1008,CRIT
      PRINT 1007,(I,VOYR(I),I=1,N)
C                  DETERMINE WHETHER NEXT LOOP SHOULD BE EXECUTED
      IF (LOOP.EQ.3) GO TO 1
      DO 34 I=1,N
      IF (VOYR(I) .GT. CRIT) GO TO 35
   34 CONTINUE
      GO TO 1
C                                          PREPARE TO DO NEXT LOOP
   35 LOOP=LOOP+1
      DO 18 I=1,N  $  DO 18 J=1,N
   18 P(I,J)=SAVE(I,J)
      GO TO 14
C
```

```
1000 FORMAT(9A8)
1001 FORMAT (I3,F6.4,6A8)
1002 FORMAT(////,20X,*MAXIMUM POWER*,5X,I4)
1003 FORMAT (///20X*STARTING MATRIX*,10X1H(,A8,* MATRIX)* //
    . (10X,10(A1,I2,7X)) )
1004 FORMAT(3X, A1,I2,2X,10(F6.4,4X) / (8X,10(F6.4,4X)))
1005 FORMAT (///15X*POWER*I5//(10X,10(A1,I2,7X)))
1007 FORMAT (///20X*HI/LO COLUMN RATIOS*/
    .  20X*(FORCED TO 5 DECIMAL PLACES FOR TEST)*//(20XI2,F9.5))
1008 FORMAT (///20X*STOP CRITERION=*F9.4//)
        END
```

Program NO MEM

This program represents a slightly modified version of MARCHAIN, adapted for independent-trials experiments. The only difference between the two programs is that whereas a transition probability matrix is used as input for MARCHAIN, the input for NO MEM is a single row vector of probabilities that add to 1.0. It is designed mainly for use with the fixed probability vector derived from STOCHEX by raising the transition probability matrix to the power at which it stabilizes. This fixed probability vector, as stated in the text, represents equilibrium proportions among the states of the system. Interesting comparisons can be made between the output of say 500 simulations from MARCHAIN with a transition probability matrix, and 500 independent-events drawings from the fixed probability vector. Usually thickness of the rock layers will be noticeably thinner in NO MEM, although relative total thicknesses will be about the same.

In addition to use of NO MEM for comparisons with MARCHAIN output, NO MEM can be used with a row vector of probabilities from a process that has no memory (i.e., that does not disclose a Markov property by the test in TESTMARK). For example, by feeding in a row vector with six data words, each 0.1667 (forced to add to 1.0000 by using the value 0.1666 for two words at random), simulation will represent successive rolls of a six-sided die, in which the probabilities of each state (A = 1, B = 2, etc.) are equal.

Program NO MEM operates internally by the simple expedient of generating a matrix with as many rows as there are columns in the fixed probability vector. It thus goes through the same operations as MARCHAIN, except that all rows are the same, and hence the "transitions" depend only on the fixed probabilities. Some readers may find it interesting to modify the internal structure of the program so that it draws successively from the single row vector.

Input for NO MEM starts with 4 title cards, followed by one control card, and by the fixed probability vector. The same two choices of output are available as in MARCHAIN, which means that if the lithologic section is desired, rock symbol cards representing each state must be inserted between the control card and the fixed probability vector input. The form of the control card for NO MEM is:

Cols. 1-6	Project number, A6. This is for record, and is not read in.
Cols. 7-10	Size of input vector, I4. This is simply the number of states in the fixed probability vector. Thus for a vector with six states the entry is 0006.
Cols. 11-14	Number of random drawings in simulation run, I4. Normally 500 or 1000 are called for, i.e., 0500 or 1000.
Cols. 15-18	Leave blank, unless a specific starting state is desired. In that case, the remarks under the control card for MARCHAIN apply.
Cols. 19-22	Type of output desired. Code 1 = "position output" and code 2 = "litho output". For the second choice, rock symbol cards are required for each state.
Cols. 23-70	Format statement, 6A8. The format for a fixed probability vector with six states punched to 3 decimal spaces is (10X, 6F3.3), where the 10X omits the first ten columns that usually have project number and other information.

Tables A-9 and A-10 give examples of NO MEM output, which has the same general form of MARCHAIN output. As with the other program, summary values are presented at the end of the run. The program itself is listed in Table A-11.

Interesting comparisons can be made between MARCHAIN output from a transition probability matrix, and NO MEM output from the corresponding fixed probability vector obtained with STOCHEX. Table 2 of the text summarizes one simulation experiment (500 transitions) from MARCHAIN, and Table A-10 gives part of the output from NO MEM. Total output of 500 draws was summarized in terms of the occurrences of each rock type, and their average thicknesses. The following summary compares these values. It is clear that the output from NO MEM contains many more separate occurrences of each rock type, so that the average thicknesses are significantly smaller. The average percentage thickness of each lithology in the total simulations are roughly the same, however, as is shown by the summary at the bottom of Table A-10 and the fourth column of Table 2 in the text.

MARCHAIN EXPERIMENT

Rock Type	Number of Occurrences	Average Bed Thickness
Sandstone (A)	28	33.7 ft.
Shale (B)	75	23.1
Limestone (C)	67	19.8

NO MEM EXPERIMENT

Rock Type	Number of Occurrences	Average Bed Thickness
Sandstone (A)	91	11.3 ft.
Shale (B)	116	14.1
Limestone (C)	102	13.1

These results are typical of comparative experiments with transition probability matrices structured as first-order Markov chains, as against the use of the fixed probability vector that arises from the transition matrix. It is sometimes useful to compare such outputs with the observational data, especially when the test for the Markov property is indecisive or marginal.

TABLE A-9

KRUMBEIN PROJECT 01 0275

CHESTER DATA, 8-FOOT INTERVALS

3 STATES, SAND,SHALE,LIME

P(I,J) MATRIX BASED ON 309 TRANSITIONS

FIXED PROBABILITY VECTOR INPUT

A	B	C
0.230	0.450	0.320

CUMULATED PROBABILITY VECTOR

0.230	0.680	1.000

CLOCK= 550.697

DRAWS= 500

```
CCCC          C   CC     C C      CC     C            C C  CCC C         C   C
    BBBB B    B             BB B    BB B BBBB BBBBB B BB    B B BBBB   BBB
         A AA   AA   AAAA A    A A     A        A                    A     A
```

FIXED PROBABILITY COUNTS FROM OUTPUT

115 220 165

PROBABILITY VECTOR FROM OUTPUT

A	B	C
0.230	0.440	0.330

	COUNTS	PERCENT
A	115	23.00
B	220	44.00
C	165	33.00

TABLE A-10

KRUMBEIN PROJECT 01 0275

CHESTER DATA, 8-FOOT INTERVALS

3 STATES, SAND,SHALE,LIME

P(I,J) MATRIX BASED ON 309 TRANSITIONS

		SYMBOLS
A	SANDSTONE	******
B	SHALE	------
C	LIMESTONE	LLLLLL

FIXED PROBABILITY VECTOR INPUT

A	B	C
0.230	0.450	0.320

CUMULATED PROBABILITY VECTOR

0.230	0.680	1.000

CLOCK= 550.715

DRAWS= 500

500	B	------
499	A	******
498	B	------
497	C	LLLLLL
496	C	LLLLLL
495	C	LLLLLL
494	C	LLLLLL
493	A	******
492	B	------
491	A	******
490	B	------
489	C	LLLLLL
488	B	------
487	A	******
486	C	LLLLLL
485	C	LLLLLL

TABLE A-10, CONT.

```
30   B   ------
29   C   LLLLLL
28   C   LLLLLL
27   C   LLLLLL
26   B   ------
25   C   LLLLLL
24   B   ------
23   A   ******
22   A   ******
21   A   ******
20   A   ******
19   C   LLLLLL
18   A   ******
17   A   ******
16   C   LLLLLL
15   A   ******
14   B   ------
13   A   ******
12   B   ------
11   B   ------
10   C   LLLLLL
 9   B   ------
 8   B   ------
 7   B   ------
 6   A   ******
 5   A   ******
 4   A   ******
 3   B   ------
 2   B   ------
 1   B   ------
```

```
FIXED PROBABILITY COUNTS FROM OUTPUT
------------------------------------
128   205   167

PROBABILITY VECTOR FROM OUTPUT
------------------------------
   A      B      C
0.256  0.410  0.334


          COUNTS     PERCENT
          ------     -------
A          128        25.60
B          205        41.00
C          167        33.40

TIME IN MINUTES 000.61 ASSEMBLY, 000.12 EXECUTE
```

TABLE A-II

```
      PROGRAM NO MEM
C                        FEB 1967
C                  FOR W.C. KRUMBEIN.......... BETTY BENSON
      DIMENSION X(20,20),TRANS(20,20),TITL(36),INGA(20,100),PILLAR(2000)
      DIMENSION LSYM(20), KRISTIE(100),LNAME(20,2)
      DIMENSION LET(20),KOUNT(20),PC(20),FMT(6)
C
      INTEGER PILLAR
      DATA (LET=1HA,1HB,1HC,1HD,1HE,1HF,1HG,1HH,1HI,1HJ,1HK,1HL,1HM,1HN,
     .  1HO,1HP,1HQ,1HR,1HS,1HT)
C           LTYPE=1 FOR 'POSITION' OUTPUT
C           LTYPE=2 FOR LITHO
C
C                                          READ CONTROLS
    2 READ 100,TITL
  100 FORMAT (9A8)
      IF (EOF,60) 3,4
    3 STOP
    4 CONTINUE
      PRINT 100,TITL
      READ 101,N,NDRAW,ISTART,LTYPE,FMT
  101 FORMAT (6X,4I4,6A8)
      IF (LTYPE.EQ.1) GO TO 1
      READ 119,((LNAME(I,J),J=1,2),LSYM(I),I=1,N)
      PRINT 120,(LET(I),(LNAME(I,J),J=1,2),LSYM(I),I=1,N)
    1 CONTINUE
C                                                READ,PRINT FIXED PROB.VECTOR
      READ FMT,(X(1,J),J=1,N)
      DO 5 I=2,N  $  DO 5 J=1,N
    5 X(I,J) = X(1,J)
      PRINT 102
      PRINT 114,(LET(J),J=1,N)
      PRINT 103,(X(1,J),J=1,N)
C
C                     INITIALIZE...CONVERT TO CUMULATIVE VECTOR, PRINT
      DO 8 I=1,N
      KOUNT(I)=0
      DO 8 J=1,N
    8 TRANS(I,J)=0
      DO 7 I=1,N
      DO 7 J=2,N
    7 X(I,J) = X(I,J) + X(I,J-1)
      PRINT 104
      PRINT 103,(X(1,J),J=1,N)
C
      T = TIMEF(0)   $  CALL RANFSET(T)  $  PRINT 111,T
      PRINT 116,NDRAW
      I=1
C                                          MAKE SPECIFIED NO. OF DRAWS
      DO 50 L=1,NDRAW
      IF (LTYPE.EQ.2) GO TO 14
C                                  INITIALIZE EVERY 100 DRAWS FOR TYPE 1
      IF (MOD(L,100) .EQ.1) 13,14
   13 LL=0
      DO 10 M=1,N
      DO 10 J=1,100
```

```
   10 INGA(M,J) = 1H
   14 LL=LL+1
      RANDA = RANF(-1)
      DO 11 J=1,N
   11 IF (RANDA .LE. X(I,J)) GO TO 12
   12 TRANS(I,J) = TRANS(I,J) +1.0
      KOUNT(J) = KOUNT(J) +1
      I=J
      GO TO (15,17) LTYPE
   17 PILLAR(L)=J
      GO TO 50
   15 INGA(J,LL) = LET(J)
C                                          PRINT OUT EVERY 100 DRAWS ON TYPE 1
   25 IF (LL.LT.100) GO TO 50
      PRINT 113 $  PRINT 115
      DO 40 M=1,N
      MREV=N+1-M
   40 PRINT 106,(INGA(MREV,K),K=1,100)
   49 PRINT 115
   50 CONTINUE
C                                             PRINT OUT SINGLE LITHO-TYPE PILLAR
      IF (LTYPE .EQ.1) GO TO 30
      DO 28 L=1,NDRAW
      LREV=NDRAW +1 -L
      J=PILLAR(LREV)
   28 PRINT 118,LREV,LET(J),LSYM(J)
C
C                                       PRINT OUT END VECTOR, KOUNTS
   30 PRINT 107
      PRINT 108,(KOUNT(I),I=1,N)
      DRAW=NDRAW
      DO 18 I=1,N
   18 PC(I) = KOUNT(I) / DRAW
      PRINT 112
      PRINT 114,(LET(J),J=1,N)
      PRINT 103,(PC(I),I=1,N)
C
      PRINT 109
      DO 19 I=1,N
   19 PRINT 110,LET(I),KOUNT(I),PC(I)
      GO TO 2
C
  102 FORMAT (//10X30HFIXED PROBABILITY VECTOR INPUT/10X,30(1H-))
  103 FORMAT (8X,20F6.3)
  104 FORMAT (//10X28HCUMULATED PROBABILITY VECTOR /10X,28(1H-))
  106 FORMAT (10X,100A1)
  107 FORMAT (//10X,36HFIXED PROBABILITY COUNTS FROM OUTPUT/10X,3((1H-)/
     .  1H))
  108 FORMAT (7X,20I6)
  109 FORMAT (//10X6HCOUNTS,4X7HPERCENT/10X6H------,4X7H-------)
  110 FORMAT (3XA1,I10,2PF12.2)
  111 FORMAT (//30X*CLOCK=*F10.3)
  112 FORMAT (/10X*PROBABILITY VECTOR FROM OUTPUT* / 10X30(1H-))
  113 FORMAT (1H-)
  114 FORMAT (8X,20(3XA3))
  115 FORMAT (10X,100(1H-))
  116 FORMAT (//10X*DRAWS=*I6)
  117 FORMAT (X)
  118 FORMAT (50X15,3XA1,3XA6)
  119 FORMAT (2A8,A6)
  120 FORMAT (1H0,39X*SYMBOLS*//(20XA1,2X,2A8,XA6))
      END
```

KANSAS GEOLOGICAL SURVEY COMPUTER PROGRAM
THE UNIVERSITY OF KANSAS, LAWRENCE

PROGRAM ABSTRACT

Title (If subroutine state in title):

Four short programs, TESTMARK, MARCHAIN, STOCHEX, and NOMEM (no subroutines)

Computer: CDC 3400 Date: Final versions February, 1967

Programming language: FORTRAN IV (3400)

Author, organization: W.C. Krumbein, Department of Geology,
Northwestern University, Evanston, Illinois.

Direct inquiries to: Author, or

Name: D.F. Merriam Address: Kansas Geological Survey,
University of Kansas, Lawrence

Purpose/description: These programs are used (1) to test for the Markov property, (2) to simulate stratigraphic or other sequences from a transition probability matrix, (3) to estimate equilibrium state of system, and (4) to simulate independent-events processes.

Mathematical method: Mainly randomized drawings from a Markov chain transition probability matrix, or operations on the matrix.

Restrictions, range: The programs accept matrices up to 20 x 20.

Storage requirements:

Equipment specifications: Memory 20K 40K 60K K

Automatic divide: Yes No Indirect addressing Yes No

Other special features required

Additional remarks (include at author's discretion: fixed/float, relocatability; optional: running time, approximate number of times run successfully, programming hours)

COMPUTER CONTRIBUTIONS

Kansas Geological Survey
University of Kansas
Lawrence, Kansas

Computer Contribution

1. Mathematical simulation of marine sedimentation with IBM 7090/7094 computers, by J.W. Harbaugh, 1966. . . . . $1.00
2. A generalized two-dimensional regression procedure, by J.R. Dempsey, 1966 . . . . . $0.50
3. FORTRAN IV and MAP program for computation and plotting of trend surfaces for degrees 1 through 6, by Mont O'Leary, R.H. Lippert, and O.T. Spitz, 1966 . . . . . $0.75
4. FORTRAN II program for multivariate discriminant analysis using an IBM 1620 computer, by J.C. Davis and R.J. Sampson, 1966 . . . . . $0.50
5. FORTRAN IV program using double Fourier series for surface fitting of irregularly spaced data, by W.R. James, 1966 . . . . . $0.75
6. FORTRAN IV program for estimation of cladistic relationships using the IBM 7040, by R.L. Bartcher, 1966 . . . . . $1.00
7. Computer applications in the earth sciences: Colloquium on classification procedures, edited by D.F. Merriam, 1966 . . . . . $1.00
8. Prediction of the performance of a solution gas drive reservoir by Muskat's Equation, by Apolonio Baca, 1967 . . . . . $1.00
9. FORTRAN IV program for mathematical simulation of marine sedimentation with IBM 7040 or 7094 computers, by J.W. Harbaugh and W.J. Wahlstedt, 1967 . . . . . $1.00
10. Three-dimensional response surface program in FORTRAN II for the IBM 1620 computer, by R.J. Sampson and J.C. Davis, 1967. . . . . $0.75
11. FORTRAN IV program for vector trend analyses of directional data, by W.T. Fox, 1967 . . $1.00
12. Computer applications in the earth sciences: Colloquium on trend analysis, edited by D.F. Merriam and N.C. Cocke, 1967 . . . . . $1.00
13. FORTRAN IV computer program for Markov chain experiments in geology, by W.C. Krumbein, 1967. . . . . $1.00

Reprints (available upon request)

Finding the ideal cyclothem, by W.C. Pearn (reprinted from Symposium on cyclic sedimentation, D.F. Merriam, editor, Kansas Geological Survey Bulletin 169, v. 2, 1964)

Fourier series characterization of cyclic sediments for stratigraphic correlation, by F.W. Preston and J.H. Henderson (reprinted from Symposium on cyclic sedimentation, D.F. Merriam, editor, Kansas Geological Survey Bulletin 169, v. 2, 1964)

Geology and the computer, by D.F. Merriam (reprinted from New Scientist, v. 26, no. 444, 1965)

Quantitative comparison of contour maps, by D.F. Merriam and P.H.A. Sneath (reprinted from Journal of Geophysical Research, v. 71, no. 4, 1966)

Geologic model studies using trend-surface analysis, by D.F. Merriam and R.H. Lippert (reprinted from Journal of Geology, v. 74, no. 5, 1966)

Geologic use of the computer, by D.F. Merriam (reprinted from Wyoming Geological Association, 20th Field Conf., 1966)

Computer aids exploration geologists, by D.F. Merriam (reprinted from the Oil and Gas Journal, v. 65, no. 4, 1967)

Comparison of cyclic rock sequences using cross-association, by D.F. Merriam and P.H.A. Sneath (reprinted from Essays in Paleontology and Stratigraphy: R.C. Moore commemorative volume, edited by C. Teichert and E. Yochelson, Dept. Geology, Univ. Kansas, 1967)