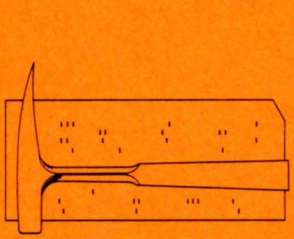
DANIEL F. MERRIAM, Editor

FORTRAN IV PROGRAM FOR VECTOR TREND ANALYSES OF DIRECTIONAL DATA

By

WILLIAM T. FOX
Williams College



COMPUTER CONTRIBUTION 11
State Geological Survey
The University of Kansas, Lawrence
1967

The Kansas Survey is the only geological organization now known to be distributing computer program decks as well as data decks. The programs are written in ALGOL, FORTRAN II, and FORTRAN IV, and sold for a limited time at a nominal cost. The programs are for Burroughs B5500, CDC 3400, GE 625, IBM 1620, 7040, and 7090/1401 or 7094/1401 computer systems. A list of available decks is given below.

	ALGOL	FORTRAN II	FORTRAN IV
*Marine Simulation (CC 1)			
2D Regression (CC 2)	$10.00	$10.00	$10.00
Trend-6 (CC 3)			$25.00
Discrim (CC 4)		$ 5.00	
Nongridded Double Fourier (CC 5)			$10.00
Cladistics (CC 6)			$25.00
Muskat (CC 8)	$10.00		
Marsim (CC 9)			$20.00
Response surface (CC 10)		$10.00	
Vector trend (CC 11)			$15.00
*Trend-3 (SDP 3)			
Match-Coeff (SDP 4)		$ 2.00	
*Correlation and distance coeff (SDP 9)			
Time-trend (SDP 12)		$ 5.00	$ 5.00
Covap (SDP 13)		$15.00	
Trend-3 (SDP 14)		$25.00	
Cross-Association (SDP 23)			$10.00
*Single and * double Fourier (SDP 24)			
Precambrian wells (SDP 25)			
List of about 2,650 Precambrian wells $50.00			
Trend-4 (SDP 26)		$ 7.50	
Sediment analysis (SDP 28)		$10.00	
4D Trend (B 171)			$10.00
Conversion of T&R to Cartesian coordinates (B 170-3)		$ 5.00	$ 5.00
Hydrodynamic oil-trap mapping (reprint, Colo. Sch. Mines)			$10.00

*Out of print, therefore not available.

Comments and suggestions concerning the Computer Contribution Series are welcome and should be addressed to the editor. An up-to-date list of publications is available on request.

FORTRAN IV PROGRAM FOR VECTOR TREND ANALYSES OF DIRECTIONAL DATA

By

WILLIAM T. FOX

ABSTRACT

A FORTRAN IV computer program for the IBM 7090/7094 uses orthogonal polynomial trend-surface mapping techniques in the analysis of regional trends in directional data. Direction cosines are used for each cell in the grid to compute the vector mean azimuth, mean dip angle, length of resultant vector, and spherical radius of the circle of confidence around the resultant vector. Linear, quadratic, cubic, quartic, and quintic orthogonal polynomial response surfaces are computed and plotted as isoazimuth and vector trend maps to aid in interpreting regional flow patterns. Residuals from the trend-surface maps emphasize local topographic anomalies on the depositional surface.

The first example output consists of 20 azimuth and dip measurements of cross-bedding from the Wasatch Formation of Wyoming. A second example is based on a study of the orientation of drumlins south of Lake Ontario in New York State. The third example uses cross-bedding measurements from the Pocono Formation in Pennsylvania. In the last two examples, linear, quadratic, and cubic isoazimuth and vector trend maps are used to interpret the major current pattern, and residual maps to outline topographic highs or lows.

INTRODUCTION

As a wave breaks over an offshore bar or a glacier molds boulder clay into a drumlin, current direction is recorded. By plotting a large number of recorded directions as vectors with a constant magnitude, where the magnitude has no geologic significance, it is possible to study the distribution of currents which existed when for example a sediment was deposited. Because major currents move down a regional slope under the influence of gravity or are carried parallel to a shoreline by longshore drift, a study of the trends of ancient currents provides a means of paleogeographic interpretation. In this computer program, techniques developed for orthogonal polynomial trend-surface mapping of gridded data are applied to directional data, and the method is called vector trend analysis.

Vector trend analysis makes use of techniques explained by Grant (1957) for fitting a trend surface using orthogonal polynomials. The program is limited to gridded data but can be expanded to accept nongridded data through the use of nonorthogonals as described by Krumbein (1959). To handle vectorial data, direction cosines are used for the computation of the resultant vector within each cell. Steinmetz (1962) discusses the use of direction cosines for the analysis of dip directions and dip angles in cross-bedding. Direction cosines are used for three-dimensional vector summation to determine both direction of the dip and dip angle. Magnitude of the resultant vector, R, is computed for each grid location along with the spherical radius of the circle of confidence, theta, around the resultant vector. The spherical radius of the circle of confidence gives a measure of dispersion of observed vectors around the resultant vector.

In vector trend analysis, the linear, quadratic, or cubic trend surfaces can be used to represent trends of regional flow pattern, and residuals to show local anomalies or deflections from the regional trend. Vector trend maps show the overall flow pattern that is parallel to the paleoslope, and residuals are used to interpret local topographic highs or lows which would deflect the major flow pattern.

The total sum of squares of deviation from the mean azimuth is computed for the resultant vector map. For each trend surface, the sum of squares of deviations from the trend is computed. Reduction in the percentage of the total sum of squares indicates the amount of total variability accounted for by the trend.

Acknowledgments.-The author thanks several people for assistance in different phases of the development of this program. Dr. L. Nobles and Dr. W. C. Krumbein at Northwestern University introduced the author to the use of trend-surface analysis for studying directional data. Robert Elwell of Williams College collected and analyzed data for the drumlin example using a desk calculator. The Massachusetts Institute of Technology and Stanford University kindly allowed use of their computer facilities. Dr. J. W. Harbaugh of Stanford University read the manuscript and made several useful suggestions.

---

[1] Temporary address, Department of Geology, Stanford University

PROGRAM DESCRIPTION

The program listed in Table 1 is written in FORTRAN IV for use on an IBM 7090/7094 computer system. The program is subdivided into a MAIN program and four subroutines, DIRCOS, TREND, PRTMAP, and MAPVEC, which are used to calculate the polynomial coefficients and plot the vector trend maps.

The MAIN program is used to read in the data and control cards that handle options available with the program. The arrangement of data cards and different options are discussed in the section on data input preparation.

In preparing data for the vector trend program, a rectangular grid is placed over a map and azimuth and dip measurements are recorded for each grid cell. The program is dimensioned to handle a grid with a maximum of eight rows and columns, and fifty azimuth readings per cell. Each observation includes either an azimuth and dip reading or an azimuth. If the number of azimuth and dip readings differs for each grid cell, a data card with the number of observations preceeds the azimuth and dip measurements. If the number of observations is the same for each location, the constant, NAZM for the number of azimuths is read in on the control card. The number of rows, NROW, and number of columns, NCOL, are limited to eight by the plotting subroutine, MAPVEC. If more than eight rows or columns are included, the program will compute the azimuth of the resultant vector and dip angle for each grid cell, but will not compute the corresponding trend-surface maps.

The north azimuth, ANORTH, is used to orient the grid on the original map. The north arrow's azimuth on the original map with respect to the vertical direction of the grid is recorded as the azimuth ANORTH. For example, if the top of the grid is parallel to top of the map so that rows extend east and west and columns north-south, the value for ANORTH is zero. On the other hand, if the top of the grid lies along the western margin of the map so that rows extend north and south and columns in an east-west direction, orientation of north arrow on the map with respect to top of the grid is 270 degrees. In the last example used in this paper, ANORTH is 270 degrees.

Subroutine DIRCOS is called by the MAIN program to compute the vector mean azimuth and dip angle for each grid cell. Direction cosines are used for calculating the azimuth and dip angle according to the equations outlined by Steinmetz (1962, p. 806-809). The method is based on a series of equations developed by Fisher (1953) for measuring dispersion of points on a sphere. Azimuth directions are measured from north in a clockwise direction and range from 0 to 360 degrees. Dip angles, which are measured in the direction of the azimuth, range from 0 to 90 degrees. In using cross-bedding measurements, readings must be corrected for regional dip before being processed by the program. As a measure of dispersion, magnitude of the resultant vector, R, is computed for each grid cell. If all angles in the cell are equal, R equals N, the number of observations in the cell. If the azimuths in the cell are not equal, the corresponding value of R is less than N, and if the azimuths within a cell diverge widely, the value of R is small. Significance points of R are available in tables to test data for random orientation (Watson, 1956). Theta, the spherical radius of the circle of confidence around the resultant vector, R, is computed for 0.05 and 0.01 levels of significance. The spherical radius of the circle of confidence resembles a confidence interval on a two-dimensional plot or a trend surface (Krumbein and Graybill, 1965, p. 137). As an example of meaning of the radius of the circle of confidence, at a significance level of 0.05 the probability that the true azimuth direction and dip angle deviates more than theta degrees from calculated resultant dip direction and angle is 5 percent. If the constant NOPRT is read in as a -1 on the control card, values for the direction cosines A, B, and C; magnitude of the resultant vector, R; vector mean azimuth direction and dip angle; and theta, the radius of the circle of confidence around the resultant vector, are printed for each grid cell.

Subroutine TREND is called by the MAIN program to compute the polynomial coefficients that are used to plot the linear, quadratic, cubic, quartic, and quintic vector trend maps. Methods described by DeLury (1950) and Krumbein and Graybill (1965, p. 308) using matrix algebra to compute orthogonal polynomials trend-surface maps are incorporated into this program to determine the polynomials coefficients and Z-squared array. Orthogonal polynomials for n = 3 to n = 8 based on DeLury (1950) are included as a data statement in subroutine TREND. Polynomial coefficients are used to compute trend values for each grid cell. Residual values are calculated for each trend by subtracting the trend value in each cell from the corresponding value for the resultant vector.

Subroutine PRTMAP is called by the MAIN program to print a series of maps representing the resultant vectors based on observed data in each cell, the different combinations of linear, quadratic, cubic, quartic, and quintic trends, and residual maps. The title, which is printed at the top of each map, is placed at the end of the data deck and read in by subroutine PRTMAP. The first map in the series gives values for resultant vectors in each grid cell. The map is followed by a plot of the azimuths of the resultant vectors, which are printed by subroutine MAPVEC. The next map in the sequence gives residual values from the mean azimuth with the total sum of squares printed at the bottom of the page. Three maps are printed for each trend, with cumulative maps for linear through quintic. For each trend, the first map gives trend-surface values for each grid cell. These maps are manually contoured to give the isoazimuth trend maps. The second map is a plot of the azimuths, which is followed by a map of residuals for the trend. At the bottom of the residual map, the sum of squares for the map is printed with percentage reduction in the total sum of squares. The sum of squares gives the amount of variability not accounted for by the trend.

Subroutine MAPVEC is called by subroutine PRTMAP to plot directional arrows or vectors. In subroutine MAPVEC, the origin of the vector is plotted as an X and the shaft of the vector is plotted as a series of dots. If the azimuth is located in the upper half of the circle meaning that it is less than or equal to 90 degrees or greater than 270 degrees, the arrowhead is plotted as an A. If the azimuth is greater than 90 degrees and less than or equal to 270 degrees, the arrowhead of the vector is plotted as V. Because letters are printed closer together in the rows than in columns, there is a better resolution for vectors which trend in a general north-south direction than those which trend east-west. The north arrow is plotted on the right side of the map with X at the origin and the letter N at the north end of the arrow.

INPUT DATA PREPARATION

Data cards for the vector trend program include a program control card, variable format card, set of cards with azimuth and dip readings, and title format card. A sample listing of data cards for example 1 is given in Table 2.

The first data card, which is the program control card, is as follows:

Col.		
1-4	NAZM=0	The number of azimuth and dip readings is different for each location.
	NAZM=N	The number of azimuth readings is the same for each location and is equal to N.
8	IFDIP=0	No dip measurements are included with the data.
	IFDIP=1	A dip measurement is included with each azimuth reading.
12	NROW=N	N is equal to the number of rows in the grid. Maximum number of rows = 8.
16	NCOL=N	N is equal to the number of columns in the grid. Maximum number of columns = 8.
20	NOPRT=-1	Print out the direction cosines, length of resultant vector, mean azimuth and dip, and circle of confidence for each grid cell.
	NOPRT=1	Do not print the above information for each cell.
21-28	ANORTH	The azimuth reading of north with respect to the right margin of the grid is measured clockwise from vertical on grid.
30	IFTRD	A single digit is used to indicate the highest order trend map to be computed; 1 for linear, 2 for quadratic, 3 for cubic, 4 for quartic, and 5 for quintic.

The second card in the data deck is for the variable format used in reading the azimuth and dip measurements. The variable format must be enclosed in parentheses and arranged on data cards according to the following description.

The arrangement of data cards that follow the variable format card is set by the program control card and variable format card. If the number of azimuth readings is constant (NAZM=N) and no dip measurements are included (IFDIP=0), azimuth readings for each location are arranged across a single card according to the variable format. If the number of azimuth readings is constant (NAZM=N) and dip measurements are included with each azimuth (IFDIP=1), one azimuth and one dip reading are included on each card. If the number of azimuths differs for each location (NAZM=0), the number of azimuths is punched on a separate card in columns 1-4 and placed before the azimuth and dip readings for each location. Also if the number of azimuths differs, the azimuth readings must be punched with one azimuth per card according to the variable format. If azimuth and dip readings are included, the azimuth reading must preceed the dip reading on the data card.

The title card using columns 1-72 is read in by subroutine PRTMAP and placed at the end of the data deck. The title is printed at the top of all maps and used to identify the output.

PROGRAM OUTPUT

Three examples of output from the vector trend program are used to illustrate how the data cards are set up and how different program options operate. In the first example, a series of azimuth and dip readings for cross-bedding are taken from a single outcrop. In the second example, orientation measurements, taken on the long axes of drumlins, are used to study the direction of flow of a continental glacier across an area in western New York State. The third example is based on a map of cross-bedding directions from the Pocono Formation in Pennsylvania.

Example 1 - Cross-bedding from Wasatch Formation

In the first example, data consist of 20 azimuth and dip measurements (Table 2) from a cross-bedded sandstone in the Cathedral Bluffs Member of the Wasatch Formation (Eocene) in southwestern Carbon County, Wyoming (Steinmetz, 1962, p. 808). Computer output (Table 3) includes a listing of the azimuth and dip readings. Values for direction cosines A, B, and C which are used to compute the azimuth of resultant vector and dip angle are given with the magnitude of the resultant vector (R). If all azimuth and dip readings are the same, length of the resultant vector equals the number of azimuths. In the Cathedral Bluffs example, magnitude of the resultant vector equals 12.99 and number of observations equals 20. The computed azimuth of the resultant vector, which is measured in a clockwise direction from north, equals 10.2 degrees and the mean dip angle is 15.4 degrees. The spherical radius of the circle of confidence around the resultant vector equals 24.8 degrees with a level of significance of 0.05. Radius of the circle of confidence of 24.8 indicates that probability of true direction of cross-bedding deviating more than 24.8 degrees from the calculated resultant vector is 5 percent. For the 0.01 level of significance, the radius of the circle of confidence equals 31.58 degrees. Because this example included only one location, vector trend maps were not printed and the program terminated after the above parameters were computed and printed.

In the second and third examples, all information included in the first example also could be printed for each grid cell. In a complete analysis, it is useful to compute the magnitude of the resultant vector and radius of the circle of confidence for each grid cell in order to note the variability within each cell.

Example 2 - Drumlins from New York State

The second example is based on a study of drumlin orientation south of Lake Ontario in western New York State. The area extends 32 miles in an east-west direction and 24 miles north-south including 768 square miles. A rectangular grid with 6 rows and 8 columns was placed over topographic maps of the area with the upper left corner of the grid touching the southern tip of Sodus Bay, New York. Each grid cell is 4 miles on a side enclosing an area of 16 square miles. Five drumlins were selected within each cell, using a random numbers table. The azimuth of the long axis of each drumlin was measured in a clockwise direction from north. A listing of the data cards is given in Table 4.

The polynomial coefficients array, Z-squared array, and array of the percentage of total corrected sum of squares for each coefficient are given in Table 5. The arithmetic mean azimuth, which is given by the A_{00} regression coefficient, is 161.237. In testing the program, the mean azimuth also was computed by vector summation using subroutine DIRCOS giving a value of 161.263. Because there is less than 0.03 degrees difference in the means computed using the different methods, the arithmetic mean, which is the A_{00} regression coefficient in the coefficients array, is used in the program. The B_{01}, C_{02}, D_{03}, E_{10}, F_{20}, and G_{30} regression coefficients also were computed using vector summation, and they agreed within 0.01 degrees with results obtained using matrix algebra. Therefore, vector summation with direction cosines is used for computing the mean azimuth within each cell, and matrix algebra is used for computing the coefficients array and Z-squared array.

Maps of the mean azimuth within each cell, which are based on the computed resultant vectors, are given in Figure 1. Figure 1A, which is a contour map of the azimuth of resultant vectors, is defined as an isoazimuth map. Contour lines that connect points of equal azimuth are defined as isoazimuth lines. Figure 1B is a plot of the azimuths contoured in Figure 1A. It is possible to note the significance of isoazimuth lines in Figure 1A by referring to the plot of azimuths in Figure 1B. The residual map in Figure 1C was derived by subtracting the mean azimuth (161.24) from the resultant vector in each cell of the grid. Because a constant value was subtracted from each grid cell, the map for the resultant vectors in Figure 1A corresponds closely to the contour map of residual values in Figure 1C. The total sum of squares for the residuals (4543.83) is printed at the bottom of the residual map. It can be seen from the isoazimuth contour map and the plot of the vectors that the general trend of glacier flow was deflected to the east in the northeastern part of the map. Minor deflections in the glacier flow are emphasized by areas enclosed by contour lines on the residual map.

Linear trend-surface and residual maps of glacial flow are given in Figure 2. Isoazimuth lines for the linear trend-surface are shown on Figure 2A with the mean (161) represented by a darker line. The azimuth for each cell is plotted as an arrow on the vector trend map (Fig. 2B). From the vector trend map, it can be seen that the glacier spread from north to a general southerly direction fanning out southeastward. Major deviations from the trend are presented on the residual map (Fig. 2C) with the dark line indicating zero deviation. Deviations in the area are small, but a cluster of large negative or counter-clockwise deviations is present in the northeastern corner. A line of positive or clockwise deviations extends in a southeasterly direction from the center of the map. An area of negative deviations also is found across the western side and along the southern margin of the map. The sum of squares for the residual map is 1362.03, which is 29.98 percent of the total sum of squares. The linear trend therefore accounts for 70.02 percent of the variability on the original map.

The quadratic trend-surface maps are given in Figure 3. In Figure 3A, the isoazimuth lines form a dome with high azimuth values in the center and lower azimuth readings around the margin. The long axis of the dome extends in a northwest-southeast direction with the azimuth values ranging from 164.0 to 154.4 degrees. The quadratic vector trend map (Fig. 3B) shows a general twisting of vectors in a clockwise direction toward the center of the map and counter-clockwise toward the edges. The quadratic vector trend map could be interpreted as a major clockwise deflection of the glacier flow in the center of the map area. Residuals for the quadratic trend map resemble residuals from the original with contour lines generally pulled in toward the center. The zero deviation line in the center divides the map into positive deviations in the west and negative ones in the east. For the quadratic residual map, the sum of squares is 4289.65 which is 94.41 percent of the total sum of squares. Therefore, the quadratic trend map accounts for only 5.59 percent of the total sum of squares.

Trend and residual maps for linear plus quadratic are given in Figure 4. On the isoazimuth map (Fig. 4A) contour lines bend around the high values in the northwestern corner of the map with the lowest values in the northeastern corner. The mean azimuth line (161) extends as a curve in a north-south direction through the middle of the map. On the vector trend map (Fig. 4B) the vectors spread out to the southeast. In contrast to the linear map, linear plus quadratic vectors are farther apart in the southern part of the map area. For the residual map from the linear plus quadratic trend surface, an area of high negative or counter-clockwise residuals truncates the northeastern corner of the map. Negative residuals also are present along the southern tier of cells and near the northwestern corner. The center of the map generally has low positive deviations. For the linear plus quadratic trend, the sum of squares equals 1107.85, which represents a 75.62 percent reduction in the total sum of squares.

The cubic trend and residual maps are presented in Figure 5. The isoazimuth lines of the cubic trend (Fig. 5A) show a pocket of low azimuth readings in the northeast with a corresponding high area in the southwest. The cubic vector trend map (Fig. 5B) shows the slight deflections which result from the cubic trend. Residuals from the cubic trend (Fig. 5C) show high negative residual values in the eastern part of the map and positive residuals to the west. The sum of squares for the cubic residuals is 4320.36, which is 95.08 percent of the total sum of squares. Therefore, the cubic vector trend accounts for 4.92 percent of the total sum of squares.

Combined maps for the linear, quadratic, and cubic components are given in Figure 6. The isoazimuth contour lines of the combined trend (Fig. 6A) show a general pattern that is similar to the isoazimuth lines for the resultant vectors in Figure 1A. The vector trend map (Fig. 6B) is dominated by the linear trend, and to the southeast is modified by the quadratic and cubic components. The combined linear, quadratic, and cubic map has a sum of squares of 884.38, which is 19.46 percent of the total sum of squares. Therefore, the combined map accounts for 80.54 percent of the variability, leaving only 19.46 percent for the quartic and higher trends.

Trend-surface and residual maps in Figure 1-6 are useful in interpreting the glacier flow pattern and the effect which the preglacial topographic relief had on the movement of the glacier. The major control of the glacier comes from the buildup of ice to the north of the drumlin field in Lake Ontario. As ice became thicker in the source area, it spread southward molding the ground moraine and lake-bed clay into drumlins. Where the glacier flow was unimpeded, it spread in a fan-shaped mass to the south and southeast. The glacier took the path of least resistance and flowed around and eventually over hills which stood in its path. Where the glacier encountered a stream valley, it tended to follow the valley course. Record of the path followed by the glacial ice is preserved in the spoon-shaped drumlins that were formed as the glacier bulldozed over the boulder clay.

The linear and quadratic trend-surface map (Fig. 4) is used to interpret the course the glacier would have taken over a smooth featureless plain. The linear trend alone accounts for 70 percent of the total sum of squares and exhibits a fan shape of a spreading glacier. By adding the quadratic trend to the linear, the twist inherent in the linear trend is partially removed. The linear plus quadratic trend accounts for 75.6 percent of the total sum of squares, leaving only 24.4 percent for the cubic and higher trends. Residuals from the linear plus quadratic (Fig. 4C) are used to interpret the preglacial surface. In the northeastern corner of the map, the glacier was deflected in a clockwise direction toward the zero line as indicated by the positive deviation. To the west of the zero line, the glacier was deflected in a counter-clockwise direction to the east. Therefore, the zero line lies along a preglacial valley into which the glacier flowed from both sides. In the middle of the map, the low deviation values represent gently rolling topography, which did not exert strong influence on the glacier. Near the northwestern corner, the high negative deviation is the result of a topographic high that deflected the glacier to the east. Along the southern margin, high deviation values may represent waning of the glacier or may be caused by the edge effect of the trend-surface map.

Example 3 - Cross-bedding in Pocono Formation

A series of cross-bedding measurements collected by Pelletier (1958, p. 1047) from the Pocono Formation in Pennsylvania are used for the third example of the vector trend program. A listing of the data cards is given in Table 6. Because the number of observations varies with each location, zero is read in for NAZM on the control card. No dip measurements are included, so zero is read in for IFDIP. The number of rows (NROW) equals 6 and the number of columns (NCOL) equals 3. NOPRT was set equal to 1 to suppress the intermediate output. The azimuth reading for north (ANORTH) is 270; therefore, the north arrow points to the left margin of the map. Subroutine MAPVEC plots the azimuth direction from the center to the margin of a 2-inch square. Because a 1-inch square has 10 letters in the horizontal direction and 6 letters in the vertical direction, there is a higher resolution along the bottom or top than the right or left margin. Most azimuths point in a western direction, so the grid was flipped on its side by setting the north azimuth at 270. The coefficients array, Z-squared array, and percentage of the total corrected sum of squares for each coefficient are printed in Table 7.

The isoazimuth map of resultant vectors for cross-bedding direction is given in Figure 7A with a plot of resultant vectors in Figure 7B. From the plot of vectors, it can be seen that the general trend of cross-bedding is to the west. Figure 7C represents the residuals with the mean azimuth (290.33) subtracted from each resultant vector. The residual map shows a trough of negative residuals in the middle of the map with positive residuals along the southern margin and in the northeastern (upper left) corner.

Maps for the linear trend are given in Figure 8. The linear trend-surface map (Fig. 8A) shows an increase in azimuth readings from the northeastern to southwestern corner of the map. This trend is shown in the vector trend map (Fig. 8B), where vectors converge on the northwestern corner of the map (lower left corner of the plot). Residuals for the linear trend (Fig. 8C) closely resemble residuals from the resultant vectors in Figure 7C. Although the linear trend appears strong, it only accounts for 20.88 percent of the total sum of squares. The high percentage of the total sum of squares not accounted for by the linear trend (79.12) is reflected in the large values of the linear residuals.

The quadratic trend-surface map (Fig. 9A) has the shape of an elongated basin with the highest azimuth values on the margin and in the northeastern and southwestern corners, and the lowest values in the center. For the quadratic vector trend map (Fig. 9B), the primary change occurs in a north-south direction with little change from east to west. Residuals from the quadratic map contoured (Fig. 9C) account for 22.5 percent of the total sum of squares.

The combined maps for the linear plus quadratic vector trend surfaces are presented in Figure 10A. The isoazimuth trend-surface map for linear plus quadratic (Fig. 10A) resembles a plunging syncline with highest azimuth values in the southwestern corner and lowest values at the center of the eastern margin. The vector trend map (Fig. 10B) shows the southern column of vectors converging on the middle column and to the west. The northern column of vectors spreads from the middle column and to the northwest. The residual map for the linear plus quadratic (Fig. 10C) has an S-shaped area of negative residual values extending from the southeast toward the northwest. The combined linear plus quadratic vector trend map accounts for 43.37 percent of the total sum of squares, leaving 56.63 percent for trends of cubic or higher degree.

By inspection of the Z-squared array (Table 7), it can be seen that the quartic cross-product coefficient (3, 1) accounts for a higher percentage of the total sum of squares (26.76) than the linear (20.88) or quadratic surface (22.50). With the program, it is possible to compute and plot up to the quintic surface. According to Whitten (1966, p. 134), however, it is desirable to have at least three times the number of data points as number of coefficients for the degree of trend surface. For the quartic trend surface, the number of coefficients is 15 and the minimum number of evenly distributed data points desirable would be 45. Because there are only 18 data points on the Pocono map, the quadratic trend with 6 coefficients would be the highest degree trend justified by the number of data points.

REFERENCES

DeLury, D. B., 1950, Values and intergals of the orthogonal polynomials up to n = 26: Univ. Toronto Press, Toronto, Canada, 33 p.

Fisher, R. A., 1953, Dispersion on a sphere: Proc. Roy. Soc. London, Ser. A., v. 217, p. 295-306.

Grant, F. 1957, A problem in the analysis of geophysical data: Geophysics, v. 22, p. 309-344.

Krumbein, W. C., 1956, Regional and local components in facies maps: Am. Assoc. Petroleum Geologists Bull., v. 40, no. 9, p. 2163-2194.

Krumbein, W. C., 1959, Trend surface analysis of contour-type maps with irregular control-point spacing: Jour. Geophysical Res., v. 64, p. 823-834.

Krumbein, W. C., and Graybill, F. A., 1965, An introduction to statistical models in geology: McGraw-Hill Co., New York, 475 p.

Pelletier, B. R., 1958, Pocono paleocurrents in Pennsylvania and Maryland: Geol. Soc. America Bull., v. 69, no. 8, p. 1033-1064.

Steinmetz, R., 1962, Analysis of vectorial data: Jour. Sed. Pet., v. 32, no. 4, p. 801-812.

Watson, G. S., 1956, A test for randomness of directions: Monthly Notices Roy. Astron. Soc., Geophysical Supplement, v. 7, p. 160-161.

Whitten, E. H. T., 1966, The general linear equation in prediction of gold content in Witwatersrand Rocks, South Africa; in Proceedings of the symposium on mathematical statistics and computer applications in ore valuation: Jour. South Africa Inst. Min. Met., Johannesburg, p. 124-156.

Table 1.-Listing of FORTRAN IV program for vector trend analysis.

```
$IBJOB
$IBFTC MAIN    DECK
C
C    FORTRAN IV PROGRAM FOR VECTOR TREND ANALYSIS OF DIRECTIONAL DATA.
C
C     WILLIAM T. FOX
C     DEPARTMENT OF GEOLOGY          AND        DEPARTMENT OF GEOLOGY
C     WILLIAMS COLLEGE                          STANFORD UNIVERSITY
C     WILLIAMSTOWN, MASS. 01267                 STANFORD, CALIF. 94305
C
C     FIRST CARD READS IN NAZM,IFDIP,NROW,NCOL,NOPRT,ANORTH,IFTRD.
C     COL. 1-4, 0 - NUMBER OF AZIMUTH OBSERVATIONS VARIES AT EACH LOCATION
C                   FIRST DATA CARD FOR EACH LOCATION IS USED TO GIVE THE
C                   NUMBER OF OBSERVATIONS AT THAT LOCATION.
C               N - THE NUMBER OF OBSERVATIONS WHICH IS CONSTANT FOR
C                   ALL LOCATIONS.
C     COL. 8    0 - NO DIP OBSERVATIONS MADE.
C               1 - DIP MEASUREMENTS INCLUDED WITH EACH AZIMUTH ON A CARD.
C     COL. 12     NUMBER OF ROWS FOR VECTOR TREND ANALYSIS
C     COL. 16     NUMBER OF COLUMNS FOR VECTOR TREND ANALYSIS.
C     COL. 20      1- DO NOT PRINT INTERMEDIATE OUTPUT.
C                 -1 - PRINT INTERMEDIATE OUTPUT.
C     COL. 21-28, AZIMUTH READING OF NORTH FROM VERTICAL ON MAP
C                 WITH ONE DECIMAL PLACE
C     COL. 30     HIGHEST TREND MAP TO OE COMPUTED
C                 1 - LINEAR TREND
C                 2 - QUADRATIC TREND
C                 3 - CUBIC TREND
C                 4 - QUARTIC TREND
C                 5 - QUINTIC TREND
C
C     SECOND DATA CARD IS FOR THE VARIABLE FORMAT, FMT, FOR READING IN
C     AZIMUTH AND DIP DATA.
C
C     THE THIRD CARD IS THE FIRST OF THE AZIMUTH AND DIP READINGS.
C
C     THE TITLE CARD IS THE LAST CARD IN THE DATA DECK.
C
      DIMENSION AZ(50), AZBAR(10,10),AZMAP(10,10), AZI(100),DIP(50),
     1 DIP1(50),DIPBAR(10,10),NMAP(10,10),RES(10,8,8),TRD(10,8,8),
     2 THETA(2),FMT(2),FLN(10),TITLE(12),P(8,8,8),SS(8,8),DNAZ(8,8),
     3 G(8,8),COEF(8,8),ZSQ(8,8),DIV(8,8),SUMSQ(10),D(8,8),PCTSQ(8,8)
       COMMON ABAR, AZ, AZBAR, AZMAP, AZI, DBAR, DIP, DIP1, DIPBAR,D,
     1 I,J,K,M,NAZ,NCOL,NROW,N1,NP,NOPRT,ANORTH,AVGA,AVGD,IFTRD,TRD,
     2 RES,SUMSQ,TOTSQ,G,COEF,ZSQ,DIV,DNAZ,MD,PCTSQ
C
      READ (5,900) NAZM,IFDIP,NROW,NCOL,NOPRT,ANORTH,IFTRD
      WRITE (6,928) NROW
      WRITE (6,929) NCOL
      IF (NAZM) 100,105,101
  101 NAZ = NAZM
      WRITE (6,930) NAZ
      GO TO 106
  105 WRITE (6,931)
  106 IF (IFDIP) 100,107,108
  107 WRITE (6,932)
      GO TO 3
```

```
  108 WRITE(6,933)
    3 READ (5,901)  (FMT(M),M=1,12)
      WRITE(6,922) (FMT(M),M=1,12)
      DO 50 I=1,NROW
      DO 48 J=1,NCOL
      IF (NOPRT .EQ. -1) WRITE (6,903) I,J
      IF (IFDIP) 1,1,12
C
    1 IF (NAZM) 2,8,2
    2 IF (NOPRT .EQ. -1) WRITE (6,904) NAZ
      IF (NOPRT .EQ. -1) WRITE (6,905)
      READ(5,FMT) (AZ(K),K=1,NAZ)
      DO 4 K = 1,NAZ
    4 IF (NOPRT .EQ. -1) WRITE (6,907) K,AZ(K)
      GO TO 30
    8 READ(5,909) NAZ
      IF (NOPRT .EQ. -1) WRITE (6,904) NAZ
      IF (NOPRT .EQ. -1) WRITE (6,905)
      DO 10 K=1,NAZ
      READ(5,FMT) AZ(K)
   10 IF (NOPRT .EQ. -1) WRITE (6,907) K,AZ(K)
      GO TO 30
C
   12 IF (NAZM) 15,14,15
   14 READ(5,909) NAZ
   15 IF (NOPRT .EQ. -1) WRITE (6,904) NAZ
      IF (NOPRT .EQ. -1) WRITE (6,911)
      DO 18 K=1,NAZ
      READ(5,FMT) AZ(K),DIP1(K)
   18 IF (NOPRT .EQ. -1) WRITE (6,913) K,AZ(K),DIP1(K)
      GO TO 34
   30 DO 32 K=1,NAZ
   32 DIP1(K)=0.0
      IF (NAZ .EQ. 1) GO TO 46
   34 N1=NAZ
      DO 36 K=1,NAZ
      AZI(K)=AZ(K)
   36 DIP(K)=DIP1(K)
      CALL DIRCOS
      AZBAR(I,J)=ABAR
      DIPBAR(I,J)=DBAR
      GO TO 48
   46 AZBAR(I,J) = AZ(K)
      DIPBAR(I,J) = DIP1(K)
   48 CONTINUE
   50 CONTINUE
C
      IF (NROW .LE. 2 .AND. NCOL .LE. 2) GO TO 100
      IF (NROW .GT. 8 .OR. NCOL .GT. 8) GO TO 100
      CALL TREND
C
  900 FORMAT (5I4,F8.1,I2)
  901 FORMAT (12A6)
  903 FORMAT(1H1,10X,22HOBSERVED DATA FOR ROW ,I4,2X,10HAND COLUMN,I4)
  904  FORMAT(1H0,10X,24HNUMBER OF OBSERVATIONS =,I4)
  905 FORMAT(1H0,10X,17HNUMBER    AZIMUTH)
  907 FORMAT (1H ,11X,I3,4X,F8.1)
  909 FORMAT(I4)
  911 FORMAT(1H0,10X,25HNUMBER    AZIMUTHS    DIP)
  913 FORMAT (1H ,11X,I3,5X,F8.1,F9.1)
```

```
  922 FORMAT(1H0,9X,36HVARIABLE FORMAT FOR READING DATA IS ,12A6)
  928 FORMAT (1H1,9X,16HNUMBER OF ROWS =,I4)
  929 FORMAT (1H0,9X,19HNUMBER OF COLUMNS =,I4)
  930 FORMAT(1H0,9X,20HNUMBER OF AZIMUTHS =,I4)
  931 FORMAT (1H0,9X,48HNUMBER OF AZIMUTHS IS DIFFERENT AT EACH LOCATION
     1 )
  932 FORMAT(1H0,9X,24HNO DIP MEASUREMENTS MADE)
  933 FORMAT (1H0,9X,43HDIP MEASUREMENTS INCLUDED WITH EACH AZIMUTH)
  100 RETURN
      END
$IBFTC SDIRCOS DECK
      SUBROUTINE DIRCOS
C
C     SUBROUTINE DIRCOS IS USED TO CALCULATE THE VECTOR MEAN AZIMUTH
C     AND DIP ANGLE FOR EACH CELL IN THE GRID AND FOR THE GRAND MEAN.
C
      DIMENSION AZ(50), AZBAR(10,10),AZMAP(10,10), AZI(100),DIP(50),
     1 DIP1(50),DIPBAR(10,10),NMAP(10,10),RES(10,8,8),TRD(10,8,8),
     2 THETA(2),FMT(2),FLN(10),TITLE(12),P(8,8,8),SS(8,8),DNAZ(8,8),
     3 G(8,8),COEF(8,8),ZSQ(8,8),DIV(8,8),SUMSQ(10),D(8,8),PCTSQ(8,8)
     4 ,PR(2)
       COMMON ABAR, AZ, AZBAR, AZMAP, AZI, DBAR, DIP, DIP1, DIPBAR,D,
     1 I,J,K,M,NAZ,NCOL,NROW,N1,NP,NOPRT,ANORTH,AVGA,AVGD,IFTRD,TRD,
     2 RES,SUMSQ,TOTSQ,G,COEF,ZSQ,DIV,DNAZ,MD,PCTSQ
C
      PR(1) = 0.05
      PR(2) = 0.01
      SUMA=0.0
      SUMB=0.0
      SUMC=0.0
      RADIAN = 57.2958
      FLN1=N1
C
C     CALCULATE DIRECTION COSINES A,B AND C.
C
      DO 10 K=1,N1
      IF (AZI(K)) 55,5,4
    4 X=AZI(K)/RADIAN
      GO TO 6
    5 X = 0.0
    6 IF (DIP(K)) 55,8,7
    7 Y = DIP(K)/RADIAN
      GO TO 9
    8 Y = 0.0
    9 SUMA = SUMA + COS(Y) * SIN(X)
      SUMB = SUMB + COS(Y) * COS(X)
      SUMC = SUMC + SIN(Y)
   10 CONTINUE
C
C     R IS THE MAGNITUDE OF THE RESULTANT VECTOR.
C
      R=SQRT (SUMA**2+SUMB**2+SUMC**2)
      IF (SUMA) 62,61,62
   61 A = 0.0
      GO TO 70
   62 A = SUMA / R
   70 IF (SUMB) 72,71,72
   71 B = 0.0
      GO TO 80
   72 B = SUMB / R
```

```
   80 IF (SUMC) 82,81,82
   81 C = 0.0
      GO TO 11
   82 C = SUMC / R
C
C     ABAR IS THE AVERAGE AZIMUTH DIRECTION.
C
   11 ABAR=RADIAN*ATAN(A/B)
      IF(B) 12,20,15
   12 ABAR=ABAR+180.0
      GO TO 35
   15 IF(A) 17,25,35
   17 ABAR=ABAR+ 360.0
      GO TO 35
   20 IF (A) 22,35,21
   21 ABAR = 90.0
      GO TO 35
   22 ABAR = 270.0
      GO TO 35
   25 ABAR = 0.0
C
C     DBAR IS THE AVERAGE DIP ANGLE
C
   35 DBAR = RADIAN * ARSIN(C)
   37 IF (NOPRT .EQ. 1) GO TO 55
      DO 45 N=1,2
      X1 = (1./PR(N))**(1./(FLN1-1.))-1.
      CX = 1. - ((FLN1-R)*X1)/R
      IF (CX .GT. 1.0 .OR. CX .LT. -1.0) GO TO 46
C
C     THETA IS THE SPHERICAL RADIUS OF THE CIRCLE OF CONFIDENCE
C     AROUND THE RESULTANT VECTOR.
C
      THETA(N) = RADIAN * ARCOS (CX)
      GO TO 45
   46 THETA (N) = -1.
   45 CONTINUE
C
   48 WRITE(6,902) A
      WRITE(6,903) B
      WRITE(6,904) C
      WRITE(6,905) R
      WRITE(6,910) ABAR
      WRITE(6,911) DBAR
      DO 50 N=1,2
      WRITE(6,912) PR(N)
      IF (THETA(N) .EQ. -1.) GO TO 52
      WRITE (6,913) THETA (N)
      GO TO 50
   52 WRITE (6,915)
   50 CONTINUE
C
  902 FORMAT(1H0,10X,20HDIRECTION COSINE A =,F9.5)
  903 FORMAT(1H0,10X,20HDIRECTION COSINE B =,F9.5)
  904 FORMAT(1H0,10X,20HDIRECTION COSINE C =,F9.5)
  905 FORMAT(1H0,10X,32HLENGTH OF RESULTANT VECTOR (R) =,F6.2)
  910 FORMAT(1H0,10X,21HVECTOR MEAN AZIMUTH = ,F8.3,8H DEGREES )
  911 FORMAT(1H0,10X,27HVECTOR MEAN DIP ANGLE =     ,F8.1,8H DEGREES )
  912 FORMAT(1H0,10X,23HLEVEL OF SIGNIFICANCE = F7.3)
  913 FORMAT (1H0,10X,64HRADIUS OF THE CIRCLE OF CONFIDENCE AROUND THE R
```

```
     1ESULTANT VECTOR =,F7.2,8H DEGREES )
  915 FORMAT (1H0,10X,55HRADIUS OF CIRCLE OF CONFIDENCE GREATER THAN 180
     1 DEGREES)
   55 RETURN
      END
$IBFTC STREND  DECK
      SUBROUTINE TREND
C
C     SUBROUTINE TREND IS USED TO CALCULATE THE COEFFICIENTS ARRAY
C     AND THE Z-SQUARED ARRAY USING ORTHOGONAL POLYNOMIALS AND MATRIX
C     ALGEBRA. TREND AND RESIDUAL VALUES FOR SURFACES UP TO QUINTIC
C     CAN BE COMPUTED.
C
      DIMENSION AZ(50), AZBAR(10,10),AZMAP(10,10), AZI(100),DIP(50),
     1 DIP1(50),DIPBAR(10,10),NMAP(10,10),RES(10,8,8),TRD(10,8,8),
     2 THETA(2),FMT(2),FLN(10),TITLE(12),P(8,8,8),SS(8,8),DNAZ(8,8),
     3 G(8,8),COEF(8,8),ZSQ(8,8),DIV(8,8),SUMSQ(10),D(8,8),PCTSQ(8,8)
       COMMON ABAR, AZ, AZBAR, AZMAP, AZI, DBAR, DIP, DIP1, DIPBAR,D,
     1 I,J,K,M,NAZ,NCOL,NROW,N1,NP,NOPRT,ANORTH,AVGA,AVGD,IFTRD,TRD,
     2 RES,SUMSQ,TOTSQ,G,COEF,ZSQ,DIV,DNAZ,MD,PCTSQ
C
C     ORTHOGONAL POLYNOMIALS FOR N = 3 TO N = 8 ( DELURY, 1950, P 18-19 )
C
      DATA (P(2,3,L),L=1,3)/-1.,0.,1./,(P(2,4,L),L=1,4)/-3.,-1.,1.,3./,
     1(P(2,5,L),L=1,5)/-2.,-1.,0.,1.,2./,
     2(P(2,6,L),L=1,6)/-5.,-3.,-1.,1.,3.,5./,
     3 (P(2,7,L),L=1,7)/-3.,-2.,-1.,0.,1.,2.,3./,
     4(P(2,8,L),L=1,8)/-7.,-5.,-3.,-1.,1.,3.,5.,7./,
     5(SS(2,L),L=3,8)/2.,20.,10.,70.,28.,168./,
     6(P(3,3,L),L=1,3)/1.,-2.,1./,(P(3,4,L),L=1,4)/1.,-1.,-1.,1./,
     7(P(3,5,L),L=1,5)/2.,-1.,-2.,-1.,2./,
     8(P(3,6,L),L=1,6)/5.,-1.,-4.,-4.,-1.,5./,
     9(P(3,7,L),L=1,7)/5.,0.,-3.,-4.,-3.,0.,5./,
     1(P(3,8,L),L=1,8)/7.,1.,-3.,-5.,-5.,-3.,1.,7./,
     2(SS(3,L),L=3,8)/6.,4.,14.,84.,84.,168./,
     3(P(4,3,L),L=1,3)/0.,0.,0. /,(P(4,4,L),L=1,4)/-1.,3.,-1.,3./,
     4(P(4,5,L),L=1,5)/-1.,2.,0.,-2.,1./,
     5(P(4,6,L),L=1,6)/-5.,7.,4.,-4.,-7.,5./,
     6(P(4,7,L),L=1,7)/-1.,1.,1.,0.,-1.,-1.,1./,
     7(P(4,8,L),L=1,8)/-7.,5.,7.,3.,-3.,-7.,-5.,7. /,
     8(SS(4,L),L=3,8)/0.,20.,10.,180.,6.,264./
       DATA (P(5,5,L),L=1,5) / 1.,-4.,6.,-4.,1./,
     1 (P(5,6,L),L=1,6) / 1.,-3.,2.,2.,-3.,1. / ,
     2 (P(5,7,L),L=1,7) / 3.,-7.,1.,6.,1.,-7.,3. / ,
     3 (P(5,8,L),L=1,8) / 7.,-13.,-3.,9.,9.,-3.,-13.,7. / ,
     4 (SS(5,L),L=5,8) / 70.,28.,154.,616. / ,
     5 (P(6,6,L),L=1,6) / -1.,5.,-10.,10.,-5.,1. / ,
     6 (P(6,7,L),L=1,7) / -1.,4.,-5.,0.,5.,-4.,1. / ,
     7 (P(6,8,L),L=1,8) / -7.,23.,-17.,-15.,15.,17.,-23.,7. / ,
     8 (SS(6,L),L=6,8) / 252.,84.,2184. / ,
     9 (P(7,7,L),L=1,7) / 1.,-6.,15.,-20.,15.,-6.,1. / ,
     1 (P(7,8,L),L=1,8) / 1.,-5.,9.,-5.,-5.,9.,-5.,1. / ,
     2 (SS(7,L),L=7,8) / 924.,264. / ,
     3 (P(8,8,L),L=1,8) / -1.,7.,-21.,35.,-35.,21.,-7.,1. / ,
     4 SS(8,8) / 3432. /
       SUMSS = 0.0
       DO 10 I = 1,8
       SUMSS = SUMSS + 1.0
       SS(1,I) = SUMSS
       DO 10 J = 1,8
```

```
   10 P(1,I,J) = 1.0
C
C     CALCULATE THE VECTOR MEAN AZIMUTH USING DIRECTION COSINES AND
C     TRANSFORM RESULATNT VECTORS TO STANDARD AZIMUTHS WITH A MEAN OF
C     180 DEGREES. STANDARD AZIMUTHS ARE USED IN THE MATRIX MULTIPLICATION
C     TO COMBAT THE PROBLEM OF CROSSING THE 360 DEGREE BARRIER WITH
C     DIRECTIONAL DATA.
C
      K=0
      DO 30 I=1,NROW
      DO 29 J=1,NCOL
      K=K+1
      AZI(K)=AZBAR(I,J)
      DIP(K)=DIPBAR(I,J)
   29 CONTINUE
   30 CONTINUE
      N1=K
      IF (NOPRT .EQ. -1) WRITE (6,915)
      CALL DIRCOS
      AVGA=ABAR
      AVGD=DBAR
      IF (NOPRT .EQ. -1) WRITE (6,900) AVGA,AVGD
      STAN = 180.0 - AVGA
C
C     CALCULATE COEFFICIENTS ARRAY AND Z-SQUARED ARRAY USING MATRIX
C     ALGEBRA AND ORTHOGONAL POLYNOMIALS.
C
      MD = 1
      DO 40 I = 1,NROW
      DO 40 J = 1,NCOL
      D(I,J) = AZBAR(I,J) + STAN
      IF (D(I,J) .GT. 360.0) D(I,J) = D(I,J) - 360.0
      IF (D(I,J) .LT. 0.0 ) D(I,J) = D(I,J) + 360.0
   40 CONTINUE
   42 IF (MD .EQ. 1) WRITE (6,910)
      IF (MD .EQ. 2) WRITE (6,911)
      DO 43 I = 1,10
   43 TRD (I,1,1) = 999.
      DO 50 I = 1,NROW
      DO 50 K = 1,NCOL
      SUMX = 0.0
      DO 45 J = 1,NCOL
      SUMX = SUMX + D(I,J) * P(K,NCOL,J)
   45 CONTINUE
      DNAZ (I,K) = SUMX
   50 CONTINUE
      SUMZ = 0.0
      DO 60 KI = 1,NROW
      DO 60 KJ = 1,NCOL
      SUMX = 0.0
      DO 55 I = 1,NROW
   55 SUMX = SUMX + P(KI,NROW,I) * DNAZ(I,KJ)
      D(KI,KJ) = D(KI,KJ) - STAN
      G(KI,KJ) = SUMX
      DIV(KI,KJ) = SS(KI,NROW) * SS(KJ,NCOL)
      COEF(KI,KJ) = G(KI,KJ) / DIV(KI,KJ)
      ZSQ(KI,KJ) = G(KI,KJ)**2 / DIV(KI,KJ)
      SUMZ = SUMZ + ZSQ(KI,KJ)
   60 CONTINUE
      COEF(1,1) = COEF(1,1) - STAN
```

```
      TOTSQ = SUMZ - ZSQ(1,1)
      ZSQ(1,1) = 0.0
      WRITE (6,903)
      DO 80 KI = 1,NROW
   80 WRITE (6,905) (COEF(KI,KJ),KJ=1,NCOL)
      WRITE (6,904)
      DO 90 KI = 1,NROW
   90 WRITE (6,902) (ZSQ (KI,KJ),KJ=1,NCOL)
      WRITE (6,914)
      DO 100 I = 1,NROW
      DO 95 J = 1,NCOL
   95 PCTSQ(I,J) = 100. * (ZSQ(I,J) / TOTSQ )
  100 WRITE (6,902) (PCTSQ(I,J),J=1,NCOL)
      DO 120 I = 1,NROW
      DO 120 J = 1,NCOL
C
C     ARITHMETIC MEAN
C
      TRD(1,I,J) = COEF(1,1)
      RES(1,I,J) = D(I,J) - TRD(1,I,J)
C
C     LINEAR TREND AND RESIDUAL VALUES
C
      IF(IFTRD .LT. 1) GO TO 120
      TRD   (2,I,J) = COEF(1,1) + COEF(2,1)*P(2,NROW,I) +
     1 COEF(1,2) * P(2,NCOL,J)
      RES(2,I,J) = D(I,J) - TRD(2,I,J)
      SUMSQ(2) = ZSQ(2,1) + ZSQ(1,2)
C
C     QUADRATIC TREND AND RESIDUAL VALUES
C
      IF (NCOL .LT. 3 .AND. NROW .LT. 3) GO TO 120
      IF (IFTRD .LT. 2) GO TO 120
        TRD (3,I,J) = COEF(1,1) + COEF(3,1)*P(3,NROW,I) +
     1 COEF(1,3)*P(3,NCOL,J)+ COEF(2,2)*P(2,NCOL,J)*P(2,NROW,I)
      RES(3,I,J) = D(I,J) - TRD(3,I,J)
      SUMSQ(3) = ZSQ(3,1) + ZSQ(2,2) + ZSQ(1,3)
C
C
C     LINEAR AND QUADRATIC TREND AND RESIDUAL VALUES
C
      TRD (4,I,J) = TRD(2,I,J) + TRD(3,I,J) - COEF(1,1)
      RES(4,I,J) = D(I,J) - TRD(4,I,J)
      SUMSQ(4) = SUMSQ(2) + SUMSQ(3)
C
C     CUBIC TREND AND RESIDUAL VALUES
C
      IF (IFTRD .LT. 3) GO TO 120
      IF (NCOL .LT. 4 .AND. NROW .LT. 4) GO TO 120
        TRD (5,I,J) = COEF(1,1) + COEF(4,1)*P(4,NROW,I) +
     1 COEF(1,4)*P(4,NCOL,J) + COEF(3,2)*P(3,NROW,I)*P(2,NCOL,J) +
     2 COEF(2,3)*P(2,NROW,I)*P(3,NCOL,J)
      RES(5,I,J) = D(I,J) - TRD(5,I,J)
      SUMSQ(5) = ZSQ(4,1) + ZSQ(3,2) + ZSQ(2,3) + ZSQ(1,4)
C
C     LINEAR, QUADRATIC AND CUBIC TREND AND RESIDUAL VALUES
C
      TRD (6,I,J) = TRD(4,I,J) + TRD(5,I,J) - COEF(1,1)
      RES(6,I,J) = D(I,J) - TRD(6,I,J)
      SUMSQ(6) = SUMSQ(4) + SUMSQ(5)
```

```
C
C     QUARTIC TREND AND RESIDUALS VALUES
C
      IF (IFTRD .LT. 4) GO TO 120
      IF (NCOL .LT. 5 .AND. NROW .LT. 5) GO TO 120
        TRD (7,I,J) = COEF(1,1) + COEF(5,1)*P(5,NROW,I)  +
     1 COEF(4,2)*P(4,NROW,I)*P(2,NCOL,J) + COEF(3,3)*P(3,NROW,I)*
     2 P(3,NCOL,J) + COEF(2,4)*P(2,NROW,I)*P(4,NCOL,J) +
     3 COEF(1,5)*P(5,NCOL,J)
      RES(7,I,J) = D(I,J) - TRD(7,I,J)
      SUMSQ(7) = ZSQ(5,1) + ZSQ(4,2) + ZSQ(3,3) + ZSQ(2,4) + ZSQ(1,5)
C
C     LINEAR, QUADRATIC, CUBIC AND QUARTIC TREND AND RESIDUALS
C
      TRD (8,I,J) = TRD(6,I,J) + TRD(7,I,J) - COEF(1,1)
      RES(8,I,J) = D(I,J) - TRD(8,I,J)
      SUMSQ(8) = SUMSQ(6) + SUMSQ(7)
C
C     QUINTIC TREND AND RESIDUAL VALUES
C
      IF (IFTRD .LT. 5) GO TO 120
      IF (NCOL .LT. 6 .AND. NROW .LT. 6) GO TO 120
      TRD(9,I,J) = COEF(1,1) + COEF(6,1)*P(6,NROW,I) +
     1 COEF(5,2)*P(5,NROW,I)*P(2,NCOL,J) + COEF(4,3)*P(4,NROW,I)*
     2 P(3,NCOL,J) + COEF(3,4)*P(3,NROW,I)*P(4,NCOL,J) + COEF(2,5)*
     3 P(2,NROW,I)*P(5,NCOL,J) + COEF(1,6)*P(6,NCOL,J)
      RES(9,I,J) = D(I,J) - TRD(9,I,J)
      SUMSQ(9) = ZSQ(6,1) + ZSQ(5,2) + ZSQ(4,3) + ZSQ(3,4) +
     1     ZSQ(2,5) + ZSQ(1,6)
C
C     LINEAR THROUGH QUINTIC TREND AND RESIDUAL VALUES
C
      TRD (10,I,J)= TRD(8,I,J) + TRD(9,I,J) - COEF(1,1)
      RES(10,I,J) = D(I,J) - TRD(10,I,J)
      SUMSQ(10) = SUMSQ(8) + SUMSQ(9)
C
  120 CONTINUE
C
      CALL PRTMAP
      IF (AVGD .LT. 0.01 ) GO TO 200
      IF (MD .EQ. 2) GO TO 200
C
C     CALCULATE DIP VALUES
C
      MD = 2
      STAN = 0.0
      DO 180 I = 1,NROW
      DO 180 J = 1,NCOL
  180 D(I,J) = DIPBAR(I,J)
      GO TO 42
  900 FORMAT (1H0,10X,47HAVERAGE AZIMUTH COMPUTED BY DIRECTION COSINES =
     1  , F8.3 // 11X,43HAVERAGE DIP COMPUTED BY DIERCTION COSINES =  ,
     2 F8.3 )
  901 FORMAT (1H1,10X,22HVECTOR MEAN DIRECTIONS   //)
  902 FORMAT (1H ,10X,8F10.2)
  903 FORMAT (1H0,10X,19HCOEFFICIENTS ARRAY   /)
  904 FORMAT (1H0,10X,15HZ-SQUARE ARRAY  /)
  905 FORMAT (1H ,10X,8F10.5)
  910 FORMAT (1H1,10X,14HAZIMUTH ARRAYS /)
  911 FORMAT (1H1,10X,10HDIP ARRAYS /)
```

```
  914 FORMAT (1H0,10X,44HPERCENTAGE OF TOTAL CORRECTED SUM OF SQUARES /)
  915 FORMAT (1H1,10X,40HVECTOR MEAN AZIMUTH OF RESULTANT VECTORS  /)
  200 RETURN
      END
$IBFTC SPRTMAP DECK
      SUBROUTINE PRTMAP
C
C     SUBROUTINE PRTMAP IS USED TO PRINT MAPS FOR OUTPUT DISPLAY.
C
      DIMENSION AZ(50), AZBAR(10,10),AZMAP(10,10), AZI(100),DIP(50),
     1 DIP1(50),DIPBAR(10,10),NMAP(10,10),RES(10,8,8),TRD(10,8,8),
     2 THETA(2),FMT(2),FLN(10),TITLE(12),P(8,8,8),SS(8,8),DNAZ(8,8),
     3 G(8,8),COEF(8,8),ZSQ(8,8),DIV(8,8),SUMSQ(10),D(8,8),PCTSQ(8,8)
       COMMON ABAR, AZ, AZBAR, AZMAP, AZI, DBAR, DIP, DIP1, DIPBAR,D,
     1 I,J,K,M,NAZ,NCOL,NROW,N1,NP,NOPRT,ANORTH,AVGA,AVGD,IFTRD,TRD,
     2 RES,SUMSQ,TOTSQ,G,COEF,ZSQ,DIV,DNAZ,MD,PCTSQ
C
      DATA
     1(NMAP(2,J),J=1,5)/6H      ,6H      ,6H      ,6H      ,6HLINEAR/,
     2(NMAP(3,J),J=1,5)/6H      ,6H      ,6H    PU,6HRE QUA,6HDRATIC/,
     3(NMAP(4,J),J=1,5)/6H      ,6H   LIN,6HEAR PL,6HUS QUA,6HDRATIC/,
     4(NMAP(5,J),J=1,5)/6H      ,6H      ,6H      ,6H  PURE,6H CUBIC/,
     5(NMAP(6,J),J=1,5)/6H   LIN,6HEAR, Q,6HUADRAT,6HIC AND,6H CUBIC/,
     6(NMAP(7,J),J=1,5)/6H      ,6H      ,6H      ,6HPURE Q,6HUARTIC/,
     7(NMAP(8,J),J=1,5)/6HLINEAR,6H QUAD,,6H CUBIC,6H AND Q,6HUARTIC/,
     8(NMAP(9,J),J=1,5)/6H      ,6H      ,6H      ,6HPURE Q,6HUINTIC/,
     9(NMAP(10,J),J=1,5)/6H      ,6H  LINE,6HAR THR,6HOUGH Q,6HUINTIC/
C
      READ (5,902) (TITLE(M1),M1=1,12)
C
C     PRINT MAPS OF THE MEAN VECTOR DIRECTION OR DIP IN EACH CELL OF THE GRID.
C
      WRITE (6,901) (TITLE(M1),M1=1,12)
      IF (MD .EQ. 1) WRITE (6,903)
      IF (MD .EQ. 2) WRITE (6,904)
      DO 50 I=1,NROW
      WRITE (6,906) (D(I,J),J=1,NCOL)
      DO 50 J=1,NCOL
      AZMAP (I,J) = D(I,J)
   50 CONTINUE
      IF (MD .EQ. 2) GO TO 45
      WRITE (6,901) (TITLE(M1),M1=1,12)
      WRITE(6,903)
      CALL MAPVEC
   45 WRITE (6,901) (TITLE(M1),M1=1,12)
      IF (MD .EQ. 1) WRITE (6,920)
      IF (MD .EQ. 2) WRITE (6,930)
      DO 40 I=1,NROW
   40 WRITE (6,906) (RES(1,I,J),J=1,NCOL)
      WRITE (6,921) TOTSQ
C
C     PRINT TREND MAPS FOR LINEAR THROUGH QUINTIC
C
      DO 160 NP =2,10
      IF (TRD(NP,1,1) .GT. 998. ) GO TO 200
      WRITE (6,901) (TITLE(M1),M1=1,12)
      IF (MD .EQ. 1) WRITE (6,915) (NMAP(NP,L),L=1,5)
      IF (MD .EQ. 2) WRITE (6,917) (NMAP(NP,L),L=1,5)
      DO 60 I=1,NROW
      WRITE (6,906)(  TRD(NP,I,J),J=1,NCOL)
```

```
      DO 60 J=1,NCOL
      AZMAP(I,J) =   TRD(NP,I,J)
   60 CONTINUE
      IF (MD .EQ. 2) GO TO 69
      WRITE (6,901) (TITLE(M1),M1=1,12)
      WRITE(6,915) (NMAP(NP,L),L=1,5)
      CALL MAPVEC
C
C     PRINT RESIDUAL MAPS FOR LINEAR THROUGH QUINTIC
C
   69 WRITE (6,901) (TITLE(M1),M1=1,12)
      IF (MD .EQ. 1) WRITE (6,916) (NMAP(NP,L),L=1,5)
      IF (MD .EQ. 2) WRITE (6,918) (NMAP(NP,L),L=1,5)
      DO 70 I=1,NROW
      WRITE(6,906)(RES   (NP,I,J),J=1,NCOL)
   70 CONTINUE
      SS1 = TOTSQ-SUMSQ(NP)
      WRITE (6,923) SS1
      PCTSS = 100. * (SUMSQ(NP)/TOTSQ)
      WRITE (6,924) PCTSS
  160 CONTINUE
C
  901 FORMAT (1H1,10X,12A6)
  902 FORMAT(12A6)
  903 FORMAT(1H0,10X,70HMAP OF VECTOR MEAN AZIMUTH DIRECTIONS IN EACH CE
     1LL BY ROWS AND COLUMNS)
  904 FORMAT(1H0,10X,69HMAP OF VECTOR MEAN DIP ANGLES IN EACH CELL BY RO
     1WS AND COLUMNS          )
  906 FORMAT(1H0////10X,15F10.1)
  915 FORMAT(1H0,10X,5A6,40H TREND SURFACE MAP OF AZIMUTH DIRECTION )
  916 FORMAT(1H0,10X,5A6,35H RESIDUALS MAP OF AZIMUTH DIRECTION)
  917 FORMAT(1H0,10X,5A6,33H TREND SURFACE MAP OF DIP ANGLES )
  918 FORMAT(1H0,10X,5A6,28H RESIDUALS MAP OF DIP ANGLES)
  920 FORMAT(1H0,10X,58HRESIDUALS - OBSERVED AZIMUTH DIRECTIONS MINUS ME
     1AN AZIMUTH)
  921 FORMAT(1H0///10X,22HTOTAL SUM OF SQUARES =, F12.2)
  923 FORMAT(1H0///11X,24HSUM OF SQUARES FOR MAP =,F12.2)
  924 FORMAT (1H0,10X,47HPERCENTAGE REDUCTION IN TOTAL SUM OF SQUARES =
     1   , F10.3 )
  930 FORMAT(1H0///10X,47HRESIDUAL - OBSERVED DIP ANGLE MINUS AVERAGE DI
     1P )
  200 RETURN
      END
$IBFTC SMAPVEC DECK
      SUBROUTINE MAPVEC
C
C
C     SUBROUTINE MAPVEC IS USED TO PLOT VECTOR TREND MAPS WITH AZIMUTH
C     DIRECTION PLOTTED AS A VECTOR.
C
      DIMENSION AZ(50), AZBAR(10,10),AZMAP(10,10), AZI(100),DIP(50),
     1 DIP1(50),DIPBAR(10,10),NMAP(10,10),RES(10,8,8),TRD(10,8,8),
     2 THETA(2),FMT(2),FLN(10),TITLE(12),P(60,120),SS(8,8),DNAZ(8,8),
     3 G(8,8),COEF(8,8),ZSQ(8,8),DIV(8,8),SUMSQ(10),D(8,8),PCTSQ(8,8)
       COMMON ABAR, AZ, AZBAR, AZMAP, AZI, DBAR, DIP, DIP1, DIPBAR,D,
     1 I,J,K,M,NAZ,NCOL,NROW,N1,NP,NOPRT,ANORTH,AVGA,AVGD,IFTRD,TRD,
     2 RES,SUMSQ,TOTSQ,G,COEF,ZSQ,DIV,DNAZ,MD,PCTSQ
C
      DATA BLANK,AA,EX,DOT,VEE,FN/ 1H  , 1HA , 1HX , 1H. , 1HV , 1HN /
      WRITE (6,901)
```

```
      IFNOR = 0
      NR = NROW
      NC = NCOL
C
    2 DO 3 LX = 1,60
      DO 3 MX = 1,119
    3 P(LX,MX) = BLANK
      L = 7
    5 DO 60 IROW = 1,NR
      M = 12
      DO 50 K=1,NC
      IF (IFNOR) 4,4,6
    4 BNORTH = 360.-ANORTH
      AZMAP(IROW,K) = AZMAP(IROW,K) - BNORTH
      IF (AZMAP(IROW,K) .LT. 0.) AZMAP(IROW,K) = AZMAP(IROW,K)+360.
      GO TO 7
    6 M = MNOR
      L = 10
    7 X = AZMAP(IROW,K)
      IF (X .LE. 180.0) GO TO 22
      M1 = M-1
      M2 = M-2
      M3 = M-3
      M4 = M-4
      M5 = M-5
      M6 = M-6
      M7 = M-7
      M8 = M-8
      M9 = M-9
      GO TO 24
   22 M1 = M+1
      M2 = M+2
      M3 = M+3
      M4 = M+4
      M5 = M+5
      M6 = M+6
      M7 = M+7
      M8 = M+8
      M9 = M+9
C
   24 L1 = L
      IF ( X .LE. 270.0 .AND. X .GT. 90.0) GO TO 26
      L2 = L-1
      L3 = L-2
      L4 = L-3
      L5 = L-4
      L6 = L-5
      STAR = AA
      IF (IFNOR .EQ. 1) STAR = FN
      GO TO 27
   26 L2 = L+1
      L3 = L+2
      L4 = L+3
      L5 = L+4
      L6 = L+5
      STAR = VEE
      IF (IFNOR .EQ. 1) STAR = FN
C
   27 IF (X .LE. 90.0) X=180.0-X
      IF (X .GT.180.0 .AND. X .LE. 270.0) X = 360.0 - X
```

```
      IF (X .GT. 270.0) X=X-180.0
      P(L,M) = EX
      IF(X.LE.106.0.AND.X.GE.090.0) P(L1  , M3)=DOT
      IF(X.LE.095.0.AND.X.GE.090.0) P(L1  , M6)=DOT
      IF(X.LE.095.0.AND.X.GE.090.0) P(L1  , M9)=STAR
      IF(X.LE.180.0.AND.X.GT.166.0) P(L2  , M )=DOT
      IF(X.LE.166.0.AND.X.GT.143.0) P(L2  , M1)=DOT
      IF(X.LE.143.0.AND.X.GT.124.0) P(L2  , M2)=DOT
      IF(X.LE.124.0.AND.X.GT.106.0) P(L2  , M3)=DOT
      IF(X.LE.116.0.AND.X.GT.095.0) P(L2  , M6)=DOT
      IF(X.LE.106.0.AND.X.GT.095.0) P(L2  , M9)=STAR
      IF(X.LE.180.0.AND.X.GT.171.0) P(L3  , M )=DOT
      IF(X.LE.171.0.AND.X.GT.161.0) P(L3  , M1)=DOT
      IF(X.LE.161.0.AND.X.GT.147.0) P(L3  , M2)=DOT
      IF(X.LE.147.0.AND.X.GT.138.0) P(L3  , M3)=DOT
      IF(X.LE.138.0.AND.X.GT.124.0) P(L3  , M4)=DOT
      IF(X.LE.124.0.AND.X.GT.116.0) P(L3  , M6)=DOT
      IF(X.LE.116.0.AND.X.GT.106.0) P(L3  , M9)=STAR
      IF(X.LE.180.0.AND.X.GT.177.0) P(L4  , M )=DOT
      IF(X.LE.177.0.AND.X.GT.166.0) P(L4  , M1)=DOT
      IF(X.LE.166.0.AND.X.GT.156.0) P(L4  , M2)=DOT
      IF(X.LE.156.0.AND.X.GT.151.0) P(L4  , M3)=DOT
      IF(X.LE.151.0.AND.X.GT.143.0) P(L4  , M4)=DOT
      IF(X.LE.143.0.AND.X.GT.138.0) P(L4  , M5)=DOT
      IF(X.LE.138.0.AND.X.GT.124.0) P(L4  , M6)=DOT
      IF(X.LE.124.0.AND.X.GT.116.0) P(L4  , M9)=STAR
      IF(X.LE.180.0.AND.X.GT.177.0) P(L5  , M )=DOT
      IF(X.LE.177.0.AND.X.GT.171.0) P(L5  , M1)=DOT
      IF(X.LE.171.0.AND.X.GT.161.0) P(L5  , M2)=DOT
      IF(X.LE.161.0.AND.X.GT.156.0) P(L5  , M3)=DOT
      IF(X.LE.156.0.AND.X.GT.151.0) P(L5  , M4)=DOT
      IF(X.LE.151.0.AND.X.GT.147.0) P(L5  , M5)=DOT
      IF(X.LE.147.0.AND.X.GT.138.0) P(L5  , M6)=DOT
      IF(X.LE.138.0.AND.X.GT.132.0) P(L5  , M7)=DOT
      IF(X.LE.132.0.AND.X.GT.124.0) P(L5  , M9)=STAR
      IF(X.LE.180.0.AND.X.GT.177.0) P(L6  , M )=STAR
      IF(X.LE.177.0.AND.X.GT.171.0) P(L6  , M1)=STAR
      IF(X.LE.171.0.AND.X.GT.166.0) P(L6  , M2)=STAR
      IF(X.LE.166.0.AND.X.GT.161.0) P(L6  , M3)=STAR
      IF(X.LE.161.0.AND.X.GT.156.0) P(L6  , M4)=STAR
      IF(X.LE.156.0.AND.X.GT.151.0) P(L6  , M5)=STAR
      IF(X.LE.151.0.AND.X.GT.147.0) P(L6  , M6)=STAR
      IF(X.LE.147.0.AND.X.GT.143.0) P(L6  , M7)=STAR
      IF(X.LE.143.0.AND.X.GT.138.0) P(L6  , M8)=STAR
      IF(X.LE.138.0.AND.X.GT.132.0) P(L6  , M9)=STAR
      M = M+10
   50 CONTINUE
      L = L + 6
   60 CONTINUE
      IF (IFNOR .EQ. 1) GO TO 70
   90 NC = 1
      NR = 1
      MNOR = M+13
      AZMAP(1,1) = ANORTH
      IFNOR = 1
      GO TO 5
   70 IX = 7 + 6*NROW
      DO 80 IP = 1,IX
      WRITE (6,905) (P(IP,J),J=1,119)
   80 CONTINUE
```

```
C
  901 FORMAT (1H0)
  905 FORMAT (1H ,119A1)
  100 RETURN
      END
```

Table 2.-Listing of input data cards used in example 1 (Steinmetz, 1962).

```
  20   1   1   1  -1     0.0 0
(F5.1,3X,F5.1)
321.0    19.0         81.0    12.0
322.0    24.0         36.0    05.0
324.0    10.0         40.0    12.0
310.0     8.0        338.0    10.0
302.0    06.0         60.0    13.0
307.0    08.0         58.0    06.0
267.0    16.0         37.0    07.0
 43.0    04.0         70.0    04.0
 34.0    08.0         81.0    05.0
346.0    10.0         18.0    13.0
```

Table 3.-Output from example 1, cross-bedding measurements from the Wasatch Formation, Eocene, Wyoming.

```
OBSERVED DATA FOR ROW    1  AND COLUMN    1

NUMBER OF OBSERVATIONS =  20

NUMBER    AZIMUTHS    DIP          10        346.0     10.0
    1       321.0     19.0         11         81.0     12.0
    2       322.0     24.0         12         36.0      5.0
    3       324.0     10.0         13         40.0     12.0
    4       310.0      8.0         14        338.0     10.0
    5       302.0      6.0         15         60.0     13.0
    6       307.0      8.0         16         58.0      6.0
    7       267.0     16.0         17         37.0      7.0
    8        43.0      4.0         18         70.0      4.0
    9        34.0      8.0         19         81.0      5.0
                                   20         18.0     13.0
DIRECTION COSINE A =  0.17134

DIRECTION COSINE B =  0.94857

DIRECTION COSINE C =  0.26619

LENGTH OF RESULTANT VECTOR (R) = 12.99

VECTOR MEAN AZIMUTH =  10.239 DEGREES

VECTOR MEAN DIP ANGLE =        15.4 DEGREES

LEVEL OF SIGNIFICANCE =  0.050

RADIUS OF THE CIRCLE OF CONFIDENCE AROUND THE RESULTANT VECTOR =  24.80 DEGREES

LEVEL OF SIGNIFICANCE =  0.010

RADIUS OF THE CIRCLE OF CONFIDENCE AROUND THE RESULTANT VECTOR =  31.58 DEGREES
```

Table 4.-Listing of input data cards used in example 2, drumlins from western New York State.

```
  5    0    6    8    1        0.0 3
(5X,5F6.1)
  11 178.0 182.0 175.0 168.0 173.0
  12 172.0 169.0 168.5 166.0 170.0
  13 165.5 167.5 176.5 173.0 171.0
  14 157.5 155.5 165.5 161.5 162.0
  15 158.5 158.5 158.5 158.0 158.5
  16 149.0 159.0 149.0 140.0 147.0
  17 157.0 157.0 147.5 145.5 152.5
  18 146.5 146.5 150.5 138.5 140.0
  21 120.0 166.5 169.0 175.0 167.0
  22 176.0 166.0 173.0 158.0 175.0
  23 169.0 165.5 168.5 165.0 176.0
  24 159.5 165.5 161.0 166.0 178.5
  25 156.5 159.5 152.5 160.0 166.5
  26 152.0 138.5 145.5 148.5 154.0
  27 137.5 137.5 137.5 147.5 151.5
  28 143.0 143.0 143.0 162.5 162.5
  31 176.5 176.0 174.0 174.0 170.5
  32 178.0 174.0 175.0 166.5 169.0
  33 174.0 172.5 175.5 173.0 168.0
  34 168.5 166.0 166.0 169.0 167.5
  35 164.5 167.0 154.0 170.5 165.5
  36 155.5 148.5 155.0 164.5 162.5
  37 138.5 138.5 136.5 139.0 136.5
  38 148.5 143.0 148.5 143.5 143.5
  41 174.0 172.0 170.5 169.0 167.0
  42 172.5 172.0 167.0 171.5 173.0
  43 166.5 172.5 171.5 178.0 180.0
  44 171.0 156.0 169.5 166.5 169.0
  45 147.0 173.5 153.0 172.0 166.0
  46 166.0 160.5 164.5 162.5 153.5
  47 156.0 155.5 160.0 174.0 139.0
  48 132.5 142.5 142.0 157.5 148.5
  51 170.0 167.5 163.0 170.5 169.0
  52 171.5 167.5 169.5 170.0 172.5
  53 169.0 170.0 175.5 170.0 172.5
  54 169.5 165.5 169.5 163.5 173.0
  55 167.0 152.0 172.5 157.0 164.0
  56 162.5 155.5 151.5 155.0 245.0
  57 135.5 153.5 161.5 165.0 155.0
  58 162.0 137.0 158.5 170.0 147.0
  61 173.0 173.0 172.0 170.0 174.0
  62 168.5 168.0 174.5 164.0 165.5
  63 159.5 157.5 170.0 160.0 164.0
  64 157.5 161.5 164.0 156.5 159.5
  65 151.5 150.5 159.5 149.5 154.5
  66 144.0 156.5 152.0 158.7 152.0
  67 169.0 164.5 166.5 145.0 157.5
  68 152.5 168.5 152.5 161.0 110.0
ORIENTATION OF DRUMLINS IN WESTERN NEW YORK STATE
```

Table 5.-Coefficients array, Z-squared array, and percentage of total sum of squares array for example 2, drumlins from western New York State.

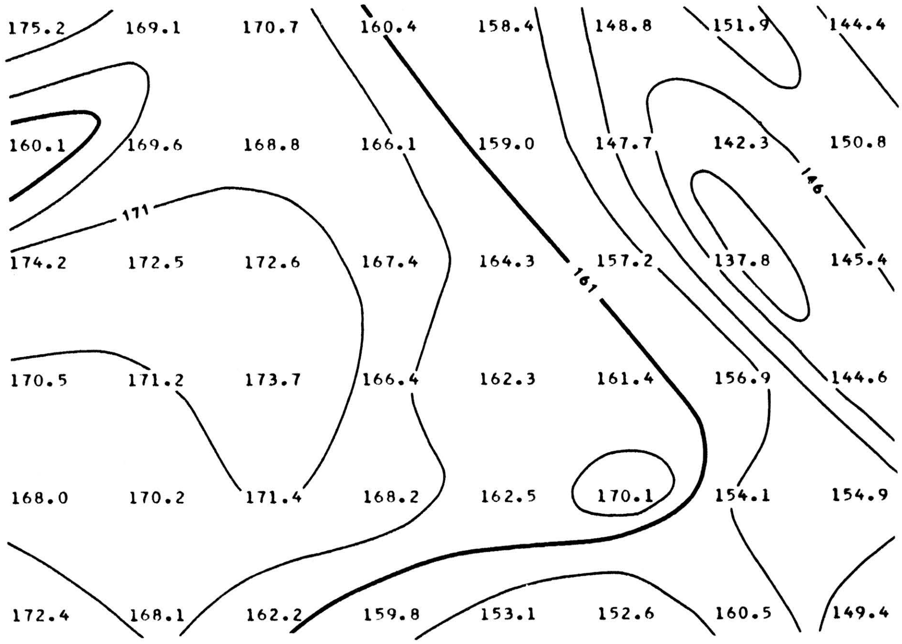
```
AZIMUTH ARRAYS

COEFFICIENTS ARRAY
 161.23739  -1.76089  -0.31518   0.20594   0.00355  -0.00289   0.10369  -0.00095
   0.31692   0.09177   0.02546  -0.05985  -0.02841  -0.00217   0.00911   0.01097
  -0.28608   0.02192   0.10155  -0.02690  -0.02569  -0.01794  -0.03947  -0.00442
  -0.31390  -0.01593   0.01444   0.03477   0.01193   0.00255  -0.02412  -0.00002
   0.01068  -0.14443   0.03981  -0.08342  -0.01549  -0.01550  -0.01277  -0.01142
  -0.05958   0.02332  -0.00116   0.00086  -0.01720  -0.00818  -0.01642  -0.00169

Z-SQUARE ARRAY
       0.    3125.55    100.14     67.18      0.05      0.11     17.03      0.02
     56.25     99.05      7.62     66.19     34.81      0.72      1.53     28.93
     55.00      6.78    145.54     16.04     34.14     59.02     34.54      5.62
    141.88      7.68      6.30     57.44     15.77      2.56     27.65      0.00
      0.03     98.12      7.46     51.44      4.14     14.68      1.21     12.53
      7.16     23.03      0.06      0.05     45.95     36.82     17.93      2.46

PERCENTAGE OF TOTAL CORRECTED SUM OF SQUARES
       0.      68.79      2.20      1.48      0.00      0.00      0.37      0.00
      1.24      2.18      0.17      1.46      0.77      0.02      0.03      0.64
      1.21      0.15      3.20      0.35      0.75      1.30      0.76      0.12
      3.12      0.17      0.14      1.26      0.35      0.06      0.61      0.00
      0.00      2.16      0.16      1.13      0.09      0.32      0.03      0.28
      0.16      0.51      0.00      0.00      1.01      0.81      0.39      0.05
```

Figure 1.-Isoazimuth map, vector trend map, and deviations from mean azimuth based on orientation of drumlins in western New York State. Area 32 miles east-west by 24 miles north-south.

ORIENTATION OF DRUMLINS IN WESTERN NEW YORK STATE

MAP OF VECTOR MEAN AZIMUTH DIRECTIONS IN EACH CELL BY ROWS AND COLUMNS

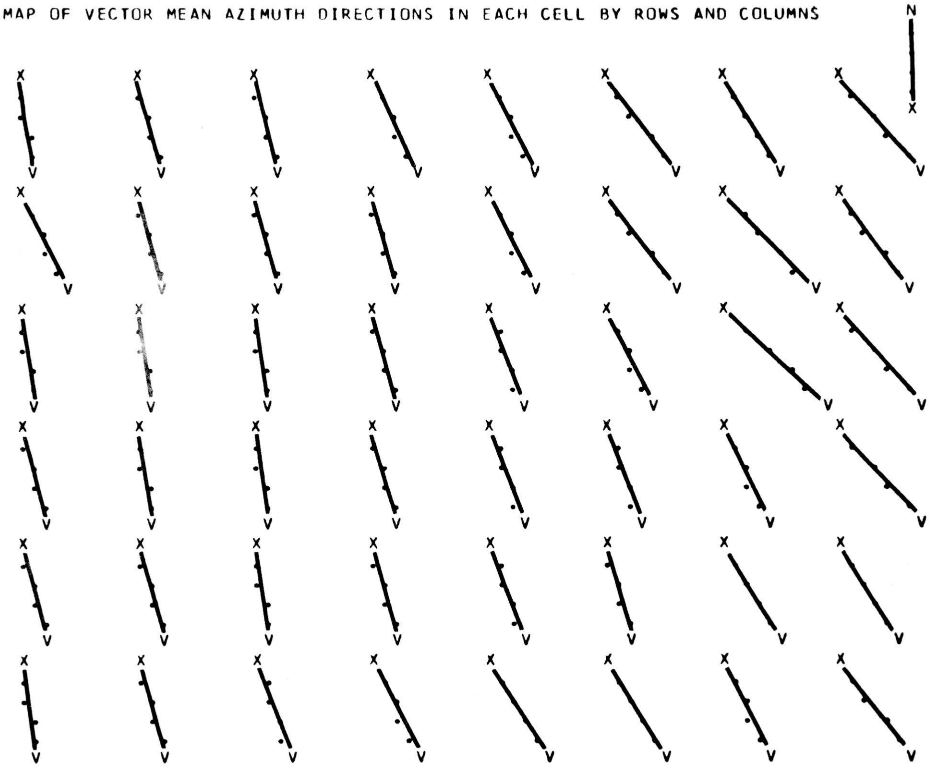
1A



ORIENTATION OF DRUMLINS IN WESTERN NEW YORK STATE

MAP OF VECTOR MEAN AZIMUTH DIRECTIONS IN EACH CELL BY ROWS AND COLUMNS

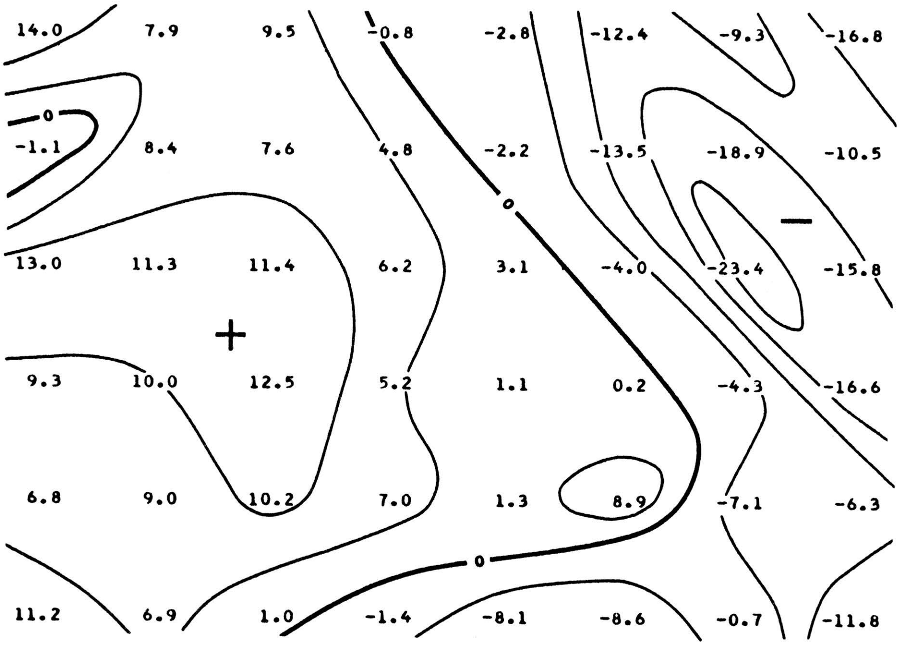
1B



ORIENTATION OF DRUMLINS IN WESTERN NEW YORK STATE

RESIDUALS - OBSERVED AZIMUTH DIRECTIONS MINUS MEAN AZIMUTH

1C



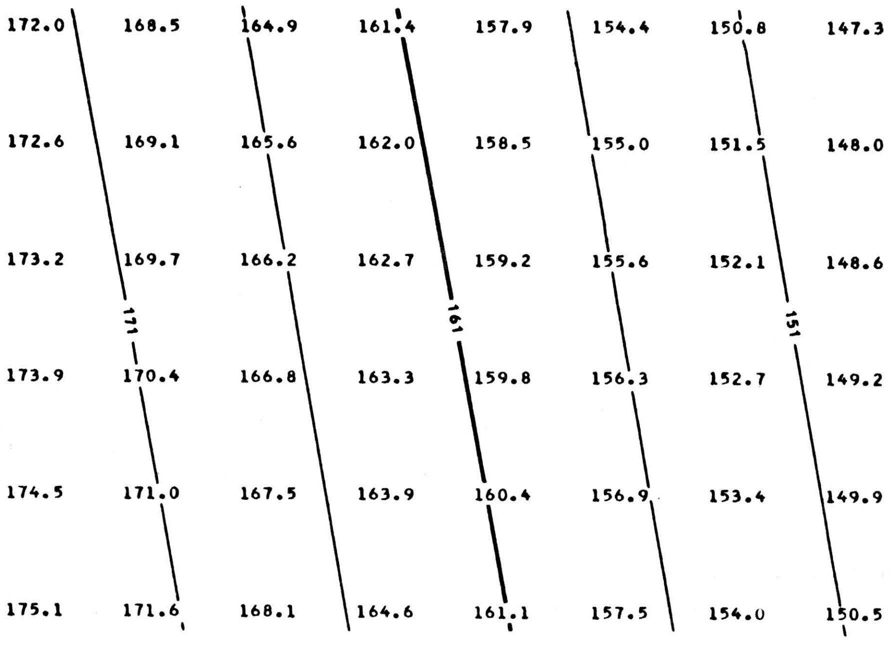
TOTAL SUM OF SQUARES = 4543.83

Figure 2.-Linear trend surface map, linear vector trend map and residuals from linear for drumlin orientation.

ORIENTATION OF DRUMLINS IN WESTERN NEW YORK STATE

LINEAR TREND SURFACE MAP OF AZIMUTH DIRECTION

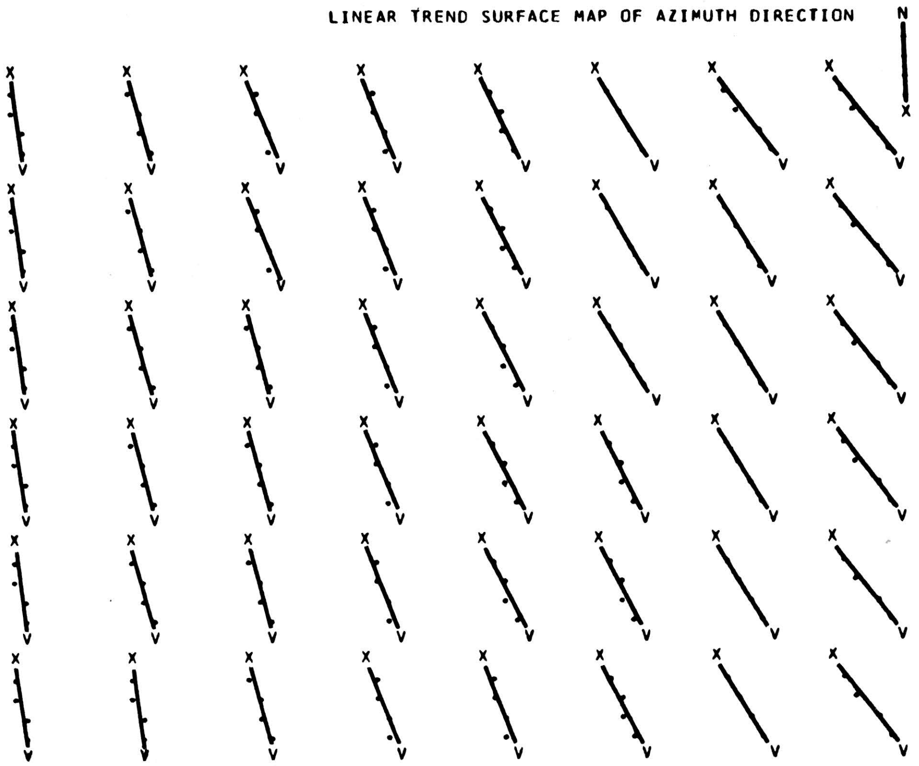
2A



ORIENTATION OF DRUMLINS IN WESTERN NEW YORK STATE

LINEAR TREND SURFACE MAP OF AZIMUTH DIRECTION

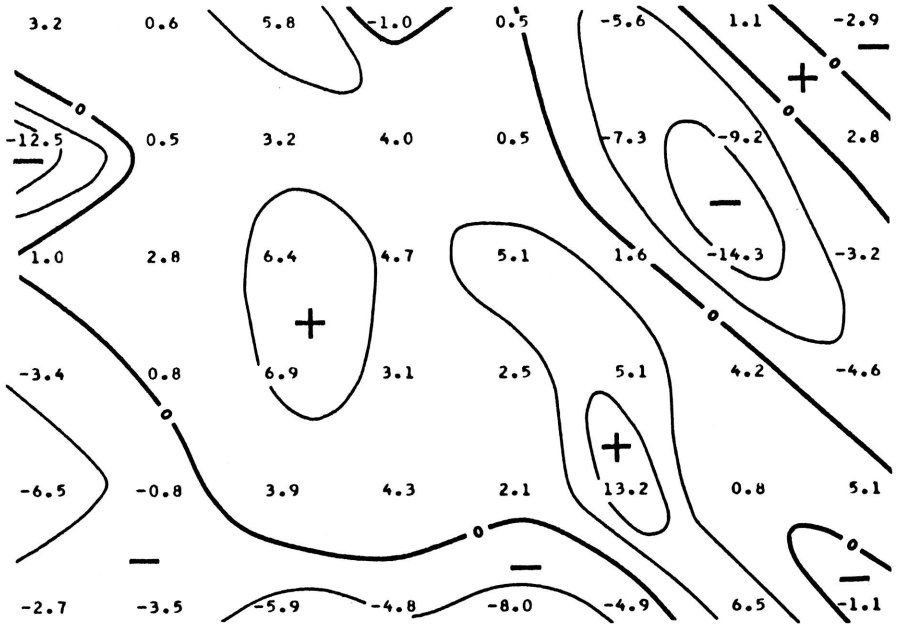
2B



ORIENTATION OF DRUMLINS IN WESTERN NEW YORK STATE

LINEAR RESIDUALS MAP OF AZIMUTH DIRECTION

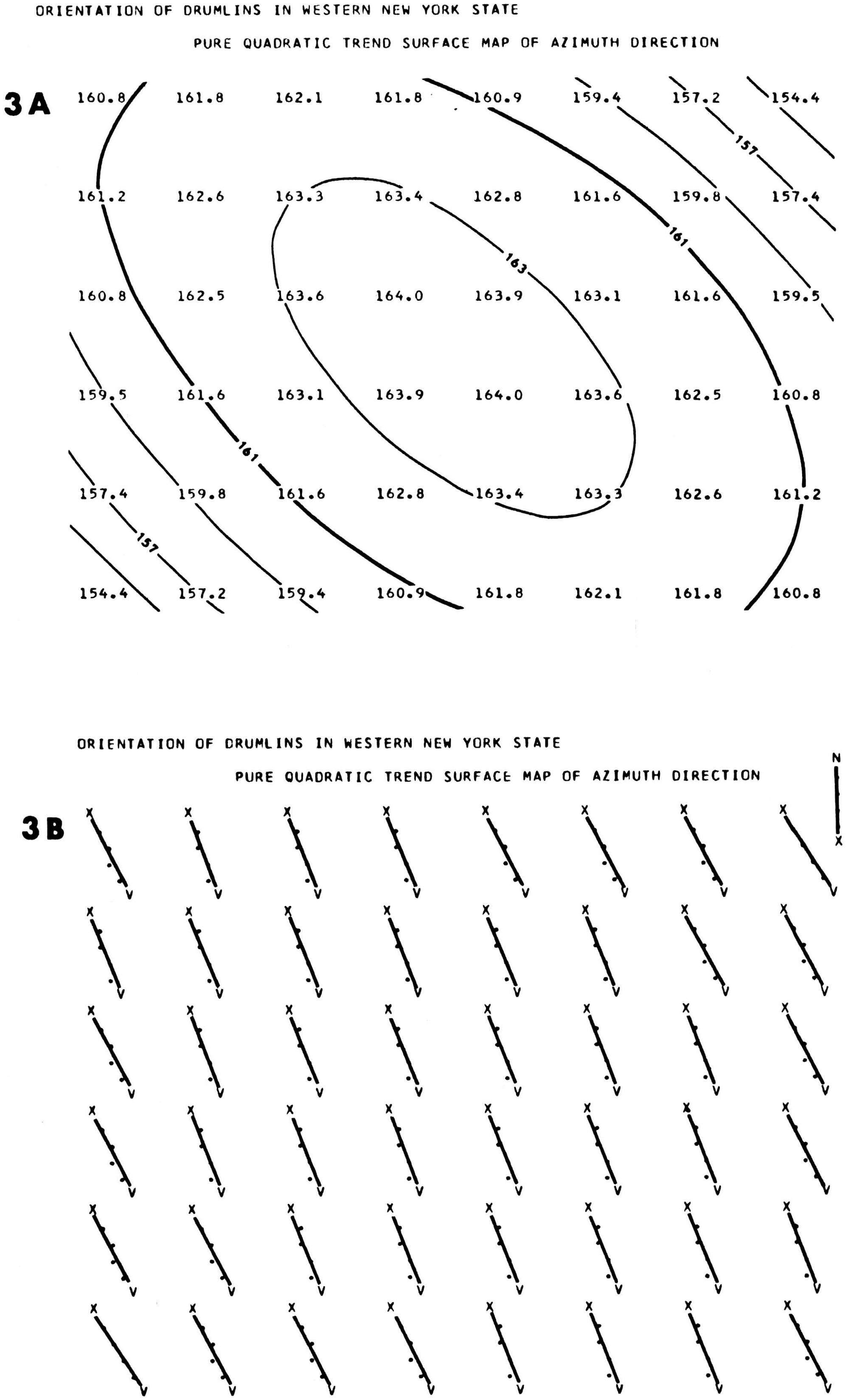
2C



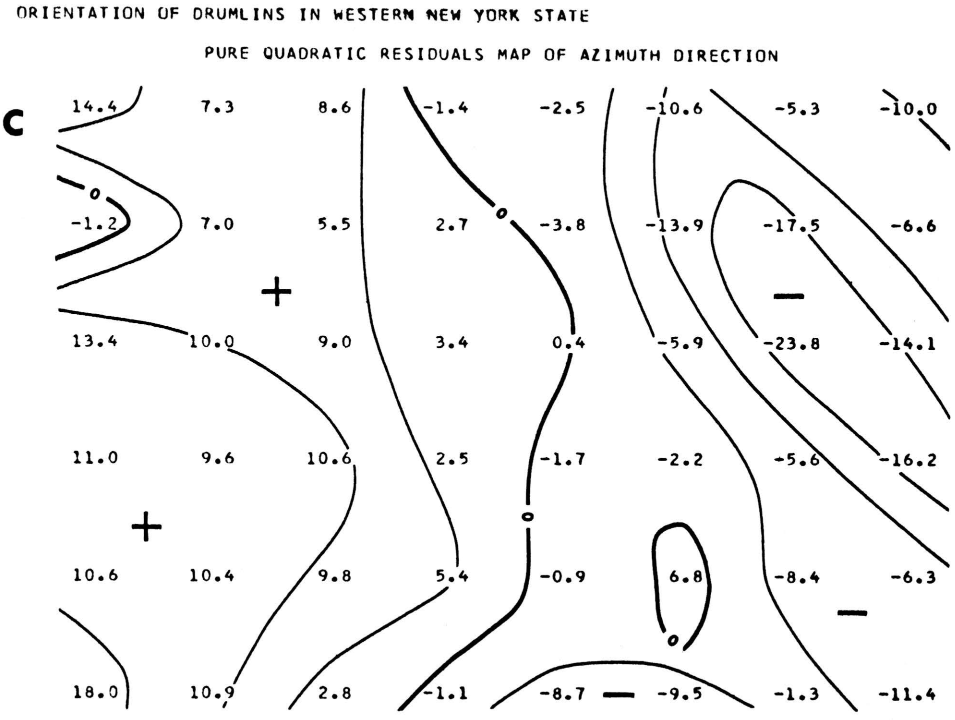
SUM OF SQUARES FOR MAP = 1362.03

PERCENTAGE REDUCTION IN TOTAL SUM OF SQUARES = 70.025

Figure 3.-Quadratic trend-surface map, vector trend map and residuals from quadratic trend for drumlin orientation.



3C

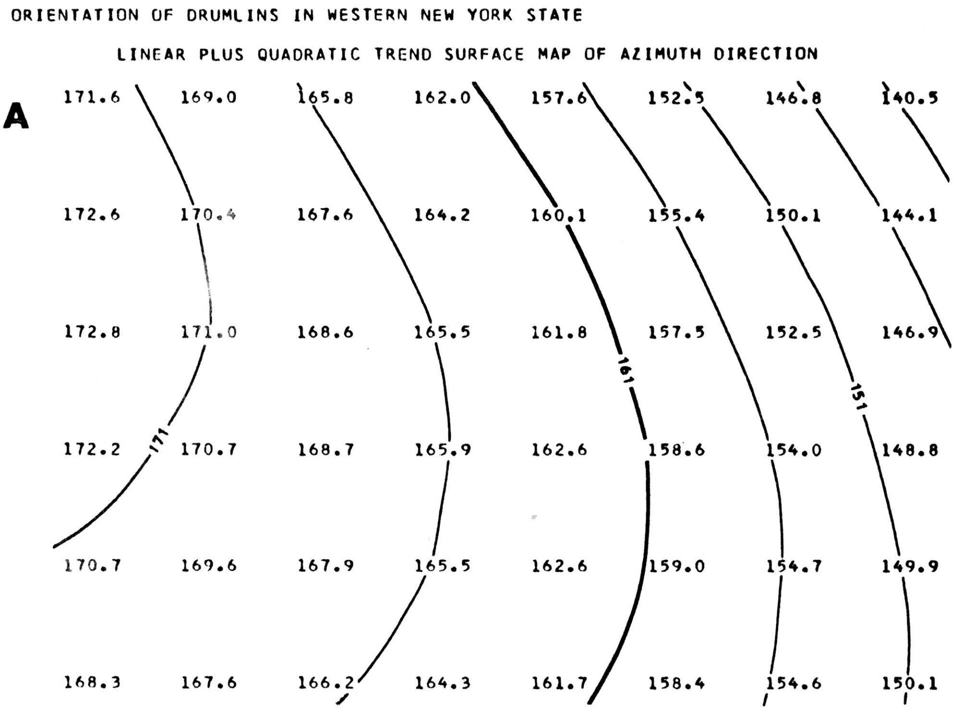


SUM OF SQUARES FOR MAP = 4289.65

PERCENTAGE REDUCTION IN TOTAL SUM OF SQUARES = 5.594

Figure 4.-Linear plus quadratic trend surface map, vector trend map and residuals from linear plus quadratic trend for drumlin orientation.

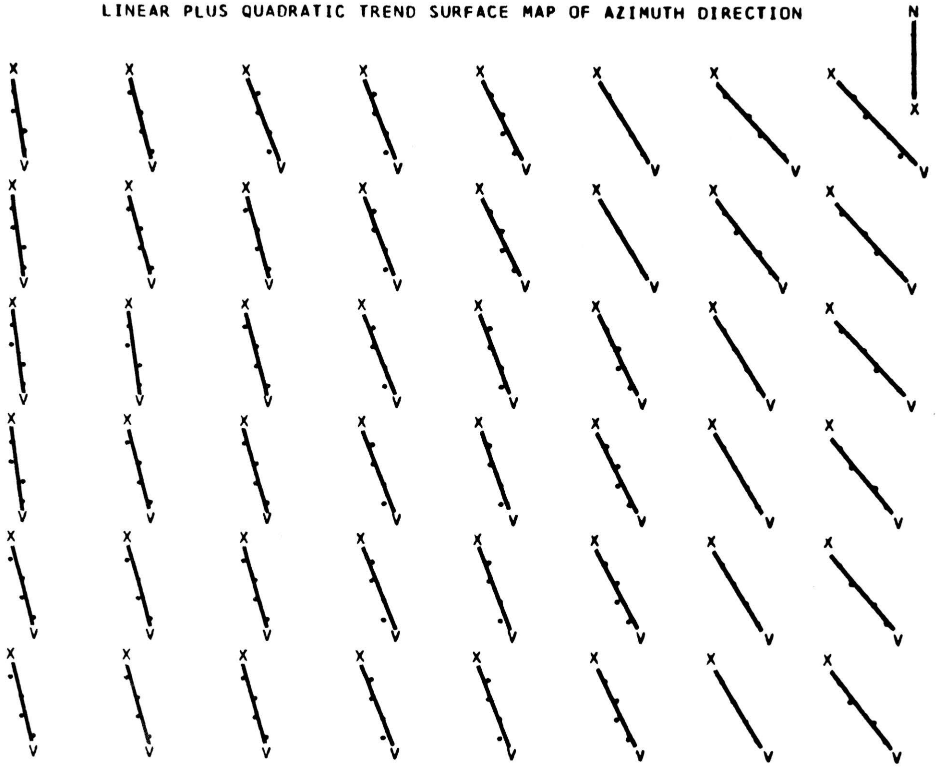
4A



ORIENTATION OF DRUMLINS IN WESTERN NEW YORK STATE

LINEAR PLUS QUADRATIC TREND SURFACE MAP OF AZIMUTH DIRECTION

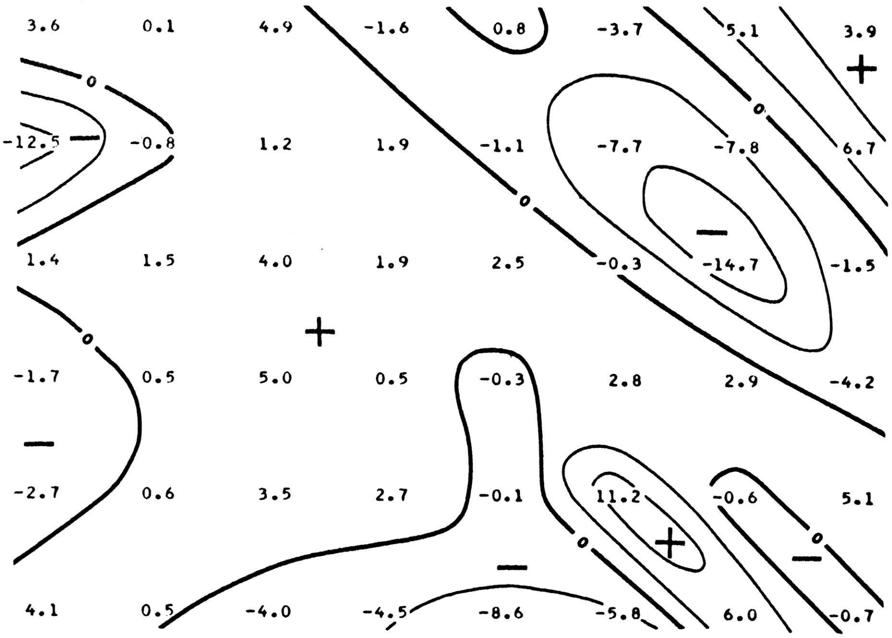
4B



ORIENTATION OF DRUMLINS IN WESTERN NEW YORK STATE

LINEAR PLUS QUADRATIC RESIDUALS MAP OF AZIMUTH DIRECTION

4C



SUM OF SQUARES FOR MAP = 1107.85

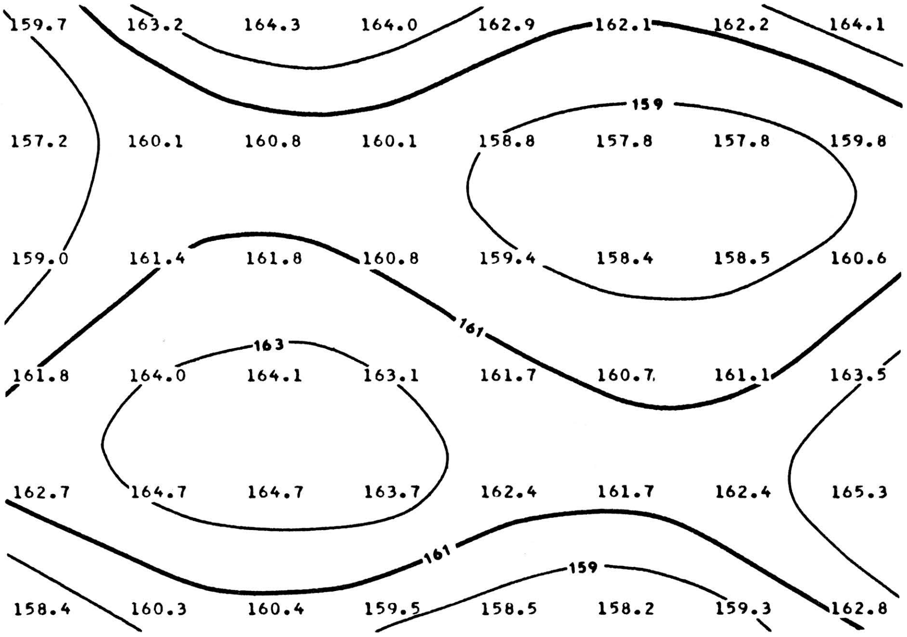
PERCENTAGE REDUCTION IN TOTAL SUM OF SQUARES = 75.619

Figure 5.-Cubic trend surface map, vector trend map and residuals from cubic trend for drumlin orientation.

ORIENTATION OF DRUMLINS IN WESTERN NEW YORK STATE

PURE CUBIC TREND SURFACE MAP OF AZIMUTH DIRECTION

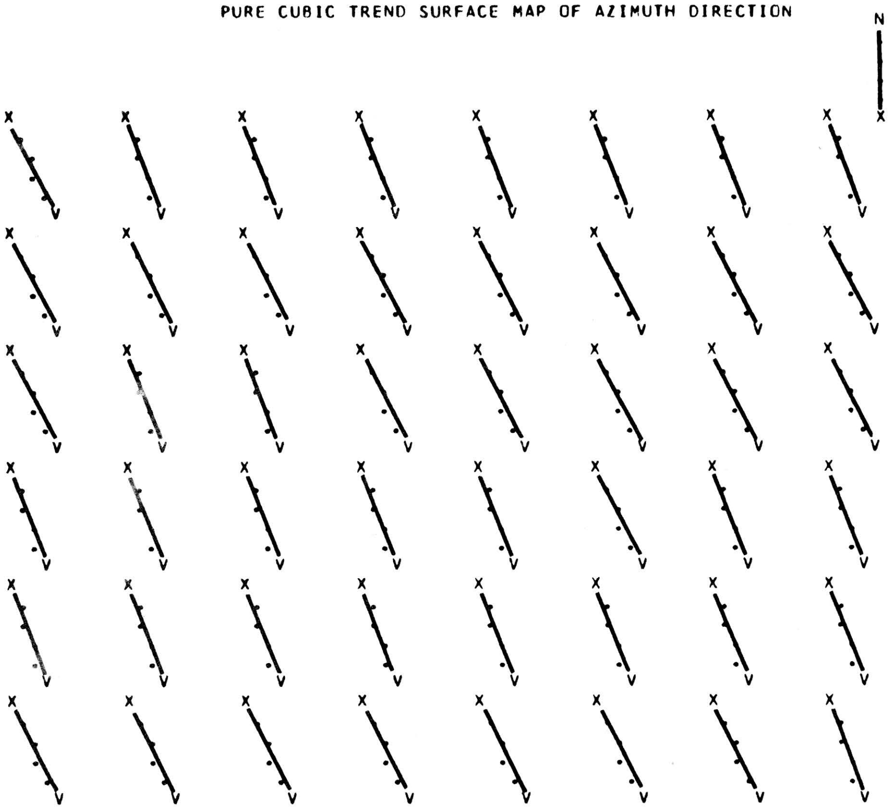
5A



ORIENTATION OF DRUMLINS IN WESTERN NEW YORK STATE

PURE CUBIC TREND SURFACE MAP OF AZIMUTH DIRECTION

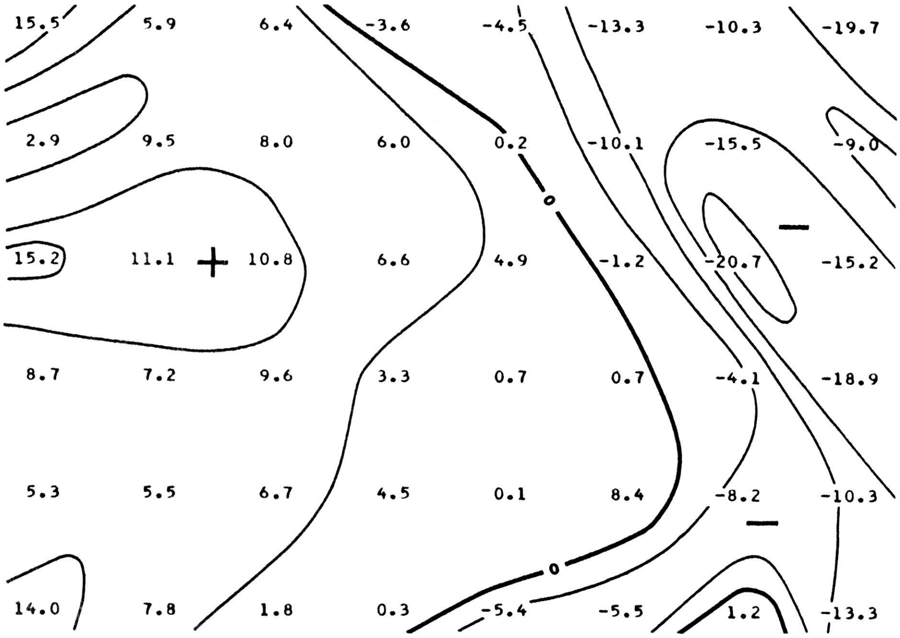
5B



ORIENTATION OF DRUMLINS IN WESTERN NEW YORK STATE

PURE CUBIC RESIDUALS MAP OF AZIMUTH DIRECTION

5C



SUM OF SQUARES FOR MAP = 4320.36

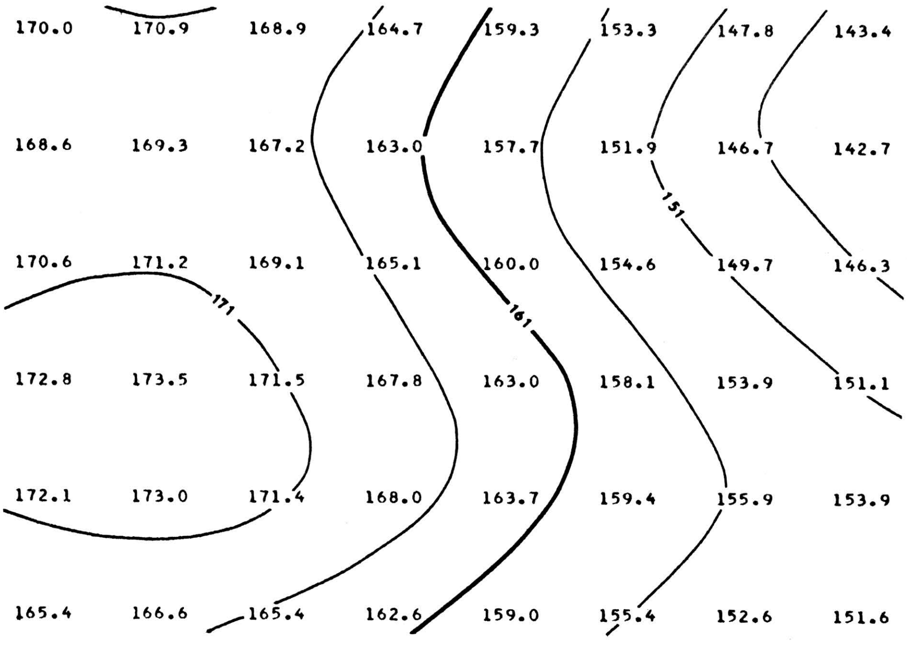
PERCENTAGE REDUCTION IN TOTAL SUM OF SQUARES = 4.918

Figure 6.-Linear, quadratic, and cubic trend surface map, vector trend map and residuals from linear, quadratic, and cubic for drumlin orientation.

ORIENTATION OF DRUMLINS IN WESTERN NEW YORK STATE

LINEAR, QUADRATIC AND CUBIC TREND SURFACE MAP OF AZIMUTH DIRECTION

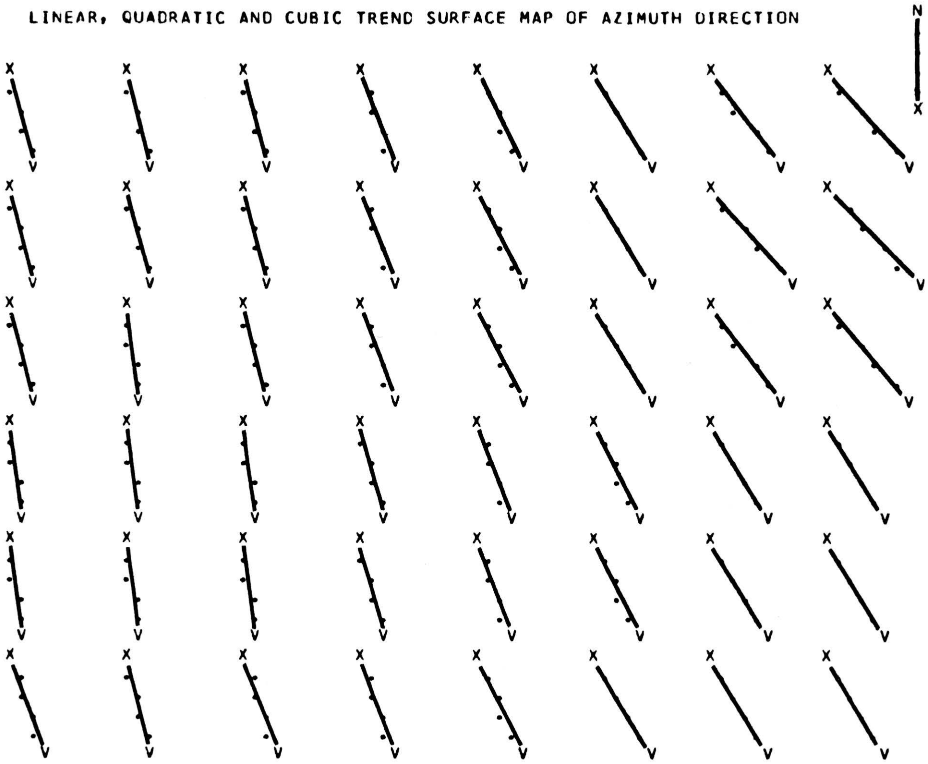
6A



ORIENTATION OF DRUMLINS IN WESTERN NEW YORK STATE

LINEAR, QUADRATIC AND CUBIC TREND SURFACE MAP OF AZIMUTH DIRECTION

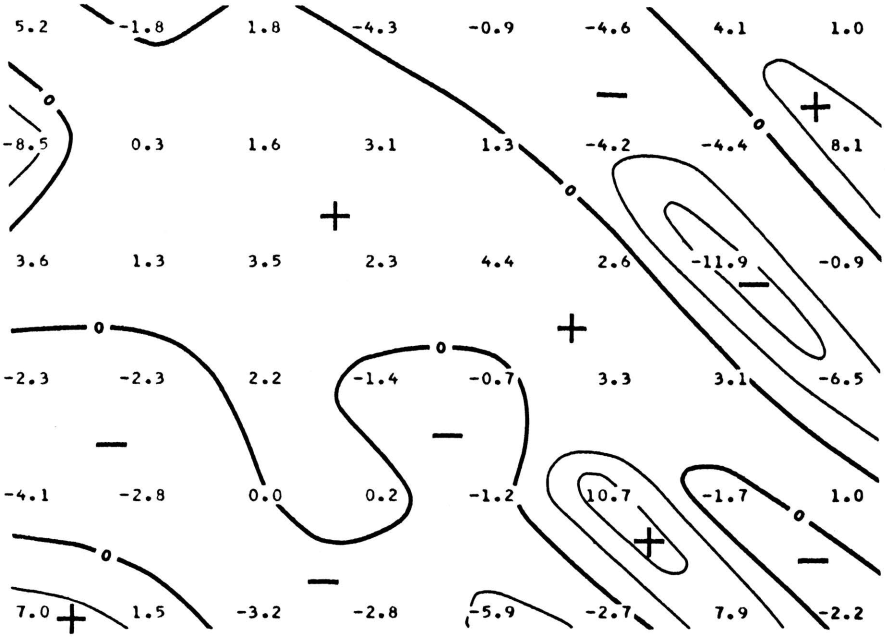
6B



ORIENTATION OF DRUMLINS IN WESTERN NEW YORK STATE

LINEAR, QUADRATIC AND CUBIC RESIDUALS MAP OF AZIMUTH DIRECTION

6C



SUM OF SQUARES FOR MAP = 884.38

PERCENTAGE REDUCTION IN TOTAL SUM OF SQUARES = 80.537

Table 6.-Listing of input cards used in example 3, cross-bedded sandstone in Pocono Formation (Pelletier, 1958).

```
  0   0   6   3   1   270.0 2
(F8.1)
  3                  343.0            301.0             303.0
  292.0              290.0              6.0             259.0
     .0              300.0            311.0             296.0
  323.0              277.0            201.0             224.0
  6                  267.0            303.0             3
  289.0              5                4                 248.0
  245.0              320.0            320.0             329.0
  266.0              308.0            330.0             215.0
  248.0              329.0            270.0             1
  269.0              305.0            322.0             317.0
  202.0               18.0            6                 2
  5                  5                323.0              16.0
  294.0              203.0            357.0             205.0
  218.0              258.0            246.0             4
  308.0              254.0            323.0             261.0
  191.0              232.0            210.0             293.0
  258.0              294.0            245.0             273.0
  5                  4                3                 351.0
  324.0              330.0            326.0             1
  316.0              246.0            298.0             345.0
  186.0              245.0            304.0
  254.0              270.0            6
  158.0              6                350.0
  5                  297.0            298.0
CROSS BEDDING - POCONO FM., PELLETIER (1952), P 1047
```

Table 7.-Coefficients array, Z-squared array, and percentage of total sum of squares array for Pocono Formation in example 3.

```
AZIMUTH ARRAYS

COEFFICIENTS ARRAY
 290.33019  11.64035   7.13742
   2.56305   3.03923   0.53284
   0.66410  -2.31410   0.58008
   0.33561   3.27245  -0.88107
  -0.06009  -3.99322   0.13387
   1.27850  -0.38499  -0.04299

Z-SQUARE ARRAY
        0.    1625.97   1833.94
   1379.53    1293.17    119.25
    111.14     899.65    169.59
     60.82    3855.21    838.38
      0.30     892.96      3.01
   1235.73      74.70      2.79

PERCENTAGE OF TOTAL CORRECTED SUM OF SQUARES
        0.      11.29     12.74
      9.58       8.98      0.83
      0.77       6.25      1.18
      0.42      26.78      5.82
      0.00       6.20      0.02
      8.58       0.52      0.02
```

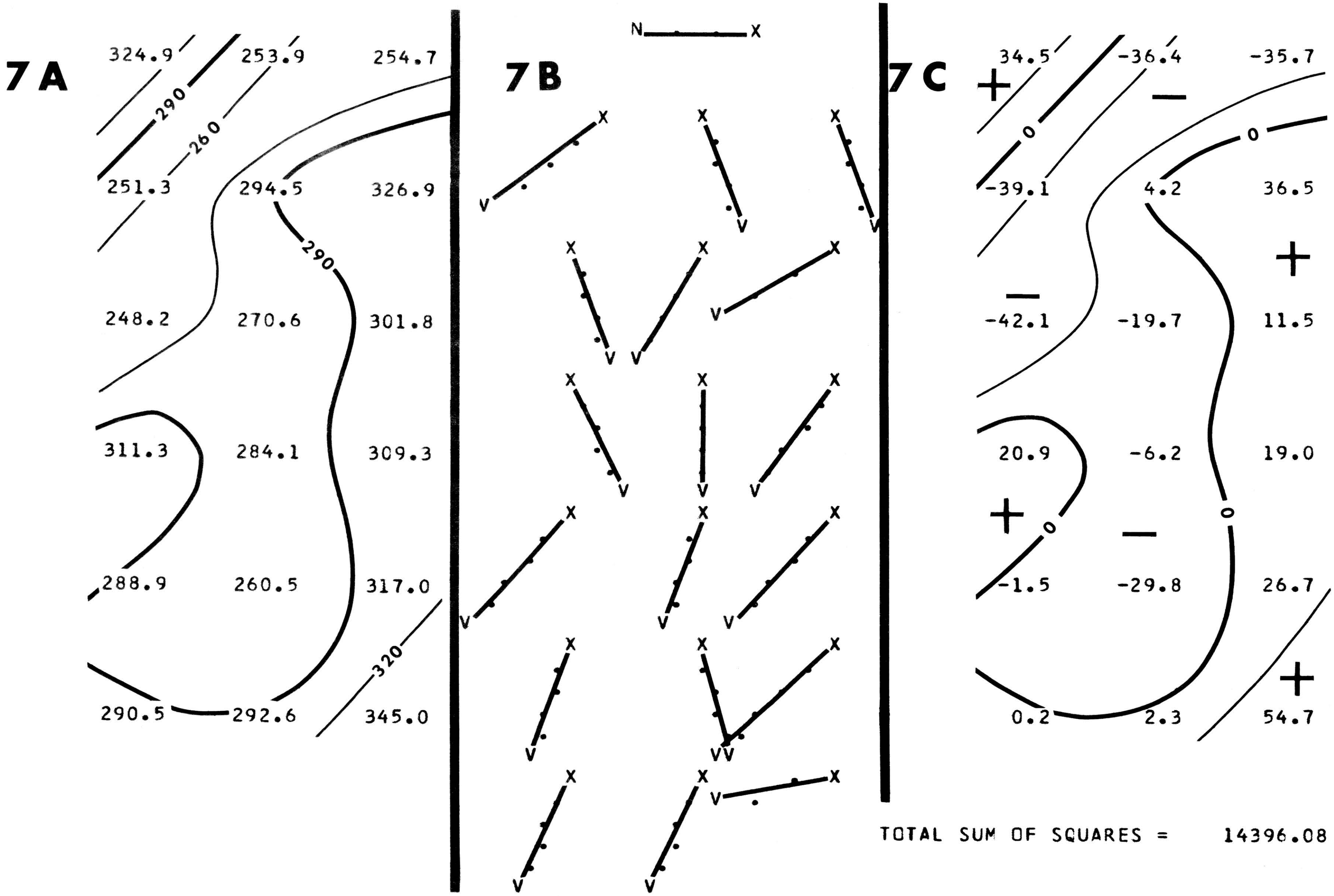


Figure 7.-A, Map of vector mean azimuth directions in each cell by rows and columns; B, Map of vector mean azimuth directions in each cell by rows and columns; C, Residuals - observed azimuth directions minus mean azimuth (Pelletier, 1952).

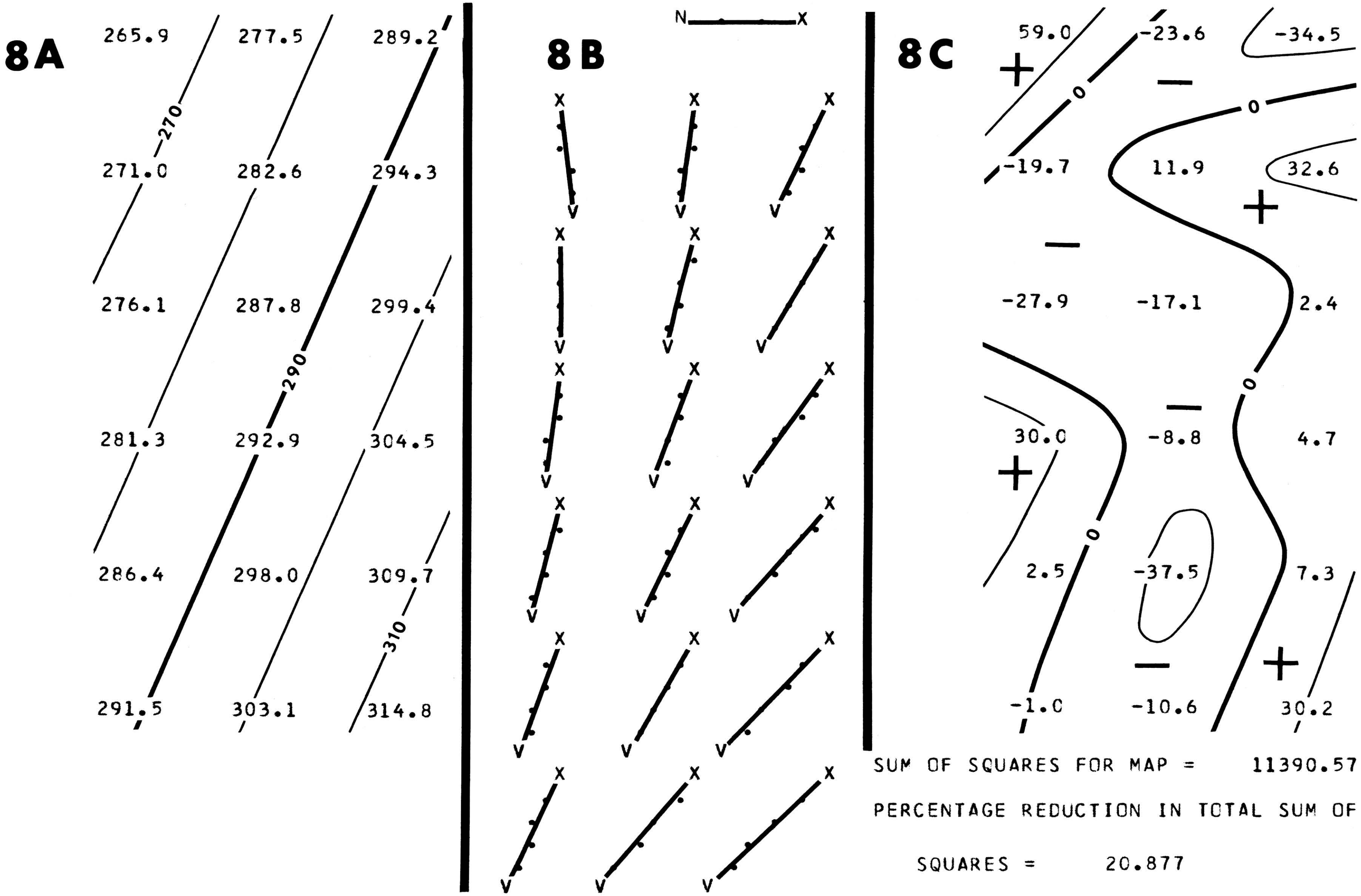
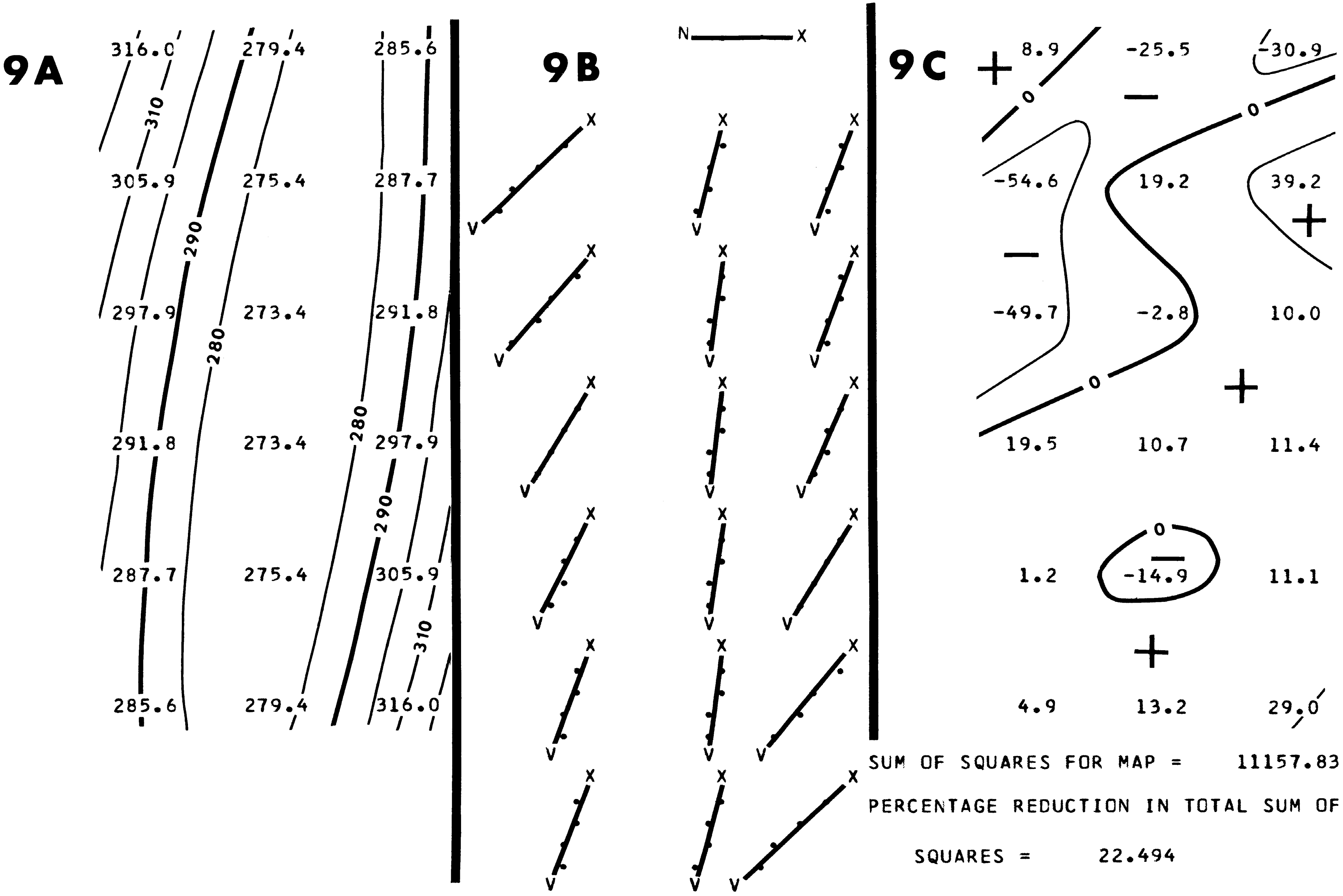


Figure 8.-A, Linear trend-surface map of azimuth direction; B, Linear trend-surface map of azimuth direction; C, Linear residuals map of azimuth direction (Pelletier, 1952).

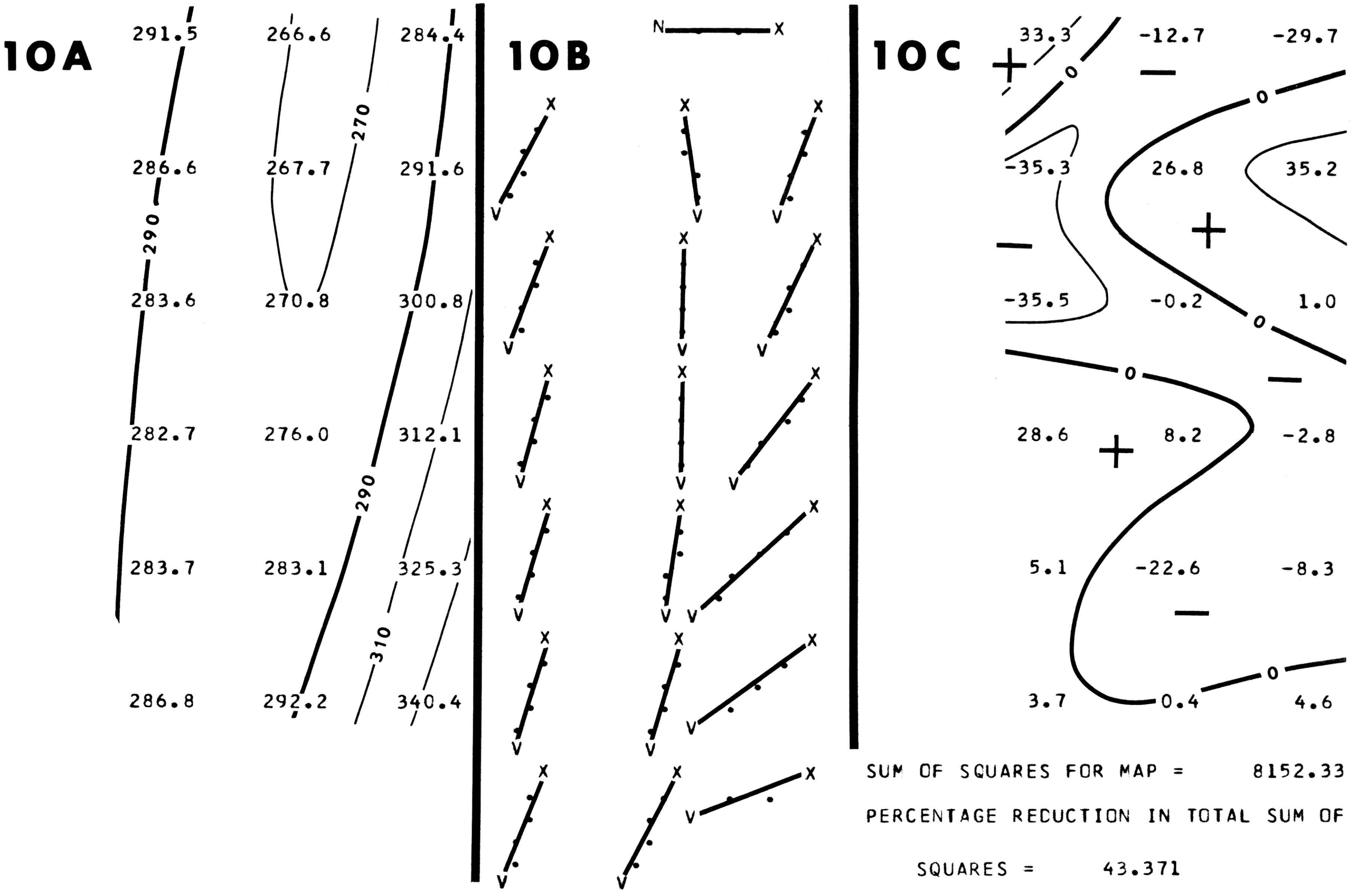
Figure 9.-A, Pure quadratic trend-surface map of azimuth direction; B, Pure quadratic trend-surface map of azimuth direction; C, Pure quadratic residuals map of azimuth direction (Pelletier, 1952).



SUM OF SQUARES FOR MAP = 11157.83

PERCENTAGE REDUCTION IN TOTAL SUM OF SQUARES = 22.494

Figure 10.-A, Linear plus quadratic trend-surface map of azimuth direction; B, Linear plus quadratic trend-surface map of azimuth direction; C, Linear plus quadratic residuals map of azimuth direction (Pelletier, 1952).



KANSAS GEOLOGICAL SURVEY COMPUTER PROGRAM
THE UNIVERSITY OF KANSAS, LAWRENCE

PROGRAM ABSTRACT

Title (If subroutine state in title):

FORTRAN IV program for vector trend analysis of directional data

Computer: IBM 7090/7094

Date: February 1, 1967

Programming language: FORTRAN IV

Author, organization: W. T. Fox, Department of Geology, Williams College
Williamstown, Massachusetts

Direct inquiries to: Author, or

Name: D. F. Merriam

Address: Kansas Geological Survey, University of
Kansas, Lawrence

Purpose/description: Analyze regional trends in directional data

Mathematical method: Linear, quadratic, cubic, quartic, and quintic orthogonal polynomial response surfaces computed and plotted as isoazimuth and vector trend maps.

Restrictions, range: Limited to gridded data.

Storage requirements:

Equipment specifications: Memory 20K 40K 60K K

Automatic divide: Yes No Indirect addressing Yes No

Other special features required

Additional remarks (include at author's discretion: fixed/float, relocatability; optional: running time, approximate number of times run successfully, programming hours) Program readily adaptable to other types of problems.

COMPUTER CONTRIBUTIONS

Kansas Geological Survey
University of Kansas
Lawrence, Kansas

Computer Contribution

1.	Mathematical simulation of marine sedimentation with IBM 7090/7094 computers, by J.W. Harbaugh, 1966.	$1.00
2.	A generalized two-dimensional regression procedure, by J.R. Dempsey, 1966	$0.50
3.	FORTRAN IV and MAP program for computation and plotting of trend surfaces for degrees 1 through 6, by Mont O'Leary, R.H. Lippert, and O.T. Spitz, 1966	$0.75
4.	FORTRAN II program for multivariate discriminant analysis using an IBM 1620 computer, by J.C. Davis and R.J. Sampson, 1966	$0.50
5.	FORTRAN IV program using double Fourier series for surface fitting of irregularly spaced data, by W.R. James, 1966	$0.75
6.	FORTRAN IV program for estimation of cladistic relationships using the IBM 7040, by R.L. Bartcher, 1966	$1.00
7.	Computer applications in the earth sciences: Colloquium on classification procedures, edited by D.F. Merriam, 1966	$1.00
8.	Prediction of the performance of a solution gas drive reservoir by Muskat's Equation, by Apolonio Baca, 1967	$1.00
9.	FORTRAN IV program for mathematical simulation of marine sedimentation with IBM 7040 or 7094 computers, by J.W. Harbaugh and W.J. Wahlstedt, 1967	$1.00
10.	Three-dimensional response surface program in FORTRAN II for the IBM 1620 computer, by R.J. Sampson and J.C. Davis, 1967	$0.75
11.	FORTRAN IV program for vector trend analyses of directional data, by W.T. Fox, 1967	$1.00

Reprints (available upon request)

Finding the ideal cyclothem, by W.C. Pearn (reprinted from Symposium on cyclic sedimentation, D.F. Merriam, editor, *Kansas Geological Survey Bulletin* 169, v. 2, 1964)

Fourier series characterization of cyclic sediments for stratigraphic correlation, by F.W. Preston and J.H. Henderson (reprinted from Symposium on cyclic sedimentation, D.F. Merriam, editor, *Kansas Geological Survey Bulletin* 169, v. 2, 1964)

Geology and the computer, by D.F. Merriam (reprinted from *New Scientist*, v. 26, no. 444, 1965)

Quantitative comparison of contour maps, by D.F. Merriam and P.H.A. Sneath (reprinted from *Journal of Geophysical Research*, v. 71, no. 4, 1966)

Trend-surface analysis of stratigraphic thickness data from some Namurian rocks east of Sterling, Scotland, by W.A. Read and D.F. Merriam (reprinted from *Scottish Journal of Geology*, v. 2, pt. 1, 1966)

Geologic model studies using trend-surface analysis, by D.F. Merriam and R.H. Lippert (reprinted from *Journal of Geology*, v. 74, no. 5, 1966)

Geologic use of the computer, by D.F. Merriam (reprinted from *Wyoming Geological Association*, 20th Field Conf., 1966)

Computer aids exploration geologists, by D.F. Merriam (reprinted from the *Oil and Gas Journal*, v. 65, no. 4, 1967)

Comparison of cyclic rock sequences using cross-association, by D.F. Merriam and P.H.A. Sneath (reprinted from Essays in Paleontology and Stratigraphy: R.C. Moore commemorative volume, edited by C. Teichert and E. Yochelson, *Dept. Geology, Univ. Kansas*, 1967)