Minimum Saturated Thickness Calculator

Method Overview and Spreadsheet Description

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1 Introduction

The saturated thickness of an aquifer is one factor that can limit well yields. When a well is pumped, water levels in the vicinity of the well decline and a local cone of depression forms. As water levels decline in an unconfined aquifer, the thickness of the aquifer decreases. The thickness may decrease to a point at which the aquifer can no longer yield water to the well at the desired rate of pumping.

Hecox et al. (2002) outlined a simple approach for calculating the minimum saturated thickness required to sustain a particular pumping rate and duration. The objective of this report is to further describe the Hecox et al. (2002) method for calculation of the minimum saturated thickness and to present a spreadsheet tool for its implementation. This spreadsheet tool expands the Hecox et al. (2002) approach to include the Jacob correction (Sections 2.2 and 2.5) to account for the decrease in aquifer thickness due to pumping in an unconfined aquifer. The spreadsheet was developed to be broadly applicable, allowing user-specified aquifer and well characteristics.

2 Method

The minimum saturated thickness is calculated using the Cooper-Jacob (1946) approximation of the equation derived by Theis (1935) for drawdown produced by pumping at the well of interest (henceforth, target well) and a polynomial approximation (Abramowitz and Stegun, 1972) of the Theis equation for the drawdown at the target well produced by pumping at four nearby wells. The summation of these two is then corrected for changes in saturated thickness using an approach developed by Jacob (Brown et al., 1963). The drawdown calculation is completed by adding the additional drawdown produced by well inefficiencies in the target well to the corrected drawdown. To calculate minimum saturated thickness, the total drawdown is then compared against the specified aquifer thickness. If the total drawdown is greater than the specified aquifer thickness plus the user defined increment, then this increment is added to the aquifer thickness and the calculations are repeated. If the total drawdown is less than the specified aquifer thickness, then the specified aquifer thickness is output as the minimum saturated thickness. The following subsections further outline the steps involved in calculating the minimum saturated thickness.

2.1 Drawdown due to pumping in the target well ($s_{aquifer}$)

The Cooper-Jacob approximation, which is used to calculate the drawdown due to pumping in the target well, is

$$s_{aquifer} = \frac{Q}{4\pi T}\left[-0.5772 - \ln\left(\frac{r^2 S}{4Tt}\right)\right]$$

where $s_{aquifer}$ is the calculated drawdown in the target well (ft), Q is the pumping rate (gpm), T is the aquifer transmissivity ($ft^2 day^{-1}$), S is the specific yield (-), r is the effective radius of the well (ft), and t is the duration of pumping (days). Transmissivity is defined as the average hydraulic conductivity of the aquifer (K [$ft day^{-1}$]) multiplied by the saturated thickness (b [ft]). This equation is used for calculating the drawdown produced by pumping at the target well in the absence of well losses.

2.2 Jacob correction for drawdown due to pumping in the target well ($s_{targ\text{-}jacob}$)

The previously calculated drawdown in the target well produced by pumping in that well is based on the assumption that the aquifer thickness does not change with drawdown (confined aquifer assumption). In unconfined aquifers, such as the High Plains aquifer in Kansas, the thickness of the aquifer decreases with pumping. The drawdown in the target well must therefore be corrected to account for this decrease in aquifer thickness. This is done in the spreadsheet using the Jacob correction (Brown et al., 1963). The corrected drawdown ($s_{targ\text{-}jacob}$) is a function of the previously calculated drawdown and the initial saturated thickness:

$$s_{targ-jacob} - \left(\frac{s_{targ-jacob}^2}{2b}\right) = \left(s_{aquifer}\right)$$

This equation is solved for $s_{targ\text{-}jacob}$ in the spreadsheet tool.

2.3 Drawdown due to well losses in the target well (s_{eff})

Additional drawdown will occur as a result of energy losses produced by well inefficiencies. Even new, properly designed wells will only be 70–80% efficient (Driscoll, 1986). The additional drawdown in the target well (s_{eff}) produced by well inefficiencies is

$$s_{eff} = \left(\frac{100}{Eff} - 1\right) s_{targ-jacob}$$

where Eff is the well efficiency as a percentage (ratio of drawdown in the aquifer to drawdown in the well multiplied by 100). Well inefficiencies only arise from pumping in the target well.

2.4 Drawdown due to neighboring pumping wells ($s_{neighbor}$)

The drawdown in the target well produced by pumping at neighboring wells can also be calculated and is incorporated as an option within the spreadsheet. A scenario of four additional wells pumping at the same rate and located equidistant from the target well is the only configuration currently available (fig. 1). In this multi-well scenario, the Abramowitz and Stegun (1972) approximation (truncation) of the infinite series representation of the Theis (1935) equation is used to account for the additional drawdown produced by pumping at these four neighboring wells:

$$s_{neighbor} = \frac{Q_n}{4\pi T}\left[-0.5772 - \ln u + u - 0.25u^2 + 0.05556u^3 - 0.01042u^4 + 0.001667u^5\right]$$

where $u = \left(\frac{{r_n}^2 S}{4Tt_n}\right)$, Q_n is the pumping rate of the neighboring wells (gpm), r_n is the distance from the target well to the neighboring wells (ft), and t_n is the duration of pumping at the neighboring wells (days). The truncated infinite series representation is required because the Cooper-Jacob approximation may not always be appropriate for calculating drawdown produced by pumping at the neighboring wells.

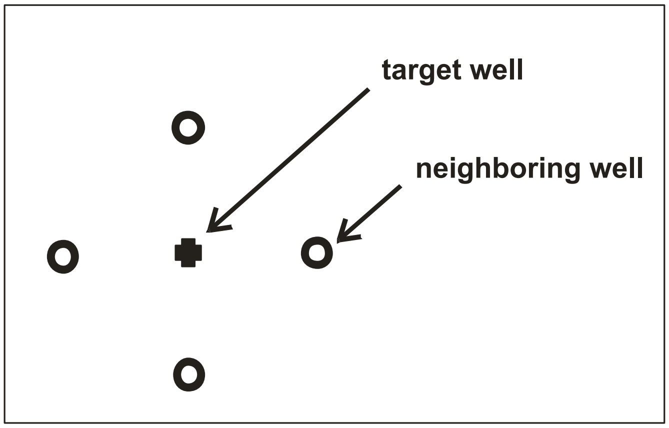

Figure 1. Arrangement of neighboring wells around target well.

2.5 Jacob correction ($s_{all\text{-}jacob}$)

The previously calculated components of drawdown in the target well produced by pumping in the target and neighboring wells are based on the assumption that the aquifer thickness does not change with drawdown (confined aquifer assumption). In unconfined aquifers, such as the High Plains aquifer in Kansas, the thickness of the aquifer decreases with pumping. The drawdown in the target well must therefore be corrected to account for this decrease in aquifer thickness by pumping in the target well and the neighboring wells. As previously described in section 2.2, this is done in the spreadsheet using the Jacob correction (Brown et al., 1963). The corrected drawdown ($s_{jacob\text{-}all}$) is a function of the previously calculated drawdowns and the initial saturated thickness:

$$s_{all-jacob} - \left(\frac{s_{all-jacob}^2}{2b}\right) = \left(s_{aquifer} + s_{neighbor}\right)$$

This equation is solved for $s_{all\text{-}jacob}$ in the spreadsheet tool.

2.6 Total drawdown (s_{total})

The total drawdown in the target well is the sum of the corrected drawdown ($s_{all\text{-}jacob}$) and the drawdown produced by well inefficiencies (s_{eff}):

$$s_{total} = s_{all-jacob} + s_{eff}$$

2.7 Determining the minimum saturated thickness

The spreadsheet tool determines the minimum saturated thickness through an iterative process. The tool iterates through a user-specified range of initial saturated thickness values (b), from a lower-bound (b_{min}) to an upper-bound (b_{max}), at a user-specified increment (b_{inc}). The spreadsheet calculates total drawdown for each initial saturated thickness and subtracts the total drawdown from the initial aquifer thickness:

$$b - (s_{total} + b_{inc}) = b_{res}$$

If the residual aquifer thickness (b_{res}) is less than zero (s_{total} + b_{inc} is greater than b), the spreadsheet increases the initial saturated thickness:

$$b = b + b_{inc}$$

and repeats the process. The calculation is finished when either b_{res} is equal to or greater than zero (s_{total} + b_{inc} is less than or equal to b) or when b is greater than b_{max}. If b_{res} is greater than or equal to zero, the current value of b is reported as the minimum saturated thickness. If b is greater than b_{max}, then "Max Thickness Not Sufficient" will be reported, meaning that the upper limit of the range of aquifer thickness specified by the user (*maximum thickness to test*, section 3.2) was not large enough to meet the specified pumping requirements at the target well. Note that the initial saturated thickness (b) is compared against the sum of the total drawdown and an additional increment (s_{total} + b_{inc}). This is to ensure that the minimum saturated thickness reported allows for some submerged screen length. If total drawdown (s_{total}) is equal to the initial saturated thickness (b), there would be no submerged screen length and thus no way for water to continue entering the well.

3 Spreadsheet Tool Description

The spreadsheet tool consists of two separate sheets within one Excel file (*Min_Sat_Thickness.xlsm*). The first sheet (*MST Calculation*) is where the user enters parameters, the calculations are initiated, and the results are presented. The second sheet (*Parameter Explanation*) provides a definition of each parameter the user is required to enter and other pertinent information for determining reasonable values for each parameter. The remainder of this document describes the parameters required for the MST Calculation sheet.

The spreadsheet enables numerous target wells to be assessed simultaneously. Each row of data represents an individual target well. The rows are split into five sections: well information, aquifer information, pumping information for the target well, pumping information for the neighboring wells, and results.

3.1 Well Information

A unique *well ID* can be entered for each row. This allows users to maintain links to other databases.

The *effective radius of the well* (ft) is the distance from the center of the well to the outer edge of the gravel pack. A typical value is 1 ft.

The *well efficiency* (%) parameter accounts for the additional energy losses produced by well design and/or construction factors. Some of these factors are further described in the Parameter Explanation sheet of the spreadsheet tool; values typically range from 50 to 80 percent.

3.2 Aquifer Information

Hydraulic conductivity (ft day^{-1}) characterizes the ease with which groundwater flows in an aquifer. The input parameter should be the average hydraulic conductivity of that portion of the aquifer. Example hydraulic conductivity ranges are provided on the Parameter Explanation sheet of the spreadsheet tool.

Specific Yield (-) is the drainable porosity of the aquifer. It is the ratio of the volume of water that drains from a certain portion of the aquifer to the total volume of that portion of the aquifer. Example specific yield values are provided on the Parameter Explanation sheet of the spreadsheet tool.

The *minimum thickness to test* (ft) is the smallest aquifer thickness that will be assessed in the spreadsheet, b_{min} in section 2.6.

The *maximum thickness to test* (ft) is the largest aquifer thickness that will be assessed in the spreadsheet, b_{max} in section 2.6.

The *thickness increment* (ft) is the interval at which the spreadsheet will calculate total drawdown between the minimum and maximum thicknesses, b_{inc} in section 2.6. For example: with b_{min} equal to 5 ft, b_{max} equal to 15 ft, and b_{inc} equal to 5 ft, the spreadsheet will calculate total drawdown using aquifer thicknesses of 5, 10, and 15 ft.

3.3 Pumping Information for the Target Well

The *pumping rate* (gpm) is the desired rate at the target well. This rate is held constant throughout the calculation.

The *duration of pumping* (days) is the number of days the target well will run at the given pumping rate.

3.4 Pumping Information for the Neighboring Wells

Distance from target well (ft) is the distance to the four neighboring wells from the target well. This distance is the same for all four neighboring wells.

The *pumping rate* (gpm) is the desired pumping rate at the neighboring wells. This rate is the same for all four neighboring wells and is held constant throughout the calculation. **If there are no neighboring wells, this rate should be zero**.

The *duration of pumping* (days) is the number of days the neighboring wells will run at the given pumping rate. This duration is the same for all four neighboring wells.

3.5 Results

Once parameters have been entered into all columns for each desired target well, the spreadsheet tool can be run using **Ctrl+Shift+t**. The run time depends on the number of target wells and the number of iterations needed to reach a solution. Calculations typically are completed within seconds.

Once the calculations have finished, the *minimum saturated thickness (ft)* will be displayed in the column with that name. This will be the smallest thickness required, given the increments entered, to meet the user-specified pumping conditions. If the maximum thickness used in the calculations is insufficient to yield the desired pumping rate, “Max Thickness Not Sufficient” will be displayed.

In addition to the minimum saturated thickness, the spreadsheet tool will also output the *total drawdown (ft)* calculated using an initial saturated thickness equal to the minimum saturated thickness displayed in the previous column.

Five other calculation products are also output: *drawdown due to pumping in target well (ft) ($s_{aquifer}$), drawdown due to pumping in target well with Jacob correction (ft) ($s_{targ\text{-}jacob}$), additional drawdown due to well inefficiencies (ft) (s_{eff}), drawdown due to pumping in neighboring wells (ft) ($s_{neighbor}$),* and*, drawdown due to pumping in target and neighboring wells with Jacob correction (ft) ($s_{all\text{-}jacob}$).* All parameter names in parentheses are consistent with those presented in section 2 of this report. These calculation products are given to provide the user with a better understanding of each component of the calculated drawdown.

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