CAPTURE ZONES FOR SIMPLE AQUIFERS

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ABSTRACT

The protection and cleanup of aquifers is a matter of high priority for all states and the federal government. One concept that is receiving increased attention is that of wellhead protection. Capture zones showing the area influenced by a well within a certain time are useful for both aquifer protection and cleanup. If hydrodynamic dispersion is neglected, a deterministic curve defines the capture zone. Analytical expressions for the capture zones can be derived for simple aquifers. However, the capture zone equations are transcendental and cannot be explicitly solved for the coordinates of the capture zone boundary. Fortunately, an iterative scheme allows the solution to proceed quickly and efficiently even on a modest personal computer. Three forms of the analytical solution must be used in an iterative scheme to cover the entire region of interest, after the extreme values of the x coordinate are determined by an iterative solution. The resulting solution is a discrete one and usually 100-1000 intervals along the x-axis are necessary for a smooth definition of the capture zone. The presented program is written in FORTRAN and has been used in a variety of computing environments. No graphics capability is included with the program; it is assumed the user has access to a commercial package. The superposition of capture zones for multiple wells is expected to be satisfactory if the spacing is not too close. Because this program deals with simple aquifers, the results will rarely be the final word in a real application.

However, the program is useful as a first phase in developing wellhead protection or aquifer cleanup schemes.

INTRODUCTION

The protection and cleanup of aquifers is a matter of high priority for all states and the federal government, as evidenced by the large number of laws and regulations that have been established in recent years. One concept that is receiving increased attention is wellhead protection, where certain potentially polluting activities are banned or regulated within an area that would affect a well within a certain time period. In terms of aquifer cleanup, one would like to know what area of an aquifer will be influenced by a discharge well within a certain time period. These areas are commonly referred to in the literature as capture zones. If hydrodynamic dispersion is neglected, a deterministic curve (sharp front) can be used to define the capture zone. Because real world aquifers are very complex, exhibiting heterogeneity, anisotropy, and other complicating factors, the calculation of realistic capture zones is difficult. Possible techniques range from simple analytical methods to complex numerical procedures.

The calculation of sharp front movement for wells in infinite aquifers dates at least to Muskat (1937). More recently Bear and Jacobs (1965) have investigated the movement of water bodies injected into isotropic homogeneous aquifers with uniform regional flow by analytical methods. Most ground water texts present a steady-state analytical solution for the ground water divide in an isotropic homogeneous aquifer with one pumping well located in a uniform regional flow field (see for example Todd, 1980, pp. 121-123); this corresponds to an infinite-time capture zone. Javandel et al. (1984, pp. 175-204) present semi-analytical methods for calculating pathlines and time related capture zones for multiple wells in simple aquifers (isotropic, homogeneous, uniform thickness, uniform regional flow, and steady state). However, their computer program is rather complex. EPA (1990) has recently sponsored development of a program to calculate wellhead protection areas (WHPA); but again the program is fairly complex. Javandel and Tsang (1986) propose infinite-time capture zone curves as a tool for aquifer cleanup; again, they use analytical methods for simple aquifers. A few authors have utilized numerical methods to calculate time-related capture zones in the presence of aquifer heterogeneity. Kinzelback (1986, pp. 227-230) presents the formalism for considering a heterogeneous velocity distribution. Shafer (1987) presents the formalism and gives examples of capture zones in heterogeneous aquifers.

The purpose of the present paper is to present a program for calculating time-related capture zones in simple aquifers. The program is short and efficient and adaptable to a range of computing environments from personal computers to mainframes. Because the program assumes simple aquifer conditions (isotropic, homogeneous, uniform thickness, uniform regional flow, and steady state), it should be used with care in a real-world situation. However, the program should be a useful initial planning tool for aquifer protection or cleanup.

BASIC EQUATIONS

The basic equations that are used to describe the capture-zone curves are taken from Bear and Jacobs (1965). As mentioned in the introduction, this formulation assumes an aquifer with a constant regional hydraulic conductivity (K). A regional flow direction and magnitude (q_o) also is assumed constant and given by the Darcy equation.

$$q_o = -K \frac{\partial h}{\partial s} \tag{1}$$

h is the regional hydraulic head (without the pumping well) and s is the direction of the head gradient perpendicular to lines of constant head. In what follows, it will be assumed the x axis is parallel (or antiparallel) to the regional flow direction. The aquifer is assumed to be of constant thickness (B) and constant effective porosity (n). At this point, a well pumping at a rate Q is superimposed upon the regional system and a new steady-state head configuration is established. The object is to calculate the area of the aquifer that will contribute water to the well during a specified time period; or alternatively, to calculate the area affected by injection for a given time interval, after the well is in steady state with the regional system. The curves surrounding these affected areas are loosely called capture curves for a given time period. It is convenient to define three dimensionless parameters:

$$\bar{x} = \frac{2\pi q_o B}{Q} x \tag{2}$$

$$\bar{y} = \frac{2\pi q_o B}{Q} y \tag{3}$$

$$\bar{t} = \frac{2\pi q_o^2 B}{nQ} t \tag{4}$$

q_o, B, Q, and n are the previously defined Darcy velocity, aquifer thickness, pumpage rate and effective porosity, respectively. x, y, and t are the space and time coordinates in the real world; whereas, $\bar{x}$, $\bar{y}$, and $\bar{t}$ are their dimensionless counterparts. Using these dimensionless variables, Bear and Jacobs (1965) show that the capture curves are given by the following equation.

$$\exp(\bar{x} - \bar{t}) = \cos \bar{y} + \frac{\bar{x}}{\bar{y}} \sin \bar{y} \tag{5}$$

Unfortunately, equation 5 is a transcendental equation which cannot be solved explicitly for either $\bar{x}$ or $\bar{y}$.

There are two cases where equation 5 can be simplified somewhat. When $\bar{t} \to \infty$, equation 5 reduces to

$$\bar{x} = -\frac{\bar{y}}{\tan \bar{y}} , \tag{6}$$

which is the familiar form for the ground water divide (Todd, 1980, pp. 121-123). From equation 6, it is seen that as $\bar{x} \to \infty$ the limiting value of $\bar{y}$ is $\pm\pi$. The stagnation point occurs at $\bar{y} = 0$ and $\bar{x} = -1$; this result can be obtained from equation 6 by taking the limit as $\bar{y} \to 0$ (See figure 1.). Another useful simplification of equation 5 results when $\bar{y} = 0$; this corresponds to the two points where the capture curve crosses the $\bar{x}$ axis. Taking the limit as $\bar{y} \to 0$ of equation 5 gives the extreme values $\bar{x}_e$,

$$\exp(\bar{x}_e - \bar{t}) = 1 + \bar{x}_e. \tag{7}$$

(See figure 1 for examples of $\bar{x}_e$.) A slight rearrangement of equation 7 gives the form

$$\bar{t} = \bar{x}_e - \ln(1 + \bar{x}_e). \tag{8}$$

Equations 5 and 8 will form the basis for calculating capture curves at a given normalized time $\bar{t}$. Unfortunately, both are transcendental equations, so iterative techniques will be used to obtain their solution. The resulting curves will enclose the area of the aquifer containing water either injected or discharged by the well up to time $\bar{t}$. The curves represent sharp fronts (deterministic curves) because hydrodynamic dispersion has been neglected. Those who are not interested in the mathematical details of solution may wish to skip to the section describing the computer program.

ITERATIVE SOLUTION

The capture curves given by equation 5 are symmetric about the $\bar{x}$ axis; and the x axis is assumed to be parallel to the regional hydraulic gradient with its origin at the well. The requirement that the origin be at the well is relaxed in the computer program. From the discussion in the previous section, the limits on the coordinates are $-1 \leq \bar{x} \leq \infty$ and $-\pi \leq \bar{y} \leq \pi$. The solution that we shall obtain is a numerical one at discrete values of $\bar{x}$. The approach that we shall take involves solving equation 8 for the extreme values of $\bar{x}$ at a certain $\bar{t}$. The region bounded by these two extremes will be discretized to give a certain number of discrete values of $\bar{x}$ (usually between 100 and 1000). Let $\bar{x}_i$ represent one of these values. At that point with $\bar{t}$ and $\bar{x}$ known, equation 5 will be solved to obtain $\bar{y}$. Unfortunately, no single form of equation 5 seems to work well for the full range of coordinates. (Three forms will be used later.)

The extreme values of $\bar{x}$ are found by solving equation 8. Rearranging equation 8 slightly allows an iterative solution scheme to be developed (one-point method, Atkinson, 1989, pp. 76-

83).

$$\bar{x}_e^{(m+1)} = \bar{t} + \ln[1 + \bar{x}_e^{(m)}] \tag{9}$$

The m in equation 9 is an iteration index. An initial guess for $\bar{x}_e$ must be known, but $\bar{x}_e = 0$ always seems to work well. Iteration continues on equation 9 until convergence occurs. If the initial guess for $\bar{x}_e$ is zero and $\bar{t}$ is positive, it is clear that equation 9 will converge on a positive value. If $\bar{t}$ is small then $\bar{x}_e$ also will be small and the logarithmic term of equation 8 can be written as a series expansion to yield

$$\bar{t} = \frac{\bar{x}_e^2}{2} - \frac{\bar{x}_e^3}{3} + \frac{\bar{x}_e^4}{4} - \cdots \tag{10}$$

Solving for the lowest power of $\bar{x}_e$ gives

$$\bar{x}_e^{(m+1)} = \sqrt{2}\left[\bar{t} + \frac{\bar{x}_e^{(m)3}}{3} - \frac{\bar{x}_e^{(m)4}}{4} + \cdots\right]^{1/2} \tag{11}$$

Iterating equation 11 works well for small values of $\bar{t}$ and $\bar{x}_e$.

Equations 9 and 11 work well for the positive value of the

$\bar{x}$ extremes; however, a slightly different version is needed to find the negative extreme value. Rearranging equation 7 slightly gives the following iterative solution.

$$\bar{x}_e^{(m+1)} = \exp(\bar{x}_e^{(m)} - \bar{t}) - 1 \tag{12}$$

Clearly, if $\bar{t} \to \infty$ equation 12 gives an extreme value of -1. If the initial guess for $\bar{x}_e$ is zero and $\bar{t}$ is positive the result for the first iteration will be negative. Experience has shown that equation 12 converges rapidly on the negative value of $\bar{x}_e$.

Now that the extreme values of $\bar{x}$ are known for a particular $\bar{t}$, we can pick a discrete value $\bar{x}_i$ located between these two extremes. The only unknown in equation 5 is now $\bar{y}$ and an iterative solution can be set up. The most obvious iterative form is obtained from equation 5 by multilpying by $\bar{y}$ and $\exp(\bar{t} - \bar{x})$ to obtain

$$\bar{y}_i^{(m+1)} = \exp(\bar{t} - \bar{x}_i) \cdot \left[\bar{y}_i^{(m)} \cos \bar{y}_i^{(m)} + \bar{x}_i \sin \bar{y}_i^{(m)} \right] . \tag{13a}$$

However, numerical experiments show that equation 13a does not have as wide a region of convergence as we would like. The convergence properties of equaion 13a can be changed by adding $\bar{y}_i$

to each side of the equation (Atkinson, 1989, pp. 76-83). The resulting equation which we shall use is

$$\bar{y}_i^{(m+1)} = \frac{\bar{y}_i^{(m)}}{2} + \frac{1}{2}\exp(\bar{t} - \bar{x}_i) \cdot \left[\bar{y}_i^{(m)} \cos \bar{y}_i^{(m)} + \bar{x}_i \sin \bar{y}_i^{(m)} \right] \qquad (13b)$$

As long as $|\bar{y}_i| \le \pi/2$ and $\bar{x} \ge 1$ equation 13b works well.

An alternate form of equation 5 can be obtained by solving for cos $\bar{y}$ and then taking the inverse cosine function.

$$\bar{y}_i^{(m+1)} = \cos^{-1}\left[\exp(\bar{x}_i - \bar{t}) - \frac{\bar{x}_i}{\bar{y}_i^{(m)}} \sin \bar{y}_i^{(m)} \right] \qquad (14)$$

Numerical experiments show that this form works well for all values of $\bar{x}$ and $\bar{y}$ as long as $\bar{t} \le 1$. For $\bar{t} \ge 1$ equation 14 can be used only for $\bar{x} \le 1$.

The final form of equation 5 needed to fill in all remaining values of $\bar{x}$, $\bar{y}$, and $\bar{t}$ is given by rearranging and solving for the tangent of $\bar{y}_i$.

$$\tan \bar{y}_i = \left(\frac{\bar{y}_i}{\bar{x}_i}\right) \cdot \left(\frac{\exp(\bar{x}_i - \bar{t})}{\cos \bar{y}_i} - 1\right) \tag{15}$$

Using the trigonometric identity $\tan(-\theta) = \tan(\pi - \theta)$ allows us to rewrite equation 15 in iterative notation.

$$\bar{y}_i^{(m+1)} = \pi - \tan^{-1}\left[\left(\frac{\bar{y}_i^{(m)}}{\bar{x}_i}\right) \cdot \left(1 - \frac{\exp(\bar{x}_i - \bar{t})}{\cos \bar{y}_i^{(m)}}\right)\right] \tag{16}$$

Numerical experiments show that this equation works well for $\bar{t} > 1$ and $\bar{x} > 1$ if $|\bar{y}_i| > \pi/2$. Clearly, equation 16 has a problem at $\bar{y} = \pi/2$ because the cosine function is zero. Therefore, special provision must be made to prevent equation 16 from being used too near the region where $\bar{y} = \pi/2$.

The iterative equations 13, 14, and 16 for $\bar{y}$ require an initial guess for the m = 0 iteration. That question was avoided in the above paragraphs where the equations were developed. However, in practice this presents no problem. Using the extreme values of $\bar{x}$, a discrete set of $\bar{x}_i$'s are calculated by dividing the region between the extremes into an integral number of steps (usually between 100 and 1000). Solution then proceeds sequentially from the negative $\bar{x}$ extreme to the positive $\bar{x}$ extreme. At each of the extreme values of $\bar{x}$ we know that $\bar{y} = 0$. Therefore, as we step through the solution we will always know the value of $\bar{y}$ at the previous $\bar{x}$ value and we can use this as the initial guess for $\bar{y}$ at the current value of $\bar{x}$. If at least 100 steps in $\bar{x}$ are used, the value of $\bar{y}$ does not change dramatically in one step and the above procedure is very efficient. As the solution proceeds, the appropriate equation (13, 14, or 16) is selected depending on the values of $\bar{t}$, $\bar{x}$, and the current value of $\bar{y}$.

COMPUTER PROGRAM

A computer program to calculate capture curves based on the material presented here has been written in FORTRAN and a listing is included in appendix I. We commonly run it on an IBM AT compatible computer; however, it can be easily adapted to a wide variety of computer environments, usually only the OPEN command for the output file will need modification. The program consists of three parts, the main program and two subroutines. The three parts are briefly described in the following paragraphs.

The main program interacts with the user via the screen to read in some necessay information. The user interface is very simple and does not do input error checking or write the input to a file. If a permanent record of the input is desired, the screen print command should be used. The intent is to keep the program as simple as possible so that users may modify it to suit their own needs. All input should be in any consistent set of units (gallons are not allowed for example). An example data input session is shown in appendix II. The first request is for the aquifer thickness (B), followed by requests for the effective porosity (n) and the regional Darcy velocity (q_o). The flow rate of the well (Q) is requested next along with its location coordinates (x,y). The pumping well may be located anywhere in the flow field; however, the x axis must be parallel (or antiparallel) to the regional flow direction. The next request is for the user to supply a value of t. Also, the user must specify the number of intervals between the two $\bar{x}$ extremes (N). If N is specified, there will be N+1 values of $\bar{x}_i$ calculated, including the two extreme values. The x and y values outlining a capture zone for a given t will always be written to a computer file; however, the user may have the values echoed to the screen if one responds with YES at the prompt. The data file to receive the calculated values must be specified next. As currently written, the prompt asks for the path name in the usual IBM format; that can easily be changed. The output file should be given a descriptive name to identify it, since only x and y values are written by the program. After opening the file the program calls the subroutine CAPCUR to calculate the values of the generic coordinates $\bar{x}$ and $\bar{y}$ for a given $\bar{t}$. CAPCUR only calculates the positive values of $\bar{y}$, so the main program fills in the negative values for all the $\bar{x}$'s. Because the capture curves are symmetric about the $\bar{x}$-axis, that does not involve more calculations. The real world coordinates are written to the file after calculation using equations 2 and 3 solved for x and y and after correction for pumping well location. After closing the output file, the main program asks if a new capture zone calculation for a new value of t is desired. If the answer is no, the program terminates execution. Those people who are not interested in the details of the subroutines may skip to the section Results and Application.

The subroutine CAPCUR has five arguments: T, N, X, Y, and ECHO. T, N, and ECHO are user supplied and are passed from the main program. T is the value of $\bar{t}$ and N is the number of intervals to be used in calculating the $\bar{x}_i$. If ECHO is set to YES, the calculated values of $\bar{x}$ and $\bar{y}$ (generic coordinates) will be written to the screen. Currently, the arrays for $\bar{x}$ and $\bar{y}$ are dimensioned to a maximum of 1001, which means that N can not be larger than 1000. That can easily be changed; however, we have found 1000 to be sufficient even for large values of $\bar{t}$ up to about 1600. For larger values of $\bar{t}$, the arrays will need to be dimensioned larger. A good rule is that the dimension of the arrays should be at least .6 * max $\bar{t}$ (* means multiplication). One should not set N less than 100, because we have found that the capture curves may not have an adequate sample to be represented smoothly. CAPCUR calls ENDPTS which calculates the extreme values of $\bar{x}$ from equations 9, 11, and 12. (Of course $\bar{y}$ =0at each of these extreme values of $\bar{x}$.) The interval (DX) between various $\bar{x}_i$ is calculated by dividing the region between the extremes by N. The DO loop ending on statement 150 calculates the current $\bar{x}_i$ by adding DX and then calculates the corresponding $\bar{y}_i$ by using the appropriate iterative equation (13, 14, or 16). The program requires the first $\bar{x}_i$ in the loop (i = 2) to be less than 1.0 so that equation 14 can be used for the first calculation. If this is not the case, the program states that N is too small and stops. The initial guess for $\bar{y}_{i+1}$ is simply the value of $\bar{y}_i$ at the preceding point. The appropriate iterative equation is selected by testing $\bar{t}$, $\bar{x}$, and $\bar{y}$. If the iterative procedure does not converge in 2000 iterations a message will be printed and the program stops. Failure to converge will rarely occur, but could happen if N is chosen too small. Once all $\bar{y}_i$ are calculated for all $\bar{x}_i$, the subroutine returns control to the main program.

The subroutine ENDPTS calculates the extreme values of $\bar{x}$ for a given $\bar{t}$. It is a straight forward application of the iterative equations 9, 11, or 12. An initial guess of $\bar{x} = 0$ is used. Equation 9 or 11 is used for the positive extreme value of $\bar{x}$, depending on the value of $\bar{t}$. For the negative solution, equation 12 is used. These extreme values are then returned to the calling program.

RESULTS AND APPLICATION

The results of using the algorithms discussed here are shown in figure 1 for $\bar{t}$ values of 1, 3, 5, and ∞. The $\bar{t} = \infty$ curve corresponds to the normal ground water divide. Equations 13, 14, and 16 can only be applied in certain regions of $\bar{t}$, $\bar{x}$, and $\bar{y}$ as discussed earlier. These various regions are shown on figure 1, each with a different background pattern. Figure 1 was produced with a commercially available graphics package directly from output files of the program. No graphics capability is included in the program, it is assumed that the user has access to a similar package. A sample input data set and the resulting output files, for producing the $\bar{t}$ equal 1, 3, and 5 curves of figure 1, are shown in appendix II. Note that if q_o, B, and n are all set equal to one when Q is set to 2π then $\bar{t}$ and t are equal. This has been done in appendix II. N equal 100 was used to keep the output files manageable; however, for curves to plot smoothly more points may be desired. Appendix II may be useful for users to insure the program is working properly on their system.

In a real world application, one will not be dealing with the dimensionless quantities $\bar{t}$, $\bar{x}$, and $\bar{y}$ but with actual time and distances. However, equations 2, 3, and 4 provide the necessary conversions, so the simple user interface in the main program provides the connection to a specific application.

Parameters may be given in any consistent set of units. To begin, one must know the average value of hydraulic conductivity and the regional hydraulic gradient vector (direction and magnitude). These quantities are used in equation 1 to calculate the specific discharge or Darcy velocity (q_o). Knowing the average regional thickness of the aquifer (B), the effective porosity (n), and the discharge (or injection) rate of the well (Q), the program can calculate $\bar{t}$ from equation 4 for the actual time of interest. The program then calculates the $\bar{x}_i$ and $\bar{y}_i$ of the capture curve of interest. These values of $\bar{x}$ and $\bar{y}$ are used with equations 2 and 3 to solve for the real world coordinates x and y, which can then be plotted on an appropriate map base. Currently, the program assumes that the x axis is parallel to the regional hydraulic gradient; but, the well may be located at arbitrary coordinates. If the x axis assumption is not true, an appropriate rotation of coordinates will be needed before plotting on the desired map base.

DISCUSSION

Strictly speaking, the program presented here only deals with one well in a uniform, homogeneous, isotropic aquifer with uniform, steady, regional flow. In practice these conditions are rarely satisfied. However, the type of analysis presented here can be very useful as a first phase in developing wellhead protection or aquifer cleanup schemes (Javandel and Tsang, 1986).

If conservative aquifer parameters are used, the analysis presented here should outline a maximum capture zone. The program presented here only deals with one well; however, the approximate result for several wells can be obtained by applying the program once for each well and superimposing the results. As long as the capture zones do not overlap, the approximate result should be very good. As the well spacing gets smaller and the capture zones overlap, the approximate results will deviate more from the correct solution; as long as the well spacing is greater than or equal to $Q/\pi q_o B$ the results are expected to be acceptable (see Javandel and Tsang for details of superimposing multiple wells). For the final analysis, if heterogeneity and nonuniform flow are very important a more complex program such as that presented by Shafer (1987) should be used.

The program presented here is useful for planning wellhead protection and aquifer cleanup schemes. However, the user must always be mindful of its limitations. The presented program is simple and can be embedded in many computing environments, including personal computers, work stations, and mainframes. We have used the program on a work station interfaced with a geographical information system (GIS) to plot capture zones for several wells in Kansas (Woods et al., 1987; Whittemore et al., 1987; and Merchant et al., 1988). The program is presented here in the hope that it will be useful to others.

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Figure 1. Capture zones and regions of application.

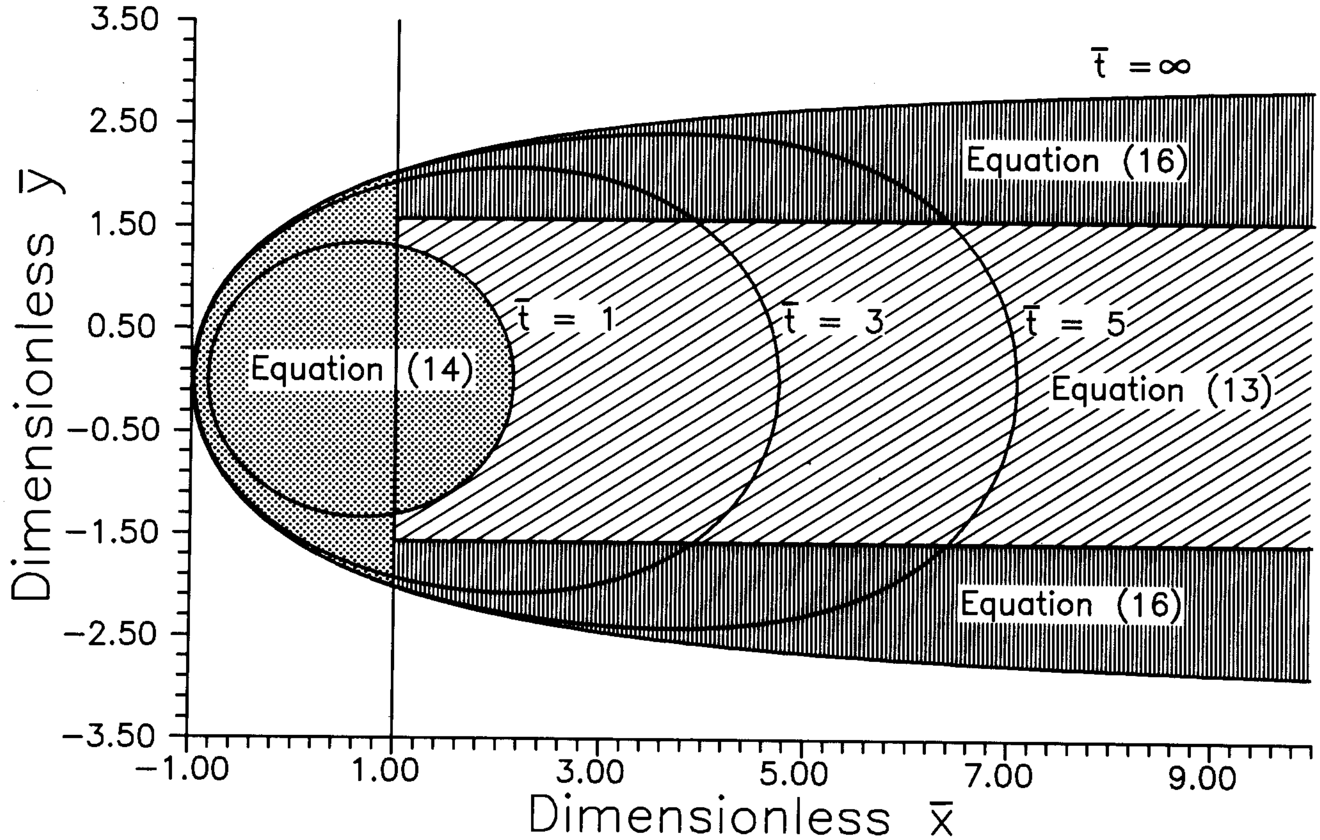

Figure 1. Capture zones and regions of equation application.

APPENDIX I

PROGRAM LISTING

FORTRAN LISTING

```
      CHARACTER*40 NAME
      CHARACTER*3 ECHO, ANS
      DIMENSION X(1001), Y(1001), XR(1001), YR(1001)
      PI = 3.1415926536
      WRITE (*,102)
  102 FORMAT (' READ IN THE PARAMETERS FOR CAPTURE ZONE CALCULATIONS ',/,
     + ' PLEASE BE SURE TO USE CONSISTENT UNITS.')
      WRITE (*,103)
  103 FORMAT (' READ IN THE THICKNESS OF THE AQUIFER, B (L).')
      READ (*,*) B
      WRITE (*,104)
  104 FORMAT (' READ IN THE EFFECTIVE POROSITY OF THE AQUIFER, POR.')
      READ (*,*) POR
      WRITE (*,105)
  105 FORMAT (' READ IN THE AVERAGE REGIONAL DARCY VELOCITY, q (L/T).'
     + /,' IT IS ASSUMED PARALLEL TO THE X AXIS, BUT MAY BE + OR -.')
      READ (*,*) q
      WRITE (*,109)
  109 FORMAT (' READ IN THE PUMPING RATE FOR THE WELL, QW (L**3/T),'
     + /, ' POSTIVE FOR INJECTION, NEGATIVE FOR WITHDRAWAL.')
      READ (*,*) QW
      WRITE (*,205)
  205 FORMAT (' READ IN THE COORDINATES OF THE PUMPING WELL, (X,Y) (L)'
     + /, ' REMEMBER THE X-AXIS IS PARALLEL TO REGIONAL FLOW.')
      READ (*,*) XP, YP
   10 WRITE (*,101)
  101 FORMAT (' READ IN THE TIME FOR CAPTURE ZONE CALCULATIONS, t(T).'
     + / ' IF TIME IS NEGATIVE THE PROGRAM WILL TERMINATE.')
      READ (*,*) TIME
      IF (TIME .LT. 0.0) STOP
      T = ABS( 2.*PI*q**2*B*TIME/(POR*QW) )
      WRITE (*,106)
  106 FORMAT (' READ IN THE NUMBER OF CALCULATION POINTS, N.',/
     + ,' N SHOULD BE BETWEEN 100 AND 1000.')
      READ (*,*) N
      WRITE (*,107)
  107 FORMAT (' DO YOU WANT TO ECHO THE RESULTS TO THE SCREEN?')
      READ (*,203) ECHO
  203 FORMAT (A3)
      WRITE (*,108)
  108 FORMAT (' READ IN THE FILE NAME TO SAVE DATA IN.'
     + ,' MUST BE OF THE FORM [C:\DIRECT\NAME.DAT].')
      READ (*,202) NAME
  202 FORMAT (A40)
      OPEN (99,FILE=NAME,STATUS='UNKNOWN')
      CALL CAPCUR(T,N,X,Y,ECHO)
      IF (ECHO .EQ. 'YES') WRITE (*,402)
  402 FORMAT (' THE FOLLOWING ARE THE ACTUAL COORDINATES.')
      DO 300 I = 1, N+1
      XR(I) = X(I)*QW/(2.*PI*q*B)
      YR(I) = Y(I)*QW/(2.*PI*q*B)
      WRITE (99,201) XR(I)+XP, YR(I)+YP
  201 FORMAT (2E20.8)
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      IF (ECHO .EQ. 'YES')  WRITE (*,401) XR(I)+XP,ABS(YR(I))+YP
  401 FORMAT (' FOR X =',E20.8,' THE Y = ',E20.8 )
  300 CONTINUE
      DO 400 I = N, 1, -1
      WRITE (99,201) XR(I)+XP, -YR(I)+YP
  400 CONTINUE
      CLOSE (99)
      WRITE (*,501)
  501 FORMAT (' DO YOU WANT TO CALCULATE A CAPTURE ZONE ',/
     + ,' FOR A NEW TIME? (YES/NO)')
      READ (*,203) ANS
      IF (ANS .EQ. 'YES') GO TO 10
      STOP
      END

      SUBROUTINE CAPCUR(T,N,X,Y,ECHO)
      CHARACTER*3 ECHO
      DIMENSION X(1001), Y(1001)
C  SET CONVERGENCE CRITERIA
      CONVER = .00001
      PID2 = 1.5707963
      PID2P = PID2 + .01
      CALL ENDPTS(T,XMIN,XMAX)
      X(1) = XMIN
      Y(1) = 0.0
      IF (ECHO .EQ. 'YES') WRITE (*,106)
  106 FORMAT (' THE FOLLOWING ARE DIMENSIONLESS GENERIC COORDINATES.')
      IF (ECHO .EQ. 'YES')  WRITE (*,103) XMIN, 0.0, 0
  103 FORMAT (' FOR X =',E20.8,' THE Y = ',E20.8
     + ,' NEQ = ',I2)
      NEQ = 1
      DX = (XMAX - XMIN)/N
      YN = 0.1
      DO 150 I = 2, N
      XI = XMIN + DX*(I-1)
      IF (XI .GT. 1.0 .AND. I .EQ. 2 ) THEN
      WRITE (*,101)
  101 FORMAT( '  N HAS BEEN CHOSEN TOO SMALL.')
      STOP
      ENDIF
      II = 0
      IF ( T .LE. 1.0) GO TO 19
      IF (XI .LE. 1.0) GO TO 19
      IF (NEQ .EQ. 3) GO TO 40
      IF ((XI .GT. 1.0) .AND. (YN .GT. PID2)) GO TO 30
      IF ((XI .GT. 1.0) .AND. (YN .LT. PID2)) GO TO 40
   19 NEQ = 1
   20 YI = YN
      II = II + 1
      IF (II .GT. 2000) GO TO 200
      YN = ACOS(EXP(XI-T)-(XI/YI)*SIN(YI))
      YN = ABS(YN)
      IF (ABS((YN-YI)/(YN+.00001)) .GT. CONVER) GO TO 20
      IF (ECHO .EQ. 'YES')  WRITE (*,103) XI, YN, NEQ
```

```
      GO TO 130
   30 NEQ = 2
   35 YI = YN
      II = II + 1
      IF (II .GT. 2000) GO TO 200
      YN = 3.14159-ATAN((YI/XI)*(1.0-EXP(XI-T)/COS(YI)))
      YN = ABS(YN)
      IF (ABS((YN-YI)/(YN+.00001)) .GT. CONVER) GO TO 35
      IF (ECHO .EQ. 'YES')  WRITE (*,103) XI, YN, NEQ
      IF ( YN .LT. PID2P) NEQ = 3
      GO TO 130
   40 NEQ =3
   45 YI = YN
      II = II + 1
      IF (II .GT. 2000) GO TO 200
      YN = EXP(T-XI)*(YI*COS(YI) + XI*SIN(YI))/2.0 + YI/2.0
      YN = ABS(YN)
      IF (ABS((YN-YI)/(YN+.00001)) .GT. CONVER) GO TO 45
      IF (ECHO .EQ. 'YES')  WRITE (*,103) XI, YN, NEQ
      GO TO 130
  200 WRITE (*,105)
  105 FORMAT (' WARNING, DID NOT CONVERGE IN 2000 ITERATIONS.')
      IF (ECHO .EQ. 'YES')  WRITE (*,103) XI, YN, NEQ
  130 X(I) = XI
      Y(I) = YN
  150 CONTINUE
      X(N+1) = XMAX
      Y(N+1) = 0.0
      IF (ECHO .EQ. 'YES')  WRITE (*,103) XMAX, 0.0, 0
      RETURN
      END

      SUBROUTINE ENDPTS(T, XN1, XN2)
      XN = 0.0
C POSITIVE X SOLUTION
      IF (T .LT. .10) GO TO 90
   30 X = XN
      XN = T + LOG(1.0 +X)
      IF (ABS((XN - X)/XN) .GT. .00001) GO TO 30
      XN2 = XN
      GO TO 140
   90 CONTINUE
   95 X = XN
      XN=SQRT(2.*(T+X**3/3.-X**4/4.+X**5/5.-X**6/6.
     * +X**7/7.-X**8/8.+X**9/9.-X**10/10.))
      IF (ABS((XN-X)/(XN+.00001)) .GT. .00001) GO TO 95
      XN2 = XN
C NEGATIVE X SOLUTION
  140 XN = 0.0
  160 X = XN
      XN = EXP(X-T) -1.
      IF (ABS((XN-X)/(XN+.00001)) .GT. .00001) GO TO 160
      XN1 = XN
      RETURN
      END
```

APPENDIX II

SAMPLE DATA INPUT AND OUTPUT

SAMPLE DATA INPUT

```
READ IN THE PARAMETERS FOR CAPTURE ZONE CALCULATIONS
PLEASE BE SURE TO USE CONSISTENT UNITS.
READ IN THE THICKNESS OF THE AQUIFER, B (L).
1.
READ IN THE EFFECTIVE POROSITY OF THE AQUIFER, POR.
1.
READ IN THE AVERAGE REGIONAL DARCY VELOCITY, q (L/T).
IT IS ASSUMED PARALLEL TO THE X AXIS, BUT MAY BE + OR -.
1.
READ IN THE PUMPING RATE FOR THE WELL, QW (L**3/T),
POSTIVE FOR INJECTION, NEGATIVE FOR WITHDRAWAL.
6.2831853
READ IN THE COORDINATES OF THE PUMPING WELL, (X,Y) (L)
REMEMBER THE X-AXIS IS PARALLEL TO REGIONAL FLOW.
0.,0.
READ IN THE TIME FOR CAPTURE ZONE CALCULATIONS, t(T).
IF TIME IS NEGATIVE THE PROGRAM WILL TERMINATE.
1.
READ IN THE NUMBER OF CALCULATION POINTS, N.
N SHOULD BE BETWEEN 100 AND 1000.
100
DO YOU WANT TO ECHO THE RESULTS TO THE SCREEN?
NO
READ IN THE FILE NAME TO SAVE DATA IN. MUST BE OF THE FORM [C:\DIRECT\NAME.DAT].
D:\CAPCURVE\T1N100.DAT
DO YOU WANT TO CALCULATE A CAPTURE ZONE
FOR A NEW TIME? (YES/NO)
YES
READ IN THE TIME FOR CAPTURE ZONE CALCULATIONS, t(T).
IF TIME IS NEGATIVE THE PROGRAM WILL TERMINATE.
3.
READ IN THE NUMBER OF CALCULATION POINTS, N.
N SHOULD BE BETWEEN 100 AND 1000.
100
DO YOU WANT TO ECHO THE RESULTS TO THE SCREEN?
NO
READ IN THE FILE NAME TO SAVE DATA IN. MUST BE OF THE FORM [C:\DIRECT\NAME.DAT].
D:\CAPCURVE\T3N100.DAT
DO YOU WANT TO CALCULATE A CAPTURE ZONE
FOR A NEW TIME? (YES/NO)
YES
READ IN THE TIME FOR CAPTURE ZONE CALCULATIONS, t(T).
IF TIME IS NEGATIVE THE PROGRAM WILL TERMINATE.
5.
READ IN THE NUMBER OF CALCULATION POINTS, N.
N SHOULD BE BETWEEN 100 AND 1000.
100
DO YOU WANT TO ECHO THE RESULTS TO THE SCREEN?
NO
READ IN THE FILE NAME TO SAVE DATA IN. MUST BE OF THE FORM [C:\DIRECT\NAME.DAT].
D:\CAPCURVE\T5N100.DAT
DO YOU WANT TO CALCULATE A CAPTURE ZONE
FOR A NEW TIME? (YES/NO)
NO
Stop - Program terminated.

A:\>
```

OUTPUT FILE D:\CAPCURVE\T1N100.DAT

-.84140500E+00	.00000000E+00
-.81152910E+00	.26302210E+00
-.78165320E+00	.37010310E+00
-.75177720E+00	.45098800E+00
-.72190130E+00	.51809930E+00
-.69202540E+00	.57627080E+00
-.66214950E+00	.62799250E+00
-.63227360E+00	.67475360E+00
-.60239760E+00	.71752490E+00
-.57252170E+00	.75698420E+00
-.54264580E+00	.79362580E+00
-.51276980E+00	.82782790E+00
-.48289390E+00	.85988100E+00
-.45301800E+00	.89001810E+00
-.42314200E+00	.91842860E+00
-.39326610E+00	.94526770E+00
-.36339020E+00	.97066920E+00
-.33351420E+00	.99474170E+00
-.30363830E+00	.10175800E+01
-.27376240E+00	.10392660E+01
-.24388650E+00	.10598710E+01
-.21401050E+00	.10794550E+01
-.18413460E+00	.10980760E+01
-.15425870E+00	.11157800E+01
-.12438270E+00	.11326110E+01
-.94506820E-01	.11486060E+01
-.64630900E-01	.11637990E+01
-.34754960E-01	.11782230E+01
-.48790350E-02	.11919030E+01
.24996890E-01	.12048660E+01
.54872820E-01	.12171330E+01
.84748750E-01	.12287250E+01
.11462470E+00	.12396600E+01
.14450060E+00	.12499550E+01
.17437650E+00	.12596240E+01
.20425250E+00	.12686820E+01
.23412840E+00	.12771400E+01
.26400430E+00	.12850090E+01
.29388030E+00	.12923000E+01
.32375620E+00	.12990230E+01
.35363210E+00	.13051820E+01
.38350800E+00	.13107860E+01
.41338400E+00	.13158420E+01
.44325990E+00	.13203530E+01
.47313580E+00	.13243250E+01
.50301180E+00	.13277600E+01
.53288770E+00	.13306630E+01
.56276360E+00	.13330360E+01
.59263960E+00	.13348780E+01
.62251540E+00	.13361930E+01
.65239140E+00	.13369790E+01
.68226730E+00	.13372340E+01
.71214320E+00	.13369650E+01

.74201920E+00	.13361580E+01
.77189510E+00	.13348190E+01
.80177100E+00	.13329480E+01
.83164700E+00	.13305330E+01
.86152290E+00	.13275670E+01
.89139880E+00	.13240540E+01
.92127470E+00	.13199840E+01
.95115070E+00	.13153540E+01
.98102660E+00	.13101500E+01
.10109030E+01	.13043660E+01
.10407790E+01	.12979880E+01
.10706540E+01	.12910130E+01
.11005300E+01	.12834270E+01
.11304060E+01	.12752160E+01
.11602820E+01	.12663740E+01
.11901580E+01	.12568720E+01
.12200340E+01	.12466960E+01
.12499100E+01	.12358340E+01
.12797860E+01	.12242630E+01
.13096620E+01	.12119670E+01
.13395380E+01	.11989110E+01
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.13992900E+01	.11704130E+01
.14291650E+01	.11549140E+01
.14590410E+01	.11385410E+01
.14889170E+01	.11212390E+01
.15187930E+01	.11029620E+01
.15486690E+01	.10836720E+01
.15785450E+01	.10633110E+01
.16084210E+01	.10418020E+01
.16382970E+01	.10190700E+01
.16681730E+01	.99504630E+00
.16980490E+01	.96961680E+00
.17279250E+01	.94266380E+00
.17578010E+01	.91406150E+00
.17876770E+01	.88364950E+00
.18175530E+01	.85121150E+00
.18474290E+01	.81652680E+00
.18773040E+01	.77927260E+00
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.19370560E+01	.69541040E+00
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.19968080E+01	.59455840E+00
.20266840E+01	.53483050E+00
.20565600E+01	.46579820E+00
.20864360E+01	.38245740E+00
.21163120E+01	.27195080E+00
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.20864360E+01	-.38245740E+00
.20565600E+01	-.46579820E+00
.20266840E+01	-.53483050E+00
.19968080E+01	-.59455840E+00
.19669320E+01	-.64757430E+00
.19370560E+01	-.69541040E+00
.19071800E+01	-.73907070E+00

.18773040E+01	-.77927260E+00
.18474290E+01	-.81652680E+00
.18175530E+01	-.85121150E+00
.17876770E+01	-.88364950E+00
.17578010E+01	-.91406150E+00
.17279250E+01	-.94266380E+00
.16980490E+01	-.96961680E+00
.16681730E+01	-.99504630E+00
.16382970E+01	-.10190700E+01
.16084210E+01	-.10418020E+01
.15785450E+01	-.10633110E+01
.15486690E+01	-.10836720E+01
.15187930E+01	-.11029620E+01
.14889170E+01	-.11212390E+01
.14590410E+01	-.11385410E+01
.14291650E+01	-.11549140E+01
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.13694140E+01	-.11850720E+01
.13395380E+01	-.11989110E+01
.13096620E+01	-.12119670E+01
.12797860E+01	-.12242630E+01
.12499100E+01	-.12358340E+01
.12200340E+01	-.12466960E+01
.11901580E+01	-.12568720E+01
.11602820E+01	-.12663740E+01
.11304060E+01	-.12752160E+01
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.10706540E+01	-.12910130E+01
.10407790E+01	-.12979880E+01
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.98102660E+00	-.13101500E+01
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.38350800E+00	-.13107860E+01
.35363210E+00	-.13051820E+01
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.29388030E+00	-.12923000E+01
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.20425250E+00	-.12686820E+01
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.14450060E+00	-.12499550E+01
.11462470E+00	-.12396600E+01
.84748750E-01	-.12287250E+01
.54872820E-01	-.12171330E+01
.24996890E-01	-.12048660E+01
-.48790350E-02	-.11919030E+01
-.34754960E-01	-.11782230E+01
-.64630900E-01	-.11637990E+01
-.94506820E-01	-.11486060E+01
-.12438270E+00	-.11326110E+01
-.15425870E+00	-.11157800E+01
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-.21401050E+00	-.10794550E+01
-.24388650E+00	-.10598710E+01
-.27376240E+00	-.10392660E+01
-.30363830E+00	-.10175800E+01
-.33351420E+00	-.99474170E+00
-.36339020E+00	-.97066920E+00
-.39326610E+00	-.94526770E+00
-.42314200E+00	-.91842860E+00
-.45301800E+00	-.89001810E+00
-.48289390E+00	-.85988100E+00
-.51276980E+00	-.82782790E+00
-.54264580E+00	-.79362580E+00
-.57252170E+00	-.75698420E+00
-.60239760E+00	-.71752490E+00
-.63227360E+00	-.67475360E+00
-.66214950E+00	-.62799250E+00
-.69202540E+00	-.57627080E+00
-.72190130E+00	-.51809930E+00
-.75177720E+00	-.45098800E+00
-.78165320E+00	-.37010310E+00
-.81152910E+00	-.26302210E+00
-.84140500E+00	.00000000E+00

OUTPUT FILE D:\CAPCURVE\T3N100.DAT

-.98133940E+00	.00000000E+00
-.92403570E+00	.40632300E+00
-.86673200E+00	.57110110E+00
-.80942830E+00	.69515280E+00
-.75212470E+00	.79775180E+00
-.69482090E+00	.88641880E+00
-.63751730E+00	.96503100E+00
-.58021350E+00	.10359150E+01
-.52290990E+00	.11005910E+01
-.46560620E+00	.11601230E+01
-.40830250E+00	.12152950E+01
-.35099880E+00	.12666980E+01
-.29369510E+00	.13147950E+01
-.23639140E+00	.13599590E+01
-.17908770E+00	.14024900E+01
-.12178410E+00	.14426420E+01
-.64480360E-01	.14806230E+01
-.71766750E-02	.15166140E+01
.50127010E-01	.15507660E+01
.10743070E+00	.15832140E+01
.16473440E+00	.16140720E+01
.22203810E+00	.16434440E+01
.27934180E+00	.16714190E+01
.33664550E+00	.16980750E+01
.39394920E+00	.17234810E+01
.45125280E+00	.17477040E+01
.50855650E+00	.17707990E+01
.56586020E+00	.17928220E+01
.62316390E+00	.18138110E+01
.68046760E+00	.18338050E+01
.73777130E+00	.18528480E+01
.79507500E+00	.18709770E+01
.85237870E+00	.18882130E+01
.90968230E+00	.19045780E+01
.96698610E+00	.19201070E+01
.10242900E+01	.19348220E+01
.10815930E+01	.19487370E+01
.11388970E+01	.19618710E+01
.11962010E+01	.19742360E+01
.12535050E+01	.19858480E+01
.13108080E+01	.19967160E+01
.13681120E+01	.20068500E+01
.14254160E+01	.20162560E+01
.14827190E+01	.20249420E+01
.15400230E+01	.20329110E+01
.15973270E+01	.20401660E+01
.16546300E+01	.20467080E+01
.17119340E+01	.20525380E+01
.17692380E+01	.20576540E+01
.18265410E+01	.20620530E+01
.18838450E+01	.20657310E+01
.19411490E+01	.20686820E+01
.19984520E+01	.20709000E+01
.20557560E+01	.20723770E+01
.21130600E+01	.20731000E+01

.21703630E+01	.20730670E+01
.22276670E+01	.20722590E+01
.22849710E+01	.20706590E+01
.23422750E+01	.20682590E+01
.23995780E+01	.20650360E+01
.24568820E+01	.20609790E+01
.25141860E+01	.20560650E+01
.25714890E+01	.20502740E+01
.26287930E+01	.20435750E+01
.26860970E+01	.20359550E+01
.27434000E+01	.20273850E+01
.28007040E+01	.20178250E+01
.28580080E+01	.20072590E+01
.29153120E+01	.19956410E+01
.29726150E+01	.19829480E+01
.30299190E+01	.19691280E+01
.30872230E+01	.19541530E+01
.31445260E+01	.19379620E+01
.32018300E+01	.19205160E+01
.32591340E+01	.19017600E+01
.33164370E+01	.18816370E+01
.33737410E+01	.18600860E+01
.34310450E+01	.18370260E+01
.34883480E+01	.18123930E+01
.35456520E+01	.17861060E+01
.36029560E+01	.17580500E+01
.36602590E+01	.17281540E+01
.37175630E+01	.16962630E+01
.37748670E+01	.16622870E+01
.38321700E+01	.16261060E+01
.38894740E+01	.15878950E+01
.39467780E+01	.15712890E+01
.40040820E+01	.15015650E+01
.40613850E+01	.14541500E+01
.41186890E+01	.14032300E+01
.41759920E+01	.13483800E+01
.42332960E+01	.12890910E+01
.42906000E+01	.12246770E+01
.43479040E+01	.11542750E+01
.44052080E+01	.10766820E+01
.44625110E+01	.99019240E+00
.45198150E+01	.89218650E+00
.45771180E+01	.77830490E+00
.46344220E+01	.64009590E+00
.46917260E+01	.45590360E+00
.47490300E+01	.00000000E+00
.46917260E+01	-.45590360E+00
.46344220E+01	-.64009590E+00
.45771180E+01	-.77830490E+00
.45198150E+01	-.89218650E+00
.44625110E+01	-.99019240E+00
.44052080E+01	-.10766820E+01
.43479040E+01	-.11542750E+01
.42906000E+01	-.12246770E+01
.42332960E+01	-.12890910E+01
.41759920E+01	-.13483800E+01

.41186890E+01	-.14032300E+01
.40613850E+01	-.14541500E+01
.40040820E+01	-.15015650E+01
.39467780E+01	-.15712890E+01
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.38321700E+01	-.16261060E+01
.37748670E+01	-.16622870E+01
.37175630E+01	-.16962630E+01
.36602590E+01	-.17281540E+01
.36029560E+01	-.17580500E+01
.35456520E+01	-.17861060E+01
.34883480E+01	-.18123930E+01
.34310450E+01	-.18370260E+01
.33737410E+01	-.18600860E+01
.33164370E+01	-.18816370E+01
.32591340E+01	-.19017600E+01
.32018300E+01	-.19205160E+01
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.30872230E+01	-.19541530E+01
.30299190E+01	-.19691280E+01
.29726150E+01	-.19829480E+01
.29153120E+01	-.19956410E+01
.28580080E+01	-.20072590E+01
.28007040E+01	-.20178250E+01
.27434000E+01	-.20273850E+01
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