

Lamb Wave Patterns

Inverse velocity trends, i.e., phase velocity increasing with frequency.

Typically, Lamb-wave patterns can be observed when there is a high velocity stiff layer at the top of your section. In such instances some parts of the dispersion curves may exhibit features that resemble Lamb wave (plate wave) patterns and not Rayleigh wave, "... free Rayleigh waves can only propagate at phase velocities slower than the shear-wave velocity of half space"(Thrower, 1965).

When applying the MASW method on structures with reverse velocity trends (i.e., velocities decreasing with depth), then for wavelengths up to five times that of the top stiff layer (Martinček, 1994) the dispersion-curve images may exhibit Lamb-wave patterns. In such cases the overall trend consists of a sequence of multi-mode events observable within narrow frequency ranges (i.e., small portions of higher modes), almost node-like, which when viewed together can shape a different trend. With such images mode identification becomes difficult and even if accurate using the conventional Rayleigh wave inversion algorithms (which assumes that half space has the highest velocity) will not be numerically appropriate.

In the lack of accurate numerical solution a qualitative approach has been used in practice using half-wavelength (or smaller; e.g., 30%) approximation, which can be used as a "qualitative" tool to observe lateral Vs variations, which for some purposes could be sufficient. With SurfSeis this can be achieved by using the "Approximate" inversion button.

Recent research (Pan et al., 2013b) continue previous efforts (Phinney, 1961; Schroder and Scott, 2001; Ryden et al., 2004) in using the real part of the complex roots of the Rayleigh-wave equation (a.k.a. leaky-waves) as phase-velocities of Rayleigh waves (Cf) and showed that a subset of multi-mode dispersion-curve points (events) is numerically equivalent to that of a fundamental mode dispersion curve from a substitute model that uses only the top 2 layers of the original model (consisting of 3 or more layers).

Beginning with SurfSeis 4 version, users can obtain such a simple 2-layer reverse velocity trend solutions, i.e., a high-velocity layer on top of a lower-velocity half space. However, the conventional Rayleigh-wave initial model creation (default in SurfSeis) will most likely be unsuitable and some inversion tune-up may be useful. Following is an example of the inversion steps needed when dealing with Lamb-wave data patterns.

We used a three-layer model similar to the one studied by Ryden et al. (2004) (but at a greater scale) (Table 1) to calculate synthetic seismic data using KGS' software (SeisSynth, soon to be released).

H	Vs	Vp	Density
5	1000	2000	2.0
10	419	1050	1.9
HS	96	318	1.8

Table 1. Three-layer model used to calculate synthetic seismic data, where H is layer thickness, V_s and V_p – shear- and compressional velocities, accordingly.

Next a dispersion-curve image was obtained and the first 6 modes were calculated (Figure 1). Following the overall reverse velocity trend with the intent of treating it as resulting from a 2-layer model a reverse-trend (i.e., velocities increasing with frequencies) dispersion curve was picked (Figure 2).

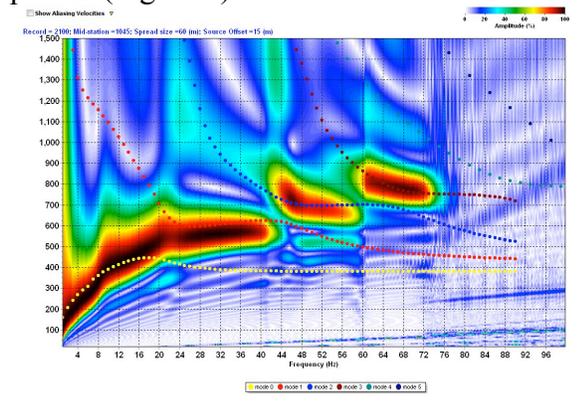


Figure 1. Dispersion-curve image obtained and 6 calculated dispersion-curve modes.

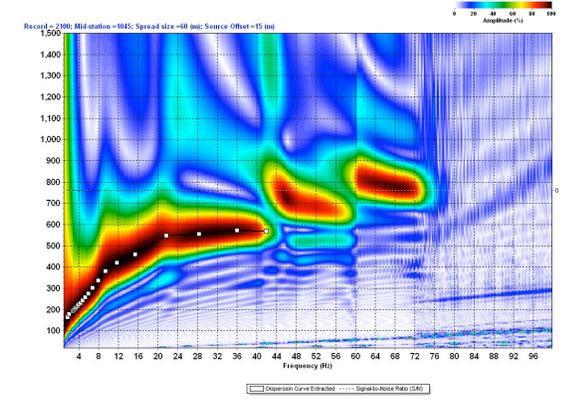


Figure 2. Dispersion-curve estimation.

The conventional Rayleigh-wave initial model generation algorithm using half wavelength depth and $C_f = 0.92 V_s$ assumptions did not provide adequate inversion results. Another approach was needed for coming up with an initial 2-layer model.

The main trend asymptotic velocity at 40 Hz is about 600 m/s and corresponds to ~ 15 m wavelength. Knowing that the synthetic model thickness and V_s are 5 m and 1000 ms accordingly, the initial model algorithm for this case can be adjusted depth = $\sim 33\%$ wavelength depth and $V_s = \sim C_f/0.6$.

However, following Ryden et al. (2004) another reference point can be used, such as ~ 720 m/s at 72 Hz when considering the trend formed from all modes. The corresponding estimates would be 10 m wavelength, top layer thickness = $\sim 50\%$ wavelength and $V_s = \sim C_f/0.72$.

One way to run the inversion in SurfSeis 4 with this type of initial model tune-up can be accomplished by following the steps:

1. Copy an existing layer-model file (.lyr) and edit it into a 2-layer model with top layer thickness and Vs parameters estimated with an algorithm similar to the one above.
2. Open a .dc file from “Invert Dispersion curves”
3. Click the “Controls” button, “Initial Layer Model” tab, and at the “Initial S-Velocity” radio group select “Fixed data in layer table”.
4. Click the “Layer” button, import Model (*.LYR)” button and read the specially created 2-layer model file in step 1.
5. Adjust iteration parameter to your judgment, if necessary (“Controls” button, “Iterations” tab).

Using the estimated dispersion curve (Figure 2) and following the above approach a simple 2-layer Lamb-wave inversion solution was obtained.

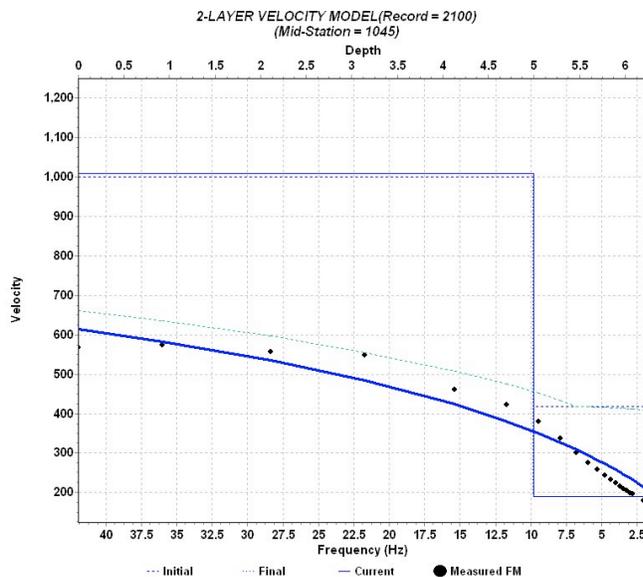


Figure 3. 2-layer model inversion results and corresponding dispersion curve fit to the observed data points.

The calculated dispersion curve fit is not perfect but still provides an overall match to the data from a 3-layer model. A better match can be obtained if using fewer lower-frequency points, e.g., above 10 Hz, etc. (Pan et al., 2013a). The velocity of the top layer was estimated within ~1% error ~1010 m/s and the velocity of half space (~190 m/s) was estimated in-between the values of second layer and half space of the synthetic model (Table 1).

The initial model generation parameters (H, Cf, etc.) may be different for other cases (velocity models, source offset and spread size, observe frequency range, etc.) and should be regarded only as an example of the complexity when dealing with Lamb-wave patterns.

Pan et al. (2013a) suggest a more complicated algorithm for dealing with high-velocity surface layer models that have 3 or more layers. If proven robust, such an algorithm might be implemented in future versions of SurfSeis (i.e., 5 and higher).

References:

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