FORTAN IV PROGRAM FOR THE GE 625 TO COMPUTE THE POWER SPECTRUM OF GEOLOGICAL SURFACES

By
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and
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Editor's Remarks

A recent world public opinion inquiry was made among earth scientists for information in regard to the forthcoming International Geological Congress in Prague. It was emphasized by Jiří Krupička of Czechoslovakia, that we are accumulating knowledge at an incredibly fast pace. Because of the nature of the acquired data, much of which is difficult to compartmentalize, more questions in geology are being raised than answered. More than ever a synthesis of available material is needed. It would seem that many old convictions now are being replaced by new and more up-to-date ideas. In other words, we are attempting better interpretations based on more complete information.

New information, however, is being generated at an ever increasing rate and it is almost impossible to keep up. As a result, there is needless duplication of effort. This situation, according to Krupička, demands a new approach. He states, "The new methods and techniques born by the modern scientific and technical revolution are being vehemently employed for the acquisition of new special data. Should they not, especially the most recent methods of the computer technique, be even preferentially used for a better evaluation of the existing data, for a more convenient storage, a much faster distribution, and above all, for an objective pretreatment in such a way that the highest possible quantity of information could be treated simultaneously by one mind?" Of course, these new methods are being utilized by earth scientists world-wide, as shown by the distribution of the COMPUTER CONTRIBUTION Series.

COMPUTER CONTRIBUTIONS are one of many publications making the latest results available to scientists. It is an important outlet because it is the only series featuring computer techniques and applications. As aptly stated by W.C. Krumbein and F.A. Graybill (An introduction to statistical models in geology, McGraw-Hill Book Co., 1965), "...the high-speed computer has opened new doors in preliminary data evaluation and in the selection of appropriate models for analyzing sets of geological observations...."

With the readily available third-generation computers, effort is now being directed towards formation and maintenance of geological information files. Various techniques are being developed and tested to utilize this vast amount of information. The program reported here, "FORTRAN IV program for the GE 625 to compute the power spectrum of geologic surfaces," by J.E. Esler and F.W. Preston, is just such a technique. This program is designed to test for periodicity in geologic data. Geologists and other earth scientists should find an immediate use for the program in analyzing data in a variety of problems. It will be useful especially to those who are concerned with describing and analyzing topographic or structural surfaces.

For a limited time, the Kansas Geological Survey will make available this program on magnetic tape for $10.00.

An up-to-date listing of available COMPUTER CONTRIBUTIONS and computer programs available can be obtained by writing Editor, COMPUTER CONTRIBUTION Series, Kansas Geological Survey, the University of Kansas, Lawrence, Kansas 66044. Your comments and suggestions in regard to this series will be most appreciated.
FORTRAN IV PROGRAM FOR THE GE 625 TO COMPUTE THE
POWER SPECTRUM OF GEOLOGICAL SURFACES

By

JAMES E. ESLER and FLOYD W. PRESTON

NUMERICAL CHARACTERIZATION OF
SURFACES

Geologists and geographers have always been
interested in the description and characterization
of surfaces. Methods used to date mainly have
been written descriptions and mathematical approx-
imations. In the first category are discussions of
landforms by early geologists and geographers.
These were poetic and pictorial, but not useful in
an operational sense. Descriptions by one inves-
tigator could not, in general, be used by another
investigator to reconstruct a three-dimensional
likeness of the surface.

To overcome the inadequacies of verbal descrip-
tions, earth scientists have turned to numerical ap-
proximations of surfaces. These possess a degree of
objectivity not present in verbal accounts, and are
consistent and reproducible by other investigators.
Statistics of numerically defined surfaces can be
computed, and the surfaces can be compared by
numerical techniques.

Least squares is the most widely used surface-
fitting criterion, although there are others such as
minimax and absolute deviation criteria. Math-
ematical models fitted to observed surfaces by least
squares include polynomial and Fourier series ap-
proximations. Excellent reviews of these forms are
given by Krumbein (1966a, 1966b) and James (1966).

All presently used mathematical representations
of surfaces built around the least-squares criterion
are based on the general linear regression model. A
gologic surface is considered to be composed of a
regional component (trend) and a residual component.
The residual component in turn is composed of a local
variation confounded with random error. Compo-
nents are separated by the regression as a means of
estimating variances in the surface due to regional
effects and local effects, random fluctuation or error
being neglected. In some geologic studies this as-
sumption seems reasonable, as in trend-surface map-
ning of well-log data where random measurement
errors are extremely small compared to other factors
influencing the surface. If the residuals do not con-
tain systematic variations, the trend is assumed to
contain all geologically significant variation
(regional and local) and represents an appropriate
generalization of the surface.

Utility of the general linear regression model
for description of geologic surfaces depends upon
the ability of the investigator to determine the point
where regional and residual components are most
effectively separated.

Well-conceived methods have been presented to
determine the order of an equation necessary to
describe major elements of a surface. Despite im-
pressive efforts in trend analysis there is growing
uneasiness as to the appropriate interpretation of
"trend," particularly when surfaces of high order
are generated. The basic problem is distinguishing
between "trend" and "random" components. In the
phraseology of electronics engineering, these elements
would be called "signal" and "noise." In this regard,
an admonition from the electronics engineer may be
cited: if one wishes to extract signal from noise, one
must know, a priori, the inherent nature of either
the signal or the noise. This a priori insight does not
seem to be available for most geological and geo-
graphical surfaces.

Another model, which assumes the surface to
have little or no "trend" above a first- or possibly a
second-degree polynomial, may be considered. The
observed surface essentially is noise superimposed
upon this weak trend. The noise or variability is a
fundamental characteristic of the surface and is con-
sidered both necessary and sufficient for description
of the surface. Surfaces that are similar are assumed
to have similar variabilities; surfaces with different
variabilities are assumed to be different. This sup-
poses that judgments of similarity based on variability
agree with judgments of similarity made by conven-
tional visual inspection. The measure of variability
used is the power spectrum.

In this model, the surface is considered to be
oscillatory or undulating in two perpendicular direc-
tions such as east-west and north-south. Within a
given frequency band for these oscillations, all
frequencies are considered present. The power spec-
trum measures the relative contribution of each fre-
quency to the variability or oscillatory nature of the
surface.
USE OF POWER SPECTRUM IN DESCRIBING GEOLOGICAL SURFACES

Geologists are accustomed to thinking of a map as a scaled replica of the original region. More properly, maps should be considered as generalizations created from information that is available concerning the original surface. If the data are surface elevations plotted as a function of position in an east-west, north-south coordinate system, the result is a topographic map. Two sets of data at the same location but representing elevations of top and bottom of one particular geological unit can be used to prepare an isopach map. The actual surface or thickness of the unit is only approximately represented by the diagram. The map is neither the surface itself nor a true scaled representation, but is only a generalization.

A less conventional method of map generalization has been developed by Krumbein (1966b), who introduced the concept of "coefficient space." If the method of least squares is used to fit a polynomial or Fourier series in two independent directions to surface data, the resulting trend surface is a generalization of the original surface. As Krumbein points out, however, the same "information" concerning the trend surface also is contained in the set of polynomial or Fourier coefficients. These coefficients may be displayed as a two-dimensional array. If an investigator is searching for regularity or order in the surface, this may be more readily found in the array of coefficients than in the surface itself. The power-spectrum method of surface description, as discussed by Preston (1966) and by Rayner (1967), can be considered as surface representation in a "coefficient space" sense. However, the power-spectrum technique utilizes a "frequency space." As in "coefficient space" techniques, points on the power-spectrum map do not correspond to points in the sample space.

THE POWER SPECTRUM

The concept of the power spectrum has been developed by electrical engineers and used to analyze signals that are largely random in nature. The method has been successfully used to detect signals contaminated by random noise (Blackman and Tukey, 1959; Diamantides, 1961), to analyze sea waves (Pierson, 1960), and to characterize and classify porous media (Preston, Green, and Aldenderfer, 1966). Most work has been concerned with one-dimensional random data (Lee, 1960; Blackman and Tukey, 1959). A few workers have extended their investigations to two dimensions, especially W. J. Pierson, who studied the configuration of a wind-generated oceanic surface. The program described here follows his method of development. If a geological profile or cross section can be expressed as an unknown function, f(x), then various approximations can be used to represent f(x). One common method of approximating the function f(x) is through the use of Fourier series, here given in its one-dimensional trigonometric form:

\[ f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos \frac{2\pi nx}{L} + b_n \sin \frac{2\pi nx}{L} \right) \]

where

- \( a_n, b_n = \) Coefficients of the zeroth through the nth harmonic
- \( L = \) Fundamental period or sample length
- x = Distance along measurement axis

Using Euler's identity, equation (1) can be represented in exponential form:

\[ f(x) = \sum_{n=-\infty}^{\infty} F(n) e^{i \frac{2\pi nx}{L}} \]

where

\[ F(n) = \frac{1}{2} \sqrt{a_n - ib_n} \]

The magnitude of the function F(n) is the absolute value \( |F(n)| \) of the amplitude of the nth harmonic in the function f(x). The set of values of \( |F(n)| \) for all n is called the amplitude spectrum. The square of the amplitude spectrum, \( S = |F(n)|^2 \), is the power spectrum of the function f(x).

In analysis of stationary random functions, another method of deriving the power spectrum has been developed. It begins by defining the autocorrelation function Q as:

\[ Q(\tau) = \frac{1}{T_1} \int_{-\frac{T_1}{2}}^{\frac{T_1}{2}} f(x) f(x+\tau) \, dx \]

where

- \( \tau = \) Value of lag
- \( T_1 = \) Period of the fundamental angular frequency, \( \omega_1 \)

The relationship between \( T_1 \) and \( \omega_1 \) is:

\[ T_1 = 2 \pi / \omega_1 \]

Lee (1960) has shown that Q is an even function, that is, \( Q(\tau) = Q(-\tau) \). Q is also the Fourier transform of the power spectrum. Since the autocorrelation function is even, the sine term in the transform is zero. Therefore, the power spectrum S is given by:
Applications discussed in this paper use the weighting function given in Equation 9a.

Each of the values of $SS_h$ can be considered as an estimate of the contribution to the total sample variance made by frequencies in the range between $2\pi (h-1/2)/2\tau \Delta t$ and $2\pi (h+1/2)/2\tau \Delta t$, where $\Delta t$ represents the time or space interval between successive measurements.

Equations for the two-dimensional case are similar to those described above and can be developed by analogy. The autocorrelation function is given by the formula:

$$Q(p,q) = \frac{1}{(n-p) (m-q)} \sum_{k=|q|}^{m} \sum_{j=|p|}^{n} X_{i,k} X_{i+p, k+q}$$

for $p = 0, 1, 2, \ldots \tau$

$q = -\tau, -(\tau-1), \ldots -1$ \hspace{1cm} (10)

and

$$Q(p,q) = \frac{1}{(n-p) (m-q)} \sum_{k=|q|}^{m-|q|} \sum_{j=|p|}^{n-|p|} X_{i,k} X_{i+p, k+q}$$

for $p = 0, 1, 2, \ldots \tau$

$q = 0, 1, 2, \ldots \tau$

where $m$ and $n$ represent the number of data points along the two coordinates.

In order to weight values of $Q$ properly, the following adjustments are made:

$$Q^*(p,q) = 2Q(p,q) \hspace{1cm} p = 1 \text{ to } (\tau-1)$$

$q = -(\tau-1) \text{ to } (\tau-1)$

$$Q^*(0,q) = Q(0,q) \hspace{1cm} q = -(\tau-1) \text{ to } (\tau-1)$$

$$Q^*(p,\tau) = Q(p,\tau) \hspace{1cm} p = 1 \text{ to } (\tau-1)$$

$$Q^*(p,-\tau) = Q(p,-\tau) \hspace{1cm} p = 1 \text{ to } (\tau-1)$$

$$Q^*(0,\tau) = 1/2 (0,\tau)$$

$$Q^*(0,-\tau) = 1/2 (0,-\tau)$$

$$Q^*(\tau,\tau) = 1/2 Q(\tau,\tau)$$

$$Q^*(\tau,-\tau) = 1/2 Q(\tau,-\tau)$$
These formulas assume that the maximum lag used in each of the two dimensions is the same. This assumption greatly simplifies calculation of the Fourier transform. For most problems, this seems a reasonable assumption and does not greatly decrease the general-ity.

Unsmoothed estimates of the power spectrum are then given by:

\[
S(r,s) = \frac{1}{2\pi^2} \sum_{q=-\tau}^{\tau} \sum_{p=0}^{\tau} \mathcal{Q}^*(p,q) \cos \frac{\pi}{\tau} (rp + sq) \tag{11}
\]

\[r = 0, 1, 2, \ldots \tau \]

\[s = -\tau, -(\tau-1), \ldots, -1, 0, 1, \ldots \tau\]

The following conditions are used so that the smoothing process may be continued to the edge of the data set:

\[
\begin{align*}
S(-1,b) &= S(1,b) \quad b = -\tau \text{ to } \tau \\
S(\tau+1,b) &= S(\tau-1,b) \quad b = -\tau \text{ to } \tau \\
S(a,\tau+1) &= S(a,\tau-1) \quad a = 0 \text{ to } \tau \\
S(a,-\tau-1) &= S(a,-\tau+1) \quad a = 0 \text{ to } \tau \\
S(-1,\tau+1) &= S(1,\tau-1) \\
S(\tau+1,\tau+1) &= S(\tau-1,\tau-1) \\
S(\tau+1,-\tau-1) &= S(\tau-1,-\tau+1) \\
S(-1,-\tau-1) &= S(-1,-\tau+1)
\end{align*}
\]

The smoothing function used is an extension of the one-dimensional case (Equation 9a), a product of two linear smoothing functions which gives a square array of nine values. This function is a two-di-men-sional moving average with .25, .50, and .25 weightings for points in each direction.

<table>
<thead>
<tr>
<th>0.25</th>
<th>0.50</th>
<th>0.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>0.0625</td>
<td>0.125</td>
</tr>
<tr>
<td>0.50</td>
<td>0.125</td>
<td>0.250</td>
</tr>
<tr>
<td>0.25</td>
<td>0.0625</td>
<td>0.125</td>
</tr>
</tbody>
</table>

The smoothed spectrum is given by:

\[
SS(r,s) = 0.0625 \left[ S(r-1,s-1) + S(r-1,s+1) + S(r+1,s-1) + S(r+1,s+1) \right] + 0.125 \left[ S(r-1,s) + S(r+1,s) \right] + S(r,s-1) + S(r,s+1) \left[ S(r,s) \right] \tag{13}
\]

Final values of the smoothed power spectrum, \(SS(r,s)\), represent contributions to total variance made by frequencies between \(2\pi (r-1/2)/2\pi \Delta t\) and \(2\pi (r+1/2)/2\pi \Delta t\) in the \(r\) direction, and between \(2\pi (s-1/2)/2\pi \Delta t\) and \(2\pi (s+1/2)/2\pi \Delta t\) in the \(s\) direction.

**Removal of Linear Trend**

Data often will contain a linear trend which distorts the power spectrum. This trend can easily be removed using standard least-squares techniques, by fitting a plane of the form \(X'(i,j,k) = A + Bk + C\) to the data points and using residuals for calculating the power spectrum. Pierson (1960) has shown that for equally spaced data the constants \(A\), \(B\), and \(C\) can be found from the equation:

\[
\begin{bmatrix}
\frac{mn(n+1)}{6} & \frac{n(n+1)m(m+1)}{4} & \frac{mn(n+1)}{2} \\
\frac{n(n+1)m(m+1)}{4} & \frac{mn(m+1)(2m+2)}{6} & \frac{mn(m+1)}{2} \\
\frac{mn(n+1)}{2} & \frac{mn(m+1)}{2} & \frac{mn}{1}
\end{bmatrix}
\begin{bmatrix}
A \\
B \\
C
\end{bmatrix}
\]

\[
\begin{bmatrix}
\sum_{i=1}^{n} \sum_{k=1}^{m} iX_{i,k} \\
\sum_{j=1}^{m} \sum_{k=1}^{m} kX_{j,k} \\
\sum_{j=1}^{n} \sum_{k=1}^{m} X_{i,k}
\end{bmatrix}
\]

Solving for \(A\), \(B\), and \(C\) and subtracting \(X'(i,j,k)\) from the data matrix will then yield a new data matrix better adapted to power spectrum analysis.

**Choice of Parameters**

Accuracy and usefulness of the power spectrum depends upon the choice of certain parameters, such as the sampling interval \(\Delta t\) and the maximum value of lag \(\tau\). Proper choices for these parameters will minimize aliasing, or failure to recognize higher
harmonics, and will maximize degrees of freedom and resolution. Bendat (1966) suggested the sampling interval $\Delta t$ should be given by:

$$\Delta t = \frac{1}{2f_c}$$  \hspace{1cm} (15)

where $f_c$ is the lowest frequency of interest in the record along either of the two axes. This gives two points per cycle at the cutoff frequency $f_c$. Wherever possible, more points should be used for improved results. An accurate autocorrelation function, $Q$, can be formed by taking

$$\Delta t = \frac{1}{4f_c}$$  \hspace{1cm} (16)

If the power spectrum is of prime concern, a spacing of

$$\Delta t = \frac{2}{5f_c}$$  \hspace{1cm} (17)

is sufficient. Values of $\Delta t$ as close as possible to $1/2f_c$ are, of course, most economical, as fewer points are needed.

The maximum value of lag, $\tau$, should be limited by the smaller of the two dimensions of the data matrix. Several limits have been suggested for $\tau$ in the one dimensional case. For instance, Granger and Hatanaka (1964) recommended $\tau \leq m/3$ and Blackman (1965) recommends $\tau \leq m/10$, as does Crowson (1963) who performed a very thorough error analysis. Because a two dimensional problem involves a larger number of data points a value of $\tau \leq m/4$ should provide adequate results.

Another important aspect to consider in the choice of $\tau$ is the equivalent resolution bandwidth, $\beta_c$, desired for power spectrum calculations. Bendat has determined $\beta_c$ to be given by:

$$\beta_c = \frac{1}{\tau \Delta t^2}$$  \hspace{1cm} (18)

For a given $\Delta t$, $\beta_c$ will decrease as $\tau$ increases.

The degrees of freedom of each spectral estimate in the one-dimensional case have a Chi Square distribution with

$$d.f. = 2 \left( \frac{n}{\tau} - \frac{1}{4} \right).$$  \hspace{1cm} (19)

Pierson (1960) has expanded this formula to two dimensions to give:

$$d.f. = 1.58 \left( \frac{n}{\tau} - \frac{1}{2} \right) \left( \frac{m}{\tau} - \frac{1}{2} \right)$$  \hspace{1cm} (20)

Equation (20) may underestimate the true degrees of freedom. Values of $1/4$ instead of $1/2$ therefore may weight equation (20) more properly.

**PROGRAM DESCRIPTION**

The program to compute two-dimensional power spectra consists of a mainline routine, PSMAIN, and two subroutines, PS and PLOT. PSMAIN reads raw data and processes it according to instructions on the control cards. PS levels the data if necessary, and computes autocorrelation functions and power spectra. It also produces any punched or printed output requested. Printed output is plotted by subroutine PLOT, which can be called by PS or PSMAIN, according to options on the control cards.

Two forms of printed output for the autocorrelation function and rough and smoothed power function are provided by subroutine PLOT. The first option is a numerical listing of the functions as a matrix of three digit integers with a scaling factor giving their order of magnitude. This plotting system does not depict symmetry associated with evenness of either the autocorrelation function or the power spectrum. Alternately, a contour plot may be produced, in which each symbol represents tenths of the maximum value. A scaling factor indicates approximate values of contours. This plotting system shows symmetry of the functions about their origins.

**OPERATIONAL INSTRUCTIONS FOR PSMAIN**

**CARD 1**

<table>
<thead>
<tr>
<th>Col. 1</th>
<th>NMBR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>- Number of data sets to be run</td>
</tr>
</tbody>
</table>

**CARD 2**

<table>
<thead>
<tr>
<th>Col. 1</th>
<th>KALL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 = calculate power spectrum</td>
</tr>
<tr>
<td></td>
<td>2 = plot previously punched autocorrelation function or power spectrum</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Col. 2</th>
<th>IPLOTA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>- Plot autocorrelation function*</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Col. 3</th>
<th>IPLOTR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>- Plot rough power spectrum*</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Col. 4</th>
<th>IPILOTS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>- Plot smoothed power spectrum*</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Col. 5</th>
<th>IPNCHA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>- Punch autocorrelation function#</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Col. 6</th>
<th>IPNCHS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>- Punch smoothed power spectrum#</td>
</tr>
</tbody>
</table>

5
CARD 3
Col. 1-78 IDENT - Any alphanumeric title

CARD 4
Col. 1-3 LAG - Maximum value of lag
Col. 4 IBID - If KALL = 1, then
  0 = begin with autocorrelation
  1 = begin with power spectrum
  2 = begin with removed linear trend
  3 = begin with removed linear trend and punch leveled data
If KALL = 2, then
  1 = plot autocorrelation function
  2 = plot smoothed power spectrum
  3 = rough plot spectrum

Col. 5-7 N - Number of data points along horizontal axis
Col. 8-10 M - Number of data points

Col. 11-70 IFMT - Format to read data.
  Left parenthesis must be in column 11. Right parenthesis follows format statement

Col. 71 IFIRST - 1 = vary N first
  2 = vary M first

*For plotting options, 0 represents omit, +1 is numerical plot, +2 is contour plot, and +3 is both.
#For punching options, 0 represents omit and +1 is punch.

DATA CARDS

There are N x M data points which are arranged on the cards in the manner specified by the variable FORMAT statement on card 4.

Note: Card No. 1 appears only once. Cards No. 2, 3, and 4 are placed before each data set to be processed. Cards No. 3 and 4 are included in any punched output for identification purposes and need not be changed when used for input.

SYMBOLIC DICTIONARY

Subroutine PS

The following calling sequence is used for subroutine PS:

PS (D, M, N, LAG, IBEGIN, IPLOTA, IPLOT, IPLOTS, IDENT, IPNCHA, IPNCHS, Q)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Type</th>
<th>S/A</th>
<th>I/O</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>R</td>
<td>A</td>
<td>1</td>
<td>The data matrix, D(N, M)</td>
</tr>
<tr>
<td>M</td>
<td>I</td>
<td>S</td>
<td>1</td>
<td>Second (horizontal) dimension of D</td>
</tr>
<tr>
<td>N</td>
<td>I</td>
<td>S</td>
<td>1</td>
<td>First (vertical) dimension of D</td>
</tr>
<tr>
<td>LAG</td>
<td>I</td>
<td>S</td>
<td>1</td>
<td>Maximum value of lag, τ</td>
</tr>
<tr>
<td>IBEGIN</td>
<td>I</td>
<td>S</td>
<td>1</td>
<td>Switch to control starting point of calculation (0 to begin with autocorrelation, +1 with power spectrum, +2 or +3 with removed linear trend. +3 will also punch leveled data).</td>
</tr>
<tr>
<td>IPLOTA</td>
<td>I</td>
<td>S</td>
<td>1</td>
<td>Switch for plotting autocorrelation function (0 is off, +1 is numerical plot, +2 is contour plot, +3 is both).</td>
</tr>
<tr>
<td>IPLOT</td>
<td>I</td>
<td>S</td>
<td>1</td>
<td>Switch for plotting rough power spectrum. (Same as for IPLOTA).</td>
</tr>
<tr>
<td>IPLOTS</td>
<td>I</td>
<td>S</td>
<td>1</td>
<td>Switch for plotting smoothed power spectrum (Same as for IPLOTA).</td>
</tr>
<tr>
<td>IDENT</td>
<td>I</td>
<td>S</td>
<td>1</td>
<td>78 character identifying title</td>
</tr>
<tr>
<td>IPNCHA</td>
<td>I</td>
<td>S</td>
<td>1</td>
<td>Switch to punch autocorrelation function (0 is off, +1 is on)</td>
</tr>
</tbody>
</table>
IPNCHS  I   S  I  
Q     R   A  I/O

Switch to punch power spectrum (0 is off,  
+1 is on)  
Input for autocorrelation function, output  
for power spectrum.

Subroutine PLOT

The following calling sequence is used for subroutine PLOT:

PLOT (Q, LAG, METH, ID, IDENT)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Type</th>
<th>S/A</th>
<th>I/O</th>
<th>Comments</th>
</tr>
</thead>
</table>
| Q        | R    | A   | I   | Autocorrelation function or power spectrum  
to be plotted. |
| LAG      | I    | S   | I   | Maximum value of lag |
| METH     | I    | S   | I   | Switch to determine plotting method (+1  
is numerical list, +2 is contour map, +3  
is both). |
| ID       | I    | S   | I   | Identifies Q as autocorrelation function  
(with +1), smoothed power spectrum (with  
+2), or rough spectrum (with +3) |
| IDENT    | I    | A   | I   | 78 character identifying title |

S = single variable    A = subscripted variable    R = real    I = integer

Note: Symbols in contour map may be changed by changing card No. 9 in the plotting routine.

INTERPRETATION OF SAMPLE OUTPUT

Sea Surface Example

The two-dimensional power spectrum is primarily  
a method for study of the frequency components or  
"wave forms" present in an undulating surface. Such  
a surface could be topography in an area of rolling  
hills, a geologic datum in a folded area, or as in  
this example, the undulating surface of the sea.  
Stereophotographs of a region of the sea surface  
permit measurement of water elevation at every  
intersection on an imaginary grid. Sample data are  
from Data Set 2A of the Stereo Wave Observation  
Project (SWOP) described by Pierson (1960). Data  
are elevations in feet above an arbitrary datum close  
to mean sea level. Elevations were read at all inter-
sections on a 50 x 70 grid covering an area approx-
imately 1800 by 2700 feet. By the method of least  
squares, a plane may be passed through the data and  
the deviation of each data point from this plane de-
termined. Deviations correspond to elevations of  
points on the surface of the sea under the condition  
that there is no planar trend in the elevations. These  
leveled data appear as Table 1 (p. 15).

Following removal of the trend, the autocorre-
lation function is computed. Results are given in Figure  
1. Numbers in the body of Figure 1 represent ampli-
tude of the autocorrelation function at various lags in  
two perpendicular directions. One direction was  
chosen to correspond to direction of the prevailing  
wind, the other is at right angles. Rows and columns  
in the illustration are indexed according to "lags" of  
the autocorrelation function. The data may also be  
contoured as shown in Figure 2. Integers represent  
values of the autocorrelation function as fractional  
parts of the maximum value. For example, the digit  
1 represents 0.1 of the maximum value; dashes repre-
sent negative values.

The power spectrum of the SWOP data is shown in  
Figure 3 in raw or unsmeothed form, in Figure 4 as a  
smoothed spectrum, and in Figure 5 as a scaled contour  
map. The scales for these figures correspond to those  
of Figure 2. Each "lag" corresponds to a given frequen-
cy (or period) contribution to the overall undulating  
character of the surface. To convert these lag values  
to period, the equation shown below is used. Here  
period has a distance rather than time connotation
because the surface is undulating in space.

$$\lambda = \frac{2\tau_m \Delta L}{(\tau \pm 1/2)}$$ (21)

where

$$\lambda = \text{wave length or period in feet represented by lag } \tau; \text{ frequency is the reciprocal } 1/\lambda.$$

$$\Delta L = \text{grid interval in feet.}$$

$$\tau_m = \text{maximum number of lags, not to exceed } 1/10 \text{ to } 1/3 \text{ of total sampling interval.}$$

Here, $$\tau_m = 20 \text{ for each direction.}$$

Note that one direction has a lag of -20 to +20.

$$\tau = \text{a particular lag.}$$

The term $$\pm 1/2$$ indicates that the value of the power spectrum at each lag does not correspond to a precise period or frequency, but rather to all frequencies in a band one lag in width.

In the example given for this data, $$\Delta L = 30$$ feet and the maximum lag is 20. A lag of +3 in the $$x$$ direction (column heading +3) and +5 in the $$y$$ direction (row heading +5) shows a power spectrum value of 13. This corresponds to contributions made by oscillations with periods of

$$\lambda_x = \frac{(2)(20)(30)}{(3 \pm 1/2)} = 343 \text{ ft. to } 480 \text{ ft.}$$

$$\lambda_y = \frac{(2)(20)(30)}{(5 \pm 1/2)} = 218 \text{ ft. to } 267 \text{ ft.}$$

The presence of a range of values for a period (or frequency) arises from the fact that data were obtained from a sampled grid rather than from a continuous function.

Figure 4 can be regarded as a map, but in a frequency ($$\lambda_x$$, $$\lambda_y$$) rather than a distance ($$x$$, $$y$$) domain. In a sense, it represents a transformation of the original data so that frequency information is displayed rather than elevation information. Thus, there is no correspondence between a point in the original $$x$$, $$y$$ coordinate space and the frequency space of Figure 4.

The spectral characterization of a dendritic drainage pattern may be considered as a geological example. The original area is a region taken from the Lone Star 7-1/2 minute Quadrangle, Douglas County, Kansas (Preston and Harbaugh, 1965). The topographic map of this area is shown as Figure 6. Figure 7 shows data from a 25 x 50 grid on this map printed and hand contoured. For this example, only the scaled power spectrum map is shown (Fig. 8). It is possible that a similar pattern is characteristic of all dendritic drainages. If this technique is to be used to characterize and compare other areas, some map orientation convention must be developed, such as aligning the primary grid parallel to the main or average stream direction.

In the Lone Star Quadrangle data, significant values in the power spectrum are concentrated vertically. This reflects greater variation of the data along a vertical axis, as is evident in the topographic map. If the grid axes are rotated, the power spectrum also rotates. If an adjacent area in this physiographic provenance were analyzed, the spectrum would be nearly identical. It is this similarity in the spectra of similar landforms that could provide a basis for landform classification.

REFERENCES


Listing of FORTRAN IV statements in power-spectrum program.

PSMAIN IS THE MAINLINE ROUTINE FOR PS AND PLOT SUBROUTINES. IT WILL PROCESS UP TO NINE DATA DECKS ACCORDING TO OPTIONS ON THE CONTROL CARDS.

PROGRAMMED BY J. E. ESLER, KANSAS UNIVERSITY

AUGUST 1966

DIMENSION D(100,100),Q(25,50),IDENT(13),IFMT(10)
READ(5,8) NMBR
DO 100 J=1,NMBR
  READ(5,8)KALL,IPLOTA,IPLOTR,IPLOTS,IPNCHA,IPNCHS,IDENT,LAG,IBID
100 IN=M,IFMT,IFIRST
8 FORMAT(611/13A6/I3,I1,2I3,10A6/I1)
  LP1=LAG+1
  LLP1=LAG+LP1
  IF(KALL.EQ.2) GO TO 10
  IF(IBID.NE.1) GO TO (30,40),IFIRST
  READ(5,14)(Q(K,L),K=1,LP1),L=1,LLLP1
  GO TO 20
30 READ(5,IFMT)((D(K,L),K=1,N),L=1,M)
  GO TO 20
40 READ(5,IFMT)((D(K,L),L=1,M),K=1,N)
20 CALL PS(D,M,N,LAG,IBID,IPLOTA,IPLOTR,IPLOTS,IDENT,IPNCHA,IPNCHS,Q)
  GO TO 100
10 IF(IBID.EQ.0) WRITE(6,6)J
  READ(5,IFMT)((Q(K,L),K=1,LP1),L=1,LLLP1)
6 FORMAT(1H1,35H ERROR IN CONTROL CARD FOR DATA SET 13,4BH, RAW OR L 1EVELED DATA CANNOT BE PLOTTED BY PLOT .1X,35H WILL ATTEMPT TO SKIP 2 TO NEXT DECK.)
290
50 KK=IPLOTA
   IF(IBID.EQ.2) KK=IPLOTS
   CALL PLOT(Q,LAG,KK,IBID,IDENT)
100 CONTINUE
   CALL EXIT

PS IS A SUBROUTINE TO COMPUTE THE TWO DIMENSIONAL POWER SPECTRUM. OPTIONS IN THE ARGUMENT LIST DETERMINE MATERIAL TO BE PLOTTED OR PUNCHED ON CARDS.

ALL PLOTTING IS DONE BY THE SUBROUTINE PLOT, WHICH IS CALLED INTERMS OF THE PROGRAM IS RUN.

PROGRAMMED BY J. E. ESLER, KANSAS UNIVERSITY

AUGUST 1966

SUBROUTINE PS(D,M,N,LAG,IBEGN,IPLOTA,IPLOTR,IPLOTS,IDENT,IPNCHA,IPNCHS,Q)
  DIMENSION D(100,100),Q(25,50),SR(27,52),IDENT(13)
  IF(LAG.GT.0) GO TO 11
  WRITE(6,12)LAG
12 FORMAT(1X,43HILLEGAL VALUE OF LAG ENTERED INTO PS. LAG =,I1)
   RETURN
11 IF(IRFGN.LE.1) GO TO 21
C

OPTIONAL DATA LEVELING

MN=M*N
A11=FLOAT((N+1)*(2*N+2))/6
A12=FLOAT((N+1)*(M+1))*0.25
A13=FLOAT((N+1))*0.5
A22=FLOAT((M+1)*(2*M+2))/6
A32=FLOAT((M+1))*0.5
A33=MN
DO 10 J=1,N
DO 10 K=1,M
A21=A21+FLOAT(J)*D(J,K)
A23=A23+FLOAT(K)*D(J,K)
10 A31=A31+D(J,K)
T1=A12*A13-A11*A32
T2=A13**2-A11*A33
T3=A21*A13-A11*A31
T4=A12**2-A11*A22
C3=((A21*A12-A23*A11)*T1-T3*T4)/(T1**2-T2*T4)
C2=(T3-C3*T2)/T1
C1=(A31-A33*C3-A32*C2)/A13
DO 20 J=1,N
DO 20 K=1,M
20 D(J,K)=D(J,K)-C1*FLOAT(J)-C2*FLOAT(K)-C3

C

OPTIONAL PUNCH OF LEVELED DATA

IF(IEQG^NE.3) GO TO 21
WRITE(7*82) IDENT
WRITE(7*22) LAG*M,N
WRITE(7*B1)((D(J,K),J=1,N),K=1,M)
22 FORMAT(13,H0*213,3H0*5E16,9)*52X,1H1)
21 LP1=LAG+1
LLP1=LP1+LAG
IF(IEQG^EQ.1) GO TO 71
KK=1
IQU=LP1
IQL=1
C

COVARIANCE MATRIX

30 DO 40 IQ=IQL,IQU
IT=LP1-IQ
IF(IT^LT.0) IT=IT
KU=M-IT
KL=1
IF(IQ^GT.LP1) GO TO 50
KU=M
KL=IT+1
50 DO 40 IP=1,LP1
NMP=N-IP+1
SUM=0.
DO 60 K=KL,KU
KPQ=K+IT*KK
DO 60 J=1,NMP
JPP=J+IP-1
60 SUM = SUM+D(J,K)*D(JPP,KPQ)
40 Q(IQ,IQ)=SUM/FLOAT(NMP*(M-IT))
LP2=LAG+2
IF(IQU^NE.LP1) GO TO 70
IQU=LLP1
IQL=LP2
KK=1
GO TO 30
70 LPL=LAG+LAG

11
OPTIONAL PLOT OF COVARIANCE MATRIX
IF (IPLOTA .NE. 0) CALL PLOT(Q,LAG,IPLOTA,1,IDENT)

OPTIONAL PUNCH OF COVARIANCE MATRIX
IF (IPNCHA .NE. 1) GO TO 71
WRITE (7,82) IDENT
WRITE (7,84) LAG
WRITE (7,81) ((Q(IP, IQ), IP=1,LP1), IQ=1,LLP1)

71 DO 80 IP=2,LAG
DO 80 IQ=2,LP1
80 Q(IP, IQ)=Q(IP, IQ)*2.
Q(1, LLP1) = 5*Q(1, LLP1)
Q(1,1) = 5*Q(1,1)
Q(LP1, LLP1) = 5*Q(LP1, LLP1)
Q(LP1,1) = 5*Q(LP1,1)

81 FORMAT (5E16.9)
82 FORMAT (13A6)
83 RLAG=LAG
84 FORMAT (13,1H1,6X,8H(5E16.9))
PI=3.1415926
RC=COS(PI*RLAG)
RS=SIN(PI*RLAG)
CRP=1.
SRP=0.
CT=RC
ST=RS
CI=COS(PI/RLAG)
SI=SIN(PI/RLAG)

ROUGH POWER SPECTRUM BY FOURIER TRANSFORM
DO 90 IR=1,LP1
CSQ=-1.
SSQ=0.
DO 100 IS=1,LLP1
CIS=CT
SIS=ST
SUM=0.
DO 110 IQ=1,LLP1
CII=CT
SIQ=ST
DO 120 IP=1,LP1
SUM=SUM+Q(IP, IQ)*CT
TEMP=CT
CT=CT*CRP-ST*SRP
120 ST=ST*CRP+TEMP*SRP
CT=CIQ*CSQ-SIQ*SSQ
110 ST=SIQ*CSQ+CIQ*SSQ
SR(IR+1, IS+1)=SUM/FLOAT(2*LAG**2)
CT=-CIS
ST=-SIS
TEMP=CSQ
CSQ=CSQ*CI-SSQ*SI
100 SSQ=SSQ*CI+TEMP*SI
CT=RC
ST=RS
TEMP=CRP
CRP=CRP*CI-SRP*SI
90 SRP=SRP*CI+TEMP*SI
LLP3=LLP1+2
DO 130 IR=2,LP2
SR(IR,1)=SR(IR,3)
130 SR(IR,LLP1+2)=SR(IR,LLP1)
DO 140 IS=1,LLP3
SR(1,IS) = SR(3,IS)
140 SR(LP1+2,IS) = SR(LP1,IS)
IF(IPLotr EQ 0) GO TO 142
DO 141 IR=1,LP1
DO 141 IS=1,LLP1
141 Q(IR,IS) = SR(IR+1,IS+1)
CALL PLOT(Q,LAG,IPLotr+3,IDENT)
C SMOOTHING OF POWER SPECTRUM
142 DO 150 IR=1,LP1
DO 150 IS=1,LLP1
150 Q(IR,IS) = 0.625*(SR(IR+2,IS+2)+SR(IR+2,IS)+SR(IR,IS)+SR(IR,IS+2))
1+0.125*(SR(IR+1,IS+2)+SR(IR+1,IS)+SR(IR+2,IS+1)+SR(IR,IS+1))
2*SR(IR+1,IS+1)
C OPTIONAL PUNCH OF SMOOTH SPECTRUM
IF(IPNCHS NE 1) GO TO 151
WRITE(7,82) IDENT
WRITE(7,152) LAG
152 FORMAT(13,1H2,6X,8H(5E16.9))
WRITE(7,81)(I(IP,IP),IP=1,LP1,IP=1,LLP1)
C OPTIONAL PLOT OF SMOOTH SPECTRUM
151 IF(IPLOTS NE 0) CALL PLOT(Q,LAG,IPLots+2,IDENT)
RETURN
END

C PLOT IS THE OUTPUT SUBROUTINE FOR THE POWER SPECTRUM SUBROUTINE
C PS* IT IS CALLED INTERNALLY FROM PS ACCORDING TO OPTIONS IN THE PS PLOT
C ARGUMENT LIST* PLOT CAN ALSO BE USED SEPARATELY TO PLOT DATA PUNCHED
C OUT BY PS* BY CALLING IT FROM THE MAINLINE ROUTINE, MAIN*
C PROGRAMMED BY J. E. ESSLER, KANSAS UNIVERSITY
C AUGUST 1966*
C
SUBROUTINE PLOT(Q,LAG,METH,IDENT)
DIMENSION IP(12),Q(25,50),IS(97),IHOLD(97,24),IDENT(13)
DATA IP1H1,1H1,1H2,1H3,1H4,1H5,1H6,1H7,1H8,1H9,1H10,1H-
RMAX=Q(1,1)
M=LAG+1
N=M+LAG
F=4.9999999
DO 10 J=1,M
DO 10 K=1,N
10 IF(Q(J,K) GT RMAX) RMAX=Q(J,K)
IF(METH EQ 2) GO TO 100
C NUMERICAL PLOT
IF(ID EQ 1) WRITE(6,1) IDENT
IF(ID EQ 2) WRITE(6,2) IDENT
IF(ID EQ 3) WRITE(6,9) IDENT
1 FORMAT(1H1,29H AUTOCORRELATION FUNCTION FOR*1X,13A6)
2 FORMAT(1H1,19H POWER SPECTRUM FOR*1X,13A6)
9 FORMAT(1H1,25H ROUGH POWER SPECTRUM FOR*1X,13A6)
WRITE(6,5)
5 FORMAT(1X,52H NUMERICAL LISTING, SYMMETRY ABOUT ORIGIN NOT SHOWN*
WRITE(6,6) LAG
6 FORMAT(1X,24H MAXIMUM VALUE OF LAG IS*13)
PWR=100000.
20 R=RMAX*PWR+F
IF(R LT 1000) GO TO 30
PWR=PWR*1
PLOT 010
C
PLOT 020
C
PLOT 030
C
PLOT 040
C
PLOT 050
C
PLOT 060
C
PLOT 070
C
PLOT 080
C
PLOT 090
C
PLOT 100
C
PLOT 110
C
PLOT 120
C
PLOT 130
C
PLOT 140
C
PLOT 150
C
PLOT 160
C
PLOT 170
C
PLOT 180
C
PLOT 190
C
PLOT 200
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PLOT 210
C
PLOT 220
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PLOT 230
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PLOT 240
C
PLOT 250
C
PLOT 260
C
PLOT 270
C
PLOT 280
C
PLOT 290
C
PLOT 300
C
PLOT 310
C
PLOT 320
C
PLOT 330
C
PLOT 340

13
GO TO 20
30 WRITE(6,4)PWR
   DO 40 K=1,N
      KI=N-K+1
   DO 50 J=1,M
      JS(J)=Q(J,KI)*PWR+F
50 IF(Q(J,KI)>0.) JS(J)=Q(J,KI)*PWR-F
3 FORMAT(1X,25I5)
   IF(METH.EQ.1) RETURN
   CONTINUE
C   MAP PLOT
100 S=RMAX*1
   IF(ID.EQ.1) WRITE(6,1)IDENT
   IF(ID.EQ.2) WRITE(6,2)IDENT
   IF(ID.EQ.3) WRITE(6,9) IDENT
   WRITE(6,7)
6 FORMAT(1X,35H CONTOUR MAP PLOT SHOWING SYMMETRY)
   WRITE(6,6)LAG
   SC=1/S
   WRITE(6,4) S
   4 FORMAT(1X,18H SCAlING FACTOR IS*F12.5//)
   L2=47+N
   NDX2=M
   L=49-N
   F=1.4999999
   DO 110 K=1,M
      NMK=N-K+1
      NDX1=M
      L1=L
      NDX3=48
      DO 120 J=1,M
         NN=Q(NDX1+K)*SC+F
         IF(Q(NDX1+K)>0.) NN=12
         IS(L1)=IP(NN)
         IF(NDX1.EQ.1) GO TO 140
         NN=HOLD*SC+F
         IF(HOLD>LT.0.) NN=12
         IS(L1+1)=IP(NN)
   140 NN=Q(J,NMK)*SC+F
   IF(Q(J,NMK)>0.) NN=12
   IS(NDX3)=IP(NN)
   IF(J.EQ.M) GO TO 160
   NN=HOLD*SC+F
   IF(HOLD>LT.0.) NN=12
   IS(NDX3+1)=IP(NN)
   160 NDX1=NDX1-1
   NDX3=NDX3+2
   120 L1=L1+2
   IS48=IS48
   IS47=IS47
   NN=HOLD*SC+F
   IF(HOLD>LT.0.) NN=12
   IS(47)=IP(NN)
   NN=Q(1,NMK)*SC+F
   IF(Q(1,NMK)>0.) NN=12
   IS(48)=IP(NN)
   WRITE(6,8) IS(LL),LL=L,L2
   8 FORMAT(2X,130A1)
   PLOT 350
   PLOT 360
   PLOT 370
   PLOT 380
   PLOT 390
   PLOT 400
   PLOT 410
   PLOT 420
   PLOT 430
   PLOT 440
   PLOT 450
   PLOT 460
   PLOT 470
   PLOT 480
   PLOT 490
   PLOT 500
   PLOT 510
   PLOT 520
   PLOT 530
   PLOT 540
   PLOT 550
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   PLOT 570
   PLOT 580
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   PLOT 600
   PLOT 610
   PLOT 620
   PLOT 630
   PLOT 640
   PLOT 650
   PLOT 660
   PLOT 670
   PLOT 680
   PLOT 690
   PLOT 700
   PLOT 710
   PLOT 720
   PLOT 730
   PLOT 740
   PLOT 750
   PLOT 760
   PLOT 770
   PLOT 780
   PLOT 790
   PLOT 800
   PLOT 810
   PLOT 820
   PLOT 830
   PLOT 840
   PLOT 850
   PLOT 860
   PLOT 870
   PLOT 880
   PLOT 890
   PLOT 900
   PLOT 910
   PLOT 920
   PLOT 930
   PLOT 940
   PLOT 950
1.61 1.49 1.47 1.45 1.43 1.41 1.39 1.37 1.35 1.33 1.31 1.29 1.27 1.25 1.23 1.21 1.19 1.17 1.15 1.13 1.11 1.09 1.07 1.05 1.03 1.01 0.99 0.97 0.95 0.93 0.91 0.89 0.87 0.85 0.83 0.81 0.79 0.77 0.75 0.73 0.71 0.69 0.67 0.65 0.63 0.61 0.59 0.57 0.55 0.53 0.51 0.49 0.47 0.45 0.43 0.41 0.39 0.37 0.35 0.33 0.31 0.29 0.27 0.25 0.23 0.21 0.19 0.17 0.15 0.13 0.11 0.09 0.07 0.05 0.03 0.01 0.01 0.03 0.05 0.07 0.09 0.11 0.13 0.15 0.17 0.19 0.21 0.23 0.25 0.27 0.29 0.31 0.33 0.35 0.37 0.39 0.41 0.43 0.45 0.47 0.49 0.51 0.53 0.55 0.57 0.59 0.61 0.63 0.65 0.67 0.69 0.71 0.73 0.75 0.77 0.79 0.81 0.83 0.85 0.87 0.89 0.91 0.93 0.95 0.97 0.99 1.01 1.03 1.05 1.07 1.09 1.11 1.13 1.15 1.17 1.19 1.21 1.23 1.25 1.27 1.29 1.31 1.33 1.35 1.37 1.39 1.41 1.43 1.45 1.47 1.49 1.51 1.53 1.55 1.57 1.59 1.61 1.63 1.65 1.67 1.69 1.71 1.73 1.75 1.77 1.79 1.81 1.83 1.85 1.87 1.89 1.91 1.93 1.95 1.97 1.99 2.01 2.03 2.05 2.07 2.09 2.11 2.13 2.15 2.17 2.19 2.21 2.23 2.25 2.27 2.29 2.31 2.33 2.35 2.37 2.39 2.41 2.43 2.45 2.47 2.49 2.51 2.53 2.55 2.57 2.59 2.61 2.63 2.65 2.67 2.69 2.71 2.73 2.75 2.77 2.79 2.81 2.83 2.85 2.87 2.89 2.91 2.93 2.95 2.97 2.99 3.01 3.03 3.05 3.07 3.09 3.11 3.13 3.15 3.17 3.19 3.21 3.23 3.25 3.27 3.29 3.31 3.33 3.35 3.37 3.39 3.41 3.43 3.45 3.47 3.49 3.51 3.53 3.55 3.57 3.59 3.61 3.63 3.65 3.67 3.69 3.71 3.73 3.75 3.77 3.79 3.81 3.83 3.85 3.87 3.89 3.91 3.93 3.95 3.97 3.99 4.01 4.03 4.05 4.07 4.09 4.11 4.13 4.15 4.17 4.19 4.21 4.23 4.25 4.27 4.29 4.31 4.33 4.35 4.37 4.39 4.41 4.43 4.45 4.47 4.49 4.51 4.53 4.55 4.57 4.59 4.61 4.63 4.65 4.67 4.69 4.71 4.73 4.75 4.77 4.79 4.81 4.83 4.85 4.87 4.89 4.91 4.93 4.95 4.97 4.99 5.01 5.03 5.05 5.07 5.09 5.11 5.13 5.15 5.17 5.19 5.21 5.23 5.25 5.27 5.29 5.31 5.33 5.35 5.37 5.39 5.41 5.43 5.45 5.47 5.49 5.51 5.53 5.55 5.57 5.59 5.61 5.63 5.65 5.67 5.69 5.71 5.73 5.75 5.77 5.79 5.81 5.83 5.85 5.87 5.89 5.91 5.93 5.95 5.97 5.99 6.01 6.03 6.05 6.07 6.09 6.11 6.13 6.15 6.17 6.19 6.21 6.23 6.25 6.27 6.29 6.31 6.33 6.35 6.37 6.39 6.41 6.43 6.45 6.47 6.49 6.51 6.53 6.55 6.57 6.59 6.61 6.63 6.65 6.67 6.69 6.71 6.73 6.75 6.77 6.79 6.81 6.83 6.85 6.87 6.89 6.91 6.93 6.95 6.97 6.99 7.01 7.03 7.05 7.07 7.09 7.11 7.13 7.15 7.17 7.19 7.21 7.23 7.25 7.27 7.29 7.31 7.33 7.35 7.37 7.39 7.41 7.43 7.45 7.47 7.49 7.51 7.53 7.55 7.57 7.59 7.61 7.63 7.65 7.67 7.69 7.71 7.73 7.75 7.77 7.79 7.81 7.83 7.85 7.87 7.89 7.91 7.93 7.95 7.97 7.99 8.01 8.03 8.05 8.07 8.09 8.11 8.13 8.15 8.17 8.19 8.21 8.23 8.25 8.27 8.29 8.31 8.33 8.35 8.37 8.39 8.41 8.43 8.45 8.47 8.49 8.51 8.53 8.55 8.57 8.59 8.61 8.63 8.65 8.67 8.69 8.71 8.73 8.75 8.77 8.79 8.81 8.83 8.85 8.87 8.89 8.91 8.93 8.95 8.97 8.99 9.01 9.03 9.05 9.07 9.09 9.11 9.13 9.15 9.17 9.19 9.21 9.23 9.25 9.27 9.29 9.31 9.33 9.35 9.37 9.39 9.41 9.43 9.45 9.47 9.49 9.51 9.53 9.55 9.57 9.59 9.61 9.63 9.65 9.67 9.69 9.71 9.73 9.75 9.77 9.79 9.81 9.83 9.85 9.87 9.89 9.91 9.93 9.95 9.97
**Figure 1.** Autocorrelation function for SWOP data set 2A numerical listing, symmetry about origin not shown. Maximum value of lag is 20; scaling factor is 100,000,000.

**Figure 2.** Autocorrelation function for SWOP data set 2A contour map plot showing symmetry. Maximum value of lag is 20; scaling factor is 0.42040.
Figure 3. Rough power spectrum for SWOP data set 2A numerical listing, symmetry about origin not shown. Maximum value of lag is 20; scaling factor is 999.9999.

Figure 4. Power spectrum for SWOP data set 2A numerical listing, symmetry about origin not shown. Maximum value of lag is 20; scaling factor is 999.9999.

20
Figure 5. -Power spectrum for SWOP data set 2A contour map plot showing symmetry. Maximum value of lag is 20; scaling factor is 0.01949.

Figure 6. -Topographic map of northern part of Lone Star Quadrangle, Kansas; area is about ten miles southwest of Lawrence (from Preston and Harbaugh, 1965).
Figure 7. -Data values taken from topographic map (Fig. 6) and recontoured manually for comparison purposes. Data array contains 49 rows and 25 columns. Grid points are spaced about 1/10 mile apart (from Preston and Harbaugh, 1965).

Figure 8. -Power spectrum for C 6 BALGOL data dendritic topo data - Lone Star Quadrangle. Maximum value of lag is 8; scaling factor is 94165.27488.
PROGRAM ABSTRACT

Title (If subroutine state in title):

PSMAIN (mainline program title)

Subroutines PS and PLOT

Computer: GE 625 Date: October 9, 1967
Programming language: FORTRAN IV

Author, organization: James E. Esler and Floyd W. Preston
University of Kansas, Lawrence, Kansas, 66044

Direct inquiries to: Authors or

Name: D. F. Merriam Address: Kansas Geological Survey
University of Kansas, Lawrence

Purpose/description: Two-dimensional power spectrum can be used for numerical description of land
forms and possibly their classification.

Mathematical method: Fourier transform and autocorrelation function.

Restrictions, range: Data must be gridded, maximum is 100 by 100 points.

Storage requirements:

Equipment specifications: Memory 20K 40K 60K K 32

Automatic divide: Yes No Indirect addressing Yes No

Other special features required

Additional remarks (include at author's discretion: fixed/float, relocatability; optional: running time,
approximate number of times run successfully, programming hours) Running time increases rapidly as
the size of data array is increased. Maximum array will run in about 20 minutes. Program has also
been run on the IBM 7040.
COMPUTER CONTRIBUTIONS
Kansas Geological Survey
University of Kansas
Lawrence, Kansas

Computer Contribution

1. Mathematical simulation of marine sedimentation with IBM 7090/7094 computers, by J.W. Harbaugh, 1966 ........................................... $1.00
2. A generalized two-dimensional regression procedure, by J.R. Dempsey, 1966 ........................................... $0.50
3. FORTRAN IV and MAP program for computation and plotting of trend surfaces for degrees 1 through 6, by Mont O'Leary, R.H. Lippert, and O.T. Spitz, 1966 ........................................... $0.75
4. FORTRAN II program for multivariate discriminant analysis using an IBM 1620 computer, by J.C. Davis and R.J. Sampson, 1966 ........................................... $0.50
5. FORTRAN IV program using double Fourier series for surface fitting of irregularly spaced data, by W.R. James, 1966 ........................................... $0.75
6. FORTRAN IV program for estimation of cladistic relationships using the IBM 7040, by R.L. Bartcher, 1966 ........................................... $1.00
7. Computer applications in the earth sciences: Colloquium on classification procedures, edited by D.F. Merriam, 1966 ........................................... $1.00
8. Prediction of the performance of a solution gas drive reservoir by Muskat's Equation, by Apolonia Baca, 1967 ........................................... $1.00
9. FORTRAN IV program for mathematical simulation of marine sedimentation with IBM 7040 or 7094 computers, by J.W. Harbaugh and W.J. Wahlstedt, 1967 ........................................... $1.00
10. Three-dimensional response surface program in FORTRAN II for the IBM 1620 computer, by R.J. Sampson and J.C. Davis, 1967 ........................................... $0.75
11. FORTRAN IV program for vector trend analyses of directional data, by W.T. Fox, 1967 ........................................... $1.00
12. Computer applications in the earth sciences: Colloquium on trend analysis, edited by D.F. Merriam and N.C. Cocke, 1967 ........................................... $1.00
13. FORTRAN IV computer programs for Markov chain experiments in geology, by W.C. Krumbein, 1967 ........................................... $1.00
14. FORTRAN IV programs to determine surface roughness in topography for the CDC 3400 computer, by R.D. Hobson, 1967 ........................................... $1.00
15. FORTRAN II program for progressive linear fit of surfaces on a quadratic base using an IBM 1620 computer, by A.J. Cole, C. Jordan, and D.F. Merriam, 1967 ........................................... $1.00
16. FORTRAN IV program for the GE 625 to compute the power spectrum of geological surfaces, by J.E. Esler and F.W. Preston, 1967 ........................................... $0.75