Computing the Location of Shallow Seismic Event

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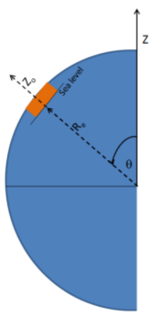
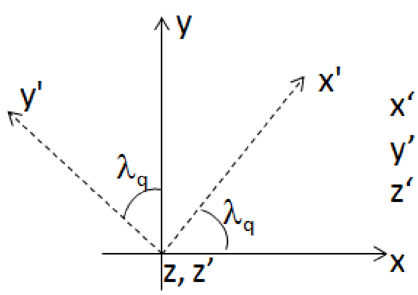
This analysis is in support of the South-central Kansas CO_2 Project, Small Scale Field Test Demonstrating CO_2 Sequestration. This document will derive the equations that will predict the location of shallow seismic events below a 15 sensor array, located around the Wellington KGS 2-32 Mississippian Injection Well. This analysis is designed to predict the location of any seismic event using 3 sensors to triangulate the position of the event. The solution to predicting the location of the seismic event is to translate from earth coordinate system to a shallow event coordinate system, using simple algebraic equations and trigonometry to create a series of equations that will give the location of the seismic event as latitude, longitude and depth with respect to the elevation of the sensors.



Using our Sensor array it should be possible to predict the depth of the seismic event using simple algebraic equations from three seismic sensors around and above the seismic event. The algebraic equations are three equations of a sphere, which theoretically can be reduced to give you latitude, longitude and depth of the seismic event.

Rotate the earth coordinates through the seismic event so the z-axis is the depth axis and then translation to the sea level, the latitude is the x-axis and the longitude becomes the y-axis and the depth is computed above or below the sea level.

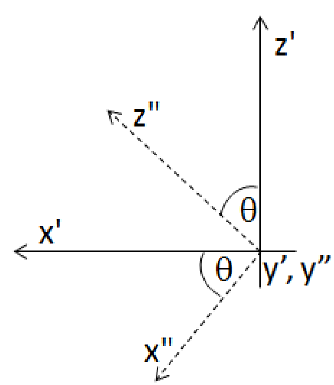
Rotate the x-y axis about the earth z-axis to the longitude of the seismic event (λ_q).



$$x' = x\cos\lambda q + y\sin\lambda_q$$
$$y' = -x\sin\lambda_q + y\cos\lambda_q$$
$$z' = z$$

Then rotate around the y'-axis down to the latitude of the seismic event. Since the latitude is measured from the equator the angle θ is related to the latitude (ϕ) as $\theta = \pi/2 - \phi$.

$$x'' = x'\cos\theta - z'\sin\theta$$
$$y'' = y'$$
$$z'' = x'\sin\theta + z'\cos\theta$$



Now combine the rotations to find x", y", z" coordinate to the earth's x, y, z coordinate.

$x'' = x \cos\theta \cos\lambda_q + y \cos\theta \sin\lambda_q - z \sin\theta$
$y'' = -x \sin\lambda_q + y \cos\lambda_q$
$z'' = x \sin\theta \cos\lambda_q + y \sin\theta \sin\lambda_q + z \cos\theta$

Now replace θ for latitude as $\theta = \pi/2 - \phi_q$ as follows,

$x'' = x \sin\phi_q \cos\lambda_q + y \sin\phi_q \sin\lambda_q - z \cos\phi_q$
$y'' = -x \sin\lambda_q + y \cos\lambda_q$
$z'' = x \cos\phi_q \cos\lambda_q + y \cos\phi_q \sin\lambda_q + z \sin\phi_q$

Now translate the z-axis up to sea level $z''' = z'' - Re$, where Re is the radius of the earth at sea level, which is approximately equal to 6371 km. Rename the x''', y''', z''' to x_o, y_o, z_o.

The coordinate of the sensors and seismic event is in the earth coordinate systems.

n^{th} sensor
$x_n = (R_e + h_n) \cos\phi_n \cos\lambda_n$
$y_n = (R_e + h_n) \cos\phi_n \sin\lambda_n$
$z_n = (R_e + h_n) \sin\phi_n$

seismic event
$x_q = (R_e + z_o) \cos\phi_q \cos\lambda_q$
$y_q = (R_e + z_o) \cos\phi_q \sin\lambda_q$
$z_q = (R_e + z_o) \sin\phi_q$

where
R_e = Radius of earth at sea level
h_n = Altitude of n^{th} -sensor above sea level
ϕ_n = latitude of n^{th} -sensor
λ_n = longitude of n^{th} -sensor

z_o = the depth above or below sea level
ϕ_q = latitude of seismic event
λ_q = longitude of seismic event

The n^{th} sensor x_n, y_n, z_n coordinates in the new rotation/translation coordinates is as follows,
$x_{no} = x_n \sin\phi_q \cos\lambda_q + y_n \sin\phi_q \sin\lambda_q - z_n \cos\phi_q$
$y_{no} = -x_n \sin\lambda_q + y_n \cos\lambda_q$
$z_{no} = x_n \cos\phi_q \cos\lambda_q + y_n \cos\phi_q \sin\lambda_q + z_n \sin\phi_q - R_e$

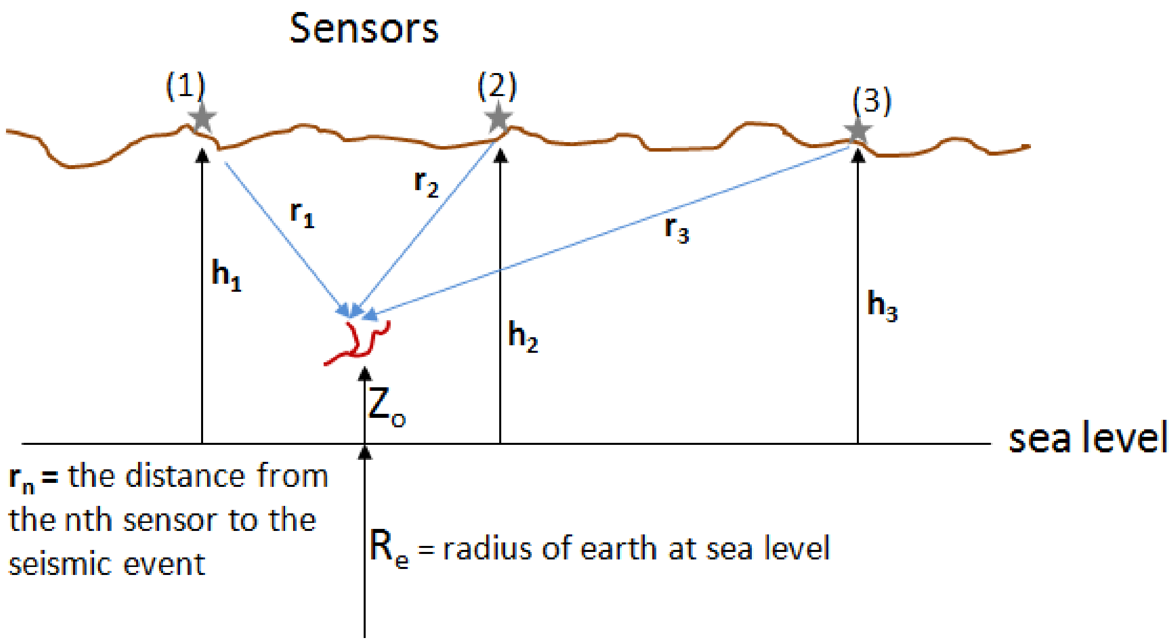
Substituting the trigonometric relations for x_n, y_n, z_n into the rotation/translation coordinates x_{no}, y_{no}, z_{no} is as follows,

$x_{no} = (R_e + h_n) [\sin\phi_q \cos\phi_n \cos(\lambda_n-\lambda_q) + \cos\phi_q \sin\phi_n]$
$y_{no} = (R_e + h_n) \cos\phi_n \sin(\lambda_n-\lambda_q)$
$z_{no} = (R_e + h_n) [\cos\phi_q \cos\phi_n \cos(\lambda_n-\lambda_q) + \sin\phi_q \sin\phi_n] - R_e$

The latitude and longitude angles differences are extremely small, i.e. $\lambda_n-\lambda_q$ and $\theta_n-\theta_q$ are on the order of 0.1 degrees or 0.00174533 radians. The cosine of 0.1 degrees is essentially 1 and can be set to 1. The sine on the other hand with the product of the earth radius can't be eliminated. Also notice that the altitude above sea level h_n is much less than the earth radius R_e ($h_n << R_e$) for x_{no} and y_{no}, $(R_e + h_n) \sim R_e$. z_{no} has been translated up to the sea level so the radius of the earth R_e is subtracted out so only the elevation of the nth sensor above sea level is left. The above equations reduces to

$x_{no} = R_e \sin(\phi_q - \phi_n)$
$y_{no} = R_e \cos\phi_n \sin(\lambda_n - \lambda_q)$
$z_{no} = h_n$

The rotation of the earth coordinate system is centered on the seismic event, which reduces the x_{qo}, y_{qo}, z_{qo} coordinates to 0.0, 0.0, z_o. z_o is the depth above or below the sea level, + or – value.



The equation of the location of the seismic event with respect to each of the sensors can be found from the equation of a sphere $x^2 + y^2 + z^2 = R^2$. For the n^{th} sensor the equation is,

$$(x_{qo} - x_{no})^2 + (y_{qo} - y_{no})^2 + (z_{qo} - z_{no})^2 = r_n^2$$

Where x_{qo}, y_{qo}, z_{qo} is the coordinates of the seismic event and x_{no}, y_{no}, z_{no} is the coordinates of the n^{th} sensor, $x_{qo} = y_{qo} = 0$ and $z_{qo} = z_o$. Also substituting x_{no}, y_{no}, z_{no} trigonometric relationships, reduces the above equation to,

$$R_e^2 \sin^2(\phi_q - \phi_n) + R_e^2 \cos^2\phi_n \sin^2(\lambda_n - \lambda_q) + (z_o - h_n)^2 = r_n^2 \qquad \text{(a)}$$

The above equation can be combined for all three sensors to find the latitude (ϕ_q), longitude (λ_q), and depth (z_o) of the seismic event. To make the equations manageable a number of approximations must be made. The first is to set $\cos^2\phi_n$ to an average of the sensors latitudes, i.e. $\phi = (\phi_1 + \phi_2 + \phi_3)/3$, but note that

$\cos\phi_n = \cos(\phi_n-\phi+\phi)$ or $\cos\phi_n = \cos(\phi_n-\phi)\cos\phi - \sin(\phi_n-\phi)\sin\phi$

In this case $\cos(\phi_n-\phi) \sim 1$ and $\sin(\phi_n-\phi) = 0$, so $\cos\phi_n \sim \cos\phi$.

The other variable is h_n which will be replaced with the average altitude above sea level, i.e. $h = (h_1 + h_2 + h_3)/3$. This last approximation can only be justified from the point of view that the sensors are basically nearly at the same elevation, but could be a real factor in zeroing in on the exact depth of the seismic event, which could be alleviated in the computer with some iterative process. The next thing is to redefine the center of the n^{th} sensor with respect to the 1^{st} sensor, this will help in reducing the three equations, do the following,

$\sin(\phi_q-\phi_n) = \sin(\phi_q-\phi_1+\phi_1-\phi_n)$
$\sin(\phi_q-\phi_n) = \sin(\phi_q-\phi_1)\cos(\phi_1-\phi_n) + \cos(\phi_q-\phi_1)\sin(\phi_1-\phi_n)$

Again assume that $\cos(\phi_q-\phi_1)$ and $\cos(\phi_1-\phi_n)$ are effectively equal to one then,

$\sin(\phi_q-\phi_n) \sim \sin(\phi_q-\phi_1) + \sin(\phi_1-\phi_n)$

The same can be done for the longitude, i.e.

$\sin(\lambda_n-\lambda_q) \sim -\sin(\lambda_q-\lambda_1) + \sin(\lambda_n-\lambda_1)$

Substituting all back into the n^{th} sensor equation (a),

$$R_e^2 (\sin(\phi_q-\phi_1) + \sin(\phi_1-\phi_n))^2 + R_e^2 \cos^2\phi (\sin(\lambda_q-\lambda_1) - \sin(\lambda_n-\lambda_1))^2 + (z_o - h)^2 = r_n^2$$

Expanding and rearranging the equation,

$$\begin{aligned}&R_e^2 \sin^2(\phi_q-\phi_1) + R_e^2 \cos^2\phi \sin^2(\lambda_q-\lambda_1) + (z_o - h)^2 \\ &\quad - 2 R_e^2 \sin(\phi_q-\phi_1) \sin^2(\phi_n-\phi_1) - 2 R_e^2 \cos^2\phi \sin(\lambda_q-\lambda_1) \sin^2(\lambda_n-\lambda_1) \\ &\qquad + R_e^2 \sin^2(\phi_n-\phi_1) + R_e^2 \cos^2\phi \sin^2(\lambda_n-\lambda_1) = r_n^2\end{aligned}$$

Now setting the equation for each sensor, understanding that $R_e^2 \sin^2(\phi_q-\phi_1) + R_e^2 \cos^2\phi \sin^2(\lambda_q-\lambda_1) + (z_o - h)^2 = r_1^2$. The equations for each sensor is as follows,

(1) $R_e^2 \sin^2(\phi_q-\phi_1) + R_e^2 \cos^2\phi \sin^2(\lambda_q-\lambda_1) + (z_o - h)^2 = r_1^2$

(2) $$\begin{aligned}&- 2 R_e^2 \sin(\phi_q-\phi_1) \sin^2(\phi_2-\phi_1) - 2 R_e^2 \cos^2\phi \sin(\lambda_q-\lambda_1) \sin^2(\lambda_2-\lambda_1) \\ &\quad + R_e^2 \sin^2(\phi_2-\phi_1) + R_e^2 \cos^2\phi \sin^2(\lambda_2-\lambda_1) = r_2^2 - r_1^2\end{aligned}$$

(3) $$\begin{aligned}&- 2 R_e^2 \sin(\phi_q-\phi_1) \sin^2(\phi_3-\phi_1) - 2 R_e^2 \cos^2\phi \sin(\lambda_q-\lambda_1) \sin^2(\lambda_3-\lambda_1) \\ &\quad + R_e^2 \sin^2(\phi_3-\phi_1) + R_e^2 \cos^2\phi \sin^2(\lambda_3-\lambda_1) = r_3^2 - r_1^2\end{aligned}$$

Now define $x_{nm} = R_e \sin(\phi_n-\phi_m)$ and $y_{nm} = R_e \cos\phi \sin(\lambda_n-\lambda_m)$ and substituting and rearranging the equations

(1) $x_{q1}^2 + y_{q1}^2 + (z_o - h)^2 = r_1^2$
(2) $2\, x_{q1}\, x_{21} + 2\, y_{q1}\, y_{21} = x_{21}^2 + y_{21}^2 - r_2^2 + r_1^2$
(3) $2\, x_{q1}\, x_{31} + 2\, y_{q1}\, y_{31} = x_{31}^2 + y_{31}^2 - r_3^2 + r_1^2$

Solving for x_{q1}, y_{q1} and z_o,

$$x_{q1} = \frac{[\ x_{31}^2 + y_{31}^2 - r_3^2 + r_1^2\ ]\, y_{21} - [\ x_{21}^2 + y_{21}^2 - r_2^2 + r_1^2\ ]\, y_{31}}{x_{31}\, y_{21} - x_{21}\, y_{31}}$$

$$y_{q1} = \frac{[\ x_{21}^2 + y_{21}^2 - r_2^2 + r_1^2\ ]\, x_{31} - [\ x_{31}^2 + y_{31}^2 - r_3^2 + r_1^2\ ]\, x_{21}}{x_{31}\, y_{21} - x_{21}\, y_{31}}$$

$$z_o = h - [\ r_1^2 - x_{q1}^2 - y_{q1}^2\ ]^{1/2}$$

Where the latitude and longitude can be found from x_{q1} and y_{q1} as follows

$$\phi_q = \phi_1 + \text{arcsine}(\ x_{q1} / R_e\ )$$

and

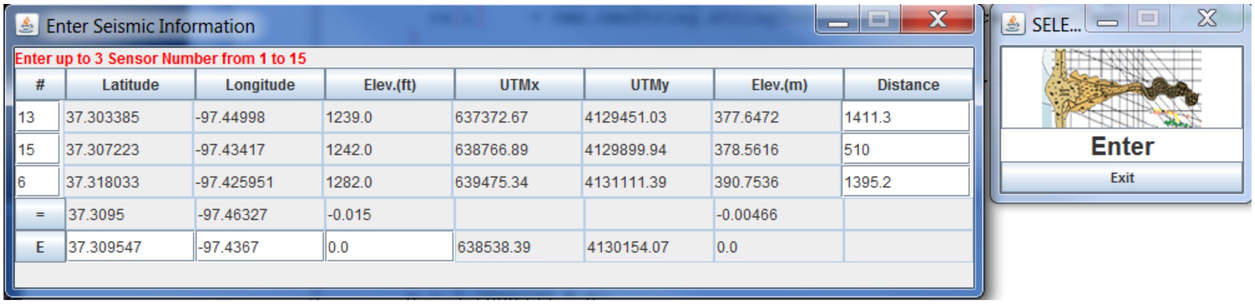
$$\lambda_q = \lambda_1 + \text{arcsine}(\ y_{q1} / (R_e \cos\phi)\ )$$

A simple program was created to predict a seismic event from the computed distances of the seismic event from 3 sensors. The test was performed on a hypothetical event under the 3 sensors at sea level or $z_o = 0.0$ [m].

Seismic Event: Latitude (ϕ_q) = 37.309547°, Longitude (λ_q) = -97.4367°, Depth (z_o) = 0.0 [m]

n	**latitude (ϕ_n)**	**longitude (λ_n)**	**Elevation (h_n) [km]**	**Distance (r_n) [m]**
13	37.303385	-97.449980	0.3776472	1411.3
15	37.307223	-97.434170	0.3785616	510.0
6	37.318033	-97.425951	0.3907536	1395.2

Inserting the latitude, longitude and expected depth of the seismic event for equation (a) at each sensor selected above as well as the distances expected with the selected sensors if there were no errors in the measured data. The Java program displayed the following output.



#	Latitude	Longitude	Elev.(ft)	UTMx	UTMy	Elev.(m)	Distance
13	37.303385	-97.44998	1239.0	637372.67	4129451.03	377.6472	1411.3
15	37.307223	-97.43417	1242.0	638766.89	4129899.94	378.5616	510
6	37.318033	-97.425951	1282.0	639475.34	4131111.39	390.7536	1395.2
=	37.3095	-97.46327	-0.015			-0.00466	
E	37.309547	-97.4367	0.0	638538.39	4130154.07	0.0	

Figure: The # column is the sensor number, = is the latitude, longitude and depth of the seismic event with the approximations applied to equation (a) and E is the actual values expected.

The results were close to the expected values, but in some cases it could be round off error with Java since Java's BigDecimal Math was not used. There was no actual event under the 3 sensors to show how good the program would do in predicting a seismic event's depth. The above algorithm is sensitive to the actual distances from the sensors to the seismic event, so large errors may not even be able to be solved with the present algorithm. The program can be modified to assume error and with some iteration zero in on the depth.

The same experiment was performed on an actual earthquake approximately 30 km from the sensor arrays. The test was to see just how "good" the approximation would be for an event away from the sensors with very little parallax. The USGS identified a 3.1 magnitude earthquake that occurred at 9:33 pm on 28 January 2015 at latitude of 37.093^{o} and longitude of -97.637^{o}. I used Swarm to get the time difference from all the sensors and picked 3 sensors to predict the location of the depth, 13, 15, and 6.

3.1 Magnitude Earthquake: Latitude (ϕ_q) = 37.093^{o}, Longitude (λ_q) = -97.637^{o}

n	latitude (ϕ_n)	longitude (λ_n)	Elevation (h_n)	Distance (r_n)	ΔT from swarm
13	37.303385	-97.449980	0.3776472 [km]	28.652 [km]	3.639 [sec]
15	37.307223	-97.434170	0.3785616 [km]	29.820 [km]	3.808 [sec]
6	37.318033	-97.425951	0.3907536 [km]	31.216 [km]	3.959 [sec]

The first experiment was to set the depth at 0.0 with the above earthquake latitude and longitude and the distance (r_n). The result for depth z_o was -1136.3 m, not even close. Inserting the above Δt from swarm and I wasn't able to get a result at all, because the above Δt's will not define a single point.

The present algorithm with the applied approximations can predict the location and depth of the seismic events under the seismic arrays. The accuracy will depend on how well the Δt's for the selected sensors can be determined and the average velocities of the shear (s) and compression (p) waves can be determined. We have "Davies" (Kansas Sample Log Service Company) Cuttings Report and Sonic Log for the Wellington KGS 1-28 that can be used to compute average velocities V_s and V_p under the sensors.