# The Role of Specific Yield in Ground-Water Recharge Estimations: A Numerical Study

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# The Role of Specific Yield in Ground-Water Recharge Estimations: A Numerical Study

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## ABSTRACT

This paper numerically demonstrates and quantifies the importance of capillary-fringe and variable specific yield phenomena in ground-water recharge estimations. A one-dimensional numerical experiment consisting of a soil either with a capillary fringe or without it was set up using a finite-element code. A prescribed infiltrating flux was superimposed on top of the soil column, and the resulting unsaturated-saturated water flow was observed. By assuming a single value for specific yield, recharge is usually overestimated. For two cases considered in this study, the errors involved in ground-water recharge estimations by using such an assumption ranged from 88 to 330 times the simulated recharge, when variable specific yield is considered. This study also clearly demonstrates the effect of a constant specific yield value on the behavior of the water table rise, the total amount of which as well as the rate of such rise are underestimated under that assumption. In addition, under that assmption, the timing of the water table rise is out of phase with the timing of recharge events.

## INTRODUCTION

The concept that the specific yield as defined in operational hydrology is not a unique porous medium property is well established (Childs, 1960; Youngs and Smiles, 1963; Dos Santos, Jr. and Youngs, 1969; Duke, 1972). It is also well understood that the shape of the water characteristic curve (that is, the relationship between capillary pressure head and soil-moisture content), the depth to the water table, and its rate of change affect the apparent values of specific yield obtained experimentally; and that any value of specific yield obtained under drainage or recharge conditions will be less than the maximum or ultimate specific yield which can be estimated from an equilibrium soil water profile (Duke, 1972). Discrepancies between observed and predicted shallow water table responses to recharge events based on constant specific yield values are often attributed to variable specific yield concepts and in particular to capillary fringe effects, the hydrologic significance of which has been recognized recently (Ragan, 1968; Sklash and Farvolden, 1979; O'Brien, 1982; Gillham, 1984). However, the above-mentioned advances have not yet precipitated into praxis in contemporary operational hydrology. Also, consideration of such ideas in the inverse problem in hydrology (that is, the problem of determining a set of hydrogeologic parameters, including specific yield, given some information about the system and its behavior) has not yet surfaced. The purpose of this paper is to numerically demonstrate and quantify the importance of capillary-fringe and variable specific yield phenomena in ground-water recharge-related studies.

In order to avoid any possible confusion, specific yield, capillary fringe and related concepts are defined as follows (Bear, 1972, 1979). *Specific yield* is defined as the average amount of water per unit volume of soil drained from a soil column extending from the water table to the ground surface, per unit lowering of the water table. The corresponding amount of water retained in the soil

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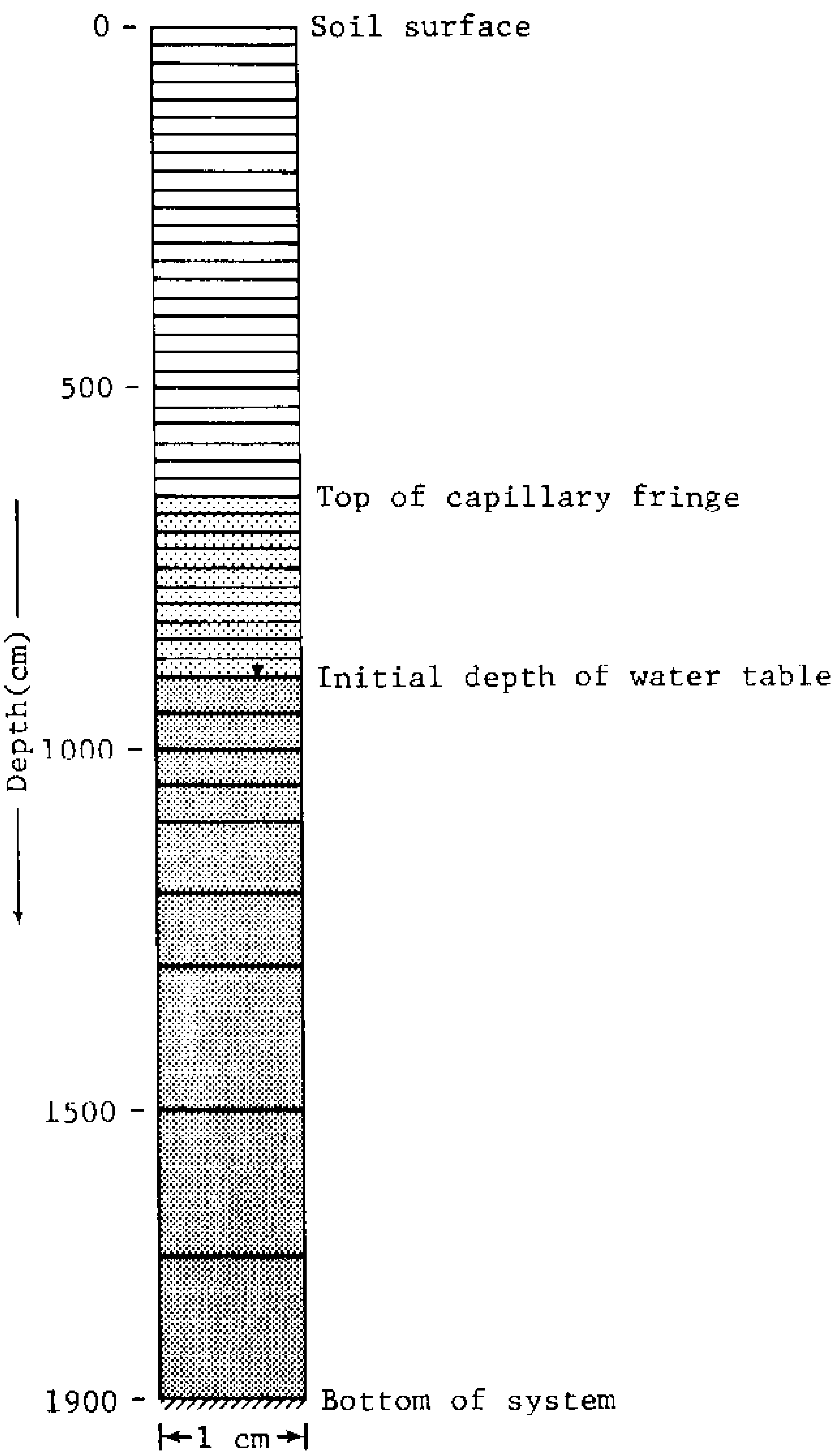

**Fig. 1. Soil column with superimposed finite-element network.**

against gravity when the water table is lowered is called *specific retention.* For a homogeneous isotropic soil and very deep water table, the specific retention is identical to the *field capacity* or to the *irreducible water saturation,* and the *effective porosity* is identical to the *specific yield.* The nearly saturated region immediately above the water table which extends from zero water pressure head to approximately the air entry value (if most of the reduction in water content takes place at the air entry value) is referred to as the *capillary fringe* or *zone of tension saturation.* The *air entry value* is the pressure head of the pore water where air begins to displace water, as evidenced by the first decrease in water content in the water characteristic (or desaturation) curve.

## NUMERICAL SIMULATIONS

### Numerical Experiment Setup

In order to gain more insight into the role of the capillary fringe in ground-water recharge-related phenomena, the following one-dimensional numerical experiment was run. A soil column 19 meters long was subdivided into a network of 45 rectangular elements as shown in Figure 1. All unsaturated elements had a uniform length of 0.25 m. The soil was initially saturated from a depth of 9 m to the bottom of the column, which is treated as a no-flow boundary. The physical properties of the soil, considered homogeneous and isotropic, include a saturated hydraulic conductivity, K, of 0.25 m/day, a uniform porosity value, $\phi$, of 0.30, a specific storativity value $(S_S)$ of $7.6 \times 10^{-5}\ m^{-1}$, and two water characteristic curves to be tested, one with a capillary fringe height corresponding to the air-entry value of 2.5 m of water and the other without a capillary fringe, as shown in Figure 2. For discussion purposes the first case will be referred to as the "fringe" case and the second as the "no fringe" case. Another "no-fringe" characteristic curve, C, shown in Figure 2 will be

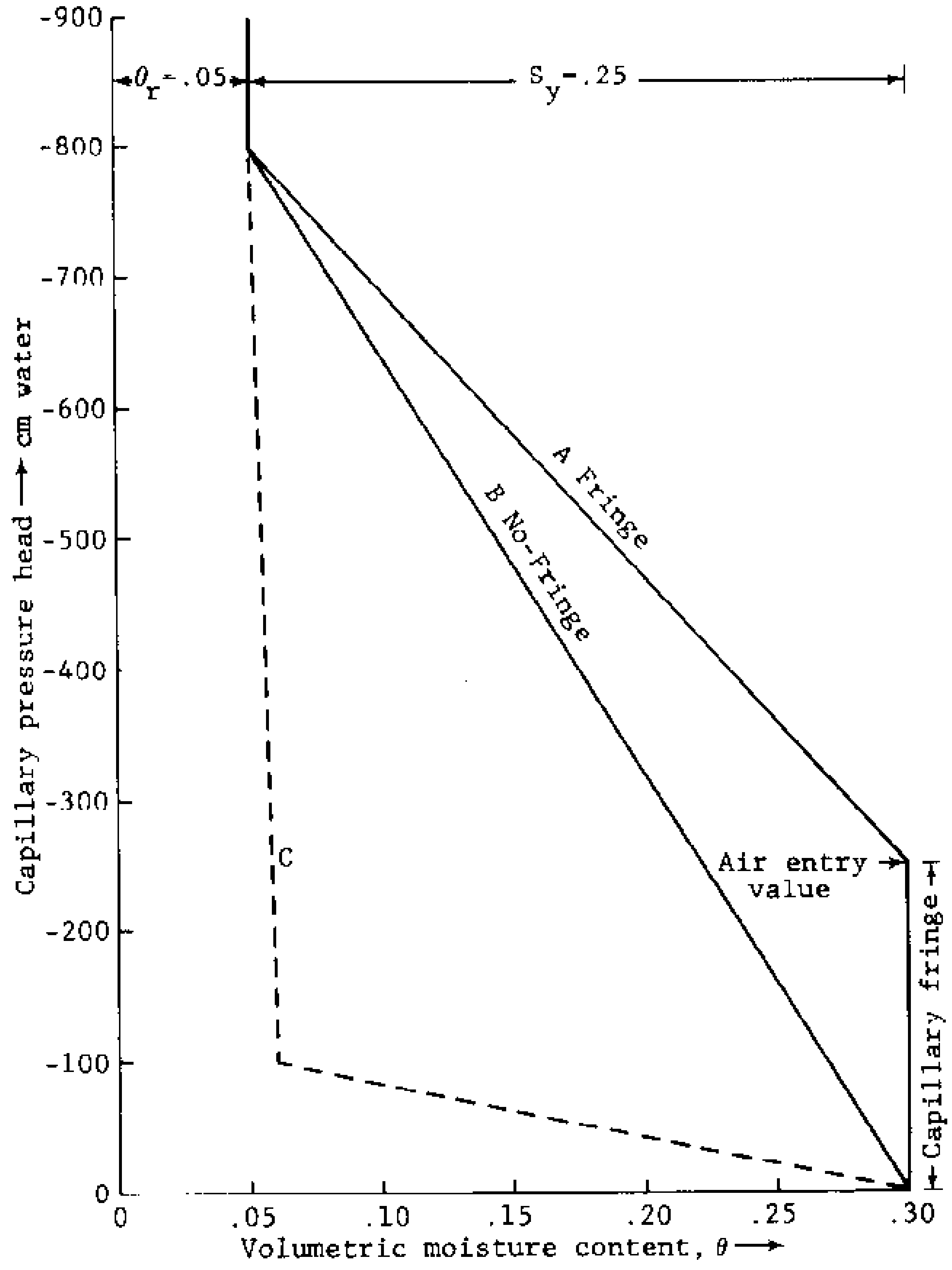

**Fig. 2. Water characteristic curves.**

discussed further on. The unsaturated hydraulic conductivity function for the above-mentioned two cases was obtained by employing the modified Millington-Quirk method (Millington and Quirk, 1959; Jackson *et al.*, 1965). The resulting unsaturated hydraulic conductivity functions are shown in Figure 3.

The soil-water profile is initially considered to be in static equilibrium with the water table, which in this case is sufficiently deep so that the water content at the soil surface is reduced to the specific retention (field capacity or irreducible water content) value, $\theta_r$, of 0.05 (Figure 2). Under such conditions then, the effective or drainable porosity, which is identical to the (ultimate) specific yield, $S_y$, is 0.25, and the capillary pressure head at any point is numerically equal to the elevation of that point above the water table. Therefore, the characteristic curves considered in this report represent the ***minimum*** water content to which the soil will drain by gravity as a function of height above the water table.

A constant flux top-boundary condition is superimposed on the equilibrium profile. Because the emphasis in this paper will be on ground-water recharge-related phenomena, an infiltration flux is employed. For one set of experiments this flux was held constant at the rate of 0.02 m/day until the entire profile was saturated. In another set of experiments, the same flux was applied for a certain time period after which it was stopped and the moisture redistribution in the profile observed.

The finite-element saturated-unsaturated flow model UNSAT2 (Neuman *et al.*, 1974) was employed to numerically simulate the above-mentioned experiment. This model outputs, among other things, the pressure and total hydraulic head for every node, as well as the water content at each unsaturated node for every time step. The UNSAT2 program was suitably modified for this study so as to output several additional items such as the exact position of the water table as depth to water, the unsaturated hydraulic conductivity at appropriate nodes, and the Darcian fluxes entering and leaving every element. Also, in order to reduce the size of the output in cases of lengthy simulations, the program was modified to display results at selected times.

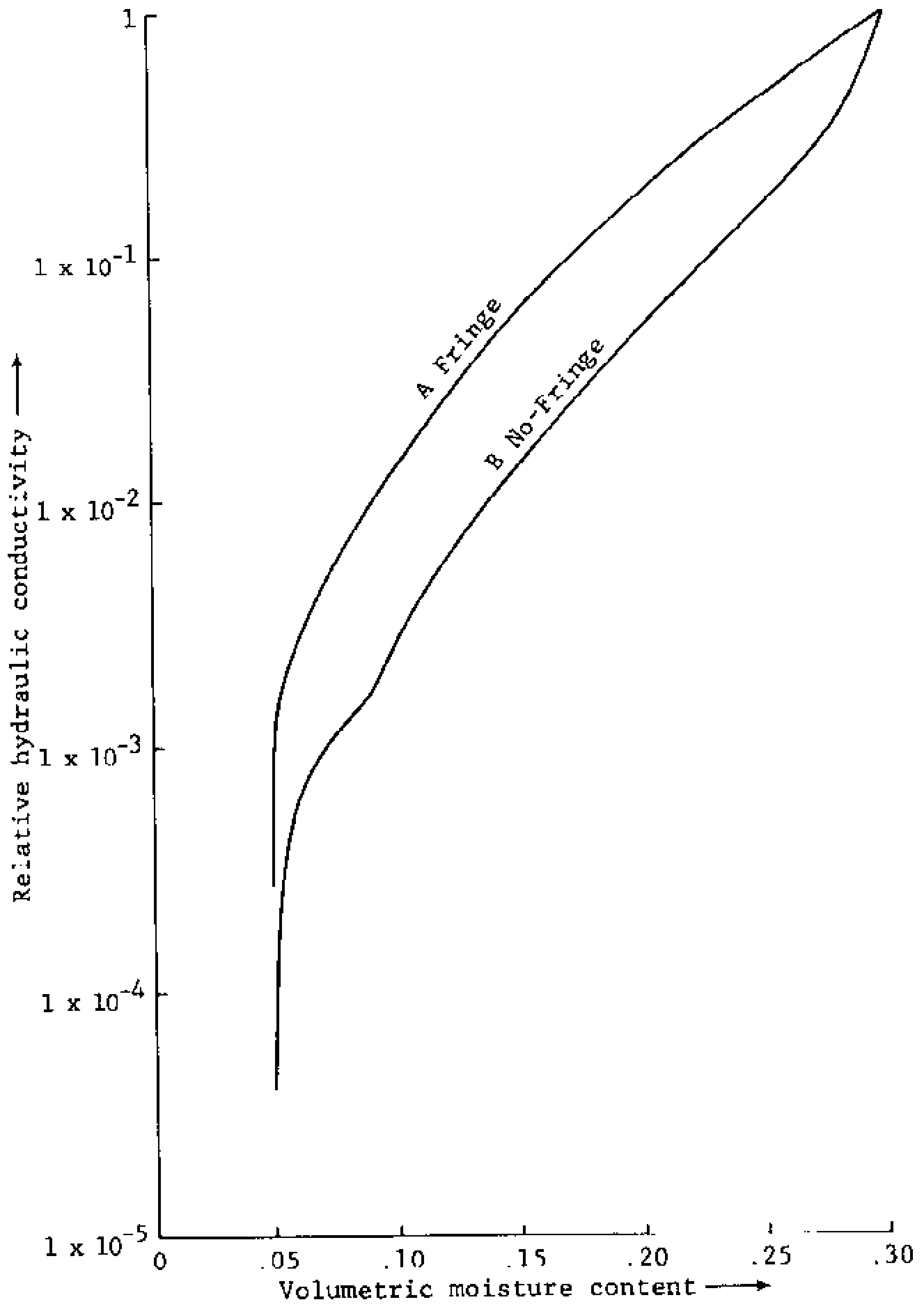

**Fig. 3. Relative hydraulic conductivity versus moisture content for the "fringe" and "no fringe" cases.**

A fully implicit backward difference scheme for time integration of the finite-element flow equations (Neuman *et al.*, 1974) was employed in all simulations. Preliminary runs of this model indicated that the numerical results from such simulations were very sensitive to the size of the time steps. Therefore, variable time steps depending on the magnitude of the water fluxes and ranging from 0.00002 to a maximum of 0.05 days were used. Initially, very small time steps were employed in the computations. This was necessary to insure convergence at small times because large hydraulic gradients were present near the soil surface. The time steps were lengthened as the computations progressed and the hydraulic gradients in the system became smaller. The time steps were again shortened whenever the fluxes at the water table were nearing their peak and large water-table rises were observed. Convergence was assumed whenever the maximum change in pressure head at all nodes in the finite-element network between two consecutive iterations during a time step did not exceed a specified tolerance of 0.005 m. In fact, this maximum change was much smaller than the specified tolerance in all simulations reported herein.

**Table 1. Simulated Results for the "Fringe" Case with a Continuous Surface Influx of 2 cm/day**

| 1<br>*Elapsed time (day)* | 2<br>*Depth to water (cm)* | 3<br>*Water table rise/day $\Delta h/\Delta t$ (cm/day)* | 4<br>*Cumulative water table rise (cm)* | 5<br>*Incoming flux in element where water table is located, $Q_{in}$ (cm/day)* | 6<br>*Customarily calculated recharge, R col. 3 * $S_y$ (cm/day)* | 7<br>*Calculated over simulated recharge, $R/Q_{in}$ col. 6 ÷ col. 5* |
|---|---|---|---|---|---|---|
| 0 | 900.0 | | | | | |
| 5 | 900.0 | 0.0 | 0.0 | 0.0000 | | |
| 10 | 897.7 | 1.4 | 2.3 | 0.0011 | 0.36 | 327.3 |
| 15 | 866.2 | 9.8 | 33.9 | 0.0075 | 2.44 | 325.3 |
| 20 | 803.1 | 13.6 | 97.0 | 0.0114 | 3.41 | 299.1 |
| 25 | 730.9 | 14.3 | 169.2 | 0.0130 | 3.57 | 274.6 |
| 30 | 655.8 | 14.8 | 244.3 | 0.0144 | 3.70 | 256.9 |
| 35 | 575.7 | 16.5 | 324.3 | 0.0168 | 4.13 | 245.8 |
| 40 | 482.7 | 20.5 | 417.3 | 0.0226 | 5.13 | 227.0 |
| 45 | 325.7 | 47.2 | 574.4 | 0.0630 | 11.80 | 187.3 |
| 46 | profile saturated | | | | | |

**Table 2. Simulated Results for the "No Fringe" Case with a Continuous Surface Influx of 2 cm/day**

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 900.0 | | | | | |
| 5 | 900.0 | 0.0 | | | | |
| 10 | 900.0 | 0.0 | | | | |
| 15 | 900.0 | 0.0 | | | | |
| 20 | 900.0 | 0.0 | | | | |
| 25 | 897.0 | 1.5 | 3.0 | 0.0033 | 0.37 | 112.1 |
| 30 | 871.4 | 8.5 | 28.6 | 0.0190 | 2.13 | 112.1 |
| 35 | 804.0 | 16.4 | 95.9 | 0.0385 | 4.10 | 106.5 |
| 40 | 703.4 | 22.1 | 196.6 | 0.0484 | 5.53 | 114.2 |
| 45 | 588.1 | 23.5 | 311.9 | 0.0568 | 5.88 | 103.5 |
| 50 | 468.0 | 24.3 | 432.0 | 0.0645 | 6.07 | 94.1 |
| 55 | 342.5 | 26.0 | 557.5 | 0.0725 | 6.50 | 89.7 |
| 60 | 194.9 | 34.2 | 705.1 | 0.0971 | 8.55 | 88.0 |
| 63 | profile saturated | | | | | |

## Numerical Results and Discussion

The "fringe" case using the water characteristic curve A (Figure 2) was run first with a continuous infiltration flux of 0.02 m/day. Some results from this simulation are shown in Table 1. The flux reaching the water table, $Q_{in}$, shown in column 5, is an average over a number of time steps during each daily period.

Comparison of the simulated influx at the water table $Q_{in}$ (column 5) to the customarily calculated recharge $R = (\Delta h/\Delta t)\ S_y$ (column 6) shows that the ratio $R/Q_{in}$ (column 7) ranges up to approximately 330 in this case. That is, the rate of rise of the water table, $\Delta h/\Delta t$, is much higher than the rate of influx to the water table, a phenomenon clearly affected by the existence of the capillary fringe. The percolating water readily converts the tension-saturated capillary fringe into a pressure-saturated zone, which results in a disproportionate rise in water level compared to the magnitude of the recharging flux. For further explanation of the physical basis of this capillary fringe phenomenon the reader is referred to O'Brien (1982) and Gillham (1984).

The "no fringe" case using the water characteristic curve B (Figure 2) was subsequently run under the same conditions as the "fringe" case. Some results from this run are shown in Table 2. Note that because of the drier initial profile (curve B in Figure 2), the profile took longer to saturate in comparison with the "fringe" case. Despite the fact that no capillary fringe is present in this case, the rate of rise of the water table, $\Delta h/\Delta t$, is still higher than the rate of influx to the water table; the ratio $R/Q_{in}$ ranges up to approximately 115 in this case. This rate of water table rise is, however, much lower compared to the one in the "fringe" case. The reason for the ratio $R/Q_{in}$ being as high as 115, even without the presence of a capillary fringe, lies in the fact that the available water content for drainage or recharge (i.e., the apparent specific yield) near the water table, where the above calculations are performed, is very small compared to the customarily assumed constant value of (ultimate) specific yield employed in calculating column 6 of Table 2. This phenomenon could be recognized easily by examining the characteristic curves A and B in Figure 2 which indicate that for a considerable distance above the water table, the water that will drain by gravity is significantly less than the ultimate specific yield.

In order to amplify the last point, a third simulation was run where the characteristic curve, C, indicated by the dashed line in Figure 2 is employed. This characteristic curve indicates that above a short distance from the water table, the water that will drain by gravity is not significantly less than the ultimate specific yield. This situation may correspond to a purely sandy soil as opposed to the characteristic curves A and B which may correspond to a loamy sand or sandy loam. The results of this simulation indicate that the ratio $R/Q_{in}$ is much smaller than both previous cases, ranging up to approximately 25. In this case, at least for some time period during the simulation, the available water for drainage or recharge is not as far away from the ultimate specific yield value as the previous two cases.

In all three cases considered above, the apparent specific yield ($Sy_a = \phi - \theta(z)$, where $\theta(z)$ is the volumetric water content as a function of depth) is variable and smaller than the ultimate specific yield ($S_y = \phi - \theta_r$), becoming negligibly small in the immediate vicinity of the water table where the water flux, $Q_{in}$, was calculated. This variance is why the calculated recharge based on the ultimate specific yield is much higher than the actual flux to the water table. The apparent specific yield decreases progressively as the water table approaches the ground surface until it becomes effectively zero when the top of the capillary fringe intersects the ground surface.

Another set of simulations involves the same "fringe" and "no fringe" cases examined previously but with the surface flux applied for a duration of 35 days, before complete saturation of the soil profile occurred, after which the surface flux is reduced to zero. The objective of this set of simulations is to examine the effect of considering the specific yield as a constant or variable hydrogeologic property on the response of the water table to temporal recharge events. The results of these simulations, presented as daily (D1) and cumulative (C1) water table rises and daily water table recharging influxes ($Q_{in}$), are shown in Figure 4 for the "fringe" case and in Figure 5 for the "no fringe" case. Also shown in the same figures are the expected daily (D2) and cumulative (C2) water table rises, based only on the knowledge of the ultimate specific yield (a constant) and the soil surface influx.

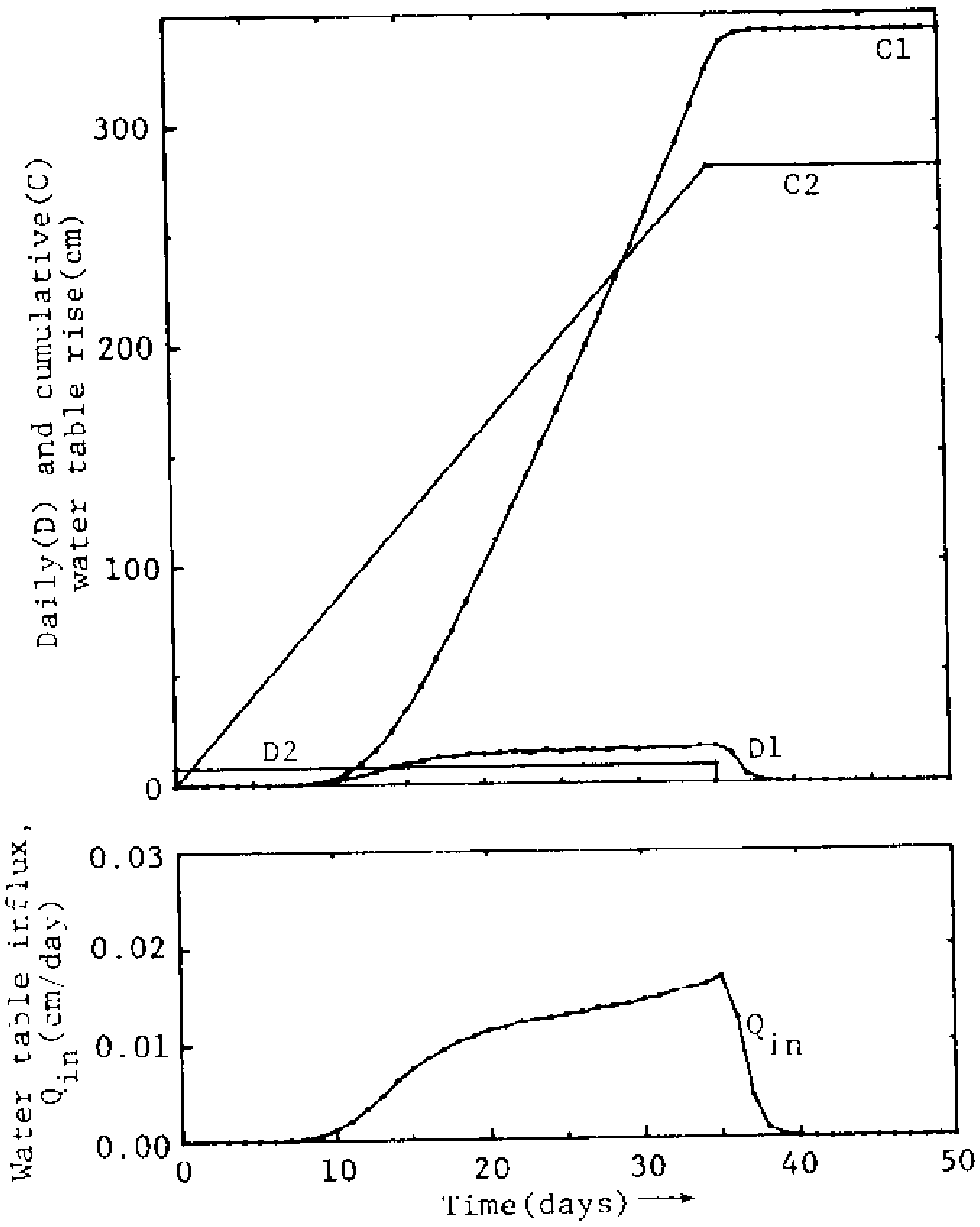

**Fig. 4. Water table influx, and daily and cumulative water table rises for the "fringe" case.**

Comparison of the cumulative rate of simulated water table rise (C1) with the cumulative rate of expected water table rise when employing a constant specific yield (C2; Figures 4 and 5) shows (a) different slope; (b) different total value in the cumulative rate of water table rise due to a recharge event; and (c) time lag. The simulated cumulative rate of water table rise when considering the rising response of the water table is steeper, of higher total value, and occurs later than the expected value based on a constant specific yield; these differences are more pronounced in the "fringe" case (Figure 4) as compared to the "no fringe" case (Figure 5). Note also that the simulated response (D1) of the water table to the considered infiltrating surface flux or the recharging flux, $Q_{in}$, is different (Figures 4 and 5) than the one expected on the basis of a constant

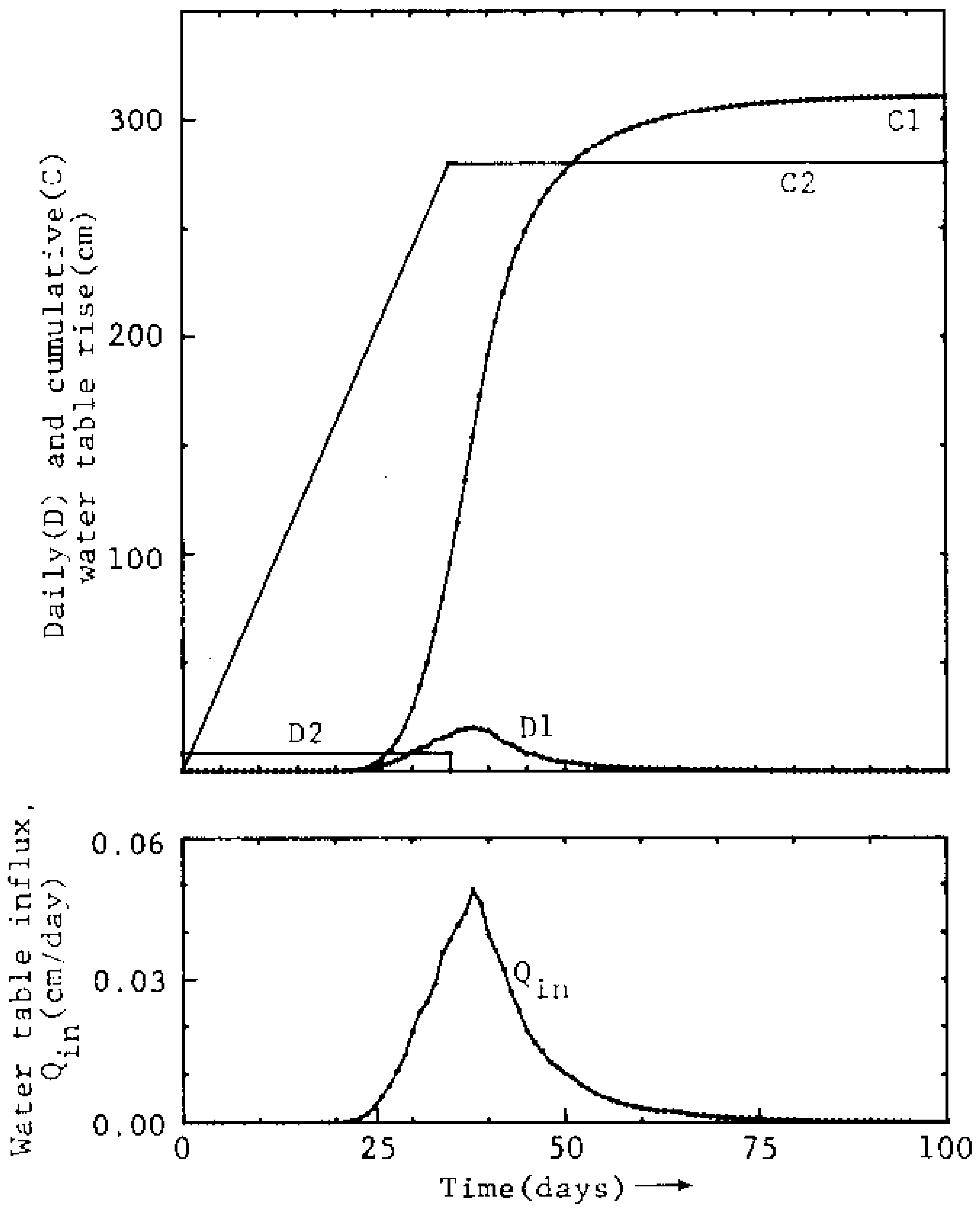

**Fig. 5. Water table influx, and daily and cumulative water table rises for the "no fringe" case.**

specific yield value (D2), the area under the D1 curve being larger than the corresponding one under D2.

All previous simulations considered only homogeneous and isotropic media. However, stratification effects on the water table response to recharge events also can be understood on the basis of the homogeneous cases described above. This relationship was verified by carrying out a set of two-layer simulations, one with a layer, A, characterized by the soil-water characteristic curve A in Figure 2 overlying another layer, B, characterized by the soil-water characteristic curve B in Figure 2, and another simulation with the reverse situation to the previous case. The interface between the two layers in both cases occurred at a depth of 6.5 m. The results of both sets of simulations indicate that the profile, where layer A was overlying layer B, was saturated earlier than the reverse case as expected. This earlier saturation is due to the fact that there was less unsaturated pore space available for the A/B layering than in the reverse case. This difference can be seen in Figure 6 where successive moisture profiles resulting from a continuous infiltrating flux of 0.1 m/day in the layered cases are shown.

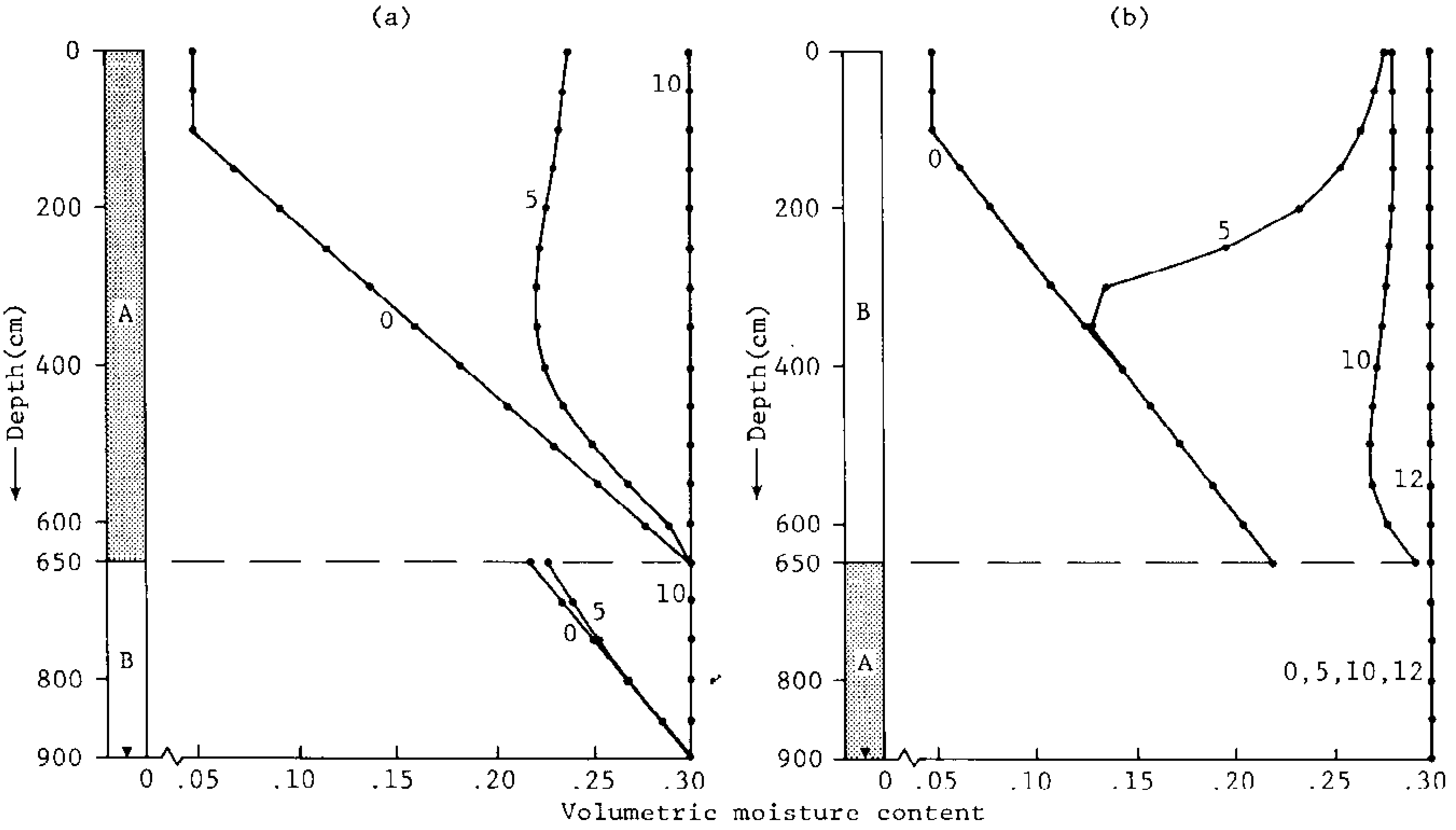

**Fig. 6. Soil column layering and moisture profiles during different times. Water characteristic curves A and B of Figure 2 characterize corresponding soil layers. Numbers by the moisture curves represent different times, in days, since the infiltration flux of 0.1 m/day was applied. (a) A/B layering; (b) B/A layering.**

## CONCLUSIONS

In light of the results of the numerical simulations presented in this paper, the following conclusions can be derived:

1. Calculation of a specific yield value by any inverse procedure based on water level changes and an assumed or estimated recharge value may be erroneous. Calculation of recharge values based on observed water level fluctuations and the ultimate specific yield value usually are overestimated. This study was able to quantify the errors involved in such recharge calculations for the cases considered here (Tables 1 and 2). Thus, by assuming a constant specific yield value, the estimated recharge ranged from 330 ("fringe" case) to 88 ("no fringe" case) times higher than the simulated recharge using the variable yield concept.

2. Estimated responses of the water table to recharge events based on soil surface infiltration fluxes and ultimate specific yield values not only are out of phase with the timing of recharge events, but also underestimate the rate and total amount of water table rise (Figures 4 and 5). Although the fact that the water table rise is out of phase with the timing of recharge events when considering constant specific yield is well known, the fact that the total amount of water table rise is underestimated and that the rate of cumulative water table rise is generally steeper than the one estimated by using a constant value of specific yield, is clearly demonstrated in this study.

3. Finally, capillary-fringe effects on groundwater recharge estimations may be significant, especially in shallow, fine-grained water table aquifers, and therefore should not be ignored.

## ACKNOWLEDGMENT

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