Ice-Core Data from Greenland and Near-Term Climate Prediction

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Abstract

Records from the GISP2 Greenland ice core are considered in terms of dynamical systems theory and nonlinear prediction. Dynamical systems theory allows us to reconstruct some properties of a phenomenon based only on past behavior without any mechanistic assumptions or deterministic models. A short-term prediction of temperature, including a mean estimate and confidence interval, is made for 800 years into the future. The prediction suggests that the present short-time global warming trend will continue for at least 200 years and be followed by a reverse in the temperature trend.

Keywords: climate, model, chaos, dynamical system, phase space.

Introduction

Predicting the global climate is one of the major unresolved challenges facing 21st century applied science. Most previous efforts in climate prediction have required constructing deterministic models of global climate systems based on equations from mathematical physics. These models consist of a complex of relationships in the form of diffusion equations, mass transfer equations, and mass balance conditions (Hansen and others, 1983; Russell, Miller and Rind, 1995). To be successful, such a deterministic approach must be founded on strict conceptual grounds and the resulting models must be implemented on extremely powerful computers.

Over the past 20 years, fundamentally new approaches to the prediction of time series have been developed, based on dynamical systems theory (general discussions of this approach are given in recent texts on time series analysis such as Weigend and Gershenfeld, 1994). These new procedures allow us to estimate certain fundamental properties that are required in theoretical models of nonlinear phenomena (such as the number of degrees of freedom in a system or its fundamental dimensions). More importantly from the viewpoint of climate modeling, these new procedures also provide a way to predict the near-term state of a complex system that exhibits chaotic behavior. Such approaches have already proven useful in scientific fields as diverse as physics, psychology, economy, and medicine, and seem sufficiently general and powerful enough to be useful for global climate modeling as well.

Records of climate change from Greenland ice core

Here, we will apply procedures of dynamical systems theory to two natural climate records and, for illustrative purposes, to one artificial record. The first natural record was produced by the Greenland Ice Sheet Project Two (GISP2), which investigated climatic and environmental changes over the past 250,000 years by analyzing a core drilled completely through the ice in the central part of the Greenland continental glacier (NSIDC, 1997) .

The ratio of the two isotopes of oxygen, ^{16}O and ^{18}O, varies according to temperature in water because the lighter isotope is more volatile. This ratio can be used as a proxy measure of atmospheric temperature because the relative amount of ^{18}O is greater in snow that has precipitated in warmer air. The oxygen isotope ratio is conventionally expressed as $\delta^{18}O$, which is standardized relative to the standard mean oxygen isotope ratio in sea water (SMOW). The relationship between atmospheric temperature and $\delta^{18}O$ in Greenland has been empirically determined (Johnsen, Dansgaard and White, 1992) to be $\delta^{18}O=0.67T\ (°C)-13.7$. Figure 1 shows the distribution of $\delta^{18}O$ averaged over 20 year intervals, back to 10,000 years B.P. (For conventions used in the GISP2 ice core record, see Davis and Bohling, 2000).

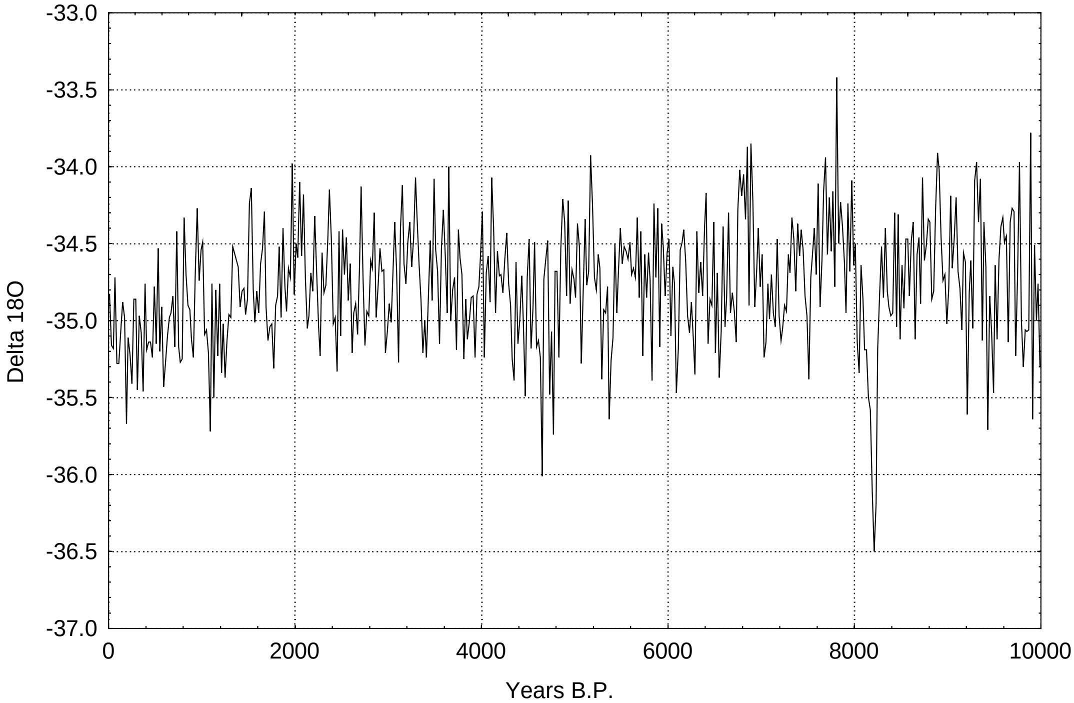

Fig. 1. 20-year average record of $\delta^{18}O$ for the period of time 0-10 kyr BP from the GISP2 ice core.

On cursory examination, the temperature record throughout the Holocene appears chaotic, but closer examination shows trends of increasing or decreasing average temperatures over specific intervals of time, such as during the last 200 years. Davis and Bohling (2000) have characterized the GISP2 record of 20-year average $\delta^{18}O$ values from a stochastic viewpoint. In contrast, we will examine the same record regarding it as the output from a dynamic, nonlinear system complicated by random influences.

The climate dynamics of the Pleistocene prior to the start of the Holocene differ greatly from the dynamics that have been in operation since the collapse of continental ice sheets in the northern hemisphere. This is apparent in the dramatic change in the $\delta^{18}O$ record that occurs about 10,000 years B.P. (Figure 2). The possible causes of this change are beyond the scope of this paper; interested readers can find an extensive discussion in Lowe and Walker (1998).

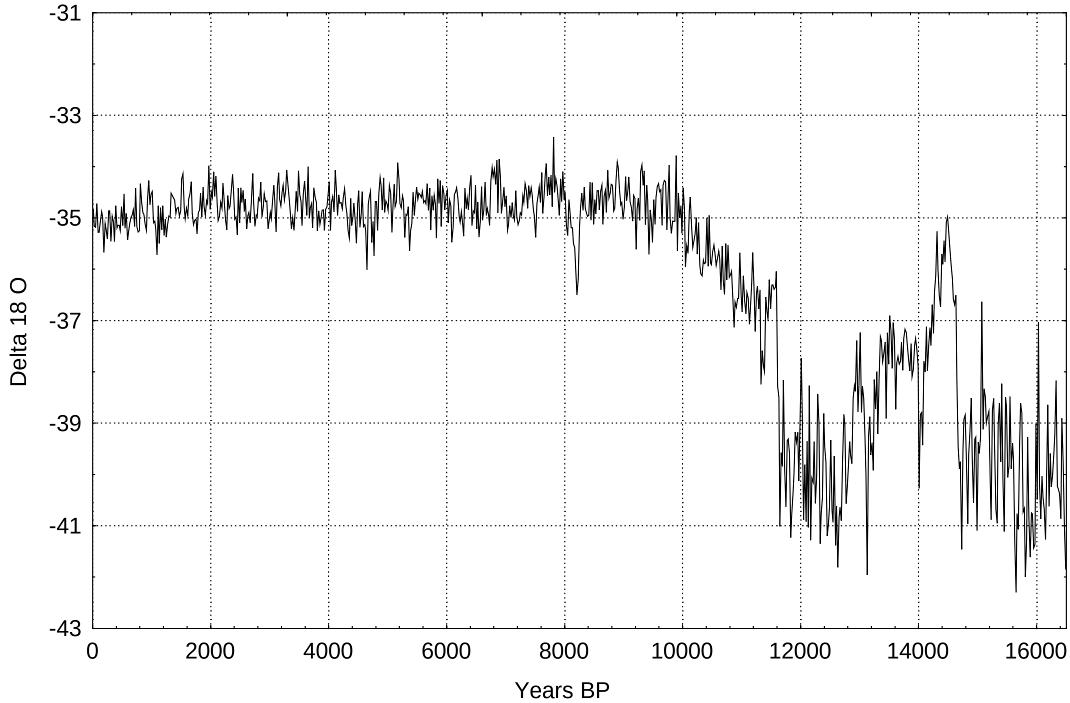

Fig. 2. 20-year average record of $\delta^{18}O$ for the period of time 0-16 kyr BP from the GISP2 ice core.

The GISP2 record of 20-year average $\delta^{18}O$ extends back only a short time into the Pleistocene (to 16,490 years B.P.) and consequently is not long enough to allow us to assess climate dynamics of the pre-Holocene interval. However, there are more extensive records of other constituents extracted from the GISP2 core, including Na, NH_4, K, Mg, Ca, Cl, NO_3, and SO_4. These variables can be combined into a single composite variable by principal component analysis (Gorsuch, 1983), yielding a new composite variable that is highly correlated with most of the measured constituents and which expresses more than 76% of the variation in all of the original variables. The record over time of this component is shown in Figure 3.

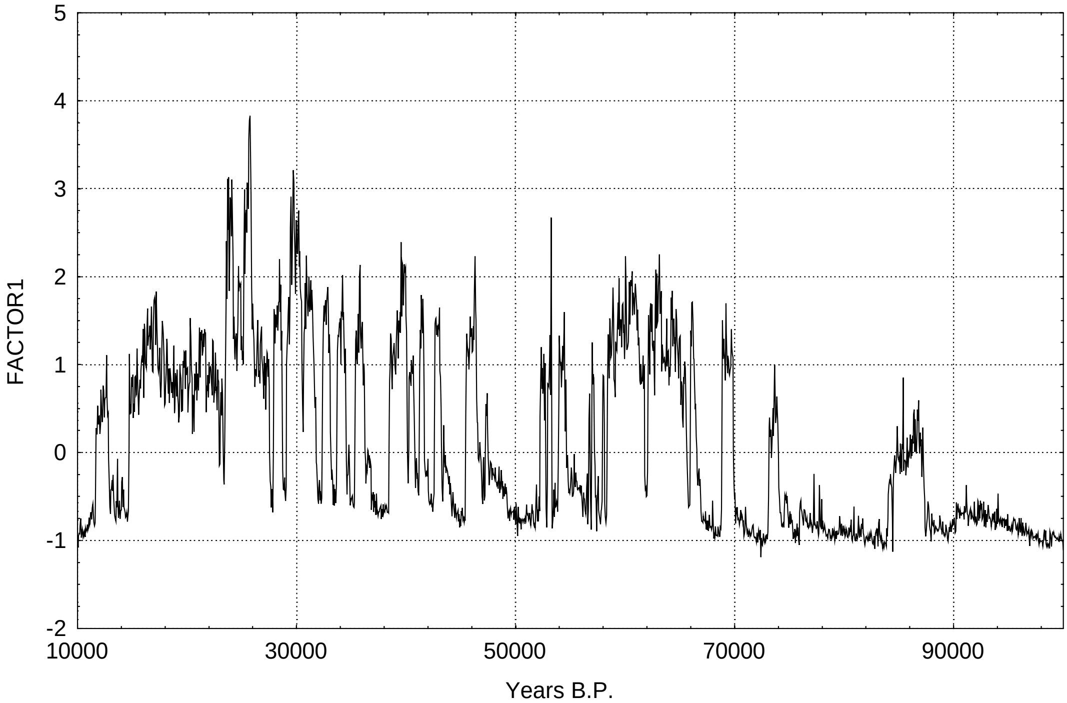

Fig. 3. Distribution of first principal component of chemical constituents from GISP2 ice core for the period of time 10-100 kyr B.P.

By combining the different variables measured on the GISP2 ice core into a single dominant component, we not only avoid the problem of choosing the most appropriate variable for analysis, we also suppress superfluous noise which is relegated to other, lesser components. An analysis by Mayewski and others (1997) yielded an essentially identical composite variable they called the Polar Circulation Index (PCI), which was interpreted as reflecting increased continental dust and marine aerosols during cold intervals. We note that the behavior of the first principal component strongly reflects climatic conditions, because it is approximately inversely proportional to the temperature at which ice forms. This has been determined by correlating the component to $\delta^{18}O$. The linear correlation coefficient over the last 16,000 years is $r = -0.91$, which is significantly different from zero. In any case, strict dependence between temperature and component is not important for our purpose, which is to examine the dynamics of climatic behavior.

The main characteristic of the GISP2 component record is its intermittent behavior, with intervals representing periods that were relatively warm (on average) that changed abruptly to periods that were cold. Within this general pattern, the record is characterized by high-frequency, low-amplitude oscillations. Causes of the major episodic alterations from relatively warm to cold and vice versa remain to be established. Most likely, these changes are a consequence of both external and internal influences that operated at a planetary scale. Possibly these long-term variations in climatic temperature were due to orbital forcing (Imbrie and others, 1993) modulated by changes in circulation within the oceanic and atmospheric covers of the Earth (Lowe and Walker, 1998).

The third record to be considered in this paper is artificial, the *x*-coordinate of the realization of mathematical equations that describe the Lorenz system. This system of equations was developed by Edward Lorenz by simplifying and linearizing hydrodynamic equations as part of his research into weather patterns (Lorenz, 1963).

$$\begin{aligned} \frac{dX}{dt} &= -\sigma X + \sigma Y; \\ \frac{dY}{dt} &= -XZ + rX - Y; \\ \frac{dZ}{dt} &= XY - bZ. \end{aligned} \qquad (1)$$

The Lorenz system operates in three-dimensional phase space – the space in which variables describing the behavior of a dynamic system are entirely confined. The parameter σ is a Prandtl number, the parameter *r* is the ratio of the Rayleigh number and critical Rayleigh number. The third parameter *b* is related to the horizontal wave number of the system.

For σ=10, *b*=8/3 and *r*>24.74, the Lorenz system exhibits chaotic behavior and any trajectory is attracted to a subset of phase space having a fractal dimension (Mandelbrot, 1977). The moving path described by this system (Figure 4) is wandering; that is, the trajectory follows several right-hand coils, then abruptly switches and follows several left-hand coils, then switches again, and so on. The trajectory is very sensitive to small variations in the initial parameters, making it extremely difficult to predict how many successive coils will be completed during some period of time before abruptly switching to the alternative state. This behavior is now popularly known as the "butterfly effect"— the idea that a butterfly flapping its wings in Saint Petersburg can set in motion a complicated chain of events that ultimately affects the weather in Kansas City.

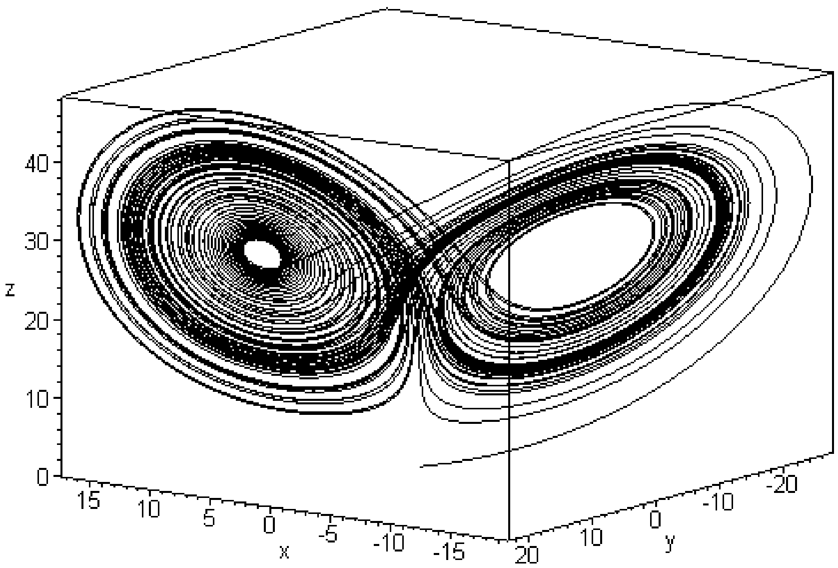

Fig. 4. Trajectory of the Lorenz system.

When viewed in its full three dimensions as in Figure 4, the Lorenz system seems to have no resemblance to the measures of climate recorded in the ice cores. However, if the wandering locus of the system of equations is projected onto a single dimension, its record appears quite different, as can be seen in Figure 5. This one-dimensional section of the Lorenz trajectory has, in a certain sense, features similar to those that appear in Figure 3: the trace of the Lorenz system consists of high-frequency, low-amplitude oscillations centered around local averages, with unpredictable "snaps" from one average state to the other. We will now consider how closely the dynamics of the Lorenz system matches records from the Pleistocene ice core, and how this resemblance can be used for climate prediction.

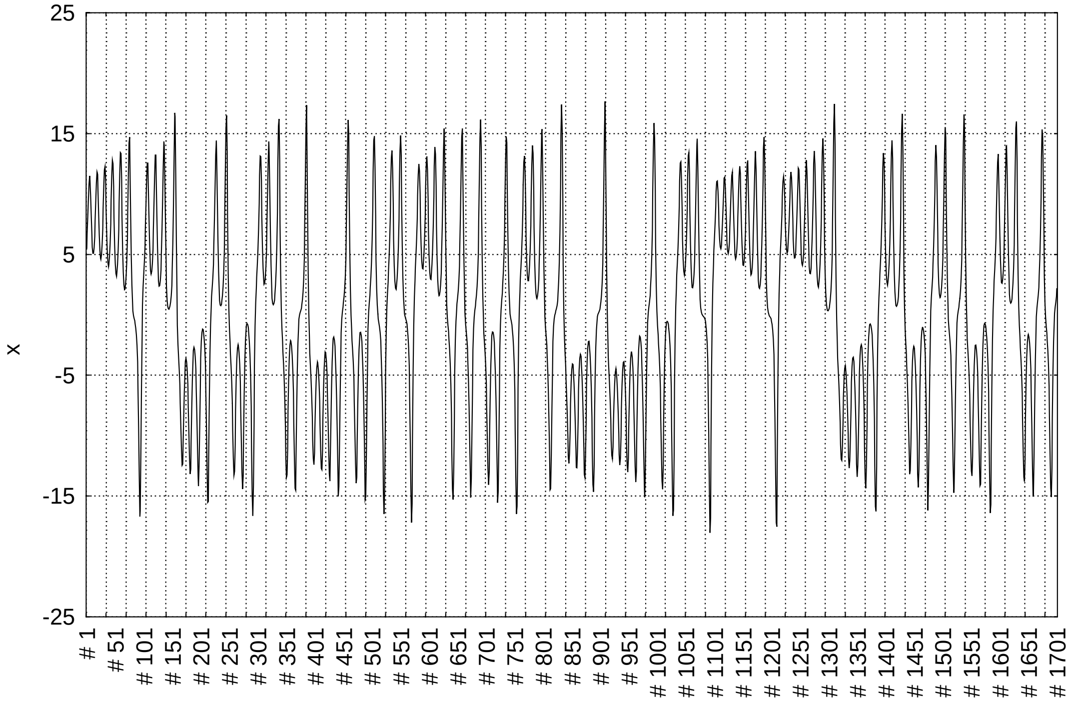

Fig. 5. *x*-coordinate of the Lorenz system. Horizontal axis is a discrete non-dimensional time (δt=0.05).

Simple nonlinear prediction

Approaches stemming from dynamical systems theory allow us to make predictions both in strictly deterministic systems that exhibit chaotic behavior (such as the Lorenz system) and in systems which contain superimposed random noise. Different methods of prediction are used in such systems (Weigend and Gershenfeld, 1994), but most of them are based on the idea of the time decomposition of a single time series followed by phase space reconstruction (Grassberger and Procassia, 1983).

First, following this approach we create a sequence of state vectors $X(i)$ from the available one-dimensional sequence, $x(i)$:

$$X(i) = \{x(i), x(i-L), ..., x(i-L(M-1))\}\,. \qquad (2)$$

Here, L is the "lag," or number of sampling intervals between successive components of the delay vectors and M is the dimension of the delay vector. In other words, from a one-dimensional sequence of measured values we construct a new sequence of M-dimensional vectors $X(i)$ which define some trajectory in M-dimensional space. A theorem by Takens (1981) and by Sauer and others (1991), states that if the sequence $x(i)$ consists of a scalar measurement of the state of a dynamical system, then under certain assumptions the time delay procedure provides a one-to-one image of the original sequence, provided M is sufficiently large.

Next, we must determine M, the dimension of the phase space. This dimensional parameter is very important because it specifies the number of degrees of freedom in the system. Recall that the trajectory of the Lorenz system lies within a three-dimensional space (Figure 4). Using the Grassberger-Procassia algorithm (Grassberger and Procassia, 1983), it is possible to estimate this dimension using only the record of the observed x-coordinate shown in Figure 5. To do this, we used the so-called correlation integral in the form

$$C(M,r,w) = \frac{1}{N_{pairs}} \sum_{j=1}^{N} \sum_{k<j-w} \Theta(r - |X(j) - X(k)|) \qquad (3)$$

where N is the number of observations, $X(j)$ is a M –dimensional vector defined by (2), $N_{pairs} = ((N-w)^2-N+w)/2$ is the number of pairs of points covered by the sums, Θ is Heaviside step function, $|\cdot|$ is the suitable M-norm (Euclidian norm in our case) and w is the so-called Theiler window (Theiler, 1990). The w allows us to exclude false correlation due to samples close it time in highly sampled flow data. In such data subsequent delay vectors can be highly correlated. Normally the choice of w is defined by the first zero of the autocorrelation function. Another important characteristic of a dynamical system reflecting geometrical features of a dynamical system trajectory is the correlation dimension D_C (Hentschel, Procaccia, 1983). On sufficiently small and appropriate scales r (the so-called "scaling intervals"), $C(r) \propto r^{D_C}$, so we can estimate the correlation dimension D_C as the slope of the log-log plot of $C(r)$ versus r. According to Grassberger and Procaccia's algorithm, D_C will not change from some value M corresponding to the correlation dimension of the entire trajectory D^* (Grassberger, Procaccia, 1983). Moreover, the first integer number greater than D^* will define the embedding dimension, i.e. the quantity of degrees of freedom excited in the system.

So, for an arbitrary one-dimensional sequence it is possible to estimate the dimensionality of an entire dynamical system, if such a system exists. The estimate of the dimension of the phase space is also useful as a measure of the complexity of the system.

The last step is prediction itself. Figure 6 shows, for illustrative purposes, a record of the Lorenz system. To predict $x(i+j)$ from the record at point i we first impose a metric on the M-dimensional state space (in this instance, we have used simple Euclidean distance as the metric) and find the k nearest neighbors of $X(i)$ from the past $X(l): l < i, l \in S$, where S is the set of indices of the k nearest neighbors. The prediction is simply the average over the "future" $X(l+j)$ of the neighbors $X(l)$, $l = l_1, l_2, ..., l_k$. In other words, we must consider that part of the phase space around the predicted point and see what happens within this domain during the evolution of the system (Figure 6). To obtain a prediction in the one-dimensional space of the original data, we need only consider the first components of the delay-time vectors. The prediction is then simply

$$x_{pred}(i, j) = \frac{1}{k} \sum_{l \in S} x(l + j), \qquad (4)$$

i.e., the average over the first components of “future” of the neighbors. The procedure is described in detail in Farmer and Sidorovich (1987) and Hegger and others (1999).

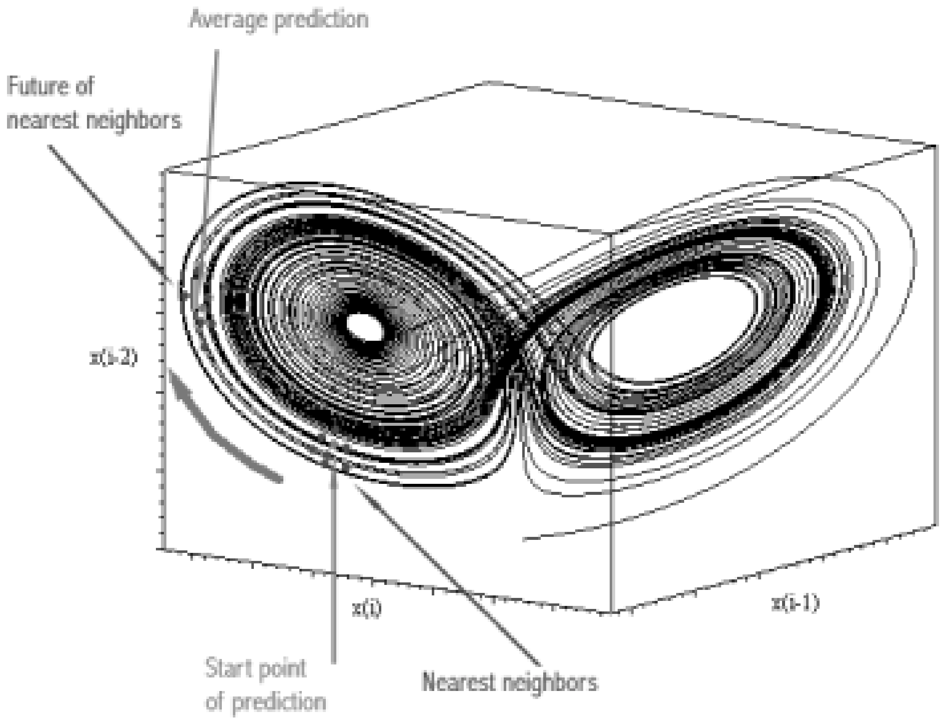

Fig. 6 Prediction scheme in reconstructed space for the Lorenz system.

These predictions can be made more useful by enclosing them in estimated confidence intervals for a specified level of probability. We may presume that inevitably a theoretical natural dynamic system is confounded with independent random processes (random noise). Thus, we can consider the observed system trajectory to represent a random cloud of points surrounding an imaginary theoretical trajectory in multidimensional space. It is well known that the sums of a large number of independent random processes will form a normal (Gaussian) distribution of values. So, we may assume that a normal distribution (representing random noise) has been superimposed on the response of the dynamic system. Student's criterion, a standard method for estimating confidence intervals around the average of a small number of random values, can be used to construct intervals around the prediction within which the true estimate will fall with specified probability.

It is important to emphasize that this methodology is intended for prediction only over a short time into the future. If a phenomenon truly exhibits chaotic behavior, the actual and predicted trajectories will diverge with time in an unpredictable manner because of the sensitivity of the system to the initial parameters. Long-term predictions should be made only with extreme caution.

Examples of predictions

The first illustration of the method uses artificial data, the *x*-coordinate of Lorenz system shown in Figure 5. The results of applying the short-time prediction technique are shown in Figure 7. Note that the Lorenz trace progresses from the "past" on the right side of the illustration to the "present" on the left, corresponding in orientation with the ice core records. Predictions begin at $N = 300$ and are based only on the characteristics of the prior record. Predictions and their confidence intervals have been made up to $N = 0$ and can be compared to the actual values of the Lorenz system over this interval. There is very good correspondence between observations and predictions over the short time, and a gradual divergence between observations and predictions at longer times. Note that this artificial record is purely deterministic and free of random noise.

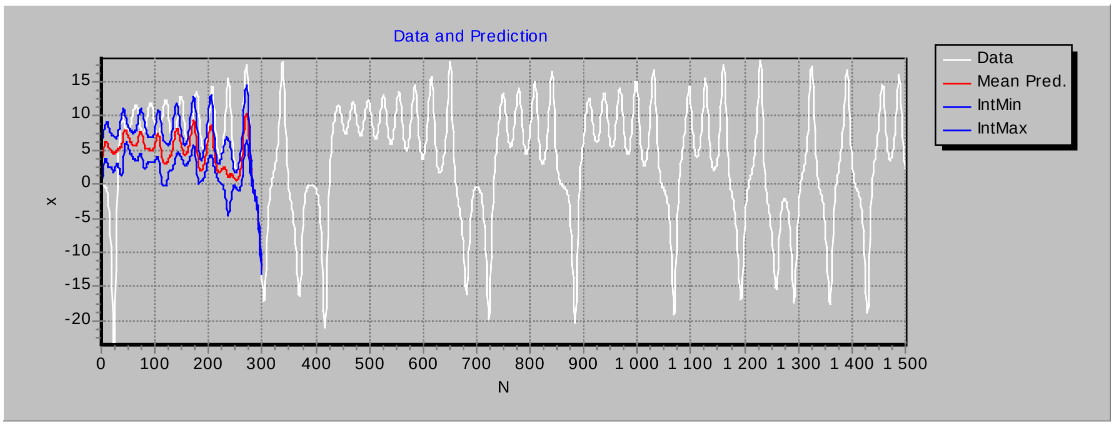

Fig. 7. Prediction for trace of Lorenz system: white curve – *x*-coordinate of the Lorenz system (discrete time is on horizontal axis, δt =0.25), middle dark curve is average prediction, upper and lower dark curves represent 99% confidence interval. $\boldsymbol{M} = 3$, $L = 1$, $k = 25$.

The approach is not restricted to deterministic systems, as we can see in an application to data from the GISP2 core for the interval prior to the Holocene (10,000-100,000 years B.P.) shown in Figure 3. Previous analyses have determined that the record of the first component for this period can be regarded as a dynamic system having a phase space dimension of $M = 3$ (Kotov, 2000). Figure 8 shows a portion of the distribution of the first component as a function of time as well as the mean prediction and the confidence interval for the prediction ($M = 3$, $L = 1$, $k = 25$, 99% confidence interval). In this illustration, the record goes from the distant past (40,000 years B.P.) on the right side to the start of the Holocene (10,000 years B.P.) on the left side. Predictions and their confidence intervals begin 25,000 years B.P. and extend to the start of the Holocene. As in the previous example, the predictions are based only on the part of the record prior to 25,000 years B.P.

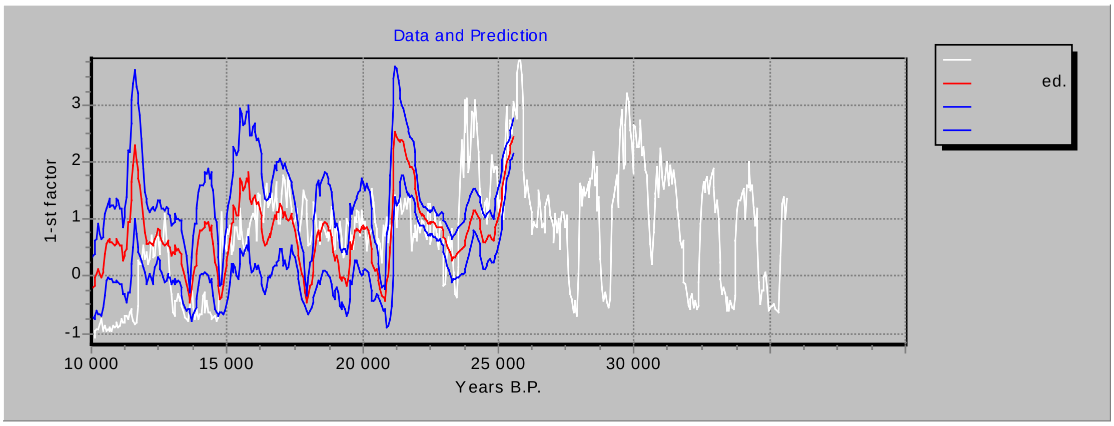

Fig. 8. Distribution of first principal component for the period 10-40 kyr BP and prediction for the period 10-25 kyr BP.

A good correspondence between prediction and real data is apparent; although the confidence interval does not always cover the data, the tendencies in data and prediction are coincident. This result can be taken as supporting evidence for the notion that the northern hemisphere Pleistocene climate can be regarded as a type of low-dimensional dynamic system.

Next, we will examine the results from analyzing the $\delta^{18}O$ ice-core record for the last 10,000 years (Holocene). In dynamic systems terms, the behavior of the trajectory for this record is more complicated – the estimated phase space dimension M is 8. The results of testing the predictions are shown on Figure 9 ($M = 8$, L=1, k=25, 99% confidence interval). The prediction begins at 1040 years B.P. and extends through the end of the ice core record (-30 years B.P., or 1980 AD) and beyond into the future. There is good correspondence between prediction and observation for a short period of time (approximately 300 years) and a general correspondence between prediction and data for the rest of the time interval.

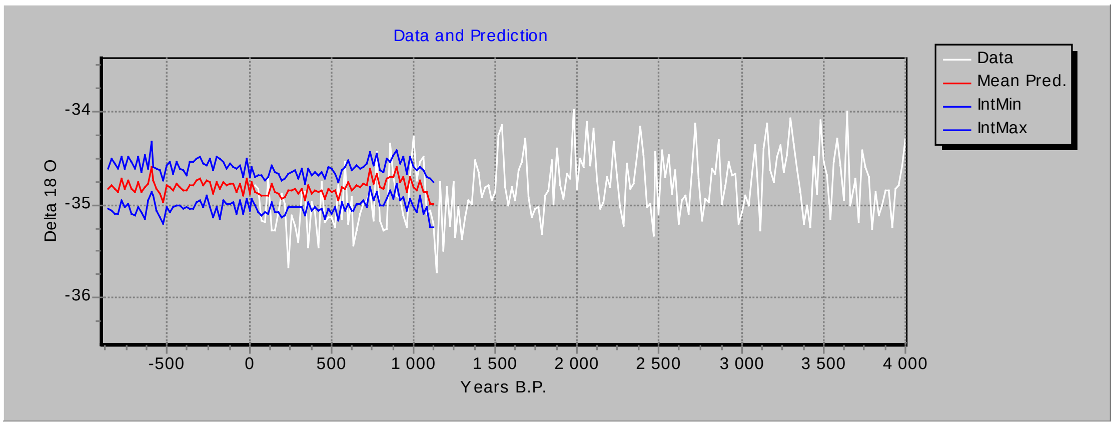

Fig. 9. Distribution of $\delta^{18}O$ for the period 0-4 kyr BP and prediction for the period 1140 years BP to 2800 AD.

Figure 10 shows the mean prediction and the confidence interval for prediction over the next 800 years into the future ($M = 8$, $L=1$, $k = 25$, 99% confidence interval). This prediction is based on characteristics of the entire ice core record since the start of the Holocene (10,000 years B.P.); predictions begin at the latest date in the 20-year average $\delta^{18}O$ record (1980 AD) and extend into the future. Note that the most recent 200-year time interval is characterized by a pattern of increasing temperatures. The onset of such a pattern has been noted over the past decades and often is ascribed to global warming caused by a greenhouse effect associated with the Industrial Revolution. It is true that this pattern coincides with the duration of the Industrial Revolution, which began approximately 150 years ago. On the other hand, if the analytical method has been chosen appropriately, then from these results it follows that this positive trend in temperature is not an extraordinary one for the Holocene. Moreover, this short-time warming is predicted to continue for about 200 years and to be followed by a reversal in the temperature trend.

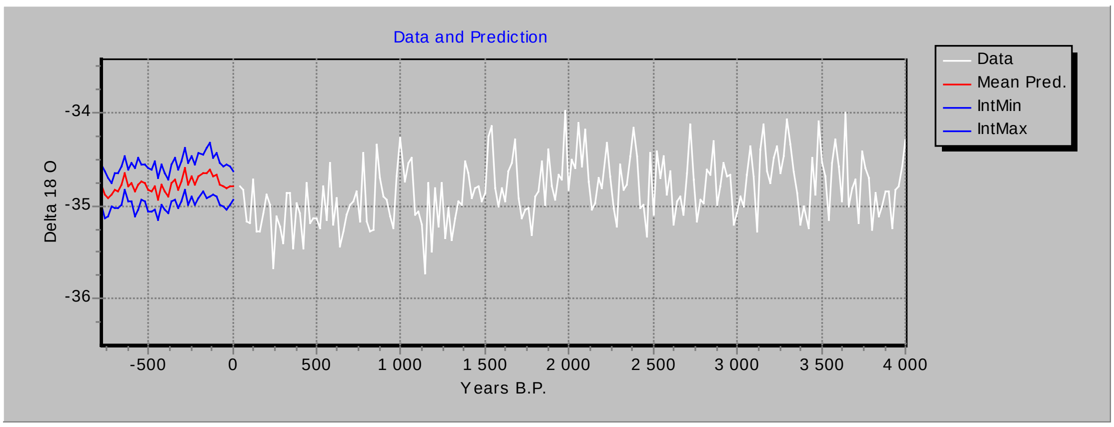

Fig. 10. Distribution of $\delta^{18}O$ for the period 0-4 kyr BP and prediction for 800 years into the future.

Conclusions

Traditional models of global climate change are extremely complicated, rely on a number of fundamental assumptions, and are difficult and costly to operate. Perhaps most troublesome is the brevity of the historical record of climate on which the models are conditioned. However, records of various constituents measured in Greenland ice cores provide information about past variations in climate over thousands of years before the present which can be used to help understand better the processes of global climate change. Methods for reconstructing the entire multidimensional trajectory of a process from its one-dimensional sequence of measured values allows us to estimate important characteristics of dynamic systems such as the number of degrees of freedom or the true dimensionality of the system. The dimensionality reflects the number of independent variables in a system and must be taken into account in the construction of a global climate model. In addition, this approach permits us to make short-term predictions of important climatic variables.

The main characteristic of the 90,000 years that preceded the Holocene is the intermittent behavior of the climate. Periods that were relatively warm (on average) were abruptly followed by periods of intense cold. Within these relatively warm or cold intervals, temperatures varied through high-frequency, low-amplitude oscillations. Described in dynamic system terms, the climate behaved as a dynamical system with 3 degrees of freedom. We obtain a good correspondence between predicted behavior and reality by modeling climate as a low-dimensional dynamical system over this period of time. The same characteristics are exhibited by a Lorenz system – the simplest nonlinear deterministic model that can be applied. Such a system has 3 degrees of freedom and demonstrates high-frequency oscillations with unpredictable "snaps" from one space domain to another.

In terms of dynamical systems, the behavior of the climatic record in the Holocene is more complicated – the dimensionality of the phase space M is 8. Estimates of temperature based on a dynamical system model indicate that a positive trend in temperature over a 200-year period is not unexpected for the Holocene. Predictions based on the characteristics of the Holocene and extending into the future indicate that the present short-term warming trend may continue for at least 200 years and be followed by a reverse in the temperature trend.

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