NUMERICAL INVESTIGATIONS OF FLUID FLOW THROUGH SOME GEOLOGICAL DISCONTINUITIES


Department of Applied Mathematics,
University of Leeds, Leeds LS2 9JT, UK

*email: amtaka@amsta.leeds.ac.uk

Abstract
The present analysis relates to the study of two-dimensional fluid flow through porous media, incorporating the geological features of both faults and fractures. A geometrical configuration which involves several discontinuities in the channel height, and where the entry and exit sections of the channel are composed of several layers of porous material, is considered and hence the effects on the fluid flow of different permeabilities are demonstrated. Numerical solutions are obtained by utilising a computational fluid dynamics (CFD) approach based on the Control Volume Method (CVM). This technique is used to solve the full two-dimensional Brinkman equation, representing the fluid flow through porous media, and involves the parameter, $\alpha = \Gamma/Da$, where $Da$ is the Darcy number which symbolises the type of medium in which the fluid flows and $\Gamma$ is the ratio of the fluid viscosity $\mu_f$ to the effective viscosity $\mu$. In the fully developed region, the numerical solutions are compared with the available analytical solutions in order to validate the accuracy of the numerical technique. The streamline pattern is presented for each case, together with the pressure field in the area corresponding to the fault region.

Keywords
Fluid flow, Brinkman equation, Porous media, Numerical solutions, Control volume method.
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<td>$A$</td>
<td>area of the faces of the control volume, $A_x, A_w, A_n, A_z$</td>
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<td>$b_m$</td>
<td>mass residual</td>
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<td>$C$</td>
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<td>$Da$</td>
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<td>$p'$</td>
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<td>$P$</td>
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<td>$Q$</td>
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\( X_0, X_1 \) dimensional distances
\( X_L, X_R \) dimensional distances
\( Y_0, Y_1, Y_2, Y_3 \) dimensional distances
\( Y_D, Y_U \) dimensional distances

**Greek Symbols**

\( \alpha \) Darcy parameter, \( \Gamma / Da \)
\( \alpha_i \) Darcy parameter in region \( i \), \( i = 1, 2, \ldots \)
\( \alpha^* \) typical value of \( \alpha \) from within geological structure
\( \gamma \) constant, \( (\bar{\mu}/\mu_f)^{1/2} \)
\( \mu_f \) fluid viscosity, \( kgm^{-1}s^{-1} \)
\( \bar{\mu} \) effective viscosity, \( kgm^{-1}s^{-1} \)
\( \rho \) non-dimensional fluid density, \( \mathcal{P}/\rho_0 \)
\( \rho_0 \) reference fluid density
\( \Gamma \) constant, \( \mu_f/\bar{\mu} \)
\( \Omega \) volume of fluid

**Subscripts**

\( e \) east face of the control volume
\( E \) east grid point where \( v \) is stored
\( n \) north face of the control volume
\( nb \) summation of the grid points where \( v \) is stored
\( N \) north grid point where \( v \) is stored
\( s \) south face of the control volume
\( S \) south grid point where \( v \) is stored
\( P \) main/central grid point where \( v \) is stored
\( u \) \( u \)-control volume
\( v \) \( v \)-control volume
\( w \) west face of the control volume
\( W \) west grid point where \( v \) is stored

**Superscripts**

\( ' \) corrected value
1 Introduction

Fluid flows through porous media have gained considerable attention during the past decade and it has become a subject of permanent interest to numerous researchers. In particular, the process of fluid flow through composite channels is of interest to a wide range of engineers and scientists, in addition to politicians and economists who recognise the importance of ground-water flows and the variety of oil recovery processes. Also, the prediction of the transport of fluid in composite channels is one of the most important problems currently being addressed by groundwater hydrologists and chemical and mechanical engineers, and whilst their individual goals differ somewhat, depending on their discipline, the basic questions and the problems are universal.

In addition, most of the studies in modern engineering applications of porous media have been performed on the basis of the Darcy flow model, which is an empirical law for creeping flow through a uniform medium. However, researchers now generally recognise that non-Darcian effects are quite important in certain applications. A generalised model has been used by different researchers for studying non-Darcian effects, such as inertia, the impermeable boundary and porosity variation, see, for example, Bejan and Poulakakos (1984), Vafai and Kim (1990), and many others. Vafai and Kim (1990) have briefly discussed the effects of the Darcy number, $Da$, which was first investigated by Beavers and Joseph (1967), and concluded that a decrease in the Darcy number results in a lower mass flow rate through the porous region. A similar problem was investigated by Vafai and Thiyagaraja (1987), who obtained an analytical solution based on matched asymptotic expansions for the fluid velocity and the temperature distributions.

Most of the previous researchers, for example, Ochoa-Tapia and Whitaker (1995a,b), Al-Nimr and Alkam (1998) and Kuznetsov (1999), have restricted their investigations to solving the fully developed fluid flow in two-dimensional composite channels which are partially occupied by a clear fluid and partially by a fluid saturated porous medium. However, Al-Hadhrami et al. (2000, 2001a,b) have presented analytical studies for fully developed flows in two and three composite channels which are occupied by one, two and three regions of different uniform
porosity. They have also confirmed that the Darcy number acts as a resistance to the fluid flow and that for the same flux of fluid through the channel the non-dimensional pressure gradient increases as the Darcy number decreases.

However, the most well-known examples of fluid flow in porous media tend to be geological in nature, such as groundwater flows and oil recovery processes. The heterogeneous and complex nature of these systems requires that several numerical investigations be presented in order to obtain a basic understanding of the fluid flow in such structures. Thus, the main focus of this work is to consider numerical investigations of fluid flow through geological configurations involving several discontinuities in the channel height and where different sections of the channel are composed of several layers of different porous material. The effects of these discontinuities on the fluid flow through the regions of different permeabilities are demonstrated by presenting for each situation both the streamline pattern and the change of the overall pressure.

The numerical results are obtained by solving the governing equation for the fluid flow through porous media, namely, the Brinkman equation (1947a,b), using a CFD technique which is based on a control volume method (CVM), where the solution domain is divided into a number of control volumes surrounding each grid point, see, for example, Patankar and Spalding (1972) and Patankar (1980). In the control volume algorithms the pressure-correction equations are derived from the continuity equation for each control volume and the solution of the Brinkman equation is achieved by successively predicting and correcting the fluid velocity components and pressure. Convergence and stability are better understood than for other computational discretisation methods, such as the finite-difference and the finite element methods. In addition, the CVM is physically flexible, but it can be quite time consuming in obtaining convergence when large resistances are present in the flow due to small values of the Darcy numbers, $Da$. Hence, we introduce briefly some of the techniques which can be implemented in order to improve the rate of convergence of the iterative scheme.

The present research investigation, as outlined in Figure 1, models the physical situation observed in typical laboratory experiments, see, for example, Lesnic et al. (1998). When due to an imposed pressure gradient, fluid flows within a horizontal channel composed of several layers of
Figure 1: A schematic diagram of the problem when fluid flows through a two-dimensional geometrical configuration composed of the inlet and outlet composite channels which are linked by a vertical channel.

porous material, each layer having a thickness $H_i$ and a permeability $k_i$, where $i = 1, 2, 3$, and the width of the channel $2H = \sum_{i=1}^{3} H_i$. The horizontal channel and its layers then link with a vertical channel of width $2W$ and permeability $k_7$ in which the permeability of a central block is $k_8$, where $k_8$ will be assumed to be either equal to $k_7$ or to be greatly different, both much larger and much smaller. Fluid leaves from the vertical channel in a similar manner to that in which it enters, i.e. along another horizontal layered channel, but, this outlet channel is offset from the inlet channel by a distance $(Y_D - Y_U)$. The widths of the individual layers, namely $H_4$, $H_5$ and $H_6$, which have permeabilities $k_4$, $k_5$ and $k_6$, respectively, can assume different values to those in the inlet channel as can their combined width, but, for the present work these parameters are set to be the same as in the entry channel. The upper and lower surfaces of the horizontal channels are assumed to be impermeable and the length of the channels to be sufficiently long for a fully developed velocity profile to be observed, hence corresponding to the semi-infinite channel situation. The vertical sides and the upper and lower ends of vertical channel are taken as closed and the length of the channel sufficient for the flow details throughout the majority of this channel to be independent of its length.
Various geometrical configurations of Figure 1 are numerically investigated, especially where the fluid flows through the vertical channel. The vertical channel is considered as a fault region, where in some investigations a central barrier will be linked to the horizontal layers from the inlet and outlet horizontal channels. A situation analogous to the present one can be seen in oil reservoirs composed of layers of sandstone and impermeable shales in which faults and fractures are located. The values assigned to the various parameters during the numerical calculations have been prescribed in order to generate an understanding of the general processes occurring in the composite channel as shown in Figure 1.

2 Mathematical Formulation

The general equation governing the steady incompressible fluid flow through porous media was given by Brinkman (1947a,b) and takes the following form:

\[ P(V, \nabla) V = -\nabla P + \tilde{\mu} \nabla^2 V - \frac{\mu_f}{k_i} V \]

(1)

where \( P \) is the fluid density, \( P \) is the fluid pressure, \( \tilde{\mu} \) the effective viscosity for the porous region and \( \frac{\mu_f}{k_i} V \) is the traditional Darcy term with \( \mu_f \) the dynamic viscosity of the fluid and \( k_i \) the permeability of the porous region \( i, i = 1, 2, \ldots \). We now introduce the non-dimensional variables as follows:

\[ V = U_0 \nu, \quad P = \frac{\tilde{\mu} U_0}{H} p, \quad X = Hx, \quad T = \frac{H}{U_0} t, \quad \rho = \rho_0 \rho, \quad Da_i = \frac{k_i}{H^2} \]

(2)

where \( U_0 \) is the average inlet velocity, \( H \) represents a characteristic length scale, which has been taken to be half of the channel height in the \( Y \)-direction, \( \rho_0 \) is the reference density of the fluid, and \( Da \) is the Darcy number. We hence obtain the non-dimensional form of the Brinkman equation by substituting these non-dimensional variables into equation (1), namely,

\[ \rho \Gamma (V, \nabla) V = \frac{1}{\mathcal{R}} \left[ -\nabla p + \nabla^2 V - \alpha_i V \right] \]

(3)

where \( \mathcal{R} = \rho_0 U_0 H / \mu_f \) is the Reynolds number, \( \alpha_i = \Gamma / Da_i \), represents the different materials within the channel, and \( \Gamma = \mu_f / \tilde{\mu} \), which in many papers, e.g. Kuznetsov (1999), is set to be \( 1/\gamma^2 \). For an incompressible fluid flow we also have the continuity equation which is given by

\[ \nabla . V = 0 \]

(4)
The control volume method (CVM) is used to discretise the governing equations (3) and (4) on a uniform staggered grid system, where the fluid velocity components and the pressure are stored at different grid points. Physically this system satisfies the conservation principle, since the pressure difference between two adjacent grid points drives the fluid located between them. The staggered grid concept locates the fluid pressure at the main grid points and offsets by half a grid spacing to the left and below the grid points for the $u$ and $v$ components of the fluid velocity, respectively. We present briefly the formulation of the control volume method for the governing equations (3) and (4), but for more information about the technique in the general form, see, for example, Patankar (1980).

We now consider an arbitrary volume of fluid $\Omega$ within a surface $S$ which has a unit outward normal $n$. Applying the Gauss theorem to equations (3) and (4), yields the integral forms

$$\nabla \int_S (\rho \mathbf{v} \cdot n) \mathbf{v} \, dS = - \int_S p \, n \, dS + \int_S (\nabla \mathbf{v}) \cdot n \, dS - \alpha_t \mathbf{v} \Omega$$

(5)

$$\int_S \mathbf{v} \cdot n \, dS = 0$$

(6)

The discretised equations for the $u$ and $v$ fluid velocity components are obtained by using equation (5) for every control volume in the computational domain. Hence they may be expressed in the form

$$a_e u_e = \sum a_{n \hat{b}} u_{n \hat{b}} + b_u + A_e (p_P - p_E)$$

(7)

$$a_n v_n = \sum a_{n \hat{b}} v_{n \hat{b}} + b_v + A_n (p_P - p_N)$$

(8)

where, in general,

$$\sum a_{n \hat{b}} \phi_{n \hat{b}} = a_E \phi_E + a_W \phi_W + a_N \phi_N + a_S \phi_S$$

(9)

$$a_e = \sum a_{n \hat{b}} + \alpha_t \Omega_e, \quad a_n = \sum a_{n \hat{b}} + \alpha_t \Omega_n$$

(10)

Here $P, E, W, N$ and $S$ are the grid points where the pressure and the fluid velocity components are stored, $u_{n \hat{b}}$ and $v_{n \hat{b}}$ are the values of the $u$- and $v$-components of the fluid velocity at the nearest neighbouring points, $a_{n \hat{b}}$ are the standard matrix coefficients which are obtained by
using the upwind difference scheme, the coefficients $A$ are the areas of the faces of the control volume, the terms $A_e(p_F - p_E)$ and $A_n(p_F - p_N)$ are the pressure forces acting on the $u$- and $v$-control volume, respectively, and $\Omega_e$ and $\Omega_n$ are the volumes of the fluid on the east and north faces, respectively.

In the SIMPLE (Semi-Implicit Method for the Pressure Linked Equation) algorithm the pressure correction can then be obtained by substituting all the velocity components into the continuity equation for the control volume. This yields

$$a_P p_F' = a_E p_E' + a_W p_W' + a_N p_N' + a_S p_S' + b_m$$  \hspace{1cm} (11)$$

where

$$a_P = \sum a_n, \quad a_E = d_e A_e, \quad a_W = d_w A_w, \quad a_N = d_n A_n, \quad a_S = d_s A_s,$$  \hspace{1cm} (12)$$

and

$$d_e = \frac{A_e}{a_e}, \quad d_w = \frac{A_w}{a_w}, \quad d_n = \frac{A_n}{a_n}, \quad d_s = \frac{A_s}{a_s}$$  \hspace{1cm} (13)$$

The mass residual of every control volume is given by

$$R_{\text{mass}}^i = C_e - C_w + C_n - C_s$$  \hspace{1cm} (14)$$

where $C_e, C_w, C_n$ and $C_s$ are the masses of fluid convected through each of the faces of the control volume surrounding the point where the pressure is located. Then the source term in equation (11) is given by

$$b_m = -R_{\text{mass}}^i$$  \hspace{1cm} (15)$$

If the mass residual, $b_m$, is zero then the fluid velocity satisfies the continuity equation and the solution to the problem has been obtained. A measure of the rate of convergence of the iterative procedure used in this work is the sum of the mass residuals over all the control volumes, namely,

$$R_{\text{mass}} = \sum_i |R_{\text{mass}}^i|/Q$$  \hspace{1cm} (16)$$

where $Q$ is the flux of fluid across the channel. The resulting finite difference forms of the Brinkman equation and the pressure-correction equation are solved by using a line-by-line tridiagonal-matrix algorithm with one iteration and with four or less iterations, respectively.
4 Numerical Accuracy

In order to establish the accuracy of the numerical solutions from utilising the control volume method, the numerical solutions are compared with analytical solutions for the fully developed situations which are already available in Al-Hadhrami et al. (2000). The only boundary conditions required are on the external boundaries of the computational domain (no conditions are required on the interfaces between the different media). These external boundaries relate to the input conditions, the no slip condition on the walls of the channel, which are assumed to be impermeable, and the fully developed solution far downstream, namely, $\partial u / \partial z = 0$. Figure 2 represents a comparison between the numerical and analytical fully developed solutions of the fluid velocity for (a) two porous media where $Da_1 = 1.0$ and $Da_2 = 10^{-3}$, and (b) three porous media where $Da_1 = 10^{-2}$, $Da_2 = 1.0$ and $Da_3 = 10^{-3}$. The two solutions are graphically indistinguishable and hence this validates the accuracy of the control volume method in the present situation.

Figure 2: Comparison between the analytical and numerical solutions of the fully developed fluid velocity for (a) two porous media, and (b) three porous media.
5 Numerical Investigations

In any fluid flow problem, the Reynolds number, $R$, is an indicator of the flow mode. The linear Brinkman equation holds for flow at low Reynolds numbers, $R \lesssim 10$, in which the driving forces are balanced by both the viscous forces and the media resistance. In the present situation when $0 < R \lesssim 1$, we have concluded from experimenting with different lengths that it is sufficient for the horizontal inlet and outlet channels in the composite channel of Figure 1 to have dimensional distances $(X_0 - X_L) \gtrsim 3H$ and $(X_R - X_1) \gtrsim 4H$, and for the vertical sides of the vertical channel to have dimensional distances $(Y_0 - Y_D) = (Y_U - Y_1) \gtrsim 5H$. These distances are sufficient in order for the fully developed solutions to have been reached and for the flow details throughout the majority of the vertical channel to be independent of its length. In addition, in all the results presented in this paper the dimensional width of the vertical channel is taken to be equal to half the width of the inlet and outlet channels, namely, $H$, and the outlet channel is offset from the inlet channel by a distance $5H$.

The permeability $k$, which has the dimensions of area, is independent of the nature of the fluid and depends only on the structure of the porous medium. Values of $k_i$ are represented in the numerical calculations by using different Darcy numbers, $Da_i = (k_i/H^2)$. The Darcy numbers used in the results presented in this paper are $Da = 10^{-4}$, for a low permeable porous material, and $Da = \infty$, for a clear fluid region. Although in geological applications, the physical values of $Da$ are in the range $10^{-10} - 10^{-16}$, the streamline patterns in this range of values are similar to when $Da = 10^{-4}$. However, on implementing these small values of $Da$ in the non-dimensional Brinkman equation (3) then the numerical results were found to converge extremely slowly as a result of the high resistance provided by the small values of $Da$. Therefore, a modification to the present technique is required to enhance the convergence of the iterative scheme, and this investigation is already in the process of being developed. However, in this paper our aim is to show the effect that the Darcy parameter has on the solution, and therefore it is sufficient to take $Da$ to be as large as $10^{-4}$.

In this paper, several numerical investigations are performed in order to generate an understanding of the general flow processes through some of the geological structures likely to be
observed in the field. We first consider the fluid flow around a partial barrier positioned in the
centre of a vertical channel. Then we change to fluid flow to be through a restricted central
clear passage way within the vertical channel. Finally, the effect of placing complete barriers
in both the vertical and horizontal directions, and complete barriers, but for the presence of
small gaps in the horizontal barriers, are considered.

All the numerical results presented in this paper are for $\rho = 1$, $\Gamma = 1$, $R = 1$ and for the same
non-dimensional flux of fluid through the channel, namely, $q = 2$. The measure of the rate
of convergence criteria is based on the sum of the mass residuals, $R_{\text{mass}}$, over all the control
volumes, and in all the results presented $R_{\text{mass}}$ is taken to be $10^{-5}$. Further, all the results
presented in this paper are for mesh sizes $\Delta x = \Delta y = 0.05$, since the results obtained for any
smaller mesh are graphically indistinguishable.

6 Results and Discussion

In order to illustrate the results we have obtained, we consider the situation in which the inlet
and outlet channels are composed of two outer layers of thickness 0.5 and $Da = 10^{-4}$ on their
upper and lower impermeable surfaces, together with an inner core of height 1.0 and $Da = \infty$,
since such a layered system is a more physical realistic representation of the geological nature
in groundwater and oil reservoirs. Then all the investigations are related to changes that
occur in the vertical channel, which physically can be considered as a fault region. For each
situation the streamline pattern is presented at regular values of the streamfunction, namely,$\psi = 0.1, 0.2, 0.3, \ldots, 1.9$, together with pressure contours to show the variation that occurs in
the vertical channel.

The first investigation considered is as indicated in Figure 3, which shows the streamline pattern
corresponding to the flow through a composite channel that undergoes a large abrupt change
in height over a vertical channel in which a partial barrier is positioned. Here, and later,
the shaded regions of the channel have $Da = 10^{-4}$, whilst elsewhere $Da = \infty$, and the non-
dimensional values associated with the vertical block are height 6, width 0.4 and distance 0.5
from the upper and lower surfaces of the vertical channel. It can be observed from Figure
The streamlines where the light shaded regions have $Da = 10^{-4}$, whilst elsewhere $Da = \infty$, and the non-dimensional values associated with the vertical barrier are height 6 and width 0.4 and positioned at $3.3 < x < 3.7$ and $0.5 < y < 6.5$.

Figure 3: The streamlines across the inlet and outlet channels are symmetrical about the lines, $y = 6$, and 1, respectively, since the horizontal layers are themselves symmetrically distributed about these lines. The fact that the streamlines are very close in the centre of the main stream path indicates that there is an increase in the fluid velocity in the central layer as a result of the inclusion of low permeable layers, namely $Da = 10^{-4}$, onto the channel walls, since in such regions the flow is severely restricted and rapidly migrates into the central layer. All the other numerical investigations outlined in Figures 4, 6 and 7 exhibit similar behaviour in the layered system domains. Further, the streamline pattern in Figure 3 shows that the barrier, $Da = 10^{-4}$, acts a resistance to the fluid flow, with most of the fluid diverted around it to flow across a clear fluid region, $Da = \infty$, and only a very small amount actually passing through the barrier.

Clearly as we have vertical, and equally spaced, contours of constant pressure across these layered systems in the inlet and outlet channels, apart from at their entrances, they are not presented. However, the pressure contours in the vertical channel of each situation, which show much greater variations depending on the different structures, are presented in Figure
Figure 4: The streamlines where the light shaded regions have $Da = 10^{-4}$, whilst elsewhere $Da = \infty$, and the non-dimensional values associated with the two thin vertical barriers, which are of distance 0.2 apart, are height 4.5, width 0.1 and positioned at $3.3 < x < 3.4$ and $3.5 < z < 3.6$, respectively.

5. Thus, it can be seen in Figure 5(a), the lack of flow across the barrier, $Da = 10^{-4}$, in Figure 3 occurs despite a large pressure gradient existing across the barrier. Furthermore, in the vertical regions, on either side of the barrier, where $Da = \infty$, the same uniform constant vertical pressure gradient has been clearly established.

In the next numerical investigations the horizontal layers from the inlet and outlet channels are linked with some vertical barriers and hence we consider the effects on both the flow pattern and the distribution of the pressure contours when the fluid passes through these horizontal and vertical barriers. Initially, in Figure 4, we consider the effect of connecting the layered system from the inlet and outlet channels to two thin vertical barriers, $Da = 10^{-4}$, where the non-dimensional values associated with these barriers are height 4.5 and width 0.1, and at a distance 0.2 apart. The streamlines indicate that the fluid avoids the regions of the vertical channel where parts of the horizontal layers with $Da = 10^{-4}$ are present and distributes itself almost uniformly between the two vertical barriers and between the barriers and the walls of the channel where $Da = \infty$. Although both the horizontal layers and the vertical barriers consist of
Figure 5: The pressure contours in the vertical channels of (a) Figure 3, (b) Figure 4, (c) Figure 6, and (d) Figure 7.
identical low permeable porous material, namely $Da = 10^{-4}$, the non-dimensional thicknesses of the horizontal layers, namely, 0.5, are much greater than those of the vertical barriers, namely, 0.1. As a result of having thin vertical barriers then the total resistance exhibited to the flow in the vertical direction in the region of the vertical channel is low, and hence the fluid flows relatively easily down this part of the channel. This result is also confirmed from Figure 5(b), which shows almost uniform horizontal contours of constant pressure across the central region. In addition, although large pressure gradients exist across the horizontal layers, compared to across the vertical barriers, there is no flow of fluid across the horizontal barriers as a result of their greater thicknesses.

The effect of having complete restrictions of fluid flow in both the horizontal and vertical directions is now investigated in Figure 6, where the horizontal layers, $Da = 10^{-4}$, from the inlet and outlet channels are linked to the vertical barrier, $Da = 10^{-4}$. The non-dimensional values associated with the barrier are width 0.2 and the same height as that of the vertical channel. The streamline pattern indicates that there is virtually no flow across the horizontal layers and that most of the fluid passes through sections of the vertical barrier in the central regions between the horizontal layers where $Da = \infty$. As in the previous investigation displayed in Figure 4, although both the vertical and horizontal barriers are made of the same low permeable material, namely $Da = 10^{-4}$, the majority of the fluid flows across that of the smaller width, which is that of the vertical barrier, namely 0.2. As the geometrical configurations in Figure 6, at the downstream end of the inlet channel and at the upstream end of the outlet channel, are simply reflections of each other in the vertical barrier then we have a similar streamline pattern down the vertical channel, where the fluid crosses the vertical barrier in order to avoid the thick regions of low permeable material, $Da = 10^{-4}$, in the horizontal layers. Again, despite large horizontal pressure gradients across the central region of the vertical barrier as shown in Figure 5(c), the flow is much reduced compared to that in the clear fluid regions. Whereas due to the high permeable media in the vertical direction either side of the barrier, namely $Da = \infty$, a relatively small vertical pressure gradient ensures that a considerable flow is observed in that direction. In addition, to ensure the same flux of fluid through the channel, it is necessary that there is an increase in the overall pressure gradient compared with previous situations since maintaining the same overall pressure gradient would result in a corresponding flow reduction.
Figure 6: The streamlines where the light shaded regions have $Da = 10^{-4}$, whilst elsewhere
$Da = \infty$, and the non-dimensional values associated with the vertical block are height 7, width
0.2, and positioned at $3.4 < x < 3.6$.

Figure 7: The streamlines for geometry in Figure 4 except at the horizontal layers where there
are two vertical discontinuities with non-dimensional values of width 0.1 and heights 1.0 and
0.5, respectively.
In the final investigation presented in Figure 7 we consider a similar configuration to that in Figure 6, but with the presence of small passage ways created by discontinuous regions between the vertical barrier and the horizontal layers. In this case the resistance imposed by the complete barrier in the horizontal direction has been reduced to two small regions, resulting in the fluid finding itself easy passage ways through the discontinuous regions. Although the vertical barrier around these regions is of low permeable material, $Da = 10^{-4}$, approximately half of the fluid in the vertical channel continues to flow across the vertical barrier. In addition, despite the fact that the combined widths of the gaps and the small vertical discontinuous barriers, $Da = 10^{-4}$, are of the same thickness, namely, 0.2, the presence of irregular placed material in the discontinuous region results in the flow across these vertical barriers. However, there is virtually no flow across the vertical barrier in the central part of the vertical channel, as this region is of high resistance, and the fluid has already found easier passage ways of $Da = \infty$ between the walls and the vertical barrier, unlike the pattern seen in Figure 6. The results from Figure 5(d) show that much larger values of the pressure gradient are required for flow to occur across the upper and lower parts of the vertical barrier, $Da = 10^{-4}$, compared to through the gaps, $Da = \infty$, in the horizontal barriers.

7 Technique for Accelerating CVM Convergence Rate

As we have mentioned in section 5, the values of the permeability vary widely for natural materials and typical values of $k$, in units of $m^2$, are as follows: sand $1.8 \times 10^{-10}$, sandstone $1.0 \times 10^{-12}$, limestone $4.5 \times 10^{-14}$ and shale $1.0 \times 10^{-16}$, see Scheidegger (1974). Unfortunately, when implementing these small values of the permeabilities in the non-dimensional Brinkman equation (3) then the numerical results are found to converge extremely slowly. For example, a composite channel, which is filled with sandstone, $Da = 1.0 \times 10^{-12}$, takes approximately 2 weeks CPU time to converge into a fully developed solution (where all the CPU times stated in this paper are based using a SUN workstation with an ULTRA 10 processor). This is because of the high resistance provided by the Darcy parameter, $\alpha_i = \Gamma/Da_i$, and hence fixing the flux of fluid through the channel requires extremely large values of pressure gradient in order to balance the Darcy term, $\alpha_i v$. 

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A faster convergence rate of the solution procedure may be established by using an alternative modified version of the non-dimensional form of the Brinkman equation to that given in equation (3). Thus, some of the non-dimensional quantities in the Brinkman equation are scaled in a different manner in order to find a direct balance between the most important terms which arise in the situation when very small values of $Da$ are employed. Hence, we introduce similar non-dimensional variables to those given in equation (2) for all the quantities, except for the fluid pressure which contains an additional factor $\alpha^*$, where $\alpha^*$ is a typical value of the parameter $\alpha_i$ from within the geological structure, i.e.

$$P = \alpha^* \frac{\bar{U}_0}{H} p$$  \hspace{1cm} (17)

On substituting this quantity, together with the rest of the quantities in equation (2), into equation (1) then we obtain a modified version of the non-dimensional Brinkman equation, namely,

$$\frac{\rho}{\alpha^*} (\nabla \cdot \mathbf{v}) \mathbf{v} = \frac{1}{R} \left[ -\nabla p + \frac{1}{\alpha^*} \nabla^2 \mathbf{v} - \frac{\alpha_i}{\alpha^*} \mathbf{v} \right]$$ \hspace{1cm} (18)

Thus we see that at large values of $\alpha_i$, i.e. small Darcy numbers, the dominant terms in equation (18) are the pressure gradient and the Darcy term, $\frac{\alpha_i}{\alpha^*} \mathbf{v}$, which are both $O(1)$ in the region where $\alpha_i = \alpha^*$. In addition, the magnitudes of the convection and diffusion terms are negligible, as compared to the magnitudes of these dominant terms and the non-dimensional Brinkman equation (18) reduces effectively to the Darcy equation for $Da \ll 1$. Therefore, we have found that the Brinkman equation in the form given by equation (18) enables us to consider very small values of $Da$, together with much improved rates of convergence. In particular, the CPU time required for the solution of equation (18), for any value of the Darcy number in the range $10^{-16} \leq Da \leq 10^{-5}$, when using the above scaling is only about 2% of the CPU time consumed when solving equation (3) when $Da = 10^{-10}$.

However, a difficulty arises in securing a converged numerical solution when implementing the modified version of the Brinkman equation (18) for a composite channel where the ratio of the lowest permeability to the highest permeability of the different media is less than about $10^{-4} m^2$, i.e. ratio of the largest to the smallest Darcy numbers is $\gtrsim 10^4$. The presence of a low
relative permeability, by several orders of magnitude relative to elsewhere in the channel, can result in a large and rapid pressure drop and hence again a very slow rate in the convergence of the solution procedure. This slow convergence results from the global mass conservation being obtained from the local mass conservation through the pressure correction equation which is not sensitive to changes in the fluid velocity caused by rapid changes in the permeability of the porous media. Hence, the iterative procedure requires many iterations to establish the large pressure gradients.

8 Conclusions

In this paper, a CFD approach which is based on the CVM with the staggered grid system has been implemented to solve the Brinkman equation in various geological configurations. The accuracy of the numerical calculations has been confirmed by comparing the analytical and numerical results, which are seen to be graphically indistinguishable. The control volume method has been shown to be an extremely accurate and efficient technique for solving problems related to fluid flow through composite channels. However, when a large resistance is present in the flow, as a result of large values of the Darcy parameter, \( \alpha = \Gamma / Da \), then the numerical results are found to converge extremely slowly. Although, we have introduced a modification to the present technique in order to enhance the convergent of the iterative scheme, a difficulty arises in securing the convergent solution where the relative permeability of the different media is very small, \( (\lesssim 10^{-4}) \), a problem which is already in hand.

In addition, several numerical investigations have been considered in order to add to our understanding of the general processes which model certain physical situations which occur in different geological structures of composite channels. The streamline patterns within the layered inlet and outlet channels are much closer in the regions of high permeability, and hence the fluid velocity is much faster, as result of the inclusion of the layers of low values of \( Da \) into the channel walls.

As all the investigations presented in this paper are related to changes that occur within the vertical channel, which is considered as a fault region, the results obtained have established
that a high pressure gradient in a particular direction, for example, across the barriers, does not necessary indicate fast fluid velocities in that direction. Instead, it all depends on both the value of the permeability and the dimension of the porous medium, a feature which is clearly displayed in Figures 6 and 7, since in Figure 6, close to the location (3.8,1.8), there is a large vertical pressure gradient compared with the same location in Figure 7, yet the flow in this direction is small in the former configuration compared with that in the latter configuration. Likewise, in Figure 7 the horizontal pressure gradient around the location (3.5,3.5) greatly exceeds the vertical pressure gradient in the neighbouring locations (3.3,3.5) and (3.7,3.5), yet the corresponding flow in the former situation is negligible compared with that in the latter situation.

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References


