FRACTURE-DISTRIBUTION MODELING IN ROCK MASS USING BOREHOLE DATA AND GEOSTATISTICAL SIMULATION

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ABSTRACT- Understanding spatial distribution of rock fractures is significant to various fields in geosciences. It is, however, difficult to obtain a realistic fracture model because the amount of fracture data is small and their locations are strongly biased in a study area. Data are usually taken by borehole investigations and then, spatial correlation structures of fractures among boreholes with different directions are needed to be clarified. For this problem, we focus on the fracture density along a borehole (number of fractures per 1-meter interval) and appearance relation of azimuths (strikes and dips) between a fracture pair. Semivariogram of the density, \( \gamma_1(h) \), and cross-semivariograms of the two indicators for two directions, \( \gamma_2(h) \), are produced. Firstly, fracture density map is produced using the \( \gamma_1(h) \) and a sequential gaussian simulation. Next, a direction of each simulated fracture is assigned using the data set, \( \gamma_2(h) \), and ordinary kriging combined with the principal component analysis. Finally, length of each fracture is defined considering the distance and difference of azimuths between a fracture pair located closely. This proposed method was applied to 629 investigation data by several boreholes in a granitic site situated in northeastern Japan. The size of study area was defined as 60×60 m². Horizontal distribution of fractures and continuities of them along the vertical direction were successfully estimated. The most noteworthy features are found in that high-density zones are located in no data areas and continuous fractures striking along the boreholes are inferred. In addition, permeability of rock mass was calculated to be about \( 10^{-17} \) m² from this distribution model and a permeability tensor analysis.

Key Words: Fracture, Granite, Semivariogram, Stochastic simulation, Permeability tensor

INTRODUCTION

Fractures with diverse origins and different scales from microcracks to faults are generally developed in rock masses. They were formed attributable to emplacement and cooling of rock bodies, crustal movements, and regional stress fields at different geologic stages. Understanding spatial distribution of rock fractures is significant to various fields in geosciences, including hydrogeology for fracture-affected flow channels, and resource exploration for vein-type mineral deposits and fluids in fractured reservoirs (e.g., National Research Council, 1996; Coward et al., 1998; Adler and Thovert, 1999). In special, characterization of hydraulic properties of rock masses has become very important recently, because it is closely related to environmental problems.

For this purpose, a general view of fracture distribution in a study area is required to be drawn using stochastic methods. There are many papers that have tried to clarify scaling laws of fractures using fractal theory (e.g., Vignes-Adler et al., 1991; Davy et al., 1992; Dawers et al. 1993; Watterson et
al., 1996; Koike and Kaneko, 1999), but most of them deal with length distribution over wide scales. In addition to the lengths, locations of fractures should be considered to reveal spatial correlation structures of fracture attributes. Geostatistical techniques that can incorporate the structures have also been applied to fracture-distribution modeling (Long and Billaux, 1987; Young, 1987; Chilès, 1988; Gringarten, 1996). It is, however, difficult to obtain a realistic model because the amount of fracture data is small and their locations are strongly biased in a study area. Fracture attributes that are applicable to distribution modeling considering length, appearance pattern, and azimuth of fracture should be firstly identified.

As the first step to a fracture-distribution modeling over various scales, this paper studies the fracture data obtained by boreholes in granitic rocks. We focus on the fracture density and appearance relation of azimuths between a fracture pair, and aims at inferring three-dimensional distribution of joint-sized fractures.

**VARIOGRAM ANALYSIS**

**Borehole Fracture Data**

The fracture data used in this study were obtained by eight horizontal boreholes in the Kamaishi mine, northeastern Japan. The mine is located in an Early Cretaceous dioritic granite (partly by diorite). The boreholes were dilled over 215-m length in total and toward two perpendicular directions, N15°W and N75°E, at almost the same level, 250 m above the sea. Orientations (strikes and dips), in addition to kinds of interstitial minerals, width of alteration halo, and fracture width were measured for 629 fractures, by the examination of the borehole TV images and drilling cores. Figure 1 shows the arrangement of boreholes and segments that represent the locations and strikes of the fractures.

**Semivariograms**

Clarifying whether the spatial distributions of fracture attributes are perfectly random or have certain spatial correlation structures is the most fundamental process for fracture-distribution modeling. By producing experimental semivariograms of several attributes, it was found that the widths of alteration halos have almost a pure
nugget effect, whereas the widths of interstitial minerals contain a weak correlation structure. As compared to these attributes, fracture densities, which are defined by the number of fractures per 1-m interval along each borehole, showed a more clear feature. It can be demonstrated by the experimental semivariogram in Figure 2. A spherical model drawn by the broken line is applicable to approximate the semivariogram.

Another attribute considered here is the spatial relation of fracture orientations which means directional similarity between a fracture pair. We did not use directly the difference of strike angles or dip angles between two fractures, but attached great importance to the distribution relation of strike or dip data classified into several groups. This idea was proposed to consider continuities of the same fracture set and corporate conjugate-pattern features into fracture-distribution modeling. For this purpose, the four directional sectors for the EW, NW-SE, NS, and NE-SW are defined, and each fracture strike is transformed into an indcitor set ($I_1, I_2, I_3, I_4$) that represents presence ($I_i=1$) or absence ($I_i=0$) of strike in the (EW, NW-SE, NS, NE-SW). For example, N45°W strike defines the set as (0, 1, 0, 0).

This set requires four semivariograms for the same sector and six cross-semivariograms for the different sector pairs. To reduce calculation amount, the principal component analysis was adopted and the set data were transformed into four principal values. Since the fourth principal values become constant for the above indicator set, only three experimental semivariograms are needed for the first to third principal values. Figure 3 shows the experimental semivariogram of the first principal values. While the values are scattered, the semivariogram can be approximated by an exponential model (the broken line in Fig. 3). The same model is applicable to the semivariograms of second and third principal values.

To validate the spatial correlation structures clarified in the two semivariograms, satellite image-derived lineaments were investigated as another fracture-related geologic element. Figure 4 depicts

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Figure 4. Lineaments around the Kamaishi mine extracted from the Landsat TM band 4 image using the Segment Tracing Algorithm (STA).

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Figure 5. Semivariograms of A, the densities expressing number of lineament centers in unit mesh; and B, the first principal values for indicator sets of four strike sectors.
the lineaments around the Kamaishi mine extracted from the Landsat TM band 4 image. The Segment Tracing Algorithm (STA: Koike \textit{et al.}, 1995) was adopted to extract the lineaments. Using the centers of lineaments, two experimental semivariograms of the densities expressing number of centers in unit mesh and the first principal values for the indicator sets of strike sectors were produced as shown in Figure 5. Since they can be approximated by the same semivariogram models as the case of borehole fractures, the semivariograms shown in Figures 2 and 3 are considered to allow to estimate fracture distribution in the study area.

As for the fracture dips, only the dip directions indicated a spatial correlation structure. The dip data were simply transformed into indicator data, 1 for northward dip and 0 for southward dip. The experimental semivariogram of these indicators is shown in Figure 6, which can be approximated by a spherical model.

**TWO-DIMENSIONAL MODELING OF FRACTURE DISTRIBUTION**

**Production of Fracture-Density Map**

The fracture density was chosen as an important key to sow the seeds of fractures which are bases on simulating fracture distribution. It is necessary to extend the one-dimensional fracture densities along the boreholes to two-dimensional densities over the study area. The study area, defined as 60×60 m$^2$ in size, was superimposed on a grid with 1×1 m$^2$ meshes. Calculation of fracture density in each mesh is the first step for the seeds, to which the semivariogram of fracture densities (Fig. 2) is available.

We used ordinary kriging (OK) for the calculation, but could not obtain an appropriate result as shown in Figure 7A. It is obvious that the densities are strongly affected by the nearest data and there are many linear, unnatural boundaries of densities. To overcome this problem, a sequential gaussian simulation (SGS: Journel, 1989) was examined. It consists of transformation of the data into normal scores, determination of random path, Monte-Carlo simulation using kriging variance at each grid point to draw an estimation value, addition
of the value to the data set, and repetition of the calculation. The validness of SGS can be confirmed from the result (Fig. 7B) in that high-density zones are estimated in no data areas in the northwest and unnatural boundaries are not formed. In addition, the density distribution shows large spatial variation as compared to that produced through OK.

**Fracture-Distribution Modeling**

A method of fracture-distribution modeling proposed here utilizes the fracture-density map and consists of the following five steps.

1. In each mesh, fracture centers are generated by the same number as the fracture density estimated for the mesh and their locations are given using random numbers.
2. The strike data expressed in the above indicator set are transformed into the principal values. OK uses these values and interpolates them at each fracture center localized by the step (1). The interpolated principal values are then converted to values in the original coordinate system corresponding to the indicator set. The strike sector with the highest value is chosen as the estimated fracture direction. For example, a calculated set (0.8, 0.3, 0.1, 0.4) assumes the strike to be EW.
3. With respect to dip direction, OK interpolates the indicator-transformed dip directions of the measured fractures at each fracture center. The interpolated value above 0.5 assumes the dip direction of fracture to be northward, otherwise southward.
4. Strike and dip angles at each fracture center are stochastically drawn by combining the cumulative distribution functions of those angles in the selected strike sector and dip direction with Monte-Carlo method.
5. Connection of fracture centers considering both the center distance and directional difference is carried out at the final step. If an arbitrary fracture center (A) can find out another center (B) whose differences of strike and dip angles from the A are under 10° and 20°, respectively, the two centers are connectable. Then, the B is used as a start point for the next search. Isolated fracture that has no connectable center is eliminated.

Figure 8A depicts the result of the fracture-distribution modeling. To clarify distribution characteristics in detail, only fractures estimated longer than 10 m are extracted as shown in Figure 8B. The conspicuous directions of continuous fractures (N20°-40°E and N20°-50°W) agree with those of the measured fractures. Usefulness of the proposed method can be proved in that it can estimate the ENE-WSW striking fractures almost parallel to the boreholes, which are hardly appeared in the boreholes, in the northern area and continuous.

![Figure 8. A, Result of two-dimensional fracture-distribution modeling; and B, distribution of extracted fractures estimated longer than 10 m.](image-url)
fractures in the zones without data. Another interesting feature is existence of the zones that contain the concatenated fractures passing through the study area.

VALIDATION OF THE PROPOSED METHOD

The proposed method of fracture-distribution modeling must be validated whether it allows to estimate practical distribution and characterize hydraulic properties of the rock mass. For this purpose, a rock specimen over which fracture distribution is entirely observed is desirable. We prepared a thin-section of the Inada granite with 1.5×1.5-cm² size. The Inada granite situated in central Japan is known to contain many microcracks. A modified version of the STA suitable for the microcrack analysis was used to extract linear features from the digital image of thin-section. This non-filtering technique can specify the portions in an image where the brightness largely change attributable to cracks and trace the portions along the cracks.

Figure 9A shows the extracted 3559 linear features that partly correspond to microcracks. On the image, virtual lines were set by modeling after the arrangement of boreholes in the study area (Fig. 1), and the intersections of the lines and linear features (Fig. 9B) were used to reconstruct the linear-feature distribution. Unit mesh size for calculating the linear-feature density was defined as 0.25×0.25 mm² so that the number of meshes, 60×60, is equal to that used for the fracture-density map in Figure 7.

As seen in the final result (Fig. 9C), the reconstructed pattern is similar to the linear-feature distribution in that it highlights three dominant directions, about 30°, 120°, and 160° counterclockwise from the x-axis. The validness of the proposed method is demonstrated because these dominant continuous line features are estimated in the zones with no data.

EXTENSION TO THREE-DIMENSIONAL FRACTURE-DISTRIBUTION MODELING

For more practical modeling of fracture distribution, it is necessary to extend the proposed
method to be available in the three-dimensional space. Information on vertical continuities of the fractures cannot be obtained because the boreholes are located at almost the same level. Therefore, an assumption is needed for a three-dimensional modeling. We assumed that the fractures observed at the 250-m level may be continuous only within the ±5-m depth range.

The fracture locations at the 245 to 255-m levels were given at 1-m depth interval by extending the observed fractures along their strikes and dips. At each level, virtual boreholes were set on the same locations at the 250-m level, and the intersection points of the virtual boreholes and the extended fractures were used for calculating the fracture densities. Two-dimensional distribution modelings were at first executed by the above-mentioned method and then, fracture planes were constructed by connecting simulated fractures at adjacent levels based on the similarity of strike and dip and the nearness of locations. For example, the distributions conjectured at the 248 and 252-m levels are shown in Figure 10, from which different patterns can be found with the level.

The modeling result is visualized in Figure 11, which represents the locations of fracture planes longer than 5 m along the vertical direction. The constructed planes are slightly undulate, but can be considered to steeply dip at larger than 70°. Figure 12 shows the azimuthal frequencies of the planes using the lower hemisphere projection of the Schmidt’s net. There are two sets of the simulated continuous fracture planes, the most remarkable NW-SE striking set and the NNE-SSW striking set subordinate to it.

**APPLICATION TO CHARACTERIZATION OF HYDRAULIC PROPERTIES**

The three-dimensional fracture-distribution model can be linked to the permeability tensor analysis to estimate permeability of the rock mass. According to Oda et al. (1987), the permeability tensor, \( k_{ij} \), of degree three is expressed by

\[
\begin{align*}
\mathbf{k}_{ij} = \lambda \left( \mathbf{P}_{ij} \delta_{ij} - \mathbf{P}_i \right) \\
\mathbf{P}_{ij} = \frac{\pi \rho}{4} \iiint_\Omega r^2 t^3 \mathbf{n}_i \mathbf{n}_j E(n, r, t) d\Omega dr dt
\end{align*}
\]

where \( \delta_{ij} \): delta function, \( \lambda \): a constant expressing the continuity of fractures, \( r \): diameter of fracture, \( t \): hydraulic aperture of fracture, \( \mathbf{n} \): normal vector of fracture plane, \( E(n, r, t) \): probabilistic density function with respect to \( n, r \), and \( t \), \( \rho \): volumetric fracture density, and \( \Omega \): solid angle. We defined the \( \lambda \) as 1/12 and supposed that the \( t \) is \( 10^{-6} \) times the \( r \). The \( r \) was determined by converting a fracture plane into a disc with the equivalent area.

Discretization of Eq. (1) was carried out and three principal values \( (k_1, k_2, k_3) \) and directions of the principal axes were calculated as shown in Table...
1. The principal axes generally correspond to the two dominant strikes for the $k_1$ and $k_2$ and the dip direction of the predominant fractures for the $k_3$. The magnitude of principal values cannot be validated because there is no measured permeability data in the study area. However, hydraulic well tests were executed in other areas in the Kamaishi mine compared to the present study area. Considering the data, condition of the rock mass, and the permeability data reported for hard granitic rocks in many areas (e.g., Brace, 1980; Brace, 1984; Clauser, 1992), the first principal value, $10^{-17} \text{ m}^2$, can be regarded as a plausible magnitude.

**CONCLUSION**

By analyzing the borehole fracture data in the Kamaishi mine, spatial correlations were clarified on the three attributes of fractures, fracture densities along the boreholes, appearance relations of fracture strikes, and dip directions with respect to northward or southward dip. The experimental semivariograms...
of these attributes could be approximated by spherical, exponential, and spherical models, respectively. The SGS was proved to be effective for estimating fracture densities in the sparse data zones. To consider the azimuthal correlation of fracture pairs, we transformed the strike and dip data into the indicators. Applying the principal component analysis and ordinary kriging to the indicators, two- and three-dimensional models of fracture distributions, which incorporate both the azimuthal and positional information of the fracture data, could be constructed.

The three-dimensional model was effectively linked to the permeability tensor analysis for estimating the permeability and principal axes. Two characteristics were detected by it in that the permeability along the first principal axis is about $10^{-17}$ m$^2$ and the major axes correspond to the dominant azimuths of simulated fracture planes. The estimated permeability is plausible by considering the condition of rock mass and the hydraulic test data obtained at other sites in the mine.

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REFERENCES


