All Models Are Wrong, but Some Models Are Useful: "Solving" the Simandoux Equation

John H. Doveton,
Kansas Geological Survey, University of Kansas, Lawrence, Kansas USA

Abstract

Log analysis solutions for water saturation in shaly sandstone hydrocarbon reservoirs are elaborations of the Archie equation, with extra terms that accommodate volumes of shale or bound water and their associated electrical properties. There are a large number of alternative shaly sandstone equations that are used today because no uniquely satisfactory solution has been reached. Simpler equation forms run the danger of becoming simplistic, but are robust, comprehensible, and can perform surprisingly well when applied thoughtfully. More complex equations are better functional representations, but involve additional terms which are often difficult to estimate, while introducing more error interactions. With the typical situation of limited subsurface information and the variety of shaly sandstones, the comparative performance of any equation model is debatable. However, if models are used from the point of view of utility, then the calibration within a shaly sandstone reservoir can be made as an optimization problem based on a (provisional) recognition of water zones. By this means, a petrophysical optimum is honored rather than a purely mathematical one. The approach applies a statistical derivation of parameter values from the shaly sandstone reservoir under analysis within the framework of classic shaly sandstone equations that have been proposed from theoretical and laboratory considerations.

Prolog: The Archie Equation

From empirical observations, Archie (1942) established that the ratio of the resistivity of a completely brine-saturated rock (Ro) to the resistivity of its contained brine (R_W) was a constant for any given rock sample, and gave the name, resistivity formation factor (F) to this proportionality constant,

$$\text{i.e. } F = \frac{R_0}{R_w}$$

Archie (1942) further showed that there was a strong linear relationship between the logarithmic transforms of the formation factor (F) and porosity (Φ) in sandstones which took the form of:

$$F = \frac{1}{\Phi^m}$$

where *m* took different values according to the relative consolidation of the sandstone sample. Unconsolidated sandstones showed low values of m, as contrasted with higher values in consolidated sandstones. This observation led to the informal name for *m* as the “cementation factor”. In later work, particularly following the lead of Winsauer et al (1952), the numerator was generalized from 1 to the term *a*, so that the relationship could accommodate a variety of sandstone types.

In the same paper, Archie (1942), showed that the ratio of the resistivity of a rock that was partially saturated with brine (R_t) to the resistivity of the same rock that was completely saturated with brine (R_O) could be related to the fractional water saturation (Sw) by the relationship:

$$\frac{R_t}{R_0} = S_w^{-n}$$

where he stated that *n* took a value close to 2. In subsequent work, *n* has been called the "saturation exponent".

Combining these equations to solve for water saturation led to what is now known as the Archie equation:

$$S_w = \sqrt[n]{\frac{a \; R_w}{R_t \quad \Phi^m}}$$

which remains the most widely applied equation in petrophysics, in spite of all attempts to replace this empirical relationship with a more rigorous formulation made up of terms identified with distinctive physical properties.

Although the origins of the Archie equation are empirical, Bussian (1982) showed that the Archie equation was equivalent to the Hannai-Bruggeman equation at the low limit of electrical frequency and where the rock framework conductivity was effectively zero. The Hannai-Bruggeman equation represents the culmination of progressive developments in physics dating back to Maxwell (1873). Other authors such as Herrick and Kennedy (1994) disagree and maintain that the Archie equation remains a useful parametric function with no physical basis. Even if the Archie equation can be reconciled with physics, the physical significance of its terms is still not fully clarified. Many authors have tried to determine what the components of the equation actually *mean* as observable descriptors of the passage of an electrical current through the pore network of a rock and its content of fluids and gas. Part of the problem is the difficulty of first finding simple but effective descriptors of the labyrinthine complexities of the pore network of a sandstone, and second doing this in a central conceptual construct that is acceptable to a variety of disciplines. Fundamentally, the problem can be related either to properties of the pore network or its mineral framework (Fig. 1). Of course, the two are complementary but relationships and interrelationships can be complex and not necessarily intuitive.

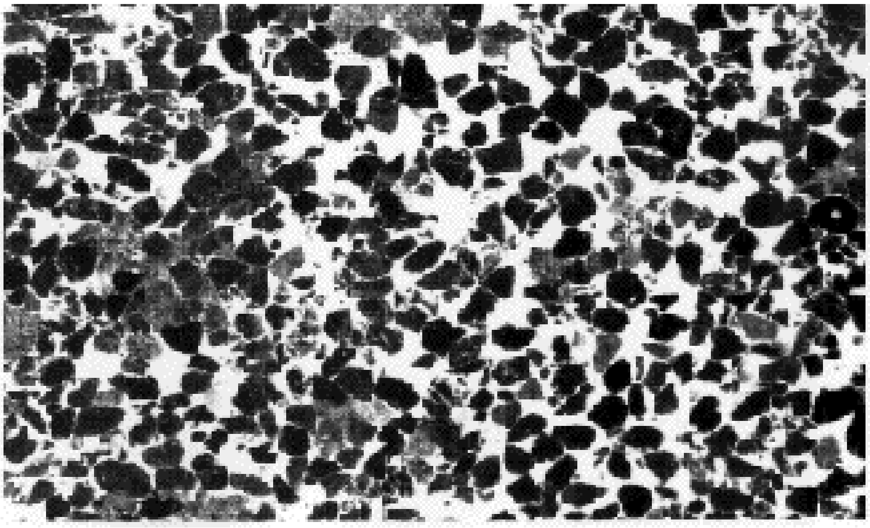

Figure 1: Thin-section of Bartlesville Sandstone (Pennsylvanian). Dark objects are quartz grains, mica, and lithic fragments; white is pore space. Approximately X20 magnification.

One of the earliest models considered the pore space to be represented by a bundle of capillary tubes. If the tubes have a constant cross-sectional area, then resistivity variation is controlled by the length of the tubes. The cementation factor, m, was then effectively a measure of pore channel tortuosity. However, capillary tubes are a restrictive representation of the elaborate branching of the pore network. Some authors such as Perez-Rosales (1982) consider the pore space to be subdivided into channels and traps, where channel pores are the only elements to contribute to electrical current flow. Under this model, the cementation factor is a measure of the relative partition between conductive channel pores and non-conductive trap pores. Still other authors, such as Etris et al (1989) have expanded the representation to a more realistic system of pore bodies connected by pore throats. In this case it is argued that the major control on the formation factor is the distribution of pore-throat areas. In a later paper that expanded this model, Ehrlich and others (1991) suggested that the cementation factor, m, was effectively a measure of the ratios of the logarithms of the pore-throat area to the pore-body area.

Finally, turning from pore-network to framework models of explication, much has been learned from laboratory measurements of electrical current flow in grain packs with varying sizes, shapes, and degrees of sorting. Jackson et al (1978) showed that size and sorting of grain-size had little effect on the cementation factor, but that this was sensitive to grain shape. The cementation factor increased as grains became less spherical, and was highest in sediments dominated by platy grains. The conclusions from these empirical data were supported independently by a mathematical derivation of Sen et al (1981). Mendelson and Cohen (1982) observed that the cementation factor was lowest in rocks where the grains had the same shape and orientation. However, cementation factor values were increased if the grains were aligned with a distribution of shapes or if they were randomly oriented. In summary, grain properties that controlled cementation factor variation are shape, shape distribution, and orientation.

Although Archie's original formulation contained no parameter termed *a*, opinion has been sharply divided on the physical significance or lack thereof of this term. For those who ascribe a physical meaning to *a*, it is often known as the "tortuosity index". Others (such as Doveton, 1986) see it simply as slippage element to accommodate changes in the cementation factor in the Archie equation for a heterogeneous set of sandstones, when the cementation factor is constrained to take a single value. This is certainly the case with the "Humble equation" (Winsauer et al, 1952) where: $F = \frac{0.62}{\Phi^{2.15}}$ and so $a = 0.62$. Apart from anything else, a value of $a \neq 1$ implies that in the limit, $R_w \neq R_w$ and so that the Archie equation is a poor representation of a true functional relationship. However, as a conclusion from a detailed laboratory study (and a vindication of Archie's original equation), Maute et al (1992) demonstrated that *a* was "a weak-fitting parameter, with no physical significance, that can generally be set to unity."

Although a considerable body of research has been devoted to the Archie equation as applied to rocks that are completely saturated with brine, less work has been directed to the consideration of the saturation exponent, n, which controls the resistivity – water saturation relationship. In part, this discrepancy reflects the extra difficulties involved in both measurement and evaluation. Archie's original suggestion (Archie, 1942) that *n* takes an approximate value of 2 is still widely accepted, and is not unreasonable for water-wet rocks, although it takes a much higher value in oil-wet rocks. However, even in water-wet systems, the saturation exponent has been observed to vary as a function of water saturation. Explanations for the meaning and behavior of the saturation exponent must be related to the modified geometry available for electrical current flow. At partial water saturations, pore bodies contain globules of oil that are an additional non-conductive phase.

Stalheim and Eidesmo (1995) suggest that changes in the saturation exponent are caused by changes in oil globule shape from spheres at high water saturation to shapes that progressively take on the pore-body shape at lower water saturations. As an alternative explanation, Etris et al (1989) point out that the mechanism of oil emplacement within pores is controlled by capillary forces. The lowering of water saturation represents the progressive breaching of successively smaller pore throats, so that at partial saturations, larger pores contain oil, while smaller pores remain fully water-saturated. If the pores are effectively filled by oil such that the residual water saturation makes a minimal contribution to conductivity, then all the electrical current is carried preferentially by smaller pores. Etris et al (1989) therefore concluded that resistivity at lower water saturations was matched by a progressive change to smaller pore-throats available for current transmission.

In summary, a variety of explanations have been offered for the *meaning* of the elements of the Archie equation, and so to account for its predictive power. Each explanation has merits within the framework of its conceptual model, but limitations that reflect the failure of the model to capture salient characteristics of the reality of the pore system. The motivation to clarify the role and meaning of the terms in the equation comes from both a healthy scepticism concerning empirical equations and the commercial need to evaluate the performance and operating limits of the equation. The success of the Archie equation can be judged by the informal use of the term “Archie rock” with the parameters a=1, m=2, n=2 which is a remarkably useful descriptor for the resistivity of rocks that are dominated by intergranular or intercrystalline porosity. However, the extension of the empirical model to the “non-Archie” rocks of shaly sandstones requires the evaluation of more elaborate (and frequently contentious) alternative models.

Shaly sandstone resistivity equations

The Archie equation presupposes that the rock framework has no electrical conductivity. However, the presence of clay minerals in sandstones adds a conductivity contribution that causes the Archie equation to overestimate the water saturation and therefore be pessimistic in the search for oil or gas. The economic implications of an ideal resistivity equation for shaly sandstones have stimulated considerable research and discussion by industrial petrophysicists over the years. A selection of a few of the many shaly sandstone equations that have been published is shown in Figure 2. Without exception, they all revert to the Archie equation in the limit of no shale content, and all are constructed with the concept of parallel resistances of the pore brine and shale components. So, the generic equation takes the form of:

$$\frac{1}{R_t} = \frac{S_w^2}{F\ R_w} + X$$

where X is the conductivity contribution of the shale element.

Historically, two distinct model families of shaly sandstone equations have been developed (Worthington, 1985). The older model considered the shale as a homogeneous conductive medium and developed resistivity equations keyed to V_{sh}, the volumetric fraction of shale in the rock. Although the physical basis of the model is incorrect, the equations often provide reasonable approximate solutions to water saturation, especially when the equation parameters are adjusted so that the results conform with local water saturation data measured from cores or production tests. Some of these equations are still widely used for this reason and also because of their relative simplicity and limited demands on additional input parameters. Examples of these equations in Figure 1 are those which use an R_{sh} term for shale resistivity.

Poupon et al (1954) : $$\frac{1}{R_t} = \frac{(1 - V_{sh})S_w^2}{F \;\; R_w} + \frac{V_{sh}}{R_{sh}}$$

Hossin (1960) : $$\frac{1}{R_t} = \frac{S_w^2}{F \;\; R_w} + \frac{V_{sh}^2}{R_{sh}}$$

Simandoux (1963) : $$\frac{1}{R_t} = \frac{S_w^2}{F \;\; R_w} + \frac{\varepsilon V_{sh}}{R_{sh}}$$

Waxman and Smits (1968) : $$\frac{1}{R_t} = \frac{S_w^2}{F^* \;\; R_w} + \frac{BQ_v S_w}{F^*}$$

Bardon and Pied (1969) :
(modified Simandoux) $$\frac{1}{R_t} = \frac{S_w^2}{F \;\; R_w} + \frac{V_{sh} S_w}{R_{sh}}$$

Poupon and Leveaux (1971) : $$\frac{1}{R_t} = \frac{S_w^2}{F \;\; R_w} + 2\sqrt{\frac{V_{sh}^{2-Vsh}}{FR_w R_{sh}}}S_w^2 + \frac{V_{sh}^{2-Vsh} S_w^2}{R_{sh}}$$

Schlumberger (1972) : $$\frac{1}{R_t} = \frac{S_w^2}{F(1 - V_{sh})R_w} + \frac{V_{sh} S_w}{R_{sh}}$$

Clavier et al (1977) : $$\frac{1}{R_t} = \frac{S_w^2}{F_o R_w} + \frac{(C_{bw} - C_w)V_Q Q_v S_w}{F_o}$$

Juhasz (1981) : $$\frac{1}{R_t} = \frac{S_w^2}{F \;\; R_w} + \left(\frac{1}{F_{sh} R_{sh}} - \frac{1}{R_w}\right)\frac{V_{sh}\phi_{sh} S_w}{\phi}$$

Figure 2: "All models are wrong; some models are useful" : A sample selection of species from the extensive collection of published shaly sandstone equations.

Probably the best known is that of Simandoux (1963) which originally was written as:

$$\frac{1}{R_t} = \frac{S_w^2}{F \;\; R_w} + \frac{\varepsilon V_{sh}}{R_{sh}}$$

where ε=1 when Sw=1 and ε<1 when Sw<1. However, the Simandoux equation used today is usually the modified version as a quadratic equation of water saturation, where ε = Sw as first introduced by Bardon and Pied (1969) and so is:

$$\frac{1}{R_t} = \frac{S_w^2}{F \quad R_w} + \frac{V_{sh} S_w}{R_{sh}}$$

However, not only has the physical model for its inspiration been discredited, but the seminal paper (Simandoux, 1963) is obscure, and even its author only vaguely recollects the equation, as recounted by Chemali (2001): “As for Simandoux, you must have heard of a visit I made to IFP Production Automation Department. I noticed that the name of the Department Manager was Simandoux. I approached him and asked him if he knew of another Simandoux who was a log analyst. He told me that he had "dabbed" in the subject during his youth, but remembered little of it. He then asked me how I did come to know of his previous job in log analysis. I explained to him that the Simandoux equation was in just about every log analysis package known to man. He could not believe it. He did remember though that it had to do with calculating saturation....”.

The second, and more recent, model is based on the ionic double-layer observed in shaly sandstones for which the classic equation is that of Waxman and Smits (1968). In reality, the conductivity of the shale component is a function of cation exchange capacities of the various types and abundances of clay minerals which are present. Since the cations are exchanged primarily at broken bonds on the edges of flakes or by lattice substitutions on cleavage surfaces, the phenomenon tends to be surface-area dependent rather than controlled simply by the volume of clay minerals. This implies that a fine grained clay has a higher exchange capacity than a coarser grained form of the same clay volume, and this observation is confirmed by experimental data. Since all the shale indicators estimate (at best) the volume of the shale component, no explicit assessment is made of the grain size or clay mineralogical variation. Although these factors are widely known among log analysts, it is difficult to design log analysis procedures to accommodate them, in the absence of a tool that measures cation exchange capacity directly. Consequently, the model equations that use cation-exchange data have been modified to variants that substitute quantities that can be measured on logs as surrogate variables, such as that of Juhasz (1981) which is suggested to be a “normalized” representation of the Waxman-Smits equation.

In summary, the clay volume resistivity equations are unrealistic and the ionic double-layer equations are usually impractical . Since none of these equations can possibly be “correct”, we can agree with the widely-quoted aphorism of the statistician, George Box, “All models are wrong; some models are useful”. But which model is the most useful? According to Einstein "A model should be as simple as possible and yet no simpler" and this advice conforms with the dictum of Occam’s Razor. Certainly, from a statistical point of view, we should choose the model that minimizes the error between the estimates of the water saturation (definitely the variable of interest) and their true values. At the same time, model equations should be favored that are simple (but not simplistic) with few parameters to estimate, thereby minimizing error terms and error propagation. Rather than being purely a philosophical ideal, the principle of parsimony has practical consequences in statistical fitting where overly complex models often perform poorly. Calibration data are readily overfitted by models with many parameters because of the loss of degrees of freedom and the characterization of quirks in variation that are peculiar to the data set. Predictive power is captured in simpler functions that generalize systematic relationships in calibration data sets which are common to other sets sampled from the same population. These fundamental concepts of model design and evaluation govern good statistical practice, where calibration trials are followed by a validation phase of testing before being used for prediction. In cases where extra data sets are not available for validation, jackknifing of the calibration data set is an acceptable substitute.

In its most basic form, the shaly sandstone equation can be written as:

$$\frac{1}{R_t} = \frac{S_w^2}{F\ R_w} + X$$

which is partitioned between an Archie equation component and a composite of clay conductivity terms, X. In the simpler (and still widely used) shaly sandstone equations, the two parameters of the X component are V_{sh}, the volume of shale, and R_{sh}, the resistivity of the shale. Both of these parameters are problematic and bring uncertainties to all shaly sandstone equations as applied to formation evaluation in the subsurface.

The volume of shale is determined typically from either the spontaneous potential, gamma ray, or the combination of neutron and density logs. In each case, a value is selected for a shale-free zone, and another as a value that best typifies shales, and V_{sh} calculated for the section by interpolating between these two extremes (see Figure 3).

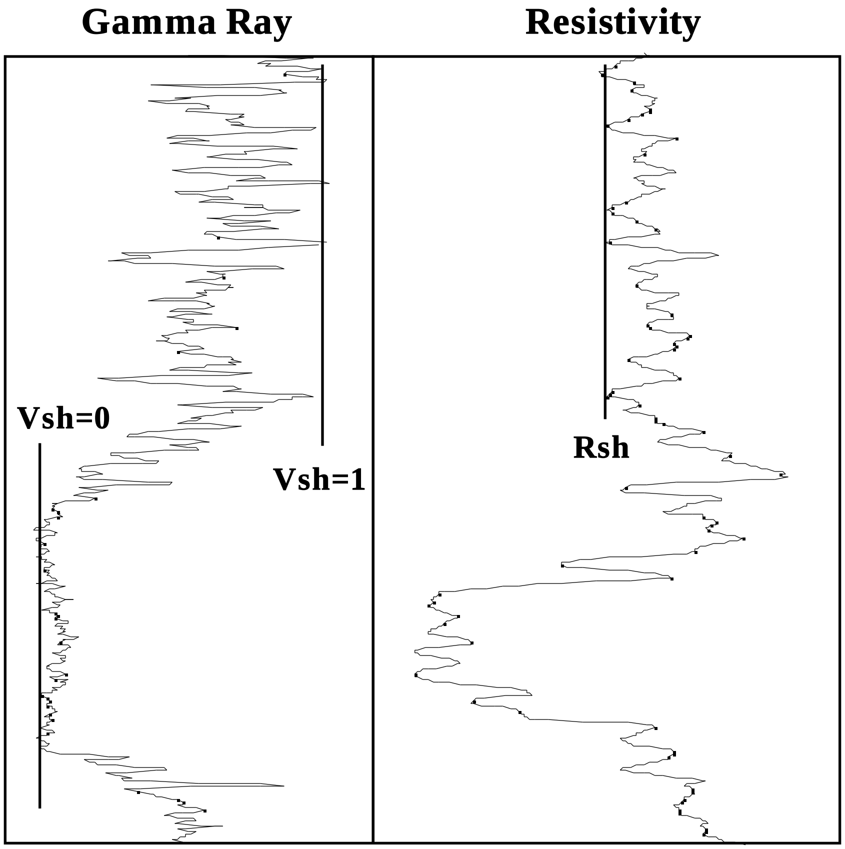

Figure 3: Simple schematic diagram of Vsh shale volume estimation (using the gamma-ray log in this example) and selection of a shale resistivity value, Rsh.

Estimates of V_{sh} calculated from different logs differ from one another to varying degrees, and the lowest estimate is usually taken to be the most valid. This decision is made on the grounds that, if anything, a shale indicator will tend to overestimate the shale content. This is a reasonable thesis, because accessory components other than shale will generally result in an increased apparent shaliness as recorded by the logging tools. Unfortunately, the shale used for calibration is located *between* the reservoir units and often differs from shaly material *within* the reservoir. These external shales are lithostatically loaded are typically a mixture of about 60% clay minerals and 40% non-conductive silt mineral accessories such as quartz and calcite. By contrast, the clay minerals within the pore network of sandstones are hydrostatically loaded and have different morphologies and often different species than those in external shales. The V_{sh} scale is therefore a provisional and apparent estimation of “shale volume” applied to clays within reservoir units. Even where the volumetric estimation is not unreasonable, the clay conductivity is likely to be more closely controlled by surface *area.*

As with the volumetric estimation procedure, the shale resistivity, R_{sh}, is selected as a low (or lowest) value from shales between the reservoir units. Again, any systematic equivalence between resistivities of external shales and that of clays within reservoir units is likely to be fortuitous. This discrepancy is widely recognized and, in practice, some adjustment may be made within the shaly sandstone analysis by, for example, adjusting the value of R_{sh} to generate good estimates of water saturation in zones that are considered to be fully water-saturated.

Clearly it would be preferable for shaly sandstone analysis to be recast as a procedure that calibrates shale effects from within the reservoir. This strategy can be implemented as an optimization problem that minimizes the sum of least-square differences between observed water saturations and values predicted by the model equation. The approach differs from the conventional "top-down" procedure, where parameters are determined either from core measurements or from values picked from logs or from a combination of these sources. This traditional approach has the merits of being petrophysically-driven but as we have already seen, the shale parameters are often drawn from shales external to the reservoir unit and so may have little or nothing to do with the clays that are within the reservoir sandstone. By contrast, the calibration approach is a "bottom-up" procedure that solves parameter values statistically. A particular benefit is that the predictive model is supervised by the calibration procedure. However, a drawback is that the terms are determined empirically and their petrophysical significance is evaluated after the analysis.

Least-squares minimization commonly applies either Newton’s method or a conjugate gradient method to the problem of solving values for equation parameters. The application of these optimization methods involves an iterative search for the best values of the terms in this quadratic that produce a minimum squared difference between computed water saturations and their true values. In this search, constraints are applied, so that values occur in petrophysically reasonable ranges. By these means, a petrophysically reasonable optimum is honored rather than a purely mathematical one. The procedure does not require specialized software, but is a standard option within many spreadsheet programs (such as SOLVER within EXCEL).

The source of observations of water saturation that are "true" values can be drawn either from core measurements or from shaly sandstone zones that are fully water saturated. Water saturations from oil-base core are generally accepted to be close to reservoir values, in contrast with those taken from holes drilled with water-base mud where invasion can be a significant problem. However, core data of this kind are relatively uncommon and so practical calibration will generally be based on water zones in shaly sandstone successions. In this case, the procedure has much in common with the philosophy of the reconnaissance water resistivity (R_{wa}) method where a *solvitur ambulando* approach is used to determine the formation water resistivity (R_w) from reservoir log analysis.

Case studies: The derivation of a least-squares minimization (LSM) shaly sandstone equation

(a) using core data

The first case study draws on data published by Smith and Rouleau (1977) who demonstrated that Simandoux equation predictions consistently underestimated water saturations when compared with oil-base core measurements. (This optimistic bias of the Simandoux equation to higher oil saturations is widely known and accounts for some of its popularity!). A comparison is shown between the Archie and Simandoux equation estimates of water saturation as related to oil-base core measurements in a Basal Cretaceous Zone section from Western Canada (Figure 4).

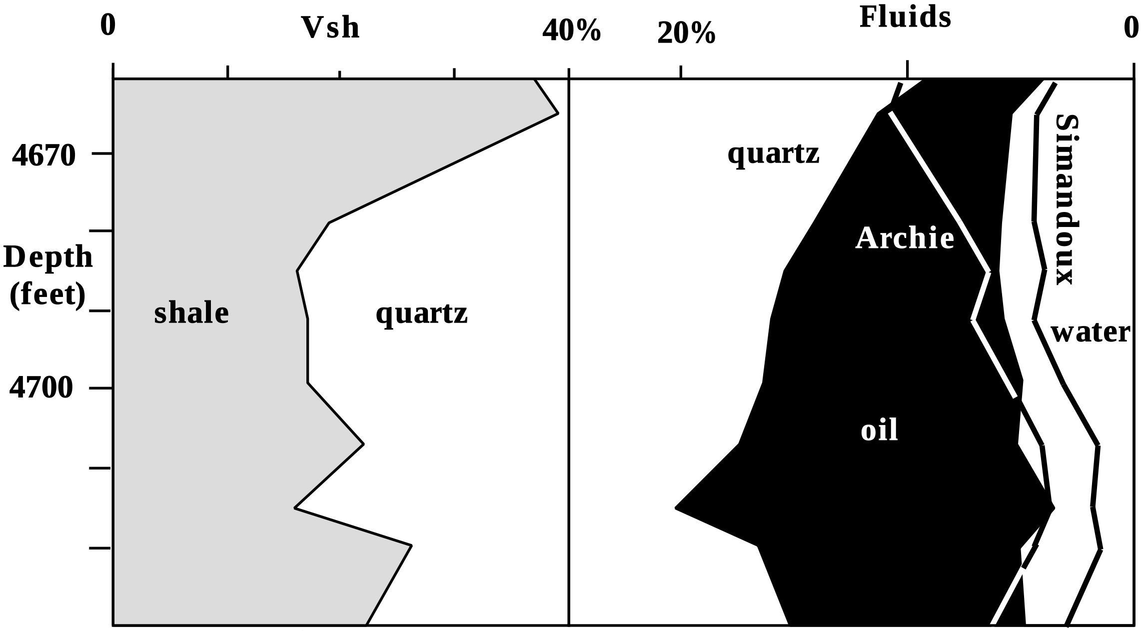

Figure 4: Comparison of water saturations computed by the Archie equation and the Simandoux equation with oil-base core measurements in a Basal Cretaceous Zone section from Western Canada.

In this case, shale volume estimates, V_{sh}, were computed from the gamma-ray log and the estimate of R_{sh} equated with the most conductive shale zone in the overlying Cretaceous section. While Simandoux estimates are biased to low water saturations throughout, the Archie equation overestimates water saturations at high shale contents, but provides a reasonable match with core values at lower shale contents. In an alternative approach, if the Simandoux equation is slightly generalized to :

$$\frac{1}{R_t} = \frac{S_w^2}{F\ R_w} + \frac{V_{sh}^d S_w}{R_{sh}}$$

then the parameters of *d* and R_{sh} can be solved in a calibration procedure internal to the shaly sandstone. The exponent *d* allows the apparent shale volume estimator to be modified to allow for differences between external shale and internal reservoir clay log responses, volume and surface area representations, and other factors. The shale resistivity term is no longer equated with an external shale but allowed to express an empirical reservoir clay resistivity (or conductivity) that is not linked with a physical process model.

The least-squares minimization between core values of water saturation and model predictions gave an LSM equation of:

$$\frac{1}{R_t} = \frac{S_w^2}{F\ R_w} + \frac{V_{sh}^{3.13} S_w}{0.25}$$

and the new estimates compared with the core values in Figure 5.

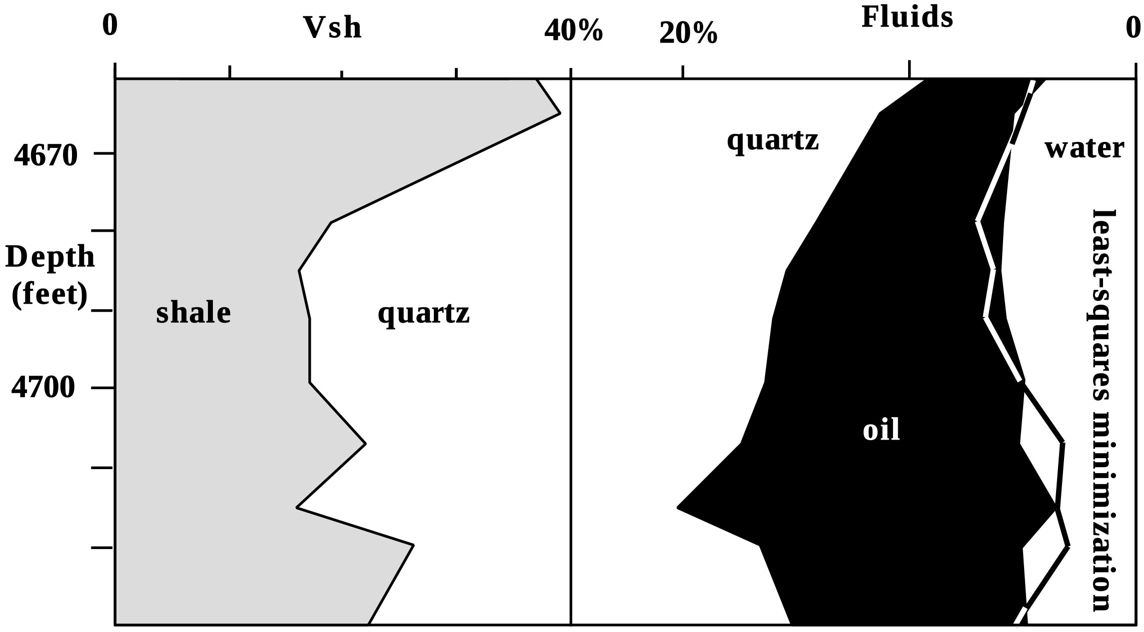

Figure 5: Comparison of water saturations fitted by least-squares minimization with oil-base core measurements in a Basal Cretaceous Zone section from Western Canada.

The increase in the exponent *d* from its implied value of unity in the conventional Simandoux equation to 3.13 reflects a preferential weighting of clay effect at higher V_{sh} values and diminished effect at lower values (where Archie equation estimates are close to core values). At the same time, the estimate of R_{sh} at 0.25 ohm-m is lower than the resistivity of external shales and is probably a closer reflection of the resistivity of bound water as would occur in a dual-water model. However, an explicit interpretation is further complicated in that part of this term may be a weighting modifier to the V_{sh} term. So the petrophysical 'meaning' of the terms is more difficult to pin down than a traditional top-down model because of its empirical nature. However, consistency is honored because the Archie component of the shaly sandstone equation is itself empirical!

(b) using water zones

Water saturation measurements from core are not widely available and. even if they were, the methodology used in the first case-study would be unnecessarily restrictive. More commonly, no core data are available and so calibration opportunities are restricted to zones that are perceived to be fully water saturated. In this case, the optimization procedure minimizes squared differences between iterative model estimates and total water saturation within water zones. Following a satisfactory analysis, the calibrated equation can then be applied to the productive interval. The procedure is described in a second case study.

In his handbook on shaly sandstone log analysis methods, Asquith (1990) describes the analysis of the Pennsylvanian Red Fork Sandstone in a well located in western Oklahoma. The Red Fork Sandstone is a fine-grained shaly sandstone, in which shale occurs as dispersed clay. In his analysis and application of the Simandoux equation, Asquith (1990) used a shale resistivity, R_{sh} of 3 ohm-m.

The Archie equation is pessimistic to the degree that irrationally high water saturations are computed in the upper section where shale effects and water saturations are highest. The profile of Simandoux water saturations are much lower throughout and probably somewhat optimistic in terms of oil. By using the upper leg as probable water zones, the optimization procedure yielded values to a LSM equation of:

$$\frac{1}{R_t} = \frac{S_w^2}{F \ R_w} + \frac{V_{sh}^{3.06} S_w}{1.26}$$

The value of *d* as 3.06 is similar to the Canadian case study and for apparently the same reason, the preferential influence of shale effects at higher rather than lower volumetric shale contents. The increase in “shale resistivity” value to 1.26 ohm-m would be consistent with a decrease in ionic mobility of Pennsylvanian clays relative to Cretaceous. At the same time, the value probably reflects a bound water resistivity term which is lower than the shale resistivity of 3 ohm-m from external shales in the section. The comparison of predictions between the Archie, Simandoux, and least-squares minimization (LSM) equations for the Red Fork Sandstone section in Figure 6 shows a similar style to the Canadian case study and therefore encouragement that water zone calibration of a shaly sandstone equation is a viable formation evaluation procedure.

Variations on the shaly sandstone equation model could (and should) be used as the basis for least squares minimization solutions. Some discussion has already been made on the incompatibility between parameters established for shales between reservoirs as representative of parameters for clays within the reservoir sandstones. In these case studies, R_{sh} was established in an independent calibration, but the V_{sh} evaluation was still linked (although modified) to a scaling anchored to an external shale. In more comprehensive LSM model, the V_{sh} upper bounding value could be set as one of the target parameters to be solved. Furthermore, the analysis could be expanded to the independent evaluation of different Vsh measures, so that the selection was keyed to prediction performance rather than the rule-of-thumb notion of selecting the most conservative shale indicator. In this paper, a generalized Simandoux equation was used as a simple model for demonstration purposes, but the philosophy of reservoir calibration by least squares minimization can be used in conjunction with any shaly sandstone log analysis procedure.

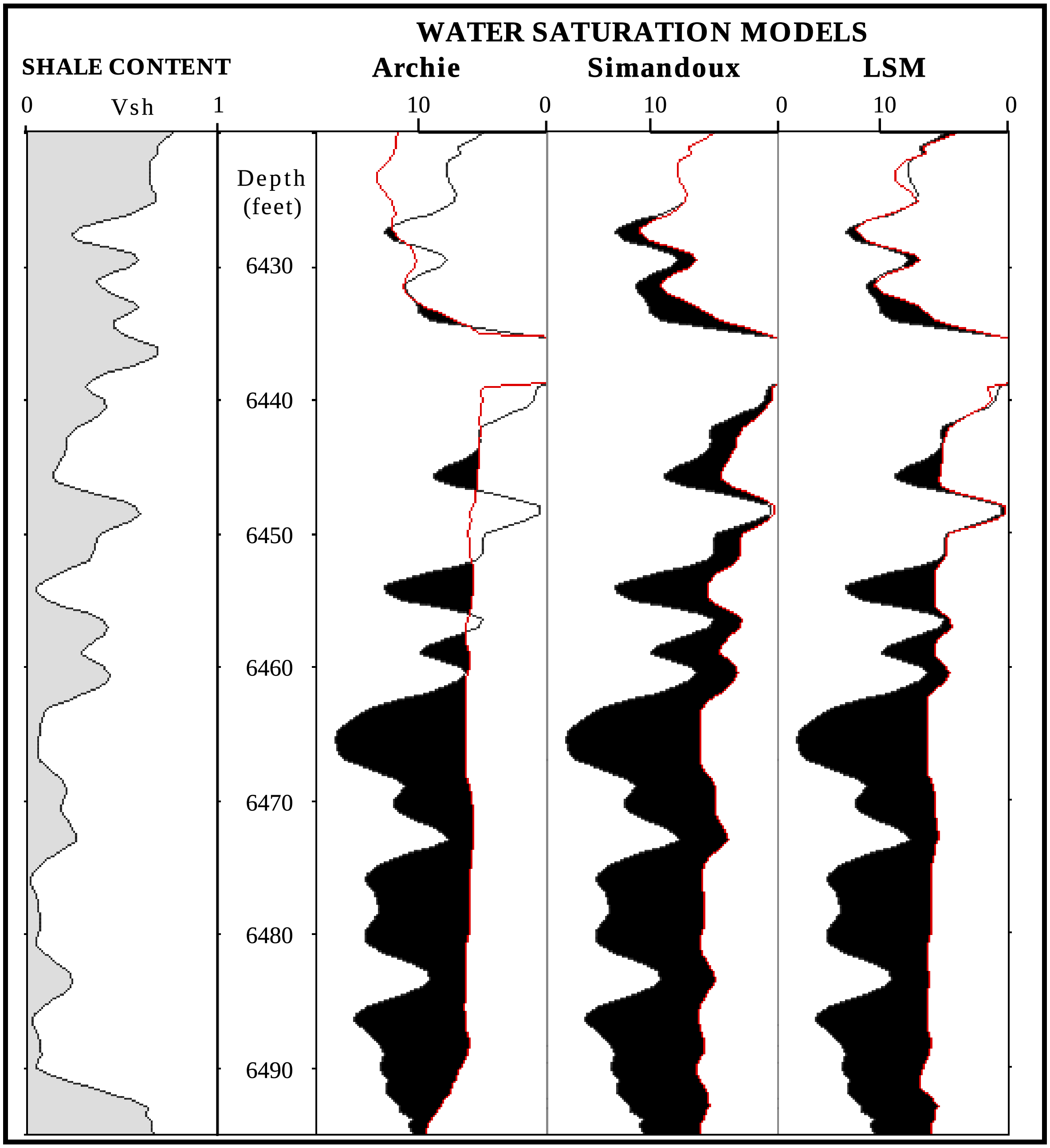

Figure 5: Comparison of alternative fluid saturations in a **Pennsylvanian Red Fork Sandstone** section from **western Oklahoma** estimated by the Archie and Simandoux equations with a **least-squares minimization** (LSM) estimation calibrated from water zones.

Conclusions

Laboratory research and physical and mathematical derivations have done much to improve our understanding of the complexities that govern the resistivities of shaly sandstones in the search for water saturation equations with predictive power. However, the failure of any single equation to triumph demonstrates the variability of shaly sandstone reservoirs and the difficult transition from the laboratory and theoretical physics to the subsurface reservoir. For practical formation evaluation, the general similarity in structure of the many alternative equations that have been published provides a framework for model solution from a utilitarian viewpoint. The Simandoux equation is an example of a simple template that can be used in an optimization procedure, where the terms are calibrated using water saturations from either core data or (more commonly) water zones within the shaly sandstone reservoir. The procedure is a standard least-squares minimization that can be run on spreadsheet software. The result is empirical, but shares this property with the Archie equation which is itself empirical. As with the Archie equation, the “meaning” of the shale terms is open to interpretation and will vary widely according to the great range of shaly sandstone facies that occur in economically important reservoirs across the world. However, in the words of Oscar Wilde, “Art is not to be taught in Academies. It is what one looks at, not what one listens to, that makes the artist. The real schools should be the streets.”

References

Archie, G.E.,1942, The electrical resistivity log as an aid in determining some reservoir characteristics: Transactions of the American Institute of Mechanical Engineers: v. 146, p. 54-62.

Asquith, G.B.,1990, *Log Evaluation of Shaly Sandstone Reservoirs: A Practical Guide*: AAPG Continuing Education Course Notes, AAPG Book store, Tulsa, Oklahoma, 59 pp.

Bardon, C., and Pied, B.,1969, Formation water saturation in shaly sands: Society of Professional Well Log Analysts10th Annual Logging Symposium Transactions: Paper Z,19 pp.

Bussian, A.E.,1982, A generalized Archie equation: Society of Professional Well Log Analysts 23rd Annual Logging Symposium Transactions: Paper E,12 pp.

Chemali, R., 2001, Personal communication.

Clavier, C., Coates, G., and Dumanoir, J., 1977, The theoretical and experimental bases for the “dual water” model for the interpretation of shaly sands: Society of Petroleum Engineers Paper No. 6859, 10 pp.

Doveton, J. H., 1986, *Log Analysis of Subsurface Geology*: Wiley Interscience, New York, 273 p.

Ehrlich, R., Etris, E.L., Brumfield, D., Yuan, L.P., and Crabtree, S.J.,1991, Petrography and reservoir physics III: Physical models for permeability and formation factor: American Association of Petroleum Geologists Bulletin, v. 75, p. 1579-1592.

Etris, E.L., Brumfield, D.S., Ehrlich, R., and Crabtree, S.J., 1989, Petrographic insights into the relevance of Archie’s equation: Formation factor without “m” and “a”: Society of

Professional Well Log Analysts 30th Annual Logging Symposium Transactions: Paper F, 18 pp.

Herrick, D.C., and Kennedy, W.D., 1994, Electrical efficiency – A pore geometric theory for interpreting the electrical properties of reservoir rocks: Geophysics, v. 59, p. 918-927.

Hossin, A.,1960, Calcul des saturations en eau par la methode du ciment argileux (formule d'Archie generalisee): Bulletin Association Francaise des Techniciens du Petrole, v. 140.

Jackson, P.D., Taylor-Smith, D., and Stanford, P.N.,1978, Resistivity-porosity-particle relationships for marine sands: Geophysics, v. 43, no. 6, p. 1250–1262.

Juhasz, I.,1981, Normalised Qv--the key to shaly sand evaluation using the Waxman-Smits equation in the absence of core data: Society of Professional Well Log Analysts 22nd Annual Logging Symposium Transactions: Paper Z, 36 pp.

Maute, R.E., Lyle, W.D., and Sprunt, E.S.,1992, Improved data-analysis method determines Archie parameters from core data: Journal of Petroleum Technology, January, p. 103-107.

Maxwell, J.C., 1873, *A Treatise on Electricity and Magnetism*, Clarendon Press, Oxford, 554 pp.

Mendelson, K.S. and Cohen, M.H.,1982, The effect of grain anisotropy on the electrical properties of sedimentary rocks: Geophysics, v. 17, no. 2, p. 257-263.

Pérez-Rosales, C.,1982, On the relationship between formation resistivity factor and porosity: Society of Petroleum Engineers Journal, p. 531-536.

Poupon, A., and Levaux, J.,1971, Evaluation of water saturation in shaly formations: Society of Professional Well Log Analysts 12th Annual Logging Symposium Transactions: Paper O, 2 pp.

Poupon, A., Loy, M.E., and Tixier, M.P.,1954, A contribution to electric log interpretation in shaly sands: Transactions of the American Institute of Mechanical Engineers: v. 201, p. 138-145.

Schlumberger,1972, *Log Interpretation, Volume 1 – Principles*: Schlumberger Ltd., New York, 113 pp.

Sen, P.N., Scala, C., and Cohen, M.H., 1981, A self-similar model for sedimentary rocks with application to the dielectric constant of fused glass beads: Geophysics, v. 46, no. 5, p. 781-796.

Simandoux, P., 1963, Dielectric measurements on porous media application to the measurement of water saturations: study of the behaviour of argillaceous formations: Revue de l'Institut Francais du Petrole 18, Supplementary Issue, p. 193-215.

Smith, W.D.M., and Rouleau, R.J., 1977, A modified shaly sand Sw equation for the Western Canada Cretaceous: Canadian Well Logging Society Sixth Formation Evaluation Conference, Paper V, 4 pp.

Stalheim, S.O., and Eidesmo, T., 1995, Is the saturation exponent *n* a constant?: Society of Professional Well Log Analysts 36th Annual Logging Symposium Transactions: Paper SSS, 12 pp.

Waxman, M.H., and Smits, L.J.M.,1968, Electrical conductivities in oil-bearing shaly sands: Society of Petroleum Engineers Journal, v. 8, p. 107-122.

Winsauer, W.O., Shearin, H.M., Jr., Masson, P.H., and Williams, M.,1952, Resistivity of brine-saturated sands in relation to pore geometry: American Association of Petroleum Geologists Bulletin, v. 36, no. 2, p. 253-277.

Worthington, P.F.,1985, The evolution of shaly-sand concepts in reservoir evaluation: The Log Analyst, v. XXVI, no. 1, p. 23-40.