The Hofmann shape entropy of natural sediments

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Much time has been expended over the last 150 years in studying the settling behaviour of particles in fluids. This is due to the fact that settling constitutes an important part of sediment transport and can be used directly, for example, to predict entrainment thresholds for unidirectional currents in water (Collins and Rigler, 1982; Komar and Clemens, 1985; Bridge and Bennett, 1992; Le Roux, 1998) and air (Le Roux, 1997a), as well as under oscillatory waves (Le Roux, 2001). In the majority of these studies, the fall velocity of spheres equivalent to the retaining sieve-size was employed, but these results can be refined by taking into account the effect of shape on the particle settling velocities.

Although the shape of pebbles can be measured by calliper, the orthogonal axis dimensions of sand-sized and smaller grains cannot be determined as readily. Measurements under a binocular microscope can only yield the intermediate and long axes, and it would be extremely time-consuming to measure the very large number of grains in even a small sand sample. This problem is approached here by employing the mean settling velocity of grains of specific sieve sizes. Using the fluid density and viscosity, as well as the grain density and size, the shape factor can be calculated from the sedimentation rate, as the effect of grain roundness appears to be almost negligible (Baba and Komar, 1981).

The effect of grain shape on settling rates has been examined extensively, employing a variety of shape indices of which the most popular have been those of Corey (1949) and Janke (1966). Le Roux (1996; 1997b) showed that the Hofmann (1994) shape entropy ($H_r$) is most suitable for predicting the settling velocity of smooth ellipsoids. The $H_r$ factor is given by:

$$H_r = -[(p_1 \cdot \ln p_1) + (p_i \cdot \ln p_i) + (p_s \cdot \ln p_s)]/1.0986 \quad [1]$$
where \( p_l \), \( p_i \) and \( p_s \) are the proportions of the long (\( D_l \)), intermediate (\( D_i \)) and short (\( D_s \)) axes, respectively. A \( H_r \) value of 1 indicates a perfect sphere. The proportion \( p_l \), for example, is expressed as:

\[
p_l = \frac{D_l}{D_l + D_i + D_s}
\]  

[2]

To determine the shape entropy of natural grains, the data sets of Zegzhda (1934), Arkhangel’skii (1935), Sarskisyan (1958) and Raudkivi (1990) as reported in Cheng (1997) were employed, yielding a total of 56 samples ranging in size from 0.0001 to 1 cm. To convert these retaining sieve-sizes into the nominal diameters of their equivalent spheres, they were multiplied by 1.32 (Komar and Cui, 1984). Water temperatures varied between 7.5 and 20ºC.

First, the dimensionless grain sizes of the equivalent spheres (\( D_{ds} \)) were determined. The dimensionless settling velocities (\( W_{ds} \)) were then calculated (Le Roux, 1992), which were converted to actual settling velocities.

A settling velocity equation for smooth ellipsoids based on the \( H_r \) factor, valid for grains of different densities in various fluids, was defined by Le Roux (1996) as:

\[
W_p = W_s [(H_r - 0.5833)/0.4167]
\]  

[3]

Using the same approach, the settling data of Baba and Komar (1981) for irregular glass particles were also reanalysed. In this case, a settling equation was defined as:

\[
W_p = W_s [(H_r - 0.23)/0.77]
\]  

[4]

The entropy of natural quartz grains can be expected to lie somewhere between that of perfectly smooth ellipsoids and the very irregular glass particles of Baba and Komar (1981). Taking the average of the values in equations [3] and [4], a tentative settling equation for natural grains can therefore be defined as:
\[ W_p = W_s[(H_r - 0.41)/0.59] \]  

From the observed settling velocity of the natural grains \((W_o)\), the mean shape entropy for each sample can be derived from equation [5], rearranged as follows:

\[ H_r = 0.59(W_o/W_s) + 0.41 \]  

The computed values of \(H_r\) for the different samples can now be plotted (Fig. 1) against the corresponding grain size \(D_n\), which produces a polynomial distribution remaining at 0.65 for grain sizes between 0.0001 to 0.007 cm, thereafter increasing to about 0.79 at a grain size of 0.063 cm and decreasing again to 0.72 between 0.1 and 0.2 cm. The curve seems to rise again for larger grain-sizes, but not enough data are available to confirm this trend.
Due to the distinct shape of the data plots, two separate polynomial equations yield the best fit for the ascending and descending parts of the curve, respectively. The two polynomial equations describing the curve are given by:

\[ H_r = 84.41D_n^3 - 14.32D_n^2 + 2.73D_n + 0.65 \]  \[7\]

For \( D_n \) exceeding 0.063 cm:

\[ H_r = -0.146D_n^3 + 0.45D_n^2 - 0.39D_n + 0.809 \]  \[8\]

In the grain-size range from clay to medium-sized pebbles, sand exhibits the highest sphericity, culminating with coarse to very coarse sand.

The Hofmann shape entropy can accordingly be employed in an improved method to determine the settling velocity of natural grains. The predicted settling velocity (\( W_p \)) is given by:

\[ W_p = W_s[(H_r - 0.41)/0.59] \]  \[9\]

where \( W_s \) is the settling velocity of the equivalent sieve-size sphere. The mean accuracy of 95.2\% is similar to that obtained for both smooth ellipsoids and angular grains, which suggests that grain roundness does not play a major role in the settling velocity of particles up to a Reynolds number of at least 4,200.

References


