Grade Tonnage Curve: How Far Can It Be Relied Upon?

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Abstract

In the mining industry, from the geology and mine planning areas to management, the grade tonnage curve has been applied in economic and financial analysis probably being one of the most important tools used in determining the volume and grade of material in face of variations of the cut-off grade.

The grade tonnage curve is usually calculated with the in-situ resources or reserves and shows always the 'best' or 'optimistic' scenario, i.e., at a given cut-off grade the respective tonnage depicted by the curve presents the ideal situation where all the material would be mined coming from the assumption that the shape of the orebody at this cut-off is continuous and every block is amenable to be mined. Usually the grade tonnage curve is not calculated with mineable reserves which depends on the mining method: geometry, spatial location and topological relations between ore and waste blocks and spatial location of main mining structures (galleries, main shaft, etc.). For a given cut-off, the difference between *in-situ* and mineable reserves is as much evident as the mining method is selective.

But to design and run a simulation of the mining method on the entire orebody for different cut-offs is a tedious and cumbersome practice. On the other hand, in most situations at the early stages of a mine, the mining method is only known in a broad sense without the details regarding the mining sequence of blocks, the mining rates, etc.

Hence in this paper it is proposed the following approximative procedure to convert the *in-situ* into mineable reserves: a given block of ore (above the threshold) is considered as mineable if it belongs to a connected set of ore blocks, otherwise it is considered waste if it is isolated by waste blocks. A stochastic simulation technique is used to generate a set of images of the deposit in order to provide the uncertainty measure of mineable reserves.

Key words: grade tonnage curve, simulation, direct sequential simulation, continuity factor, geobodies.

1. Introduction

The grade tonnage curve is widely used in the mining industry. From geology and mining planning to management and investment areas they are used for economic and financial analysis being probably one of the most important tools for representing variations in the characteristics of a deposit in function of cutoff grades.

Although the grade tonnage curve is of unquestionable importance for most economic studies they have one significant limitation which concerns the lack of information about continuity.

Depending upon the geological characteristics of the deposit and grade distribution significant changes in the geometry can occur due to variations of the cutoff grade. The grade tonnage curve calculation which is based on a block model counts every single block irrespective of its location and relation to the neighbour blocks, i.e., without any consideration of continuity. A block or group of blocks separated from the mineable areas will still be counted and added to the tonnage totals in spite of their isolation and the fact that these will have less probability of being mined.

The grade tonnage curve shows the 'best' scenario, i.e., at any cutoff the curve assumes implicitly total continuity of the mineralization and every block is considered as equally available to be mined. It is known that rarely the 'ideal' conditions apply and lack of continuity should be expected which implies reductions in the hipothetical tonnage of the curve.

At this point it is important to mention that the term continuity is used in the economic sense, i.e., meaning a combination of grade distribution and geometric characteristics which indicates a coherent mining unit feaseable to exploitation under certain technical conditions.

The approximation of the grade tonnage curve to reality depends on largely on some natural parameters as the geology and grade distribution of the deposit. In general, the more variable the grades the more complex is the geometry and the less reliable becomes the curve.

Usually the grade tonnage curve is generated by geologists or mining engineers which are familiar with the mineralization and have an expert 'feeling' of the behavior of the continuity regarding changes in the cutoff grade. Nevertheless, these curves are also used by professionals linked to the economical and financial areas which tend not to have the same perception of the deposit and likely more difficulty to evaluate its mineable characteristics in face of cutoff considerations.

This study aims at calculating the grade tonnage curve by incorporating the blocks continuity and the uncertainty attached to it. For this purpose, in a first step, a set of simulated images of the orebody provide the grade tonnage curve and an uncertainty range for the unconstrained situation, i.e., where all set of blocks above a cutoff grade is supposelly available for mining.

In a second step, the blocks are constrained by a continuity factor derived by the mining method, i.e., continuous sets of blocks above each cutoff grade are computed to give the corrected grade tonnage curve. An uncertainty range around each tonnage value above a cutoff is derived from the simulated images.

This approach, which allows a more 'realistic' view concerning the behaviour of the resource in face of variations in the cutoff grade, is applied to a case study area at the Neves Corvo copper deposit.

2. Characteristics of the Mineralization and Variography

The main economic metal present in the ore is copper and the mineralization can be described as being of the fissural or stockwork type. It is composed of veinlets and strings of sulphides and quartz which cut mainly acid volcanics rocks, concordantly or not with the schistosity. The sulphides are mainly pyrite and chalcopyrite and the thickness of the veinlets may vary from a few milimetres to a few decimetres.

The spatial distribution of the veinlets is highly irregular, as well as the grades, and does not show, in most situations, to be controlled by any particular geological feature.

Figure 1 shows a geological plan at the 815m elevation (mine reference level) through the Neves North orebody. The light grey areas represent the fissural mineralization, which is the aim of the present study, in contact with massive copper, massive pyrite and marginal fissural type mineralization.

This figure also presents the histogram of the copper data which is represented by a combination of 1m evaluation drillhole samples and 1m underground channel face samples whose lower limit values lie at 1% in accordance to the geological grade boundaries of the unit.

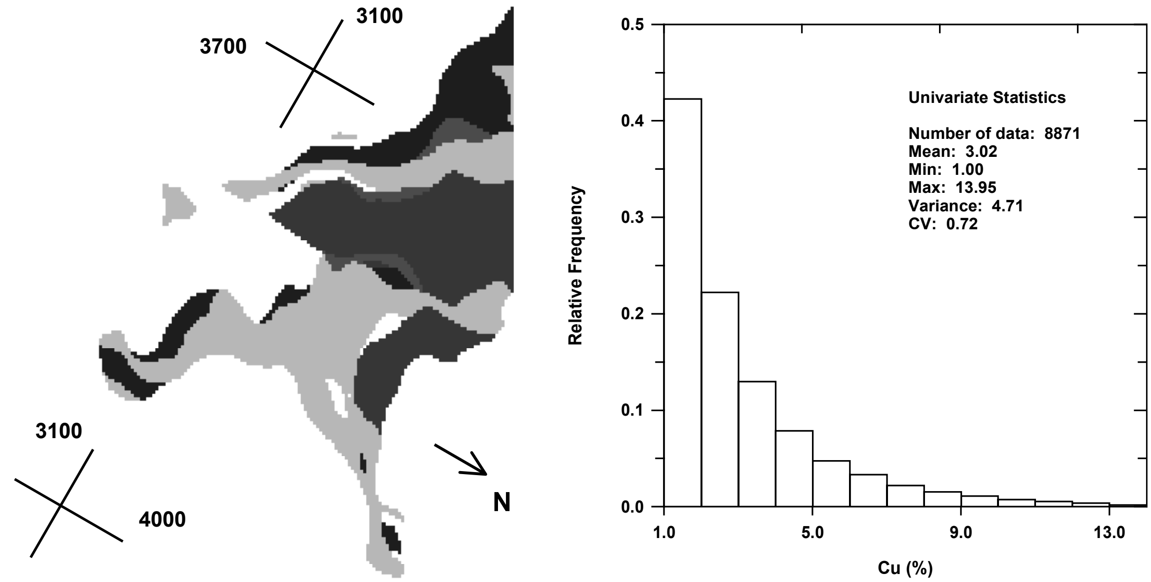

Figure 1: Geological map showing the fissural ore type (light gray) in relation to other mineralization units. The histogram of the fissural copper data (u/g face samples and u/g evaluation ddh) is also shown.

In relation to the structural analysis Figure 2 shows the variograms of the copper grades calculated using evaluation drillhole and face samples. The experimental variograms were adjusted by fitting a nested spherical model.

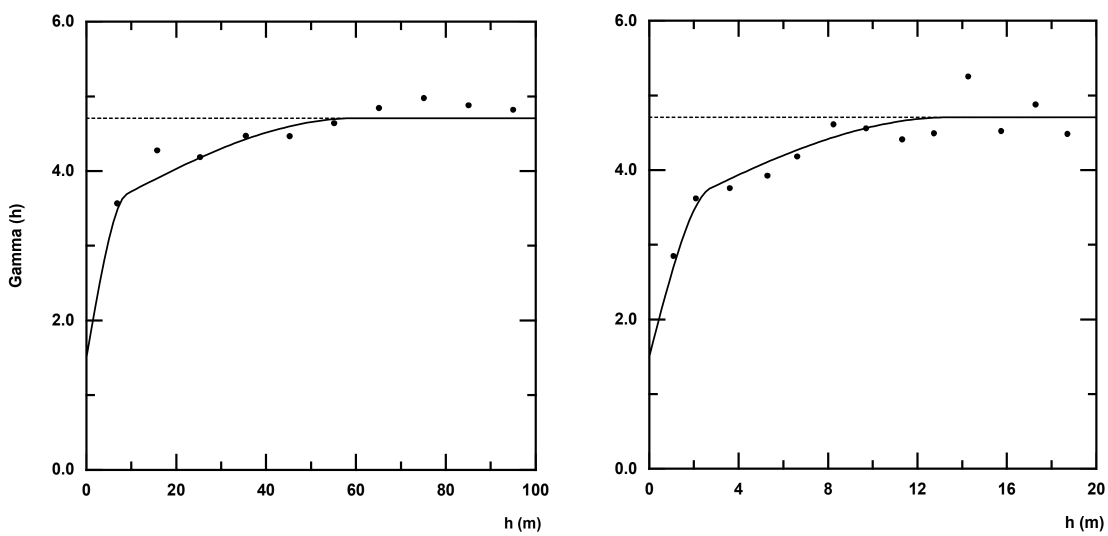

Figure 2: Experimental variograms calculated along the main direction (040º/00º) and the direction perpendicular to the mineralization with the respective fitted nested spherical models.

3. Incorporation of the Continuity Factor

Continuity is a characteristic which varies from deposit to deposit. From the economic point of view continuity means a grade distribution and geometric characteristics of the mineralization which allows mining to take place under certain technological conditions.

The traditional grade tonnage curve represents an optimistic scenario since it considers that all tonnage above one cutoff is available to be mined. It is an unconstrained grade tonnage curve which, although very common, it is based in an assumption that can hardly be considered realistic especially in underground mining methods. Several aspects play an important role in the consideration of how close the curve reproduces the reality, namely:

- spatial continuity pattern of the orebody
- type of mineralization
- mining method
- economics

With all the rigour to calculate a constrained (by the mining method) grade tonnage curve it is first necessary to calculate (estimate or simulate) the resources in blocks and then to simulate the mining method afterwards to obtain the minable reserves. The problem comes with the simulation of the mining method which, given its complexity (it depends on the geometric factors of the method, the geology and dynamic of economical factors) it becomes too cumbersome to achieve sound results.

In this paper it is proposed an approximation just by incorporating the continuity notion in the grade tonnage curves. The proposed method consists in analysing each block in relation to its immediate surrounding shell of blocks, and evaluate how it is connected to them. The concept of a geobody must be introduced: it corresponds to a set of connected blocks above a given threshold, surrounded by blocks which have grades below the cutoff grade. Hence, an entire set of geobodies can be computed for each cutoff.

4. Direct Sequential Simulation

To assess the uncertainty of the orebody reserves stochastic simulation is used as a tool to reproduce the spatial pattern and variability of the copper grades in a set of equi-probable images. The main reason for using stochastic simulated images instead of estimated grades is that when several increasing cutoffs are applied to the orebody, natural geological boundaries of different lithologies vanishes and, consequently, spatial variability of grades becomes the main issue for calculating reserves.

Sequential simulation is the most popular algorithm to generate simulated images given its simplicity of implementation (Deutch and Journel, 1998; Silva and Soares, 2000). In this paper a recent developed algorithm of Direct Sequential Simulation (Soares, 2001) was applied. This is based on a Journel´s idea (1994) that showed that the sequential simulation of a continuous variable, without any prior transformation, succeeded in reproducing the covariance model, provided that the simulated values are drawn from local distributions centered at the simple kriging estimates with a variance corresponding to the simple kriging estimation variance. Unfortunately, it does not reproduce the histogram of the original variable, which is one of the basic requirements of any simulation method. This has been the most serious limitation to the practical application of the direct simulation approach.

To overcome this drawback Nowak and Srivastava (1997) proposed an algorithm (DMESS Distribution Model Exact Sequential Simulation) to honour the global distribution. For the same purpose Caers (1999) proposed the use of post-processing and a direct indicator simulation (Caers, 2000). In general, the mentioned techniques share a common problem: in some situations the variogram deteriorates when the global histogram is matched.

Soares (2000; 2001) proposes to use the local sk estimates of the mean and variance, not to define the local cdf but to sample from the global cdf. The simulated values of the original variable are drawn from intervals of the global cdf, which are calculated with the local estimates of the mean and variance.

The method can be summarized as the following:

- Define a random path over the entire grid of nodes x_u, u=1, Ns, to be simulated.
- Estimate the local mean and variance of original variable $z(x_u)$, identified, respectively, with the simple kriging estimate $z(x_u)^*$ and estimation variance $\sigma^2_{sk}(x_u)$ conditioned to the experimental data $z(x_i)$ and previous simulated values $z^s(x_i)$.
- Define the interval of $F_Z(z)$ to be sampled, by using the Gaussian cdf: $G(y(x_u)^*, \sigma^2_{sk}(x_u))$, where $y(x_u)^*=\varphi(z(x_u)^*)$, being φ the normal score transform of original variable $z(x)$.
- Draw a value $z^s(x_u)$ from the cdf $F_Z(z)$.
- Generate a value p from a uniform distribution U(0,1).
- Generate a value y^s from $G(y(x_u)^*, \sigma^2_{sk}(x_u))$.
- $y= G^{-1}(y(x_u)^*, \sigma^2_{sk}(x_u),p)$.
- Return the simulated value $z^s(x_u)= \varphi^{-1}(y^s)$.
- Loop until all Ns nodes have been visited and simulated.

5. Incorporation of the Continuity Factor: Calculation of the Geobodies - Case Study Area at the Neves Corvo Mine

In order to have the tonnage curves for the unconstrained situation, i.e., without any consideration regarding the continuity, 10 simulated images of the deposit were produced. To simulate the copper grades the direct sequential simulation algorithm was used to obtain these realizations.

To exemplify the method three cutoff grade values were chosen namely 2%, 3% and 4% Cu where the geobodies - a set of connected blocks above a given threshold - were computed. The application of the technique shows as a result the proportion of geobodies in the total resources consisting of one single isolated blocks, two connected isolated blocks, three connected isolated blocks, and so on in consideration to a specific cutoff grade.

The cutoff grade of 2% of copper splits, in geological terms, the main mineralization of chalcopyrite from the rest (pyrite-rich rocks, schists, graywackes). Hence in relation to the geobodies calculated for the cutoff grade of 2% and shown in Figure 3 it is evident that there is one big cohesive geobody – which corresponds to the main mineralization - of 98% of the resources above 2% Cu. It can seen from this figure that the isolated single blocks of copper content above 2% represents about only 0.5%.

In relation to the geobodies calculated for the cutoff grade of 3% the main mineralization starts to lose definition giving rise to a kind of “salt and pepper” effect of the boundaries. In fact, the proportion of one single isolated blocks rise to about 4% (Figure 4) which, giving the selected mining method and grade of these blocks, may represent an unminable proportion of the resources but still counted in the traditional curve.

In Figure 5 it is shown an example of one level with the geobodies (blocks corresponding to the same geobody have the same color) calculated for the 2% cutoff. Isolated blocks above that cutoff do not represent a significant proportion. Conversely, when rising the cutoff grade to 3% (Figure 6), a different spatial pattern arises with the resulting image showing a revealing amount of blocks with grades higher than the defined threshold but clearly isolated from the main mineralized areas.

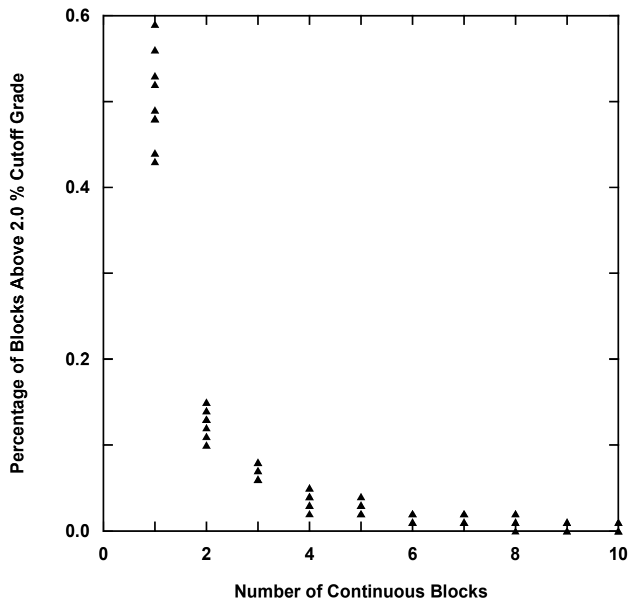

Figure 3: Geobodies calculated for the cutoff grade of 2% showing that about 0.5% of all resources correspond to single isolated blocks.

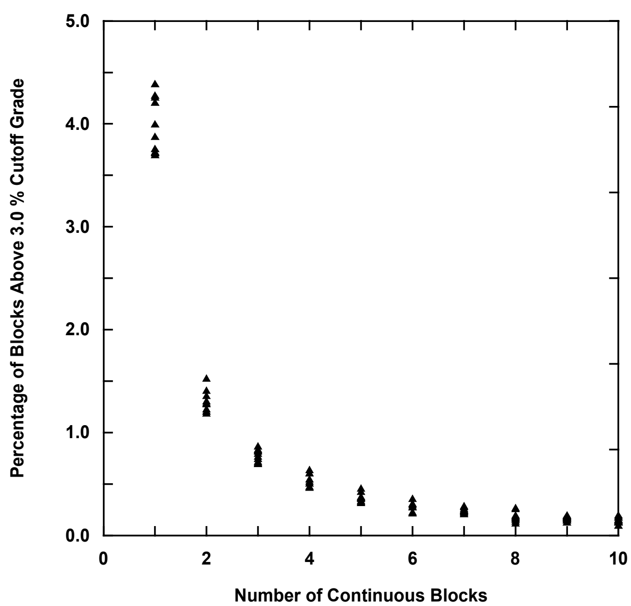

Figure 4: Geobodies calculated for the cutoff grade of 3% showing that about 4% of all resources correspond to single isolated blocks.

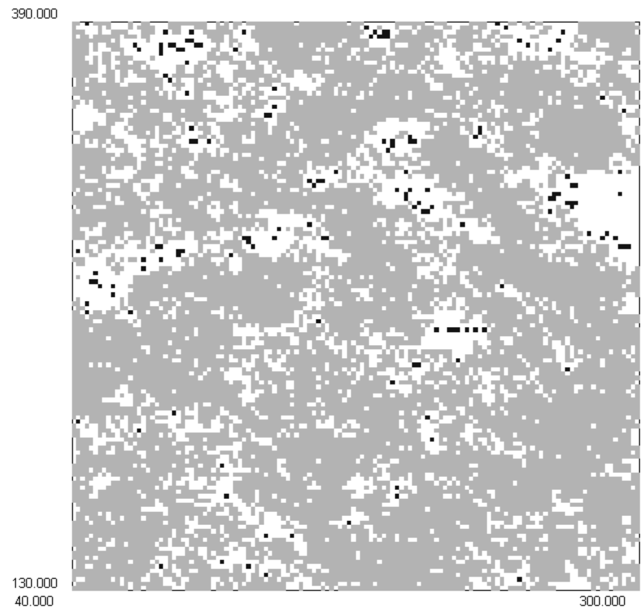

Figure 5: Image corresponding to the geobodies calculated for the cutoff grade of 2% copper. White areas show grades below the cutoff. Light gray areas represent blocks with grades above 2% copper as well as the black ones with the latter having a tendency to be isolated.

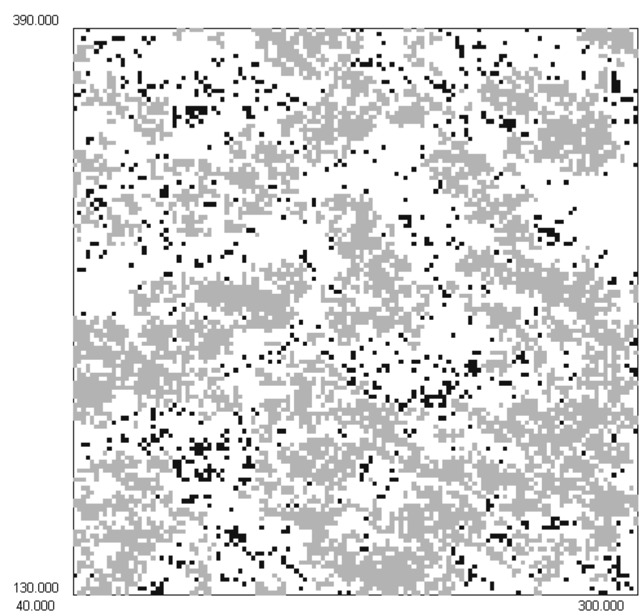

Figure 6: Image corresponding to the geobodies calculated for the cutoff grade of 3% copper. With the increase in the cutoff, and as the areas below it (white) enlarges, there is also a significant increase in the amount of blocks (in black) which, in spite of being above the cutoff, are isolated in non-mineable areas which clearly reduces their probability of being extracted.

The loss of continuity is even more evident when examining the cutoff grade of 4% where the proportion of the single isolated blocks reaches a level of about 10%.

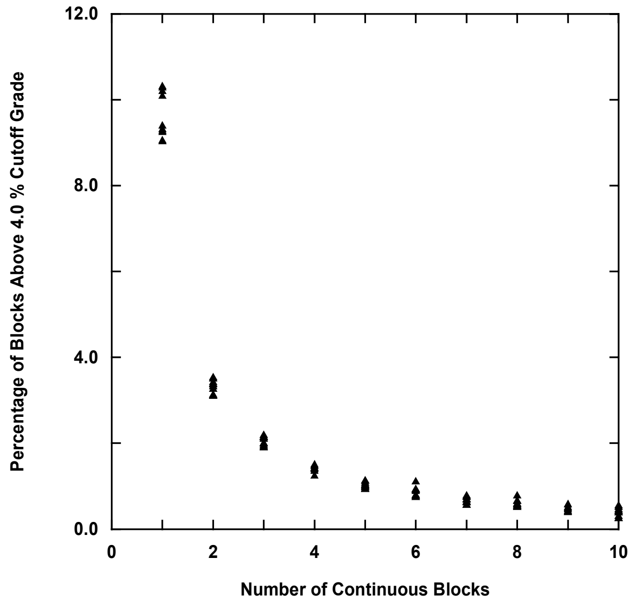

Figure 7: Geobodies calculated for the cutoff grade of 4% showing that about 10% of all resources correspond to single isolated blocks.

6. Conclusions

The traditional grade tonnage curve does not take into consideration continuity which is an important aspect in mining. The curve does regard that all blocks can be mined and gives no hint about the cohesiveness of the mineralization in face of variations in the cutoff grade.

Grade tonnage curves are usually built using kriged block models. Given the smoothing effect of kriging it generates a greater grade continuity (artificially) than the actual one. This feature contributes to make the grade tonnage curve too dependent on the smoothing effect of the estimate especially in face of significant values of the nugget effect. This characteristic helps to reinforce even more the optmistic perspective of the curve.

Simulation is regarded as an important tool in the process of attaching a continuity factor to the grade tonnage curve. Simulated images are generated and serve as the basis for the uncertainty assessment of the geobodies method which represents a cohesive set of blocks above a certain threshold. The geobodies give, in proportion, the amount of blocks which can be expected to be unminable because of their isolation. Grade tonnage curves based on direct sequential simulation are then calculated over different resource scenarios with the advantage over the traditional approach that no estimation smoothing would distort the true continuity of the block grades.

Regarding the case study shown it has become apparent that the higher the cutoff grade applied the bigger the tonnage which should be discounted (or at least not considered straitghforward mineable) on top of the tonnage already deducted due to their grade be smaller than the cutoff grade. Having as an example the cutoff grade of 3% Cu it was shown that at least 4 to 5% of all resources can be considered to be detached from the main continuous orebodies and therefore they should be regarded as a possible non-mineable asset.

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8. References

Caers, J. (1999). “Adding Local Accuracy to Direct Sequential Simulation”. Stanford Center for Reservoir Forecasting, Annual Meeting 12, v.2.

Caers, J. (2000). “Direct Sequential Indicator Simulation”. Proceedings of 6th International Geostatistics Congress, Cape Town, South Africa.

Deutch, C. and Journel, A. (1998). “GSLIB – Geostatistical Software Library and User’s Guide”. Oxford University Press, New York, 2nd Ed, 368 p.

Journel, A. (1994). “Modeling Uncertainty: Some Conceptual Thoughts”. *In* Dimitrakopoulos, R., ed., Geostatistics for the Next Century, pages 30-43.

Nowak, M. and Srivastava, R. (1997). “A Geostatistical Conditional Simulation Algorithm that Exactly Honours a Predefined Grade-Tonnage Curve”. Geostatisticas Wollongong’96, vol 2, pages 669-682.

Silva, F. and Soares, A. (2000). “Risk Assessment Calculation During Milestones of a Mine Life”. 31st International Geological Congress, Rio de Janeiro, Brazil, August 6-17, 13 p.

Soares, A. (2000). “Direct Sequential Co-simulation: A New Stochastic Modelling for Environmental Applications”. GeoENV 2000, Avignon, France.

Soares, A. (2001). “Direct Sequential Simulation and Co-simulation”. To be published in Mathematical Geology, November 2001.