On the Space-Time Variogram Models—An Application to Seasonal Precipitation

ALI M. SUBYANI
Hydrogeology Department, Faculty of Earth Sciences, P. O. Box 80206, Jeddah 21589, Saudi Arabia

e-mail {asubyani@hotmail.com}

ABSTRACT

In general, variogram is the key function in geostatistics and its determination is the basic step in spatial prediction and stochastic simulation of the natural phenomena. Therefore, the reliability of geostatistical application depends on the degree of variogram accuracy. So far, much of the cases use spatial versions of variograms. In addition, most of the natural phenomena measurements behave simultaneously in space and time. Therefore, a new space-time variogram approach will be discussed herein by considering two methods. The first method combines variograms at every tow successive season, which should have the same spatial structure, and tests whether there is a real significant cross-correlation within the station (temporal) and between the stations (spatial), using historical data. The second method is a graphical representation method, which uses a distance-time-variogram graph. In this paper, the application of these two methods is shown for the seasonal precipitation data in the southwest of Saudi Arabia.

KEY WORDS: Combined-Variograms, Distance-Time Variogram, Seasonal Precipitation Data, Cross-Correlation.
1-INTRODUCTION

Space-time phenomena deal with events that occur simultaneously in space and time. A large number of natural phenomena can be viewed as realizations of space-time random functions. In addition, these processes combine the characteristics which are met separately in the space and time domains. Geostatistical space-time models have been applied for many applied sciences that deals with spatiotemporal phenomena. Examples of such applications include in atmospheric and soil sciences, ecology, hydrology, etc. Precipitation is one of the most important spatiotemporal phenomena that develop for a wide variety of human activities and water project designs. There is a need to make temporal estimation or forecasting of the unknown value of precipitation at an unmonitored location.

Until recently, most of the published works in space-time studies were focusing on developing theoretical tools and mathematical techniques for data processing separately in time (time series) or in space (geostatistics). However, their study should not be realized by keeping space and time apart. The modern spatiotemporal geostatistics studies were involving the extension of spatial analysis tools to include the additional time dimension. This technique is not fully appreciated because the temporal and spatial scales are totally different, and cannot be compared to each other in a physical sense. Moreover, many space-time data exhibit some form of spatial heterogeneity and temporal non-stationarity. (Christakos, 1992; Bogaert, 1996; Cressie, 1996; Kyriakidis and Journel, 1999; Christakos, 2000).

In geostatistics, an integrated analysis typically uses two spatial dimensions plus the temporal one. Analysis of the actual data in the spatiotemporal domain could raise some difficulties. To resolve these problems, some space-time models and analysis methods have been proposed. Some of these solutions is to split the space-time correlation either into a product (Rodriguez-Iturbe and Mejia,1974) or a sum ( Rouhani and Hall, 1989) of space and time components, or by applying the multivariate geostatistical theory, which uses a coregionalization
of space-correlated time random functions (Rouhani and Wackernagel, 1990). It is clear that many interesting problems remain open to criticism and further studies (Rouhani and Myers, 1990).

Unfortunately, most of the articles in this area are deeply concentrated on covariance and variogram calculations either in space or in time. For example, Solow and Gorelick (1986) proposed an approach based on simple cokriging for estimating missing monthly streamflow data. Measurements from each gage were considered as separate but correlated time series, and there was no model fitting to their experimental covariance.

Rouhani and Wackernagel (1990) proposed an approach using direct and cross variograms as a linear combination of a number of hole function variograms by focusing on the dimension which has richer information (time dimension). At a further step, principle component analysis was used to determine grouping of piezometric level measurements at different temporal scales. However, in their procedure the combination of variograms in different time scales may not represent the true continuity structure of regionalized variable.

The main purpose of this study is to find two suitable approaches for modeling the space-time correlation of precipitation field. The first proposed method is the inference of spatial variogram models specific to each time instant and their subsequence combination in a (so-called) combined-variogram. The second method is the inference of space-time variogram model by contouring experimentally available variogram values for various spatial and temporal lags in a (so-called) distance-time variogram graph.
2- THE SPACE-TIME VARIOGRAM FUNCTION

Assuming that the space-time random variable $Z(\cdot)$ at location $x$ and time $t$ with $(x,t), x \in D \subseteq \mathbb{R}^n, t \in T \subseteq \mathbb{R}^1$ given by

$$Z(x,t) = \sum_{u=1}^{N} Z^u(x,t)$$

(1)

where the domain $\mathbb{R}^n$ is the Euclidean space of dimensionality $n>1$, $Z(x,\cdot)$ is a random vector at location $x \in \mathbb{R}^n$ ; $Z(\cdot,t)$ is a random vector at time $t$. The space and time components are defined as $h_{ij} = x_i - x_j$ and $\tau_{ij} = t_i - t_j$.

The problem is to predict $Z(x_0; t_0)$ at location $x_0$ at time $t_0$ from the data $\{Z(x,t), i = 1, K, m\}$.

The space-time variogram of a component is defined under a space-time intrinsic hypothesis as (Bogaert, 1996):

$$\gamma^u(h,\tau) = \frac{1}{2} Var[Z^u(x+h, t+\tau) - Z^u(x,t)]$$

(2)

In order to estimate parameters of space-time models with acceptable precision, data should be homogeneous and stationary in space and time, respectively. Generally, there are some assumptions about spatiotemporal modeling that should be satisfied (Christakos, 1992; Cressie, 1993):

1- spatial and temporal scales of variability of the natural phenomena should be intimately connected through reliable structure model. This is a prerequisite for understanding and predicting the space-time phenomena.
2- the proper model should be capable to assess quantitatively in any space-heterogeneous and time-nonstationary variability features and to provide efficient solutions to practical problem.

3- a space-time model should be tested for validity before regionalization in space and forecasting in time.

3- SPACE-TIME METHODOLOGY

Variograms are the key functions in geostatistical theory, their determination is the basic step in kriging and stochastic simulation. Therefore, the reliability of geostatistical application depends on the consistency of data and the degree of the variogram accuracy. However, the new geostatistical space-time approach will be discussed herein by considering the following methods:

3-1 Method One:

1- construct a suitable spatial variogram for every time step (monthly or seasonal), which means each time observations for the same area are considered separately in the form of monthly or seasonal variogram as

$$\gamma(h)_t = \frac{1}{2}Var[Z(x + h) - Z(x)]$$  \hspace{1cm} (3)

This classical variogram at time, t, (month or season). Therefore, we can construct 12 monthly or 4 seasonal variograms as shown in Figure 1.

2- Find the combined or aggregated variograms for successive time as a second step using weighted average method as

$$Co - \gamma(h) = \frac{\sigma_i^2 \gamma(h)_i + \sigma_{i+1}^2 \gamma(h)_{i+1}}{\sigma_i^2 + \sigma_{i+1}^2}, \hspace{1cm} t = 1, \Lambda ,n$$  \hspace{1cm} (4)
Where $C_0 - \gamma(h)$ is the combined variogram between two successive months or seasons (i.e. January and February, or winter and spring, etc.), $\sigma^2$ is the variance, and $t$ is the successive time step as shown in Figure 1. In addition, we can produce co-variogram between any two seasons like season 2 and season 3 and so on. For example, if we have four seasons, we can get 4 variograms in the first step and 3 co-variograms in the second step. However, going to further steps to combine two co-variograms (i.e. step 3 up to n-1) may produce some difficulties and the model looses its physical meaning. For instance, statistics of the first step variogram are usually preserved, yet there is no assurance that the combined or aggregate variogram statistics will be preserved. On other word, statistics for upper level may not be preserved (Salas, 1993).

3 -to avoid the meaningless of the new combined-variogram, two conditions should be satisfied prior to combination of any two successive variograms. First, the two successive variograms should have the same model structure because the same model structure indicates that the phenomena have the same behavior in space (homogeneous). Second, the phenomena have the same behavior in time (stationarity). In our example, the nature of precipitation occurrence may be change from month to month and from season to season.

4- test whether there is cross-correlation between seasons for every station, and also if there is cross-correlation between seasons for all stations. If there is a correlation we can combine every two successive seasonal variograms as mentioned in step 3.
3-2 Method Two

Another procedure is to draw contour lines showing the degree of structural continuity of the phenomena in time and space scales as follows:

1- Find the adequate variogram model from the sample variogram for every time scale (month or season).

2- Find the corresponding value of variogram for every distance.

3- Draw a graph in which the X axis represents the time scale (month or season) and Y axis represents the distance.

4- Align the variogram values for each corresponding distance and time.

5- Connect between the same variogram values by contour lines. These contour lines illustrate the degree of the spatial continuity of the phenomena with time. This graph is called “Distance-Time Variogram Graph” as shown in Figure 2.

In this research, the seasonal precipitation data in the southwest of Saudi Arabia will be analyzed and applied for these two methods.

4- STUDY AREA

The study area is located in the southwestern part of Saudi Arabia. This area receives the highest amount of precipitation compared with other regions, because it is mostly mountainous and is located within the subtropical zone. The region lies between latitudes $17^\circ.00'N$ and $22^\circ.00'N$ and longitude $40^\circ.00'E$ and $44^\circ.00'E$. (Figure 3).

The mean seasonal precipitation records in the study area were adapted from reports published by the Hydrology Division, Ministry of Agriculture and Water in Saudi Arabia, and Al-Jerash (1989). Twenty-four stations for the period 1971-1990 were selected for this study (Figure 3). These stations were chosen based on four criteria: 1) they represent the best spatial coverage of the region; 2) they maximize the same monthly precipitation records; 3)
they have continuous monthly precipitation records; and 4) they all represent different climate conditions.

Problems in the data analysis, such as non-normality, trend and outliers behavior should be fixed before developing any kind of model. However, precipitation data in arid regions, as stated by Hevesi et al. (1992), behaves as lognormal distribution. Hence, the transformation as \( y = \ln(Z(x)) \) was applied. Results indicate that the transformed annual and seasonal data are approximately normal (Subyani, 1997).

In this region, it has been stated that precipitation’s of winter and spring are under Mediterranean and orographic conditions, and summer and fall precipitation are mainly under Monsoon conditions. In this region, the network of the precipitation measuring stations is sparse and available data are insufficient to characterize the highly variable precipitation spatial distribution, especially in arid and mountain areas (Şen, 1983; Alehaideb, 1985; Nouh, 1987).

4- SEASONAL SAMPLE AND MODEL VARIOGRAMS

Figures 4 (a)-(d) present the experimental and modeled variograms and their parameters for precipitation in winter, spring, summer, and fall, respectively. In winter, the exponential variogram model was fitted with the small nugget of 0.05, the sill is 0.67, and the range of dependency is 70 Km (Fig. 4a). In spring, precipitation behaved as an exponential model with nugget of 0.1, a sill is 0.95, and the range of dependency of 90 Km. (Fig. 4b). In summer and fall, precipitation behaved as spherical models. Sill for these two seasons is around one and the ranges of dependency are 140 Km and 110 Km, respectively (Figs 4c-d). The extension of the structural continuuity technique (variogram) to the two new cases of combining or contouring these seasonal variograms are achieved by using method 1 and 2, respectively.
4-1 SPACE-TIME CO-VARIOGRAM (Method One)

As shown in Figures 4(a-b) winter and spring seasons have also more or less the same structural model (i.e. Exponential Model), and summer and fall seasons also have more or less the same structural model (i.e. Spherical Model). Based on these facts, step 2 and Eq. 4 in method one will be applied between winter and spring, and between summer and fall to find the space-time variations between the two successive seasons.

Figure 5(a) shows the combined-variogram model between winter and spring, This model consisted of a nugget value of 0.08 and exponential structure with a sill of 0.83 and a range of 85 Km. Its function was affected by a little more weight of spring function parameters. The combined-variogram model between summer and fall was developed using the same procedure shown in Figure 5(b). This model consisted of a spherical structure with a sill of 1.07 and a range of 140 Km. It was found that the variance is reduced from summer to fall when it passes the range of influence. However, this may be due to outliers of some stations affected by heavy Monsoon precipitation in summer.

4-1-1 Checking the Spatial and Temporal Correlations (Method 1)

Before combining any seasonal variograms, it is important to check if there is a real cross-correlation in time. For this purpose, the historical data are tested. Two kinds of data are selected from 24 representative stations in the study area over the period 1971-1990. The first group represents the mean seasonal of 20 years for 24 stations, and the second group represents the mean seasonal of 24 stations for 20 years record. (Salas et. al, 1980)

Finally, we estimate the lag-zero cross-correlation for these two groups (first group for spatial correlation and second group for temporal correlation) as shown in Table 1. The results of the first group shows significant seasonal cross-correlation. This data is the same
that we used for developing the seasonal variograms, and this strong correlation is due to the strong spatial correlation between stations. On the other hand, the second group shows non-significant cross-correlation (i.e. weak temporal correlation). So we cannot go farther to combine different seasonal variograms due to the spurious cross-correlation between two uncorrelated processes (Jenkins and Watts, 1968).

4-2 Distance-Time Variograms Graph (Method 2)

This graph was developed by using variogram values to obtain a contour map, showing the variations of the structural correlation for the precipitation in space and time. Figure 6 displays these variations in a simple way by plotting the time (season) on the X-axis and the distance on the Y-axis. The contour values (or variogram value) are obtained from variogram models, as explained in steps of method 2.

In winter, the contour lines gradually increase up to a certain distance, and then the contour lines go vertically with a value approximately equal to the exponential model sill (i.e. 0.67). In spring, the variation in precipitation is less than in winter, so the contour lines are smoother due to widespread precipitation distribution and the fact that the orographic factor has less effect. However, the contour lines in spring take a vertical shape when the contour value is approximately equal to the exponential model sill value (i.e. 0.95). In summer, the contour line values increase steadily up to a certain distance, and they become vertical when the value reaches or the sill of the spherical model (i.e. 1.1). In fall, this behavior occurs up to certain distance, and when the contour line becomes vertical, its value reaches the sill of the spherical model (i.e. 1.03).

In general, the contour lines or variogram model values steadily increase in a horizontal pattern up to certain distance, then the continuity gradually disappears when the
contour lines take on a vertical pattern in each season, depending on the sill and the range values.

5- SUMMARY

These methods were developed to describe the space and time continuity of precipitation. Method 1 incorporated a weighted average estimation of the seasonal variogram value at lag 1, and created a Co-variogram graph. Method 2 incorporated the distances and the seasonal variogram values. In other terms, it consisted on simultaneously taking from each seasonal variogram and for the same distance step the variogram value and plot the results on time-distance graph. The results from the checking of the seasonal cross-correlation for method 1 from historical data show that there is a spurious cross-correlation produced from two uncorrelated processes. So we can not go farther to combine different seasonal variograms. The results of Distance-Time graph from method 2 give general view of the structural variation of precipitation in time and distance Method 2 shows the climate factors which have affected precipitation occurrence in the study area. The contour lines of this graph reveal the pattern and degree of structural continuity of precipitation in space and time dimensions.

REFERENCES


Table 1. Lag-Zero Cross-Correlation Matrices for the Observed Data.

<table>
<thead>
<tr>
<th>Group</th>
<th>Winter</th>
<th>Spring</th>
<th>Summer</th>
<th>Fall</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>First Group</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Winter</td>
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<td>0.8</td>
<td>0.48</td>
<td>0.46</td>
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<tr>
<td>Spring</td>
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<tr>
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<td>0.6</td>
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<td>0.81</td>
</tr>
<tr>
<td>Fall</td>
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<td>0.44</td>
<td>0.81</td>
<td>1.0</td>
</tr>
<tr>
<td><strong>Second Group</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Winter</td>
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</tr>
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</tr>
<tr>
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<td>0.18</td>
<td>1.0</td>
<td>-0.04</td>
</tr>
<tr>
<td>Fall</td>
<td>0.14</td>
<td>0.02</td>
<td>-0.04</td>
<td>1.0</td>
</tr>
</tbody>
</table>
Figure 1. A scheme of Combined Variograms (Method 1).

Figure 2. A scheme of Distance-Time Variograms Graph (Method 2).
Figure 3. Location map of the study area.
Figure 4. Sample and fitted seasonal variogram models of precipitation.
Figure 4. Continue.

(c) SUMMER

(d) FALL

Range = 140 Km
Sill = 1.1

Range = 110 Km
Sill = 1.03
Figure 5(a). Winter and spring combined variogram model.

Nugget = 0.08
Sill = 0.83
Range = 85 Km

Figure 5(b). Summer and fall combined variogram model.

Sill = 1.07
Range = 140 Km
Figure 6. Distance-Time Variation in Seasonal Variogram Values of Precipitation (Contour lines represent $\gamma(h)$ values adopted from Figures (4a-d)).